In [1]:
```
from article_hypothesis_check import do_and_visualize_analysis,partition_
from simulate_data import simulate_fx_data, plot_price
from robustness_check import bootstrap_analysis
from identify_significant_hrs import plot_cum_mean_returns_per_hour, sr_p
import numpy as np
import pandas as pd
from matplotlib import pyplot as plt
```

# I - Data Simulation

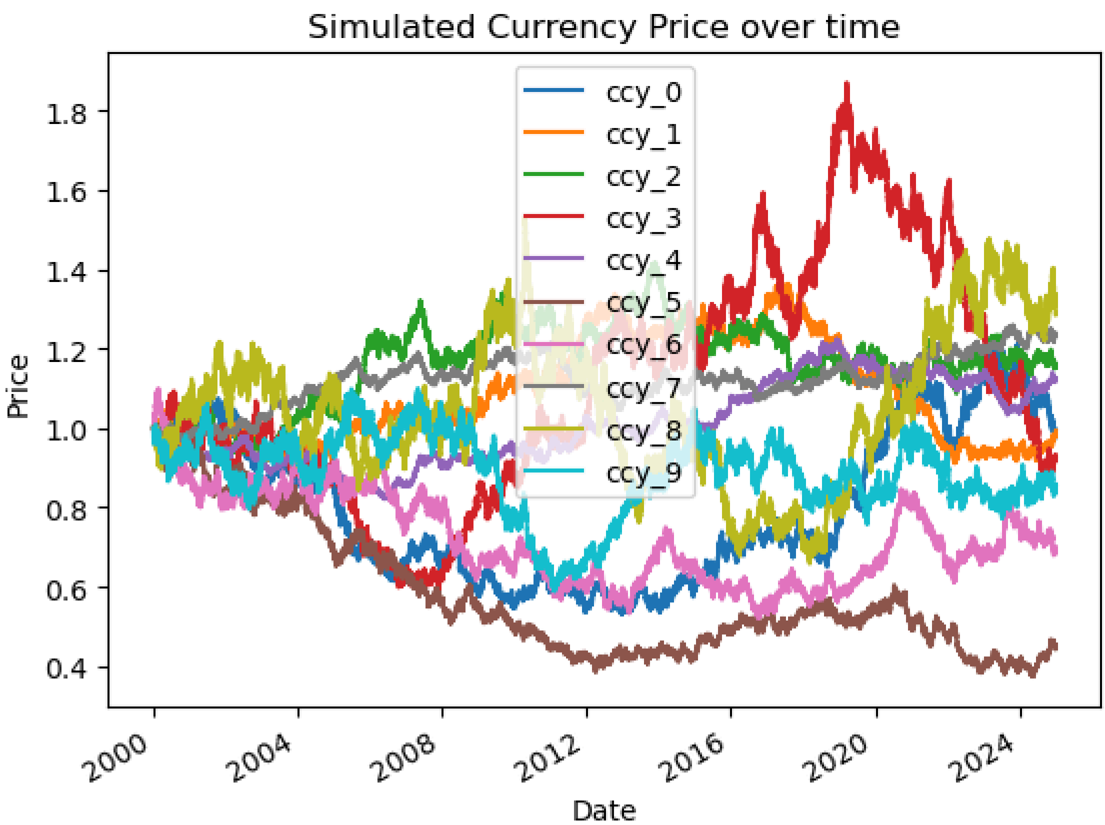
Finding free FX intraday data covering a long history proved to be challenging.

We will simulate the spot exchange rate as a geometric brownian motion.

To simulate the different behaviours between different trading sessions, we will use different drift values for foreign vs local session.

We will consider that prices are quoted as CCYUSD, so that negative returns indicate currency depreciating

In [12]:
```
# Calculate simulated data
data = simulate_fx_data(n_currencies=10)
#Compute log returns
returns = np.log(data.pct_change()+1)
# Plot Price for visualization
plot_price(data,n_plots=10)
```



# II - Trading sessions

## The article formulates the following hypothesis:

### - home currencies depreciate during their local trading hours as well as during LDN-NY overlap.

### - home currencies appreciate in the US session (between LDN 'close' and NY 'close')

### - home currencies returns are insignficant outside of those sessions

## To simplify, we will assume that all our currencies in our simulated data follow LDN time zone.

### As such we can form the following trading sessions (which is what we used for simulating the data with the corresponding pattern):

### - Non US session : 0AM to 7AM

### - Domestic session : 7AM to 2PM

### - LDN NY overlap: 2PM to 6PM

### - US session : 6PM to 10PM

# III - Identify trading sessions

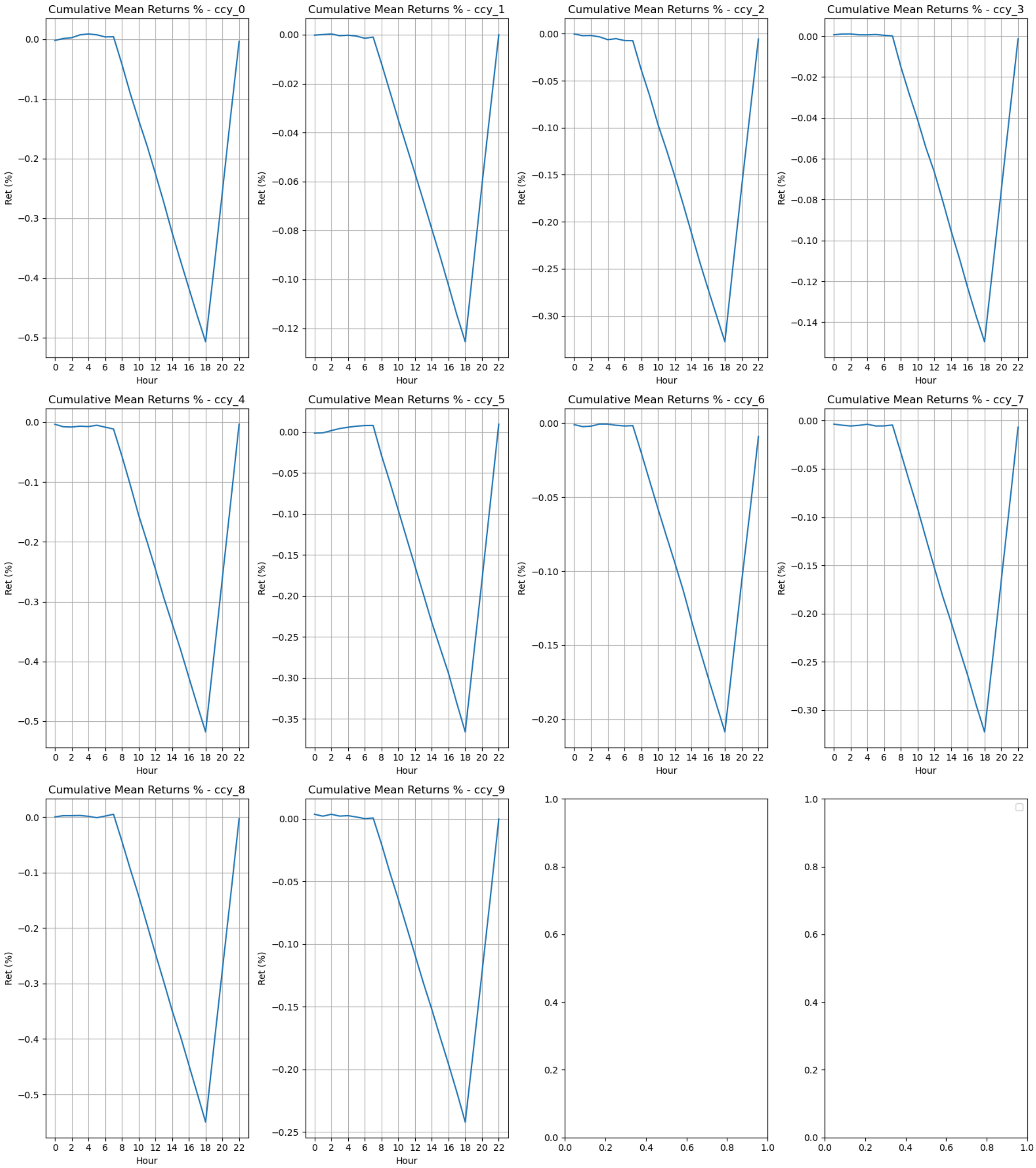
### Assuming we do not know the data is simulated, we will first attempt to confirm the pattern highlighted in the article through a graphic approach and by measuring the

### Sharpe Ratio of returns per hour accross currencies.

a - We plot the average cumulative returns per hour accross currencies. This clearly highlights intraday seasonal pattern of depreciation in domestic session and then subsequent appreciation

In [3]:
```
plot_cum_mean_returns_per_hour(returns)
```

```
No artists with labels found to put in legend.  Note that artists whose la
bel start with an underscore are ignored when legend() is called with no a
rgument.
```




b - We measure the Sharpe Ratio of returns per hour accross currencies. We see strong negative sharpe ratio in the domestic session vs strong positive sharpe ratio in the foreign session.

