# Voltage oscillations in barnacle size muscle fibre with the Morris-Lecar Equations

Rahul Mohandas, Gautham Nandakumar
[1] School of Biomedical Engineering, Drexel University, USA

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Instructor: Ahmet Sacan
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**ABSTRACT**
Barnacle muscle fibers produce constant oscillations when their internal medium consist of Ca++ ions. These oscillations can be abolished if they K+ conductance is blocked/if the Ca++ ions are removed. We aim to develop a model using Simulink that displays variability in system parameters. We use the Morris-Lecar equation to model the two systems and analyze its propensity to develop damped oscillations under varying parameters. It is concluded that a system with two no activating conductance can exhibit varying oscillatory behavior

## 1 INTRODUCTION

Previous voltage clamping studies have shown that fibers consist of simple Ca2+ and K+ channels neither of which inactivates appreciably. We use the barnacle size muscle to analyze the behavior of biological components that rely on change in action potential such as the neurons. This model will help us understand the level of oscillations observed in the neurons when there is an imbalance in the Ca+/K+ ions with the cells.

Barnacle size muscle fibers have been proven to be suitable for electrophysiological studies. They can be cannulated and their mechanical as well as electrical responses can be studied and compared to several aspects of the human body response [1].

In this paper, we focus on the voltage oscillations induced due to clamped current in two conductances in a well-spaced clamped environment. Our goal is to study a range of behavior available to a system and how it predicts the oscillations for two non-linear systems along with variations in output for several parameters.

This helps us understand how this type of system can be used in modelling biological systems that follow similar workflow such as the neurons.

When we look at the ion conductances in our model, neither of them show fact inactivation (Ca+ and K+ channels). We are varying Ca+ and K+ and observing the changes in membrane potential across the cell. This model will help us understand how our body attains equilibrium and level of resonance within our cell.

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Related Work:

In 2006, a paper was published to explain the importance of Morris–Lecar (M–L) equation in neuron model that exhibits classes I and II excitabilities when system parameters are set appropriately [2]. Also, in 2006 a mode was proposed for the design of electronic bursting neurons, based on a simple conductance neuron model. A burster is a class of neuron that displays fast spiking regimes alternating with resting periods. This method is based on the use of an electronic circuit that implements the well-known Morris–Lecar neuron model.

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## 2 METHODS

In our model, we assign two independent voltage dependent conductances g_K and g_CA. In this experiment, we are assuming that relaxation kinetics are of first order and are no very essential. The author has assumed linearity for instantaneous current-voltage across the ion channels. This case may not be practical because in reality the ion channels follow a non-linear pattern.

However, if there is a high flow of ions across a channel, it can be assumed to have some level of linearity.

<u>List of abbreviations of the model parameter:</u>

| | |
|---|---|
| I | applied current (,uA/cm2) |
| I-L, I-C, I-K | leak, Ca++, and K+ currents, respectively (ALA/cm2) |
| g-L, g-Ca, g-K | maximum or instantaneous conductance values for leak, Ca", and K+ pathways |
| V | membrane potential (mV) |
| V-L, V-Ca, V-K | equilibrium potential corresponding to leak, Ca++, and K+ conductances, |
| M | fraction of open Ca"+ channels |
| N | fraction of open K+ channels |
| M∞(V), N∞(V) | fraction of open Ca"+ and K+ channels, at steady state |
| Λm(V), Λn(V) | rate constant for opening of Ca"+ and K+ channels (s-') |
| Λm, Λn | maximum rate constants for Ca++ and K+ channel opening (s-') |
| V1 | potential at which M_ = 0.5 (mV) |
| V2 | reciprocal of slope of voltage dependence of Λm. (mV) |
| V3 | potential at which N. = 0.5 (mV) |
| V4 | reciprocal of slope of voltage dependence of Λn (mV) |
| [Ca++]i, [Ca++], | internal and external Ca++ concentration (mM) |
| K | accumulation-layer constant (cm- ') |
| F | Faraday constant |
| C | membrane capacitance (iuF/cm2) |

