

Sticky prices versus sticky information: Does it matter for policy paradoxes?^{*}

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Abstract

This paper shows that under a strict inflation targeting regime, the government spending multiplier at the zero lower bound (ZLB) is larger under sticky information than under sticky prices. Similarly, well known paradoxes, e.g., the paradox of toil and the paradox of flexibility become more severe under sticky information. For the case of sticky information it is important to assume that the fiscal policy intervention coincides with the duration of zero interest rates, while such a distinction is less important for sticky prices. We unify and clarify results that may appear to contradict each other in the literature.

JEL Codes: E52; E62; E63

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1 Introduction

A number of papers have documented that fiscal policy is extremely effective to increase demand at the ZLB. In particular the classic government spending multiplier is greater than one, while under normal circumstances it is not, for then monetary policy can do the job via interest rate cuts. Examples include Eggertsson (2011), Christiano, Eichenbaum and Rebelo (2009), Woodford (2011). The literature has also uncovered peculiar paradoxes, such as the paradox of toil (that is, distortionary labor tax cuts are contractionary, see Eggertsson (2010)) and the paradox of flexibility (that is, for a given demand shock, greater price flexibility is more contractionary, see Bhattachari, Eggertsson and Schoenle (2014) for an overview of this literature and some general results). These results can be interpreted either as a serious challenge to the conventional wisdom or reflect some fundamental flaws of the New Keynesian framework. These results are, however, derived under the assumption that prices are sticky, as in Calvo (1983).

It has long been recognized that the Calvo model of price setting has many peculiar features. This led researchers to explore alternatives, such as information frictions. One of the most prominent proposal to replace the New Keynesian Phillips curve based upon Calvo prices is the assumption of sticky information, proposed by Mankiw and Reis (2001). According to their hypothesis, firms adjust their prices slowly because they do not continuously update their information set. Mankiw and Reis argued that this alternative assumption helps explain the data better along certain dimensions. A very natural question, in the light of the radical findings documented in the Calvo model at the ZLB, is if these results carry over to a setting where information rigidities are assumed instead of sticky prices. The main conclusion of this paper is that under a strict inflation targeting rule, the answer is yes.

In an important and intriguing recent paper, Kiley (2016), documents experiments in which the fiscal policy results of the Calvo model are overturned upon assuming informational frictions as in Mankiw and Reis. The thought experiment Kiley conducts is as follows: Suppose the Central Bank follows an interest rate peg for 100 periods. What happens if the government increases spending for 1, 2, ... upto 25 periods? What happens if taxes are increased for 1 to 25 periods? Kiley documents that while under Calvo prices the predictions are in line with the existing literature at the ZLB, the predictions are different under informational frictions. In particular, the government spending multiplier is small, the tax multiplier changes sign and the paradox of flexibility disappears. Kiley's experiment is referred to in this paper as an interest rate peg experiment (PEG-EX). Kiley interprets these findings as suggesting that the sticky-information model is free from policy paradoxes, and thus to be favored over the Calvo model. Here, instead, we argue that these findings are an artifact of the thought experiment considered. The paradoxes

in the sticky information framework in fact get even stronger in policy experiments that correspond more closely to those considered in the existing literature.

This paper compares sticky prices and sticky information doing a different experiment from the PEG-EX. This experiment is identical to the one conducted in Eggertsson (2011), Christiano, Eichenbaum and Rebelo (2009), Eggertsson (2010), Woodford (2011). The ZLB is binding due to exogenous fundamental shocks. Once the shocks are over, the policy is given by a strict inflation target (which is missed for the duration of the shocks, due to the ZLB). This experiment is referred to as ZLB experiment (ZLB-EX). The paper documents that the results derived in the literature under sticky prices in the ZLB-EX are even more extreme if sticky prices are replaced with sticky information, which is the opposite of Kiley's result. The government spending multiplier becomes larger, and the paradox of toil and flexibility become more pronounced.

While this may seem to contradict Kiley's findings, it does not. Instead it clarifies that Kiley's PEG-EX is a fundamentally different experiment than done in the existing literature. What is particularly subtle – and interesting – about the comparison, and likely to trigger confusion, is that under sticky prices the ZLB-EX and PEG-EX lead to exactly the same result. It is only when assuming sticky information that the results of the ZLB-EX and the PEG-EX are different. This does not have anything to do with the nature of the nominal frictions. Instead, it is a consequence of the fact that the sticky-information model has infinite number of endogenous state variables. Meanwhile the Calvo model is purely forward looking. The presence of endogenous state variables in the sticky-information model implies that comparing the reaction of an economy assuming an exogenous interest rate peg, versus the reaction of the economy if the central bank's interest rate policy is bounded by zero due to fundamental shocks and fiscal policy is in direct response to this constraint, leads to very different results. The same does not apply for perfectly forward looking systems like the Calvo model of price stickiness.

This paper first shows analytical examples that clarify the intuition behind these findings. It then moves to numerical examples that replicate Kiley's results. These examples confirm that Kiley's results are driven by the difference in experiments being conducted rather than anything fundamental about the assumption of price stickiness. In a calibration of the two models chosen produce 10% drop in output and 2% annual deflation on impact, government spending is nearly thrice as expansionary and tax cuts are six times as contractionary under SI than SP in the ZLB-EX with a strict inflation target.

Finally, this paper shows that the distinction between PEG-EX and ZLB-EX is important even under non-strict inflation targeting regimes, as the two remain fundamentally different experiments, and one cannot be treated as a sub-case of the other. Under a general targeting rule, where the central bank stabilizes inflation as well as output deviations

from target, the fiscal multipliers in the SI model under ZLB-EX depend on the relative weight on output stabilization (the larger the weight on output stabilization, the smaller the multipliers), while those under PEG-EX are independent of the relative weight on output stabilization.¹

Arguably, the ZLB-EX is more economically relevant than PEG-EX. It seems of more limited economic interest – at least in the context of the crisis that started in 2008 – to explore the behavior of New Keynesian models if the short-term interest rate is temporarily pegged for no apparent reasons. Instead, the most economically interesting experiment appears to be when the interest rate is pegged due to the fact that the ZLB is binding on account of a fundamental recessionary shock that prevents the central bank from achieving its objective of stabilizing inflation and output.²

As a final note, let us observe that the reason fiscal policy has such a large effect in our experiments, and also at heart of the policy paradoxes, is that monetary policy is set in a sub-optimal way. For example, if monetary policy is conducted as in Eggertsson and Woodford (2003), output and inflation are almost entirely stabilized due to history dependent monetary policy (i.e. committing to lower future real interest rate). This, then, leaves much less room for fiscal policy to have a large effect on output.

2 Alternative models of nominal rigidities

The most commonly used model of nominal rigidities is based on stickiness of prices (see for example, Woodford (2003)). An alternative model of nominal rigidities is based on stickiness of information (Mankiw and Reis (2001)). In the former model, only a fraction of the firms get to reset their prices every period, while in the latter model, all firms get to reset their prices every period but only a fraction of them get to update their information sets in any given period. Both models can be described by a system of 3 equations common in the monetary economics literature - IS curve, AS curve and a policy rule.

¹ Multipliers in the SP model are independent of the relative weight on output stabilization under both ZLB-EX and PEG-EX in our setup.

² We contemplate another experiment (Coordinated PEG-EX) in which the duration of the peg is coordinated with the duration of fiscal policy (details in Appendix D). The results of this experiment are qualitatively similar to that of the ZLB-EX. These coordinated pegs have been used by other papers analyzing the effects of fiscal policy as well (see Carlstrom, Fuerst and Paustian (2014)). Exogenous interest rate pegs have been used in McKay, Nakamura and Steinsson (2016), Carlstrom, Fuerst and Paustian (2012) to analyze the forward guidance puzzle, but only in the context of forward-looking models. As we note, in perfectly forward-looking sticky-price models, it does not matter whether one assumes a passive monetary policy due to an exogenous peg or binding ZLB resulting from a fundamental shock. However, it is important to distinguish between Kiley's proposed PEG-EX and Coordinated PEG-EX in the context of sticky-information models as we will see.

In the Calvo model, firms account for the possibility of not being able to reset prices in future periods. Hence, their optimal price decision depends on their current marginal cost as well as current expectation of future marginal costs. On the other hand, in the Mankiw and Reis model, each firm gets to reset its price in every period, but firms that do not get the information update base their price decision on past expectations of current marginal cost. This introduces infinite lags into the model as firms set prices based on different vintages of information.

All of the results in this paper depend on the presence of these lagged variables in the sticky-information framework. But there is nothing special about sticky "information" versus sticky "prices". In fact, the SI model could be interpreted as an overlapping Taylor contract model. Similarly, Calvo originally interpreted the Calvo probability as an information friction rather than referring to sticky prices. The models are entirely standard, accordingly, we only report them in their log-linear form, but define all composite parameters, in terms of the underlying deep parameters in Appendix C.

Let us denote every variable x in deviation from its steady state as \hat{x} , then we can define the following approximate equilibria in the two models:

2.1 Sticky-price approximate equilibrium

A sticky-price approximate equilibrium, which is accurate up to a first order, is a collection of stochastic process for output, inflation and the short-term interest rate, $\{\hat{y}_t, \hat{\pi}_t, \hat{i}_t\}$, that solve equations (1) and (2) given (i) a path for taxes and government spending, $\{\hat{\tau}_t, \hat{g}_t\}$, determined by fiscal policy, (ii) exogenous shocks corresponding to the efficient rate of interest $\{\hat{r}_t^e\}$, and (iii) a specification of monetary policy.³

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \tilde{\sigma}^{-1} (\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{r}_t^e) + (\hat{g}_t - \mathbb{E}_t \hat{g}_{t+1}) \quad (1)$$

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa_y \hat{y}_t - \kappa_g \hat{g}_t + \kappa_\tau \hat{\tau}_t \quad (2)$$

Here, $\tilde{\sigma}, \kappa_y, \kappa_g, \kappa_\tau > 0$, $0 < \beta \leq 1$ and $\kappa_y \geq \kappa_g$.

2.2 Sticky-information approximate equilibrium

A sticky-information approximate equilibrium, which is accurate up to a first order, is a collection of stochastic process for $\{\hat{y}_t, \hat{\pi}_t, \hat{i}_t\}$ that solve equations (3) and (4) given

³Implicitly it is assumed here that there are lump-sum taxes that adjust to clear the government budget constraint, so that fiscal policy is "Ricardian", see Eggertsson (2011) for further discussion.

(i) a path for $\{\hat{\tau}_t, \hat{g}_t\}$ determined by fiscal policy, (ii) exogenous shocks $\{\hat{r}_t^e\}$, and (iii) specification of monetary policy.

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \tilde{\sigma}^{-1} (\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{r}_t^e) + (\hat{g}_t - \mathbb{E}_t \hat{g}_{t+1}) \quad (3)$$

$$\begin{aligned} \hat{\pi}_t &= \frac{\lambda}{1-\lambda} (\tilde{\kappa}_y \hat{y}_t - \tilde{\kappa}_g \hat{g}_t + \tilde{\kappa}_\tau \hat{\tau}_t) \\ &\quad + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \mathbb{E}_{t-1-j} (\hat{\pi}_t + [\tilde{\kappa}_y (\hat{y}_t - \hat{y}_{t-1}) - \tilde{\kappa}_g (\hat{g}_t - \hat{g}_{t-1}) + \tilde{\kappa}_\tau (\hat{\tau}_t - \hat{\tau}_{t-1})]) \end{aligned} \quad (4)$$

Here, $\tilde{\sigma}, \tilde{\kappa}_y, \tilde{\kappa}_g, \tilde{\kappa}_\tau > 0$, $0 < \beta \leq 1$ and $\tilde{\kappa}_y \geq \tilde{\kappa}_g$, and λ is the period-probability of updating information.

3 Analytical example

This section compares the predictions of sticky-price and sticky-information models under the two different experiments described in the introduction - ZLB-EX versus PEG-EX. The results are first summarized and then explained further using simple graphs.

3.1 ZLB Experiment (ZLB-EX)

The ZLB experiment makes the following assumptions:

A1.(Shock): $\hat{r}_1^e < 0$, $\hat{i}_1 = \frac{-\bar{i}}{1+\bar{i}}$, $\hat{r}_t^e = 0$ for $t > 1$

A2.(Fiscal policy): $(\hat{g}_1, \hat{\tau}_1) = (\hat{g}_s, \hat{\tau}_s)$, $(\hat{g}_2, \hat{\tau}_2) = (0, 0)$

A3.(Monetary policy): $\hat{\pi}_t = 0$ for $t > 1$

A4.(Perfect foresight⁴): $\mathbb{E}_t \hat{y}_{t+1} = \hat{y}_{t+1}$, $\mathbb{E}_t \hat{\pi}_{t+1} = \hat{\pi}_{t+1} \quad \forall t$

Proposition 1. (ZLB-EX) Under the assumptions of ZLB-EX, the sticky price and sticky information models have the following solutions:

⁴In the sticky information model the assumption of perfect foresight applies only to those agents that update their information set from that time on.

Sticky-price model:

$$\begin{aligned}
\hat{y}_1 &= \tilde{\sigma}^{-1} \left[\frac{\bar{i}}{1 + \bar{i}} + \hat{r}_1^e \right] + \hat{g}_1 \\
\hat{\pi}_1 &= (\kappa_y - \kappa_g) \hat{g}_1 + \kappa_y \tilde{\sigma}^{-1} \left[\frac{\bar{i}}{1 + \bar{i}} + \hat{r}_1^e \right] + \kappa_\tau \hat{\tau}_1 \\
\hat{i}_1 &= \frac{-\bar{i}}{1 + \bar{i}} \\
\hat{y}_2 &= \hat{\pi}_2 = \hat{i}_2 = 0 \\
\hat{y}_t &= \hat{\pi}_t = \hat{i}_t = 0 \quad \forall t > 2
\end{aligned} \tag{5}$$

Sticky-information model:

$$\begin{aligned}
\hat{y}_1 &= \gamma_1 \hat{g}_1 + \gamma_2 \hat{\tau}_1 + (2 - \lambda) \tilde{\sigma}^{-1} \left[\frac{\bar{i}}{1 + \bar{i}} + \hat{r}_1^e \right] \\
\hat{\pi}_1 &= \gamma_3 \hat{g}_1 + \gamma_4 \hat{\tau}_1 + \left(\frac{\lambda(2 - \lambda) \tilde{\kappa}_y \tilde{\sigma}^{-1}}{1 - \lambda} \right) \left[\frac{\bar{i}}{1 + \bar{i}} + \hat{r}_1^e \right] \\
\hat{i}_1 &= \frac{-\bar{i}}{1 + \bar{i}} \\
\hat{y}_2 &= \gamma_5 \hat{g}_1 + \gamma_2 \hat{\tau}_1 + (1 - \lambda) \tilde{\sigma}^{-1} \left[\frac{\bar{i}}{1 + \bar{i}} + \hat{r}_1^e \right] \\
\hat{\pi}_2 &= 0 \\
\hat{i}_2 &= \tilde{\sigma} (\hat{y}_3 - \hat{y}_2) \\
\hat{y}_t &= \hat{y}_{t-1} \left[\frac{\lambda \sum_{j=0}^{t-2} (1 - \lambda)^j}{\frac{\lambda}{1 - \lambda} + \lambda \sum_{j=0}^{t-2} (1 - \lambda)^j} \right] \quad \forall t > 2 \\
\hat{\pi}_t &= 0 \quad \forall t > 2 \\
\hat{i}_t &= \tilde{\sigma} (\hat{y}_{t+1} - \hat{y}_t) \quad \forall t > 2
\end{aligned} \tag{6}$$

where, $\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5 > 0$ are defined as follows:

$$\begin{aligned}
\gamma_1 &= \left[\frac{(2 - \lambda) \tilde{\kappa}_y - (1 - \lambda) \tilde{\kappa}_g}{\tilde{\kappa}_y} \right], \quad \gamma_2 = \left[\frac{(1 - \lambda) \tilde{\kappa}_\tau}{\tilde{\kappa}_y} \right], \quad \gamma_3 = \left[\frac{\lambda(2 - \lambda)}{1 - \lambda} \right] (\tilde{\kappa}_y - \tilde{\kappa}_g), \quad \gamma_4 = \left[\frac{\lambda(2 - \lambda)}{1 - \lambda} \right] \tilde{\kappa}_\tau, \\
\gamma_5 &= (1 - \lambda) \left(\frac{\tilde{\kappa}_y - \tilde{\kappa}_g}{\tilde{\kappa}_y} \right).
\end{aligned}$$

Proof. The full details of the proof are provided in Appendix A, which essentially just involves solving the models backwards using A1-A4 and keeping track of all terms. \square

These equations can be used to solve for the fiscal multipliers. As is well known in the

New Keynesian literature, policy paradoxes in the Calvo model with a one-period deterministic fundamental shock take a weaker form. Tax cuts have no effect on output and government spending increases output one-for-one. However, as shown by the following results, even in this one-period shock setting, the sticky-information model exhibits the policy paradoxes. This emphasizes that there is nothing fundamental in the Calvo price stickiness assumption that drives these paradoxes.

The following two results follow directly from solving the equations in Proposition 1.