The Sharpe Ratio allows us to have a normalized view across hours and currencies, since their volatility profile might be different.

In [4]:
```
sharpe_ratios_per_hr = sr_per_hour(returns)
sharpe_ratios_per_hr
```

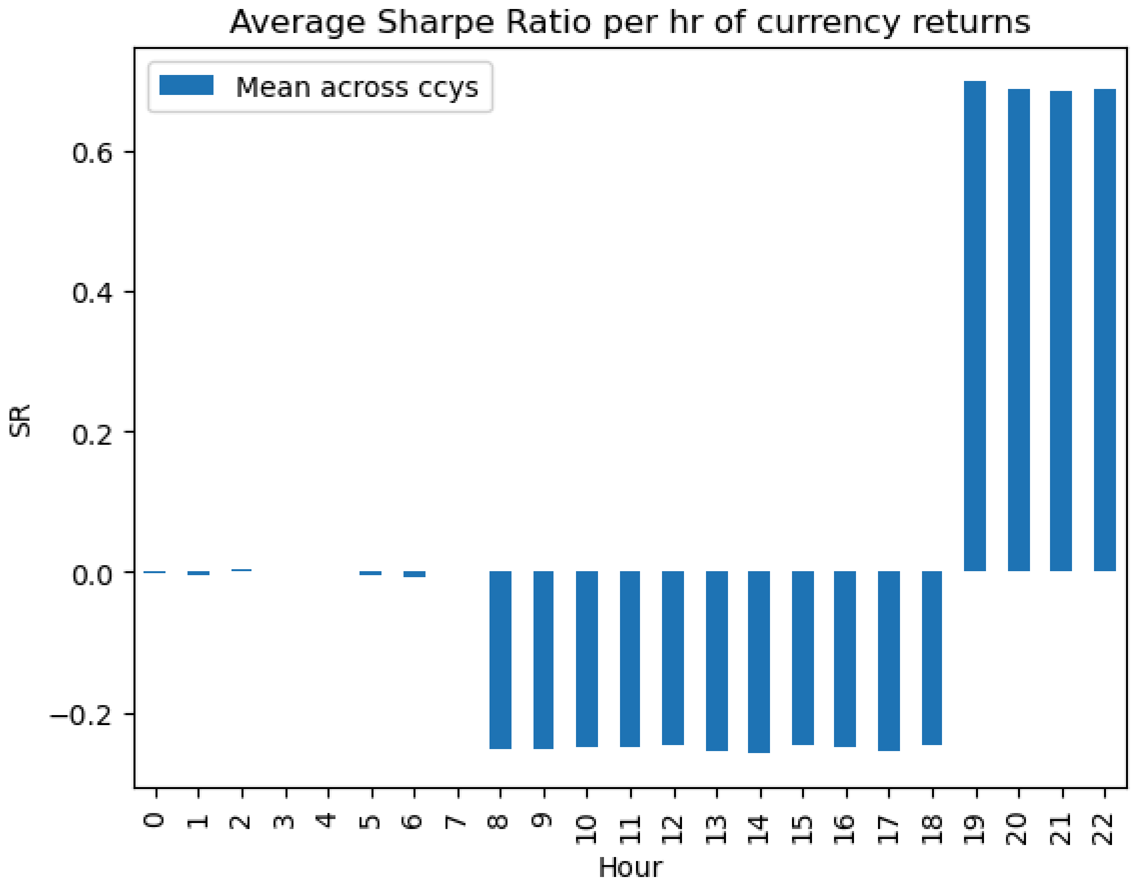
Out[4]:

| | Mean across ccys | ccy_0 | ccy_1 | ccy_2 | ccy_3 | ccy_4 | ccy_5 |
|---|---|---|---|---|---|---|---|
| **0** | -0.002727 | -0.009010 | -0.002773 | -0.001667 | 0.008520 | -0.013460 | -0.008562 |
| **1** | -0.004201 | 0.016948 | 0.005503 | -0.016098 | 0.006998 | -0.022551 | 0.002518 |
| **2** | 0.004774 | 0.007400 | 0.005335 | 0.002278 | 0.000477 | -0.002179 | 0.020264 |
| **3** | 0.002589 | 0.025606 | -0.015822 | -0.012193 | -0.007948 | 0.007407 | 0.018288 |
| **4** | 0.000174 | 0.008134 | 0.004585 | -0.025676 | -0.000355 | -0.003412 | 0.012034 |
| **5** | -0.003387 | -0.007962 | -0.007593 | 0.008957 | 0.002969 | 0.012104 | 0.009013 |
| **6** | -0.007973 | -0.018798 | -0.019660 | -0.017331 | -0.006858 | -0.017237 | 0.005827 |
| **7** | 0.002274 | 0.001719 | 0.010545 | -0.001453 | -0.005660 | -0.017125 | 0.000629 |
| **8** | -0.251807 | -0.246090 | -0.234039 | -0.262322 | -0.280371 | -0.244239 | -0.273487 |
| **9** | -0.252337 | -0.266577 | -0.248911 | -0.232696 | -0.245718 | -0.262392 | -0.239770 |
| **10** | -0.250571 | -0.239085 | -0.252693 | -0.263860 | -0.234147 | -0.280759 | -0.250814 |
| **11** | -0.247834 | -0.229445 | -0.235666 | -0.229604 | -0.258860 | -0.238653 | -0.253829 |
| **12** | -0.247344 | -0.249773 | -0.235564 | -0.243759 | -0.218736 | -0.250699 | -0.251635 |
| **13** | -0.255721 | -0.256978 | -0.246605 | -0.256242 | -0.262204 | -0.259890 | -0.251497 |
| **14** | -0.258240 | -0.277161 | -0.250951 | -0.268656 | -0.271595 | -0.237530 | -0.256760 |
| **15** | -0.247253 | -0.254114 | -0.245384 | -0.263417 | -0.248098 | -0.234768 | -0.232863 |
| **16** | -0.250358 | -0.246453 | -0.258343 | -0.242831 | -0.271654 | -0.251977 | -0.228921 |
| **17** | -0.255526 | -0.247959 | -0.260421 | -0.238963 | -0.255167 | -0.253458 | -0.275013 |
| **18** | -0.246449 | -0.233113 | -0.236651 | -0.238397 | -0.235648 | -0.244902 | -0.258047 |
| **19** | 0.698208 | 0.674556 | 0.703373 | 0.725717 | 0.703421 | 0.704096 | 0.699430 |
| **20** | 0.687376 | 0.693555 | 0.694557 | 0.701129 | 0.682010 | 0.683371 | 0.680865 |
| **21** | 0.683275 | 0.689586 | 0.671020 | 0.670406 | 0.704319 | 0.694198 | 0.704461 |
| **22** | 0.688551 | 0.666543 | 0.659663 | 0.667281 | 0.682800 | 0.707399 | 0.704052 |

In [5]:

```
sharpe_ratios_per_hr[['Mean across ccys']].plot(kind='bar')
plt.title('Average Sharpe Ratio per hr of currency returns')
plt.ylabel('SR')
plt.xlabel('Hour')
```

Out[5]: `Text(0.5, 0, 'Hour')`




# IV - Significance test

The article suggests a number of tests to identify if the returns measured are satistically significant or a product of randomness.

We will split our day into the 4 trading sessions described earlier and will measure each overall session return.

1) We will measure the mean return across session and perform an univariate t-tests, this will allow us to know if the mean is significantly different from 0.

2) We will perform a bivariate t-test and simultaneous Welch test across all our currencies to check if the means are significantly different across trading sessions.

