Relation between fundamental cutsets and circuits

Explain the relation between fundamental cutsets and circuits

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***Abstract*— This report describes about cutsets and what are their significance with respect to a graph and how they are made by similar components : branches and chords. It describes the topic for undirected graphs.**

***Keywords*— Edges, vertices, Adjacency matrix, Undirected graph, circuits, Fundamental circuits, Cut Sets , Fundamental Cut Sets, Rank, Nullity, Basic Cut Sets, Branches, Chords.**

I. INTRODUCTION

A Graph consists of vertices and edges. These vertices and edges and certain relationships between them define whether a graph is of single component or multiple components. Circuits define whether by traversing certain vertices and edges,it may lead to a closed recurrent path. Similarly Cutsets define whether by removing certain vertices and edges,multiple connected components of the parent graph can be obtained. Both circuits and cutsets involve terminology like spanning tree and co-spanning tree. In this paper we present an implementation based explanation about circuits and cutsets, and how terms like branches and chords of spanning tree signify the most.These two terms Fundamental circuits and cutsets hold their significance with respect to a particular spanning tree. Fundamental circuits and cutsets both are created by branches and chords of a spanning tree, so both terms will vary for different spanning trees. Just as a spanning tree is essential for defining a set of fundamental circuits,so is for a set of fundamental circuits.

II. MOTIVATION

Since graph is a network, it can simulate network problems better. The concept of a fundamental circuit,introduced by Kirchhoff, is of enormous significance in electric network analysis. Similarly cut-sets are of great importance in studying properties of communication and transportation networks. For finding any weak spots in the network, we look for the most vulnerable fundamental cutset. To understand and simulate above problems,first we should know about the relation holding them together.

III. BACKGROUND AND METHODOLOGY

Before proceeding to any further, we explain the terminology used throughout this paper for clarity sake, It is important to define the graph theoretic structures with which we will be working.

Definitions:

1. A vertex set, V, is a set of vertices or points.
2. The edge set on V, E_V is the set of all ordered pairs of vertices in V in the graph G.
3. A graph is an ordered pair (V, E_V) where V is the vertex set and $E_{V\ is}$ edge set on V.
4. Let G = (V, Ev) be a graph. Suppose a, b belongs to V. We define a path in G from a to b be a sequence of vertices a = v0, v1, v2, ….., v (n – 1), v(n) = b for which vi is in V, i = 0, 1, 2, ... n and for which (v (j – 1), v(j)) belong to Ev, for j = 1, 2, … n.
5. A branch set B,is a set of edges of the graph which form any particular spanning tree T.
6. A chord set C,is a set of edges which form a particular co-spanning tree(G-T).

Before moving any further,first the basic definitions of both circuits and cutsets should be cleared.The detailed explanation about both is presented in next header.

A . **Circuit and Fundamental circuit**

Circuit is a closed non-intersecting walk in which no vertex appear more than once except the starting vertex. Every vertex in a circuit has degree 2. A circuit is also called a cycle,elementary cycle,circular path, and polygon. And every self loop is a circuit but not vice versa.Before defining about fundamental circuits,first we should understand about the components involved. Every connected graph has at least one spanning tree.

a. Spanning tree

This is the property of a connected graph.Spanning tree is a subgraph of parent graph and it contains all the vertices of G.It is like the skeleton of original graph.Spanning tree is called the maximal tree because it is the largest tree in a connected graph. The most important property of spanning tree is "if a graph contains no circuits,it is it's own spanning tree". Spanning tree is made of two types of edges called branches and chords of a graph.

i) **Branches**

- Those edges of graph G which are also part of spanning tree T.
- Defined for a particular spanning tree,varies with various spanning trees of same graph.
- Branch when included with chord set C create cutsets.
- No. of Branches in a graph with respect to a given spanning tree is defined as term **Rank of a graph.**
    - **No. of branches = Rank = N - k .**
    - where N is total number of vertices in the connected graph and k is total no. of components.
    - In case of connected graph (k=1), no. of branches in graph or Rank is N - 1.

ii) **Chords**

- Those edges of graph G which are not present in spanning tree, or which are present in co-spanning tree T' which is (G-T).
- Particular to a given spanning tree.
- Chords when included with branch set B create circuits.
- No. of Chords in a graph with respect to a given spanning tree is defined as **Nullity of a graph.**
    - **No. of chords=Nullity=E-N+k.**
    - where N is total number of vertices and E is total number of edges and k is total number of components.
    - In case of connected graph (k=1), number of chords in graph or Nullity is E-N+1.

