MATLAB

LABORATORY WORK BOOK

Year	:	**2020 – 2021**
Subject Code	:	**U18MAI3201-Multivariate Calculus and Forecasting**
Regulations	:	**R18**
Class	:	**I B.E/B.Tech**
Branch	:	**AI&DS**

Certificate

This is to certify that it is a bonafied record of practical work

*done by Sri/Kum. _________**K S THRISHA VAISHNAVI**________________________ bearing the Roll*

*No. _____**20BAD047**________ of ____**2nd** ______**YEAR**_______ class*

***AI&DS**___________________________________ branch in the*

____________________________ laboratory during the academic year ___________________ under our

supervision.

Internal Examiner **Faculty Incharge**

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WORKSHEET- 1

DETERMINING MAXIMA AND MINIMA OF A FUNCTION OF TWO VARIABLES

OBJECTIVE

Evaluating Maxima and Minima

A function f(x,y) is said to have a relative minimum (or simply minimum) at x=a and y=b, if f(a,b)>f(a+h,b+k) for all small values of h and k.

A function f(x,y) is said to have a relative maximum (or simply maximum) at x=a and y=b, if f(a,b)<f(a+h,b+k) for all small values of h and k.

A maximum or a minimum value of a function is called its extreme value.

Working rule to find the extreme values of a function f(x,y):

Step 1: Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

Step 2: Solve the equations $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$ simultaneously.

Let the solutions be (a, b); (c, d); ...

The points like (a, b) at which $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$ are called stationary points of the function $f(x, y)$. The values of $f(x, y)$ at the stationary points are called stationary values of $f(x, y)$.

Step 3: For each solution in step (2), find the values of $A = \frac{\partial^2 f}{\partial x^2}, B = \frac{\partial^2 f}{\partial x \partial y}, C = \frac{\partial^2 f}{\partial y^2}$ and $\Delta = AC - B^2$.

Step 4:

(i) If $\Delta > 0$ and A (or C) <0 for the solution (a, b), then $f(x, y)$ has a maximum value at (a, b).

(ii) If $\Delta > 0$ and A (or C) >0 for the solution (a, b), then $f(x, y)$ has a minimum value at (a, b).

(iii) If $\Delta < 0$ for the solution (a, b), then $f\ (x, y)$ has neither a maximum nor a minimum value at (a, b). In this case, the point (a, b) is called a saddle point of the function $f(x, y)$.

(iv) If $\Delta = 0\ or\ A = 0$, the case is doubtful and further investigations are required to decide the nature of the extreme values of the function $f(x, y)$.

PROCEDURE

1. Open MATLAB
2. Open new M-file
3. Type the program
4. Save in current directory
5. Compile and Run the program
6. For the output see command window\ Figure window

Example 1: Find the extreme value of the function $f(x,y) = x^4 + 2y^4 - 12xy^2 - 20y^2$.

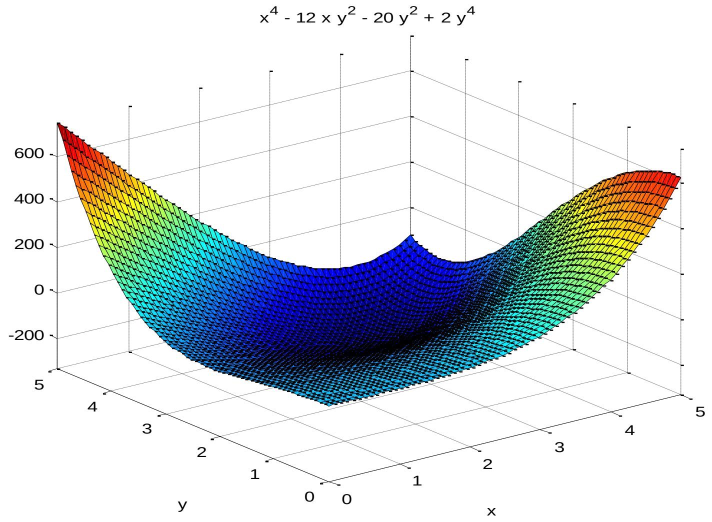
PROGRAM

```
syms x y
f=x.^4+2*y.^4-12*x*y.^2-20*y.^2
fx=diff(f,x)
fy=diff(f,y)
[a,b]=solve(fx,fy);
double([a,b])
fxx=diff(fx,x)
fxy=diff(fx,y)
fyy=diff(fy,y)
D=fxx*fyy-(fxy)^2
subs(D,[x,y],[3.6247,3.9842])
subs(fxx,[x,y],[3.6247,3.9842])
subs(D,[x,y],[3.6247,-3.9842])
subs(fxx,[x,y],[3.6247,-3.9842])
ezsurf(f,[0,5,0,5])
```

OUTPUT

```
     f =
x^4 - 12*x*y^2 + 2*y^4 - 20*y^2

fx =
 4*x^3 - 12*y^2
fy =
 8*y^3 - 24*x*y - 40*y

ans =
   0.0000 + 0.0000i   0.0000 + 0.0000i
   3.6247 + 0.0000i   3.9842 + 0.0000i
   3.6247 + 0.0000i  -3.9842 + 0.0000i
  -1.8123 - 0.9240i   1.0884 - 1.2734i
  -1.8123 + 0.9240i   1.0884 + 1.2734i
  -1.8123 - 0.9240i  -1.0884 + 1.2734i
  -1.8123 + 0.9240i  -1.0884 - 1.2734i
fxx =
12*x^2

fxy =
 -24*y
```

```
fyy =
 24*y^2 - 24*x - 40

 D =
 - 12*x^2*(- 24*y^2 + 24*x + 40) - 576*y^2

ans =
 2414018814601833021/78125000000000

ans =
 3941535027/25000000

ans =
 2414018814601833021/78125000000000

ans =
 3941535027/25000000
```

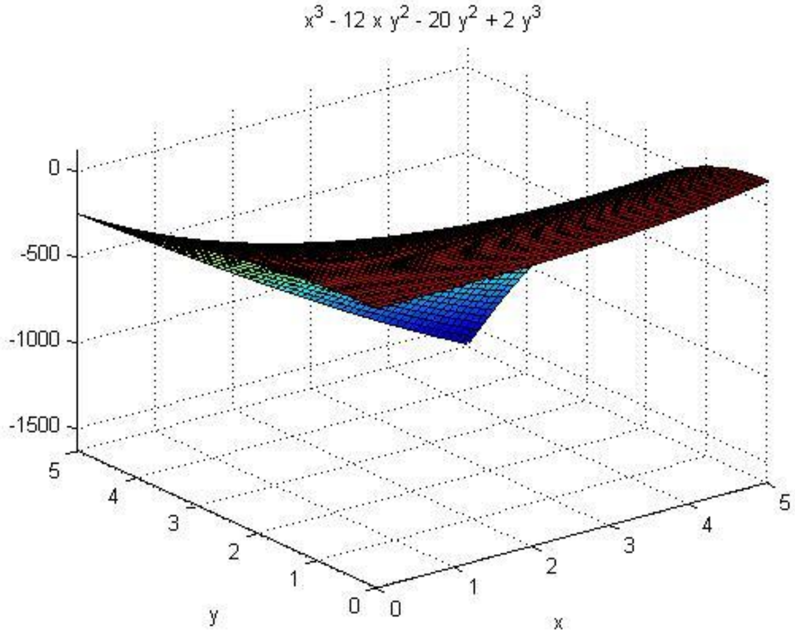


Example 2: Find the extreme value of the function $f(x,y) = x^3 + 2y^3 - 12xy^2 - 20y^2$

PROGRAM

```
syms x y
f=x.^3+2*y.^3-12*x*y.^2-20*y.^2
fx=diff(f,x)
fy=diff(f,y)
[a,b]=solve(fx,fy);
double([a,b])
fxx=diff(fx,x)
fxy=diff(fx,y)
fyy=diff(fy,y)
D=fxx*fyy-(fxy)^2
D1=subs(D,[x,y],[ -1.4815,0.7807])
A1=subs(fxx,[x,y],[ -1.4815,0.7807])
```

```
D2=subs(D,[x,y],[ -1.9048,-0.9524])
A2=subs(fxx,[x,y],[ -1.9048,-0.9524])
   if D1>0
     if A1>0
      disp('Attains Minima')
     else
      disp('Attains Maxima')
     end
   else
     if D1==0
      disp('No conclusion')
     else
      disp('Neither Minima nor Maxima')
     end
   end
   if D2>0
     if A2>0
      disp('Attains Minima')
     else
      disp('Attains Maxima')
     end
   else
     if D2==0
      disp('no conclusion')
     else
      disp('Neither Minima nor maxima')
     end
   end
   ezsurf(f,[0,5,0,5])
```



