Find out the largest complete subgraph for a given graph

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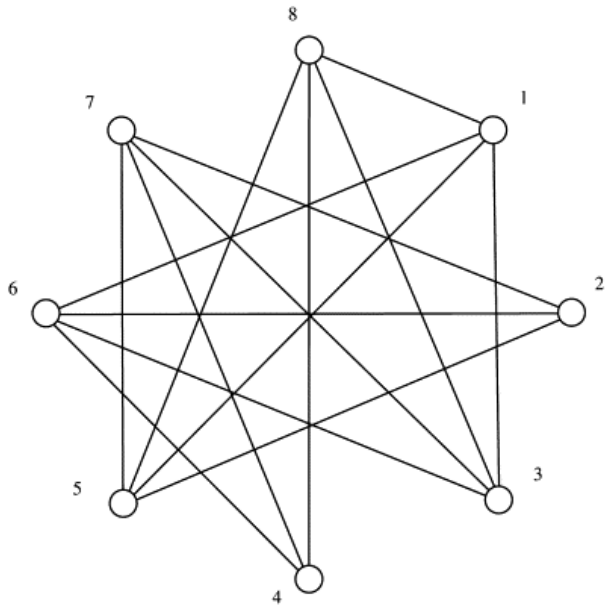
***Abstract*— This report studies about the various method to find the largest complete subgraphs using brute force and one of the standard algorithms(Bron–Kerbosch Algorithm). This paper also discusses the time complexity of all the methods.**

***Index Terms*—Graph, Subgraphs, Clique**

I. INTRODUCTION

A clique is a subset of vertices of an undirected graph such that every two distinct vertices in the clique are adjacent. In other words, a clique is a complete subgraph of the given graph. We denote an undirected graph by G=(V,E), where V is the set of vertices and E is the set of edges. Two vertices are said to be adjacent if they are connected by an edge.



II. PROBLEM DESCRIPTION

Given a graph G we need to find the clique of maximum size. We will use brute force approach and Bron Kerbosch algorithm to solve the problem.

III. APPROACH

A. *Brute Force Approach*

```
findMaxClique(Graph G)
{
        initialize max as an empty graph
        for all subgraphs=S of G
                if( isComplete(S) )
                        if( size(S) > size(max)
                                max=S
}
isComplete(Graph G)
{
            if ( size(S)<=1 ) return false;
            for i=0 to size(S)-1
                    for j=i+1 to size(S)
                        a=S[i] and b=S[j]
                        if (G[a][b]==0 OR G[b][a]==0) return false;
            return true;
}
```

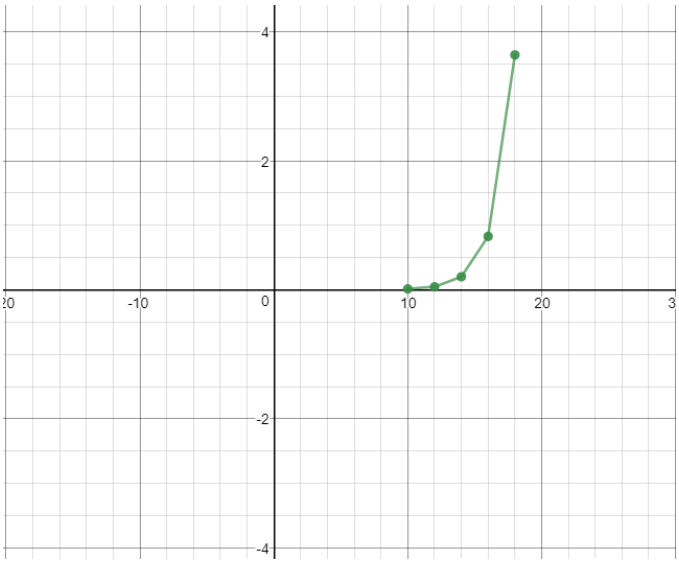
B. *Bron–Kerbosch algorithm*

```
BronKerbosch1(R, P, X):
    if P and X are both empty:
        report R as a maximal clique
    for each vertex v in P:
        BronKerbosch1(R ⋃ {v}, P ⋂ N(v), X ⋂ N(v))
        P := P \ {v}
        X := X ⋃ {v}
```

IV. TIME COMPLEXITY ANALYSIS

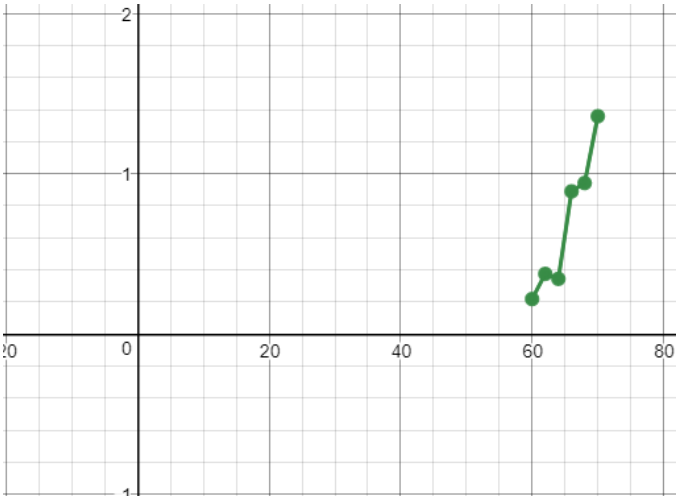
A. *Brute Force Approach*

- **Time Complexity** : $O(2^n * n^2)$



B. ***Bron–Kerbosch algorithm***

- **Time Complexity** : $O(n^k)$ (For k size clique)



V. CONCLUSION

We have presented two algorithms for finding a maximum clique in given graph. The brute force approach is better suitable for smaller size graphs but Bron–Kerbosch algorithm is suitable for larger graphs. This can be confirmed by above section.

References

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[2] https://en.wikipedia.org/wiki/Clique_(graph_theory)
[3] https://en.wikipedia.org/wiki/Complete_graph
[4] http://www.sciencedirect.com/science/article/pii/S0166218X01002906