**Table 1. List of species and parameters used in the model.**

Figure (1) shows us the electrical diagram used for this experiment

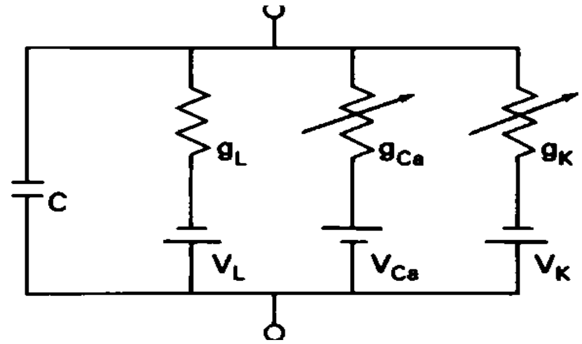


Figure (1)-Equivalent circuit for a patch of space-clamped barnacle sarcolemma. [3]

$$I = C\dot{V} + g_L(V_L) + g_{Ca}M(V - V_{Ca}) + g_K N(V - V_K)$$
$$\dot{M} = \lambda_M(V)\,[M_\infty(V) - M]$$
$$\dot{N} = \lambda_N(V)\,[N_\infty(V) - N].$$
$$M_\infty(V) = 1/2\,\{1 + \tanh\,[(V - V_1)/V_2]\}$$
$$\lambda_M(V) = \overline{\lambda_M}\cosh\,([V - V_1]/2V_2)$$
$$N_\infty(V) = 1/2\,\{1 + \tanh\,[(V - V_3)/V_4]\}$$
$$\lambda_N(V) = \overline{\lambda_N}\cosh\,([V - V_3]/2V_4).$$

Figure (2) – Hodgkin-Huxley equation [3]

The above equation represents a third-order non-linear system of Hodgkin-Huxley form which we shall use to explain the excitation behavior of our model.

We developed a Simulink model to replicate the excitation behavior of the barnacle muscle using the equations shown above. Figure (3) includes the model for Hodgkin Huxley form.

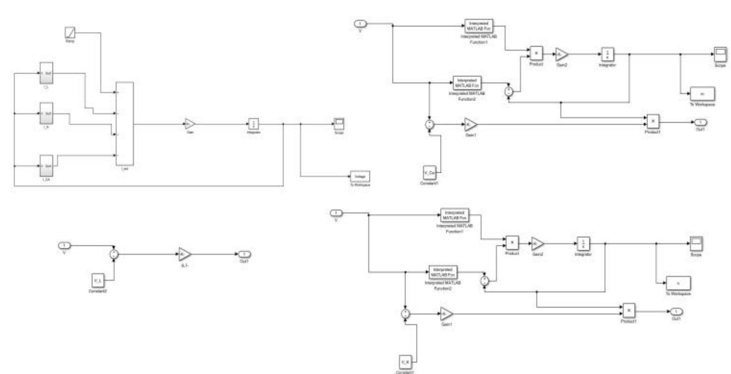

Figure (3) – Simulink Model

We replicated the Morris-Lecar equations from the article and developed a Simulink model that complies all the above parameter. Our model extends the existing model to calculate the membrane potential developed across the cell membrane due to Ca++ and K+ ion exchange. Simulink model provides flexibility in altering parameters on a fixed model. This type of system helps in plotting the output for varying input values.

Using the scope block in Simulink, we could graphically visualize the rate of change of ion concentration and membrane potential across the cell membrane for varying current values (I).

Parameter estimation was done based on hypothetical and previous experimental results. [4]

## 3 EXPERIMENTS AND RESULTS

Voltage behavior of the fiber was observed under three different conditions: Ca++-free external solution (to minimize g_Ca), with Cs/TEA-containing internal perfusion solution (to minimize g_K, and with solutions that optimize both g_K and g_Ca.

### <u>K+ Conductance</u>

In a solution with the absence of Ca++, no voltage oscillations are observed. The membranes response was not noticeable with small stimuli. But, as we increase the stimuli, a superficial response is observed. As a result, the membrane potential repolarizes producing a peak. Figure (4) shown below establishes the above explanation.