TASK 1

1. Find the Maximum value of the function $f(x,y)=x^3-3xy^2-12xy+2$

CODE:

```
    syms x y
f=x.^3-3.*x.*y.^2-12.*x.*y+2
fx=diff(f,x)
fy=diff(f,y)
[a,b]=solve(fx,fy);
double([a,b])
fxx=diff(fx,x)
fxy=diff(fx,y)
fyy=diff(fy,y)
D=fxx*fyy-(fxy)^2
D1=subs(D,[x,y],[0,-4])
A1=subs(fxx,[x,y],[0,-4])
D2=subs(D,[x,y],[0,-2])
A2=subs(fxx,[x,y],[0,-2])
   if D1>0
     if A1>0
      disp('Attains Minima')
     else
      disp('Attains Maxima')
     end
   else
     if D1==0
      disp('No conclusion')
     else
      disp('Neither Minima nor Maxima')
     end
   end
   if D2>0
     if A2>0
      disp('Attains Minima')
     else
      disp('Attains Maxima')
     end
   else
     if D2==0
      disp('no conclusion')
     else
      disp('Neither Minima nor maxima')
     end
   end
   fsurf(f,[0,5,0,5])
```

OUTPUT:

f = $x^3 - 3\,x\,y^2 - 12\,x\,y + 2$

fx = $3\,x^2 - 3\,y^2 - 12\,y$

fy = $-12\,x - 6\,x\,y$

```
ans = 4×2 complex
     0.0000 + 0.0000i   0.0000 + 0.0000i
     0.0000 + 0.0000i  -4.0000 + 0.0000i
     0.0000 - 2.0000i  -2.0000 + 0.0000i
     0.0000 + 2.0000i  -2.0000 + 0.0000i
```

fxx = $6\,x$

fxy = $-6\,y - 12$

fyy = $-6\,x$

D = $-(6\,y + 12)^2 - 36\,x^2$

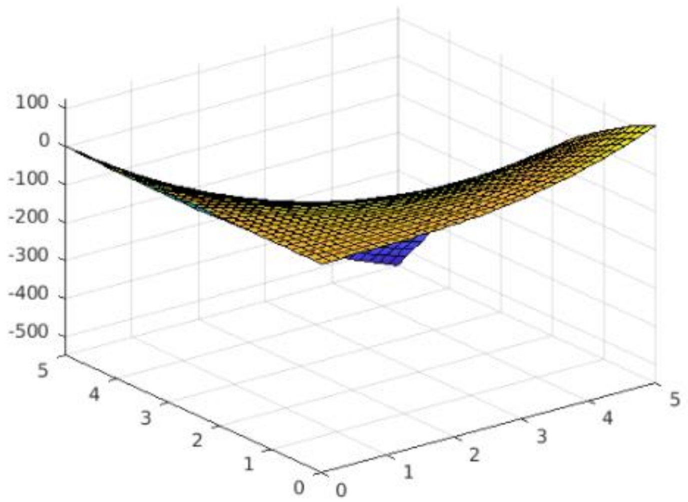
D1 = -144

A1 = 0

D2 = 0

A2 = 0

```
Neither Minima nor Maxima
no conclusion
```



TASK 2

2. Find the Minimum value of the function $f(x,y) = x^4 + 6y^3 - 36xy - 20y$

CODE:

```
syms x y
f=x.^4+6.*y.^3-36.*x.*y-20.*y
fx=diff(f,x)
fy=diff(f,y)
[a,b]=solve(fx,fy);
double([a,b])
fxx=diff(fx,x)
fxy=diff(fx,y)
fyy=diff(fy,y)
D=fxx*fyy-(fxy)^2
D1=subs(D,[x,y],[0.9603,-2.1358])
A1=subs(fxx,[x,y],[0.9603,-2.1358])
D2=subs(D,[x,y],[2.8661,2.6160])
A2=subs(fxx,[x,y],[2.8661,2.6160])
  if D1>0
     if A1>0
      disp('Attains Minima')
     else
      disp('Attains Maxima')
     end
   else
     if D1==0
      disp('No conclusion')
     else
      disp('Neither Minima nor Maxima')
     end
   end
   if D2>0
     if A2>0
      disp('Attains Minima')
     else
      disp('Attains Maxima')
     end
   else
     if D2==0
      disp('no conclusion')
     else
      disp('Neither Minima nor maxima')
     end
   end
   fsurf(f,[0,5,0,5])
```

OUTPUT:

f = $x^4 - 36\,x\,y + 6\,y^3 - 20\,y$

fx = $4\,x^3 - 36\,y$

fy = $18\,y^2 - 36\,x - 20$

```
ans = 6×2 complex

    -0.5554 + 0.0000i  -0.0190 + 0.0000i
    -2.1156 - 1.6369i   0.8374 - 1.9548i
    -2.1156 + 1.6369i   0.8374 + 1.9548i
     0.9603 + 2.6420i  -2.1358 - 1.2370i
     0.9603 - 2.6420i  -2.1358 + 1.2370i
     2.8661 + 0.0000i   2.6160 + 0.0000i
```

fxx = $12\,x^2$

fxy = -36

fyy = $36\,y$

D = $432\,x^2\,y - 1296$

D1 =

$$-\frac{67089379855797}{31250000000}$$

A1 =

$$\frac{276652827}{25000000}$$

D2 =

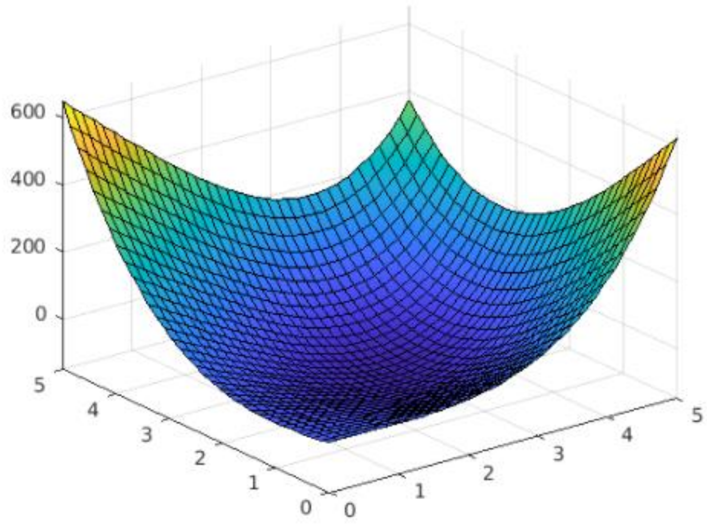
$$\frac{6240107839509}{781250000}$$

A2 =

$$\frac{2464358763}{25000000}$$

Neither Minima nor Maxima

Attains Minima



600
400
200
0
5
4
3
2
1
0
0
1
2
3
4
5

WORKSHEET-2

EVALUATING DOUBLE INTEGRAL WITH CONSTANT AND VARIABLE LIMITS

OBJECTIVES

Evaluating Iterated Integrals

Evaluating a multiple integral involves expressing it as an iterated integral, which can then be evaluated either symbolically or numerically. We begin by discussing the evaluation of iterated integrals.

Evaluating Double Integrals

To evaluate the double integral we have the syntax as
Integral1 = int(fun, variable (x), xmin, xmax)
Integral = int(Integral1, variable (y), ymin, ymax)

(Or)
Integral = int(int(fun, x, xmin, xmax),y, ymin, ymax)

PROCEDURE

7. Open MATLAB
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11. Compile and Run the program
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Double Integrals with constant limits

Example 1. Evaluate the double integral $\int_1^2\int_0^1 (x^2 + y^2)dxdy$

```
syms x y;
Integ1=int(x^2+y^2, x, 0,1);
Integ2 = int(Integ1, y,1,2);
Integral = Integ2
```

(or)

```
syms x y;
Integral=int(int(x^2+y^2,x,0,1),y,1,2)
```

Output: Integral = 8/3

Double Integrals with variable limits

Example 2. Evaluate the double integral $\int_0^1\int_0^x dxdy$

```
 syms x ;
Integ1=int(1,y,0,x);
Integ2 = int(Integ1,x,0,1);
Integral= Integ2
```

(or)

```
syms x ;
Integral=int(int(1,y,0,x),x,0,1)
```

Output: Integral = 1/2

Example 3. Evaluate $\int_0^\infty\int_0^y \frac{e^{-y}}{y} dydx$

```
syms x y  ;
Integ1=int((exp(-y))/y,x,0,y)
Integ2 = int(Integ1,y,0,inf)
Integral= Integ2
```

OUTPUT: Integral = 1

Example 4. Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2-y^2}\, dydx$

```
syms x y a ;
Integ1=int(sqrt(a^2-x^2-y^2),y,0,sqrt(a^2-x^2));
Integ2 = int(Integ1,x,0,a);
Integral= Integ2
```

OUTPUT = (pi*a^3)/6

TASK 1

Evaluate $\int_{2}^{3}\int_{1}^{2}\frac{dxdy}{xy}$

CODE:

```
syms x y;
integ1=int(1/(x*y),x,1,2)
integ2=int(integ1,y,2,3)
integral=integ2
```

