

Final Report

Predicting the first four moments in particle turbulence

Introduction

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Methodology

```
##           St           Re  Fr R_moment_1 R_moment_2 R_moment_3
## St           1.00000000 -0.03169871 NaN  0.2147681  0.1479257  0.1647465
## Re          -0.03169871  1.00000000 NaN -0.7747206 -0.3932344 -0.3844289
## Fr           NaN           NaN    1         NaN         NaN         NaN
## R_moment_1  0.21476813 -0.77472058 NaN  1.0000000  0.6298829  0.6217326
## R_moment_2  0.14792571 -0.39323445 NaN  0.6298829  1.0000000  0.9984335
## R_moment_3  0.16474648 -0.38442895 NaN  0.6217326  0.9984335  1.0000000
## R_moment_4  0.18004537 -0.37741773 NaN  0.6150484  0.9946671  0.9988414
##           R_moment_4
## St           0.1800454
## Re          -0.3774177
## Fr           NaN
## R_moment_1  0.6150484
## R_moment_2  0.9946671
## R_moment_3  0.9988414
## R_moment_4  1.0000000
```

*# We transform the variables using the sigmoid function so that this variable
will be within a finite range.*

```
train1 <- train %>%
  rename(M1 = R_moment_1, M2 = R_moment_2, M3 = R_moment_3, M4 = R_moment_4) %>%
  mutate(Fr_sigmoid = 1 / (1 + exp(-Fr)),
         Re_sigmoid = 1 / (1 + exp(-Re)),
         M1_sigmoid = 1 / (1 + exp(-M1)),
         M2_sigmoid = 1 / (1 + exp(-M2)),
         M3_sigmoid = 1 / (1 + exp(-M3)),
         M4_sigmoid = 1 / (1 + exp(-M4))) %>%
  mutate(Re_categorical = case_when(Re == 90 ~ "Low", Re == 224 ~ "Medium", Re == 398 ~ "High"),
         Fr_categorical = case_when(Fr == 0.052 ~ "Low", Fr == 0.3 ~ "Medium", Fr == Inf ~ "High"))

test1 <- test %>%
  mutate(Fr_sigmoid = 1 / (1 + exp(-Fr)),
         Re_sigmoid = 1 / (1 + exp(-Re))) %>%
  mutate(Re_categorical = case_when(Re == 90 ~ "Low", Re == 224 ~ "Medium", Re == 398 ~ "High"),
         Fr_categorical = case_when(Fr == 0.052 ~ "Low", Fr == 0.3 ~ "Medium", Fr == Inf ~ "High"))
```

