FROM THE HISTORY OF PHYSICS

# Omori's law: a note on the history of geophysics

To cite this article: A V Guglielmi 2017 *Phys.-Usp.* **60** 319

 

***FROM THE HISTORY OF PHYSICS*** PACS number: **91.30.–f**

# Omori's law: a note on the history of geophysics

A V Guglielmi

DOI: https://doi.org/10.3367/UFNe.2017.01.038039

## Contents



**Abstract. In the late nineteenth century, the Japanese seismologist Omori discovered the first law of earthquake physics, which states that the rate of aftershocks decreases hyperbolically with time. Over the years since then, there has been a vast amount of literature on this law, and the significance of its discovery has been universally recognized. There is, however, a profound division of opinion as to the interpretation of the law. Some argue that Omori just proposed a simple data-fitting formula and replace this formula by a power-law one with a negative fractional exponent, whereas for others the Omori law makes physical sense. The paper describes the history and essence of Omori's discovery, with special attention paid to interpretational questions. It is shown that Omori's original formulation of the law correlates well with the current understanding of the rock destruction mechanism at the earthquake focus.**

**Keywords:** seismology, earthquake, aftershock, tectonic fault, recombination, relaxation

## 1. Introduction. The history and essence of the discovery

Describing prominent events that took place more than a century ago would be better started from an even earlier time. In 1850, John Milne, destined to become a renowned geophysicist and one of the founders of modern seismology [1], was born in Liverpool. Milne was educated in London, worked as a mine engineer in Newfoundland and as a geologist on the Sinai Peninsula, and from 1875 to 1895, worked in Tokyo at the invitation of the government of the Empire of Japan. It is noteworthy that, longing for adventures, Milne traveled mostly overland (through Siberia), taking three months to reach Tokyo. In 1880, he designed the horizontal pendulum seismograph — the first instrument to record earthquakes that was convenient in operation and sufficiently sensitive.

In 1887, Milne was elected a member of the Royal Society of London. He managed to persuade the Society to provide means for the construction of a global network of seismic stations equipped with his instruments. (By the way, Milne decided to install three stations in Russia.) His services to the country and the world were highly appreciated in Japan. Emperor Meiji decorated him with the Order of the Rising Sun and granted him a life pension of 1000 yen. The University of Tokyo elected him an honorary professor.

Fusakichi Omori was a devoted student of John Milne and enjoyed the encouraging support of his teacher, as did all young Japanese seismologists of that time. An earthquake with the magnitude $M = 8$ took place on 28 October 1891. Milne seismographs registered numerous aftershocks. Their analysis allowed Omori to formulate a law in 1894 that bears his name [2]. It is worth mentioning that he was only 26 at that time.

The Omori law states that after a strong earthquake, the frequency of aftershocks, i.e., the underground shocks that follow the main shock, decays with time, on average, according to the hyperbolic law

$$n(t) = \frac{k}{c + t} \,. \tag{1}$$

Here $k > 0$, $c > 0$, and $t \geqslant 0$ [2]. This was the first law in the physics of earthquakes, if the chronological sequence of outstanding discoveries in seismology is meant. As the second law, the Guttenberg–Richter law [3] describing the distribution of earthquakes over magnitudes (see, e.g., Refs [4–7]) must be mentioned. We defer its discussion in order to not be diverted from our topic. A brief description of five laws of seismology can be found in Ref. [8].

**A V Guglielmi** Institute of Physics of the Earth,
Russian Academy of Sciences,
ul. B. Gruzinskaya 10, 123995 Moscow, Russian Federation
E-mail: guglielmi@mail.ru

Received 30 July 2016, revised 22 December 2016
*Uspekhi Fizicheskikh Nauk* **187** (3) 343 – 348 (2017)
DOI: https://doi.org/10.3367/UFNr.2017.01.038039
Translated by S D Danilov; edited by A M Semikhatov

