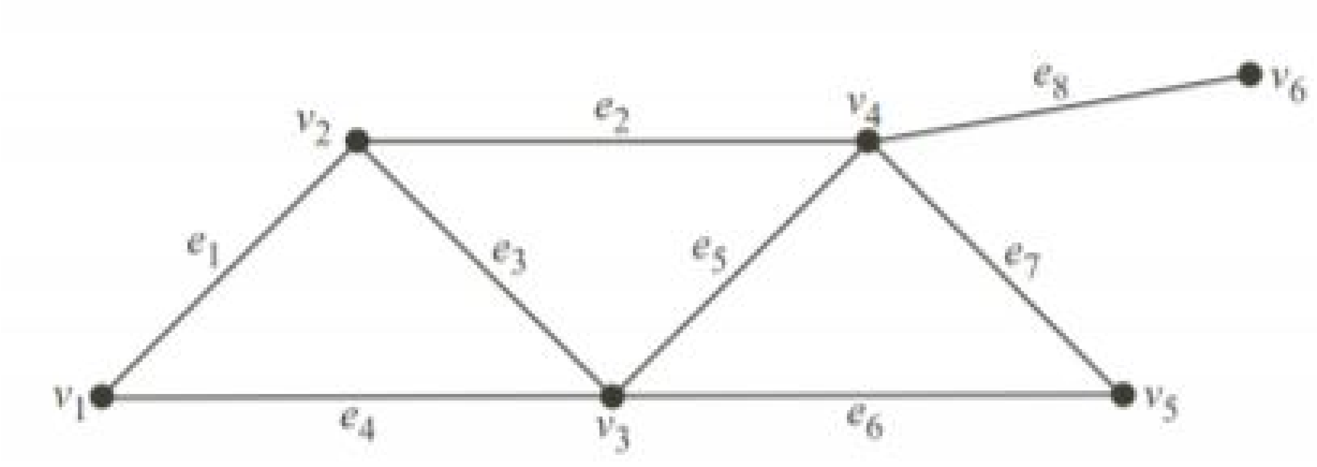
Find all fundamental cutsets in a graph

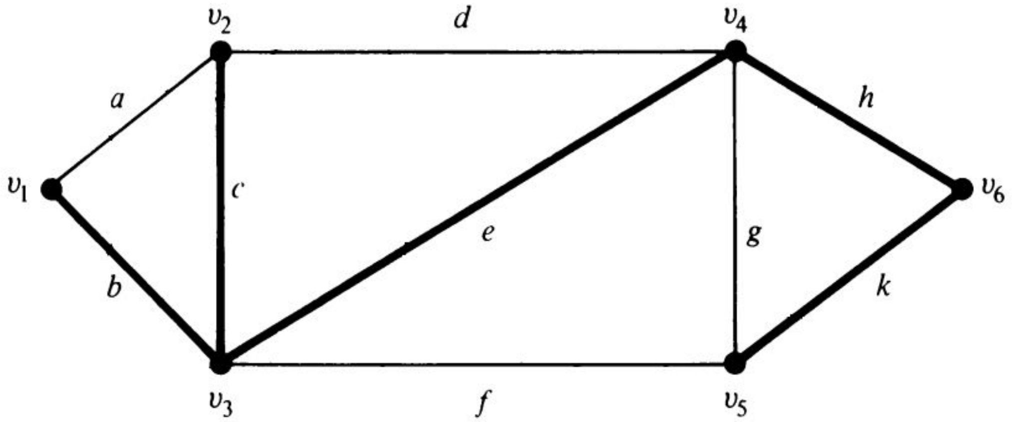
Group members
Yogesh Gupta (IRM2014004)
Akshat Aggarwal (IIM2014005)
Vishesh Middha (ISM2014007)

What is a Cut-Set?