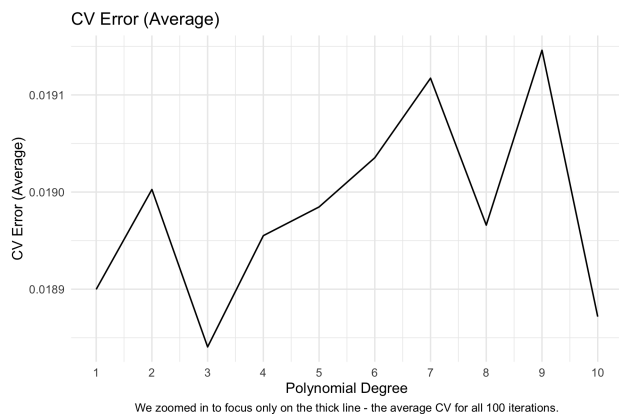
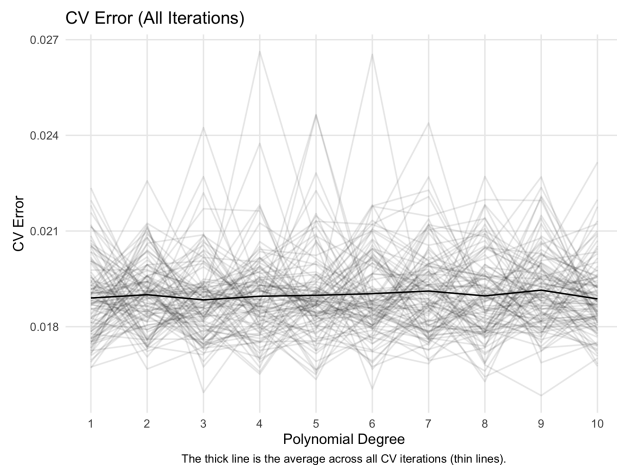
```
train1
```

```
## # A tibble: 89 x 15
##       St     Re     Fr       M1       M2       M3       M4 Fr_sigmoid Re_sigmoid
##   <dbl> <dbl> <dbl>   <dbl>   <dbl>   <dbl>   <dbl>   <dbl>   <dbl>
## 1  0.1    224  0.052 0.00216  0.130   14.4    1586.    0.513     1
## 2  3      224  0.052 0.00379  0.470   69.9    10404    0.513     1
## 3  0.7    224  Inf     0.00291  0.0435   0.822    15.6     1     1
## 4  0.05    90  Inf     0.0635   0.0907   0.467     3.27    1     1
## 5  0.7    398  Inf     0.000369 0.00622  0.126     2.57    1     1
## 6  2       90  0.3    0.148    2.01    36.2    672.    0.574     1
## 7  0.2     90  Inf     0.0813   0.324    3.04    33.0     1     1
## 8  3      224  Inf     0.00575  0.120    2.75    63.2     1     1
## 9  0.9    224  Inf     0.00302  0.0452   0.845    15.8     1     1
## 10 0.6     398  0.052 0.000314 0.00447  0.0821    1.51    0.513     1
## # ... with 79 more rows, and 6 more variables: M1_sigmoid <dbl>,
## #   M2_sigmoid <dbl>, M3_sigmoid <dbl>, M4_sigmoid <dbl>, Re_categorical <chr>,
## #   Fr_categorical <chr>
```

```
test1
```

```
## # A tibble: 23 x 7
##       St     Re     Fr Fr_sigmoid Re_sigmoid Re_categorical Fr_categorical
##   <dbl> <dbl> <dbl>   <dbl>   <dbl> <chr>         <chr>
## 1  0.05    398  0.052    0.513     1 High         Low
## 2  0.2     398  0.052    0.513     1 High         Low
## 3  0.7     398  0.052    0.513     1 High         Low
## 4  1       398  0.052    0.513     1 High         Low
## 5  0.1     398  Inf        1     1 High         High
## 6  0.6     398  Inf        1     1 High         High
## 7  1       398  Inf        1     1 High         High
## 8  1.5     398  Inf        1     1 High         High
## 9  3       398  Inf        1     1 High         High
## 10 3       224  0.3      0.574     1 Medium       Medium
## # ... with 13 more rows
```