Hence this will confirm that the patterns seen across the different sessions are statistically different

3) The article uses a GARCH(1,1) model. It takes interests in the mean equation and it then looks at the sign of the intercept and its satistical significance to confirm our findings regarding trading session returns.

### 4) Finally the article perform a regression of the session returns against the realised volatility of the returns.

### We will not investigate this further as it is simply outlining a trivial relationship from the definition of the variables.

### The realised volatility being defined in the article as the square root of the sum of the squared hourly returns within the session, it is by construction linked to the session returns. High returns will be correlated with high realised volatility

We gather all the results in one dictionary

In [6]:
```
results = do_and_visualize_analysis(data)
```

## 1) Mean Return (%) per session and ccy.

We display the mean return per session highlighted by their pvalues from the one sample t-test.

The one sample t-test tests the hypothesis H0 beta=0. If the p-value is below 0.05 it means that we have less than 5% chance of being wrong by rejecting H0. We can then conclude that the mean (beta) is significant

**Going forward all values significant at the 5% confidence level (p_value<0.05) will be displayed in a green cell vs a red cell for values not significant at the 5% level**

We see that Domestic session and LDN NY overlap exhibit significant negative returns while the US session exhibits significant positive returns.

The returns in Non US session are not significant

In [7]:
```
results['styled_mean_by_session']
```

Out[7]:

| | Domestic | LDN-NY | US | Non US |
|---|---|---|---|---|
| ccy_0 | -0.003283 | -0.001827 | 0.005032 | 0.000038 |
| ccy_1 | -0.000784 | -0.000461 | 0.001255 | -0.000010 |
| ccy_2 | -0.002044 | -0.001158 | 0.003220 | -0.000075 |
| ccy_3 | -0.000954 | -0.000543 | 0.001482 | 0.000001 |
| ccy_4 | -0.003252 | -0.001812 | 0.005146 | -0.000114 |
| ccy_5 | -0.002400 | -0.001336 | 0.003753 | 0.000077 |
| ccy_6 | -0.001316 | -0.000754 | 0.001997 | -0.000016 |
| ccy_7 | -0.002043 | -0.001140 | 0.003160 | -0.000045 |
| ccy_8 | -0.003540 | -0.002011 | 0.005473 | 0.000052 |
| ccy_9 | -0.001527 | -0.000901 | 0.002419 | 0.000007 |

## 2) Bivariate t-tests and simultaneous welch test

We will first normalize the returns by the length of the trading sessions before comparing them, otherwise the tests might lose their significance.

The Bivariate t-tests tests for each ccy the hypothesis that mean return of trading session i is equal to the mean return of the domestic trading session.

The Welch F-test tests the hypothesis that all mean returns across all trading sessions are equal.

Both tests had significant p-values, allowing us to reject the null hypothesis at the 5% confidence level, with the exception of the LDN-NY overlap session that we had already graphically identified as behaving similarly to the Domestic session.

This suggests that overall we are seeing three different behaviours/patterns across trading sessions.

In [8]:

```
results['styled_two_sample_ttest']
```

Out[8]:

| | LDN-NY | US | Non US |
|---|---|---|---|
| ccy_0 | -0.867455 | -121.715482 | -36.547188 |
| ccy_1 | 0.915915 | -118.339492 | -33.628075 |
| ccy_2 | -0.277146 | -122.810655 | -34.038338 |
| ccy_3 | -0.117188 | -122.239958 | -35.543842 |
| ccy_4 | -0.803930 | -121.739169 | -34.422305 |
| ccy_5 | -0.853847 | -121.914655 | -37.152796 |
| ccy_6 | 0.084037 | -122.338935 | -35.988960 |
| ccy_7 | -0.771808 | -118.939312 | -35.038113 |
| ccy_8 | -0.192968 | -123.364249 | -36.677881 |
| ccy_9 | 1.026810 | -118.663299 | -35.078206 |

In [9]:

```
results['styled_simultaneous_welch_test']
```

Out[9]:

| | 0 |
|---|---|
| ccy_0 | 6167.121142 |
| ccy_1 | 6035.922994 |
| ccy_2 | 6258.082146 |
| ccy_3 | 6199.689001 |
| ccy_4 | 6285.060534 |
| ccy_5 | 6321.524019 |
| ccy_6 | 6332.457196 |
| ccy_7 | 6099.420316 |
| ccy_8 | 6332.368225 |
| ccy_9 | 6124.071767 |

## 3) GARCH (1,1)

Intraday returns often exhibit heteroscedasticity and autocorrelation. For this reasons t-tests and F-tests which assume uncorrelated error and homoscedasticity might lose some of their power.

As a sanity check of our conclusions, we use the GARCH(1,1) model presented in the article with lag=2. GARCH allows to model heteroscedasticity and autocorrelation in the variance of errors.

We estimate the GARCH coefficients through Maximum Likelihood Estimation, using arch package.

We are especially interested in the constant mean from the GARCH mean equation for each session. We see that the constant mean per session is consistant with our previous finding, in terms of sign and significance.

In [10]:
```
# Jupyter Notebook seems to have some compatibility issues with arch pack
# So we will load our results csv in this cell. The results were computed
garch_means = pd.read_csv('./garch_means.csv',index_col=[0])
garch_pvals = pd.read_csv('./garch_pvals.csv',index_col=[0])
styled_means = style_df_based_on_pval(garch_means,garch_pvals)
styled_means
```

Out[10]:

| | Domestic | LDN-NY | US | Non US |
|---|---|---|---|---|
| ccy_0 | -0.001906 | -0.001091 | 0.002972 | 0.000021 |
| ccy_1 | -0.002858 | -0.001620 | 0.004263 | -0.000050 |
| ccy_2 | -0.000727 | -0.000402 | 0.001119 | 0.000006 |
| ccy_3 | -0.001562 | -0.000906 | 0.002487 | -0.000048 |
| ccy_4 | -0.001138 | -0.000662 | 0.001809 | -0.000019 |
| ccy_5 | -0.000957 | -0.000533 | 0.001540 | 0.000015 |
| ccy_6 | -0.001233 | -0.000713 | 0.001969 | -0.000007 |
| ccy_7 | -0.001680 | -0.000976 | 0.002636 | -0.000004 |
| ccy_8 | -0.001847 | -0.000987 | 0.002887 | 0.000007 |
| ccy_9 | -0.002208 | -0.001317 | 0.003539 | -0.000030 |

# V) Robustness Check through bootstrapping

To check the robustness of the results and make sure they are not sample dependant, we will perform bootstrapping.

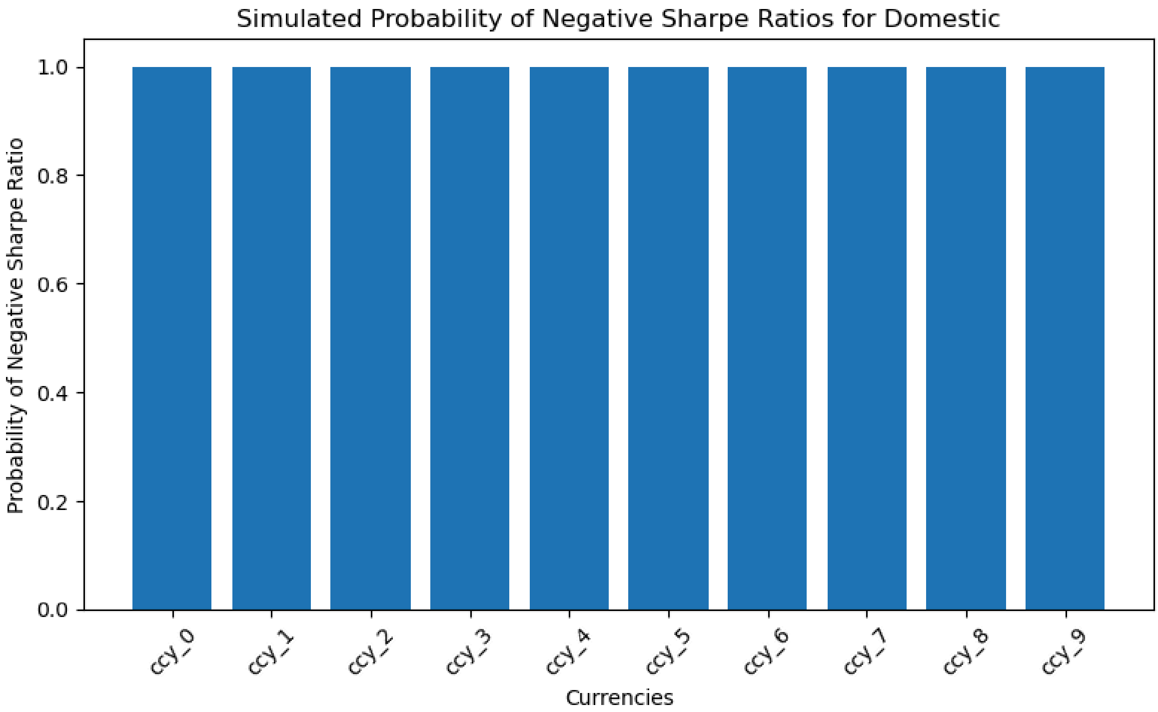
Over 1000 iterations we sample randomly a subset of 1000 returns per trading sessions.