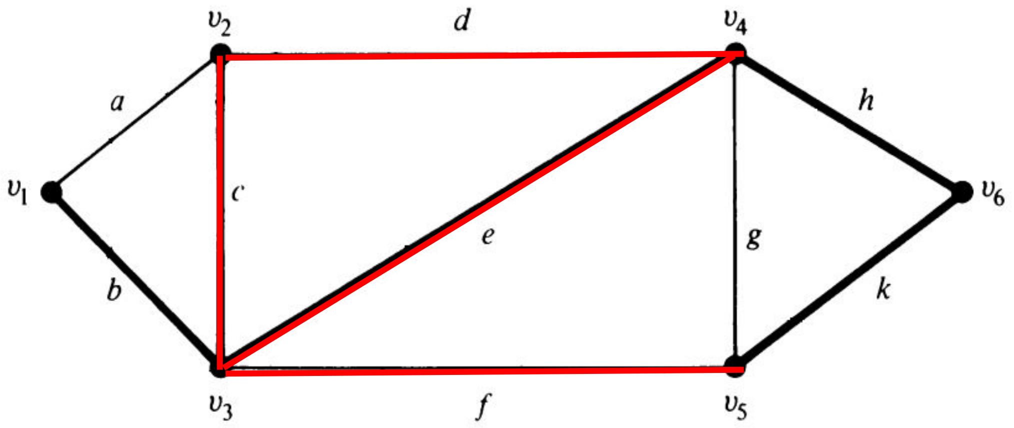


A cutset is a minimal set of edges that, when broken, breaks the graph into two completely separate parts. In the above graph $\{e_1, e_4\}$, $\{e_6, e_7\}$, $\{e_1, e_2, e_3\}$, $\{e_8\}$, $\{e_3, e_4, e_5, e_6\}$, $\{e_2, e_5, e_7\}$, $\{e_2, e_5, e_6\}$, $\{e_2, e_3, e_4\}$ are the cut-sets.

What is NOT a Cut-Set?

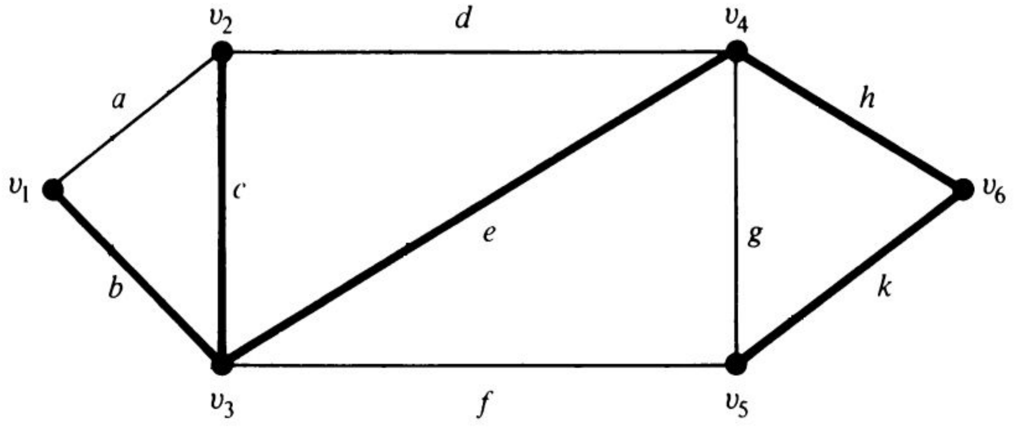


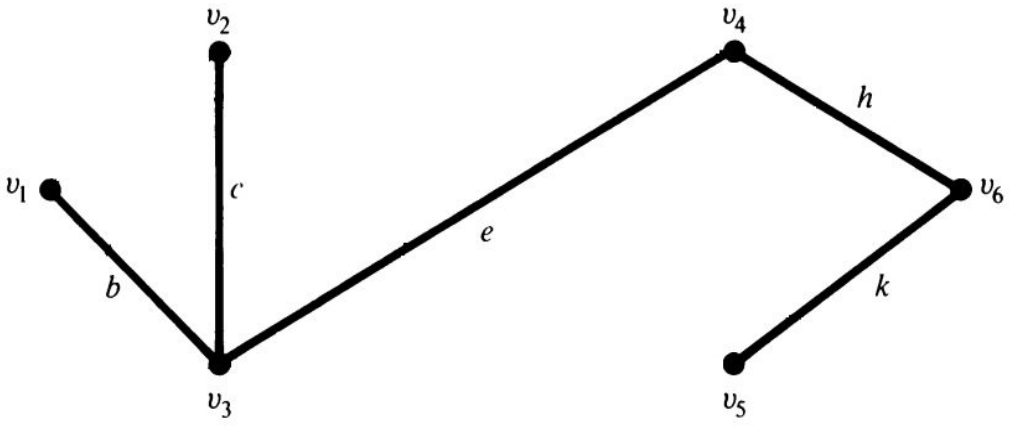
If a subset B of a set A of edges represent a cut-set already, then the set A won't be called a cut-set, but rather that subset of the initially taken set of edges will be called a cut-set.