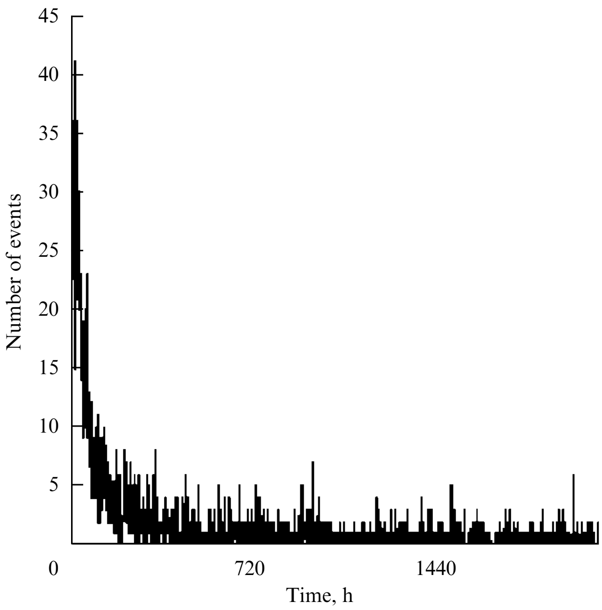


**Figure 1.** Dependence of the number of aftershocks after the main shock as a function of time on 24.11.1984 in California [9].

Omori made his discovery exactly 123 years ago. While it is not a round date, it is a remarkable date on its own, comprising a digit series — one, two, and three. The peculiarity of the present moment is that the next opportunity to celebrate a similar date will occur only in 1111 years. It is difficult to imagine what will have happened by then with Earth physics in general and seismology in particular. It is therefore reasonable not to miss the opportunity and use it to talk about the Omori law. Furthermore, we now have one more reason to turn to history, analyze the essence of law (1), discuss deviations from it, and so on.

The present situation is as follows. Over the years passed, a considerable amount of literature dealing with the Omori law has been published. It is partly reflected in review [10] published to celebrate the centennial of the discovery. The outstanding character of the discovery was never doubted. And yet, strange as it may seem, its interpretation remains not really established even at present. Furthermore, over recent years a serious disagreement has grown among geophysicists concerning the essence of what was done by Omori. Some argue that Omori only used simple formula (1) to approximate observational data and propose modifying it to improve the approximation. Others see a physical sense in the Omori law and put forward arguments that back the idea that the original formula proposed 123 yeas ago corresponds to the rupture mechanism in rocks in the earthquake source. This disagreement was the main reason for writing this work.

It is appropriate here to bring up a typical example of aftershock evolution. Figure 1 shows an aftershock sequence after the main shock with the magnitude $M = 6.6$ that took place on 24.11.1987 at 13 h 15 min 56 s world time. The main shock epicenter was located in Southern California. The hypocenter was at a depth of 10 km, and 3553 aftershocks occurred in the zone 0.5° in radius during the 90 days after the main shock. We see that on average the frequency of aftershocks decays monotonically with time. We return to the analysis of this example in Section 5.

## 2. Power-law approximation of the aftershock sequence

In seismology, when analyzing observations, Omori law (1) is commonly replaced with a power-law function

$$n(t) = \frac{k}{(c+t)^p} \, . \tag{2}$$

The power-law exponent $p$, generally speaking, varies from location to location and from case to case in wide limits. For example, according to observations in California, $p$ varies from 0.5 to 1.5, with the mean noticeably larger than unity: $p = 1.08$ [11]. Utsu, who has done much to introduce the remarkable achievement of Omori into the practice of seismic research, proposed calling formula (2) the modified Omori law [12]. Names such as 'the Omori–Utsu law' or 'power law' are also encountered in the literature. In contrast, formula (1) is commonly referred to as the hyperbolic Omori law.

Formula (2) was proposed by Hirano [13] in 1924 to approximate the activity of aftershocks after the Great Kanto Earthquake that destroyed Tokyo and claimed several hundred thousand lives. Fusakichi Omori learned about the catastrophe while at a conference in Australia and left for home immediately. During the sea trip back, his health condition deteriorated, and he died soon after returning to Tokyo.

Thus, after nearly 30 years after the discovery of the law, it was proposed that a more easily tunable formula (2) be used in place of formula (1) to describe aftershock sequences. Empirical arguments urging seismologists to widely use (2) instead of (1) are fully understandable. However, it is very likely, or to be more precise, indubitable, that Omori would not be satisfied with this replacement. This cannot be proved, but we can recall that Omori was a highly educated person and a talented researcher. He graduated from the University of Tokyo, paid visits to European scientific centers and soon became a well-known professor [1]. Strikingly naive and unjust are the words of one known seismologist that allegedly Omori did not master the methods of mathematical physics well enough. Omori, sure enough, had all the knowledge and skills needed to use (2) in place of (1) in order to tune the parameter $p$ separately for each aftershock sequence. This seems rather obvious, and careful reading of the original work [2] provides additional support. But Omori did not do it. In our opinion, he chose hyperbolic dependence (1) intentionally, based on his deep physical intuition. Interestingly, in 1938, Harold Jeffreys, a prominent mathematician and geophysicist [14], intended to use (2) to approximate the occurrence of aftershocks after the earthquake of 1927 in Tango (Japan) [15]. Jeffreys introduced the parameter $\delta = 1 - p$, but in the end set $\delta = 0$ without pointing to the reasons, i.e., simply returned to Omori formula (1). It is plausible to assume that he made this choice based on his extensive experience in physical and mathematical modeling of natural processes.