Quick Overview



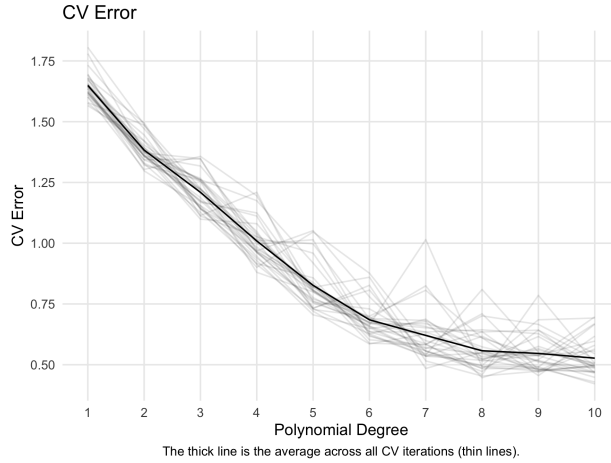


Figure 1: Predicting Raw Moment 2

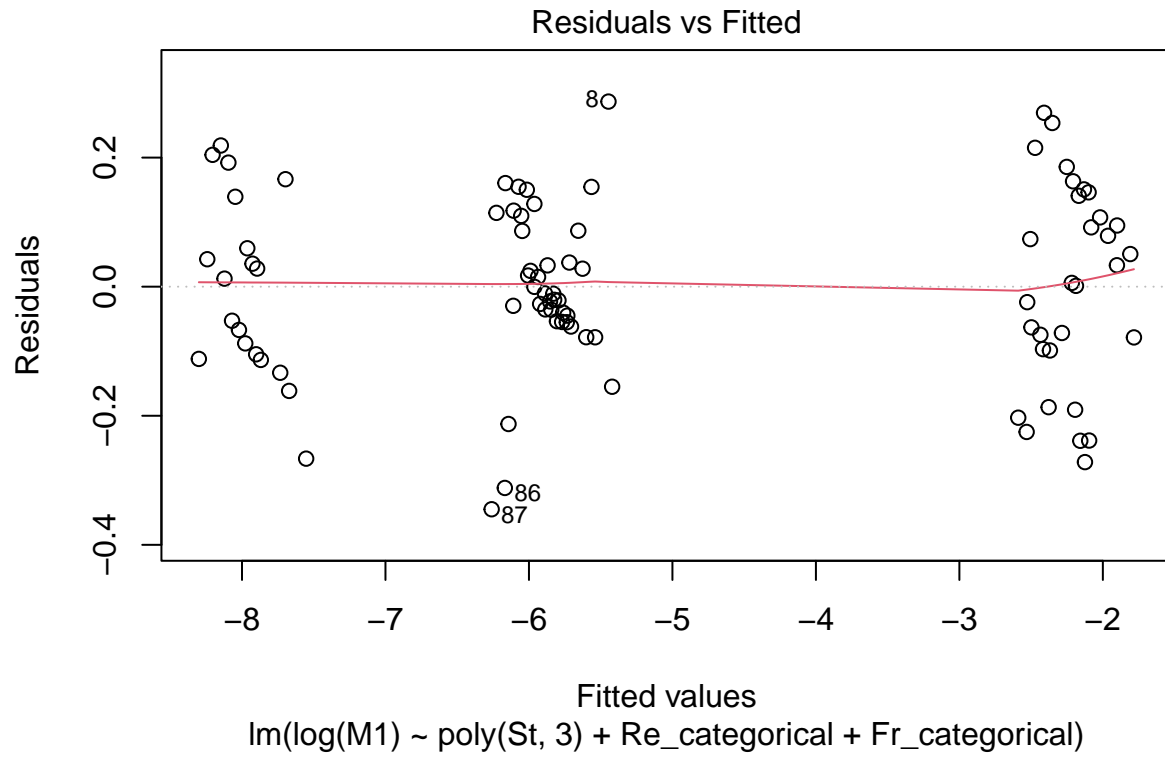
Results

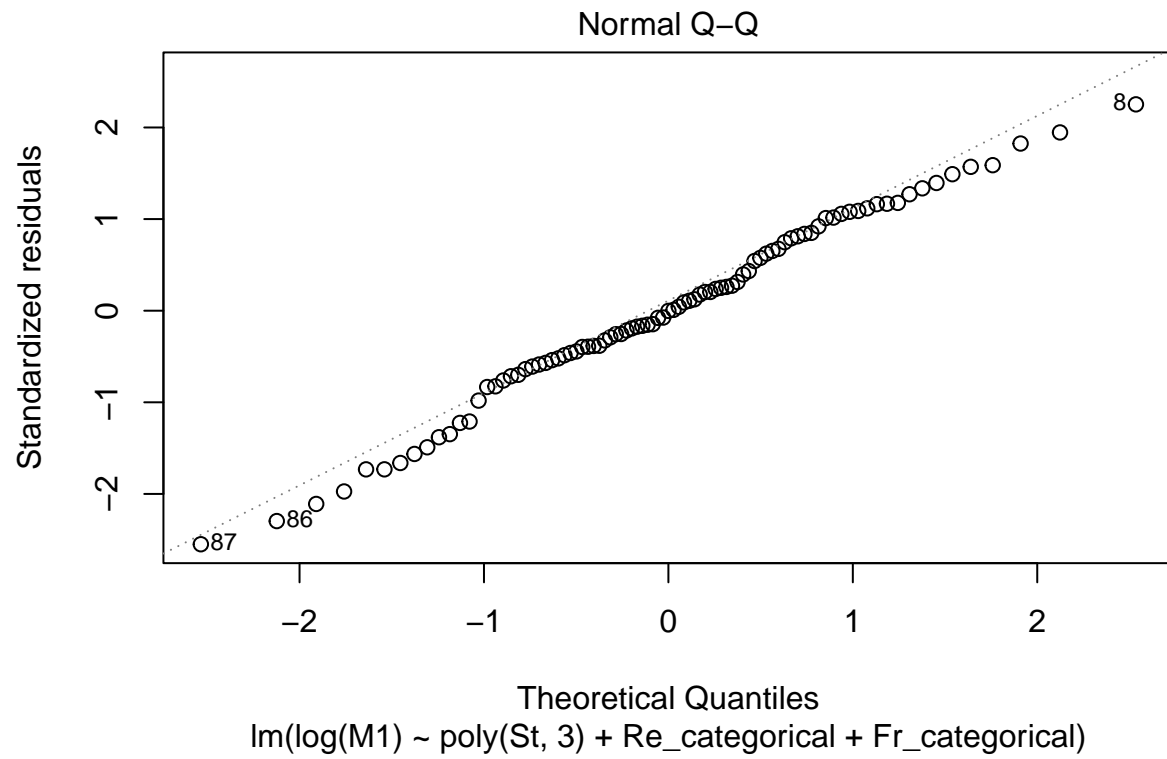