Result 1 (Government spending multipliers). *In the ZLB-EX, government spending multiplier at the ZLB is larger in the sticky-information model than in the sticky-price model.*

$$\begin{aligned}\left(\frac{d\hat{y}_1}{d\hat{g}_1}\right)^{SI} &= \frac{(2-\lambda)\tilde{\kappa}_y - (1-\lambda)\tilde{\kappa}_g}{\tilde{\kappa}_y} \geq 1 \\ \left(\frac{d\hat{y}_1}{d\hat{g}_1}\right)^{SP} &= 1\end{aligned}$$

Result 2 (Paradox of toil). *In the ZLB-EX, tax cuts at the ZLB are contractionary in the sticky-information model but have no effect in the sticky-price model.*

$$\begin{aligned}\left(\frac{d\hat{y}_1}{d\hat{\tau}_1}\right)^{SI} &= \frac{(1-\lambda)\tilde{\kappa}_\tau}{\tilde{\kappa}_y} \geq 0 \\ \left(\frac{d\hat{y}_1}{d\hat{\tau}_1}\right)^{SP} &= 0\end{aligned}$$

The spending multiplier and the paradox of toil are more extreme in the sticky information model due to the presence of endogenous state variables. One way to understand the logic of the model is to look at the dynamic paths of output, inflation, interest rates and government spending, as shown in figure 1, for illustrative parameters and shocks that have been chosen to yield a 10 percent drop in output and 2 percent deflation across the two models (parameter details in Appendix C; we do a more detailed numerical example in section 5) . After a recession at the ZLB in period 1 in the SP model, output, inflation and interest rate return to their steady state values immediately after the fundamental shock is over in period 2 (see equation (5)). In the SI model, however, the shortfall in output is persistent and it only gradually recovers back to steady state. These output dynamics in the SI model have fundamental implications for the government spending and tax multipliers. The dashed lines show the evolution of each variable in response to a government spending shock. In the SP model, the spending shock increases output one-to-one in period 1 but has no effect in period 2. The effect on output is bigger, however, in

the SI model. The reason can be understood by studying the response of interest rates. In the absence of government spending, we see that in period 2 onwards, then interest rate is high, i.e. monetary policy is contractionary. The reason for this is that low output in period 1 – the recession period – enters as a state variable in the Phillips curve in period 2 and triggers a trade-off between inflation and output in period 2 onwards (see equation (4)). Because the central bank targets zero inflation, it fully offsets this inflationary effect of the shock by raising the interest rate, which results in the prolonged recession. The reason why government spending is more expansionary in the SI model is, therefore, that by counteracting the drop in output in period 1 it also eliminates some of the negative trade-offs between inflation and output from period 2 onwards. Accordingly we see that monetary policy, in response to the government spending, is less restrictive and interest rate are lower as shown by the dashed line relative to the solid line in the SI model. A similar logic applies for the tax multiplier. The fact that the paradoxes are starker under sticky information can be understood further by contemplating \hat{y}_1 and $\hat{\pi}_1$ graphically, which is done in the next section, before we turn to clarifying Kiley's experiment, which leads to the opposite conclusion.

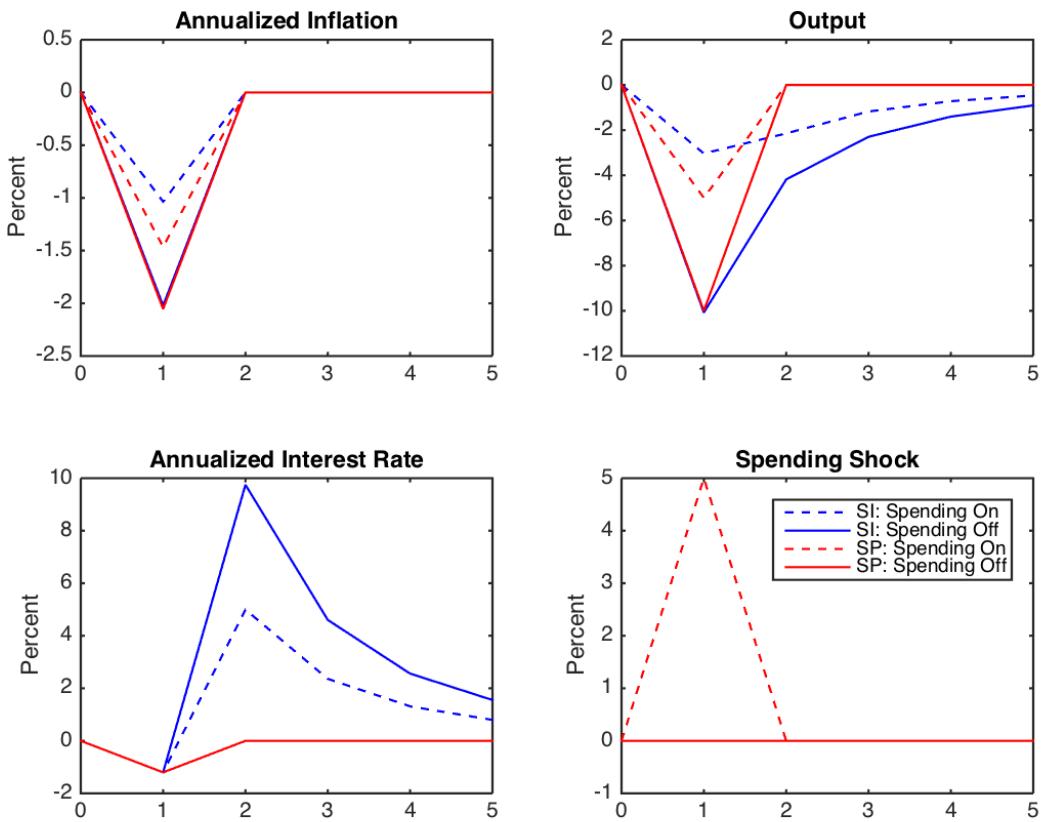


Figure 1: Impulse Responses in ZLB-EX under an Inflation Targeting Rule

3.1.1 Discussion

To simplify the analysis, consider the special case where $\lambda = \frac{\kappa}{1+\kappa}$. The system of equations becomes:

$$\begin{aligned} \text{AS: } & \left\{ \begin{array}{l} \hat{\pi}_1 = \kappa_y \hat{y}_1 - \kappa_g \hat{g}_1 + \kappa_\tau \hat{\tau}_1 , \text{SP, SI} \\ \hat{y}_1 = \hat{y}_2 + \tilde{\sigma}^{-1} \hat{\pi}_2 + \hat{g}_1 + \tilde{\sigma}^{-1} \left[\frac{\bar{i}}{1+\bar{i}} + \hat{r}_1^e \right] , \text{SP, SI} \end{array} \right. \\ \text{AD: } & \left\{ \begin{array}{l} \hat{\pi}_2 = 0 , \text{SP, SI} \\ \hat{y}_2 = \begin{cases} 0 & , \text{SP} \\ \gamma_5 \hat{g}_1 + \gamma_2 \hat{\tau}_1 + (1 - \lambda) \tilde{\sigma}^{-1} \left[\frac{\bar{i}}{1+\bar{i}} + \hat{r}_1^e \right] & , \text{SI} \end{cases} \end{array} \right. \end{aligned} \quad (7)$$

The AS curves across the two models in period 1 are the same. The AD curves in period 1, conditional on expectation of future output, are also the same. This means that the only difference between the multipliers in the two models stems from differences in the expectation of future output. While this expectation is zero in the sticky price (SP) model, it depends on lagged variables in the sticky information (SI) model.

In the $(\hat{y}_1, \hat{\pi}_1)$ space, the initial equilibrium for both the models is given by the intersection of the aggregate demand (AD) and aggregate supply (AS) curves at point E as shown in figure 2. The AD curves are vertical because demand does not depend on current inflation, but only on expected inflation. Output is completely demand determined and pinned down by the shocks \hat{r}_1^e and \hat{g}_1 , and expectation of future output \hat{y}_2 . To the extent that a tax shock does not affect \hat{y}_2 , it does not matter for current output. For a given level of output, then, inflation is determined where the AD curve intersects the AS curve. Now, consider the effects of two different policy interventions in this framework: an increase in government spending, and a tax cut. The final effect of policy on output depends on: a) the direct effect of the policy on aggregate demand (\hat{y}_1), b) the direct effect of the policy on aggregate supply ($\hat{\pi}_1$), c) the indirect effect of the policy on aggregate demand via its effect on the expectation of future output (\hat{y}_2).

The effect of government spending increase is illustrated in figure (2a). There is a direct effect on the AS curves in both the models. Government spending takes away resources from private consumption, so people want to work more to make up for the lost consumption, shifting out labor supply and reducing real wages. The lower real wages mean that the firms can produce more at any given rate of inflation, shifting the AS curve out to the right. There is also a direct effect on AD in both the models as government spending increases "autonomous" spending in the economy. In the purely forward-looking SP model, current spending leaves expectations of future output unchanged. In the SI

model, there is an additional indirect effect of spending because current government spending becomes an endogenous state variable in the model, producing expectations of a higher future output, counteracting the negative effect of the lagged output generated by the recession in period 1.⁵ The net effect is that there is a rightward shift in AD curves in both the models, but the shift is larger for the SI model. The new equilibria are given by points A' and B' for the SP and SI models, respectively. At these points, output is higher than the initial equilibrium for both SI and SP, but the increase is larger under SI.

The effect of tax cut is illustrated in figure (2b). There is a direct effect on the AS curves in both the models. People want to work more, as they get more money in their pocket for every hour worked. This reduces real wages, the benefits of which get passed onto the consumers, producing deflation in the economy. There is no direct effect on the AD curves in either model. In the purely forward-looking Calvo model, expectation of future output is independent of current fiscal policy. In the SI model, however, there is an indirect effect of tax cut on current output, as current tax cut becomes an endogenous state variable, that produces expectations of a lower future output. This in turn lowers output today. The net effect is that there is no change in AD in the Calvo model, but a leftward shift in AD in the SI model. The new equilibria are given by points A'' and B'' for the SP and SI models, respectively. At these points, output is lower in the SI model but remains unchanged in the SP model.

Hence we have just seen analytically and graphically why in the ZLB-EX the policy paradoxes are actually stronger under sticky information than under sticky prices. We now show how one can obtain the opposite result by considering an alternative thought experiment.

3.2 Exogenous Interest Rate Peg Experiment (PEG-EX)

The Exogenous-PEG experiment makes the following assumptions:

B1.(Shock): $\hat{r}_t^e = 0 \quad \forall t, \quad \hat{i}_1 = \hat{i}_2 = 0$

B2.(Fiscal policy): $(\hat{g}_1, \hat{\tau}_1) = (\hat{g}_s, \hat{\tau}_s), \quad (\hat{g}_2, \hat{\tau}_2) = (0, 0)$

B3.(Monetary policy): $\hat{\pi}_t = 0$ for $t > 2$

B4.(Perfect foresight): $\mathbb{E}_t \hat{y}_{t+1} = \hat{y}_{t+1}, \quad \mathbb{E}_t \hat{\pi}_{t+1} = \hat{\pi}_{t+1} \quad \forall t$

⁵This depends crucially on the assumption of a strict inflation target, for reasons already illustrated in figure 1.

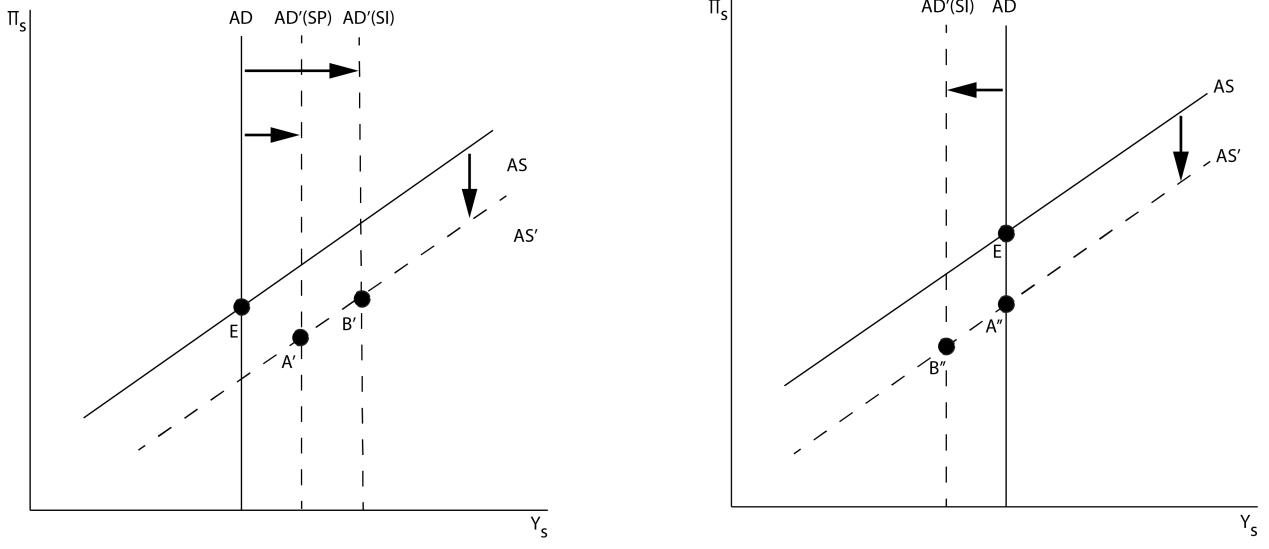


Figure 2: Policy predictions of ZLB-EX

This alternative experiment is different from the ZLB-EX in two main regards. One, there is no fundamental shock to the economy. Two, the interest rate peg lasts for two periods while the fiscal policy is active for only one period. This implies that in the second period, inflation need not be at its target in PEG-EX while it always is in the ZLB-EX (due to the policy rule assumed). This difference generates the difference between the two experiments.

Proposition 2. (PEG-EX) *Under the assumptions of exogenous PEG-EX, the sticky price and sticky information models have the following solutions:*

Sticky-price model:

$$\begin{aligned}
 \hat{y}_1 &= \hat{g}_1 \\
 \hat{\pi}_1 &= (\kappa_y - \kappa_g)\hat{g}_1 + \kappa_\tau \tau_1 \\
 \hat{i}_1 &= 0 \\
 \hat{y}_2 &= \hat{\pi}_2 = \hat{i}_2 = 0 \\
 \hat{y}_t &= \hat{\pi}_t = \hat{i}_t = 0 \quad \forall t > 2
 \end{aligned} \tag{8}$$

Sticky-information model:

$$\begin{aligned}
\hat{y}_1 &= \zeta_1 \hat{g}_s + \zeta_2 \hat{\tau}_s \\
\hat{\pi}_1 &= \zeta_3 \hat{g}_s + \zeta_4 \hat{\tau}_s \\
\hat{i}_1 &= 0 \\
\hat{y}_2 &= 0 \\
\hat{\pi}_2 &= \zeta_5 \hat{g}_s + \zeta_6 \hat{\tau}_s \\
\hat{i}_2 &= 0 \\
\hat{y}_t &= \hat{\pi}_t = \hat{i}_t = 0 \quad \forall t > 2
\end{aligned} \tag{9}$$

where, $\zeta_1, \zeta_3, \zeta_4 > 0$ and $\zeta_2, \zeta_5, \zeta_6 < 0$ are defined as follows:

$$\begin{aligned}
\zeta_1 &= \left(\frac{1+\tilde{\sigma}^{-1}\left(\frac{\lambda}{1-\lambda}\right)\tilde{\kappa}_g}{1+\tilde{\sigma}^{-1}\left(\frac{\lambda}{1-\lambda}\right)\tilde{\kappa}_y} \right), \quad \zeta_2 = \left(\frac{-\tilde{\sigma}^{-1}\left(\frac{\lambda}{1-\lambda}\right)\tilde{\kappa}_\tau}{1+\tilde{\sigma}^{-1}\left(\frac{\lambda}{1-\lambda}\right)\tilde{\kappa}_y} \right), \quad \zeta_3 = \left(\frac{-\lambda}{1-\lambda} \right) \left[\tilde{\kappa}_g - \tilde{\kappa}_y \left(\frac{1+\tilde{\sigma}^{-1}\left(\frac{\lambda}{1-\lambda}\right)\tilde{\kappa}_g}{1+\tilde{\sigma}^{-1}\left(\frac{\lambda}{1-\lambda}\right)\tilde{\kappa}_y} \right) \right], \\
\zeta_4 &= \left(\frac{-\lambda}{1-\lambda} \right) \left[\tilde{\kappa}_y \left(\frac{\tilde{\sigma}^{-1}\left(\frac{\lambda}{1-\lambda}\right)\tilde{\kappa}_\tau}{1+\tilde{\sigma}^{-1}\left(\frac{\lambda}{1-\lambda}\right)\tilde{\kappa}_y} \right) - \tilde{\kappa}_\tau \right], \quad \zeta_5 = \left(\frac{\lambda}{1-\lambda} \right) \left[\tilde{\kappa}_g - \tilde{\kappa}_y \left(\frac{1+\tilde{\sigma}^{-1}\left(\frac{\lambda}{1-\lambda}\right)\tilde{\kappa}_g}{1+\tilde{\sigma}^{-1}\left(\frac{\lambda}{1-\lambda}\right)\tilde{\kappa}_y} \right) \right], \\
\zeta_6 &= \left(\frac{\lambda}{1-\lambda} \right) \left[\tilde{\kappa}_y \left(\frac{\tilde{\sigma}^{-1}\left(\frac{\lambda}{1-\lambda}\right)\tilde{\kappa}_\tau}{1+\tilde{\sigma}^{-1}\left(\frac{\lambda}{1-\lambda}\right)\tilde{\kappa}_y} \right) - \tilde{\kappa}_\tau \right]
\end{aligned}$$

Proof. The details of the proof are shown in Appendix B. Again, it simply involves solving the model backwards, using B1-B4. The key point in the derivation is that we show that under sticky information in PEG-EX, $\hat{y}_2 = 0$ while $\hat{\pi}_2$ may be different from zero, which is in contrast with our previous example. \square

The following two results follow directly from solving the equations in Proposition 2.