The Omori discovery was highly appreciated by his contemporaries. Utsu contributed more than others to interpreting and popularizing the Omori law [10, 12, 16, 17]. In his work, he used formula (2) for $p = \text{const}$. (We note that Hirano, in contrast, allowed variable $p$ within different time intervals, even within the same series of aftershocks.) For a long time, Utsu estimated the parameter $p$ almost exclusively based on data from Japanese earthquakes.

The estimate $p = 0.9 \pm 0.1$ for earthquakes outside Japan was first proposed in 1963 [18]. It launched broad research on aftershocks based on the Utsu technique, which persists even now. At present, the parameter $p$ is measured in all seismically active regions of the planet. A rough estimate shows no fewer than 300 careful measurements of $p$. In practically all cases, $p$ is substantially different from unity. The difference between $p$ and unity is established not only for natural but also for artificial earthquakes caused by chemical [19] and nuclear [20–22] explosions.

Under these circumstances, a natural question is whether there are any grounds to discuss the hypothesis that hyperbolic dependence (1) expresses a fundamental law, whereas power-law dependence (2) is just a fitting formula. The reference to the authority of Omori, who apparently ignored dependence (2), can be invoked, but it, of course, cannot persuade people in a scientific discourse, and the same also concerns any appeal to public opinion. However, we can find a suitable analogy in the history of science and draw parallels.

For example, we consider Kepler's hypothesis on the elliptical shape of planet trajectories. It is directly related to Newton's law of universal gravitation. However, the shape of Mercury's trajectory apparently deviates from an ellipse. Nevertheless, in exploring this problem, Le Verrier did not see any reason to doubt the fundamental law of inverse squares. This story is known too well to be treated in detail here.

We leave aside any scepticism that may be caused by an apparent incomparability of the physics of Newton's and Omori's laws, and concentrate on the psychology of research. The analogy is that in both cases the researchers were facing one and the same dilemma: to abandon the simple and, in its own way, elegant mathematical expression of a fundamental law or insistently search for a rationale explaining why observations do not fully agree with expectations. From this standpoint, it cannot be ruled out that the refusal to use formula (1) to describe aftershock sequences was premature and possibly a mistake.

## 3. Interpretation of the Omori law

Soon after Omori's discovery, an opinion was established (and has been sustained in seismology for many decades now) that law (1) is purely empirical, and if it is so, it is plausible to replace (1) with fitting formula (2). For example, Karen Felzer states unequivocally: "Omori's law is empirical. No one has been able to derive it" [23]. We try to persuade the reader that both these statements are in error. In other words, following Omori [2], we propose that, the existing opinion notwithstanding, hyperbolic law (1) is a fundamental one, and derive it in the framework of a simple model.

We begin with reformulating the law [24]. First, we postulate that the Omori law represents a solution of a differential equation describing the evolution of aftershocks. Second, we assume that formula (1) is accurate in some ideal sense. It follows from these two propositions that the evolution equation has the form

$$\frac{\mathrm{d}n}{\mathrm{d}t} + \sigma n^2 = 0 \,. \tag{3}$$

Indeed, a solution of Eqn (3),

$$n(t) = n_0 (1 + \sigma n_0 t)^{-1} \,, \tag{4}$$

coincides with (1) if we set $\sigma = k^{-1}$ and $n_0 = k/c$.

At first glance, Eqn (3) is just one more representation of the hyperbolic Omori law, but it is so only at first glance. We must take into account that in modeling natural phenomena, it is easier in many cases to interpret the evolution equation than the set of its solutions. In our case, the representation of the Omori law via Eqn (3) opens up an interesting possibility to propose a physical interpretation of aftershock sequences with a frequency that monotonically decays with time.

