

Final Report

Predicting the first four moments in particle turbulence

Introduction

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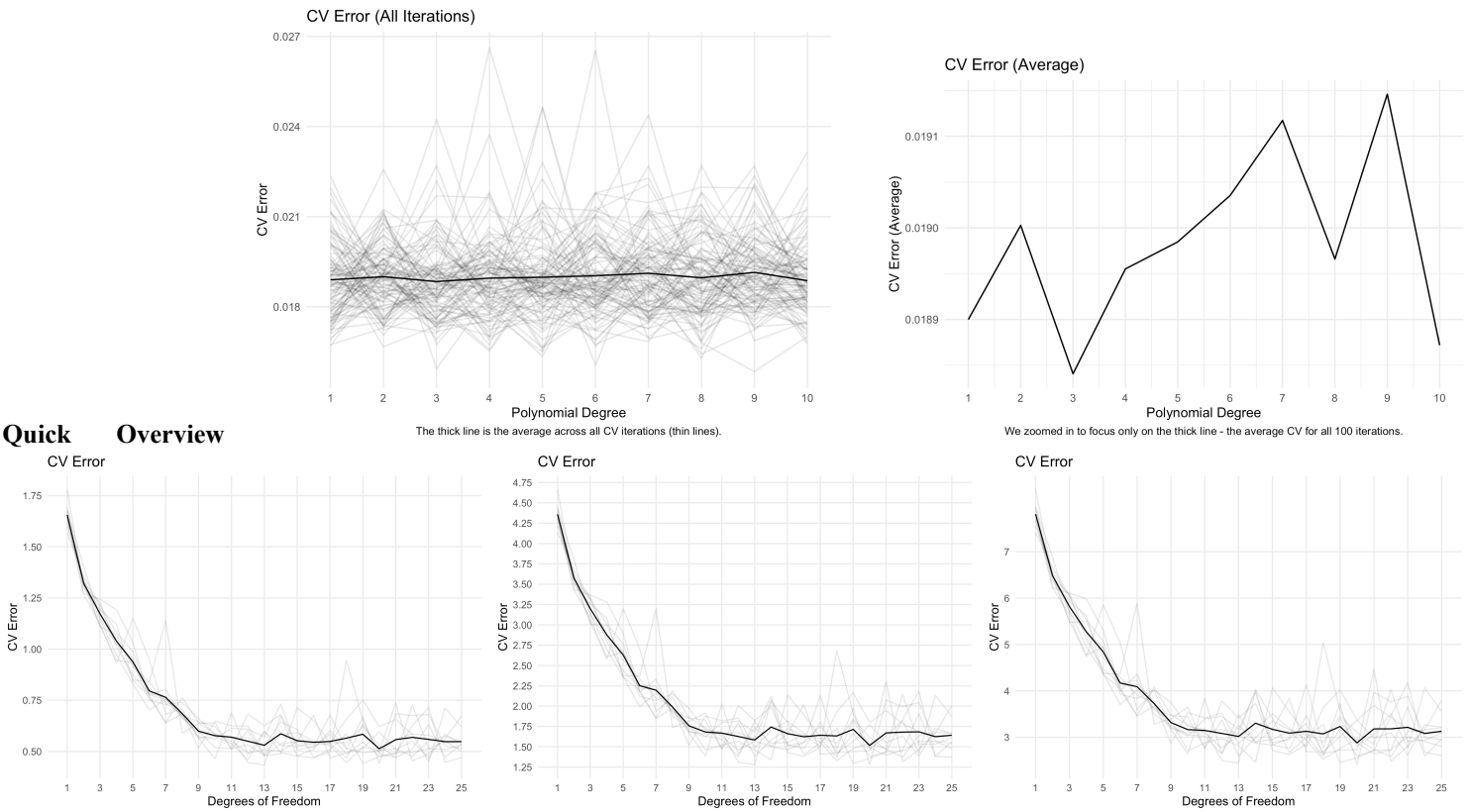
Methodology

```
##           St           Re  Fr R_moment_1 R_moment_2 R_moment_3
## St      1.00000000 -0.03169871 NaN  0.2147681  0.1479257  0.1647465
## Re      -0.03169871  1.00000000 NaN -0.7747206 -0.3932344 -0.3844289
## Fr           NaN           NaN  1           NaN           NaN           NaN
## R_moment_1 0.21476813 -0.77472058 NaN  1.0000000  0.6298829  0.6217326
## R_moment_2 0.14792571 -0.39323445 NaN  0.6298829  1.0000000  0.9984335
## R_moment_3 0.16474648 -0.38442895 NaN  0.6217326  0.9984335  1.0000000
## R_moment_4 0.18004537 -0.37741773 NaN  0.6150484  0.9946671  0.9988414
##           R_moment_4
## St      0.1800454
## Re      -0.3774177
## Fr           NaN
## R_moment_1 0.6150484
## R_moment_2 0.9946671
## R_moment_3 0.9988414
## R_moment_4 1.0000000

## # A tibble: 89 x 15
##           St      Re      Fr      M1      M2      M3      M4 Fr_sigmoid Re_sigmoid
##   <dbl> <dbl>   <dbl>   <dbl>   <dbl>   <dbl>   <dbl>   <dbl>   <dbl>
## 1  0.1    224  0.052  2.16e-3  0.130  14.4    1.59e3  0.513    1
## 2  3      224  0.052  3.79e-3  0.470  69.9    1.04e4  0.513    1
## 3  0.7    224  Inf    2.91e-3  0.0435  0.822  1.56e1  1        1
## 4  0.05    90  Inf    6.35e-2  0.0907  0.467  3.27e0  1        1
## 5  0.7    398  Inf    3.69e-4  0.00622  0.126  2.57e0  1        1
## 6  2       90  0.3    1.48e-1  2.01    36.2    6.72e2  0.574    1
## 7  0.2     90  Inf    8.13e-2  0.324   3.04    3.30e1  1        1
## 8  3      224  Inf    5.75e-3  0.120   2.75    6.32e1  1        1
## 9  0.9    224  Inf    3.02e-3  0.0452  0.845  1.58e1  1        1
## 10 0.6    398  0.052  3.14e-4  0.00447  0.0821  1.51e0  0.513    1
## # ... with 79 more rows, and 6 more variables: M1_sigmoid <dbl>,
## #   M2_sigmoid <dbl>, M3_sigmoid <dbl>, M4_sigmoid <dbl>, Re_categorical <chr>,
## #   Fr_categorical <chr>

## # A tibble: 23 x 7
##           St      Re      Fr Fr_sigmoid Re_sigmoid Re_categorical Fr_categorical
##   <dbl> <dbl>   <dbl>   <dbl>   <dbl>   <chr>         <chr>
## 1  0.05    398  0.052    0.513     1 High         Low
## 2  0.2     398  0.052    0.513     1 High         Low
## 3  0.7     398  0.052    0.513     1 High         Low
## 4  1       398  0.052    0.513     1 High         Low
## 5  0.1     398  Inf        1        1 High         High
```

```
## 6 0.6 398 Inf 1 1 High High
## 7 1 398 Inf 1 1 High High
## 8 1.5 398 Inf 1 1 High High
## 9 3 398 Inf 1 1 High High
## 10 3 224 0.3 0.574 1 Medium Medium
## # ... with 13 more rows
```