OUTPUT:

```
integ1 =
```

$$\frac{\log(2)}{y}$$

```
integ2 =
```

$$\log(2)\,\log\left(\frac{3}{2}\right)$$

```
integral =
```

$$\log(2)\,\log\left(\frac{3}{2}\right)$$

TASK 2

Evaluate $\int_{0}^{3}\int_{0}^{2}e^{2x+2y}dydx$

CODE:

```
syms x y;
integ1=int(exp(2*x+2*y),x,0,2)
integ2=int(integ1,y,0,3)
integral=integ2
```

OUTPUT:

```
integ1 =
```

$$\frac{e^{2y}\,(e^4 - 1)}{2}$$

```
integ2 =
```

$$\frac{e^4\left(\frac{e^6}{2} - \frac{1}{2}\right)}{2} - \frac{e^6}{4} + \frac{1}{4}$$

```
integral =
```

$$\frac{e^4\left(\frac{e^6}{2} - \frac{1}{2}\right)}{2} - \frac{e^6}{4} + \frac{1}{4}$$

TASK 3

Prove that $\int_0^1 \int_0^{x^2} (x^2 + y^2)\,dy\,dx = \frac{26}{105}$

CODE:

```
syms x y;
integ1=int(x^2+y^2,y,0,x^2)
integ2=int(integ1,x,0,1)
integral=integ2
```

OUTPUT:

```
integ1 =
```

$$\frac{x^4\,(x^2 + 3)}{3}$$

```
integ2 =
```

$$\frac{26}{105}$$

```
integral =
```

$$\frac{26}{105}$$

Hence proved.

TASK 4

Evaluate $\int_{0}^{1}\int_{y}^{2-y} x^3 y\,dy\,dx$

CODE:

```
syms x y;
integ1=int(x^3*y,x,y,2-y)
integ2=int(integ1,y,0,1)
integral=integ2
```

OUTPUT:

```
integ1 =
```

$$\frac{y\ (y-2)^4}{4} - \frac{y^5}{4}$$

```
integ2 =
```

$$\frac{13}{30}$$

```
integral =
```

$$\frac{13}{30}$$

TASK 5

Evaluate $\int_{1}^{\log 1}\int_{0}^{\log y} e^{x+2y}\,dx\,dy$

CODE:

```
syms x y;
integ1=int(exp(x+2*y),x,0,log(y))
integ2=int(integ1,y,1,log(1))
integral=integ2
```

OUTPUT:

```
integ1 =
```

$$e^{2y}\,(y-1)$$

```
integ2 =
```

$$\frac{e^2}{4}-\frac{3}{4}$$

```
integral =
```

$$\frac{e^2}{4}-\frac{3}{4}$$

TASK 6

Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \frac{dydx}{\sqrt{a^2-x^2-y^2}}$

CODE:

```
syms x y a;
integ1=int(1/sqrt(a^2-x^2-y^2),y,0,sqrt(a^2-x^2))
integ2=int(integ1,x,0,a)
integral=integ2
```

OUTPUT:

```
integ1 =
```

$$\int_0^{\sqrt{a^2-x^2}} \frac{1}{\sqrt{a^2-x^2-y^2}}\,dy$$

```
integ2 =
```

$$\frac{\pi\, a}{2}$$

```
integral =
```

$$\frac{\pi\, a}{2}$$

WORKSHEET-3

AREA AS DOUBLE INTEGRAL

OBJECTIVE

To sketch the region and find the area using double integral.

Formula:

$$Area = \iint_R dx\,dy \quad \text{or} \quad Area = \iint_R dy\,dx$$

Example 1 Find the area between the curves $x^2 + y^2 = 1$ and $x + y = 1$.

PROGRAM

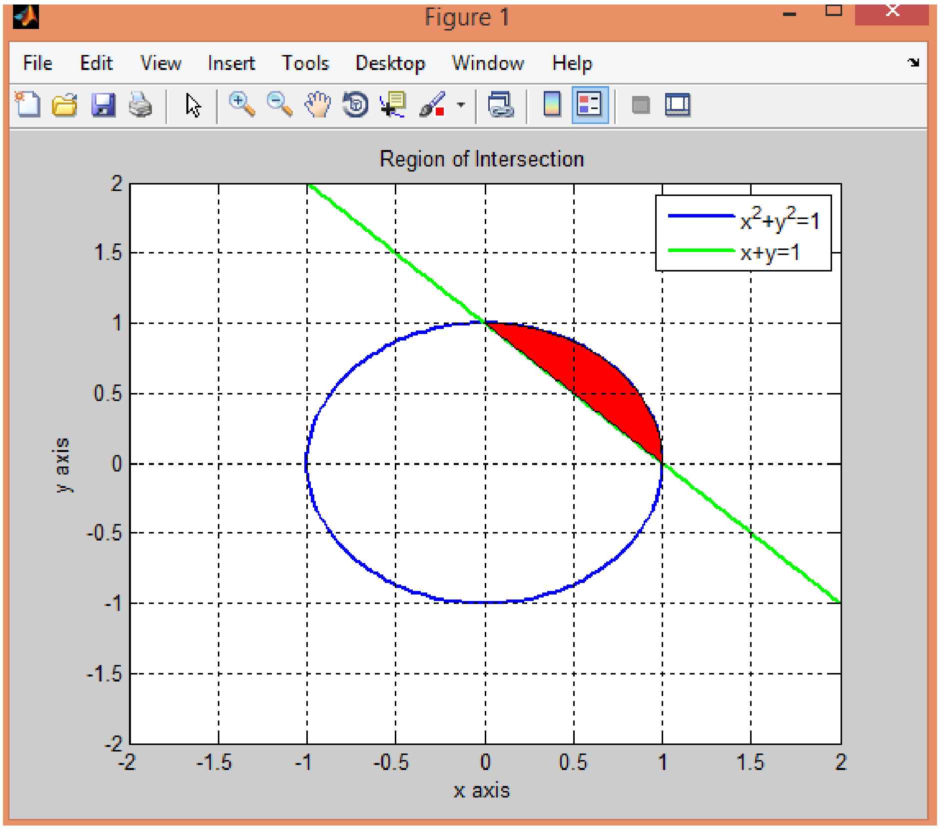
```
%Area using double integration
syms x y
h1=ezplot('x^2+y^2-1')
set(h1,'Color','b','Linewidth',2) % to give color to the curve h1
hold on
h2=ezplot('x+y-1')
set(h2,'Color','g','Linewidth',2) % to give color to the curve h2
axis([-2 2 -2 2])                 % defining the range for x,y axes
title('Region of Intersection')
legend('x^2+y^2=1','x+y=1')
xlabel('x axis')
ylabel ('y axis')
grid on
x0= solve(sqrt(1-x^2)-(1-x))    % to solve the values for x
x = linspace(x0(1),x0(2),100);
region = [x, fliplr(x)];        % common region between the curves
inBetween = [sqrt(1-x.^2), fliplr(1-x)];  % intersecting region
fill(region, inBetween, 'r');      % fill color for the intersection
hold off
syms x y
area=int(int(1,y,1-x,sqrt(1-x^2)),x,x0(1),x0(2))
```

OUTPUT

```
x0 =

 0
 1

area =
 pi/4 - 1/2
```



Example 2. Find the area bounded by the parabolas $y^2 = 4 - x \, and \, y^2 = x$ by double integration.

PROGRAM

```
syms x y
h1=ezplot('y^2-4+x')
set(h1,'Color','r','Linewidth',2)
hold on
h2=ezplot('y^2-x')
set(h2,'Color','b','Linewidth',2)
axis([-6 10 -6 10])           % defining the range for x,y axes
title('Region of Intersection')
legend('y^2=4-x', 'y^2=x')
grid on
hold off
y0= solve(4-y^2-y^2)           % to solve the values for y
syms x y
area=int(int(1,x,4-y^2,y^2),y,y0(1),y0(2))
double (area)
```