The shape of Eqn (3) hints at an analogy between the decaying aftershock sequence and the decrease in the density of ionospheric plasma due to the recombination of unlike charges. It should be recalled that radiative recombination in pairs of oppositely charged particles evolves in the ionosphere as follows:

$$\mathrm{O}_2^+ + \mathrm{e}^- \rightarrow \mathrm{O}_2 + \hbar\omega \,. \tag{5}$$

Here, $O_2$ is the oxygen molecule, $O_2^+$ is the oxygen ion, $e^-$ is an electron, and $\hbar\omega$ is a photon (see, e.g., Ref. [25]). A charge pair annihilates as a result of recombination, leaving a neutral molecule and a photon. Let $n_+$ ($n_-$) be the density of positive (negative) charges, $n = (n_+ + n_-)/2$. Then the recombination equation becomes

$$\frac{\mathrm{d}n}{\mathrm{d}t} = -\sigma n_+ n_- \tag{6}$$

and practically coincides with (3) because of the plasma quasineutrality ($n_+ \approx n_-$). Here, $\sigma$ is the recombination coefficient.

We deepen the formal analogy alluded to above in order to interpret the Omori law (1), i.e., translate it into the conceptual language of earthquake mechanics. For this, we must find a 'pair' in Earth's crust that is similar to the pair of oppositely charged particles in the ionosphere. Our hypothesis is as follows: there is nothing more natural than to consider a pair of adjacent sides of a tectonic fault for an object of that type [26]. We recall that an earthquake occurs commonly owing to a fast slip of rock along the fault plane in Earth's crust [5, 6].

Let the symbol $\uparrow\downarrow$ denote the fault with tangent stresses applied to both of its sides. Such a fault is referred to as active. A passive fault (without tangent stresses) is denoted as $\|$. There is a finite probability that rupture would take place along an active fault, entailing an earthquake. In analogy with reaction (5), we write it symbolically:

$$\uparrow + \downarrow \rightarrow \| + \text{aftershock} \,. \tag{7}$$

Let $n$ be the number of active faults in the epicentral area of the main shock. In analogy with recombination equation (6), we write

$$\frac{\mathrm{d}n}{\mathrm{d}t} = -\sigma n_\uparrow n_\downarrow \,. \tag{8}$$

Equation (3) directly follows from this one, because apparently $n_\uparrow = n_\downarrow$. The quantity $\sigma$ can be naturally called the deactivation coefficient. We draw attention to the fact that the recombination of electric charges in the ionosphere and deactivation of adjacent sides of the fault in the lithosphere lead to the evolution equation with quadratic nonlinearity and solutions that hyperbolically depend on time.

Thus, the Jeffreys parameter $\delta = 0$, because the tectonic fault presents a two-sided surface. In this sense, dependence (1) is a fundamental one. Put differently, our standpoint is that the hyperbolicity of the aftershock evolution is related to the pairing associated with opposite sides of the fault, similar to the inverse square behavior in the universal gravitation law being related to the three-dimensionality of space [27, 28]. The analogy lies in both properties being topological.

## 4. Generalized law of aftershock evolution

Perhaps we should not even mention that our arguments are incomplete and that we only built a toy model for the aftershock sequences. Models of that kind are frequently used at a certain stage in theoretical physics and astrophysics as palliatives of a certain kind. And yet if hyperbolic dependence (1) is treated as a fundamental law and relation (2) as a fitting function, then how to explain that the fitting parameter $p$ noticeably deviates from unity in experiment? It turns out that our model is 'viable' enough to propose a framework to solve this question. A plausible picture is as follows. After the main shock relaxation processes are launched in the earthquake source, speaking figuratively, the source gradually cools down. Many details here still need to be clarified, but in any case, the aftershocks are created in an apparently nonstationary geological situation. In our hypothetical model reduced to its bare essence, we can take the nonstationarity into account in only one way. Namely, the parameter $\sigma$, which we treated as fixed thus far, should be replaced with a function $\sigma(t)$. Then, instead of (4), we obtain the following solution of the aftershock equation:

$$n(t) = n_0 \left[ 1 + n_0 \int_0^t \sigma(t')\,\mathrm{d}t' \right]^{-1} . \tag{9}$$

The generalized law of aftershock evolution (9) is derived by us as a result of hypothetical, albeit plausible, reasoning. It is based on the idea of deactivation of the adjacent sides of tectonic faults. The deactivation is either enforced by external pulses or proceeds spontaneously under the action of internal fluctuations. After a strong earthquake, the system of tectonic faults in the source region undergoes a complicated relaxation process, with the decaying sequence of aftershocks being one of its manifestations. The evolution of aftershocks is described phenomenologically by nonlinear differential equation (3). The observed deviations of the decay from the strict hyperbolic law (1) is explained by the nonstationarity of the geological medium in the earthquake source. In a stationary medium, $\sigma = \mathrm{const}$, and in this ideal case Eqn (9) coincides with (1).