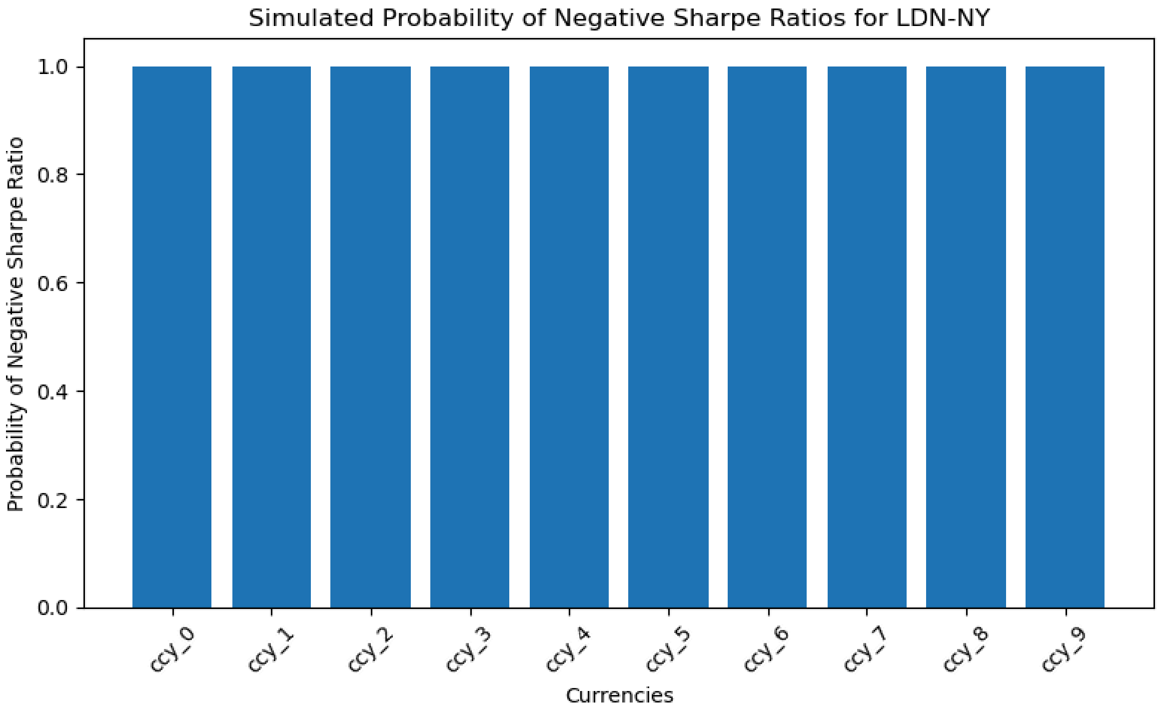
1) We infer the empirical probability of having negative sharpe ratio per trading session and it is consistent with our findings: probability close to 1 (this is expected since it is simulated data) in Domestic and overlap session, probability of 0 in US session, probability around 50% in Non US session.

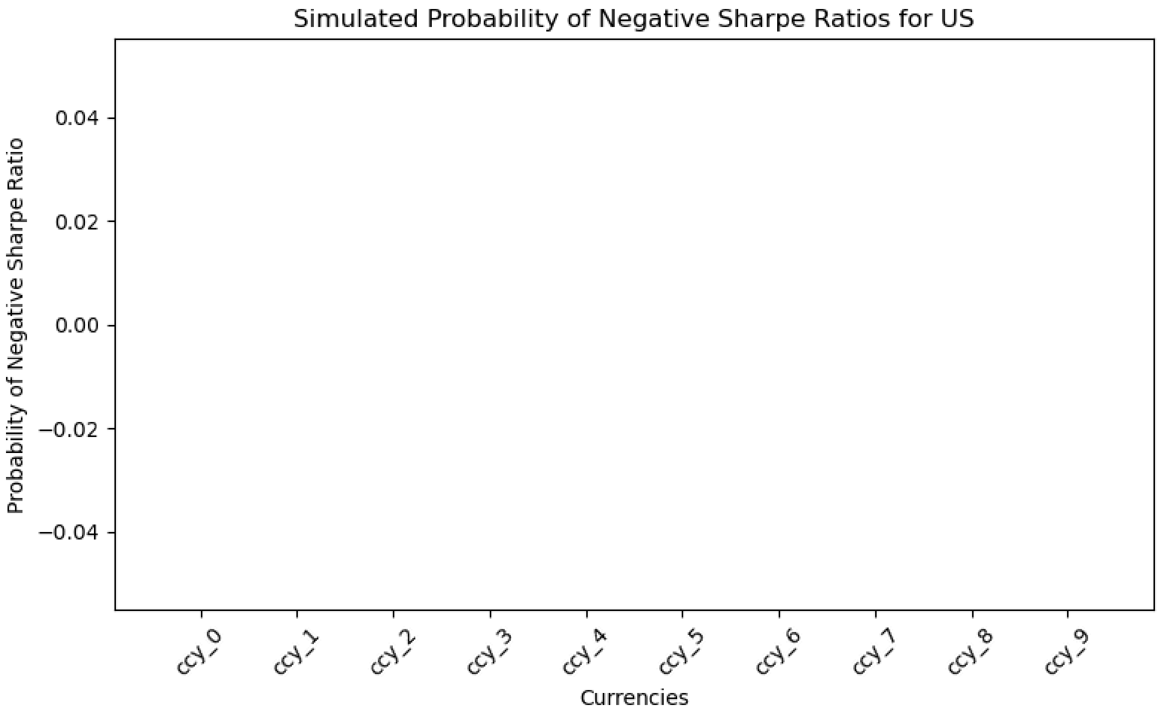
2) We plot the distribution of Sharpe Ratios across trading sessions and this confirms: a consistent positive sharpe ratio in US Session, a consistent negative sharpe ratio in Domestic session and overlap session, a random pattern in Non US Session.

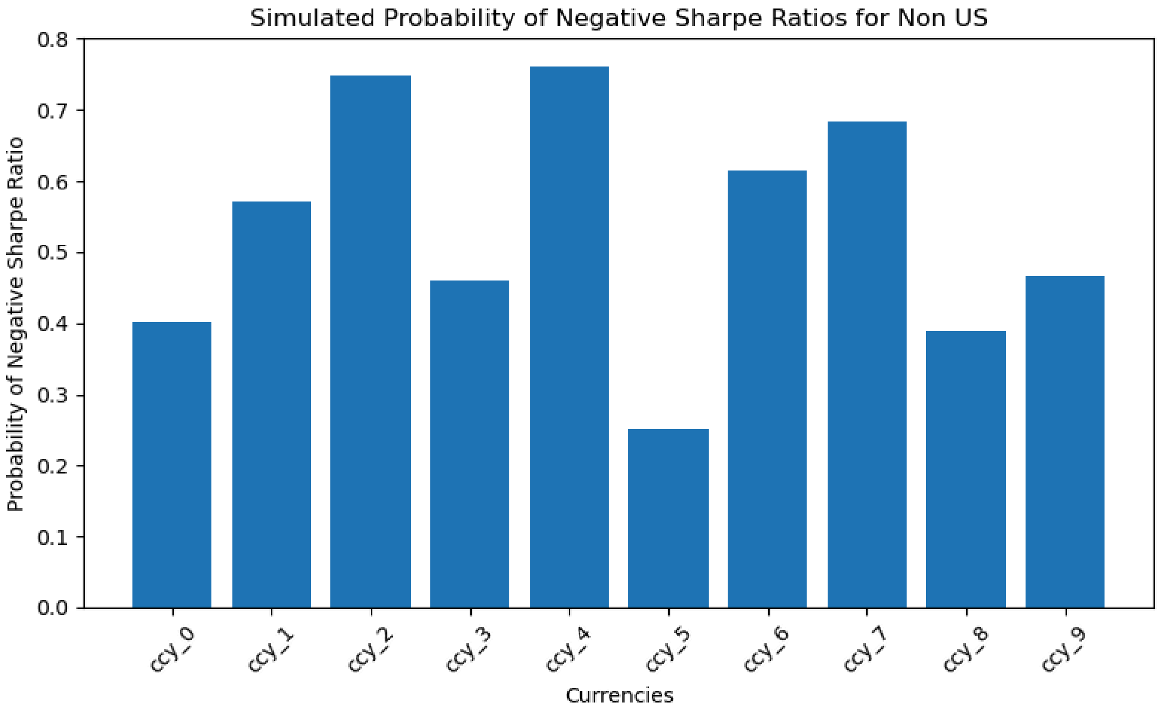
In [11]:
```
sessions = partition_returns_into_session(returns)
bootstrap_analysis(sessions, n_iterations=1000, sample_size=1000)
```