OUTPUT

y0 =

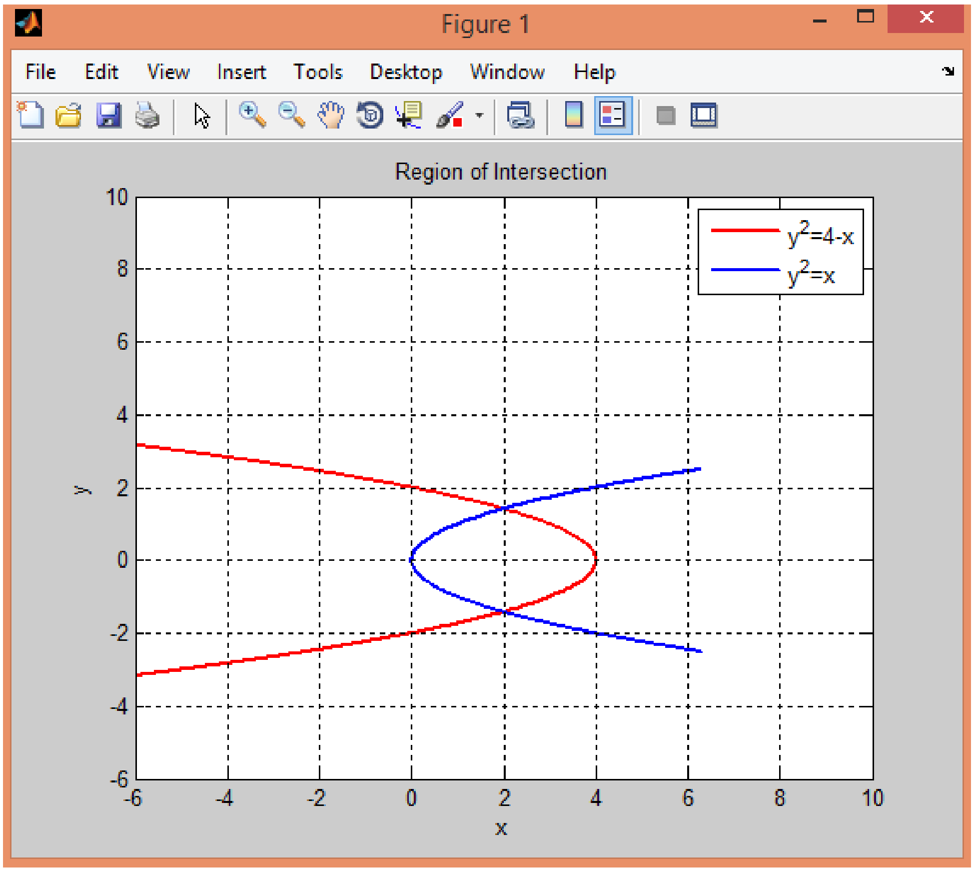
 2^(1/2)
-2^(1/2)

area =

(16*2^(1/2))/3

ans =

7.5425



Task 1

Find the area between the curves $y^2 = 4x$ and $x^2=4y$

CODE:

```
syms x y
h1=ezplot('y^2-4*x')
set(h1,'Color','b','Linewidth',2)
hold on
h2=ezplot('x^2-4*y')
set(h2,'Color','g','Linewidth',2)
axis([-2 6 -2 6])
title('Region of Intersection')
legend('y^2=4*x','x^2=4*y')
xlabel('x axis')
ylabel ('y axis')
grid on
x0= solve(sqrt(4*x)-x^2/4)
x = linspace(x0(1),x0(2),100);
region = [x, fliplr(x)];
inBetween = [sqrt(4*x), fliplr(x.^2/4)];
fill(region, inBetween, 'r');
hold off
syms x y;
area=int(int(1,y,x^2/4,sqrt(4*x)),x,x0(1),x0(2))
```

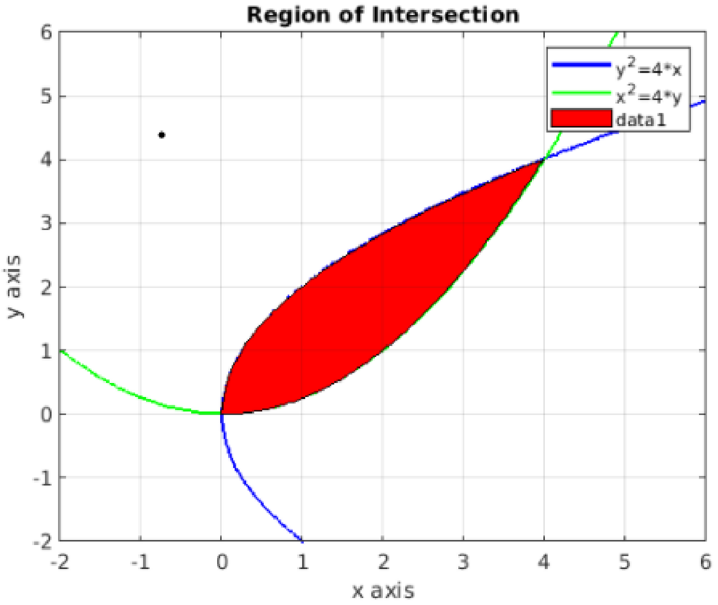
OUTPUT:

x0 =

$$\begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

area =

$$\frac{16}{3}$$



TASK 2

Find the area enclosed by the curves $y = x$ and $y^2 = x$

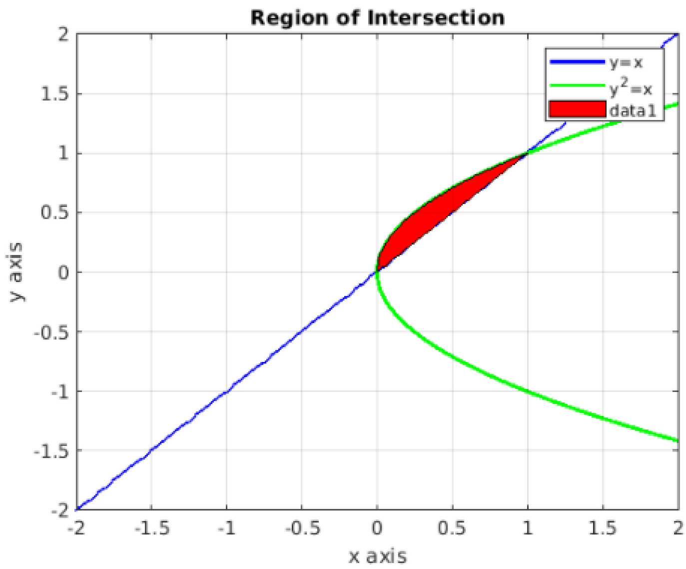
CODE:

```
syms x y
h1=ezplot('y-x')
set(h1,'Color','b','Linewidth',2)
hold on
h2=ezplot('y^2-x')
set(h2,'Color','g','Linewidth',2)
axis([-2 2 -2 2])
title('Region of Intersection')
legend('y=x','y^2=x')
xlabel('x axis')
ylabel ('y axis')
grid on
x0= solve(x-sqrt(x))
x = linspace(x0(1),x0(2),100);
region = [x, fliplr(x)];
inBetween = [x, fliplr(sqrt(x))];
fill(region, inBetween, 'r');
hold off
syms x y
area=int(int(1,y,sqrt(x),x),x,x0(1),x0(2))
```

OUTPUT:

```
x0 =
```

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$




```
area =
```

$$-\frac{1}{6}$$

TASK 3

Find area between the curves $y=x^2$ *and* $y+x-2=0$

CODE:

```
syms x y
h1=ezplot('y-x^2');
set(h1,'Color','b','Linewidth',2)
hold on
h2=ezplot('y+x-2');
set(h2,'Color','g','Linewidth',2)
axis([-4 8 -4 8])
title('Region of Intersection')
legend('y=x^2','y+x-2')
xlabel('x axis')
ylabel ('y axis')
grid on
x0= solve(x^2-(2-x))
x = linspace(x0(1),x0(2),100);
region = [x, fliplr(x)];
inBetween = [x.^2, fliplr(2-x)];
fill(region, inBetween, 'r');
hold off
syms x y
area=int(int(1,y,2-x,x^2),x,x0(1),x0(2))
```

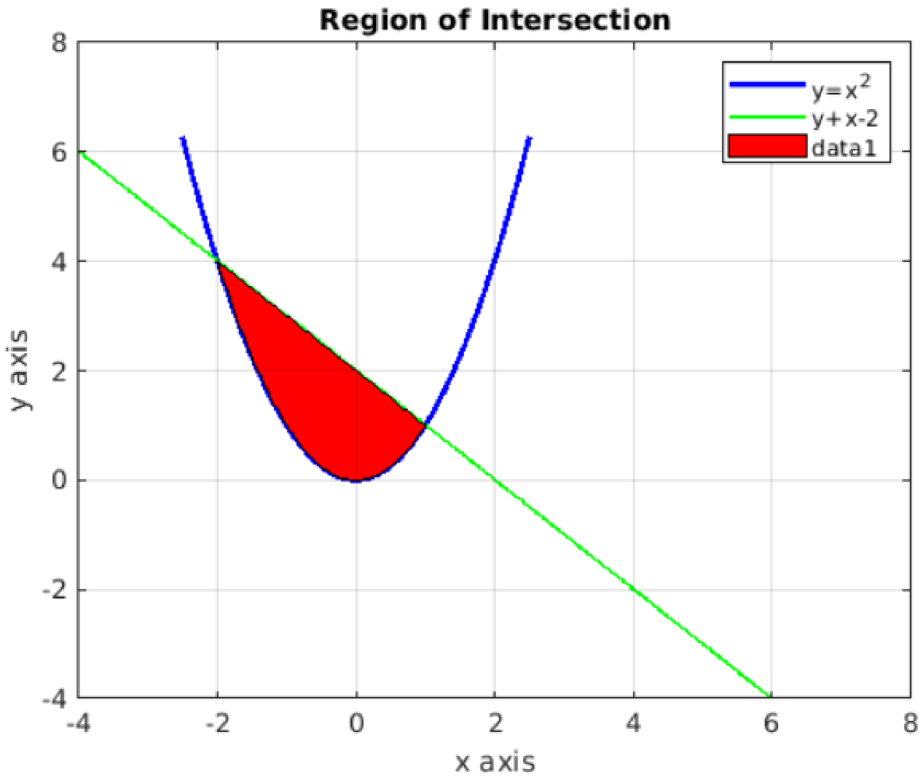
OUTPUT:

x0 =

$$\begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

area =

$$-\frac{9}{2}$$


Region of Intersection
y=x2
y+x-2
data1
y axis
x axis
-4
-2
0
2
4
6
8

TASK 4

Find the area bounded by the parabolas $y^2 = 4 - x \, and \, y^2 = 4 - 4x$ by double integration.