## 5. Aftershock equation in action

Equation (3) allows studying relaxation processes in an earthquake source that cools after the main shock. This opens up interesting perspectives. To be specific, let the task be the measurement in experiment of the deactivation coefficient $\sigma(t)$ [9]. We introduce an auxiliary function $g(t) = \int_0^t \sigma(t')\,\mathrm{d}t'$. From solution (9) of Eqn (3), it follows that

$$g(t) = \frac{1}{n(t)} - \frac{1}{n_0} . \tag{10}$$

With the help of Eqn (10), we can compute $g(t)$ based on the observational data $n(t)$, perform smooth interpolation, and finally compute $\sigma$ using the formula $\sigma = \mathrm{d}g/\mathrm{d}t$.

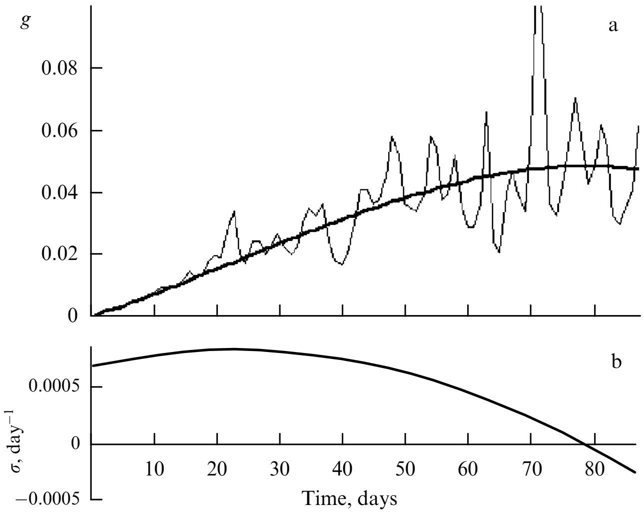


**Figure 2.** Results of analyzing the aftershocks shown in Fig. 1. (a) The time dependence of the function $g(t)$ before and after smoothing (the respective gray and black lines). (b) The time dependence of the deactivation coefficient $\sigma(t)$.

We apply this procedure to the aftershock sequence presented in Fig. 1. Figure 2 shows the quantities $g$ and $\sigma$ as functions of time. We see that the quasistationarity of the process is preserved on average for approximately 45 days after the main shock. Within this time interval (it seems natural to call it the Omori epoch), we have approximately $\sigma \approx 7.8 \times 10^{-4}$ days$^{-1}$ or $k = 1/\sigma \approx 1300$ days. Then $\sigma$ starts to decrease and decays to zero, whereas fluctuations in $g$ grow rapidly (see the gray line in Fig. 2).

We consider one more event shown in Fig. 3. It is of interest as an example of a doublet of main shocks. The first shock occurred on 23.04.1992 at 04 h 50 min 23 s (with the magnitude $M = 6.1$ and the hypocenter depth 12 km), and the second one on 28.06.1992 at 11 h 57 min 34 s (magnitude $M = 7.3$ and hypocenter depth is 1 km). The epicenters of the main shocks were approximately 30 km apart. The respective functions $g(t)$ and $\sigma(t)$ are shown in Fig. 4.

Thus, Eqn (3) and its solutions (9) offer us the possibility of approaching the analysis of aftershock data from a new direction, whereas the result of processing advocates an interesting perspective of introducing a new methodological technique: the classification of earthquake sources based on the shape of relaxation of aftershock sequences. For example, Figs 2 and 4 demonstrate two essentially different relaxation types. If we leave details aside, then in the first event we see a decaying function $\sigma(t)$, and in the second two, increasing functions $\sigma(t)$. We assume that in the future the analysis of the function $\sigma(t)$ will allow a more precise classification and help uncover the relation between the type of relaxation and the geological structure of the earthquake source.