Results

Final Models

```
##
## Call:
## lm(formula = log(M1) ~ St + Re_categorical + Fr_categorical,
##     data = train1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.47532 -0.07168  0.02101  0.10237  0.23554
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -8.19087    0.04535  -180.631  <2e-16 ***
## St              0.24652    0.02165   11.386  <2e-16 ***
## Re_categorical  5.75678    0.04826  119.287  <2e-16 ***
## Re_categoricalMedium 2.13087    0.04657   45.760  <2e-16 ***
## Fr_categoricalLow  0.02584    0.03945    0.655   0.5142
## Fr_categoricalMedium -0.08185    0.04602   -1.779   0.0789 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1593 on 83 degrees of freedom
```

```

## Multiple R-squared:  0.9952, Adjusted R-squared:  0.9949
## F-statistic:  3460 on 5 and 83 DF,  p-value: < 2.2e-16

## Warning in model.matrix.default(mt, mf, contrasts): non-list contrasts argument
## ignored

##
## Call: gam(formula = log(M2) ~ s(St, df = 9) + Re_categorical + Fr_categorical +
##   Re_categorical * Fr_categorical, data = train1)
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -3.100803 -0.185164  0.006113  0.231614  1.735705
##
## (Dispersion Parameter for gaussian family taken to be 0.5064)
##
## Null Deviance: 1212.424 on 88 degrees of freedom
## Residual Deviance: 36.461 on 72.0001 degrees of freedom
## AIC: 209.1479
##
## Number of Local Scoring Iterations: 2
##
## Anova for Parametric Effects
##
##              Df Sum Sq Mean Sq F value    Pr(>F)
## s(St, df = 9)      1  49.47    49.47  97.699 4.819e-15 ***
## Re_categorical      2 722.42   361.21  713.289 < 2.2e-16 ***
## Fr_categorical      2 205.08   102.54  202.487 < 2.2e-16 ***
## Re_categorical:Fr_categorical  3 151.24    50.41   99.550 < 2.2e-16 ***
## Residuals         72  36.46     0.51
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Anova for Nonparametric Effects
##
##              Npar Df Npar F      Pr(F)
## (Intercept)
## s(St, df = 9)           8 20.692 6.661e-16 ***
## Re_categorical
## Fr_categorical
## Re_categorical:Fr_categorical
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## Warning in model.matrix.default(mt, mf, contrasts): non-list contrasts argument
## ignored

##
## Call: gam(formula = log(M3) ~ s(St, df = 9) + Re_categorical + Fr_categorical +
##   Re_categorical * Fr_categorical, data = train1)
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -5.97775 -0.35007  0.03625  0.34668  3.06602
##
## (Dispersion Parameter for gaussian family taken to be 1.4732)
##
## Null Deviance: 2837.669 on 88 degrees of freedom
## Residual Deviance: 106.0723 on 72.0001 degrees of freedom
## AIC: 304.189
##
## Number of Local Scoring Iterations: 2
##