Final Models

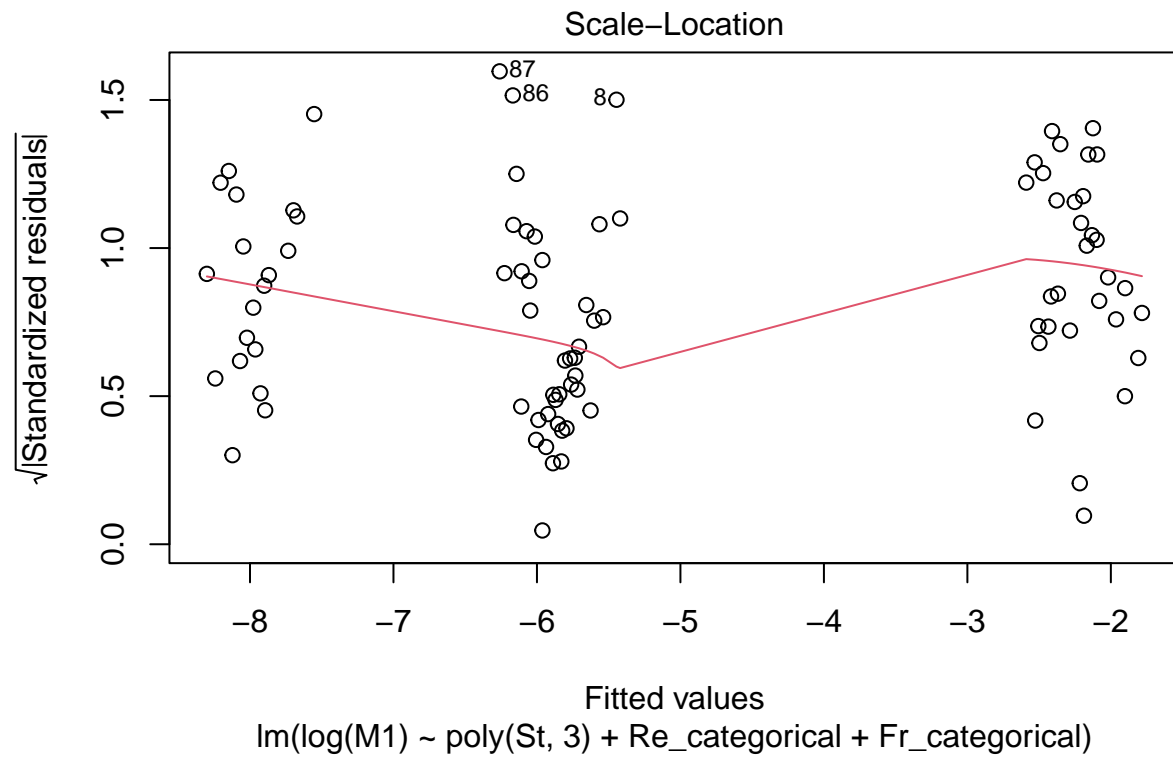
```
M1 <- lm(log(M1) ~ poly(St, 3) + Re_categorical + Fr_categorical, data = train1)
summary(M1)
```