## 6. Discussions

We have reformulated the Omori law (1) as differential equation (3) and considered a plausible interpretation. This interpretation is based on the idea of deactivation of the adjacent sides of faults in the vicinity of the main shock of an earthquake. The idea of deactivation emerged from the formal coincidence of aftershock equation (3) with the equation of radiative recombination in ionospheric plas-

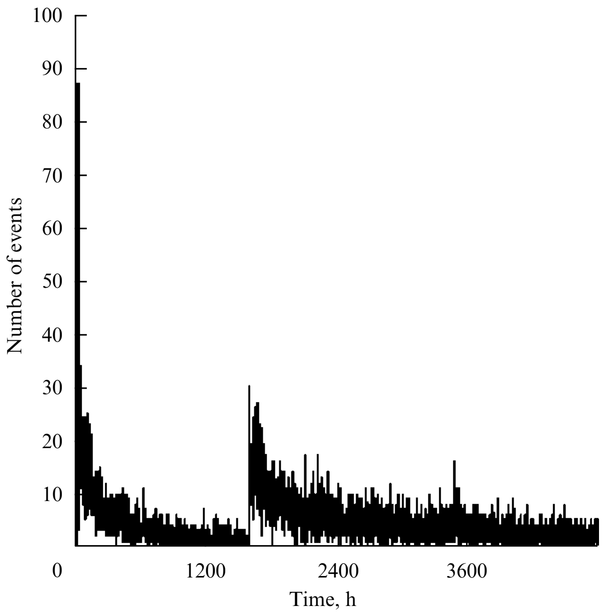


**Figure 3.** Time dependence of the number of aftershocks after two main shocks with magnitudes $M = 6.1$ and $M = 7.3$ that respectively occurred on 23.04.1992 and 28.06.1992, in southern California [9].

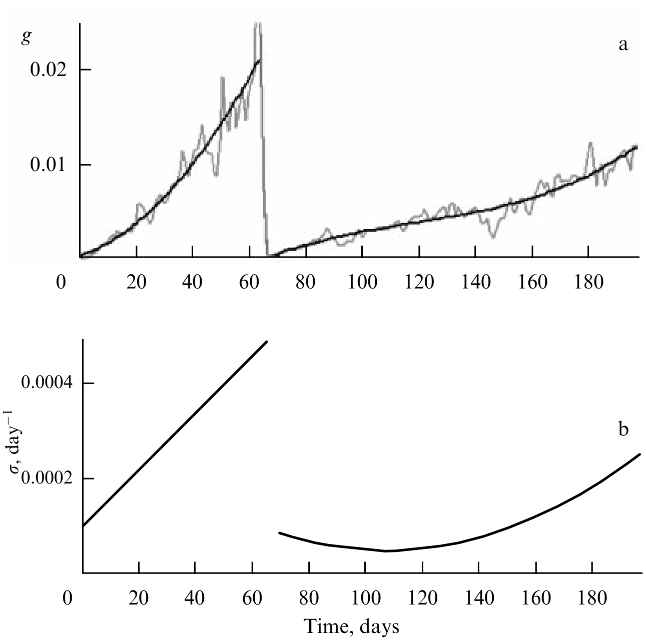


**Figure 4.** Results of the analysis of the aftershocks shown in Fig. 3. (a) The time dependence of the function $g(t)$ before and after smoothing (the respective gray and solid lines). (b) The dependence of the deactivation coefficient $\sigma(t)$ on time.

mas (6). Making all necessary reservations, we have cautiously drawn an analogy between the mean time it takes the recombination of a pair of charges to occur and the mean occurrence time for the rock failure between a pair of adjacent faces of a fault in the lithosphere. Admittedly, our arguments are incomplete. They have not thus far led to a deeper understanding of the mechanism of earthquakes. Moreover, it cannot be ruled out that we are simply dealing with a coincidence, in which case Eqn (3) should be considered just an equivalent form of writing Omori law (1). But even in this case, Eqn (3) is helpful, because it provided a hint to the shape of the generalized law of aftershock evolution (9).

Equation (3) can serve as a basis for other generalizations. Indeed, if we add a diffusive term to it, writing it as

$$\frac{\partial n}{\partial t} = -\sigma n^2 + D\nabla^2 n\,, \tag{11}$$

then we are prompted to consider the Fisher–Kolmogorov–Petrovskii–Piskunov (FKPP) equation known in mathematics and biology [29, 30]. In this manner, we can widen our horizons in searching for phenomenological models for the aftershock distribution in time and space. But we are still far from the goal, if only because the FKPP equation does not consider the anisotropy of the geological medium at an earthquake source, which is literally pierced with faults, with a dominant main fault along which the magistral rupture takes place during the main shock.