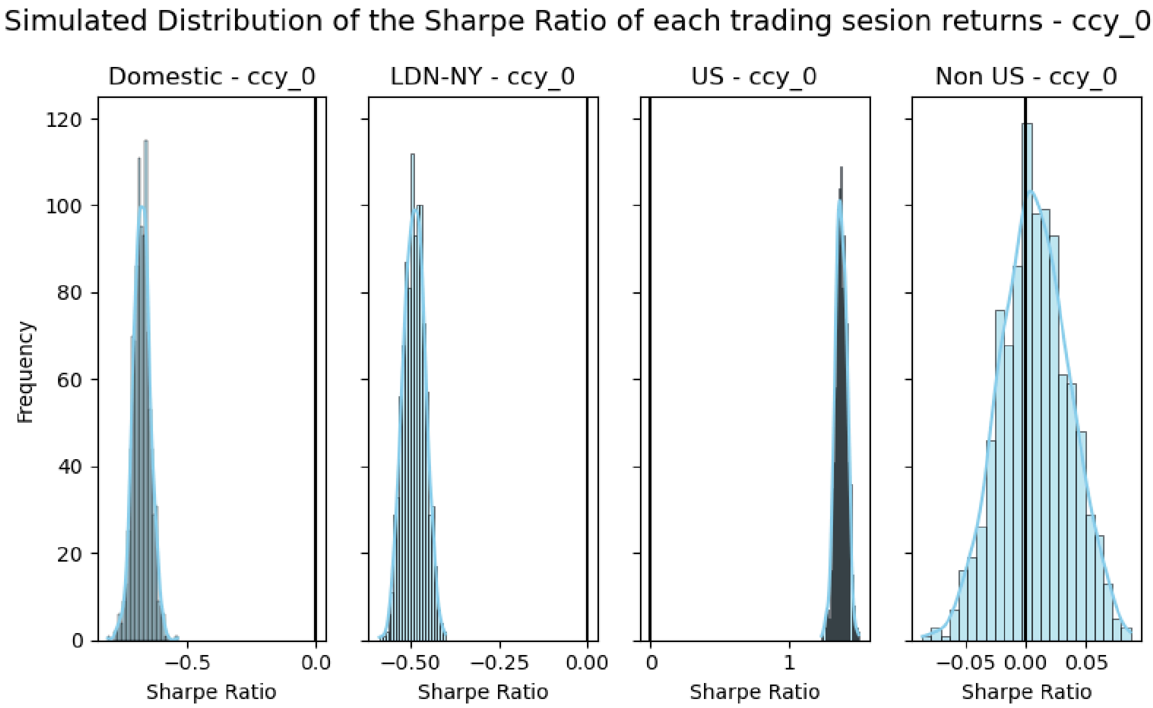
```
100%|████████████████████████████████████████████████████████████████████████
████████████████████████████████████| 1000/1000 [00:01<00:00, 690.17it/s]
```

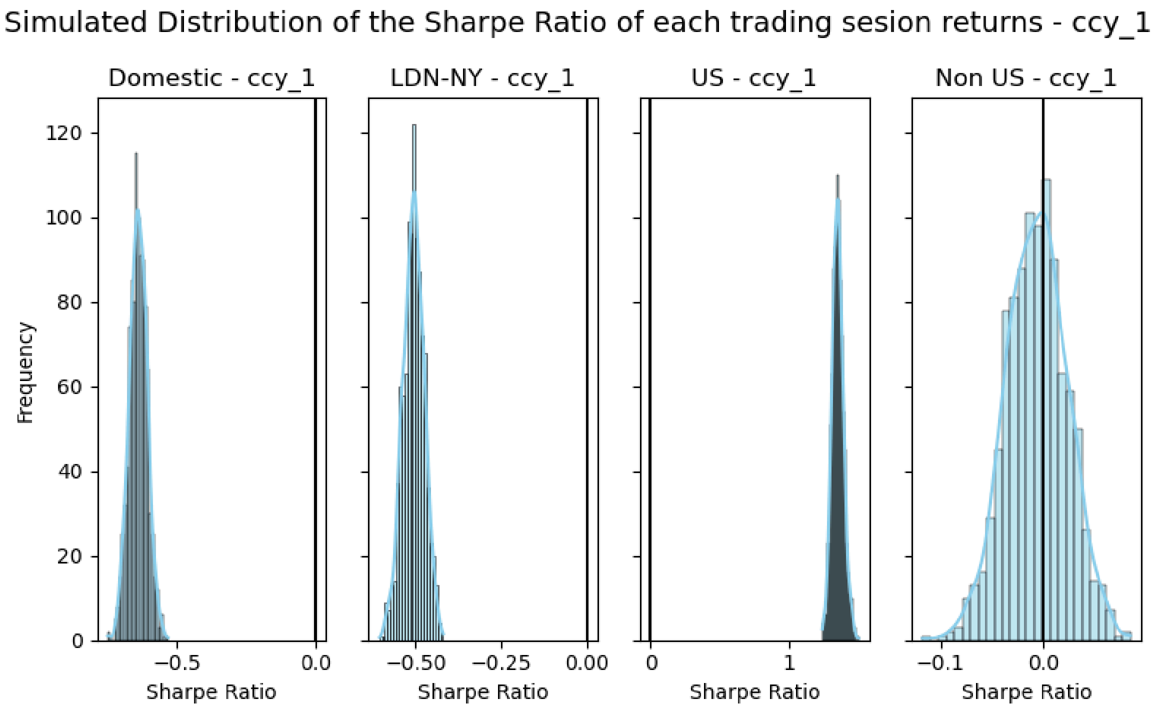

Simulated Probability of Negative Sharpe Ratios for Domestic
Probability of Negative Sharpe Ratio
1.0
0.8
0.6
0.4
0.2
0.0
ccy_0
ccy_1
ccy_2
ccy_3
ccy_4
ccy_5
ccy_6
ccy_7
ccy_8
ccy_9
Currencies