Result 3 (Standard government spending multipliers). *In the PEG-EX, the government spending multiplier in the sticky-information model is smaller than that in the sticky-price model and is bounded above by 1.*

$$\begin{aligned}
\left(\frac{d\hat{y}_1}{d\hat{g}_1} \right)^{SI} &= \frac{1 + \tilde{\sigma}^{-1}\left(\frac{\lambda}{1-\lambda}\right)\tilde{\kappa}_g}{1 + \tilde{\sigma}^{-1}\left(\frac{\lambda}{1-\lambda}\right)\tilde{\kappa}_y} < 1 \\
\left(\frac{d\hat{y}_1}{d\hat{g}_1} \right)^{SP} &= 1
\end{aligned}$$

Result 4 (No paradox of toil). *In the PEG-EX, tax cuts are expansionary in the sticky-information model but have no effect in the sticky-price model.*

$$\left(\frac{d\hat{y}_1}{d\hat{\tau}_1}\right)^{SI} = \frac{-\tilde{\sigma}^{-1}\left(\frac{\lambda}{1-\lambda}\right)\tilde{\kappa}_\tau}{1 + \tilde{\sigma}^{-1}\left(\frac{\lambda}{1-\lambda}\right)\tilde{\kappa}_y} < 0$$

$$\left(\frac{d\hat{y}_1}{d\hat{\tau}_1}\right)^{SP} = 0$$

Evidently, we obtain the opposite result from the two experiments, i.e. under PEG-EX the SI model yields a smaller spending multiplier and there is no paradox of toil. We can again see the logic of the proposition more clearly by considering the dynamic paths of inflation, output, interest rate and government spending analogous to figure 1, but for PEG-EX, see figure 3. Here the benchmark is that the interest rate is pegged at steady state for two periods, and government spending is at steady state in all periods. Hence, the benchmark for output and inflation is steady state as well, denoted by a flat line for both models. We can now, similarly to ZLB-EX, contemplate the effect of increasing government spending only in period 1, while keeping the interest rate pegged in period 1 and 2 (but allowing it to adjust in period 3 to target inflation). For the SP model we see that once again government spending increases output one-for-one. Accordingly, the multiplier of government spending is identical across the two experiments in the SP model as shown in result 3. It is in the SI model where things change. As we can see, output now increases strictly less than 1 in absolute value, much less than in ZLB-EX. The path of inflation explains the reason. In response to the government spending, output increases in period 1. The output boom in period 1, however, now triggers deflation in period 2 which – unlike in ZLB-EX – is not offset with interest rate cut in period 2 because the interest rate is pegged. Government spending thus triggers expected deflation in the SI model under PEG-EX, or equivalently, an effective monetary contraction in period 1 (because the real interest rate increases in response to government spending), precisely the opposite of what happened in ZLB-EX. Similar logic applies for the tax multiplier. As before, the logic can be further illustrated via a simple diagram.

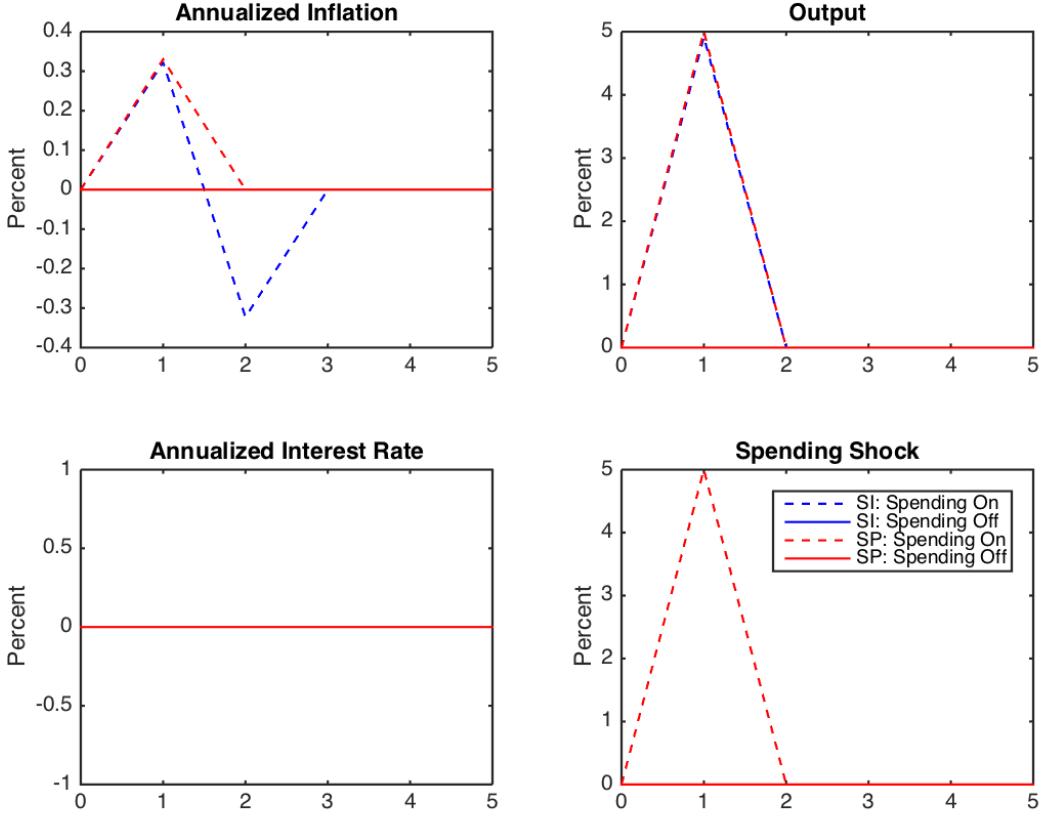


Figure 3: Impulse Responses in PEG-EX under an Inflation Targeting Rule

3.2.1 Discussion

To simplify the analysis, consider again the special case where $\lambda = \frac{\kappa}{1+\kappa}$. The system of equations becomes:

$$\begin{aligned}
 \text{AS: } & \left\{ \hat{\pi}_1 = \kappa_y \hat{y}_1 - \kappa_g \hat{g}_1 + \hat{\kappa}_\tau \hat{\tau}_1 \right. , \text{SP, SI} \\
 \text{AD: } & \left\{ \begin{array}{l} \hat{y}_1 = \hat{y}_2 + \tilde{\sigma}^{-1} \hat{\pi}_2 + \hat{g}_1 , \text{SP, SI} \\ \hat{y}_2 = 0 , \text{SP, SI} \\ \hat{\pi}_2 = \begin{cases} 0 & , \text{SP} \\ \zeta_5 \hat{g}_s + \zeta_6 \hat{\tau}_s & , \text{SI} \end{cases} \end{array} \right.
 \end{aligned} \tag{10}$$

The AS curves across the two models are the same. The AD curves, conditional on expectation of future inflation, are also the same. This means that the only difference between the multipliers in the two models stems from differences in the expectation of

future inflation. Contrast this to our previous experiment, where the only difference in AD was due to difference in output expectation, which is zero here under both models. Meanwhile, the inflation expectation is zero only in the sticky-price model, but depends on lagged variables in the sticky-information model. The final effect of policy on output depends on: a) the direct effect of the policy on aggregate demand (\hat{y}_1), b) the direct effect of the policy on aggregate supply ($\hat{\pi}_1$), c) the indirect effect of the policy on aggregate demand via its effect on the expectation of future inflation ($\hat{\pi}_2$).

As shown in figure (4a), an increase in government spending has a direct on the AS curves in that it reduces inflation in both the models, as under ZLB-EX. It also has a direct positive effect on AD in both the models as it increases "autonomous" spending in the economy. There is an additional indirect effect of government spending on output in the SI model: government spending today becomes an endogenous state variable in the model - it acts like a negative cost push shock that produces deflationary expectation in the economy. This tends to reduce output today. The net effect is that there is a rightward shift in AD curves in both the models, but the shift is smaller for the SI model. The new equilibria are given by points A' and B' for the sticky-price and sticky-information models, respectively. At these points, output is higher than the initial equilibrium for both SI and SP, but the increase is larger under SP.

As shown in figure (4b), a tax cut has a direct effect on AS in that it acts like a negative cost push shock and produces deflation in both the models, as under ZLB-EX. It has no direct effect on AD in either models. There is an indirect effect on AD in the SI model, which is absent in the Calvo model. This effect in the SI model occurs because a tax cut today becomes an endogenous state variable in the model - it acts like a cost push shock that produces inflationary expectations, which increases output today. The net effect is that there is no change in AD in the Calvo model, but a rightward shift in AD in the SI model. The new equilibria are given by points A'' and B'' for the sticky-price and sticky-information models, respectively. At these points, output is higher in the sticky-information model but remains unchanged in the sticky-price model.

In summary, under a strict inflation target, in ZLB-EX, the policy paradoxes are present in both sticky-price and sticky-information models, whereas in PEG-EX, the policy paradoxes are absent in the sticky-information model. This difference is a consequence of the intrinsic persistence of shocks in the SI model due to the presence of endogenous state variables, which is absent in the purely forward-looking SP model. The endogenous state variables become important once one artificially pegs the nominal interest rate independently of the fundamental shocks/fiscal policy. This insight will be carried over to a more dynamic setting in section 6, that replicates Kiley's quantitative experiment.

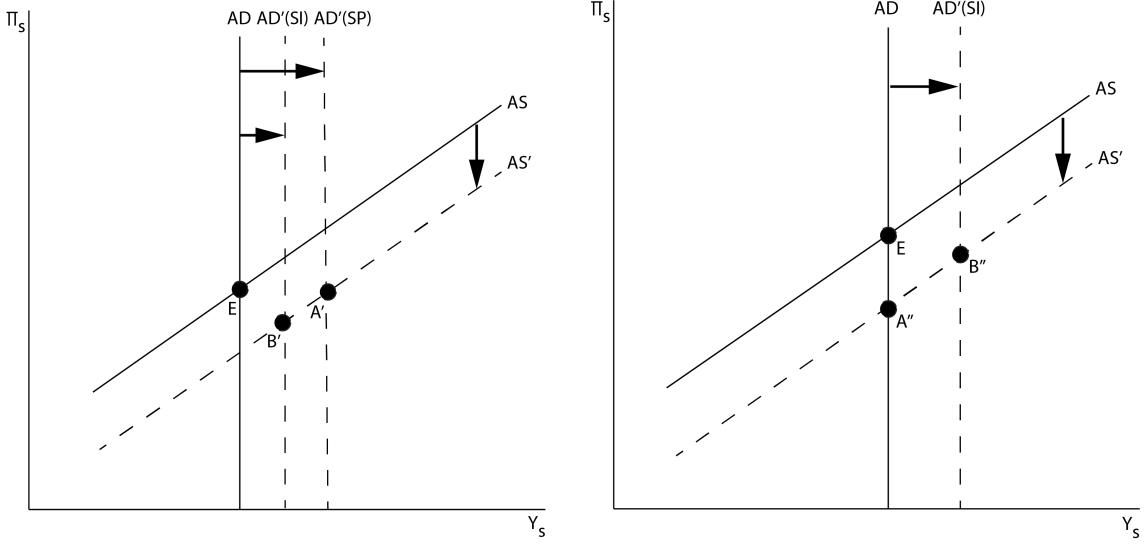


Figure 4: Policy predictions of PEG-EX

4 Inflation-Output Trade-off: General Targeting Rule

There are subtleties behind our assumption of the strict inflation target, that may not be immediately apparent, but help clarify the key mechanism.⁶ Consider that instead of a strict inflation targeting, the government implements a targeting rule, i.e., a weighted average of inflation and output given by

$$\pi_t + \phi y_t = 0 \quad (11)$$

Let us first make the following observation: In the SP model it is irrelevant whether we assume the a strict inflation target or the more general one above in either ZLB-EX or PEG-EX. The reason for this is that the SP model is perfectly forward looking, so there is no tradeoff between output and inflation in that model. Accordingly, ϕ is irrelevant. What is more subtle, however, is the SI case. Recall that there are infinite number of state variables in the SI model which can in principle generate endogenous trade-off between inflation and output. Accordingly, this different policy commitment may have substantive effect on the result, depending on the thought experiment.

Perhaps surprisingly, however, in the PEG-EX it is also irrelevant. It makes no differ-

⁶We thank an anonymous referee for the useful insight that our main result, that government spending multiplier at the zero lower bound is larger under sticky information than under sticky prices, requires qualification with respect to the policy rule. We also thank him/her for suggesting the more general targeting rule as given by equation (11), and for the result given by equation (12) that we report here.

ence assuming a strict inflation target or a general targeting rule. The dynamic paths of inflation, output, interest rate and spending are exactly the same as in figure 3. The reason for this is similar as in the SP model, even if the logic is not as obvious. A key step in the proof of Proposition 2 was that for the PEG-EX experiment $\hat{y}_2 = 0$. This result can be proved for more general interest rate peg, i.e., as long as the interest rate peg is binding for at least one period beyond the duration of the government spending, then in the period before the peg is abandoned $\hat{y}_T = 0$, where T is the last period of the peg. An immediate implication of this is that $\hat{y}_t = 0$ for all $t > T$ in the SI model. This implies that there will be no trade-off between inflation and output in the SI either once the interest rate is no longer pegged. Hence, the assumption of strict inflation targeting is not critical in understanding the difference between SP and SI in the PEG-EX.⁷

The assumption of strict inflation targeting, however, is important when thinking about the SI model in the ZLB-EX, for reasons that clarify the difference between the two experiments and the forces generating the different results. Consider now the ZLB-EX with assumptions A1, A2 and A4 from section 3.1, but replace assumption A3 with the assumption that once the fundamental shock is over, then monetary policy follows a general inflation targeting rule (11). Under this rule, the SI model has the following solution for output under ZLB-EX⁸

$$\begin{aligned}\hat{y}_1^{SI} = & \frac{\tilde{\sigma}(1-\lambda)^2 + \tilde{\sigma}\lambda\phi^{-1}[(2-\lambda)\tilde{\kappa}_y - (1-\lambda)\tilde{\kappa}_g] + \lambda(1-\lambda)\tilde{\kappa}_g}{\tilde{\sigma}(1-\lambda)^2 + \lambda\tilde{\kappa}_y(1-\lambda + \tilde{\sigma}\phi^{-1})}\hat{g}_1 \\ & - \frac{(1-\tilde{\sigma}\phi^{-1})\lambda(1-\lambda)\tilde{\kappa}_\tau}{\tilde{\sigma}(1-\lambda)^2 + \lambda\tilde{\kappa}_y(1-\lambda + \tilde{\sigma}\phi^{-1})}\hat{\tau}_1 \\ & + \frac{[(1-\lambda)^2 + \lambda(2-\lambda)\tilde{\kappa}_y\phi^{-1}]}{\tilde{\sigma}(1-\lambda)^2 + \lambda\tilde{\kappa}_y(1-\lambda + \tilde{\sigma}\phi^{-1})}\left(\frac{\bar{i}}{1+\bar{i}} + \hat{r}_1^e\right)\end{aligned}\tag{12}$$

As is evident, in the SI model, the fiscal multipliers now depend on ϕ . The spending multiplier is larger than 1 and tax cuts are contractionary if and only if $\phi < \frac{1}{\tilde{\sigma}}$.

What is the intuition of this result? It is best understood by contemplating the impulse response of the model in the ZLB-EX.

The fact that government spending in the SI model triggered a more expansionary monetary policy in the future (or more precisely, less contractionary) in figure 1 was contingent

⁷Consider the PEG-EX thought experiment with assumptions B1, B2 and B4 from section 3.2, but replace B3 with the assumption of a general target rule as in equation (11) for $t > 2$. Under this rule, the fiscal multipliers are independent of the relative weight on output stabilization, ϕ , and remain identical to the multipliers obtained under PEG-EX with assumptions (B1)-(B4). Details of the derivation are provided in Appendix F.

⁸Details of the derivation are provided in Appendix E.

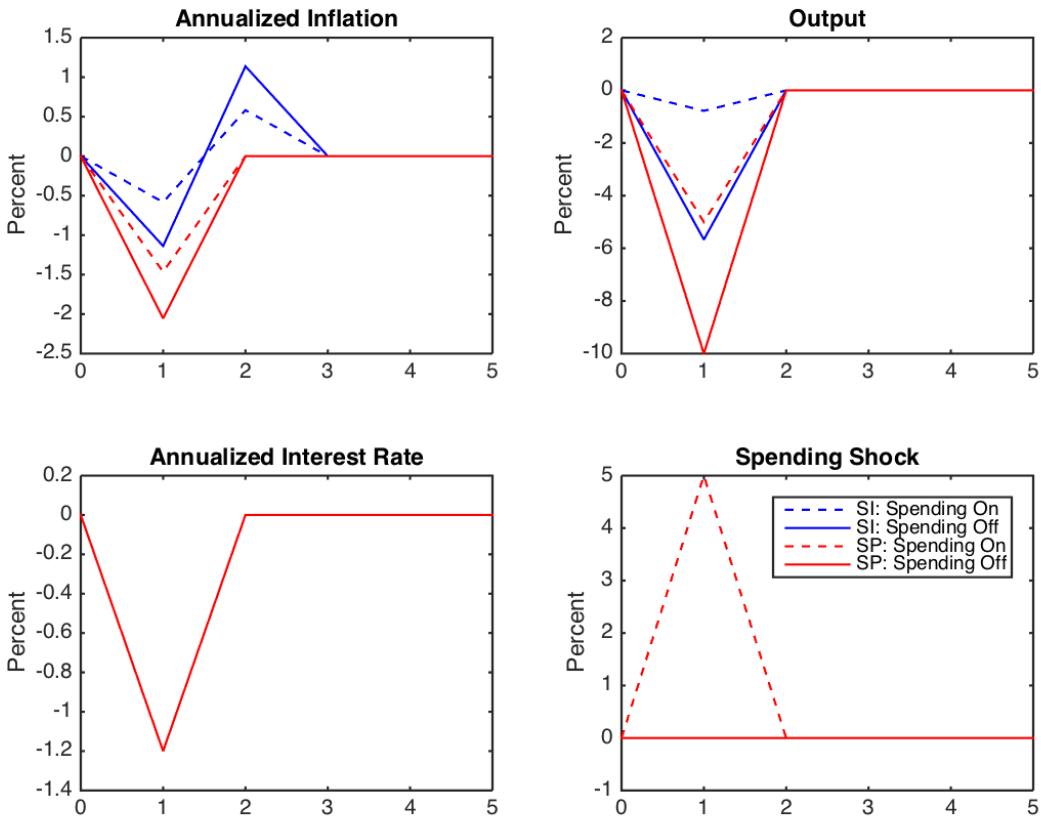


Figure 5: Impulse Responses in ZLB-EX under an Output Targeting Rule

on the fact that the central bank had a strict inflation target. Figure 5 shows the other extreme of the policy rule in equation (11) when the central bank instead targets zero output gap when the shock is over. In this case, the spending multiplier in SI is smaller. The logic is again most easily grasped by considering the dynamics of the state variables in the SI Phillips curve as in figure 5. Consider first the equilibrium when there is no government spending. We now see that once the shock is over, the central bank will fully offset the cost push effect generated by the negative output gap in period 1 by allowing excess inflation (in period 2). Accordingly the recession is much more shallow in period 1 in the SI model because the expected inflation in period 2 reduces the real interest rate during the recession — precisely what is needed at the ZLB for monetary policy to be stimulative. This also makes clear why government spending multiplier is smaller under this policy commitment. An increase in government spending reduces the output gap in period 1, which then results in a smaller cost-push shock in period 2, thus reducing the expected inflation during the trap. This tends to have a contractionary effect. Overall, the increase in government spending is thus less powerful the more the government targets output in period 2, because government spending triggers disinflationary pressures once

the ZLB is no longer binding, which makes monetary policy less expansionary during the ZLB!