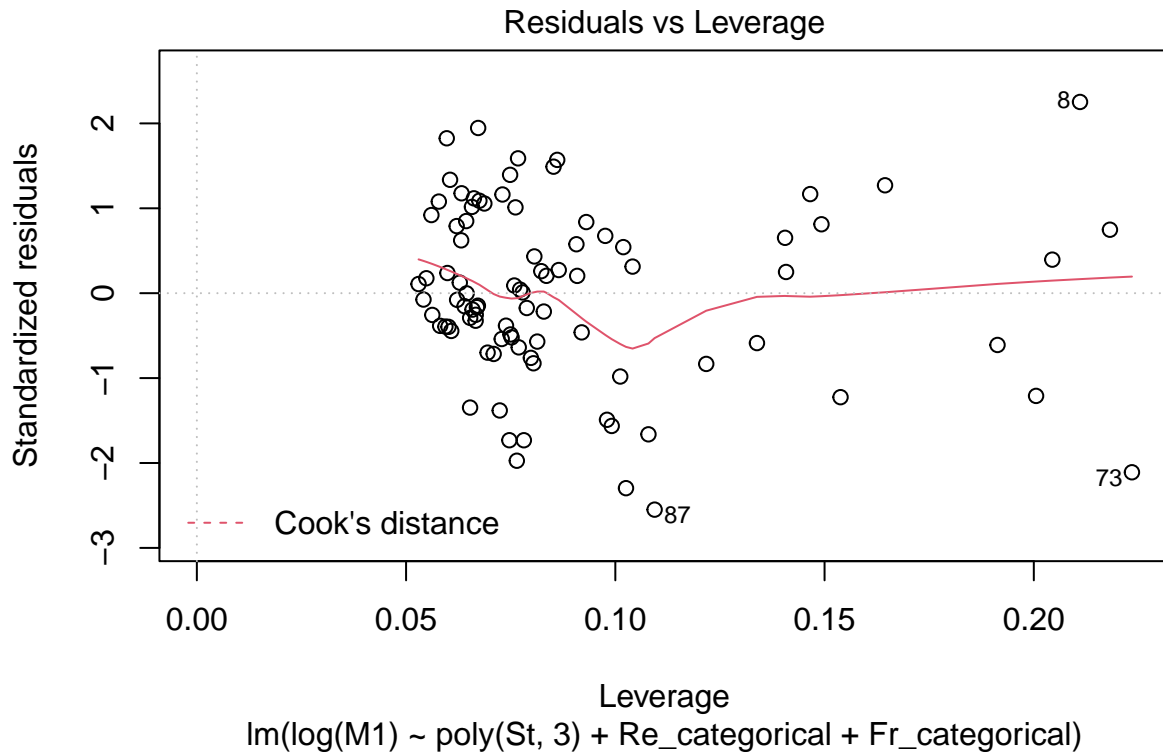
```
##
## Call:
## lm(formula = log(M1) ~ poly(St, 3) + Re_categorical + Fr_categorical,
##     data = train1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.3449 -0.0783 -0.0003  0.1073  0.2868
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -7.98153    0.03784 -210.911 < 2e-16 ***
## poly(St, 3)1    1.81526    0.14370  12.633 < 2e-16 ***
## poly(St, 3)2   -0.63184    0.14418  -4.382 3.49e-05 ***
## poly(St, 3)3    0.21344    0.14347   1.488  0.1407
## Re_categorical  5.76962    0.04355 132.489 < 2e-16 ***
## Re_categoricalMedium 2.13265    0.04191  50.881 < 2e-16 ***
## Fr_categoricalLow  0.02556    0.03552   0.720  0.4738
## Fr_categoricalMedium -0.09182    0.04155  -2.210  0.0299 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1433 on 81 degrees of freedom
## Multiple R-squared:  0.9962, Adjusted R-squared:  0.9959
## F-statistic: 3054 on 7 and 81 DF, p-value: < 2.2e-16
```

```
plot(M1)
```









```
M2 <- gam(log(M2) ~ s(St, df = 9) + Re_categorical + Fr_categorical +
           Re_categorical * Fr_categorical, data = train1)
summary(M2)
```

```