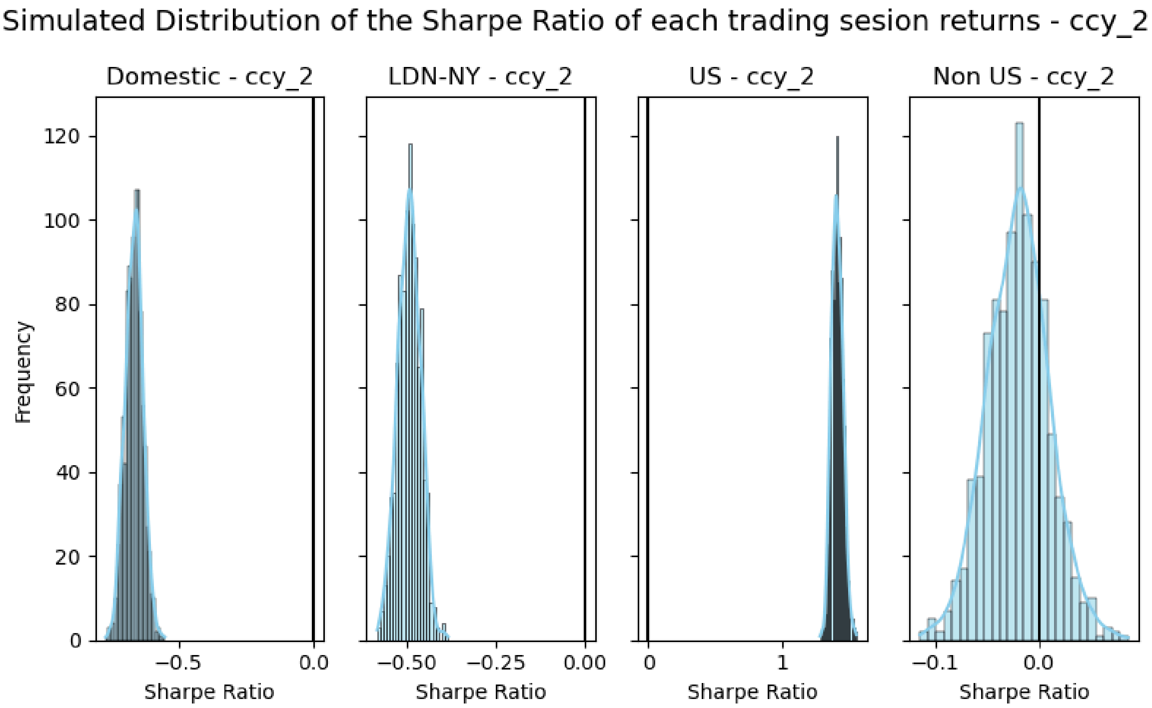

Simulated Probability of Negative Sharpe Ratios for LDN-NY
Probability of Negative Sharpe Ratio
1.0
0.8
0.6
0.4
0.2
0.0
ccy_0
ccy_1
ccy_2
ccy_3
ccy_4
ccy_5
ccy_6
ccy_7
ccy_8
ccy_9
Currencies

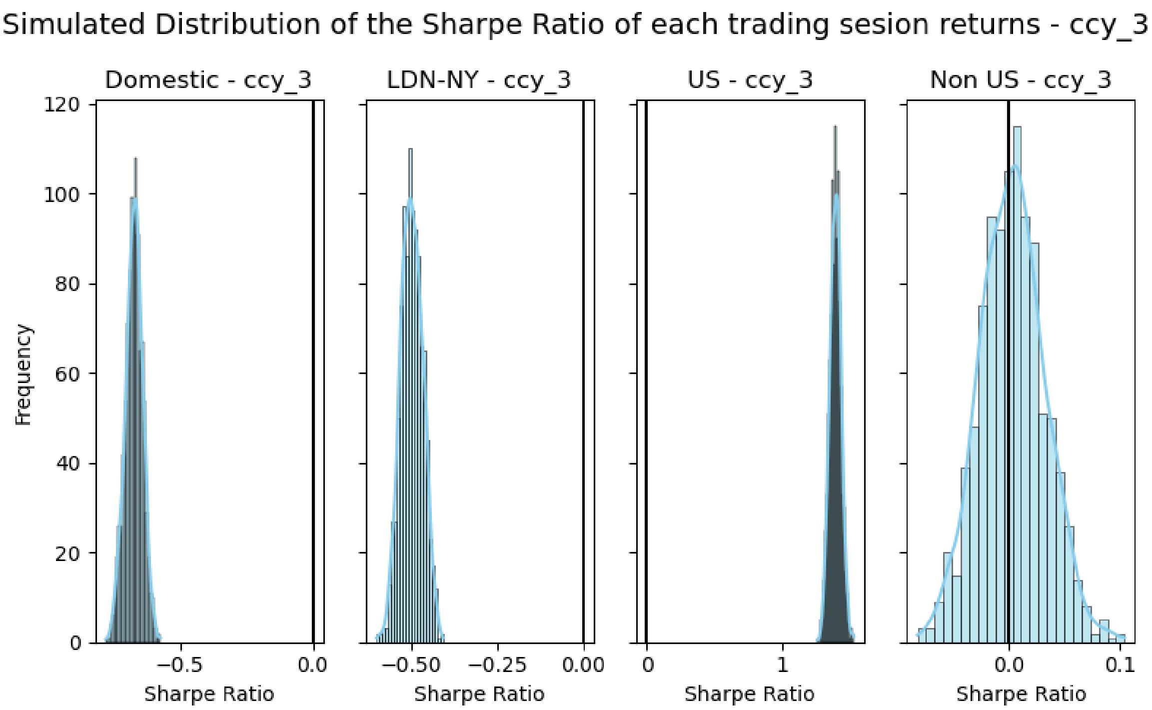

Simulated Probability of Negative Sharpe Ratios for US
Probability of Negative Sharpe Ratio
0.04
0.02
0.00
−0.02
−0.04
ccy_0
ccy_1
ccy_2
ccy_3
ccy_4
ccy_5
ccy_6
ccy_7
ccy_8
ccy_9
Currencies

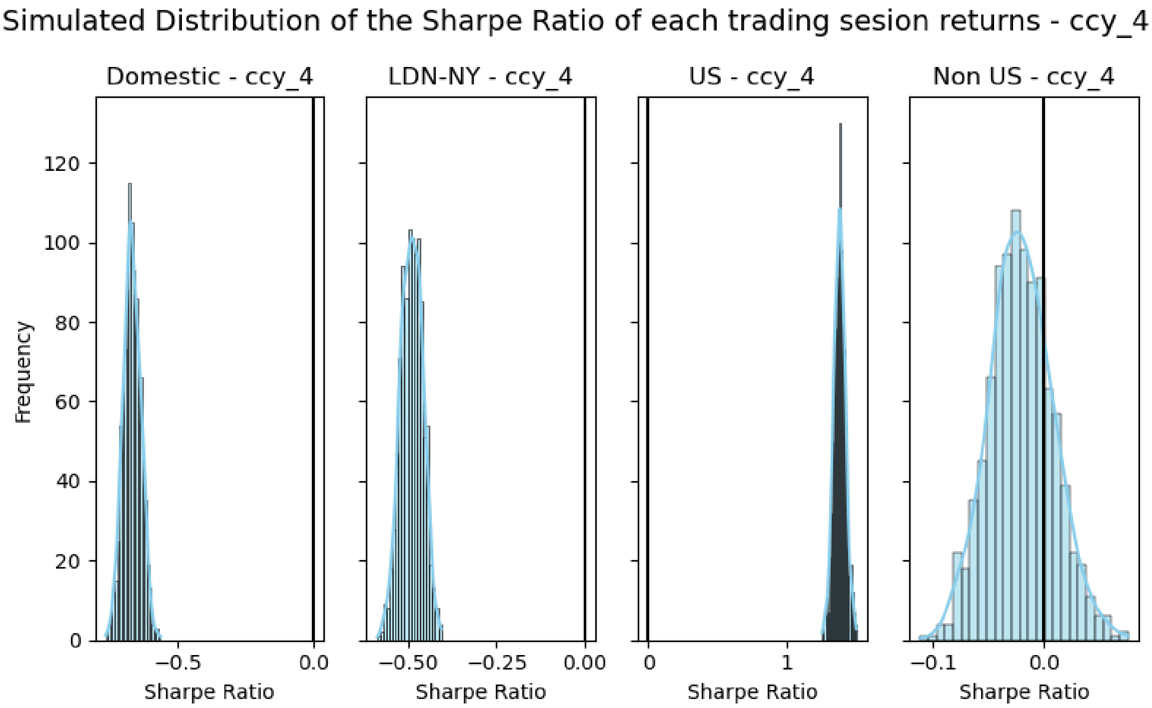

Simulated Probability of Negative Sharpe Ratios for Non US
Probability of Negative Sharpe Ratio
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
ccy_0
ccy_1
ccy_2
ccy_3
ccy_4
ccy_5
ccy_6
ccy_7
ccy_8
ccy_9
Currencies