Once we allow for the weight on output to vary, we will get a spending multiplier that can either be larger than or smaller than the spending multiplier in the SP model even if, as we will see in the quantitative example in the next section, it tends to be higher in the SI model for reasonable weights on inflation and output and quite close to the strict inflation target. The key observation is that this is because of the implication government spending has on inflation expectation in the SI model once the ZLB is no longer binding, due to the presence of state variables in the Phillips curve – a force that is absent in the SP model. It is also worth re-iterating that this force is also absent in the PEG-EX even for the SI model, for in this case the output gap is always zero in period 2 onwards, so that there is no tradeoff between inflation and output. Overall we think this extension highlights the main point of the paper, which is that in comparing results across different models, the nature of the thought experiments is of critical importance. We come back to this issue in the next section, where we consider a richer numerical example.

5 Numerical results: Sticky-price vs sticky-information under ZLB-EX

After demonstrating the main results analytically in the preceding sections, in this section we provide the results of ZLB-EX under a parameter calibration that is common in the literature (details in Appendix C).⁹ The aim is to provide a relevant magnitude of the fiscal multipliers and discuss the paradox of flexibility. An AR(1) process is chosen for the natural real interest rate shock and a deterministic transition path is contemplated. The duration of the binding ZLB is endogenously determined. The persistence and magnitude of the real interest rate shock are chosen to produce a 10% recession and 2% annual deflation on impact in the sticky-price model. The same shock process is then fed into the the sticky-information model and the information rigidity parameter is chosen to match the 10% recession and 2% annual deflation.¹⁰

This is reminiscent of the exercise in Eggertsson (2011) but the value of the multipliers is different due to a different assumption about the stochastic process – that paper assumes a two state Markov process for the real interest rate shock rather than an AR(1).

⁹The multipliers in the SI model under ZLB-EX depend on the relative weight on output stabilization, due to reasons clarified in section 4. For our baseline results in Tables 1 & 2, we assume a strict inflation target.

¹⁰Codes for this exercise are available online on our webpage.

Government spending multiplier

Table 1: Government spending multipliers

	Sticky-price	Sticky-information
Positive interest rate	0.42	0.42
Zero interest rate	1.63	4.77

At positive interest rates, government spending crowds out private consumption, and hence its effect on output is small (multiplier less than one) in both sticky price and sticky information models. At zero interest rates, however, government spending is over-expansionary in both the models, with the expansionary effect being higher in the sticky-information model.

Paradox of toil

Table 2: Tax cut multipliers

	Sticky-price	Sticky-information
Positive interest rate	0.36	0.36
Zero interest rate	-0.4	-2.41

At positive interest rates, tax cuts promote consumption spending and create an output expansion in both sticky-price and sticky-information models. At zero interest rates, however, tax cuts are contractionary. This contractionary effect is more sharply evident in the sticky-information model.

General targeting rule

In the numerical simulation above, the spending multiplier in the SI model is very high (and tax cuts very contractionary). Here, the assumption of strict inflation targeting is important for the SI model in ZLB-EX, as we have already stressed, while this assumption is irrelevant for other 3 cases as shows in section 4. As we stressed, the reason is that under SI, government spending triggers an endogenous trade-off between inflation and output stabilization once the ZLB stops binding that is deflationary, and if the central bank targets output this can mitigate the expansionary effect of the spending. If we specify a more general rule as in equation (11), the multiplier changes with ϕ as shown in figure 6. In particular, as ϕ increases, the multiplier decreases reaching 0.99 in the limit, which is below the multiplier in case of SP. When there is equal weight on inflation and output stabilization, the multiplier for the SI is still substantially higher than SP (2.79

in SI vs 1.63 in SP). A similar result can be obtained for the tax multiplier. In the Calvo model, this general targeting rule corresponds to optimal policy under discretion and the parameter ϕ can be mapped to the welfare criterion, corresponding to $\phi=0.28$. Even with the welfare weight in the targeting rule, we get higher multipliers for SI than SP (4.02 in SI vs 1.63 in SP).

Overall we conclude that for reasonable weights in the targeting rule, the multiplier is higher under SI and corresponds quite closely to a strict inflation target in our numerical experiments, which is one motivation for why it is our key benchmark.

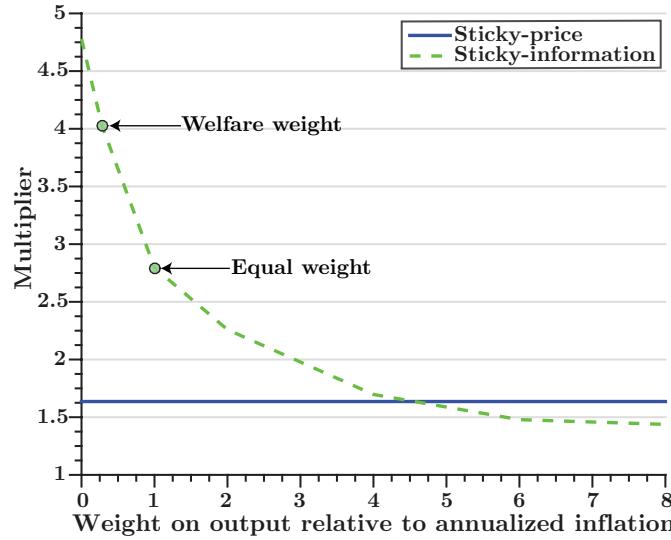


Figure 6: Spending multiplier with a general targeting rule

Paradox of flexibility

In the numerical solution, as prices become more flexible, the output contraction in response to a negative real interest rate shock becomes worse in both sticky-price and sticky-information models. Infact, the paradox of flexibility is starker under sticky-information than under sticky-prices. This is because increasing information flexibility makes inflation more responsive to the output gap. Therefore, in response to the shock, not only current deflation but also expected deflation is bigger. This expected deflation feeds into the current output and contracts it further.

It is well-known what is happening in the sticky-price model, but a brief comment upon this paradox in the sticky-information model is in order, given that it is more subtle due its dependence on the combination of information flexibility (λ) and shock persistence (ρ_{rn}). In a model where the shock persists for the first two periods and goes back to the

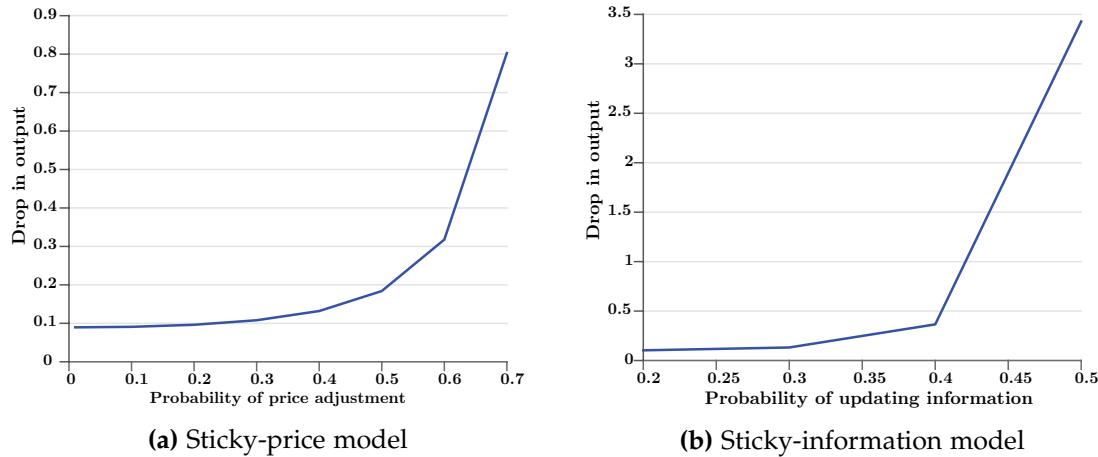


Figure 7: Effect of increasing flexibility at zero interest rates

steady state in the third period, there is a range of λ for which the paradox of flexibility holds. This can be shown analytically, but it is rather cumbersome. Similar results can be derived for variations where the shock persists for more than two periods, but it gets progressively harder to get closed form solutions. Accordingly, the finding is presented as a computational result.

Result 5 (Information threshold). Let λ^* be the threshold level of information-flexibility, beyond which the paradox of flexibility holds. That is, for $\lambda \geq \lambda^*$, $\frac{dy}{d\lambda} < 0$. And, let ρ_{rn} be the persistence of the natural interest rate shock. Then, the threshold level of information-flexibility is a decreasing function of the persistence of the shock. That is, $\frac{d\lambda^*}{d\rho_{rn}} \leq 0$.

Table 3: Paradox of flexibility and shock persistence

ρ_{rn}	λ^*
0.1	1
0.2	1
0.3	0.76
0.4	0.66
0.5	0.62
0.6	0.48
0.7	0.38
0.8	0.26
0.9	0.18

For any given probability of information updating λ , paradox of flexibility becomes more likely as persistence of the shock increases. This is because, for a given λ , a negative shock to the natural real interest has a direct effect on current output and inflation – output and inflation fall. There is an additional indirect effect on current output through changes in the expectations of future output. The expectation of future output for every period in

which the shock is present will be negative. It is only once the ZLB is no longer binding, that the inflation-targeting central bank will pursue expansionary monetary policy which would increase output. Hence, the final effect on current output depends on the relative magnitude of the cumulative negative output gap in the shock-on periods versus the positive output gap in the shock-off period. The higher the persistence of the shock, the higher is the magnitude of the cumulative negative output gap. This means that as the shock persistence increases, the expansionary effect once the shock is over, is less able to outweigh the contractionary effect while the shock is on. This is true also for the original threshold λ . Hence, the new threshold λ is lower if the persistence of the shock is higher. In the baseline calibration, $\rho_{rn} = 0.88$ and the corresponding threshold $\lambda^* = 0.16$.

6 Replicating Kiley's results

Table 4: Calibration parameters (from Kiley (2016))

Parameters	Value
λ	0.25
β	1
$\tilde{\sigma}$	1
κ_y	0.033
κ_g	0.0165
$\tilde{\kappa}_y$	0.1
$\tilde{\kappa}_g$	0.05
ϕ_π	1.5
ϕ_y	0.125

The PEG-EX is different from the ZLB-EX in two respects. The first is absence of a fundamental shock that drives the economy into the ZLB environment. The second is lack of policy coordination, that is, the duration of the peg is different from the duration of fiscal policy. It is important to understand which of these differences explains the difference in the policy results. Accordingly, we contemplate yet another experiment, which lies somewhere in between the ZLB-EX and the PEG-EX. We call this the Coordinated PEG-EX (CPEG-EX). Under this experiment, there is no fundamental shock to the economy but the duration of policy is coordinated with the duration of the peg.

This section compares the numerical results for multipliers under PEG-EX versus CPEG-EX. Both models are calibrated as per table 4 to replicate the policy results of Kiley (2016).¹¹ The only difference is in the specification of fiscal policy. In particular, under

¹¹The multiplier results for the SI model under CPEG-EX depend on the coefficients on inflation (ϕ_π) and output gap (ϕ_y) in the policy response function. The higher the relative coefficient on output, the lower the multiplier. The reason is the same as for SI in ZLB-EX as explained in section 4.

PEG-EX, duration of the fiscal policy intervention is varied from period 1 to period 25, while the interest rate is pegged to its steady state value for 100 periods. Under CPEG-EX, the interest rate peg lasts for as long as the fiscal policy shock is on. As we will see, the paradox results for the CPEG-EX are qualitatively similar to that of the ZLB-EX. This underlines the importance of policy coordination in this class of models. Quantitatively, the paradoxes are even starker under CPEG-EX than ZLB-EX.

The key result under PEG-EX is that the policy paradoxes depend on the assumptions regarding the nature of price adjustment – sticky prices versus sticky information. Kiley interprets this to mean that changes in assumptions regarding price dynamics can overturn the key policy paradoxes at the ZLB. Figure 8 (a) is a replication of Figure 2 of Kiley's paper. The graph shows that in the sticky-price model, the government expenditure multiplier – that is, the average increase in output over the period of higher government expenditure – is strictly greater than one and increasing in the duration (T) of the fiscal expansion. In the sticky-information model, the government expenditure multiplier is strictly less than one and decreasing in the duration (T) of the fiscal expansion. However, as illustrated in the preceding section, the absence of the over-expansionary effect of government spending under sticky-information here is an artifact of the exogenous interest rate peg. Under CPEG-EX, government spending is over-expansionary even under the assumption of sticky-information, as depicted in Figure 8 (b). Infact, the multiplier is higher under sticky-information than under stick-prices.

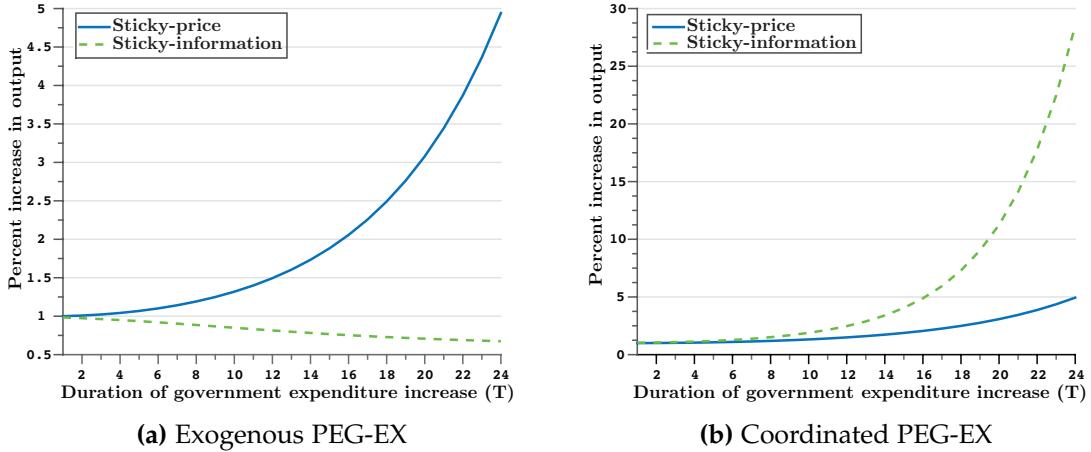


Figure 8: Government spending multiplier

Similarly, Figure 9 shows the effect a tax increase under the assumption of sticky-price and sticky-information under PEG-EX versus CPEG-EX. In the sticky-price model, tax increase creates output expansion under both experiments. However, the effect of tax increase on output in the sticky-information model depends on the experiment being performed. Under PEG-EX in Figure 9 (a), tax increase has no effect on output, whereas

under CPEG-EX in Figure 9 (b), tax increase is expansionary. Infact, the expansionary effect under CPEG-EX is higher in the sticky-information model than in the sticky-price model.

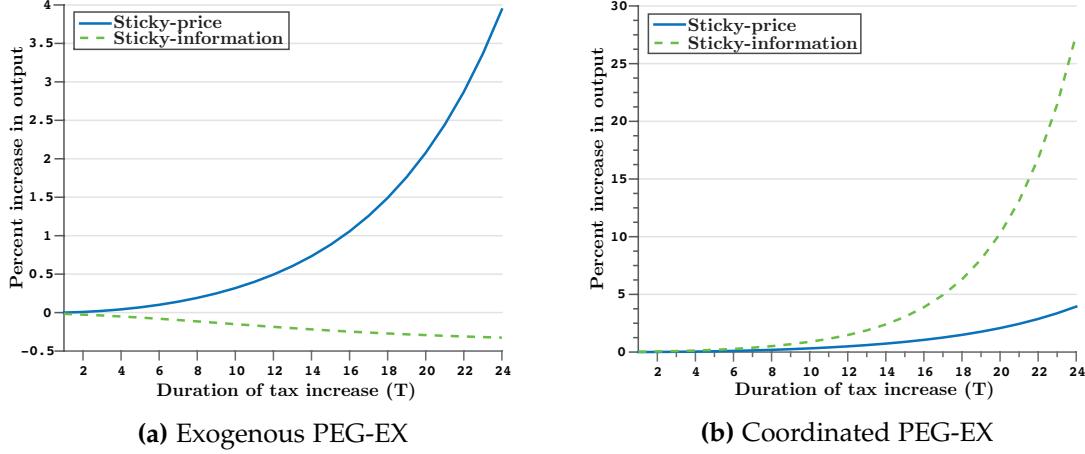


Figure 9: Tax multiplier

These graphs illustrate that keeping the interest rate pegged after the policy intervention is over (PEG-EX) reduces the expansionary power of government spending and makes the paradox of toil disappear in the sticky-information model. What is the intuition behind this? Referring back to figure 3, we see that in response to the government spending, output increases in period 1. In the SI model, the output boom in period 1, triggers deflation in period 2. In the absence of pegged interest in period 2 (as in CPEG-EX), an inflation targeting central bank lowers the interest rate, which stimulates the economy. However, in PEG-EX, the pegged period 2 policy rate constrains the central bank's ability to do so, which mechanically engineers a contractionary monetary policy in the model and prevents output from expanding. The subtlety lies in the fact that the interest rate peg produces no such contradiction, relative to ZLB-EX/CPEG-EX, in the sticky-price framework. The sticky-price model is purely forward looking, and as soon as the shock is over, output, inflation and interest rate go back to their respective steady states. Hence, holding the interest rate constant for an additional period is inconsequential in that model.

7 Conclusion

This paper demonstrates that the key policy predictions of the New Keynesian model at the ZLB are independent of the assumption about the nature of price dynamics – sticky prices or sticky information. This contradicts the recent findings of Kiley (2016) that the

policy paradoxes of the sticky-price model are fragile and disappear under the alternative sticky-information assumption of Mankiw and Reis (2001). Kiley's findings are driven by the nature of the experiment he performs, rather than anything fundamental about sticky information. He assumes an arbitrary interest rate peg independent of any fundamental shock or policy intervention (PEG-EX in the paper). This is very different from the policy experiment common in the ZLB literature, where the ZLB is binding due to a fundamental shock and policy is explicitly in response to it (ZLB-EX in the paper). It is even different from much of the literature on interest rate pegs, where it is typically assumed that fiscal intervention lasts as long as the interest rate peg (CPEG-EX).