Previously it is explained that chords along with a different branches of spanning tree create different circuits. So in order to make a graph circuit-free, minimum E-N+1 (number of chords) edges should be deleted.

b. **Fundamental circuit**

Circuit created by adding an edge of chord set C in spanning tree. Total number of fundamental circuits will be equal to total chords can be added which is E-N+1. A circuit may be fundamental wrt one spanning tree but may not be for another one.

B. **Cut-sets and Fundamental Cut-sets**

A cut-set is a set of edges whose removal from graphs leaves it disconnected. A cut-set always cuts a graph into two mutually exclusive set of edges whose removal reduces the rank of graph by one. Every edge in tree is a cut-set for it.

a. **Fundamental cut-sets**

These are also defined for a particular spanning tree. With respect to a spanning tree,a fundamental cut-set will contain exactly one branch of T,rest edges are chords.Like fundamental circuits,every chord defines a unique cut-set.

C. **Relation between fundamental cut-set and circuit**

- At least two edges must be common in fundamental circuit and cut-set.
- With respect to a given spanning tree T, a chord c° that determines a fundamental circuit T^2 occurs in every fundamental cut-set S associated with the branches in T and no other.
- With respect to a given spanning tree T,a branch b° that determines a fundamental cut-set S contained in every fundamental circuit associated with the chords in S and no other.

__**Algorithm 1**__

Input: Graph G(V, E) V is the vertex set, E is edge set

Output: Fundamental Circuits and Cutsets associated with a given spanning tree

Procedure:

1. Let V be the number of Vertices
2. Let E be the number of Edges
3. Get the Spanning Tree **T** of the Graph
4. Let C(G) be the set of chords w.r.t to **T,** chords are the edges which are in G but not in **T**
5. Take a Chord C_i and all the branches/ edges of the **T**, denote these set of edges X which denotes a fundamental circuit of the graph.
6. Take each branch of X, find the fundamental cutset associated with it, fundamental cutset consists of a branch of T and minimal number of chords w.r.t **T**.
7. To check if it is a cutset check whether the components increase in the resulting graph.

8. This method hence returns fundamental cutsets associated with each branch of a given circuit.

Algorithm 2

Input : Graph G(V, E) V is the vertex set, E is edge set
Output : Program shows that there are even number of edges(at least two) between fundamental circuits and cutsets.

Procedure :

1. Let V be the number of Vertices.
2. Let E be the number of Edges
3. Get the Spanning Tree **T** of the Graph
4. Let count be a variable which is 0 initially counts similar edges between fundamental cut-set and circuit.
5. Let C(G) be the set of chords w.r.t to **T,** chords are the edges which are in G but not in **T.**
6. Take any chord and create a fundamental circuits by adding that chord in branch set.
7. Take any branch and create a fundamental cutset by adding that branch in chord set.
8. Create two edge map E1 (for fundamental circuit) and E2 (for fundamental cut-set).
9. Mark all the edges present in Fundamental circuit in map E1 and those present in Fundamental cut-set in map E2.
10. For each edge e in edge set E
    a. if (e is present in both E1 and E2)
        i. count++;
11. return count.

IV. Results

Using both of the above implementations,it can be showed that the basic relationship between fundamental cut-set and circuit is preserved.Following are the results from the implementations,

a. Fundamental circuits and cut-sets associated with the given tree.
b. Number of edges common between both fundamental circuit and cut-set are even.

V. Conclusion

In this summary report,first we have introduced all the terminology and definitions and later highlighted the relationships among circuits and cut-sets by both explanation and implementation. It can also be understood by report that both cut-sets and circuits are creation of same constituting agents branches and edges.

VI. References

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