```

```

## Anova for Parametric Effects
##
##           Df Sum Sq Mean Sq F value    Pr(>F)
## s(St, df = 9)      1   94.67    94.67  64.263 1.429e-11 ***
## Re_categorical     2 1162.83   581.42 394.656 < 2.2e-16 ***
## Fr_categorical     2  798.47   399.24 270.994 < 2.2e-16 ***
## Re_categorical:Fr_categorical  3  542.39   180.80 122.722 < 2.2e-16 ***
## Residuals        72  106.07     1.47
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Anova for Nonparametric Effects
##
##           Npar Df Npar F      Pr(F)
## (Intercept)
## s(St, df = 9)           8 17.921 2.087e-14 ***
## Re_categorical
## Fr_categorical
## Re_categorical:Fr_categorical
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## Warning in model.matrix.default(mt, mf, contrasts): non-list contrasts argument
## ignored

##
## Call: gam(formula = log(M4) ~ s(St, df = 9) + Re_categorical + Fr_categorical +
##       Re_categorical * Fr_categorical, data = train1)
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -8.61481 -0.44595  0.04067  0.50573  4.15489
##
## (Dispersion Parameter for gaussian family taken to be 2.7642)
##
##      Null Deviance: 5248.685 on 88 degrees of freedom
## Residual Deviance: 199.0259 on 72.0001 degrees of freedom
## AIC: 360.1979
##
## Number of Local Scoring Iterations: 2
##
## Anova for Parametric Effects
##
##           Df Sum Sq Mean Sq F value    Pr(>F)
## s(St, df = 9)      1  146.72   146.72  53.076 3.281e-10 ***
## Re_categorical     2 1732.68   866.34 313.410 < 2.2e-16 ***
## Fr_categorical     2 1769.38   884.69 320.048 < 2.2e-16 ***
## Re_categorical:Fr_categorical  3 1162.78   387.59 140.217 < 2.2e-16 ***
## Residuals        72  199.03     2.76
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Anova for Nonparametric Effects
##
##           Npar Df Npar F      Pr(F)
## (Intercept)
## s(St, df = 9)           8 16.705 1.025e-13 ***
## Re_categorical
## Fr_categorical
## Re_categorical:Fr_categorical
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## Warning in model.matrix.default(mt, mf, contrasts): non-list contrasts argument
## ignored

```

```
##
## Call: gam(formula = log(M4) ~ s(St, df = 9) + Re_categorical + Fr_categorical +
##   Re_categorical * Fr_categorical, data = train1)
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -8.61481 -0.44595  0.04067  0.50573  4.15489
##
## (Dispersion Parameter for gaussian family taken to be 2.7642)
##
##      Null Deviance: 5248.685 on 88 degrees of freedom
## Residual Deviance: 199.0259 on 72.0001 degrees of freedom
## AIC: 360.1979
##
## Number of Local Scoring Iterations: 2
##
## Anova for Parametric Effects
##
##              Df Sum Sq Mean Sq F value    Pr(>F)
## s(St, df = 9)      1  146.72   146.72   53.076 3.281e-10 ***
## Re_categorical      2 1732.68   866.34  313.410 < 2.2e-16 ***
## Fr_categorical      2 1769.38   884.69  320.048 < 2.2e-16 ***
## Re_categorical:Fr_categorical  3 1162.78   387.59  140.217 < 2.2e-16 ***
## Residuals         72  199.03     2.76
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Anova for Nonparametric Effects
##
##              Npar Df Npar F      Pr(F)
## (Intercept)
## s(St, df = 9)           8 16.705 1.025e-13 ***
## Re_categorical
## Fr_categorical
## Re_categorical:Fr_categorical
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Conclusion

Appendix

Appendix 1