Analytical examples and numerical simulations are used to illustrate and explain this result. In the ZLB-EX with a strict inflation target, government spending multiplier is larger and the paradox of toil is starker under sticky-information than under sticky-prices. This is because in the purely forward looking sticky-price model, current policy leaves expectations of future variables unchanged. Hence, there is only a "direct" effect of current policy on current output. In the backward looking sticky-information model, however, current policy has an additional "indirect" effect on current output, which comes from its effect on the expectations of future output.

In the alternative experiment, PEG-EX, government spending multiplier is smaller than one and paradox of toil disappears under sticky-information. This is because an arbitrary interest rate peg is equivalent to engineering a contractionary monetary policy in that framework. Government spending triggers expected deflation in the model. In the absence of the peg, an inflation-targeting central bank would lower interest rate in order to stimulate output and inflation. The peg puts a constraint on the central bank's ability to do so. The subtle feature of this experiment is that it doesn't affect the predictions of the sticky-price model. This is because the sticky-price model is purely forward looking, and all variables revert to their steady state once the temporary fiscal policy shocks are over. Hence, pegging the interest rate to its steady state value (even after the shock is over) is inconsequential in that model.

We clarify that our assumption of strict inflation target is innocuous in the SP model in both ZLB-EX and PEG-EX and for SI under PEG-EX, but it is not so for SI in ZLB-EX. The reason for this is obvious in the SP model. The SP model is perfectly forward looking so that there is no trade-off between output and inflation. Hence, using a strict or a general target is irrelevant in that model. It is not obvious why the assumption is innocuous in the SI model under PEG-EX, but the reasoning is similar to that for the SP model. We show that the solution for this model is such that in the the last period before the peg stops binding, the output gap is zero. This implies that there is no trade-off between inflation and output in the period where the rule applies. In the SI model in ZLB-EX,

however, there is indeed a trade-off and the relative weight on inflation versus output stabilization determines the strength of the multiplier. In particular, a spending shock during the ZLB is deflationary once the ZLB is no longer binding (because last period government spending is a state variable in the model). The multiplier then depends on the accomodation of this deflationary shock by the central bank – the smaller the relative weight on output stabilization, the lower the interest rate (to counter deflation) and higher the output boom, which increases the expansionary effect today.

We further contemplate an experiment, which is a hybrid of the ZLB-EX and PEG-EX, called CPEG-EX. Under CPEG-EX, while the interest rate peg is arbitrary to the extent that there is no fundamental shock causing it, policy duration is coordinated with the peg duration. The results for the multipliers under CPEG-EX are qualitatively similar to that of the ZLB-EX. This underscores the importance of coordinating fiscal policy with the duration of monetary policy passiveness in this class of experiments.

Finally, this paper illustrates numerically that the paradox of flexibility is also starker under sticky-information. This result is harder to intuit from an analytical example, because the response of output to increased flexibility in the sticky-information model depends on the persistence of the underlying fundamental rate shock.

Overall, the paradoxical results for fiscal policy multipliers at the ZLB could be interpreted as reflecting a weakness of the Calvo model. This paper, however, clarifies that these paradoxes are not a product of the Calvo assumption, but rather are a fundamental feature of models with nominal rigidities, irrespective of its source, be it sticky prices or sticky information.

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Appendix

A Zero Lower Bound Experiment (ZLB-EX): Proof of Proposition 1

Sticky-price model

$$\begin{aligned}\hat{y}_t &= \mathbb{E}_t \hat{y}_{t+1} - \tilde{\sigma}^{-1} (\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{r}_t^e) + (\hat{g}_t - \mathbb{E}_t \hat{g}_{t+1}) \\ \hat{\pi}_t &= \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa_y \hat{y}_t - \kappa_g \hat{g}_t + \kappa_\tau \hat{t}_t\end{aligned}\tag{13}$$

Assumptions (A1)-(A4) imply that:

For t=1:

$$\begin{aligned}\hat{y}_1 &= \hat{y}_2 + \frac{\tilde{\sigma}^{-1} \bar{i}}{1 + \bar{i}} + \tilde{\sigma}^{-1} \hat{r}_1^e + \hat{g}_1 \\ \hat{\pi}_1 &= \kappa_y \hat{y}_1 - \kappa_g \hat{g}_1 + \kappa_\tau \hat{t}_1\end{aligned}\tag{14}$$

For t=2:

$$\begin{aligned}\hat{y}_2 &= \hat{y}_3 - \tilde{\sigma}^{-1} \hat{i}_2 \\ \hat{\pi}_2 &= \kappa_y \hat{y}_2\end{aligned}\tag{15}$$

For t=3:

$$\begin{aligned}\hat{y}_3 &= \hat{y}_4 - \tilde{\sigma}^{-1} \hat{i}_3 \\ \hat{\pi}_3 &= \kappa_y \hat{y}_3\end{aligned}\tag{16}$$

⋮

For any t>2:

$$\begin{aligned}\hat{y}_t &= \hat{y}_{t+1} - \tilde{\sigma}^{-1} \hat{i}_t \\ \hat{\pi}_t &= \kappa_y \hat{y}_t\end{aligned}\tag{17}$$

Applying (A3) to period 2 Phillips curve (eqn 72) allows us to solve for \hat{y}_2 as follows:

$$\begin{aligned}\hat{\pi}_2 &= \kappa_y \hat{y}_2 \\ 0 &= \kappa_y \hat{y}_2 \\ \implies \hat{y}_2 &= 0\end{aligned}\tag{18}$$

Substituting \hat{y}_2 into period 1 IS curve (eqn 71) gives:

$$\hat{y}_1 = \frac{\tilde{\sigma}^{-1} \bar{i}}{1 + \bar{i}} + \tilde{\sigma}^{-1} \hat{r}_t^e + \hat{g}_1\tag{19}$$

Substituting \hat{y}_1 in period 1 Phillips curve (eqn 71) gives:

$$\hat{\pi}_1 = (\kappa_y - \kappa_g) \hat{g}_1 + \frac{\kappa_y \tilde{\sigma}^{-1} \bar{i}}{1 + \bar{i}} + \kappa_y \tilde{\sigma}^{-1} \hat{r}_1^e + \kappa_\tau \hat{t}_1\tag{20}$$

Output for every period $t > 2$ can be solved from that period's Phillips curve using assumption (A3). We get that for t>2, $\hat{y}_t = \hat{\pi}_t = 0$.

Sticky-information model

$$\begin{aligned}\hat{y}_t &= \mathbb{E}_t \hat{y}_{t+1} - \tilde{\sigma}^{-1} (\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{r}_t^e) + (\hat{g}_t - \mathbb{E}_t \hat{g}_{t+1}) \\ \hat{\pi}_t &= \frac{\lambda}{1-\lambda} (\tilde{\kappa}_y \hat{y}_t - \tilde{\kappa}_g \hat{g}_t + \tilde{\kappa}_\tau \hat{\tau}_t) \\ &+ \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \mathbb{E}_{t-1-j} (\hat{\pi}_t + [\tilde{\kappa}_y (\hat{y}_t - \hat{y}_{t-1}) - \tilde{\kappa}_g (\hat{g}_t - \hat{g}_{t-1}) + \tilde{\kappa}_\tau (\hat{\tau}_t - \hat{\tau}_{t-1})])\end{aligned}\quad (21)$$

Assumptions (A1)-(A4) imply that:

For t=1

$$\begin{aligned}\hat{y}_1 &= \hat{y}_2 + \frac{\tilde{\sigma}^{-1} \bar{i}}{1+\bar{i}} + \tilde{\sigma}^{-1} \hat{r}_1^e + \hat{g}_1 \\ \hat{\pi}_1 &= \frac{\lambda}{1-\lambda} (\tilde{\kappa}_y \hat{y}_1 - \tilde{\kappa}_g \hat{g}_1 + \tilde{\kappa}_\tau \hat{\tau}_1)\end{aligned}\quad (22)$$

For t=2:

$$\begin{aligned}\hat{y}_2 &= \hat{y}_3 - \tilde{\sigma}^{-1} \hat{i}_2 \\ \hat{\pi}_2 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_2 + \lambda (\hat{\pi}_2 + [\tilde{\kappa}_y (\hat{y}_2 - \hat{y}_1) + \tilde{\kappa}_g \hat{g}_1 - \tilde{\kappa}_\tau \hat{\tau}_1])\end{aligned}\quad (23)$$

For t = 3:

$$\begin{aligned}\hat{y}_3 &= \hat{y}_4 - \tilde{\sigma}^{-1} \hat{i}_3 \\ \hat{\pi}_3 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_3 + [\lambda + \lambda(1-\lambda)] (\hat{\pi}_3 + \tilde{\kappa}_y (\hat{y}_3 - \hat{y}_2))\end{aligned}\quad (24)$$

For t = 4:

$$\begin{aligned}\hat{y}_4 &= \hat{y}_5 - \tilde{\sigma}^{-1} \hat{i}_4 \\ \hat{\pi}_4 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_4 + [\lambda + \lambda(1-\lambda) + \lambda(1-\lambda)^2] (\hat{\pi}_4 + \tilde{\kappa}_y (\hat{y}_4 - \hat{y}_3))\end{aligned}\quad (25)$$

\vdots

For t>2:

$$\begin{aligned}\hat{y}_t &= \hat{y}_{t+1} - \tilde{\sigma}^{-1} \hat{i}_t \\ \hat{\pi}_t &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_t \\ &+ \lambda \sum_{j=0}^{t-2} (1-\lambda)^j (\hat{\pi}_t + \tilde{\kappa}_y (\hat{y}_t - \hat{y}_{t-1}))\end{aligned}\quad (26)$$

Period 2 Phillips curve (eqn 23) along with assumption (A3) can be used to solve for \hat{y}_2 as follows:

$$\begin{aligned}\hat{\pi}_2 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_2 + \lambda (\hat{\pi}_2 + [\tilde{\kappa}_y (\hat{y}_2 - \hat{y}_1) + \tilde{\kappa}_g \hat{g}_1 - \tilde{\kappa}_\tau \hat{\tau}_1]) \\ 0 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_2 + \lambda (0 + [\tilde{\kappa}_y (\hat{y}_2 - \hat{y}_1) + \tilde{\kappa}_g \hat{g}_1 - \tilde{\kappa}_\tau \hat{\tau}_1]) \\ \hat{y}_2 &= \left(\frac{1-\lambda}{2-\lambda} \right) \left[\hat{y}_1 - \frac{\tilde{\kappa}_g}{\tilde{\kappa}_y} \hat{g}_1 + \frac{\tilde{\kappa}_\tau}{\tilde{\kappa}_y} \hat{\tau}_1 \right]\end{aligned}\quad (27)$$

Substituting \hat{y}_2 into period 1 IS curve (eqn 22) gives \hat{y}_1 :

$$\hat{y}_1 = \left[\frac{(2-\lambda) \tilde{\kappa}_y - (1-\lambda) \tilde{\kappa}_g}{\tilde{\kappa}_y} \right] \hat{g}_1 + \left[\frac{(1-\lambda) \tilde{\kappa}_\tau}{\tilde{\kappa}_y} \right] \hat{\tau}_1 + (2-\lambda) \frac{\tilde{\sigma}^{-1} \bar{i}}{1+\bar{i}} + \tilde{\sigma}^{-1} (2-\lambda) \hat{r}_1^e \quad (28)$$

Substituting \hat{y}_1 into period 1 Phillips curve (eqn 22) gives $\hat{\pi}_1$:

$$\hat{\pi}_1 = \left(\frac{\lambda(2-\lambda)}{1-\lambda} \right) \left[(\tilde{\kappa}_y - \tilde{\kappa}_g) \hat{g}_1 + \tilde{\kappa}_\tau \hat{\tau}_1 + \tilde{\kappa}_y \tilde{\sigma}^{-1} \hat{r}_1^e + \frac{\tilde{\kappa}_y \tilde{\sigma}^{-1} \bar{i}}{1+\bar{i}} \right] \quad (29)$$

To write \hat{y}_2 in terms of exogenous parameters, substitute the solution for \hat{y}_1 (eqn 28) into \hat{y}_2 (eqn 27):

$$\hat{y}_2 = (1-\lambda) \left(\frac{\tilde{\kappa}_y - \tilde{\kappa}_g}{\tilde{\kappa}_y} \right) \hat{g}_1 + (1-\lambda) \left(\frac{\tilde{\kappa}_\tau}{\tilde{\kappa}_y} \right) \hat{\tau}_1 + (1-\lambda) \frac{\tilde{\sigma}^{-1} \bar{i}}{1+\bar{i}} + (1-\lambda) \tilde{\sigma}^{-1} \hat{r}_1^e \quad (30)$$

Further, we can solve for the transition dynamics of output in each period using the Phillips curve for that period along with assumption (A3). For $t>2$, we get:

$$\begin{aligned} 0 &= \frac{\lambda}{1-\lambda} (\tilde{\kappa}_y \hat{y}_t) + \lambda \sum_{j=0}^{t-2} (1-\lambda)^j (0 + \tilde{\kappa}_y (\hat{y}_t - \hat{y}_{t-1})) \\ 0 &= \left[\frac{\lambda}{1-\lambda} + \lambda \sum_{j=0}^{t-2} (1-\lambda)^j \right] \tilde{\kappa}_y \hat{y}_t - \left[\lambda \sum_{j=0}^{t-2} (1-\lambda)^j \right] \tilde{\kappa}_y \hat{y}_{t-1} \\ \hat{y}_t &= \left[\frac{\lambda \sum_{j=0}^{t-2} (1-\lambda)^j}{\frac{\lambda}{1-\lambda} + \lambda \sum_{j=0}^{t-2} (1-\lambda)^j} \right] \hat{y}_{t-1} \end{aligned} \quad (31)$$

B Interest Rate Peg Experiment (PEG-EX): Proof of Proposition 2

Sticky-price model

$$\begin{aligned} \hat{y}_t &= \mathbb{E}_t \hat{y}_{t+1} - \tilde{\sigma}^{-1} (\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{r}_t^e) + (\hat{g}_t - \mathbb{E}_t \hat{g}_{t+1}) \\ \hat{\pi}_t &= \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa_y \hat{y}_t - \kappa_g \hat{g}_t + \kappa_\tau \hat{\tau}_t \end{aligned} \quad (32)$$

Assumptions (B1)-(B4) imply that:

For $t=1$

$$\begin{aligned} \hat{y}_1 &= \hat{y}_2 + \tilde{\sigma}^{-1} \hat{\pi}_2 + \hat{g}_1 \\ \hat{\pi}_1 &= \beta \hat{\pi}_2 + \kappa_y \hat{y}_1 - \kappa_g \hat{g}_1 + \kappa_\tau \hat{\tau}_1 \end{aligned} \quad (33)$$

For $t=2$:

$$\begin{aligned} \hat{y}_2 &= \hat{y}_3 \\ \hat{\pi}_2 &= \kappa_y \hat{y}_2 \end{aligned} \quad (34)$$

For $t=3$:

$$\begin{aligned} \hat{y}_3 &= \hat{y}_4 - \tilde{\sigma}^{-1} \hat{i}_3 \\ \hat{\pi}_3 &= \kappa_y \hat{y}_3 \end{aligned} \quad (35)$$

\vdots

For $t>2$:

$$\begin{aligned} \hat{y}_t &= \hat{y}_{t+1} - \tilde{\sigma}^{-1} \hat{i}_t \\ \hat{\pi}_t &= \kappa_y \hat{y}_t \end{aligned} \quad (36)$$

Period 3 Phillips curve along with assumption (B3) allows us to solve for \hat{y}_3 as follows:

$$\begin{aligned}\hat{\pi}_3 &= \kappa_y \hat{y}_3 \\ 0 &= \kappa_y \hat{y}_3 \\ \implies \hat{y}_3 &= 0\end{aligned}\tag{37}$$

Substituting \hat{y}_3 into period 2 IS curve (eqn 34) implies:

$$\hat{y}_2 = \hat{y}_3 = 0\tag{38}$$

Substituting \hat{y}_2 into period 1 IS curve (eqn 33) gives \hat{y}_1 :

$$\hat{y}_1 = \hat{g}_1\tag{39}$$

Finally, substituting \hat{y}_1 into period 1 Phillips curve (eqn 33) gives:

$$\hat{\pi}_1 = (\kappa_y - \kappa_g) \hat{g}_1 + \kappa_\tau \tau_1\tag{40}$$

Further, we can solve for output in each period $t > 2$ using the Phillips curve for that period along with assumption (B3). For $t > 2$, we get $\hat{y}_t = 0$ as $\hat{\pi}_t = 0$.