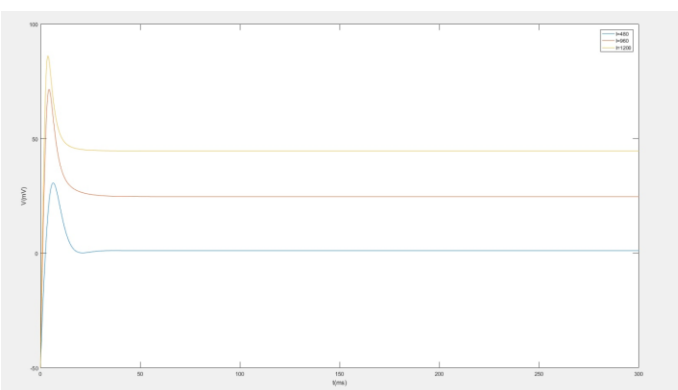

Figure 4 - Plateau potentials of the K+ system. (a) Voltage responses (in millivolts) of a fiber toincreasing current stimulus (480, 960, and 1200 gA/cm2). Other parameters were as follows: gca - 0, gK = 8, gL = 3, V_K = -70, V_L = -50, Λn = 1/15, C - 20, V3 - -1.0, V4 = 14.5.

Figure (4) shows that our model is sensitive to changing values of current (I). This is evident from the figure shown.

### <u>Ca++ Conductance</u>

The behavior of this model in absence of K+ ions is like the previous setup (g_Ca blocked). No oscillations are observed. As we increase the external stimulus a bi-stability in membrane potential is seen. As shown in figure (5)

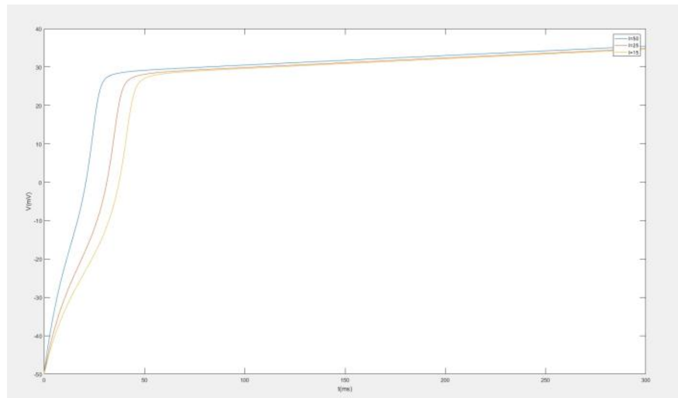

Figure 5- Voltage response (mV) of the model (nonlinear driving force) to increasing currents (I=50, 25,15). Other parameters were as follows: g_K = 0,g_C. = 40, g_L = 2, [Ca++]i = 0, [Ca++]= 100, VL = -35. AM = 0. 1, C = 20, VI = 10, V2 =15.

### <u>Oscillations: K+ and Ca++ Conductance Together</u>

In presence of both ions it is expected that a voltage plateau intermediate for all values of changing K+ and Ca++ ion concentration would result. But instead we observe damped oscillations. When the stimuli are small the system produces small passive depolarization, but when the voltage reaches its threshold and oscillations are produced.

Figure (6) results the observation explained above.

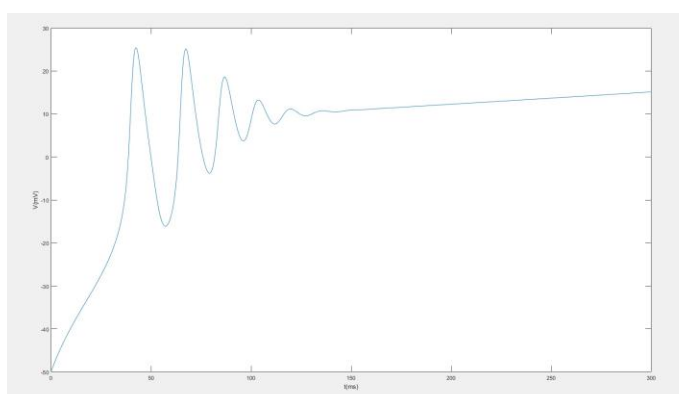

Figure (6) The parameters used for these computations are: gL = 2, V_L = - 50, V_C. = 100, V_K = -70, Λm = 1-0, Λn = 0.1, VI = 0, V2 = 15, V3 = 10, V4 = 10, C = 20.