CODE:

```
syms x y
h1=ezplot('y^2-4+x');
set(h1,'Color','r','Linewidth',2)
hold on
h2=ezplot('(y.^2)-4+4*x');
set(h2,'Color','b','Linewidth',2)
axis([-6 10 -6 10])          % defining the range for x,y axes
title('Region of Intersection')
legend('y^2-4+x', 'y^2-4+4x')
grid on
hold off
y0= solve(3*y^2-12)             % to solve the values for y
syms x y
area=int(int(1,x,(-y^2+4)/4,-y^2+4),y,y0(1),y0(2))
double (area)
```

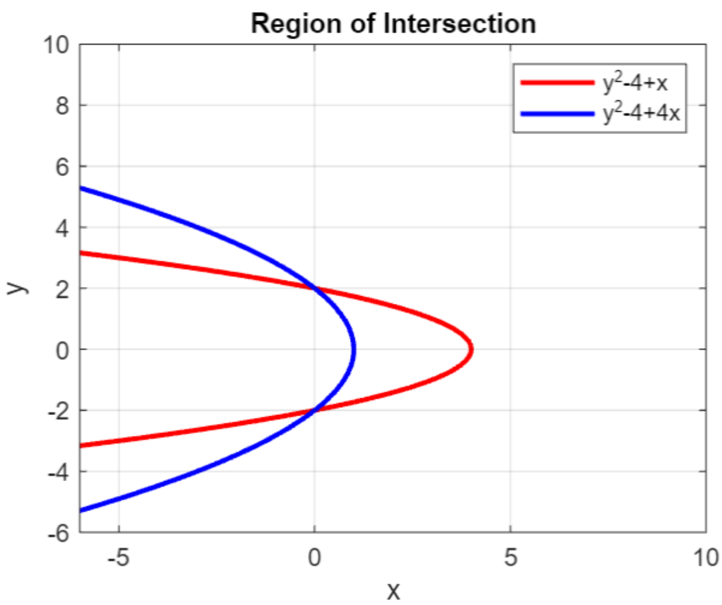
OUTPUT:

y0 =

$$\begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

area = 8

ans = 8



WORKSHEET-4

TRIPLE INTEGRAL WITH CONSTANT AND VARIABLE LIMITS

OBJECTIVES

To Evaluate the Triple integral

Evaluating Triple Integrals

Syntax for evaluating triple integral is
Integral1 = int(fun, variable (x), xmin, xmax)
Integral2 = int(Integral1, variable (y), ymin, ymax)
Integral = int(Integral2, variable (z), zmin, zmax)

(Or)
Integral = int(int(int(fun, x, xmin, xmax),y, ymin, ymax), z, zmin, zmax)

TRIPLE INTEGRAL WITH CONSTANT LIMITS

Example 1. Evaluate the Triple integral $\int_0^1\int_0^1\int_0^{\pi}(x+y)dxdydz$

PROGRAM

```
syms x y z;

Q=int(int(int(x+y,x,0,pi),y,0,1),z,0,1)
```

(or)

```
syms x y z;

Int1= int(x+y,x,0,pi);

Int2 = int(Int1,y,0,1);

Q=int(Int2,z,0,1)
```

OUTPUT : Q = (pi*(pi+1))/2

TRIPLE INTEGRAL WITH VARIABLE LIMITS

Example 2. To Evaluate the Triple integral $\int_{1}^{3}\int_{\frac{1}{x}}^{1}\int_{0}^{\sqrt{xy}} xyz\,dz\,dy\,dx$

PROGRAM

```
syms x y z;
Int1= int(x*y*z,z,0,sqrt(x*y));
Int2 = int(Int1,y,1/x,1);
Q=int(Int2,x,1,3)
```

OUTPUT Q = 13/9-1/6log3

Example 3. To Evaluate the Triple integral $\int_{0}^{c}\int_{0}^{b}\int_{0}^{a} e^{x+y+z}\,dz\,dy\,dx$

PROGRAM

```
syms x y z a b c;
Int1= int(exp(x+y+z),z,0,a);
Int2 = int(Int1,y,0,b);
Q=int(Int2,x,0,c)
```

OUTPUT: Q =

(exp(a) - 1)*(exp(b) - 1)*(exp(c) - 1)

TASK 1

Evaluate the triple integral $\int_0^3\int_0^2\int_0^1 (2x+y+z)dzdydx$

CODE:

```
syms x y z;
integ1=int(2*x+y+z,z,0,1)
integ2=int(integ1,y,0,2)
Q=int(integ2,x,0,3)
```

OUTPUT:

integ1 =

$$2\,x+y+\frac{1}{2}$$

integ2 = $4\,x+3$

Q = 27

TASK 2

Evaluate the triple integral $\int_0^3\int_0^{3-x}\int_0^{3-x-y} xyzdzdydx$

CODE:

```
syms x y z;
integ1=int(x*y*z,z,0,3-x-y)
integ2=int(integ1,y,0,3-x)
Q=int(integ2,x,0,3)
```

OUTPUT:

```
integ1 =
```

$$\frac{x\,y\,(x+y-3)^2}{2}$$

```
integ2 =
```

$$\frac{x\,(x-3)^4}{24}$$

```
Q =
```

$$\frac{81}{80}$$

TASK 3

Evaluate the triple integral $\int_0^{\log 2}\int_0^{x}\int_0^{x+\log y} e^{x+y+z}\,dz\,dy\,dx$

CODE:

```
syms x y z;
integ1=int(exp(x+y+z),z,0,x+log(y))
integ2=int(integ1,y,0,x)
Q=int(integ2,x,0,log(2))
```

OUTPUT:

integ1 = $e^{x}\,e^{y}\,(y\,e^{x}-1)$

integ2 = $e^{x}-e^{3x}+x\,e^{3x}$

Q =

$$e^{6243314768165359/9007199254740992}-\frac{17298852714467891\,e^{187299}}{8106479329}$$

TASK 4

Prove that the evaluation of the triple integral $\int_0^c \int_0^b \int_0^a (x^2 + y^2 + z^2) dxdydz = \dfrac{abc(a^2 + b^2 + c^2)}{3}$

CODE:

```
syms x y z a b c;
integ1=int(x^2+y^2+z^2,x,0,a)
integ2=int(integ1,y,0,b)
Q=int(integ2,z,0,c)
```

OUTPUT:

```
integ1 =
```

$$\frac{a^3}{3} + (y^2 + z^2)\, a$$

```
integ2 =
```

$$\frac{a\, b\, (a^2 + b^2 + 3\, z^2)}{3}$$

```
Q =
```

$$\frac{a\, b\, c\, (a^2 + b^2 + c^2)}{3}$$

Hence proved

TASK 5

Prove that the evaluation of the triple integral $\int_0^1 \int_0^{1-x} \int_0^{(x+y)^2} x dzdydx = \dfrac{1}{10}$

CODE:

```
syms x y z a b c;
integ1=int(x,z,0,(x+y)^2)
integ2=int(integ1,y,0,1-x)
Q=int(integ2,x,0,1)
```

OUTPUT:

```
integ1 =
```
$x\ (x+y)^2$

```
integ2 =
```
$$-\frac{x\ (x^3-1)}{3}$$

```
Q =
```
$$\frac{1}{10}$$

WORKSHEET - 5

EVALUATING VOLUME AS A TRIPLE INTEGRAL

OBJECTIVES

To find the volume using Triple Integration.