Sticky-information model

$$\begin{aligned}\hat{y}_t &= \mathbb{E}_t \hat{y}_{t+1} - \tilde{\sigma}^{-1} (\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{r}_t^e) + (\hat{g}_t - \mathbb{E}_t \hat{g}_{t+1}) \\ \hat{\pi}_t &= \frac{\lambda}{1-\lambda} (\tilde{\kappa}_y \hat{y}_t - \tilde{\kappa}_g \hat{g}_t + \tilde{\kappa}_\tau \hat{\tau}_t) \\ &\quad + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \mathbb{E}_{t-1-j} (\hat{\pi}_t + [\tilde{\kappa}_y (\hat{y}_t - \hat{y}_{t-1}) - \tilde{\kappa}_g (\hat{g}_t - \hat{g}_{t-1}) + \tilde{\kappa}_\tau (\hat{\tau}_t - \hat{\tau}_{t-1})])\end{aligned}\tag{41}$$

Assumptions (B1)-(B4) imply that:

For $t=1$

$$\begin{aligned}\hat{y}_1 &= \hat{y}_2 + \tilde{\sigma}^{-1} \hat{\pi}_2 + \hat{g}_1 \\ \hat{\pi}_1 &= \frac{\lambda}{1-\lambda} (\tilde{\kappa}_y \hat{y}_1 - \tilde{\kappa}_g \hat{g}_1 + \tilde{\kappa}_\tau \hat{\tau}_1)\end{aligned}\tag{42}$$

For $t=2$:

$$\begin{aligned}\hat{y}_2 &= \hat{y}_3 \\ \hat{\pi}_2 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_2 + \lambda (\hat{\pi}_2 + [\tilde{\kappa}_y (\hat{y}_2 - \hat{y}_1) + \tilde{\kappa}_g \hat{g}_1 - \tilde{\kappa}_\tau \hat{\tau}_1])\end{aligned}\tag{43}$$

For $t=3$:

$$\begin{aligned}\hat{y}_3 &= \hat{y}_4 - \tilde{\sigma}^{-1} \hat{i}_3 \\ \hat{\pi}_3 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_3 + [\lambda + \lambda(1-\lambda)] (\hat{\pi}_3 + \tilde{\kappa}_y (\hat{y}_3 - \hat{y}_2))\end{aligned}\tag{44}$$

For $t=4$:

$$\begin{aligned}\hat{y}_4 &= \hat{y}_5 - \tilde{\sigma}^{-1} \hat{i}_4 \\ \hat{\pi}_4 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_4 + [\lambda + \lambda(1-\lambda) + \lambda(1-\lambda)^2] (\hat{\pi}_4 + \tilde{\kappa}_y (\hat{y}_4 - \hat{y}_3))\end{aligned}\tag{45}$$

\vdots

For t>2:

$$\begin{aligned}\hat{y}_t &= \hat{y}_{t+1} - \tilde{\sigma}^{-1} \hat{t}_t \\ \hat{\pi}_t &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_t \\ &\quad + \lambda \sum_{j=0}^{t-2} (1-\lambda)^j (\hat{\pi}_t + \tilde{\kappa}_y (\hat{y}_t - \hat{y}_{t-1}))\end{aligned}\tag{46}$$

Period 3 Phillips curve (eqn 44) along with assumption (B3) allows us to solve for \hat{y}_3 as follows:

$$\begin{aligned}\hat{\pi}_3 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_3 + \lambda (\hat{\pi}_3 + \tilde{\kappa}_y (\hat{y}_3 - \hat{y}_2)) + \lambda(1-\lambda) (\hat{\pi}_3 + \tilde{\kappa}_y (\hat{y}_3 - \hat{y}_2)) \\ 0 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_3 + \lambda \mathbb{E}_{t-1} (0 + \tilde{\kappa}_y (\hat{y}_3 - \hat{y}_2)) + \lambda(1-\lambda) (0 + \tilde{\kappa}_y (\hat{y}_3 - \hat{y}_2)) \\ \hat{y}_3 &= \hat{y}_2 \left[\frac{\lambda + \lambda(1-\lambda)}{\frac{\lambda}{1-\lambda} + \lambda + \lambda(1-\lambda)} \right]\end{aligned}\tag{47}$$

Substituting \hat{y}_3 into period 2 IS curve (eqn 43) gives the following equation for \hat{y}_2 :

$$\hat{y}_2 = \hat{y}_2 \left[\frac{\lambda + \lambda(1-\lambda)}{\frac{\lambda}{1-\lambda} + \lambda + \lambda(1-\lambda)} \right]\tag{48}$$

Since $\lambda > 0$, the only solution to this equation is $\hat{y}_2 = 0$.

Substituting $\hat{y}_2 = 0$ into period 1 IS curve and period 2 Phillips curve gives:

$$\begin{aligned}\hat{y}_1 &= \tilde{\sigma}^{-1} \hat{\pi}_2 + \hat{g}_1 \\ \hat{\pi}_2 &= \lambda (\hat{\pi}_2 + [-\tilde{\kappa}_y \hat{y}_1 + \tilde{\kappa}_g \hat{g}_1 - \tilde{\kappa}_\tau \hat{t}_1])\end{aligned}\tag{49}$$

Solving these simultaneously gives \hat{y}_1 and $\hat{\pi}_2$:

$$\begin{aligned}\hat{y}_1 &= \left(\frac{1 + \tilde{\sigma}^{-1} \left(\frac{\lambda}{1-\lambda} \right) \tilde{\kappa}_g}{1 + \tilde{\sigma}^{-1} \left(\frac{\lambda}{1-\lambda} \right) \tilde{\kappa}_y} \right) \hat{g}_s - \left(\frac{\tilde{\sigma}^{-1} \left(\frac{\lambda}{1-\lambda} \right) \tilde{\kappa}_\tau}{1 + \tilde{\sigma}^{-1} \left(\frac{\lambda}{1-\lambda} \right) \tilde{\kappa}_y} \right) \hat{t}_s \\ \hat{\pi}_2 &= \left(\frac{\lambda}{1-\lambda} \right) \left[\tilde{\kappa}_g - \tilde{\kappa}_y \left(\frac{1 + \tilde{\sigma}^{-1} \left(\frac{\lambda}{1-\lambda} \right) \tilde{\kappa}_g}{1 + \tilde{\sigma}^{-1} \left(\frac{\lambda}{1-\lambda} \right) \tilde{\kappa}_y} \right) \right] \hat{g}_s + \left(\frac{\lambda}{1-\lambda} \right) \left[\tilde{\kappa}_y \left(\frac{\tilde{\sigma}^{-1} \left(\frac{\lambda}{1-\lambda} \right) \tilde{\kappa}_\tau}{1 + \tilde{\sigma}^{-1} \left(\frac{\lambda}{1-\lambda} \right) \tilde{\kappa}_y} \right) - \tilde{\kappa}_\tau \right] \hat{t}_s\end{aligned}\tag{50}$$

Substituting \hat{y}_1 into period 1 Phillips curve (eqn 42) gives $\hat{\pi}_1$:

$$\hat{\pi}_1 = \left(\frac{-\lambda}{1-\lambda} \right) \left[\tilde{\kappa}_g - \tilde{\kappa}_y \left(\frac{1 + \tilde{\sigma}^{-1} \left(\frac{\lambda}{1-\lambda} \right) \tilde{\kappa}_g}{1 + \tilde{\sigma}^{-1} \left(\frac{\lambda}{1-\lambda} \right) \tilde{\kappa}_y} \right) \right] \hat{g}_s + \left(\frac{-\lambda}{1-\lambda} \right) \left[\tilde{\kappa}_y \left(\frac{\tilde{\sigma}^{-1} \left(\frac{\lambda}{1-\lambda} \right) \tilde{\kappa}_\tau}{1 + \tilde{\sigma}^{-1} \left(\frac{\lambda}{1-\lambda} \right) \tilde{\kappa}_y} \right) - \tilde{\kappa}_\tau \right] \hat{t}_s\tag{51}$$

Further, from period 4 Phillips curve along with assumption (B3), and solution $\hat{y}_3 = 0$ we get:

$$\begin{aligned}0 &= \frac{\lambda}{1-\lambda} (\tilde{\kappa}_y \hat{y}_4) + [\lambda + \lambda(1-\lambda) + \lambda(1-\lambda)^2] (0 + \tilde{\kappa}_y (\hat{y}_4 - \hat{y}_3)) \\ \implies \hat{y}_4 &= \left[\frac{\lambda + \lambda(1-\lambda) + \lambda(1-\lambda)^2}{\frac{\lambda}{1-\lambda} + \lambda + \lambda(1-\lambda) + \lambda(1-\lambda)^2} \right] \hat{y}_3 \\ &= 0\end{aligned}\tag{52}$$

Similarly, we can solve for the transition dynamics of output in each period $t > 2$ using the Phillips curve for that period along with the assumption (B3). For $t > 2$, we get:

$$\begin{aligned}
0 &= \frac{\lambda}{1-\lambda} (\tilde{\kappa}_y \hat{y}_t) + \lambda \sum_{j=0}^{t-2} (1-\lambda)^j (0 + \tilde{\kappa}_y (\hat{y}_t - \hat{y}_{t-1})) \\
0 &= \left[\frac{\lambda}{1-\lambda} + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \right] \tilde{\kappa}_y \hat{y}_t - \left[\lambda \sum_{j=0}^{t-2} (1-\lambda)^j \right] \tilde{\kappa}_y \hat{y}_{t-1} \\
\implies \hat{y}_t &= \left[\frac{\lambda \sum_{j=0}^{t-2} (1-\lambda)^j}{\frac{\lambda}{1-\lambda} + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j} \right] \hat{y}_{t-1} \\
&= 0
\end{aligned} \tag{53}$$

C Calibration parameters

Table 5: Calibration parameters (Numerical Example)

Common parameters	Description	Value
β	Discount factor	0.9970
σ	Risk aversion parameter	1.032
η	Frisch elasticity of labor supply	1.7415
ω	Probability of price non-adjustment	0.66
λ	Probability of updating information	0.15
θ	CES parameter	13.6012
ϕ	Relative weight on output stabilization	0
ψ	Steady state government spending to output ratio	0.2
$\bar{\tau}$	Steady state tax rate	0.1
\hat{r}_s^e	Generates 10% drop in output, 2% deflation	-0.0225
ρ_{rn}	Makes ZLB bind for around 16 quarters	0.88
Sticky-price parameter		Value
κ		0.0841
κ_y		0.0103
κ_g		0.0044
κ_τ		0.0038
$\tilde{\sigma}$		1.29
Sticky-information parameter		Value
$\tilde{\kappa}_y$		0.1228
$\tilde{\kappa}_g$		0.0523
$\tilde{\kappa}_\tau$		0.0450
$\tilde{\sigma}$		1.29

Parameter details:

$$\begin{aligned}
\tilde{\sigma} &= \frac{\sigma}{1-\psi}, \quad \psi = \frac{G}{Y}, \quad \kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega}, \quad \kappa_y = \kappa(\tilde{\sigma} + \eta), \quad \kappa_g = \kappa\tilde{\sigma}, \quad \kappa_\tau = \frac{\kappa}{(1-\bar{\tau})}, \\
\tilde{\kappa}_y &= \frac{\tilde{\sigma} + \eta}{1+\eta\tilde{\sigma}}, \quad \tilde{\kappa}_g = \frac{\tilde{\sigma}}{1+\eta\tilde{\sigma}}, \quad \tilde{\kappa}_\tau = \frac{1}{(1+\eta\tilde{\sigma})(1-\bar{\tau})}, \quad \hat{y}_t = \frac{Y_t - \bar{Y}}{Y}, \quad \hat{g}_t = \frac{G_t - \bar{G}}{Y}, \\
\hat{\tau}_t &= \tau_t - \bar{\tau}, \quad \hat{\pi}_t = \hat{p}_t - \hat{p}_{t-1}, \quad \hat{i}_t = \frac{i_t - \bar{i}}{1+\bar{i}}, \quad \hat{r}_t^e = \hat{\xi}_t - \hat{\xi}_{t+1} = \frac{r_t^e - \bar{i}}{1+\bar{r}^e}
\end{aligned}$$

The only calibration parameters that are different in the analytical example (used to plot figures 1 and 3) are:

Sticky Information:

$$\lambda = 0.29, \quad \hat{r}_s^e = -0.079, \quad \rho_{rn} = 0$$

Sticky Price:

$$\omega = 0.53, \quad \hat{r}_s^e = -0.132, \quad \rho_{rn} = 0$$

D Coordinated Interest Rate Peg Experiment (CPEG-EX)

The Coordinated-PEG experiment makes the following assumptions:

C1.(Shock): $\hat{r}_t^e = 0 \quad \forall t, \quad \hat{i}_1 = 0$

C2.(Fiscal policy): $(\hat{g}_1, \hat{\tau}_1) = (\hat{g}_s, \hat{\tau}_s), \quad (\hat{g}_2, \hat{\tau}_2) = (0, 0)$

C3.(Monetary policy): $\hat{\pi}_t = 0$ for $t > 1$

C4.(Perfect foresight): $\mathbb{E}_t \hat{y}_{t+1} = \hat{y}_{t+1}, \quad \mathbb{E}_t \hat{\pi}_{t+1} = \hat{\pi}_{t+1} \quad \forall t$

Sticky-price model

$$\begin{aligned} \hat{y}_t &= \mathbb{E}_t \hat{y}_{t+1} - \tilde{\sigma}^{-1} (\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{r}_t^e) + (\hat{g}_t - \mathbb{E}_t \hat{g}_{t+1}) \\ \hat{\pi}_t &= \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa_y \hat{y}_t - \kappa_g \hat{g}_t + \kappa_\tau \hat{\tau}_t \end{aligned} \quad (54)$$

Assumptions (C1)-(C4) imply that:

For t=1

$$\begin{aligned} \hat{y}_1 &= \hat{y}_2 + \hat{g}_1 \\ \hat{\pi}_1 &= \kappa_y \hat{y}_1 - \kappa_g \hat{g}_1 + \kappa_\tau \hat{\tau}_1 \end{aligned} \quad (55)$$

For t=2:

$$\begin{aligned} \hat{y}_2 &= \hat{y}_3 - \tilde{\sigma}^{-1} \hat{i}_2 \\ \hat{\pi}_2 &= \kappa_y \hat{y}_2 \end{aligned} \quad (56)$$

For t=3:

$$\begin{aligned} \hat{y}_3 &= \hat{y}_4 - \tilde{\sigma}^{-1} \hat{i}_3 \\ \hat{\pi}_3 &= \kappa_y \hat{y}_3 \end{aligned} \quad (57)$$

⋮

For t>2:

$$\begin{aligned} \hat{y}_t &= \hat{y}_{t+1} - \tilde{\sigma}^{-1} \hat{i}_t \\ \hat{\pi}_t &= \kappa_y \hat{y}_t \end{aligned} \quad (58)$$

Applying (C3) to period 2 Phillips curve (eqn 56) allows us to solve for \hat{y}_2 as follows:

$$\begin{aligned} \hat{\pi}_2 &= \kappa_y \hat{y}_2 \\ 0 &= \kappa_y \hat{y}_2 \\ \implies \hat{y}_2 &= 0 \end{aligned} \quad (59)$$

Substituting \hat{y}_2 into period 1 IS curve (eqn 55) gives:

$$\hat{y}_1 = \hat{g}_1 \quad (60)$$

Substituting \hat{y}_1 in period 1 Phillips curve (eqn 55) gives:

$$\hat{\pi}_1 = (\kappa_y - \kappa_g) \hat{g}_1 + \kappa_\tau \hat{\tau}_1 \quad (61)$$

Output for every period $t > 2$ can be solved from that period's Phillips curve using assumption (C3). We get that for $t > 2$, $\hat{y}_t = \hat{\pi}_t = 0$.

Sticky-information model

$$\begin{aligned} \hat{y}_t &= \mathbb{E}_t \hat{y}_{t+1} - \tilde{\sigma}^{-1} (\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{r}_t^e) + (\hat{g}_t - \mathbb{E}_t \hat{g}_{t+1}) \\ \hat{\pi}_t &= \frac{\lambda}{1-\lambda} (\tilde{\kappa}_y \hat{y}_t - \tilde{\kappa}_g \hat{g}_t + \tilde{\kappa}_\tau \hat{\tau}_t) \\ &\quad + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \mathbb{E}_{t-1-j} (\hat{\pi}_t + [\tilde{\kappa}_y (\hat{y}_t - \hat{y}_{t-1}) - \tilde{\kappa}_g (\hat{g}_t - \hat{g}_{t-1}) + \tilde{\kappa}_\tau (\hat{\tau}_t - \hat{\tau}_{t-1})]) \end{aligned} \quad (62)$$

Assumptions (C1)-(C4) imply that:

For $t=1$

$$\begin{aligned} \hat{y}_1 &= \hat{y}_2 + \hat{g}_1 \\ \hat{\pi}_1 &= \frac{\lambda}{1-\lambda} (\tilde{\kappa}_y \hat{y}_1 - \tilde{\kappa}_g \hat{g}_1 + \tilde{\kappa}_\tau \hat{\tau}_1) \end{aligned} \quad (63)$$

For $t=2$:

$$\begin{aligned} \hat{y}_2 &= \hat{y}_3 - \tilde{\sigma}^{-1} \hat{i}_2 \\ \hat{\pi}_2 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_2 + \lambda (\hat{\pi}_2 + [\tilde{\kappa}_y (\hat{y}_2 - \hat{y}_1) + \tilde{\kappa}_g \hat{g}_1 - \tilde{\kappa}_\tau \hat{\tau}_1]) \end{aligned} \quad (64)$$

For $t=3$:

$$\begin{aligned} \hat{y}_3 &= \hat{y}_4 - \tilde{\sigma}^{-1} \hat{i}_3 \\ \hat{\pi}_3 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_3 + [\lambda + \lambda(1-\lambda)] (\hat{\pi}_2 + \tilde{\kappa}_y (\hat{y}_3 - \hat{y}_2)) \end{aligned} \quad (65)$$

For $t=4$:

$$\begin{aligned} \hat{y}_4 &= \hat{y}_5 - \tilde{\sigma}^{-1} \hat{i}_4 \\ \hat{\pi}_4 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_4 + [\lambda + \lambda(1-\lambda) + \lambda(1-\lambda)^2] (\hat{\pi}_4 + \tilde{\kappa}_y (\hat{y}_4 - \hat{y}_3)) \end{aligned} \quad (66)$$

\vdots

For $t > 2$:

$$\begin{aligned} \hat{y}_t &= \hat{y}_{t+1} - \tilde{\sigma}^{-1} \hat{i}_t \\ \hat{\pi}_t &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_t \\ &\quad + \lambda \sum_{j=0}^{t-2} (1-\lambda)^j (\hat{\pi}_t + \tilde{\kappa}_y (\hat{y}_t - \hat{y}_{t-1})) \end{aligned} \quad (67)$$

Period 2 Phillips curve (eqn 64) along with assumption (C3) can be used to solve for \hat{y}_2 as follows:

$$\begin{aligned}\hat{\pi}_2 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_2 + \lambda (\hat{\pi}_2 + [\tilde{\kappa}_y (\hat{y}_2 - \hat{y}_1) + \tilde{\kappa}_g \hat{g}_1 - \tilde{\kappa}_\tau \hat{\tau}_1]) \\ 0 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_2 + \lambda (0 + [\tilde{\kappa}_y (\hat{y}_2 - \hat{y}_1) + \tilde{\kappa}_g \hat{g}_1 - \tilde{\kappa}_\tau \hat{\tau}_1]) \\ \hat{y}_2 &= \left(\frac{1-\lambda}{2-\lambda} \right) \left[\hat{y}_1 - \frac{\tilde{\kappa}_g}{\tilde{\kappa}_y} \hat{g}_1 + \frac{\tilde{\kappa}_\tau}{\tilde{\kappa}_y} \hat{\tau}_1 \right]\end{aligned}\quad (68)$$