For instance, let us assume that in the above figure {c, d, e, f} is a cut-set. But it can be seen that {d, e, f} already form a cut-set, and therefore {c, d, e, f} will not be called as a cutset.

Fundamental Cut-sets

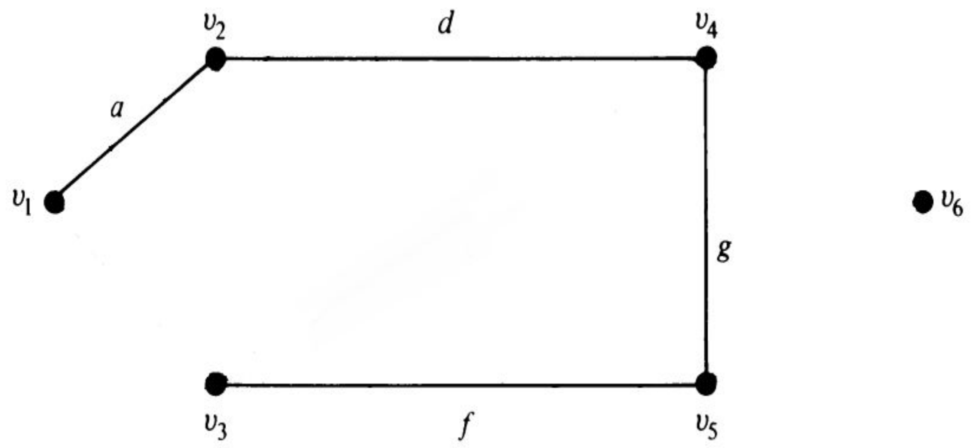




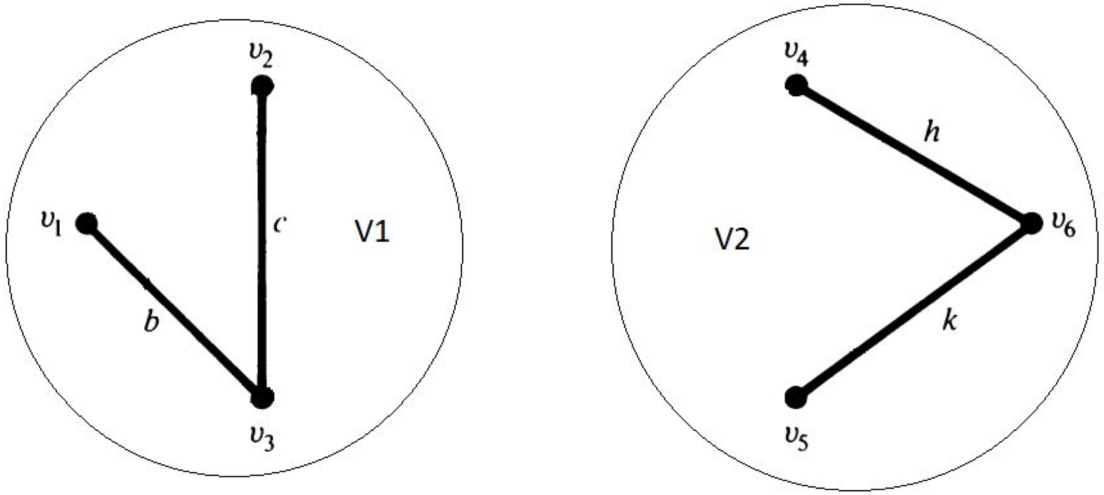
Consider, Graph, **G** and a spanning tree, **T** of **G**

- So, what if we remove a branch from the tree **T**? Say, I remove **branch e**

Fundamental Cut-sets (Contd.)