Formula:

$$\text{Volume} = \iiint_V dxdydz \quad \text{or} \quad \iiint_V dzdydx \quad \text{or} \quad \iiint_V dydxdz$$

Example1. Find the volume of sphere defined over $x^2 + y^2 + z^2 = 1$

PROGRAM

```
syms x y z;
Int1= int(1, z,- sqrt(1-x^2-y^2),sqrt(1-x^2-y^2));
Int2 = int(Int1,y,-sqrt(1-x^2),sqrt(1-x^2));
Volume=int(Int2,x,-1,1)
```

OUTPUT: Volume = (4*pi)/3

Example 2. Evaluate $\int\int\int dxdydz$ inside the tetrahedron bounded by the planes x = 0; y = 0; z = 0 and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

PROGRAM

```
syms x y z a b c;
Int1= int((1),z,0,c*(1-x/a-y/b));
Int2 = int(Int1,y, 0, b*(1-x/a));
Volume=int(Int2,x,0,a)
```

OUTPUT

Volume =

(a*b*c)/6

Example 3. Find the volume of the ellipsoid using triple integration $36x^2 + 16y^2 + 9z^2 = 144$

Converting the equation in Standard form of ellipsoid, we get,

$$\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} = 1$$

PROGRAM:

```
syms x y z ;
Integral1= int(1,z,-4*(sqrt(1-x^2/2^2-y^2/3^2)), 4*(sqrt(1-x^2/2^2-
y^2/3^2)))
Integral2 = int(Integral1,y,-3*(sqrt(1-x^2/2^2)), 3*(sqrt(1-
x^2/2^2)))
Volume= int(Integral2,x,-2,2)
```

OUTPUT

Volume =

32*pi

TASK 1

Find the volume of sphere for $x^2 + y^2 + z^2 = a^2$

CODE:

```
syms x y z a;
int1=int(1,x,-sqrt(a^2-y^2-z^2),sqrt(a^2-y^2-z^2));
int2=int(int1,y,-sqrt(a^2-z^2),sqrt(a^2-z^2));
volume=int(int2,z,-a,a)
```

OUTPUT:

volume =

$$\frac{4\,\pi\,a^3}{3}$$

TASK 2

Find the volume of that portion of the ellipsoid $\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{16}=1$ which lies in the first octant using triple integration.

CODE:

```
syms x y z ;
int1=int(1,x,-sqrt(4*(1-z^2/16-y^2/9)),sqrt(4*(1-z^2/16-y^2/9)));
int2=int(int1,y,-sqrt(9*(1-z^2/16)),sqrt(9*(1-z^2/16)));
volume=int(int2,z,-4,4)
```

OUTPUT:

volume = 32π

TASK 3

Evaluate $\int\int\int dxdydz$ inside the tetrahedron bounded by the planes x = 0; y = 0; z = 0 and

$$2x+3y+4z=12$$

CODE:

```
syms x y z;
int1=int(1,x,0,1/2*(12-3*y-4*z));
int2=int(int1,y,0,1/3*(12-4*z));
volume=int(int2,z,0,3)
```

OUTPUT:

volume = 12

WORKSHEET - 6

EVALUATING GRADIENT, DIVERGENCE AND CURL

OBJECTIVE

To evaluate

1. Gradient of a scalar function φ i.e. $\nabla\varphi$
2. Divergence of a vector function $\vec{F}$ i.e. $\nabla \cdot \vec{F}$.
3. Curl of a vector function $\vec{F}$ i.e. $\nabla \times \vec{F}$.

Example 1: Find gradient of the function $x^3 + y^3 + z^3$ at the point (1,1,1).

```
syms x y z
 f = x^3+y^3+z^3
 a = gradient(f, [x,y,z])
 G = subs(a,[x,y,z],[1,1,1])
```

OUTPUT f =

x^3 + y^3 + z^3

a =

3*x^2
3*y^2
3*z^2

G =

3
3
3

Example 2: Find divergence of $x\,\vec{i} + 2y^2\vec{j} + 3z^3\vec{k}$.

To compute the divergence of the vector, consider V(x, y, z) = [x, $2y^2$, $3z^3$] with respect to vector X = (x, y, z) in Cartesian coordinates.

PROGRAM

```
syms x y z
V=[x, 2*y^2, 3*z^3]
divergence(V, [x, y, z])
```

OUTPUT

V =

[x, 2*y^2, 3*z^3]

ans =

9*z^2 + 4*y + 1

Example 3: Find curl of the vector function $\boldsymbol{x^2z\,\vec{\imath} + 3xy\vec{\jmath} + 2xyz\vec{k}}$ **at the point (2,3,4)**

PROGRAM

```
syms x y z
 f = [x^2*z  3*y*x  2*z*y*x]
 G=curl(f, [x,y,z])
 subs(G,[x,y,z], [2,3,4])
```

OUTPUT:

```
f =

[x^2*z,3*y*x,2*z*y*x]

G=

 2*x*z
X^2 - 2*y*z
 3*y
  Ans=
      16
     -20
       9
```

(**Note:** $\vec{F}$ is a conservative field iff $\vec{F}$ is irrotational.)

Relation of Electric Field to Charge Density

The divergence of the electric field at a point in space is equal to the charge density divided by the permittivity of space.

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

E = electric field
ρ = charge density
ε_0 = permittivity

In a charge-free region of space where ρ = 0, we can say

$$\nabla \cdot E = 0$$

Example 4: Find the electric charge density (ρ) for the electric field $\boldsymbol{E = x^2\vec{\imath} + y^2\vec{\jmath}}$

Note: $\rho = \varepsilon_0\ (\nabla . E)$

PROGRAM

```
syms x y ep0 ;
E = [x^2 y^2];
rho = ep0* divergence(E,[x y])
```

OUTPUT

```
rho =
ep0*(2*x + 2*y)
```

Plotting Gradient of a function using quiver

A **quiver plot or velocity plot** displays velocity vectors as arrows with components (*u*,*v*) at the points (*x*,*y*)

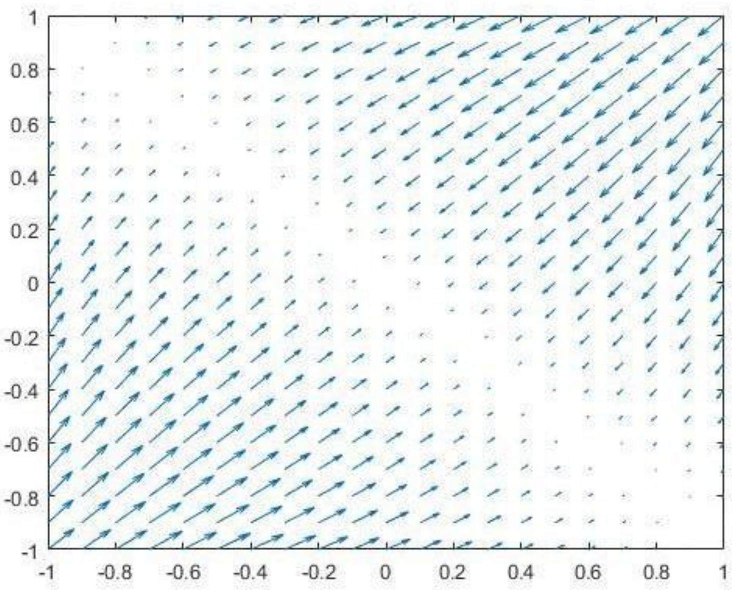
Example 5: Plot a gradient of function $-(\sin(x) + \sin(y))^2$

PROGRAM

```
syms x y
f = -(sin(x) + sin(y))^2;
g = gradient(f, [x, y])
[X, Y] = meshgrid(-1:.1:1,-1:.1:1);
G1 = subs(g(1), [x y], {X,Y});
G2 = subs(g(2), [x y], {X,Y});
quiver(X, Y, G1, G2)
```

OUTPUT

```
g =
-2*cos(x)*(sin(x) + sin(y))
-2*cos(y)*(sin(x) + sin(y))
```



TASK 1

Find the divergence of gradient of a scalar function $x^2 + y^2 + z^2$.
(Note : The result is the Laplacian [i.e (div(grad φ) = $\nabla^2\varphi$] of the scalar function φ.)

CODE:

```
syms x y z;
f=x^2+y^2+z^2;
a=gradient(f,[x,y,z])
result=divergence(a,[x,y,z])
```

OUTPUT:

a =

$$\begin{pmatrix} 2\,x \\ 2\,y \\ 2\,z \end{pmatrix}$$

result = 6

TASK 2

Find divergence of $3x^2\,\vec{\imath} + yz\vec{\jmath} + xz^2\vec{k}$ at the point $(-1,-2,6)$

CODE:

```
syms x y z;
f=[3*x^2 y*z x*z^2]
a=divergence(f,[x,y,z])
result=subs(a,[x,y,z],[-1,-2,6])
```

OUTPUT:

f = $\left(3\,x^2 \quad y\,z \quad x\,z^2\right)$

a = $6\,x + z + 2\,x\,z$

result = -12

TASK 3

Compute the curl of the vector field $x^3y^2z\,\vec{\imath} + y^3z^2x\,\vec{\jmath} + z^3x^2y\,\vec{k}$ at the point $(3,1,-4)$

CODE:

```
syms x y z;
f=[x^3*y^2*z y^3*z^2*x z^3*x^2*y]
a=curl(f,[x,y,z])
result=subs(a,[x,y,z],[3,1,-4])
```

OUTPUT:

f =

$\left(x^3\,y^2\,z \quad x\,y^3\,z^2 \quad x^2\,y\,z^3\right)$

a =

$$\begin{pmatrix} x^2\,z^3 - 2\,x\,y^3\,z \\ x^3\,y^2 - 2\,x\,y\,z^3 \\ y^3\,z^2 - 2\,x^3\,y\,z \end{pmatrix}$$

result =

$$\begin{pmatrix} -552 \\ 411 \\ 232 \end{pmatrix}$$

TASK 4

Show that $curl\,\vec{r} = 0$ and $div\,\vec{r} = 3$.

CODE:

```
syms x y z
f=[x, y, z]
a=curl(f, [x, y, z])
b=divergence(f,[x, y, z])
```

OUTPUT:

f = $\begin{pmatrix} x & y & z \end{pmatrix}$

a =

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

b = 3

TASK 5

Show that the curl of gradient of any scalar function is zero.

Let's consider the scalar function as x^2+y^2+z^2

CODE:

```
syms x y z;
f=x^2+y^2+z^2
a=gradient(f,[x,y,z])
result=curl(a,[x,y,z])
```

OUTPUT:

f = $x^2 + y^2 + z^2$

a =

$$\begin{pmatrix} 2\,x \\ 2\,y \\ 2\,z \end{pmatrix}$$

result =

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

TASK 6

Show that the divergence of the curl of any vector field is 0.

Let us consider the vector function to be x^2i+y^2j+z^2k

CODE:

```
syms x y z;
f=[x^2 y^2 z^2]
a=curl(f,[x,y,z])
result=divergence(a,[x,y,z])
```

OUTPUT:

f = $\begin{pmatrix} x^2 & y^2 & z^2 \end{pmatrix}$

a =

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

result = 0

TASK 7

For the vector $\vec{F} = x^2\vec{\imath} + y^2\vec{\jmath} + z^2\vec{k}$, check whether $\vec{F}$ is solenoidal or irrotational.

CODE:

```
syms x y z;
f=[x^2 y^2 z^2]
a=divergence(f,[x,y,z])
b=curl(f,[x,y,z])
```

OUTPUT:

f = $\begin{pmatrix} x^2 & y^2 & z^2 \end{pmatrix}$

a = $2\,x + 2\,y + 2\,z$

b =

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\vec{F}$ is irrotational.

TASK 8

Plot a gradient of the function $z = xe^{-x^2-y^2}$

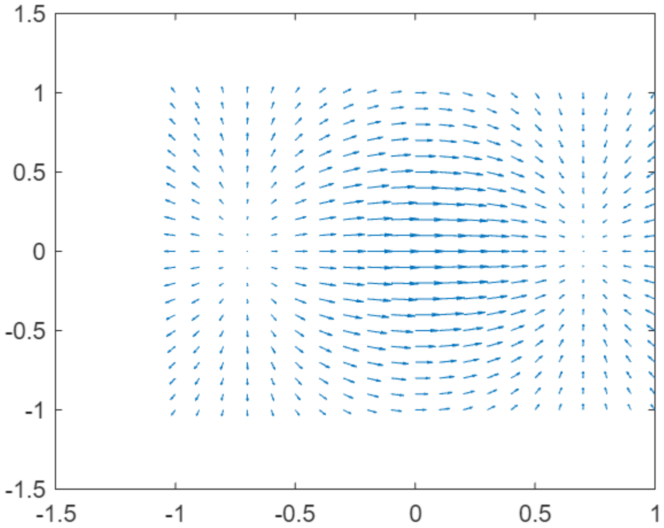
CODE:

```
syms x y
f = x*exp(-x^2-y^2);
g = gradient(f, [x, y])
[X, Y] = meshgrid(-1:.1:1,-1:.1:1);
G1 = subs(g(1), [x y], {X,Y});
G2 = subs(g(2), [x y], {X,Y});
quiver(X, Y, G1, G2)
```

OUTPUT:

g =

$$\begin{pmatrix} e^{-x^2-y^2} - 2\,x^2\,e^{-x^2-y^2} \\ -2\,x\,y\,e^{-x^2-y^2} \end{pmatrix}$$



WORKSHEET-7

Line Integral and Work done

OBJECTIVES

To evaluate

1. Line integral
2. Work done by the Force using Line Integral

Line Integral

For a vector field $\vec{F}$, the line integral along a curve C, in the direction of $\vec{r}$, is defined as

$$\int_C \vec{F}.d\vec{r} = \int_{t_1}^{t_2} \overrightarrow{F(r(t))}.\ \overrightarrow{r'(t)}\ dt$$

Work Done

The work done by a force field on the particle along a path C, is defined as

$$\int_C \vec{F}.d\vec{r} = \int_{t_1}^{t_2} \overrightarrow{F(r(t))}.\ \overrightarrow{r'(t)}\ dt$$

EXAMPLE 1: Evaluate the line integral over $\vec{F} = x^2y\,\vec{\imath} + (x-2y)\vec{\jmath}$ under the parabolic curve $y = x^2$ from (0,0) to (1,1) where $x = t$ and $y = t^2$

PROGRAM

```
syms x y z t
F=[x^2*y  x-2*y  0]
r=[t t^2 0]
FF=subs(F,[x, y z],r)
g= sum(FF.*diff(r,t))
In=int(g,t,0,1)
```

OUTPUT :

```
F =
[ x^2*y, x - 2*y, 0]
r =
[ t, t^2, 0]
FF =
[ t^4, - 2*t^2 + t, 0]
g =
t^4 + 2*t*(- 2*t^2 + t)
In =
-2/15
```

EXAMPLE 2: Find the work done by the force $\vec{F} = x^2y\,\vec{\imath} + z\vec{\jmath} + y\vec{k}$ in moving a particle along $\vec{r} = 3\,cost\,\vec{\imath} + 2\,sint\,\vec{\jmath} + sin2t\,\vec{k}$ for $0 \le t \le 2\pi$

PROGRAM

```
syms x y z t
F=[x^2*y,z,y]
r=[3*cos(t),2*sin(t),sin(2*t)]
FF=subs(F,{x, y z},r)
g= sum(FF.*diff(r,t))
work done = int(g,t,0,2*pi)
```

OUTPUT

work done =

```
-(27*pi )/2
```

EXAMPLE 3: Find the work done in moving a particle once around a circle C in the $xy -$ plane, if the circle has centre at the origin and radius 3 and if the force is given by $\vec{F} = (2x - y + z)\vec{\imath} + (x + y - z\text{^}2)\,\vec{\jmath} + (3x - 2y + 4z)\vec{k}$.

(**Note:** In xy – plane, $\vec{F} = (2x - y)\vec{\imath} + (x + y)\,\vec{\jmath} + (3x - 2y)\vec{k}$ and $\overrightarrow{dr} = dx\vec{\imath} + dy\,\vec{\jmath}$. The circle C is $x = 3cos\theta,\ y = 3sin\theta$ where θ varies from $0\ to\ 2\pi$.)

PROGRAM

```
syms x y z t
F=[2*x-y,x+y,3*x-2*y];
r=[3*cos(t),3*sin(t),0];
FF=subs(F,{x, y z},r);
g= sum(FF.*diff(r,t));
work done = int(g,t,0,2*pi)
```

OUTPUT

work done =

```
18*pi
```

EXAMPLE 4: Evaluate the line integral over $\int_C (xdx + xydy + xyzdz)$ where C is the arc of the curve $x = t;\ y = t^2;\ z = t^3,\ 0 \le t \le 1$

(**Note:** Here $\vec{F}.d\vec{r} = xdx + xydy + xyzdz = (x\vec{\imath} + xy\vec{\jmath} + xyz\vec{k}).(dx\vec{\imath} + dy\vec{\jmath} + dz\vec{k})$

So, $\vec{F} = x\vec{\imath} + xy\vec{\jmath} + xyz\vec{k}$ and $d\vec{r} = dx\vec{\imath} + dy\vec{\jmath} + dz\vec{k}$)

PROGRAM

```
syms x y z t
F=[x, x*y, x*y*z]
r=[t,t^2, t^3]
FF=subs(F,{x, y z},r)
g= sum(FF.*diff(r,t))
In=int(g,t,0,1)
```

OUTPUT

```
  F =
[x, x*y, x*y*z]
r =
[ t, t^2, t^3]
FF =
[t, t^3, t^6]
g =
3*t^8 + 2*t^4 + t
In =
37/30
```

TASK 1

Evaluate the line integral over $\vec{F} = x^2 y\,\vec{\imath} + (x - 2y)\vec{\jmath}$ over the straight line $y = x = t$ from $(0,0)$ to $(1,1)$.

CODE:

```
syms x y z t
f=[x^2*y  x-2*y 0]
r=[t t 0]
f1=subs(f,[x, y z],r);
g= sum(f1.*diff(r,t));
res=int(g,t,0,1)
```

OUTPUT:

f = $\left(x^2\,y \quad x - 2\,y \quad 0\right)$

r = $\left(t \quad t \quad 0\right)$

res =

$-\frac{1}{4}$

TASK 2

Evaluate the line integral over $\int_C (xdx + ydy + zdz)$ where C is the arc of the curve $x = t;\ y = t^2;\ z = t^2,\ 0 \le t \le 2.$

CODE:

```
syms x y z t
f=[x y z]
r=[t t^2 t^2]
ff=subs(f,[x, y z],r);
g= sum(ff.*diff(r,t));
res=int(g,t,0,2)
```

OUTPUT:

f = $(x \quad y \quad z)$

r = $(t \quad t^2 \quad t^2)$

res = 18

TASK 3

Find the total work done in moving a particle in a force field given by

$\vec{F} = 3xy\,\vec{\imath} - 5z\vec{\jmath} + 10x\vec{k}$ along $x = t^2 + 1$, $y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$

CODE:

```
syms x y z t
f=[3*x*y -5*z 10*x]
r=[t^2+1 2*t^2 t^3]
ff=subs(f,[x, y z],r);
g= sum(ff.*diff(r,t));
res=int(g,t,1,2)
```

OUTPUT:

f = $(3\,x\,y \quad -5\,z \quad 10\,x)$

r = $(t^2+1 \quad 2\,t^2 \quad t^3)$

res = 303

TASK 4

Evaluate the line integral over $\int_C (x^2y\,dx + (x-z)dy + xyz\,dz)$ where C is the arc of the parabola $y = x^2$ in the plane $z = 2$; from $(0,0,2)$ to $(1,1,2)$.

(**Note:** Here, $z = 2$ and $y = x^2$. So, $x = x,\ y = x^2, z = 2$ i.e. $\vec{r} = x\,\vec{\imath} + x^2\,\vec{\jmath} + 2\vec{k}$ and $\vec{F} = x^2y\,\vec{\imath} - (x-2)\vec{\jmath} + 2xy\vec{k}$ where x varies from 0 to 1.)

CODE:

```
syms x y z t
f=[x^2*y -(x-2) 2*x*y]
r=[x x^2 2]
ff=subs(f,[x, y z],r);
g= sum(ff.*diff(r,x));
res=int(g,x,0,1)
```

OUTPUT:

f = $\left(x^2\,y \quad 2-x \quad 2\,x\,y\right)$

r = $\left(x \quad x^2 \quad 2\right)$

res =

$\frac{23}{15}$

WORKSHEET-8

GREEN'S THEOREM IN THE PLANE

OBJECTIVE

To evaluate the integral using Green's Theorem in the plane.

Green's Theorem: $\int_C u\,dx + v\,dy = \iint_R \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dx\,dy = \iint_R (v_x - u_y)\,dx\,dy$

Area of the region using Green's Theorem: A = $\frac{1}{2}\int_C (x\,dy - y\,dx)$

EXAMPLE 1: Using Green's Theorem evaluate $\int_c (3x-8y^2)dx + (4y-6xy)dy$ where C is the boundary of the region given by $x = y = 0;\ x + y = 1$.

```
syms x y;
u=3*x-8*y^2;
v=4*y-6*x*y;
vx=diff(v,x);
uy=diff(u,y);
f=vx-uy;
```

```
Integral=int(int(f,x,0,1-y),y,0,1)
```

OUTPUT:

```
Integral =

 5/3
```

VERIFICATION OF GREEN'S THEOREM

EXAMPLE 2: Verify Green's theorem along xy plane for $\int_c (xy+y^2)dx + x^2 dy$ where C is a closed curve of the region between $y = x$ and $y = x^2$

PROGRAM

```
syms x y z
F=[x*y+y^2,x^2,0];
%Along curve 1: y=x^2
r1=[x,x^2,0];
F1=subs(F,[x, y, z],r1);
g1= sum(F1.*diff(r1,x));
INT1 = int(g1,x,0,1)
%Along curve 2 : y=x
r2=[x,x,0];
F2=subs(F,[x, y z],r2);
g2= sum(F2.*diff(r2,x));
INT2 = int(g2,x,1,0)
% LHS
INTLHS=INT1+INT2
%To evaluate by using double integral
u=x*y+y^2;
v=x^2;
uy=diff(u,y);
vx=diff(v,x);
F= vx - uy;
INTRHS=int(int(F,x,y,sqrt(y)),y,0,1)
disp('Hence Greens theorem is verified.')
```

OUTPUT

```
INT1 =
19/20

INT2 =
 -1
```

```
INTLHS =
-1/20

INTRHS =
-1/20

Hence Greens theorem is verified.
```

EXAMPLE 3: Verify green's theorem along xy plane for $\int_c (3x-8y^2)dx+(4y-6xy)dy$ where c is a boundary of region given by $y = x = 0$ and $x + y = 1$.

PROGRAM:

```
syms x y z
F=[3*x-8*y^2,4*y-6*x*y,0];
%Along curve 1: y=0
r1=[x,0,0];
F1=subs(F,[x, y, z],r1);
g1= sum(F1.*diff(r1,x));
INT1 = int(g1,x,0,1)
%Along curve 2: x+y=1
r2=[x,1-x,0];
F2=subs(F,[x, y, z],r2);
g2= sum(F2.*diff(r2,x));
INT2 = int(g2,x,1,0)
%Along curve 3:  x=0
r3=[0,y,0];
F3=subs(F,[x, y, z],r3);
g3= sum(F3.*diff(r3,y));
INT3 = int(g3,y,1,0)
% LHS
INTLHS=INT1+INT2+INT3
%To evaluate by using double integral
u=3*x-8*y^2;
v=4*y-6*x*y;
uy=diff(u,y);
vx=diff(v,x);
F= vx - uy;
INTRHS=int(int(F,x,0,1-y),y,0,1)
disp('Hence Greens theorem is verified')
```

OUTPUT:

```
INT1 =
 3/2

 INT2 =
 13/6

 INT3 =
```

```
 -2

INTLHS =
 5/3

INTRHS =
 5/3

Hence Greens theorem is verified.
```

EXAMPLE 4: Find the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ using Green's theorem.

PROGRAM:

```
syms x y t a b;
x=a*cos(t);
y=b*sin(t);
dx=diff(x,t);
dy=diff(y,t);
F=x*dy-y*dx;
AREA=1/2*(int(F,t,0,2*pi))
```

Output:

```
AREA =
pi*a*b
```

TASK 1

Using Green's Theorem evaluate $\int_c (x^2-y^2)dx+2xydy$ where C is the closed curve of the region given by $x = y \; and \; y = x^2$.

CODE:

```
syms x y;
u=x^2-y^2;
v=2*x*y;
vx=diff(v,x);
uy=diff(u,y);
f=vx-uy;
res=int(int(f,x,y,sqrt(y)),y,0,1)
```

OUTPUT:

```
res =
```

$$\frac{4}{15}$$

TASK 2

Verify Green's theorem in the plane for $\int_C (2xy - x^2)dx + (x + y^2)dy$ where C is the boundary of the region defined by $x = y^2 \, and \; y = x^2$

CODE:

```
syms x y z
f=[2*x*y,x+y^2,0];
r1=[x,x^2,0];
f1=subs(f,[x, y, z],r1);
g1= sum(f1.*diff(r1,x));
INT1 = int(g1,x,0,1)


r2=[x,sqrt(x),0];
f2=subs(f,[x, y, z],r2);
g2= sum(f2.*diff(r2,x));
INT2 = int(g2,x,1,0)
INTLHS=INT1+INT2


u=2*x*y;
v=x+y^2;
uy=diff(u,y);
vx=diff(v,x);
f= vx - uy;
INTRHS=int(int(f,x,y^2,sqrt(y)),y,0,1)
disp('Hence Greens theorem is verified')
```

OUTPUT:

```
INT1 =
```

$$\frac{3}{2}$$

```
INT2 =
```

$$-\frac{22}{15}$$

```
INTLHS =
```

$$\frac{1}{30}$$

```
INTRHS =
```

$$\frac{1}{30}$$

```
Hence Greens theorem is verified
```

TASK 3

Using Green's theorem evaluate for $\int_C (x^2+xy)dx+(x^2+y^2)dy$ where C is the square formed by the lines $x=\pm 1$ and $y=\pm 1$

CODE:

```
syms x y;
u=x^2+x*y;
v=x^2+y^2;
vx=diff(v,x);
uy=diff(u,y);
f=vx-uy;
res=int(int(f,x,-1,1),y,-1,1)
```

OUTPUT:

res = 0

TASK 4

Find the area bounded by the circle $x^2+y^2=1$ using Green's theorem.

CODE:

```
x=cos(t);
y=sin(t);
```

```
dx=diff(x,t);
dy=diff(y,t);
F=x*dy-y*dx;
area=1/2*(int(F,t,0,2*pi))
```

OUTPUT:

area = π

TASK 5

Find the area bounded by the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ using Green's theorem.
(Hint: Parametric form of Astroid: `x=a*cos³t, y=a*sin³t)`

CODE:

```
syms x y t a b;
x=a* cos(t) .^ 3 ;
y=a* sin(t) .^ 3 ;
dx=diff(x,t);
dy=diff(y,t);
F=x*dy-y*dx;
area=1/2*(int(F,t,0,2*pi))
```

OUTPUT:

area =

$$\frac{3\,\pi\,a^2}{8}$$