##
## Call: gam(formula = log(M2) ~ s(St, df = 9) + Re_categorical + Fr_categorical +
##       Re_categorical * Fr_categorical, data = train1)
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -3.100803 -0.185164  0.006113  0.231614  1.735705
##
## (Dispersion Parameter for gaussian family taken to be 0.5064)
##
## Null Deviance: 1212.424 on 88 degrees of freedom
## Residual Deviance: 36.461 on 72.0001 degrees of freedom
## AIC: 209.1479
##
## Number of Local Scoring Iterations: NA
##
## Anova for Parametric Effects
##
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
s(St, df = 9)	1	49.47	49.47	97.699	4.819e-15 ***
Re_categorical	2	722.42	361.21	713.289	< 2.2e-16 ***
Fr_categorical	2	205.08	102.54	202.487	< 2.2e-16 ***
Re_categorical:Fr_categorical	3	151.24	50.41	99.550	< 2.2e-16 ***

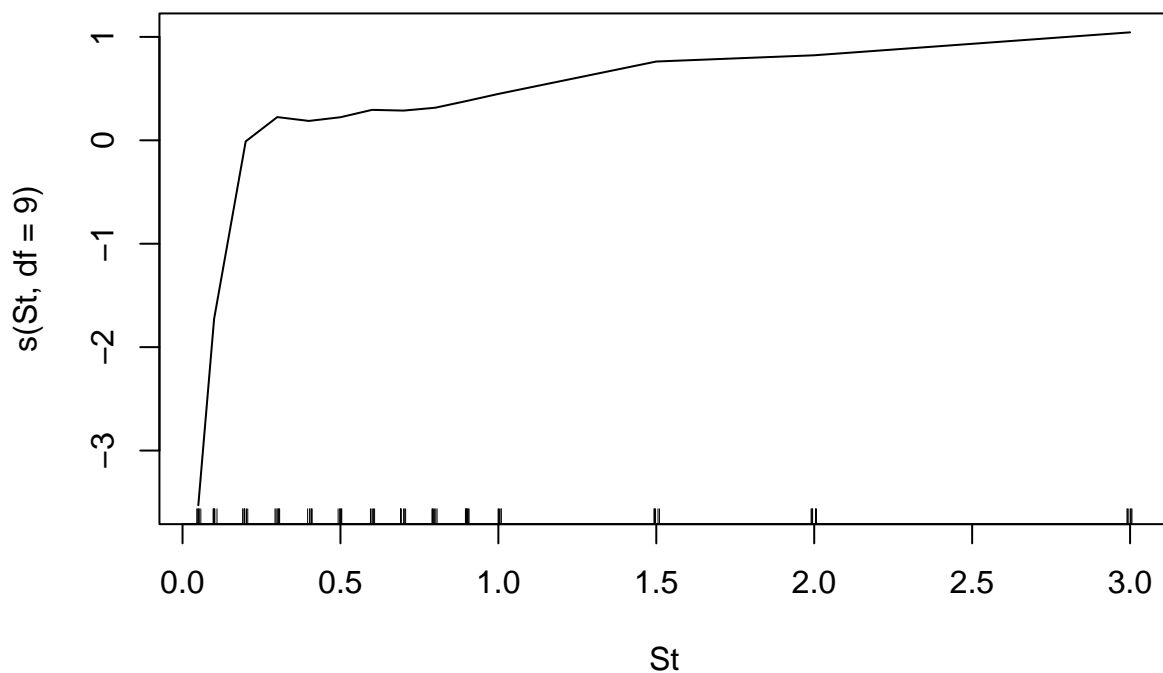
```
## Residuals              72  36.46    0.51
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Anova for Nonparametric Effects
##              Npar Df Npar F      Pr(F)
## (Intercept)
## s(St, df = 9)              8 20.692 6.661e-16 ***
## Re_categorical
## Fr_categorical
## Re_categorical:Fr_categorical
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
plot(M2)
```