Substituting \hat{y}_2 into period 1 IS curve (eqn 63) gives \hat{y}_1 :

$$\hat{y}_1 = \left[\frac{(2-\lambda)\tilde{\kappa}_y - (1-\lambda)\tilde{\kappa}_g}{\tilde{\kappa}_y} \right] \hat{g}_1 + \left[\frac{(1-\lambda)\tilde{\kappa}_\tau}{\tilde{\kappa}_y} \right] \hat{\tau}_1 \quad (69)$$

Proposition 3 (Government spending multipliers). *In the CPEG-EX, the government spending multiplier in the sticky-information model is bounded below by 1 while it is equal to 1 in the sticky-price model.*

$$\begin{aligned}\left(\frac{d\hat{y}_1}{d\hat{g}_1} \right)^{SI} &= \frac{(2-\lambda)\tilde{\kappa}_y - (1-\lambda)\tilde{\kappa}_g}{\tilde{\kappa}_y} \geq 1 \\ \left(\frac{d\hat{y}_1}{d\hat{g}_1} \right)^{SP} &= 1\end{aligned}$$

Proposition 4 (Paradox of toil). *In the CPEG-EX, tax cuts are contractionary in the sticky-information model but have no effect in the sticky-price model.*

$$\begin{aligned}\left(\frac{d\hat{y}_1}{d\hat{\tau}_1} \right)^{SI} &= \frac{(1-\lambda)\tilde{\kappa}_\tau}{\tilde{\kappa}_y} \geq 0 \\ \left(\frac{d\hat{y}_1}{d\hat{\tau}_1} \right)^{SP} &= 0\end{aligned}$$

Numerical results: ZLB-EX vs CPEG-EX

Table 6: Fiscal multipliers under sticky-prices

	Government spending	Tax cut
ZLB-EX	1.63	-0.4
CPEG-EX	1.04	-0.03

Table 7: Fiscal multipliers under sticky-information

	Government spending	Tax cut
ZLB-EX	4.77	-2.41
CPEG-EX	5.14	-2.64

E ZLB-EX: General Targeting Rule

Sticky-price model

The ZLB-EX with general targeting rule makes the following assumptions:

A1.(Shock): $\hat{r}_1^e < 0$, $\hat{i}_1 = \frac{-\bar{i}}{1+\bar{i}}$, $\hat{r}_t^e = 0$ for $t > 1$

A2.(Fiscal policy): $(\hat{g}_1, \hat{\tau}_1) = (\hat{g}_s, \hat{\tau}_s)$, $(\hat{g}_2, \hat{\tau}_2) = (0, 0)$

A3alt.(Monetary policy): $\hat{\pi}_t + \phi \hat{y}_t = 0$ for $t > 1$

A4.(Perfect foresight): $\mathbb{E}_t \hat{y}_{t+1} = \hat{y}_{t+1}$, $\mathbb{E}_t \hat{\pi}_{t+1} = \hat{\pi}_{t+1} \quad \forall t$

Sticky-price model

$$\begin{aligned}\hat{y}_t &= \mathbb{E}_t \hat{y}_{t+1} - \tilde{\sigma}^{-1} (\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{r}_t^e) + (\hat{g}_t - \mathbb{E}_t \hat{g}_{t+1}) \\ \hat{\pi}_t &= \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa_y \hat{y}_t - \kappa_g \hat{g}_t + \kappa_\tau \hat{\tau}_t\end{aligned}\tag{70}$$

Assumptions A1, A2, A3alt and A4 imply that:

For $t=1$:

$$\begin{aligned}\hat{y}_1 &= \hat{y}_2 + \frac{\tilde{\sigma}^{-1} \bar{i}}{1 + \bar{i}} + \tilde{\sigma}^{-1} \hat{r}_1^e + \hat{g}_1 \\ \hat{\pi}_1 &= \kappa_y \hat{y}_1 - \kappa_g \hat{g}_1 + \kappa_\tau \hat{\tau}_1\end{aligned}\tag{71}$$

For $t=2$:

$$\begin{aligned}\hat{y}_2 &= \hat{y}_3 - \tilde{\sigma}^{-1} \hat{i}_2 \\ \hat{\pi}_2 &= \kappa_y \hat{y}_2\end{aligned}\tag{72}$$

For $t=3$:

$$\begin{aligned}\hat{y}_3 &= \hat{y}_4 - \tilde{\sigma}^{-1} \hat{i}_3 \\ \hat{\pi}_3 &= \kappa_y \hat{y}_3\end{aligned}\tag{73}$$

\vdots

For any $t > 2$:

$$\begin{aligned}\hat{y}_t &= \hat{y}_{t+1} - \tilde{\sigma}^{-1} \hat{i}_t \\ \hat{\pi}_t &= \kappa_y \hat{y}_t\end{aligned}\tag{74}$$

Applying (A3alt) to period 2 Phillips curve (eqn 72) allows us to solve for \hat{y}_2 as follows:

$$\begin{aligned}\hat{\pi}_2 &= \kappa_y \hat{y}_2 \\ 0 &= \kappa_y \hat{y}_2 \\ \implies \hat{y}_2 &= 0\end{aligned}\tag{75}$$

Substituting \hat{y}_2 into period 1 IS curve (eqn 71) gives:

$$\hat{y}_1 = \frac{\tilde{\sigma}^{-1} \bar{i}}{1 + \bar{i}} + \tilde{\sigma}^{-1} \hat{r}_1^e + \hat{g}_1\tag{76}$$

Substituting \hat{y}_1 in period 1 Phillips curve (eqn 71) gives:

$$\hat{\pi}_1 = (\kappa_y - \kappa_g) \hat{g}_1 + \frac{\kappa_y \tilde{\sigma}^{-1} \bar{i}}{1 + \bar{i}} + \kappa_y \tilde{\sigma}^{-1} \hat{r}_1^e + \kappa_\tau \hat{\tau}_1\tag{77}$$

Output for every period $t > 2$ can be solved from that period's Phillips curve using assumption (A3). We get that for $t > 2$, $\hat{y}_t = \hat{\pi}_t = 0$.

Sticky-information model

$$\begin{aligned}\hat{y}_t &= \mathbb{E}_t \hat{y}_{t+1} - \tilde{\sigma}^{-1} (\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{r}_t^e) + (\hat{g}_t - \mathbb{E}_t \hat{g}_{t+1}) \\ \hat{\pi}_t &= \frac{\lambda}{1-\lambda} (\tilde{\kappa}_y \hat{y}_t - \tilde{\kappa}_g \hat{g}_t + \tilde{\kappa}_\tau \hat{\tau}_t) \\ &\quad + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \mathbb{E}_{t-1-j} (\hat{\pi}_t + [\tilde{\kappa}_y (\hat{y}_t - \hat{y}_{t-1}) - \tilde{\kappa}_g (\hat{g}_t - \hat{g}_{t-1}) + \tilde{\kappa}_\tau (\hat{\tau}_t - \hat{\tau}_{t-1})])\end{aligned}\tag{78}$$

Assumptions A1, A2, A3alt and A4 imply that:

For $t=1$

$$\begin{aligned}\hat{y}_1 &= \hat{y}_2 + \frac{\tilde{\sigma}^{-1} \bar{i}}{1+\bar{i}} + \tilde{\sigma}^{-1} \hat{r}_1^e + \tilde{\sigma}^{-1} \hat{\pi}_2 + \hat{g}_1 \\ \hat{\pi}_1 &= \frac{\lambda}{1-\lambda} (\tilde{\kappa}_y \hat{y}_1 - \tilde{\kappa}_g \hat{g}_1 + \tilde{\kappa}_\tau \hat{\tau}_1)\end{aligned}\tag{79}$$

For $t=2$:

$$\begin{aligned}\hat{y}_2 &= \hat{y}_3 - \tilde{\sigma}^{-1} \hat{i}_2 + \tilde{\sigma}^{-1} \hat{\pi}_3 \\ \hat{\pi}_2 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_2 + \lambda (\hat{\pi}_2 + [\tilde{\kappa}_y (\hat{y}_2 - \hat{y}_1) + \tilde{\kappa}_g \hat{g}_1 - \tilde{\kappa}_\tau \hat{\tau}_1])\end{aligned}\tag{80}$$

For $t=3$:

$$\begin{aligned}\hat{y}_3 &= \hat{y}_4 - \tilde{\sigma}^{-1} \hat{i}_3 + \tilde{\sigma}^{-1} \hat{\pi}_4 \\ \hat{\pi}_3 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_3 + [\lambda + \lambda(1-\lambda)] (\hat{\pi}_3 + \tilde{\kappa}_y (\hat{y}_3 - \hat{y}_2))\end{aligned}\tag{81}$$

For $t=4$:

$$\begin{aligned}\hat{y}_4 &= \hat{y}_5 - \tilde{\sigma}^{-1} \hat{i}_4 + \tilde{\sigma}^{-1} \hat{\pi}_5 \\ \hat{\pi}_4 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_4 + [\lambda + \lambda(1-\lambda) + \lambda(1-\lambda)^2] (\hat{\pi}_4 + \tilde{\kappa}_y (\hat{y}_4 - \hat{y}_3))\end{aligned}\tag{82}$$

\vdots

For $t > 2$:

$$\begin{aligned}\hat{y}_t &= \hat{y}_{t+1} - \tilde{\sigma}^{-1} \hat{i}_t + \tilde{\sigma}^{-1} \hat{\pi}_{t+1} \\ \hat{\pi}_t &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_t \\ &\quad + \lambda \sum_{j=0}^{t-2} (1-\lambda)^j (\hat{\pi}_t + \tilde{\kappa}_y (\hat{y}_t - \hat{y}_{t-1}))\end{aligned}\tag{83}$$

Period 2 Phillips curve (eqn 80) along with assumption (A3alt) can be used to solve for $\hat{\pi}_2$ as follows:

$$\begin{aligned}
\hat{\pi}_2 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_2 + \lambda (\hat{\pi}_2 + [\tilde{\kappa}_y (\hat{y}_2 - \hat{y}_1) + \tilde{\kappa}_g \hat{g}_1 - \tilde{\kappa}_\tau \hat{\tau}_1]) \\
(1-\lambda)\hat{\pi}_2 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_2 + \lambda [\tilde{\kappa}_y (\hat{y}_2 - \hat{y}_1) + \tilde{\kappa}_g \hat{g}_1 - \tilde{\kappa}_\tau \hat{\tau}_1] \\
(1-\lambda)\hat{\pi}_2 &= \left[\frac{\lambda}{1-\lambda} + \lambda \right] \tilde{\kappa}_y \hat{y}_2 - \lambda [\tilde{\kappa}_y \hat{y}_1 - \tilde{\kappa}_g \hat{g}_1 + \tilde{\kappa}_\tau \hat{\tau}_1] \\
(1-\lambda)\hat{\pi}_2 &= \left[\frac{\lambda}{1-\lambda} + \lambda \right] \tilde{\kappa}_y (-\phi^{-1} \hat{\pi}_2) - \lambda [\tilde{\kappa}_y \hat{y}_1 - \tilde{\kappa}_g \hat{g}_1 + \tilde{\kappa}_\tau \hat{\tau}_1] \quad (84) \\
\left[(1-\lambda) + \left(\frac{\lambda}{1-\lambda} + \lambda \right) \tilde{\kappa}_y \phi^{-1} \right] \hat{\pi}_2 &= -\lambda [\tilde{\kappa}_y \hat{y}_1 - \tilde{\kappa}_g \hat{g}_1 + \tilde{\kappa}_\tau \hat{\tau}_1] \\
\left[(1-\lambda)^2 + \lambda(2-\lambda) \tilde{\kappa}_y \phi^{-1} \right] \hat{\pi}_2 &= -\lambda(1-\lambda) [\tilde{\kappa}_y \hat{y}_1 - \tilde{\kappa}_g \hat{g}_1 + \tilde{\kappa}_\tau \hat{\tau}_1] \\
\hat{\pi}_2 &= \frac{-\lambda(1-\lambda) [\tilde{\kappa}_y \hat{y}_1 - \tilde{\kappa}_g \hat{g}_1 + \tilde{\kappa}_\tau \hat{\tau}_1]}{[(1-\lambda)^2 + \lambda(2-\lambda) \tilde{\kappa}_y \phi^{-1}]}
\end{aligned}$$

Substituting $\hat{\pi}_2$ and (A3alt) into period 1 IS curve (eqn 79) gives \hat{y}_1 :

$$\begin{aligned}
\hat{y}_1 &= -\phi^{-1} \hat{\pi}_2 + \tilde{\sigma}^{-1} \hat{\pi}_2 + \hat{g}_1 + \tilde{\sigma}^{-1} \left(\frac{\bar{i}}{1+\bar{i}} + \hat{r}_1^e \right) \\
\hat{y}_1 &= (\tilde{\sigma}^{-1} - \phi^{-1}) \left[\frac{-\lambda(1-\lambda) [\tilde{\kappa}_y \hat{y}_1 - \tilde{\kappa}_g \hat{g}_1 + \tilde{\kappa}_\tau \hat{\tau}_1]}{[(1-\lambda)^2 + \lambda(2-\lambda) \tilde{\kappa}_y \phi^{-1}]} \right] + \hat{g}_1 + \tilde{\sigma}^{-1} \left(\frac{\bar{i}}{1+\bar{i}} + \hat{r}_1^e \right) \quad (85) \\
\hat{y}_1 &= (1 - \tilde{\sigma}\phi^{-1}) \left[\frac{-\lambda(1-\lambda) [\tilde{\kappa}_y \hat{y}_1 - \tilde{\kappa}_g \hat{g}_1 + \tilde{\kappa}_\tau \hat{\tau}_1]}{\tilde{\sigma} [(1-\lambda)^2 + \lambda(2-\lambda) \tilde{\kappa}_y \phi^{-1}]} \right] + \hat{g}_1 + \tilde{\sigma}^{-1} \left(\frac{\bar{i}}{1+\bar{i}} + \hat{r}_1^e \right) \\
\hat{y}_1 \left[1 + \frac{(1 - \tilde{\sigma}\phi^{-1})\lambda(1-\lambda)\tilde{\kappa}_y}{\tilde{\sigma} [(1-\lambda)^2 + \lambda(2-\lambda) \tilde{\kappa}_y \phi^{-1}]} \right] &= \left[1 + \frac{(1 - \tilde{\sigma}\phi^{-1})\lambda(1-\lambda)\tilde{\kappa}_g}{\tilde{\sigma} [(1-\lambda)^2 + \lambda(2-\lambda) \tilde{\kappa}_y \phi^{-1}]} \right] \hat{g}_1 \\
&\quad - \frac{(1 - \tilde{\sigma}\phi^{-1})\lambda(1-\lambda)\tilde{\kappa}_\tau}{\tilde{\sigma} [(1-\lambda)^2 + \lambda(2-\lambda) \tilde{\kappa}_y \phi^{-1}]} \hat{\tau}_1 \\
&\quad + \tilde{\sigma}^{-1} \left(\frac{\bar{i}}{1+\bar{i}} + \hat{r}_1^e \right)
\end{aligned}$$

$$\begin{aligned}
\hat{y}_1 \left[\tilde{\sigma} \left[(1-\lambda)^2 + \lambda(2-\lambda) \tilde{\kappa}_y \phi^{-1} \right] + (1 - \tilde{\sigma}\phi^{-1})\lambda(1-\lambda)\tilde{\kappa}_y \right] &= \hat{g}_1 \left[\tilde{\sigma} \left[(1-\lambda)^2 + \lambda(2-\lambda) \tilde{\kappa}_y \phi^{-1} \right] \right. \\
&\quad \left. + (1 - \tilde{\sigma}\phi^{-1})\lambda(1-\lambda)\tilde{\kappa}_g \right. \\
&\quad \left. - \hat{\tau}_1 (1 - \tilde{\sigma}\phi^{-1})\lambda(1-\lambda)\tilde{\kappa}_\tau \right. \\
&\quad \left. + \tilde{\sigma}^{-1} \tilde{\sigma} \left[(1-\lambda)^2 + \lambda(2-\lambda) \tilde{\kappa}_y \phi^{-1} \right] \left(\frac{\bar{i}}{1+\bar{i}} + \hat{r}_1^e \right) \right]
\end{aligned}$$

$$\begin{aligned}\hat{y}_1 \left[\tilde{\sigma}(1-\lambda)^2 + \lambda \tilde{\kappa}_y (1-\lambda + \tilde{\sigma}\phi^{-1}) \right] &= \hat{g}_1 \left[\tilde{\sigma}(1-\lambda)^2 + \tilde{\sigma}\lambda\phi^{-1} [(2-\lambda)\tilde{\kappa}_y - (1-\lambda)\tilde{\kappa}_g] + \lambda(1-\lambda)\tilde{\kappa}_g \right] \\ &\quad - \hat{\tau}_1 (1-\tilde{\sigma}\phi^{-1})\lambda(1-\lambda)\tilde{\kappa}_\tau \\ &\quad + \tilde{\sigma}^{-1}\tilde{\sigma} \left[(1-\lambda)^2 + \lambda(2-\lambda)\tilde{\kappa}_y\phi^{-1} \right] \left(\frac{\bar{i}}{1+\bar{i}} + \hat{r}_1^e \right)\end{aligned}$$

$$\begin{aligned}\hat{y}_1 &= \hat{g}_1 \frac{\tilde{\sigma}(1-\lambda)^2 + \tilde{\sigma}\lambda\phi^{-1} [(2-\lambda)\tilde{\kappa}_y - (1-\lambda)\tilde{\kappa}_g] + \lambda(1-\lambda)\tilde{\kappa}_g}{\tilde{\sigma}(1-\lambda)^2 + \lambda\tilde{\kappa}_y(1-\lambda + \tilde{\sigma}\phi^{-1})} \\ &\quad - \hat{\tau}_1 \frac{(1-\tilde{\sigma}\phi^{-1})\lambda(1-\lambda)\tilde{\kappa}_\tau}{\tilde{\sigma}(1-\lambda)^2 + \lambda\tilde{\kappa}_y(1-\lambda + \tilde{\sigma}\phi^{-1})} \\ &\quad + \frac{[(1-\lambda)^2 + \lambda(2-\lambda)\tilde{\kappa}_y\phi^{-1}]}{\tilde{\sigma}(1-\lambda)^2 + \lambda\tilde{\kappa}_y(1-\lambda + \tilde{\sigma}\phi^{-1})} \left(\frac{\bar{i}}{1+\bar{i}} + \hat{r}_1^e \right)\end{aligned}$$

