LEAST COST HAMILTONIAN CIRCUIT

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Abstract—*Finding the least cost hamiltonian circuit is a very popular combinatorial optimization problem. The most popular application of this is travelling salesman problem.The objective is to find out a shortest possible path visiting each and every node once and returned to the origin node.Finding least cost hamiltonian circuit is one of the NP hard problems and several attempts have been done to solve it by traditional methods.In the proposed paper three approaches are presented such as Naive method ,branch and bound method and the Held Karp Algorithm which is basically a dynamic programming solution out of which held karp algorithm gives the solution faster than the other two approaches.*

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Keywords — *Hamiltonian Cycle , Branch and Bound ,Held karp, dp,np-hard*

1. Introduction

A Hamiltonian cycle, also called a Hamiltonian circuit, Hamilton cycle, or Hamiltonian circuit, is a graph cycle (i.e., closed loop) that visits each node exactly once (Skiena 1990, p. 196). A graph possessing a Hamiltonian cycle is said to be a Hamiltonian graph Least cost Hamiltonian cycles which is basically a Travelling salesman problem is one of the commonly studied and interesting problems of every researcher. Day by day evolution of nature inspired heuristic algorithms draw the attention of today's researchers and almost every new algorithm or hybrid algorithm is firstly implemented and its behavior is observed for Minimum-Cost Hamiltonian Circuit . Minimum-Cost Hamiltonian Circuit looks very simple but it can not be accurately solve by conventional mathematical techniques. Therefore various optimization techniques can be applied to obtain result. Finding least cost Hamiltonian cycle is a polynomial time (NP-Hard) problem in computer science thus need an intelligent technique to improve the produced results.

Hamiltonian Circuit : A tour that starts and ends at the same vertex (circuit definition). Visits each vertex once. (Vertices cannot be reused or revisited.) Circuits can start at any location. Use wiggly edges to show the circuit. Minimum-Cost Hamiltonian Circuit A Hamiltonian circuit with the lowest possible sum of the weights of its edges.

Adjacency Matrix : A two dimensional of array having V rows and V columns i . . Let the array be G. An entry G[i][j] represents the weight of edge from j to the *i* vertex.

V	1	2	3	4
1	0	21	4	2
2	21	0	12	0
3	4	12	0	3
4	2	0	3	0

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TABLE 1: ADJACENCY MATRIX REPRESENTATION OF A GRAPH

2. Problem Description

We have an input Graph G(V,E) in the form of Adjacency Matrix A of size [VxV], Our main objectives are :

- To find the least cost Hamiltonian circuit in the graph .
- Time Complexity Analysis of finding the least cost hamiltonian circuit .

3. Approach

A. BRUTE FORCE

In brute force approach we consider a node as starting and ending point and then generate all (n-1)! (for n number of vertices in a graph) permutations of nodes. Then we go on calculating the cost of every permutation and keep track of minimum cost permutation.Then we return the permutation with minimum cost.As it is evident from the approach that this algorithm takes too much of time in order to find the permutation and minimum cost permutation which is (n)! which is very bad for solving this problem.

B. BRANCH AND BOUND

We have divided input Graph into two kind of graphs based on the presence or absence of Hamiltonian circuit.

We will go from the first node and go through the least cost next node and calculate the cost of a hamiltonian circuit as per the brute force method. Then use the above value as a Heuristics and save the value as Min Cost.

We will go through all the permutations but we will check if the cost is ever getting greater than the value of the Min Cost, If it does we will stop and not go any further . Since all the weights are positive and the path which it leads to will be greater than Min Cost , following this path will only increase the amount of computation .

Hence by removing some redundant checks we will improve the time taken to calculate the Hamiltonian Circuits by large factor.

C. HELD KARP ALGORITHM

Let us consider a graph G = (V, E), where V is a set of cities and E is a set of weighted edges. An edge e(u, v) represents that vertices u and v are connected. Distance between vertex u and v is d(u, v), which should be non - negative. Let the given set of vertices be 1, 2, 3, 4,..n . Let us consider 1 as starting and ending point of output. For every other vertex i (other than 1), we find the minimum cost path with 1 as the starting point, i as the ending point and all vertices appearing exactly once. Let the cost of this path be cost(i), the cost of corresponding Cycle would be cost(i) + dist(i, 1) where dist(i, 1) is the distance from i to 1. Finally, we return the minimum of all [cost(i) + dist(i, 1)] values. To calculate cost(i) using Dynamic Programming, we need to have some recursive relation in terms of sub-problems. Let us define a term C(S, i) be the cost of the minimum cost path visiting each vertex in set S exactly once, starting at 1 and ending at i.We start with all subsets of size 2 and calculate C(S, i) for all subsets where S is the subset, then we calculate C(S, i) for all subsets S of size 3 and so on. Note that 1 must be present in every subset.

if size of S is 2, then S must be {1, i},
    C(S, i) = dist(1, i)
else if size of S is greater than 2
    C(S,i) = min { C(S - {i}, j) + dist(i,j)} where
    j belongs to S, j !=i and j!= 1

For a set of size n, we consider n-2 subsets each of size n-1 such that all subsets don't have nth in them.Using the above recurrence relation, we can write dynamic programming based solution.

4. Results

Finding Least cost Hamiltonian Circuit using branch and bound :

Time Complexity = O(V!)

Where V = No. of vertex in Graph

Time_Taken = Average Time taken by Algorithm on 15 different graph of V vertices(in microseconds) the values can be seen in Table 2. The plot for the one is given in the FIgure 1.

Finding Least cost Hamiltonian Circuit using Held Karp Algorithm

There are at most O(n*2n) subproblems, and each one takes linear time to solve. The total running time is therefore O(n2*2n). The time complexity is much less than O(n!), but still exponential. Space required is also exponential. So this approach is also infeasible even for slightly higher number of vertices.The space complexity for this algorithm is also O(n2*2n) as we need 2 to the power n number of space for storing the subset and n2 number of space for storing each and every vertices.

V	Time_Taken for Branch and Bound Method
3	0
3	0
5	0
7	0
8	0
8	0
10	0
11	46880
11	31287
13	516913
14	4064914
14	8605764
15	19262216

TABLE 2:VERTEX SIZE AND TIME TAKEN

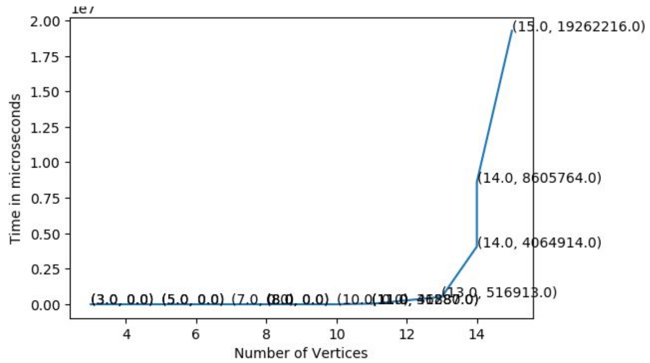

FIGURE - 1: NUMBER OF VERTICES VS TIME FOR BRANCH AND BOUND

5. CONCLUSION

We have devised three methods to calculate the number of Hamiltonian circuits in a graph. The third approach showed the best time complexity while others were in nondeterministic Polynomial in nature.

For future work one can use methods for checking if any Hamiltonian before doing the computation as an addition. Also we can add checks to see if the graph is fully connected as well before check. This would provide advancements in time taken for computation.

6. REFERENCES

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