In addition to the problem of the spatial–temporal aftershock distribution, which has local or, more precisely, regional character, there is a global aspect. It is related to Earth's sphericity and the resonant properties of Earth as a whole. The aspect lies in the interpretation of hidden periodicities discovered recently by researchers of the Schmidt Earth Physics Institute of the RAS in decaying series of aftershocks [31–34] (see also review [7]).

First, the modulation of aftershocks by spheroidal Earth oscillations ${}_0S_2$ was discovered. (We recall that the period of ${}_0S_2$ oscillations equals 54 min [35].)

Second, a cumulative action of a round-the-world seismic echo on the earthquake source was uncovered, the echo being carried by surface elastic waves, which complete a full circle around Earth in approximately 3 h.

The effects of modulation and round-the-world echo were most clearly manifested after the strongest earthquakes of the 21st century: the Sumatra–Andaman earthquake (2004, $M = 9$) and the Tohoku earthquake (2011, $M = 9$) [33]. Both effects destroy the monotonic decay in aftershock activity with time. Neither Omori law (1) nor power-law (2) describes them for this reason. Yet they can be described by evolution law (9) within the approach presented in Section 5.

Completing the discussion, we cannot avoid mentioning that in this work, devoted mainly to Omori law (1), we did not attempt an analysis of all the rich diversity of mathematical models proposed to describe aftershock sequences. For example, we skipped the compound model, in which the dependence of the aftershock frequency on time changes from a linear to a power-law one and then to an exponential one [36]. We have not considered a number of interesting models relying on ideas of self-organized criticality [37, 38]. The references given here serve to partly compensate for this gap.

## 7. Conclusions

Thus, in Japan at the end of the 19th century, modern seismology was conceived owing to a remarkable combination at that time and at that place of the demand from society, state support, and personal genius. We recalled the preceding history and recounted the history of the Omori law's discovery and discussed the present research; now it only remains to guess about the future.

The short-term outlook for aftershock research is more or less apparent. Most probably, classification of aftershocks according to the type of their frequency decay will be proposed. The study of hidden periodicities in aftershock series is already well advanced now, but much remains to be done. For example, it would be of interest to critically analyze the methods used to estimate dissipative properties of Earth's interior based on data on the $q$-factor of free Earth oscillations in connection with the fact that a part of the energy of free oscillations excited by the main shock is transmitted into the activation of aftershocks, which can then serve as sources of free oscillations. This energy transfer channel has been ignored so far. The search for phenomenological models for the spatio–temporal distribution of aftershocks will also stay in the research focus.

As concerns longer-term prospects, they remain rather vague. And yet one unsolved question related to the general theory of rupture in solid bodies will be attracting increased attention for a long time. We mean a theoretical estimate of the deactivation coefficient $\sigma$ (or the coefficient $k$ in the original formulation (1) of the Omori law).

The author is indebted to A L Kalisher for the discussions. The work was supported by Program 15 of the Presidium of the RAS and the Russian Foundation for Basic Research (project 15-05-00491).