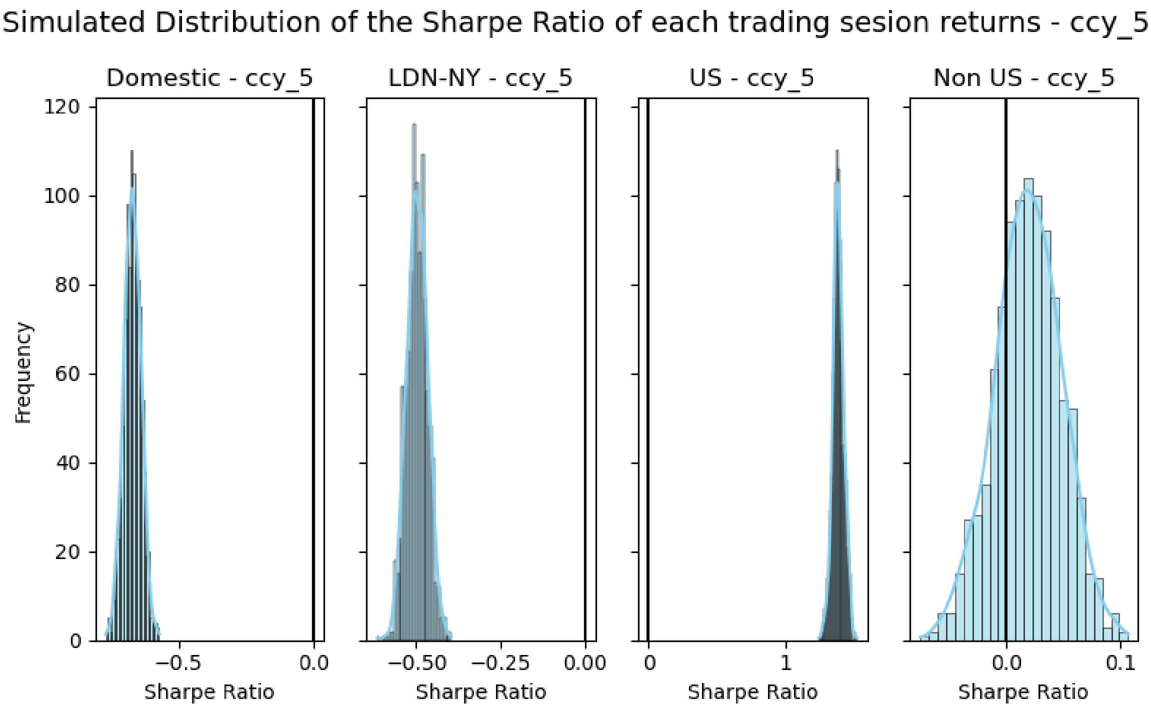

Simulated Distribution of the Sharpe Ratio of each trading sesion returns - ccy_0
Domestic - ccy_0
LDN-NY - ccy_0
US - ccy_0
Non US - ccy_0
Frequency
Sharpe Ratio

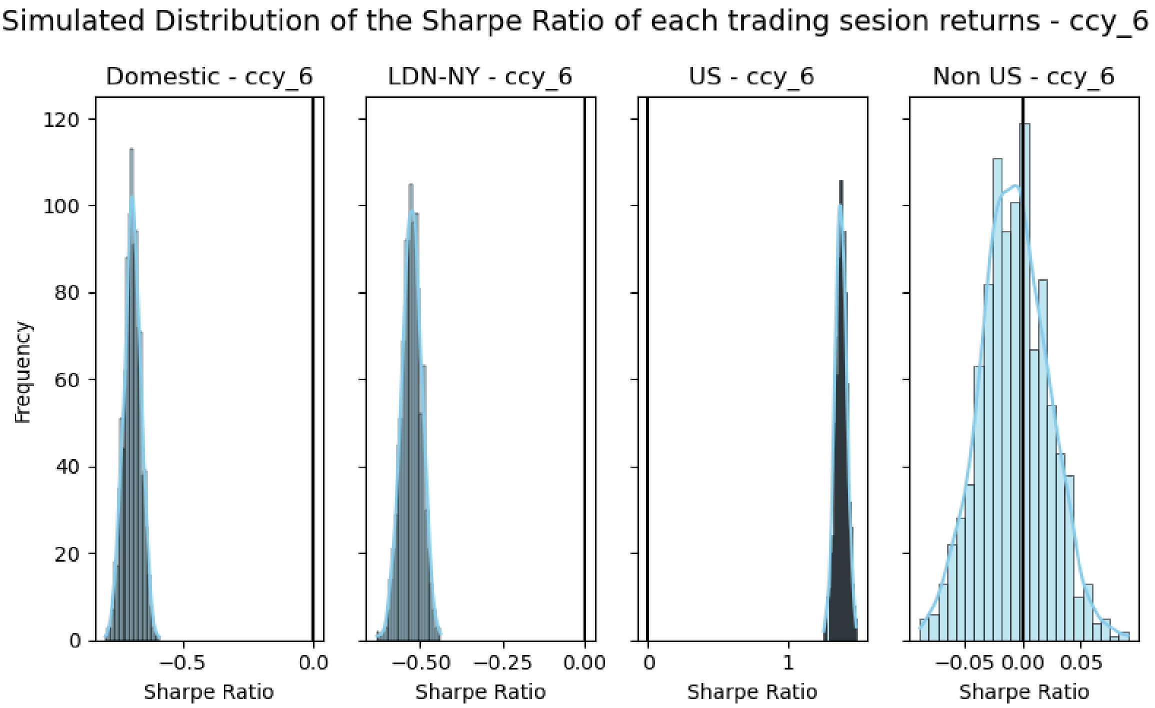

Simulated Distribution of the Sharpe Ratio of each trading sesion returns - ccy_1
Domestic - ccy_1
LDN-NY - ccy_1
US - ccy_1
Non US - ccy_1
Frequency
Sharpe Ratio

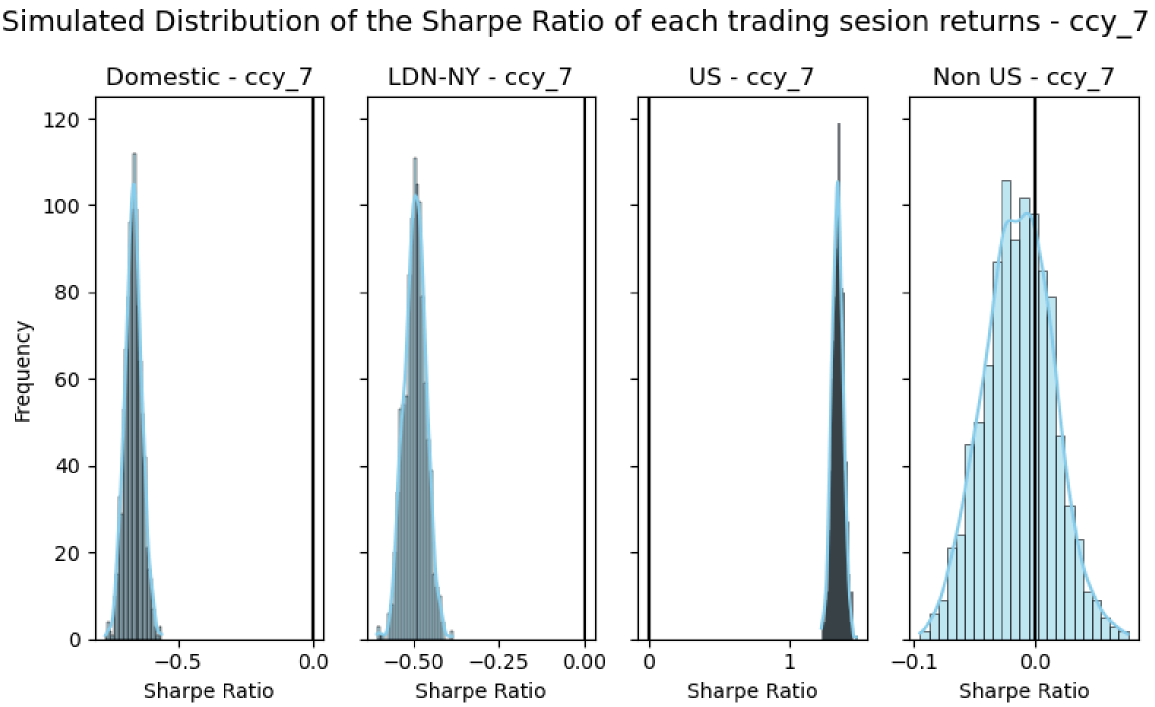

Simulated Distribution of the Sharpe Ratio of each trading sesion returns - ccy_2
Domestic - ccy_2
LDN-NY - ccy_2
US - ccy_2
Non US - ccy_2
Frequency
0
20
40
60
80
100
120
−0.5
0.0
−0.50
−0.25
0.00
0
1
−0.1
0.0
Sharpe Ratio
Sharpe Ratio
Sharpe Ratio
Sharpe Ratio