**Analysis:**

We are examining the contribution of each of the conductance system when operating in independently. This shall help us understand how the behavior of each system produces intended results.

**Response of system with single voltage conductance**

For this analysis, we solved the generalized equation shown below:

$$I = C\dot{V} + g_L(V - V_L) + g_i\mu\,(V - V_i)$$

$$\dot{\mu} = \lambda\,(V)\,[\mu_\infty(V) - \mu]$$

Where, μ can either be M or N representing the Ca and K+ ion exchange respectively.
To study the behavior of the above equations we calculated the equations for nullclines. Below are the nullcline equations.

$$(\dot{V} = 0 \text{ nullcline})\ V(\mu) = (I + g_L V_L + g_i\mu V_i)/(g_L + g_i\mu),$$

$$(\dot{\mu} = 0 \text{ nullcline})\ \mu(V) = \mu_\infty(V).$$

Figure (7) shows the graphical representation for V, N phase plane for all K+ systems for varying value of current (I= 0, 25, 100, 400)

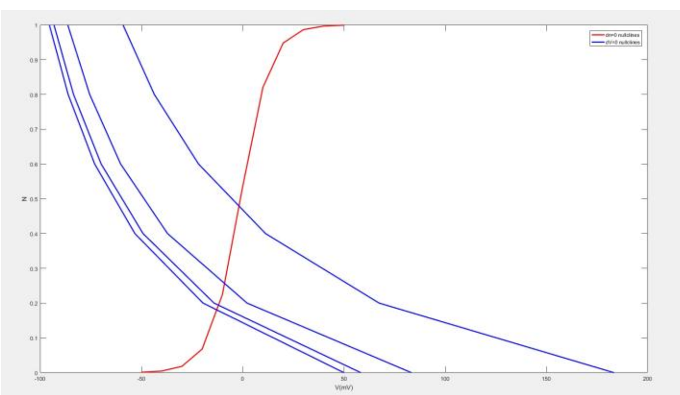

Figure 7- `Parameters used are g_L =3, g-C. = °. g-K = 8, V-L = 50. V-K =-70, V3 = - 1.0, V4 = 14.5, Λn = I//15, C = 20.`

In Figure (7) the intersection of the N = 0 and V = 0 nullcline for each value of current is the sole singular point for the all-K+ system for that set of parameters and is always a stable node.

Figure (8) shows the graphical representation for V, N phase plane for all Ca++ systems for varying value of current (I= 0, 25, 100, 400)

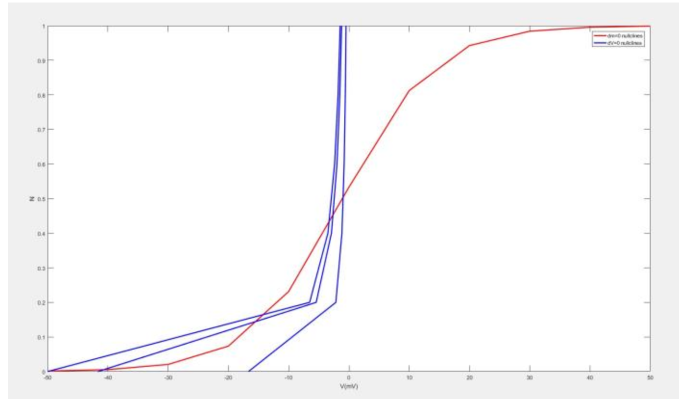

Figure 8- `Parameters used are g_L =3, g-C. = 10. g-K = 0, V-L = 50. V-K =-70, V3 = - 1.0, V4 = 14.5, Λm = I//15, C = 20.`

In Figure (8) the intersection of the N = 0 and V = 0 nullcline for each value of current is the sole singular point for the all-Ca++ system for that set of parameters and is always a stable node.

**Stability of cohesive system**

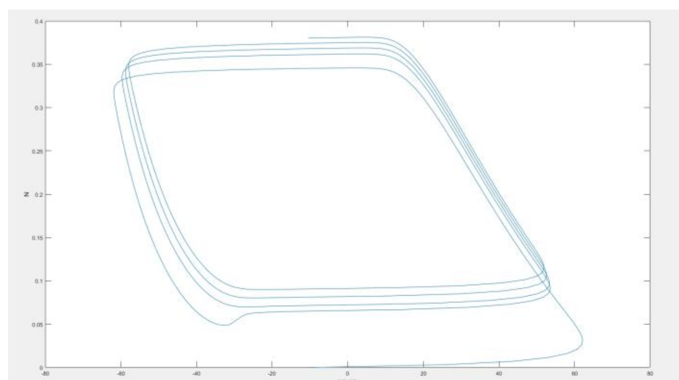

Figure 12- The phase plot of the cohesive system

## Discussion

Voltage behaviour across the cell membrane with Ca++ and K+ ions exchange showed us oscillations, which was absent in systems with Ca++ free and K+ free solutions. Hence proving that presence of both the ions exchanges are important for a stable membrane potential.
The author results a similar damped oscillation as we observed in the cohesive system. But he adds a behaviour where the system produces sustained oscillations. This observation was not established when we simulated our model. We concluded that the range of the stimuli for sustained oscillations would be too minute.
The g work to this project includes addition of two features. One, to include the extact values of the external stimuli (I) to produce sustained oscillations. Second, to simulate the same model with the third ion being Na+ and obtaining good level of stability.

## References

## 4 BIBLIOGRAPHY


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[4] T. Tateno, "Random dynamics of the Morris–Lecar neural model".

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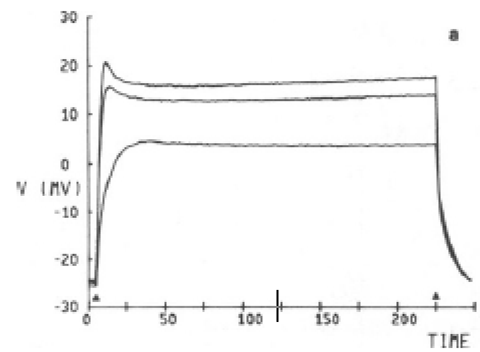
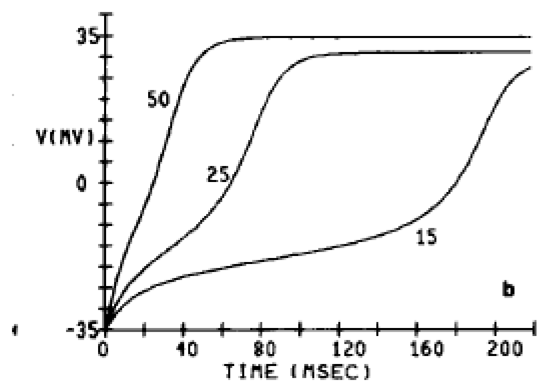
## Supplementary Reading

These are the results obtained by the Author in comparison what we obtained. Please note, that there are additional graphs in the paper we used.

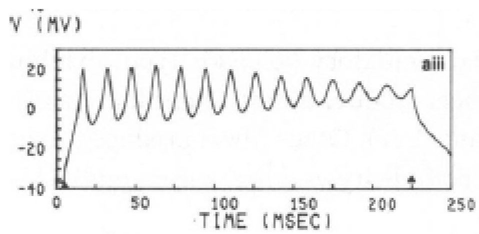
## K+ Conductance




## Ca++ Conductance




## Oscillations: K+ and Ca++ Conductance Together




## NullClines (K+ and Ca++)