```
## Warning in preplot.Gam(x, terms = terms): No terms saved for "a:b" style
## interaction terms

## Warning in gplot.default(x = c("Medium", "Medium", "Medium", "Low", "High", :
## The "x" component of "partial for Re_categorical" has class "character"; no
## gplot() methods available

## Warning in gplot.default(x = c("Low", "Low", "High", "High", "High", "Medium", :
## The "x" component of "partial for Fr_categorical" has class "character"; no
## gplot() methods available
```




```
M3 <- gam(log(M3) ~ s(St, df = 9) + Re_categorical + Fr_categorical +
           Re_categorical * Fr_categorical, data = train1)
summary(M3)
```

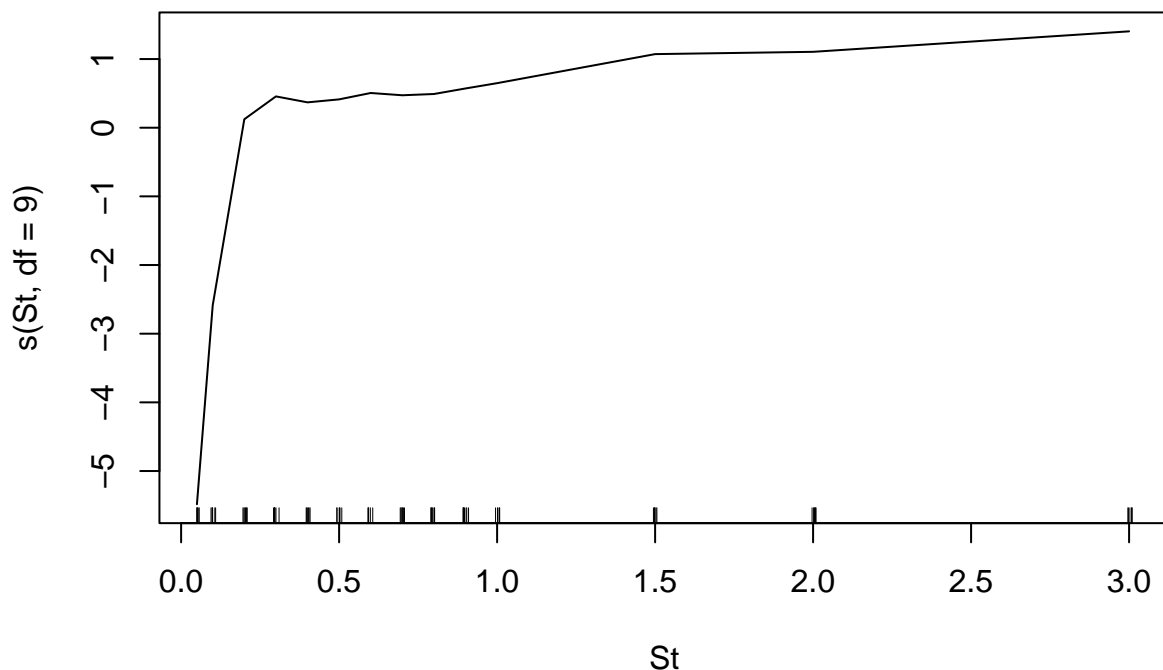
```
##
## Call: gam(formula = log(M3) ~ s(St, df = 9) + Re_categorical + Fr_categorical +
##       Re_categorical * Fr_categorical, data = train1)
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -5.97775 -0.35007  0.03625  0.34668  3.06602
##
## (Dispersion Parameter for gaussian family taken to be 1.4732)
##
## Null Deviance: 2837.669 on 88 degrees of freedom
## Residual Deviance: 106.0723 on 72.0001 degrees of freedom
## AIC: 304.189
##
## Number of Local Scoring Iterations: NA
##
## Anova for Parametric Effects
##              Df Sum Sq Mean Sq F value    Pr(>F)
## s(St, df = 9)    1   94.67    94.67  64.263 1.429e-11 ***
## Re_categorical    2 1162.83   581.42 394.656 < 2.2e-16 ***
## Fr_categorical    2   798.47   399.24 270.994 < 2.2e-16 ***
## Re_categorical:Fr_categorical 3   542.39   180.80 122.722 < 2.2e-16 ***
## Residuals       72   106.07     1.47
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Anova for Nonparametric Effects
##              Npar Df Npar F      Pr(F)
## (Intercept)
## s(St, df = 9)           8 17.921 2.087e-14 ***
## Re_categorical
## Fr_categorical
## Re_categorical:Fr_categorical
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
plot(M3)
```

```
## Warning in preplot.Gam(x, terms = terms): No terms saved for "a:b" style
## interaction terms
```

```
## Warning in gplot.default(x = c("Medium", "Medium", "Medium", "Low", "High", :
## The "x" component of "partial for Re_categorical" has class "character"; no
## gplot() methods available
```

```
## Warning in gplot.default(x = c("Low", "Low", "High", "High", "High", "Medium", :
## The "x" component of "partial for Fr_categorical" has class "character"; no
## gplot() methods available
```



```
M4 <- gam(log(M4) ~ s(St, df = 9) + Re_categorical + Fr_categorical +
             Re_categorical * Fr_categorical, data = train1)
summary(M4)
```