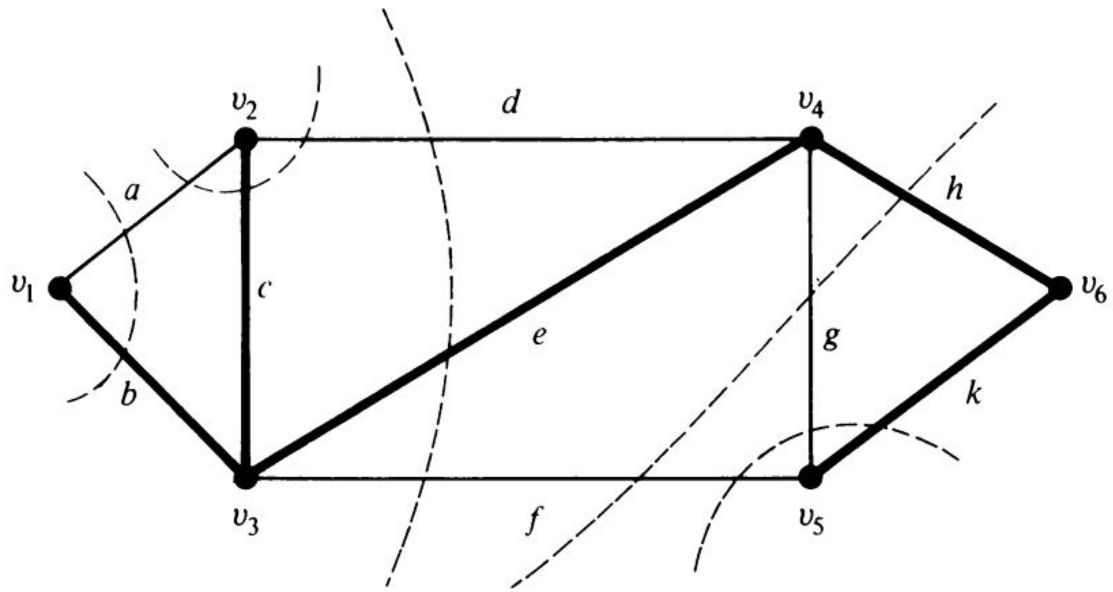
Chords with respect to **T**



V_1 , V_2 formed after removing branch **e** from **T**

- Now, we add a chord to the cutset if its endpoints lie in different sets (one in V_1 and other in V_2 or vice versa, but not both in the same set).
- This way we get a fundamental cut-set corresponding to the branch removed.

Fundamental Cut-sets (Contd.)



- Doing it for all branches, we get the fundamental cut-sets with respect to the spanning tree.

Complexity analysis

$v \leftarrow$ number of vertices in a graph
$e \leftarrow$ number of edges in a graph
$b \leftarrow$ number of branches in spanning tree T
$r \leftarrow m - b$, number of chords
for each branch b_i **do**
  Let u, v be the two vertices in spanning tree T, connected by branch b_i
  Remove branch b_i from T
  Let V_1, V_2 be the two set of vertices formed after removing b_i
  $cutset \leftarrow \{\}$
  for each chord c_i **do**
    if c_i connects two vertices v_1, v_2 such that one belongs to V_1 and the other to V_2 **then**
      Add c_i to $cutset$
    end if
  end for
  $cutset \leftarrow cutset \cup b_i$
  Add $cutset$ to $result$
  Add branch b_i to the T
end for
return $result$

O(v)

O(1)

O(*number of chords*)
i.e. O(e - v + 1)

O(v) + O(e - v + 1)

Complexity analysis

$v \leftarrow$ number of vertices in a graph
$e \leftarrow$ number of edges in a graph
$b \leftarrow$ number of branches in spanning tree T
$r \leftarrow m - b$, number of chords
for each branch b_i **do**
  Let u, v be the two vertices in spanning tree T, connected by branch b_i
  Remove branch b_i from T
  Let V_1, V_2 be the two set of vertices formed after removing b_i
  $cutset \leftarrow \{\}$
  for each chord c_i **do**
    if c_i connects two vertices v_1, v_2 such that one belongs to V_1 and the other to V_2 **then**
      Add c_i to $cutset$
    end if
  end for
  $cutset \leftarrow cutset \cup b_i$
  Add $cutset$ to $result$
  Add branch b_i to the T
end for
return $result$

$O(v * v) + O(v* (e - v + 1))$