## References

1. Davison Ch *The Founders of Seismology* (Cambridge: The Univ. Press, 1930)
2. Omori F *J. College Sci. Imperial Univ. Tokyo* **7** 111 (1894)
3. Gutenberg B, Richter C F *Bull. Seismol. Soc. Am.* **34** 185 (1944)
4. Richter C F *Elementary Seismology* (San Francisco: W.H. Freeman, 1958); Translated into Russian: *Elementarnaya Seismologiya* (Moscow: IL, 1963)
5. Bolt B A *Earthquakes* (San Francisco: W.H. Freeman, 1978); Translated into Russian: *Zemletryaseniya* (Moscow: Mir, 1981)
6. Kasahara K *Earthquake Mechanics* (Cambridge: Cambridge Univ. Press, 1981); Translated into Russian: *Mekhanika Zemletryasenii* (Moscow: Mir, 1985)
7. Guglielmi A V *Phys. Usp.* **58** 384 (2015); *Usp. Fiz. Nauk* **185** 415 (2015)
8. Costa L S et al. *Europhys. Lett.* **114** 59002 (2016)
9. Guglielmi A V, Zotov O D, in *11th Intern. Conf. and School "Problems of Geocosmos", St. Petersburg, Russia, October 3–7, 2016, Book of Abstracts* (St. Petersburg: St. Petersburg State Univ., 2016) Sect. S, p. 210; http://geo.phys.spbu.ru/geocosmos/
10. Utsu T et al. *J. Phys. Earth* **43** 1 (1995)
11. Reasenberg P A, Jones L M *Science* **243** 1173 (1989)
12. Utsu T *Geophys. Mag.* **30** 521 (1961)
13. Hirano R *Kishoshushi 2* **2** 77 (1924)
14. Jeffreys H *Gerlands Beitr. Geophys.* **56** 111 (1938)
15. Nasu N *Bull. Earthquake Res. Inst. Univ. Tokyo* **6** 245 (1929)
16. Utsu T *Zisin 2 J. Seismol. Soc. Jpn.* **10** 35 (1957)
17. Utsu T *Bull. Seismol. Soc. Am.* **52** 279 (1962)
18. Adams R D, Le Fort J H *New Zealand J. Geol. Geophys.* **6** 87 (1963)
19. Iio Y *Zisin 2 J. Seismol. Soc. Jpn.* **37** 109 (1984)
20. Ryall A, Savage W U *J. Geophys. Res.* **74** 4281 (1969)
21. Spivak A A *Geoekologiya* (6) 27 (1996)
22. Kocharyan G G, Spivak A A *Dinamika Deformirovaniya Blochnykh Massivov Gornykh Porod* (Dynamics of Deformation of Block Rock Formations) (Moscow: Akademkniga, 2003)
23. Felzer K R "The wonderful world of aftershocks", SeismoLab Seminar, January (Pasadena: CalTech, 2007); http://pasadena.wr.usgs.gov/office/kfelzer/SeisLabTalk.pdf
24. Guglielmi A V, arXiv:1604.07017
25. Ginzburg V L *The Propagation of Electromagnetic Waves in Plasmas* (Oxford: Pergamon Press, 1964); Translated from Russian: *Rasprostranenie Elektromagnitnykh Voln v Plazme* (Moscow: Fizmatgiz, 1960)
26. Guglielmi A V *Izv. Phys. Solid Earth* **52** 785 (2016); *Fiz. Zemli* (5) 165 (2016)
27. Ehrenfest P *Proc. Acad. Sci. Amsterdam* **20** 200 (1918)
28. Gorelik G E *Pochemu Prostranstvo Trekhmerno?* (Why is the Space Three-dimensional?) (Moscow: Nauka, 1982)
29. Fisher R A *Ann. Eugenics* **7** 355 (1937)
30. Kolmogorov A N, Petrovsky I G, Piskunov N S *Byull. Mosk. Gos. Univ. Ser. A. Matem. Mekh.* **1** (6) 1 (1937); in Kolmogorov A N *Selected Works* Vol. 1 (Ed. V M Tikhomirov) (Dordrecht: Kluwer Acad. Publ., 1991) p. 242; Translated from Russian: *Izbrannye Trudy. Matematika i Mekhanika* (Ed. S M Nikol'skii) (Moscow: Nauka, 1985) p. 221
31. Guglielmi A, Zotov O, arXiv:1207.0365
32. Guglielmi A V, Zotov O D *Izv. Phys. Solid Earth* **49** 1 (2013); *Fiz. Zemli* (1) 3 (2013)
33. Guglielmi A V, Zotov O D, Zavyalov A D *Izv. Phys. Solid Earth* **50** 64 (2014); *Fiz. Zemli* (1) 66 (2014)
34. Guglielmi A V *Izv. Phys. Solid Earth* **51** 920 (2015); *Fiz. Zemli* (6) 127 (2015)
35. Zharkov V N *Fizika Zemnykh Nedr* (Physics of Earth' Interior) (Moscow: Nauka i Obrazovanie, 2012)
36. Narteau C et al. *Geophys. Res. Lett.* **30** 22 (2003)
37. Bak P *How Nature Works: The Science of Self-organized Criticality* (New York: Springer-Verlag, 1999); Translated into Russian: *Kak Rabotaet Priroda: Teoriya Samoorganizovannoi Kritichnosti* (Moscow: URSS, 2013)
38. Makarov P V *Phys. Mesomech.* **13** 292 (2010); *Fiz. Mezomekh.* **13** (5) 97 (2010)