```
##
## Call: gam(formula = log(M4) ~ s(St, df = 9) + Re_categorical + Fr_categorical +
##       Re_categorical * Fr_categorical, data = train1)
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -8.61481 -0.44595  0.04067  0.50573  4.15489
##
## (Dispersion Parameter for gaussian family taken to be 2.7642)
##
## Null Deviance: 5248.685 on 88 degrees of freedom
## Residual Deviance: 199.0259 on 72.0001 degrees of freedom
## AIC: 360.1979
##
## Number of Local Scoring Iterations: NA
##
## Anova for Parametric Effects
##
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
s(St, df = 9)	1	146.72	146.72	53.076	3.281e-10 ***
Re_categorical	2	1732.68	866.34	313.410	< 2.2e-16 ***
Fr_categorical	2	1769.38	884.69	320.048	< 2.2e-16 ***
Re_categorical:Fr_categorical	3	1162.78	387.59	140.217	< 2.2e-16 ***

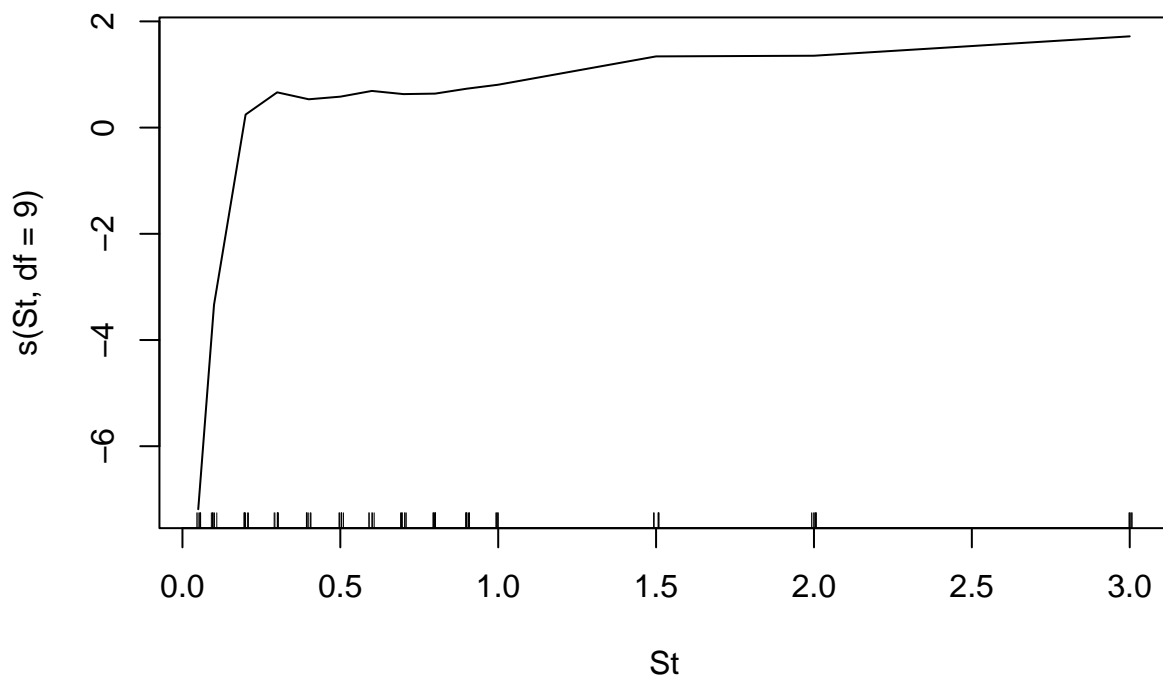
```
## Residuals              72 199.03    2.76
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Anova for Nonparametric Effects
##              Npar Df Npar F      Pr(F)
## (Intercept)
## s(St, df = 9)              8 16.705 1.025e-13 ***
## Re_categorical
## Fr_categorical
## Re_categorical:Fr_categorical
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
plot(M4)
```

```
## Warning in preplot.Gam(x, terms = terms): No terms saved for "a:b" style
## interaction terms

## Warning in gplot.default(x = c("Medium", "Medium", "Medium", "Low", "High", :
## The "x" component of "partial for Re_categorical" has class "character"; no
## gplot() methods available

## Warning in gplot.default(x = c("Low", "Low", "High", "High", "High", "Medium", :
## The "x" component of "partial for Fr_categorical" has class "character"; no
## gplot() methods available
```



Conclusion