Finally, we get:

$$\begin{aligned}\hat{y}_1 &= \frac{\tilde{\sigma}(1-\lambda)^2 + \tilde{\sigma}\lambda\phi^{-1} [(2-\lambda)\tilde{\kappa}_y - (1-\lambda)\tilde{\kappa}_g] + \lambda(1-\lambda)\tilde{\kappa}_g}{\tilde{\sigma}(1-\lambda)^2 + \lambda\tilde{\kappa}_y(1-\lambda + \tilde{\sigma}\phi^{-1})} \hat{g}_1 \\ &\quad - \frac{(1-\tilde{\sigma}\phi^{-1})\lambda(1-\lambda)\tilde{\kappa}_\tau}{\tilde{\sigma}(1-\lambda)^2 + \lambda\tilde{\kappa}_y(1-\lambda + \tilde{\sigma}\phi^{-1})} \hat{\tau}_1 \\ &\quad + \frac{[(1-\lambda)^2 + \lambda(2-\lambda)\tilde{\kappa}_y\phi^{-1}]}{\tilde{\sigma}(1-\lambda)^2 + \lambda\tilde{\kappa}_y(1-\lambda + \tilde{\sigma}\phi^{-1})} \left(\frac{\bar{i}}{1+\bar{i}} + \hat{r}_1^e \right)\end{aligned}\tag{86}$$

F PEG-EX: General Targeting Rule

The PEG experiment with general targeting rule makes the following assumptions:

D1.(Shock): $\hat{r}_t^e = 0 \quad \forall t, \quad \hat{i}_1 = \hat{i}_2 = 0$

D2.(Fiscal policy): $(\hat{g}_1, \hat{\tau}_1) = (\hat{g}_s, \hat{\tau}_s), \quad (\hat{g}_2, \hat{\tau}_2) = (0, 0)$

D3alt.(Monetary policy): $\hat{\pi}_t + \phi y_t = 0$ for $t > 2$

D4.(Perfect foresight): $\mathbb{E}_t \hat{y}_{t+1} = \hat{y}_{t+1}, \quad \mathbb{E}_t \hat{\pi}_{t+1} = \hat{\pi}_{t+1} \quad \forall t$

Sticky-price model

$$\begin{aligned}\hat{y}_t &= \mathbb{E}_t \hat{y}_{t+1} - \tilde{\sigma}^{-1} (\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{r}_t^e) + (\hat{g}_t - \mathbb{E}_t \hat{g}_{t+1}) \\ \hat{\pi}_t &= \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa_y \hat{y}_t - \kappa_g \hat{g}_t + \kappa_\tau \hat{\tau}_t\end{aligned}\tag{87}$$

Assumptions D1, D2, D3alt and D4 imply that:

For $t=1$

$$\begin{aligned}\hat{y}_1 &= \hat{y}_2 + \tilde{\sigma}^{-1} \hat{\pi}_2 + \hat{g}_1 \\ \hat{\pi}_1 &= \beta \hat{\pi}_2 + \kappa_y \hat{y}_1 - \kappa_g \hat{g}_1 + \kappa_\tau \hat{\tau}_1\end{aligned}\tag{88}$$

For t=2:

$$\begin{aligned}\hat{y}_2 &= \hat{y}_3 + \tilde{\sigma}^{-1} \hat{\pi}_3 \\ \hat{\pi}_2 &= \kappa_y \hat{y}_2\end{aligned}\tag{89}$$

For t=3:

$$\begin{aligned}\hat{y}_3 &= \hat{y}_4 - \tilde{\sigma}^{-1} \hat{i}_3 + \tilde{\sigma}^{-1} \hat{\pi}_4 \\ \hat{\pi}_3 &= \kappa_y \hat{y}_3\end{aligned}\tag{90}$$

\vdots

For t>2:

$$\begin{aligned}\hat{y}_t &= \hat{y}_{t+1} - \tilde{\sigma}^{-1} \hat{i}_t + \tilde{\sigma}^{-1} \hat{\pi}_{t+1} \\ \hat{\pi}_t &= \kappa_y \hat{y}_t\end{aligned}\tag{91}$$

Period 3 Phillips curve along with assumption (D3alt) allows us to solve for \hat{y}_3 as follows:

$$\begin{aligned}\hat{\pi}_3 &= \kappa_y \hat{y}_3 \\ \frac{-1}{\phi^{-1}} \hat{y}_3 &= \kappa_y \hat{y}_3 \\ \implies \hat{y}_3 &= 0 \left(\text{since } \tilde{\kappa}_y + \frac{1}{\phi^{-1}} \neq 0 \right)\end{aligned}\tag{92}$$

Substituting \hat{y}_3 into period 2 IS curve (eqn 89) along with (D3alt) implies:

$$\hat{y}_2 = \left(1 - \frac{\tilde{\sigma}^{-1}}{\phi^{-1}} \right) \hat{y}_3 = 0\tag{93}$$

Substituting \hat{y}_2 into period 2 Phillips'c curve (eqn 89) gives $\hat{\pi}_2$:

$$\hat{\pi}_2 = 0\tag{94}$$

Substituting \hat{y}_2 & $\hat{\pi}_2$ into period 1 IS curve (eqn 88) gives \hat{y}_1 :

$$\hat{y}_1 = \hat{g}_1\tag{95}$$

Finally, substituting \hat{y}_1 & $\hat{\pi}_2$ into period 1 Phillips curve (eqn 88) gives:

$$\hat{\pi}_1 = (\kappa_y - \kappa_g) \hat{g}_1 + \kappa_\tau \tau_1\tag{96}$$

Further, we can solve for output in each period $t > 2$ using the Phillips curve for that period along with assumption (D3). For t>2, we get $\hat{y}_t = 0$ and $\hat{\pi}_t = 0$.

Sticky-information model

$$\begin{aligned}\hat{y}_t &= \mathbb{E}_t \hat{y}_{t+1} - \tilde{\sigma}^{-1} (\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{r}_t^e) + (\hat{g}_t - \mathbb{E}_t \hat{g}_{t+1}) \\ \hat{\pi}_t &= \frac{\lambda}{1-\lambda} (\tilde{\kappa}_y \hat{y}_t - \tilde{\kappa}_g \hat{g}_t + \tilde{\kappa}_\tau \hat{\tau}_t) \\ &\quad + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \mathbb{E}_{t-1-j} (\hat{\pi}_t + [\tilde{\kappa}_y (\hat{y}_t - \hat{y}_{t-1}) - \tilde{\kappa}_g (\hat{g}_t - \hat{g}_{t-1}) + \tilde{\kappa}_\tau (\hat{\tau}_t - \hat{\tau}_{t-1})])\end{aligned}\tag{97}$$

Assumptions D1, D2, D3alt and D4 imply that:

For t=1

$$\begin{aligned}\hat{y}_1 &= \hat{y}_2 + \tilde{\sigma}^{-1} \hat{\pi}_2 + \hat{g}_1 \\ \hat{\pi}_1 &= \frac{\lambda}{1-\lambda} (\tilde{\kappa}_y \hat{y}_1 - \tilde{\kappa}_g \hat{g}_1 + \tilde{\kappa}_\tau \hat{\pi}_1)\end{aligned}\quad (98)$$

For t=2:

$$\begin{aligned}\hat{y}_2 &= \hat{y}_3 + \tilde{\sigma}^{-1} \hat{\pi}_3 \\ \hat{\pi}_2 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_2 + \lambda (\hat{\pi}_2 + [\tilde{\kappa}_y (\hat{y}_2 - \hat{y}_1) + \tilde{\kappa}_g \hat{g}_1 - \tilde{\kappa}_\tau \hat{\pi}_1])\end{aligned}\quad (99)$$

For t=3:

$$\begin{aligned}\hat{y}_3 &= \hat{y}_4 - \tilde{\sigma}^{-1} \hat{i}_3 + \tilde{\sigma}^{-1} \hat{\pi}_4 \\ \hat{\pi}_3 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_3 + [\lambda + \lambda(1-\lambda)] (\hat{\pi}_3 + \tilde{\kappa}_y (\hat{y}_3 - \hat{y}_2))\end{aligned}\quad (100)$$

For t=4:

$$\begin{aligned}\hat{y}_4 &= \hat{y}_5 - \tilde{\sigma}^{-1} \hat{i}_4 + \tilde{\sigma}^{-1} \hat{\pi}_5 \\ \hat{\pi}_4 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_4 + [\lambda + \lambda(1-\lambda) + \lambda(1-\lambda)^2] (\hat{\pi}_4 + \tilde{\kappa}_y (\hat{y}_4 - \hat{y}_3))\end{aligned}\quad (101)$$

\vdots

For t>2:

$$\begin{aligned}\hat{y}_t &= \hat{y}_{t+1} - \tilde{\sigma}^{-1} \hat{i}_t + \tilde{\sigma}^{-1} \hat{\pi}_{t+1} \\ \hat{\pi}_t &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_t \\ &\quad + \lambda \sum_{j=0}^{t-2} (1-\lambda)^j (\hat{\pi}_t + \tilde{\kappa}_y (\hat{y}_t - \hat{y}_{t-1}))\end{aligned}\quad (102)$$

Period 3 Phillips curve (eqn 100) along with assumption (D3alt) allows us to solve for \hat{y}_3 as follows:

$$\begin{aligned}\hat{\pi}_3 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_3 + \lambda (\hat{\pi}_3 + \tilde{\kappa}_y (\hat{y}_3 - \hat{y}_2)) + \lambda(1-\lambda) (\hat{\pi}_3 + \tilde{\kappa}_y (\hat{y}_3 - \hat{y}_2)) \\ \frac{-1}{\phi^{-1}} \hat{y}_3 &= \frac{\lambda}{1-\lambda} \tilde{\kappa}_y \hat{y}_3 + \lambda \mathbb{E}_{t-1} \left(\frac{-1}{\phi^{-1}} \hat{y}_3 + \tilde{\kappa}_y (\hat{y}_3 - \hat{y}_2) \right) + \lambda(1-\lambda) \left(\frac{-1}{\phi^{-1}} \hat{y}_3 + \tilde{\kappa}_y (\hat{y}_3 - \hat{y}_2) \right) \\ \hat{y}_3 &= \hat{y}_2 \left[\frac{\lambda + \lambda(1-\lambda)}{\left[\frac{\lambda}{1-\lambda} + \lambda + \lambda(1-\lambda) \right] + \frac{1}{\tilde{\kappa}_y \phi^{-1}} [1 - \lambda - \lambda(1-\lambda)]} \right]\end{aligned}\quad (103)$$

Substituting \hat{y}_3 into period 2 IS curve (eqn 99) along with (D3) gives the following equation for \hat{y}_2 :

$$\begin{aligned}\hat{y}_2 &= \hat{y}_3 + \tilde{\sigma}^{-1} \left(\frac{-1}{\phi^{-1}} \right) \hat{y}_3 \\ \hat{y}_2 &= \left(\frac{\sigma \phi^{-1} - 1}{\sigma \phi^{-1}} \right) \left[\frac{\lambda + \lambda(1-\lambda)}{\left[\frac{\lambda}{1-\lambda} + \lambda + \lambda(1-\lambda) \right] + \frac{1}{\tilde{\kappa}_y \phi^{-1}} [1 - \lambda - \lambda(1-\lambda)]} \right] \hat{y}_2\end{aligned}\quad (104)$$

For this equation to be true, either:

- $\hat{y}_2 = 0$

OR

- $\left(\frac{\sigma\phi^{-1}-1}{\sigma\phi^{-1}}\right) \left[\frac{\lambda+\lambda(1-\lambda)}{\left[\frac{\lambda}{1-\lambda} + \lambda + \lambda(1-\lambda)\right] + \frac{1}{\tilde{\kappa}_y\phi^{-1}}[1-\lambda-\lambda(1-\lambda)]} \right] = 1$. This can not hold, because:
 $\left(\frac{\sigma\phi^{-1}-1}{\sigma\phi^{-1}}\right) < 1$
& $\left[\frac{\lambda+\lambda(1-\lambda)}{\left[\frac{\lambda}{1-\lambda} + \lambda + \lambda(1-\lambda)\right] + \frac{1}{\tilde{\kappa}_y\phi^{-1}}[1-\lambda-\lambda(1-\lambda)]} \right] < 1$ because $1 - \lambda - \lambda(1 - \lambda) > 0 \equiv \lambda < 1$

Hence, we get that $\hat{y}_2 = 0$.

Substituting $\hat{y}_2 = 0$ into period 1 IS curve and period 2 Phillips curve gives:

$$\begin{aligned}\hat{y}_1 &= \tilde{\sigma}^{-1} \hat{\pi}_2 + \hat{g}_1 \\ \hat{\pi}_2 &= \lambda (\hat{\pi}_2 + [-\tilde{\kappa}_y \hat{y}_1 + \tilde{\kappa}_g \hat{g}_1 - \tilde{\kappa}_\tau \hat{\tau}_1])\end{aligned}\tag{105}$$

Solving these simultaneously gives \hat{y}_1 and $\hat{\pi}_2$:

$$\begin{aligned}\hat{y}_1 &= \left(\frac{1 + \tilde{\sigma}^{-1} \left(\frac{\lambda}{1-\lambda} \right) \tilde{\kappa}_g}{1 + \tilde{\sigma}^{-1} \left(\frac{\lambda}{1-\lambda} \right) \tilde{\kappa}_y} \right) \hat{g}_s - \left(\frac{\tilde{\sigma}^{-1} \left(\frac{\lambda}{1-\lambda} \right) \tilde{\kappa}_\tau}{1 + \tilde{\sigma}^{-1} \left(\frac{\lambda}{1-\lambda} \right) \tilde{\kappa}_y} \right) \hat{\tau}_s \\ \hat{\pi}_2 &= \left(\frac{\lambda}{1-\lambda} \right) \left[\tilde{\kappa}_g - \tilde{\kappa}_y \left(\frac{1 + \tilde{\sigma}^{-1} \left(\frac{\lambda}{1-\lambda} \right) \tilde{\kappa}_g}{1 + \tilde{\sigma}^{-1} \left(\frac{\lambda}{1-\lambda} \right) \tilde{\kappa}_y} \right) \right] \hat{g}_s + \left(\frac{\lambda}{1-\lambda} \right) \left[\tilde{\kappa}_y \left(\frac{\tilde{\sigma}^{-1} \left(\frac{\lambda}{1-\lambda} \right) \tilde{\kappa}_\tau}{1 + \tilde{\sigma}^{-1} \left(\frac{\lambda}{1-\lambda} \right) \tilde{\kappa}_y} \right) - \tilde{\kappa}_\tau \right] \hat{\tau}_s\end{aligned}\tag{106}$$

Substituting \hat{y}_1 into period 1 Phillips curve (eqn 98) gives $\hat{\pi}_1$:

$$\hat{\pi}_1 = \left(\frac{-\lambda}{1-\lambda} \right) \left[\tilde{\kappa}_g - \tilde{\kappa}_y \left(\frac{1 + \tilde{\sigma}^{-1} \left(\frac{\lambda}{1-\lambda} \right) \tilde{\kappa}_g}{1 + \tilde{\sigma}^{-1} \left(\frac{\lambda}{1-\lambda} \right) \tilde{\kappa}_y} \right) \right] \hat{g}_s + \left(\frac{-\lambda}{1-\lambda} \right) \left[\tilde{\kappa}_y \left(\frac{\tilde{\sigma}^{-1} \left(\frac{\lambda}{1-\lambda} \right) \tilde{\kappa}_\tau}{1 + \tilde{\sigma}^{-1} \left(\frac{\lambda}{1-\lambda} \right) \tilde{\kappa}_y} \right) - \tilde{\kappa}_\tau \right] \hat{\tau}_s \tag{107}$$

Further, from period 4 Phillips curve along with assumption (D3alt), and solution $\hat{y}_3 = 0$ we get:

$$\begin{aligned}\frac{-1}{\phi^{-1}} \hat{y}_4 &= \frac{\lambda}{1-\lambda} (\tilde{\kappa}_y \hat{y}_4) + [\lambda + \lambda(1-\lambda) + \lambda(1-\lambda)^2] \left(\frac{-1}{\phi^{-1}} \hat{y}_4 + \tilde{\kappa}_y (\hat{y}_4 - \hat{y}_3) \right) \\ \implies \hat{y}_4 &= \left[\frac{\lambda + \lambda(1-\lambda) + \lambda(1-\lambda)^2}{\left[\frac{\lambda}{1-\lambda} + \lambda + \lambda(1-\lambda) + \lambda(1-\lambda)^2 \right] + \frac{1}{\tilde{\kappa}_y\phi^{-1}} [1 - \lambda - \lambda(1-\lambda) - \lambda(1-\lambda)^2]} \right] \hat{y}_3 \\ &= 0\end{aligned}\tag{108}$$

Similarly, we can solve for the transition dynamics of output in each period $t > 2$ using the Phillips curve

for that period along with the assumption (D3alt). For $t > 2$, we get:

$$\begin{aligned}
\frac{-1}{\phi^{-1}} \hat{y}_t &= \frac{\lambda}{1-\lambda} (\tilde{\kappa}_y \hat{y}_t) + \lambda \sum_{j=0}^{t-2} (1-\lambda)^j \left(\frac{-1}{\phi^{-1}} \hat{y}_t + \tilde{\kappa}_y (\hat{y}_t - \hat{y}_{t-1}) \right) \\
\frac{-1}{\phi^{-1}} \left[1 - \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \right] \hat{y}_t &= \left[\frac{\lambda}{1-\lambda} + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \right] \tilde{\kappa}_y \hat{y}_t - \left[\lambda \sum_{j=0}^{t-2} (1-\lambda)^j \right] \tilde{\kappa}_y \hat{y}_{t-1} \\
\implies \hat{y}_t &= \left[\frac{\lambda \sum_{j=0}^{t-2} (1-\lambda)^j}{\frac{\lambda}{1-\lambda} + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j + \frac{1}{\tilde{\kappa}_y \phi^{-1}} \left[1 - \lambda \sum_{j=0}^{t-2} (1-\lambda)^j \right]} \right] \hat{y}_{t-1} \\
&= 0
\end{aligned} \tag{109}$$