



$$c.\text{grad} = ? \cdot \frac{dL}{dc} = ? \quad L = d \times f$$

$$L = (c + e) \times f.$$

(Introducing chain rule in calculus)

$$\left. \begin{array}{l} A = 2B \\ B = 4C \\ A = 8C \end{array} \right\} \text{Basic understanding}$$

$$\boxed{\frac{dz}{da} = \left( \frac{dz}{dy} \right) \times \left( \frac{dy}{da} \right) \Rightarrow \frac{dc}{da} = \left( \frac{dc}{db} \right) \times \left( \frac{db}{da} \right)}$$

Chain rule in calculus  
 ↪ (Heart of the Back Prop)  
 ↪ (Heart of the NN)

$$c.\text{grad} = \frac{dL}{dc}, \quad c.\text{grad} = ? \quad \text{we know, } d.\text{grad} = dL/dd$$

$$\boxed{\frac{dL}{dc} = \frac{dL}{dd} \times \frac{dd}{dc}} \Rightarrow \boxed{c.\text{grad} = d.\text{grad} \times \frac{dd}{dc}}$$

$$c.\text{grad} = \underline{d.\text{grad} \times \frac{dd}{dc}}$$

$$A) \frac{dd}{dc} = ?, \quad d = (c + e), \quad \text{slope} = f(x+h) - f(x)$$

$$\frac{dd}{dc} = ? \quad , \quad d = (c + e) \quad , \quad \text{slope } e = \frac{f(x+h) - f(x)}{h}$$

$$\text{slope } c = \frac{f(c+h+e) - f(c+e)}{h} = 1$$

$$\boxed{\frac{dd}{dc} = 1}$$

$$c \cdot \text{grad } c = d \cdot \text{grad } \cancel{\times \frac{dd}{dc}}^1 = \boxed{\begin{aligned} c \cdot \text{grad } c &= d \cdot \text{grad } d \\ \frac{dL}{dc} &= \frac{dL}{dd} \end{aligned}}$$

↓ Analytically

B)  $\frac{dd}{dc} = ? \quad d = (c + e)$

$$\frac{dd}{dc} = \frac{d}{dc}(c + e) = \frac{dc}{dc} + \frac{de}{dc} \quad (e = k)$$

$$\boxed{\frac{dd}{dc} = 1} \rightarrow \text{calculus}$$

$$c \cdot \text{grad } c = d \cdot \text{grad } \cancel{\times \left( \frac{dd}{dc} \right)^1}$$

$$\boxed{\begin{aligned} c \cdot \text{grad } c &= d \cdot \text{grad } d \\ \frac{dL}{dc} &= \frac{dL}{dd} \end{aligned}} \rightarrow \text{calculus}$$

Note:- when there is add<sup>n</sup>, the gradients flows without changing

$$e.g. \text{grad} = ?, \frac{dL}{de} = ? \quad (d = c + e)$$

We know that,  $\frac{dL}{dd}$

with chain rule in calculus,

$$\boxed{\frac{dL}{de} = \frac{dL}{dc} \times \frac{dc}{de}} \Rightarrow e.g. \text{grad} = d.g \text{rad} + \frac{dc}{de}$$

$$\frac{dc}{de} = ?, \quad d = (c + e)$$

$$A) \frac{dc}{de} = \text{slope} = \frac{f(x+h) - f(x)}{h} = \frac{(x+h)^k - x^k}{h}$$

$$\boxed{\frac{dc}{de} = 1}$$

$$e.g. \text{grad} = d.g \text{rad} + \frac{dc}{de}$$

$$\boxed{e.g. \text{grad} = d.g \text{rad}} \rightarrow \text{Analytical}$$

$$B) \frac{dc}{de} = ?, \quad d = (c + e)$$

$$\frac{dc}{de} = \frac{d}{de}(c + e) = \frac{dc}{de} + \frac{de}{de} \quad (C = k)$$

$$\boxed{\frac{dc}{de} = 1} \rightarrow \text{calculus}$$

$$\boxed{e.g. \text{grad} = d.g \text{rad}}$$

$$a.\text{grad} = ? \quad dL/da = ? \quad dL/de = \checkmark, \quad e = a+b$$

by chain rule,

$$\boxed{\frac{dL}{da} = \left( \frac{dL}{de} \right) * \left( \frac{de}{da} \right)} \rightarrow \text{chain rule}$$

$$\frac{de}{da} = ?$$

A)  $de/da = ?, \quad e = a+b, \quad \text{slope} = \frac{f(a+h) - f(a)}{h}$

$$\text{slope } e = (a+h) * b - (a+b) \underset{h}{\cancel{=}} \cancel{ab} = b$$

$$\boxed{\frac{de}{da} = b}$$

$$\frac{dL}{da} = \frac{dL}{de} * \frac{de}{da}$$

$$a.\text{grad} = e.\text{grad} * \frac{de}{da}$$

$$\boxed{a.\text{grad} = b * (e.\text{grad})}$$

$\rightarrow$  Analytically

$$\frac{d e}{da} = ?, \quad e = a+b$$

$$(b=k)$$

$$\frac{d}{da} (a+b) = a \frac{db}{da} + b \frac{da}{da} \rightarrow = b$$

$\rightarrow$  calculus

$$\boxed{de/da = b}$$

$$\boxed{a.\text{grad} = b * e.\text{grad}}$$

$$\frac{dL}{db} = ? \quad | \quad \frac{dL}{de}, \quad e = a * b$$

chain rule,

$$\boxed{\frac{dL}{db} = \frac{dL}{de} * \frac{de}{db}}$$

A)  $\frac{de}{db} = ?$ ,  $e = a * b$ , slope  $= \frac{f(n+h) - f(n)}{h}$

$$\text{slope} = \frac{(a * (b+h)) - (a * b)}{h} = \frac{ab + bh - ab}{h} = a$$

$$\boxed{\frac{de}{db} = a}$$

$$b.\text{grad} = e.\text{grad} * \frac{de}{db}$$

Analytical  $\rightarrow$

$$\boxed{b.\text{grad} = e.\text{grad} * a}$$

B)  $\frac{de}{\partial b} = ?$   $e = a * b$

$$\frac{d(a * b)}{\partial b} = \frac{ad b}{\partial b} + b \frac{da}{\partial b} = a, \quad (a = k)$$

$$\boxed{\frac{de}{\partial b} = a}$$

$$\boxed{b.\text{grad} = e.\text{grad} * a}$$

$\rightarrow$  calculus