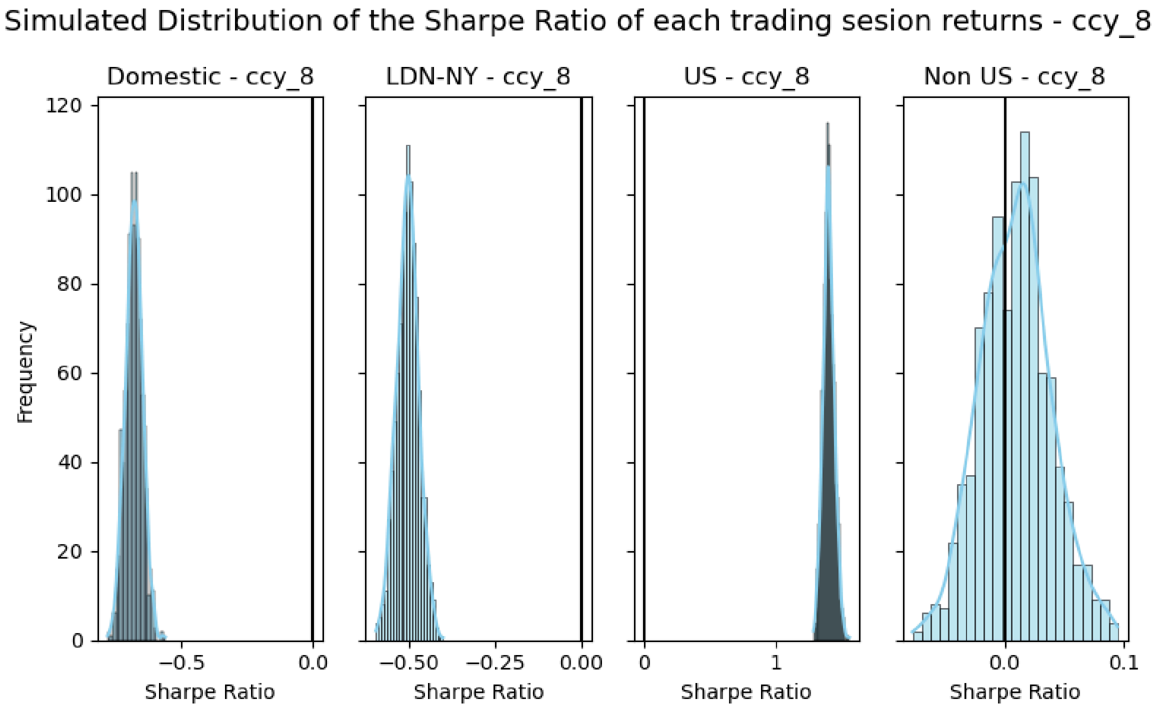

Simulated Distribution of the Sharpe Ratio of each trading sesion returns - ccy_3
Domestic - ccy_3
LDN-NY - ccy_3
US - ccy_3
Non US - ccy_3
Frequency
0
20
40
60
80
100
120
−0.5
0.0
−0.50
−0.25
0.00
0
1
0.0
0.1
Sharpe Ratio
Sharpe Ratio
Sharpe Ratio
Sharpe Ratio

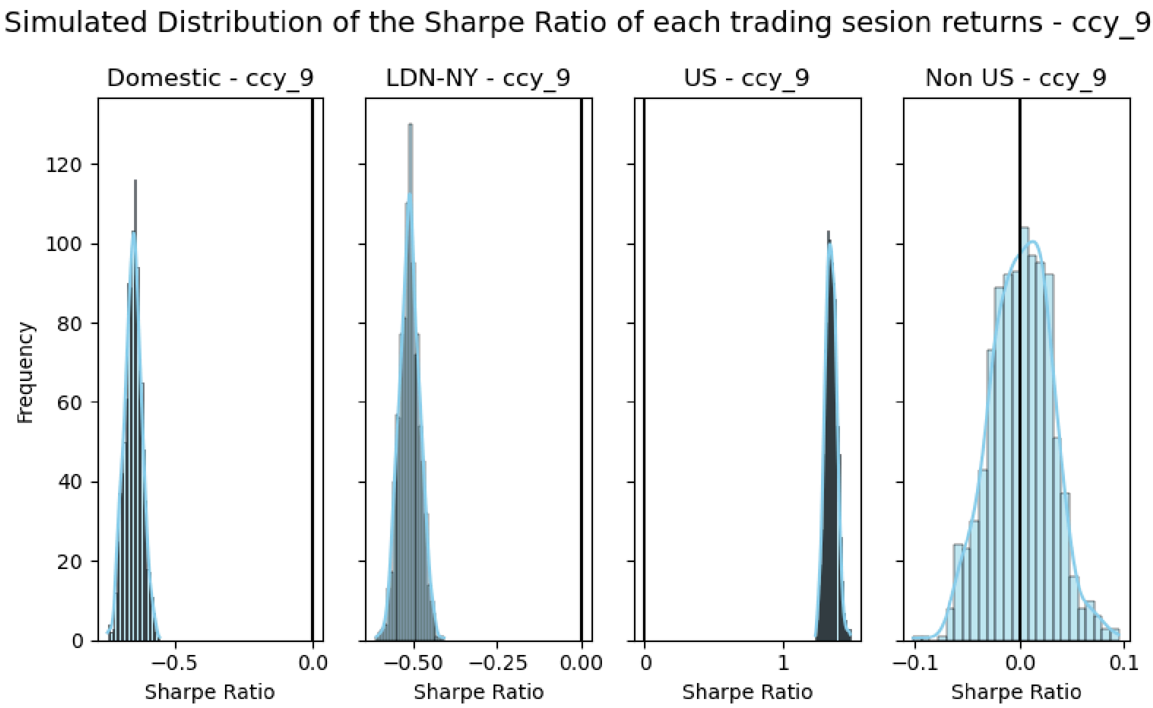

Simulated Distribution of the Sharpe Ratio of each trading sesion returns - ccy_4
Domestic - ccy_4
LDN-NY - ccy_4
US - ccy_4
Non US - ccy_4
Frequency
120
100
80
60
40
20
0
−0.5
0.0
−0.50
−0.25
0.00
0
1
−0.1
0.0
Sharpe Ratio


Simulated Distribution of the Sharpe Ratio of each trading sesion returns - ccy_5
Domestic - ccy_5
LDN-NY - ccy_5
US - ccy_5
Non US - ccy_5
Frequency
120
100
80
60
40
20
0
−0.5
0.0
−0.50
−0.25
0.00
0
1
0.0
0.1
Sharpe Ratio


Simulated Distribution of the Sharpe Ratio of each trading sesion returns - ccy_6
Domestic - ccy_6
LDN-NY - ccy_6
US - ccy_6
Non US - ccy_6
Frequency
120
100
80
60
40
20
0
−0.5
0.0
−0.50
−0.25
0.00
0
1
−0.05
0.00
0.05
Sharpe Ratio


Simulated Distribution of the Sharpe Ratio of each trading sesion returns - ccy_7
Domestic - ccy_7
LDN-NY - ccy_7
US - ccy_7
Non US - ccy_7
Frequency
120
100
80
60
40
20
0
−0.5
0.0
−0.50
−0.25
0.00
0
1
−0.1
0.0
Sharpe Ratio




# VI) Final Word

We have given strong evidence that:

1)home currencies tend to depreciate during domestic trading hours and the LDN-NY overlap session.

2) home currencies tend to appreciate during US session.

3) home currencies don't show any special pattern in Non US session.

One could profit from this effect by trading this seasonal pattern (if and only if the magnitude of this effect is greater than transaction costs), or one might adjust their execution strategy based on those patterns.

For a real world analysis it would have been interesting to analyze the volume profile of currencies during those trading sessions, to see if the patterns are exacerbated in low liquidity periods, which might not be easily tradeable. It would also have been interesting to see if this effect is persistent in the futures/forwards markets vs spot.

In [ ]: