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Demodulating Analog Signals Using GNURadio

December 18, 2016

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Training unit: Communication
Subject: Protocols for Connected objects

Contents

Introduction

Software Defined Radio (SDR) is a radio communication system where most of the components are implemented by means of software instead of hardware. Due to the flexible nature of the software, SDR allows us to explore a wide range of radio communication applications.

Amongst all the different implementation options available, we will be using GNURadio. GNURadio has several functions, from signal generation to signal modulation, with a plotting function that allows us to view the processed signal. We will use GNURadio in order to analyse a sample radio signal, from which we will extract a sound signal.

1 Presentation of Software Defined Radio

We will first consider the question from a mathematical point of view. Since we know the equation used to expressed the emitted signal, we can use it to determine the equation of the received signal. The emitted signal has the following formula:

$$s_{RF}(t) = A(t)\cos(2\pi f_0 t + \phi(t)), t \in \mathbb{R} \tag{1}$$

In order to get rid of the phase (Φ), we substitute $s_R(t) = A(t)\cos(\phi t)$ for the component in phase and $s_I(t) = A(t)\sin(\phi t)$ for the component in quadrature, to obtain equation 2.

$$s_{RF}(t) = s_R(t)\cos(2\pi f_0 t) - s_I(t)\sin(2\pi f_0 t) \tag{2}$$

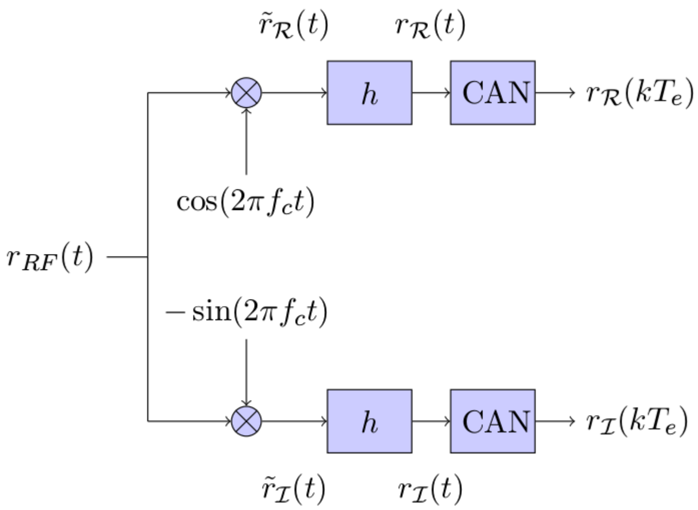

Figure 1: I/Q demodulator

Question 1

Assuming that the received signal is equal to the emitted one i,e $r_{RF}(t) = s_{RF}(t)$, deduce the formula of $\widetilde{r}_R(t)$ and $\widetilde{r}_I(t)$.

$\widetilde{r}_R(t) = r_{RF}(t)\cos(2\pi f_c t)$

$$
\begin{aligned}
&= \cos(2\pi f_c t)(s_R(t)\cos(2\pi f_0 t) - s_I(t)\sin(2\pi f_0 t)) \\
&= s_R(t)\cos(2\pi f_c t)\cos(2\pi f_0 t) - s_I(t)\cos(2\pi f_c t)\sin(2\pi f_0 t) \\
&= \frac{s_R(t)}{2}(\cos(2\pi t(f_0+f_c)) + \cos(2\pi t(f_0-f_c))) - \frac{s_I(t)}{2}(\sin(2\pi t(f_0+f_c)) + \sin(2\pi t(f_0-f_c)))
\end{aligned}
$$

Similarly , for $\widetilde{r}_I(t)$:

$$
\begin{aligned}
\widetilde{r}_I(t) &= r_{RF}(t)(-\sin(2\pi f_c t)) \\
&= \frac{s_I(t)}{2}(\cos(2\pi t(f_c-f_0)) - \cos(2\pi t(f_c+f_0))) - \frac{s_R(t)}{2}(\sin(2\pi t(f_c+f_0)) + \sin(2\pi t(f_c-f_0)))
\end{aligned}
$$

Question 2

If $f_c = f_0$, determine the filters *h* so that $r_R(t) = s_R(t)$ and $r_I(t) = s_I(t)$.

$$
\widetilde{r}_R(t) = \frac{s_R(t)}{2}(\cos(4\pi f_0 t) + 1) - \frac{s_I(t)}{2}\sin(4\pi f_0 t) \tag{3}
$$

$$
\widetilde{r}_I(t) = \frac{s_I(t)}{2}(1 - \cos(4\pi f_0 t)) - \frac{s_R(t)}{2}\sin(4\pi f_0 t) \tag{4}
$$

The Fourier transform of the functions can be to determine the form of the filter *h* (we suppose the Fourier transform of s_R and s_I are $S_R(f)$ and $S_I(f)$ respectively). The frequency spectrum will help in determining which frequency range needs to be filter to retrieve the information.

$$
\begin{aligned}
\widetilde{R}_R(f) &= \mathfrak{F}\big[\widetilde{r_R(t)}\big](f) \\
&= \mathfrak{F}\Big[\frac{s_R(t)}{2}(\cos(4\pi f_0 t) + 1) - \frac{s_I(t)}{2}\sin(4\pi f_0 t)\Big](f)
\end{aligned}
$$

As Fourier transform is a linear operation:

$$
= \mathfrak{F}\Big[\frac{s_R(t)}{2}\Big](f) + \mathfrak{F}\Big[\frac{s_R(t)}{2}\cos(4\pi f_0 t)\Big](f) - \mathfrak{F}\Big[\frac{s_I(t)}{2}\sin(4\pi f_0 t)\Big](f)
$$

As $\mathfrak{F}[a(t)b(t)](f) = \mathfrak{F}[a(t)](f) * \mathfrak{F}[b(t)](f)$:

$$
= \mathfrak{F}\Big[\frac{s_R(t)}{2}\Big](f) + \mathfrak{F}\Big[\frac{s_R(t)}{2}\Big](f) * \mathfrak{F}\big[\cos(2\pi 2f_0 t)\big](f) - \mathfrak{F}\Big[\frac{s_I(t)}{2}\Big](f) * \mathfrak{F}\big[\sin(2\pi 2f_0 t)\big](f)
$$

As $\mathfrak{F}[\cos(2\pi f_1 t)](f) = \frac{1}{2}[\delta(f+f_1) + \delta(f-f_1)]$ and $\sin(2\pi f_1 t)](f) = \frac{j}{2}[\delta(f+f_1) - \delta(f-f_1)]$:

$$
\begin{aligned}
&= \frac{S_R(f)}{2} + \frac{S_R(f)}{2}\Big(\frac{1}{2}\Big(\delta(f+2f_0) + \delta(f-2f_0)\Big)\Big) - \frac{S_I(f)}{2}\Big(\frac{j}{2}\Big(\delta(f+2f_0) + \delta(f-2f_0)\Big)\Big) \\
&= \frac{S_R(f)}{2} + \frac{S_R(f)}{4}\Big(\delta(f+2f_0) + \delta(f-2f_0)\Big) - \frac{jS_I(f)}{4}\Big(\delta(f+2f_0) + \delta(f-2f_0)\Big) \\
&= \frac{1}{4}\Big(2S_R(f) + S_R(f+2f_0) + S_R(f-2f_0) - jS_I(f+2f_0) + jS_I(f-2f_0)\Big)
\end{aligned}
$$

Hence the frequency spectrum of $r_R(t)$ is given by equation 5.

$$
\widetilde{R}_R(f) = \frac{1}{4}\Big(2S_R(f) + S_R(f+2f_0) + S_R(f-2f_0) - jS_I(f+2f_0) + jS_I(f-2f_0)\Big) \tag{5}
$$

Convolution of a signal's Fourier transform with a Dirac, $\delta(f+f_a)$, shifts that signal's frequency spectrum about $-f_a$. Hence, the right hand side of 5 can be decomposed into the following components:

- $S_R(f)$: Information to be extracted
- $S_R(f+2f_0)$: S_R shifted about $-2f_0$
- $S_R(f-2f_0)$: S_R shifted about $2f_0$
- $jS_R(f+2f_0)$: S_R shifted about $-2f_0$ and with a phase shift
- $jS_R(f-2f_0)$: S_R shifted about $2f_0$ with a phase shift

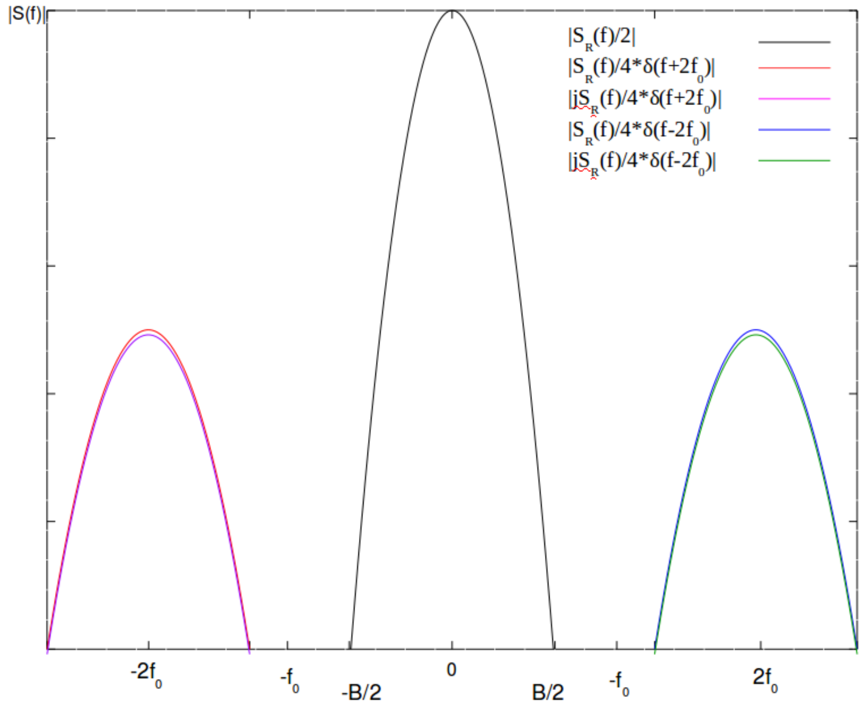

Figure 2: Sketch of $\widetilde{R_R}(f)$

Using the above information, we can deduce that to extract the signal $S_R(f)$, a low pass filter, *h* (real, thus symmetrical), must be applied to "kill" the spectrum's undesired components which are centred about $2f_0$ and $-2f_0$. The information is centred around $f = 0$, and is spread over a bandwidth of B. In order to retrieve the signal, the filter *h* will be a low-pass filter where $\frac{B}{2} < f_c < 2f_0 - \frac{B}{2}$, f_c being the cut-off frequency of *h*.

The same filter *h* can be used to extract $\widetilde{R_I}(f)$.

Question 3

Explain whether the receiver can or cannot work with large bandwidth signals ($B/2 > f_0$)?

The receiver is analogically unable to work with wide band signals because of the overlap of the signals shifted at $2f_0$ and $-2f_0$ with the centred signal (see figure 3). However, it can eventually be able to digitally process the signal using a Hilbert filter.

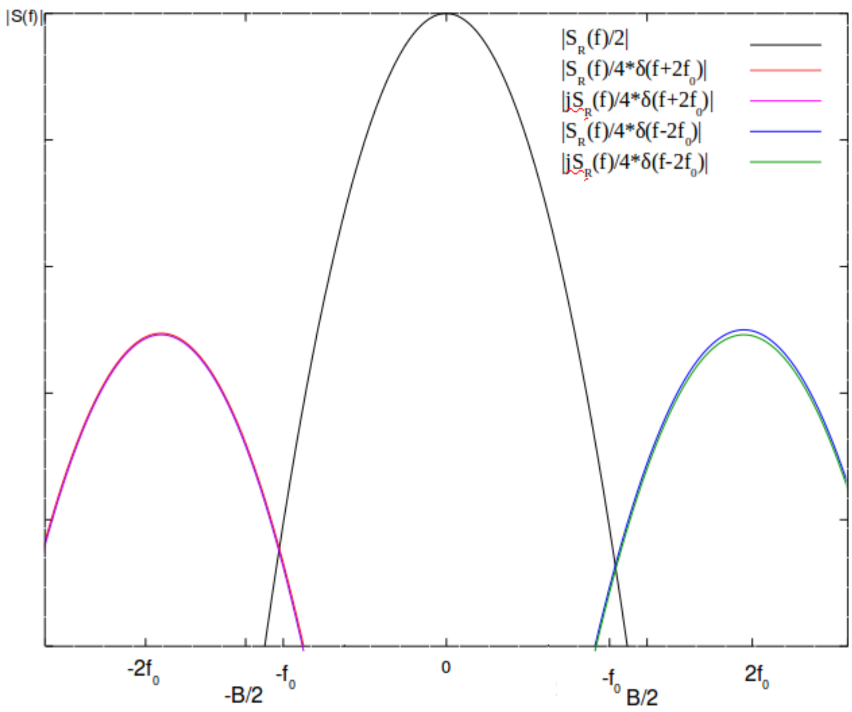

Figure 3: Sketch of $\widetilde{R_R}(f)$ overlapping when $B/2 > f_0$

Question 4

How should the sampling frequency T_e be set to be able to reconstruct $r_R(t), t \in \mathbb{R}$ from $r_R(kT_e), k \in \mathbb{Z}$?

According to Nyquist-Shannon sampling theorem, $F_e > 2f_{max}$. In practice, a sampling frequency higher than the minimum required frequency if often used.

$$F_e \geqslant 2f_{max} \implies F_e \geqslant B \tag{6}$$

With $F_e = \frac{1}{T_e}$.

Question 5

Can the frequency transposition and analog/digital conversion steps be inverted?

The inversion would theoretically be possible, but there would be more data to process because the unwanted higher frequencies are not disposed of by the filter *h* beforehand. It would cost us more time and more energy, and would not be as instantaneous as with the analog system. An additional limitation is that currently available ADCs cannot work at very high frequencies, for e.g. to process VHF we would need a ADC operating at 600 MHz and for UHF at 300 MHz - 3 GHz.

Question 6

Express the analytic signal and the complex envelope/ complex baseband of the signal $s_{RF}(t) = s_R(t)\cos(2\pi f_0 t) - s_I(t)\sin(2\pi f_0 t)$ **in terms of** f_0**.**

We will use the Fourier transform on s_{RF} to deduce the analytic signal.

$$
\begin{aligned}
S_{RF}(f) &= \frac{S_R(f)}{2} * (\delta(f-f_0)+\delta(f+f_0)) - \frac{S_I(f)}{2j} * (\delta(f-f_0)-\delta(f+f_0)) \\
&= \frac{1}{2}(S_R(f-f_0)+S_R(f+f_0)+jS_I(f-f_0)-jS_I(f+f_0)) \\
S_a(f) &= S_{RF}(f)+\mathrm{sign}(f)S_{RF}(f) \\
&= 2{S_{RF}}^{+}(f) \text{ (Because we cut the negative frequencies and we amplify the signal)} \\
&= S_R(f-f_0)+jS_I(f-f_0) \\
&= (S_R(f)+jS_I(f)) * \delta(f-f_0) \\
&\text{Applying the inverse Fourier transform to deduce analytic signal in the time domain:} \\
\mathfrak{F}^{-1}\left[S_a(f)\right] &= \mathfrak{F}^{-1}\left[S_R(f)+jS_I(f)\right) * \delta(f-f_0\right] \\
&\text{As } \mathfrak{F}[a(t)b(t)](f) = \mathfrak{F}[a(t)](f) * \mathfrak{F}[b(t)](f) \\
&= \mathfrak{F}^{-1}\left[S_R(f)+jS_I(f)\right] \times \mathfrak{F}^{-1}\left[\delta(f-f_0)\right] \\
&= (s_R(t)+js_I(t)) \times \mathfrak{F}^{-1}\left[\delta(f-f_0)\right] \\
&\text{As } \mathfrak{F}[e^{-j2\pi f_a t}](f) = \delta(f-f_a): \\
&= (s_R(t)+js_I(t))e^{-j2\pi f_0 t}
\end{aligned}
$$

$$
s_a(t) = (s_R(t)+js_I(t))e^{-j2\pi f_0 t} \tag{7}
$$

2 FM receiver

VHF comprises frequencies ranging from 30 MHz to 300 MHz, we will be focusing on the range 87.5 MHz - 108 MHz which is reserved for FM radio. The channels are 100 kHz-wide, and should thus theoretically allow transmission of 203 different channels. It is however not the case as the guard bands are required to prevent one channel overflowing on its neighbours. The signal analysed in this assignment has been recorded at Pech David, near CHU Rangueil, Toulouse, in 2015. The signal was recorded with the following characteristics: centre frequency, $f_c = 99.5$MHz, and sampling frequency, $F_e = 1.5$MHz.

Frequency analysis of the recording

Question 7

Describe the functions of the different blocks used for signal processing.

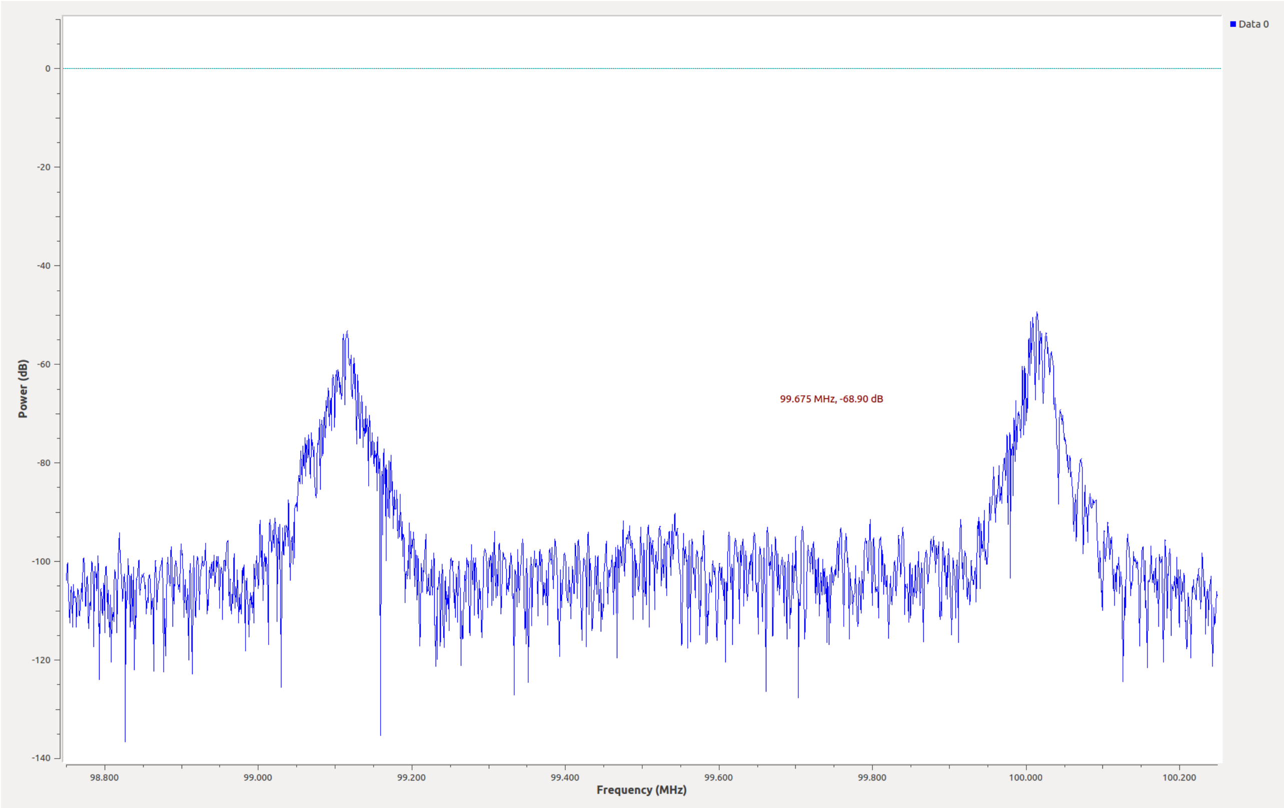

Figure 4: FFT of the received signal

File Source	Description	Retrieves data from a file
	Option: Repeat	loops the signal over and over
Throttle	Description	Restricts the input stream to limit it to value of samples_per_sec.
	Input	one stream of itemsize
	Output	one stream of itemsize
QT GUI Frequency Sink	Description	GUI to display the DFFT of the signal for 1024 samples (default value)

Table 1: Description of the GNURadio blocks used

Question 8

Give the value of the missing parameters to process the recording.

Fe is set to 1.5 MHz.

fc is set to 99.5 MHz.

Throttle's Sample Rate is set to 1.5 MHz.

QT GUI Frequency Sink's Center Frequency and Bandwidth are set to 99.5 MHz and 1.5 MHz respectively.

Question 9

How many channels are we observing? To what station do they correspond?

We observe 2 different channels: one centred about approximately 99.1MHz, the other about approximately 100.0MHz. The first signal corresponds to RFM radio station, the other is Skyrock. Every 100kHz, we do not observe a different channels are they need to be separated by guard channels to prevent spectrum overlapping.

Question 10

What is the signal to noise ratio? Is it enough to demodulate the signal?

As observed on figure 4, the average noise floor is -100dB. The maximum power of the signals' spectrum are about -60dB, giving an SNR value of about 40dB (the signal is about 10 000 times stronger than noise). This SNR should be significantly enough to demodulate the signals.

Question 11

What is the approximate bandwidth of each station's signal?

Each signal has an approximate bandwidth of 200 to 250 kHz, hence, justifying the use of guard bands in spite of the channels being limited to 100kHz.

Extraction of a channel by frequency translation then by low-pass filter

Question 12

What is the frequency shift we need to apply to centre each signals?

We need to apply a shift of 400kHz (99.5 MHz -99.1 MHz) and -500kHz (99.5 MHz - 100 MHz) respectively, centred about 99.5MHz. To apply this shift, the signal is multiplied with a cosine of the required frequency.

Question 13

What happens when we apply a frequency shift greater than F_e?

Sampling a signal corresponds to the convolution of the signal's Fourier transform with a Dirac comb of spacing F_e. As convolution of a signal's Fourier transform with a Dirac centred about a frequency F_0 results into shifting the signal's spectrum about F_0, convolution with a Dirac comb of spacing F_e produces copies of the original signal's spectrum every F_e.

Hence in our case, a shift of F_e would result into observing the same result as with a shift of 0Hz.

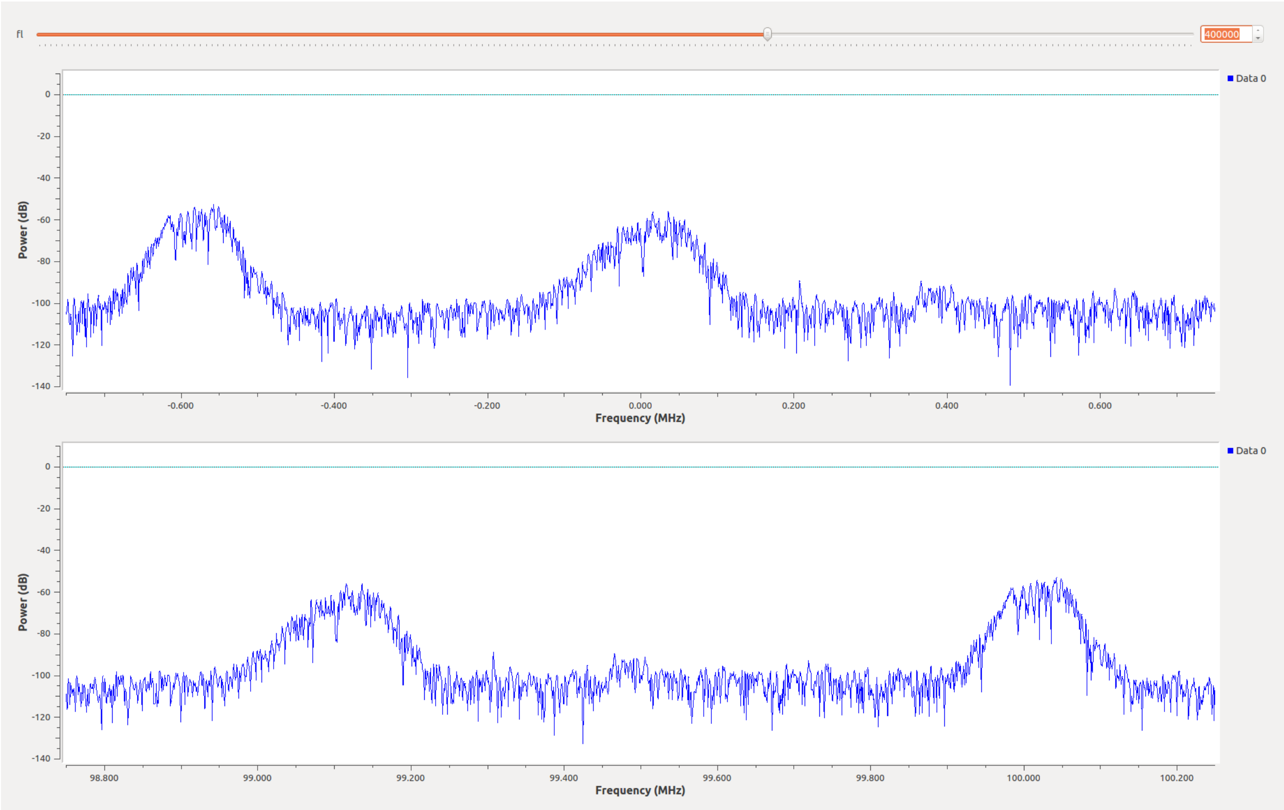

Figure 5: The top figure shows the shifted signal by 400kHz and the figure on the bottom shows the original unshifted signal

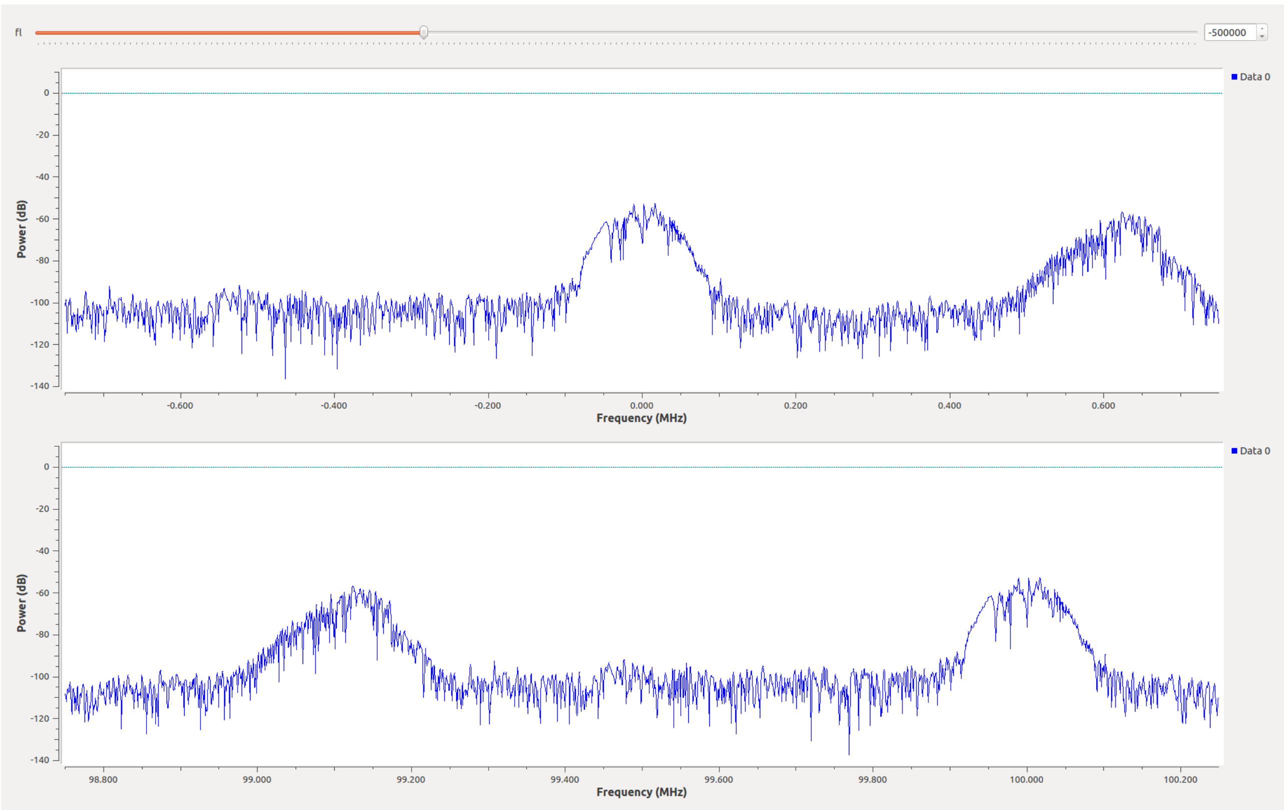

Figure 6: The top figure shows the shifted signal by -500kHz and the figure on the bottom shows the original unshifted signal

Question 14

What are the parameters of the low-pass filter and the frequency analyser at the end of the filter?

Decimation	The ratio of samples to be removed for e.g. setting the decimation parameter to 2 would cause the resulting signal to contain half the number of samples as before. It roughly corresponds to outputting a signal sampled at $\frac{F_e}{N}$, if the input signal is sampled at F_e and the decimation parameter is set to N.	1
Sample Rate	The sampling rate of the input signal.	1.5MHz
Cutoff Frequency	Cut-off frequency of the low-pass filter	125kHz
Transition width	Transition width of the low-pass filter	12.5kHz

Table 2: Low-pass filter's parameters

For the frequency analyser's GUI, most of the parameters were left at their default values. We just set the centre frequency to 0Hz and the sample frequency output by the filter.

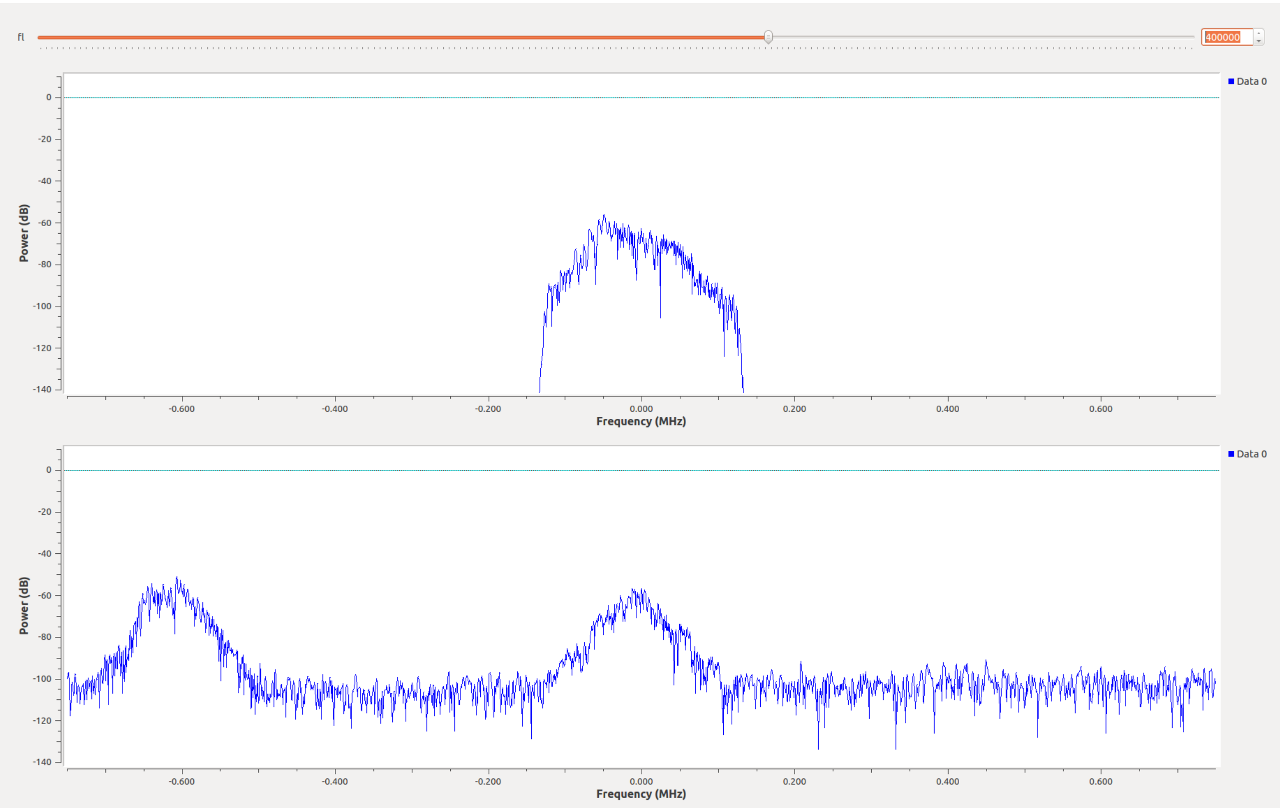

Figure 7: The shifted signal from figure 5 filtered at 125kHz

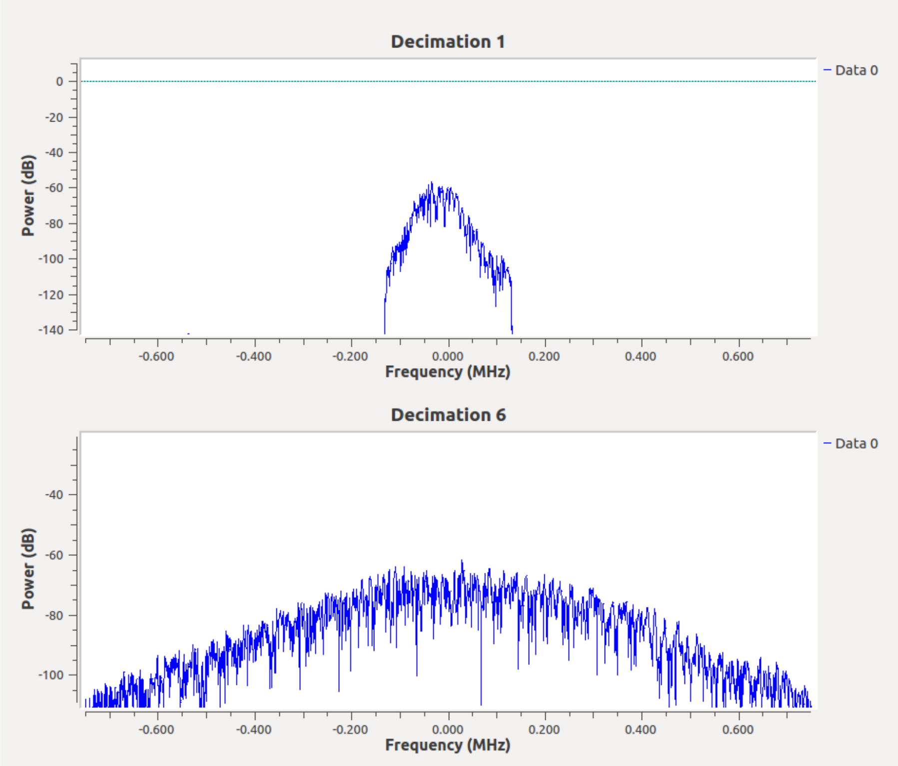

Figure 9: Comparing the result of using different decimation values, one set to 1 and the other to 6

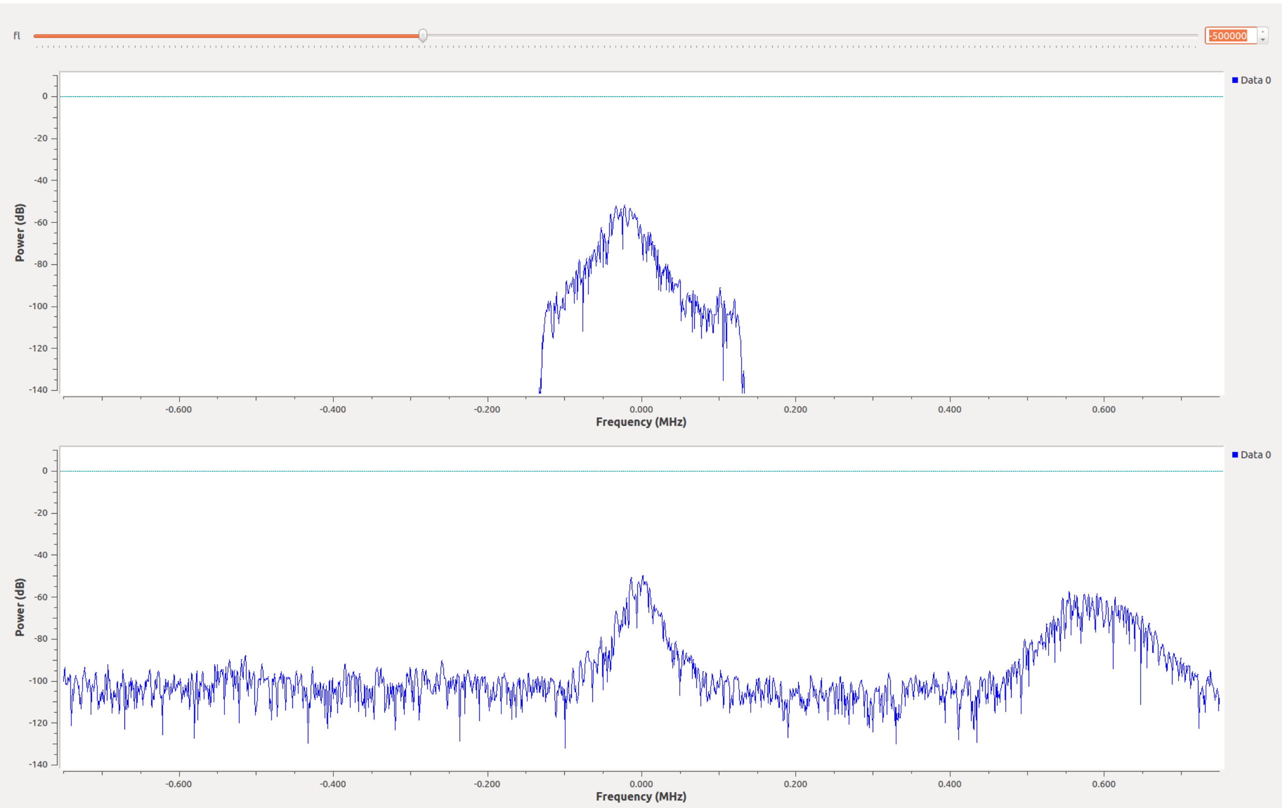

Figure 8: The shifted signal from figure 6 filtered at 125kHz

As seen on figures 7 and 8, the channels are cleanly cut. To reduced processing power needed for further processing the signal can be decimated.

With a decimation value of 6, the sampling frequency of the output signal is 250kHz which is equal to twice the maximum frequency of the considered channel (100 125 kHz). Hence we have a sufficiently higher sampling frequency according to Nyquist-Shannon sampling principle and a lower number of samples to process.

Frequency demodulation and signal restitution

To transmit monophonic sound along with stereophonic, both signals are multiplexed using equation (8), with $f_{sp} = 19\text{kHz}$.

$$m(t) = g(t) + d(t) + A_{sp}\cos\left(2\pi f_{sp} t\right) + [g(t) - d(t)]\cos\left(2\pi 2 f_{sp} t\right) \tag{8}$$

A monophonic receiver would simply have to apply a low-pass filter at 15kHz to retrieve the sound transmitted, while the stereophonic receiver would have to demodulate the signal, and then combine it to the initial signal to recover the original stereo sound. Hence, both mono and stereo sound can be transmitted using the same bandwidth.

The signal m(t) is then modulated by frequency so that the f_0 centred signal will be in the following form:

$$s_{RF}(t) = A\cos\left(2\pi f_0 t + \frac{\Delta f}{\max(|m(t)|)}\int_{-\infty}^{t} m(u)\,du\right) \tag{9}$$

Question 15

Verify that the previously measured bandwidth complies to Carson's rule.

$$B_{FM} \approx 2(\Delta f + f_m) \tag{10}$$

Applying $\Delta f = 75\text{kHz}$ and $f_m = 15\text{kHz}$ to equation 10, we get $B_{FM} \approx 180\text{kHz}$ which is close to the value previously observed.

Question 16

Using (9) **and assuming the transmitted and received signals are identical, the analytic signal,** $s_a(t)$**, of the transmission can be deduced. Show that is is represented by 11 with b being a complex noise induced by the transmission and the receiver.**

$$\begin{aligned}
s_{RF}(t) &= A\cos\left(2\pi f_0 t + \frac{\Delta f}{\max(|m(t)|)}\int_{-\infty}^{t} m(u)\,du\right)\\
&\text{Using equations 1, 2, and substituting } \frac{\Delta f}{\max(|m(t)|)}\int_{-\infty}^{t} m(u)\,d = \phi(t)\\
&= A\cos(2\pi f_0 t + \phi(t))\\
&= A\cos(\phi(t))\cos(2\pi f_0 t) - A\sin(\phi(t))\sin(2\pi f_0 t)\\
&\text{Substituting } A\cos(\phi(t)) = s_R(t)\text{, and } A\sin(\phi(t)) = s_I(t)\\
&= s_R(t)\cos(2\pi f_0 t) - s_I\sin(2\pi f_0 t)
\end{aligned}$$

Hence using equation 7,

$$\begin{aligned}
s_a(t) &= (s_R(t) + js_I(t))e^{-j2\pi f_0 t} \\
&= (A\cos(\phi(t)) + jA\sin(\phi(t)))e^{-j2\pi f_0 t} \\
&= Ae^{j\phi(t)}e^{-j2\pi f_0 t} \\
&= Ae^{j\phi(t)-j2\pi f_0 t}
\end{aligned}$$

To extract the complex baseband with respect to f_0 from the analytic signal, the analytic signal must be shifted by f_0 in the frequency domain which corresponds to a multiplication of $e^{j2\pi f_0 t}$ in time domain. Hence,

$$\begin{aligned}
s_l(t) &= Ae^{j\phi(t)-j2\pi f_0 t} \times e^{j2\pi f_0 t} \\
&= Ae^{j\phi(t)} \\
&\text{Discretisation of } \phi(t) \text{ gives } \frac{\Delta f}{\max(|m(t)|)}\sum_{i=0}^{k} m[i] \text{ with } i,k \in \mathbb{N}.\text{Hence,} \\
s_l[k] &= Ae^{j\frac{\Delta f}{\max(|m(t)|)}\sum_{i=0}^{k} m[i]} \\
&\text{Substituting, } k_f = \frac{\Delta f}{\max(|m(t)|)}: \\
s_l[k] &= Ae^{jk_f\sum_{i=0}^{k} m[i]}
\end{aligned}$$

The signal is received with some complex noise b[k].

$$\text{Received signal, } y_l[k] = Ae^{jk_f\sum_{i=0}^{k} m[i]} + b[k] \tag{11}$$

Using equation **??**, it can be shown that the signal can be demodulated according to equation **??**.

$$\widetilde{m}_l[k] = \arg(y_l[k]y_l^*[k-1]) \tag{12}$$

Using equations 11, 12, and neglecting the complex noise to simply the demonstration:

$$\begin{aligned}
\widetilde{m}_l[k] &= \arg(y_l[k]y_l^*[k-1]) \\
&= \arg(Ae^{jk_f\sum_{i=0}^{k} m[i]}Ae^{-jk_f\sum_{i=0}^{k-1} m[i]}) \\
&= \arg(A^2e^{jk_f\sum_{i=0}^{k} m[i]-jk_f\sum_{i=0}^{k-1} m[i]}) \\
&= \arg(e^{jk_f\sum_{i=0}^{k} m[i]-jk_f\sum_{i=0}^{k-1} m[i]}) \\
&= \arg(e^{jk_f\sum_{i=0}^{k} m[i]-jk_f\sum_{i=0}^{k-1} m[i]}) \\
&= \arg(e^{jk_f m[k]}) \\
&= jk_f\mathbf{m[k]}
\end{aligned}$$

The output of the WBFM Receiver is then filtered at 15 kHz to retain only the monophonic signal. To output the filtered signal to the computer's sound card a Rational Resampler is used to adapt the sampling frequency of the output signal to a frequency compatible with the sound card.

Question 17

Display the spectrum of a demodulated channel and interpret it.

To demodulate the signal, a WBFM Receive bloc is used. It demodulates its input signal using equation 12.

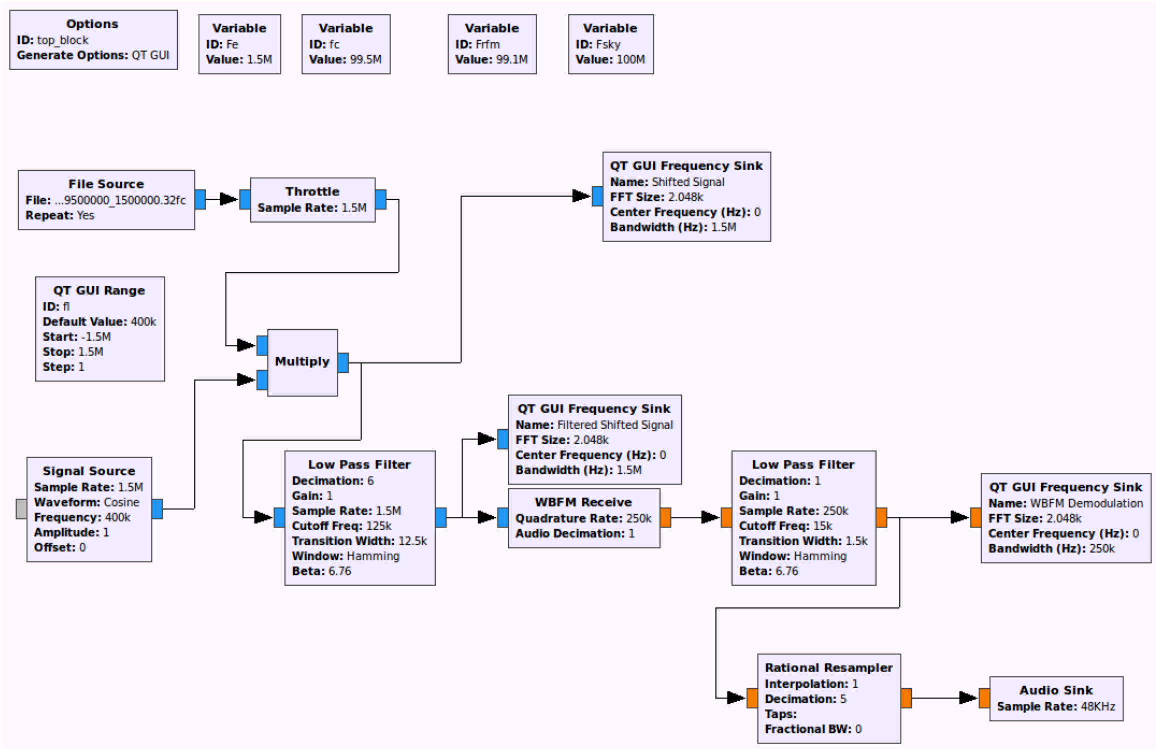

Figure 10: Signal processing used to retrieve the signal

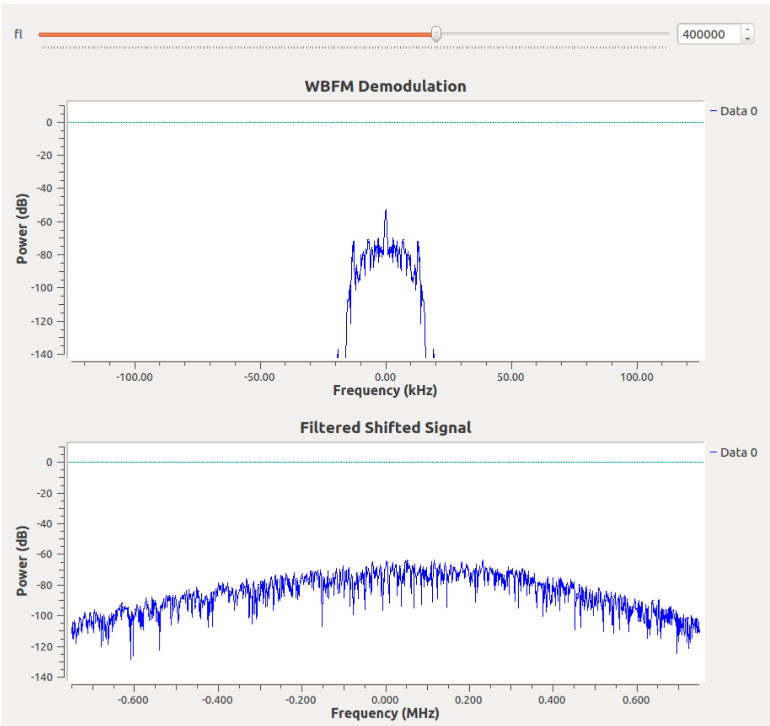

Figure 11: Spectrum of the demodulated signal

Question 18

Who won Sam Smith's album?

After listening to the demodulated signal, we can hear that a radio-listener named Jordi won the album.

Implementing a demodulator with an USRP

The previous data processing system is used, but by replacing the data source. The file source is replaced with an UHD USRP Source bloc to acquire data directly from the USRP. The frequency shifting bloc is also muted due the fact that the UHD USRP Source integrates a frequency shifting capability. The USRP and our signal processing can thus be used to receive and demodulate FM radio.

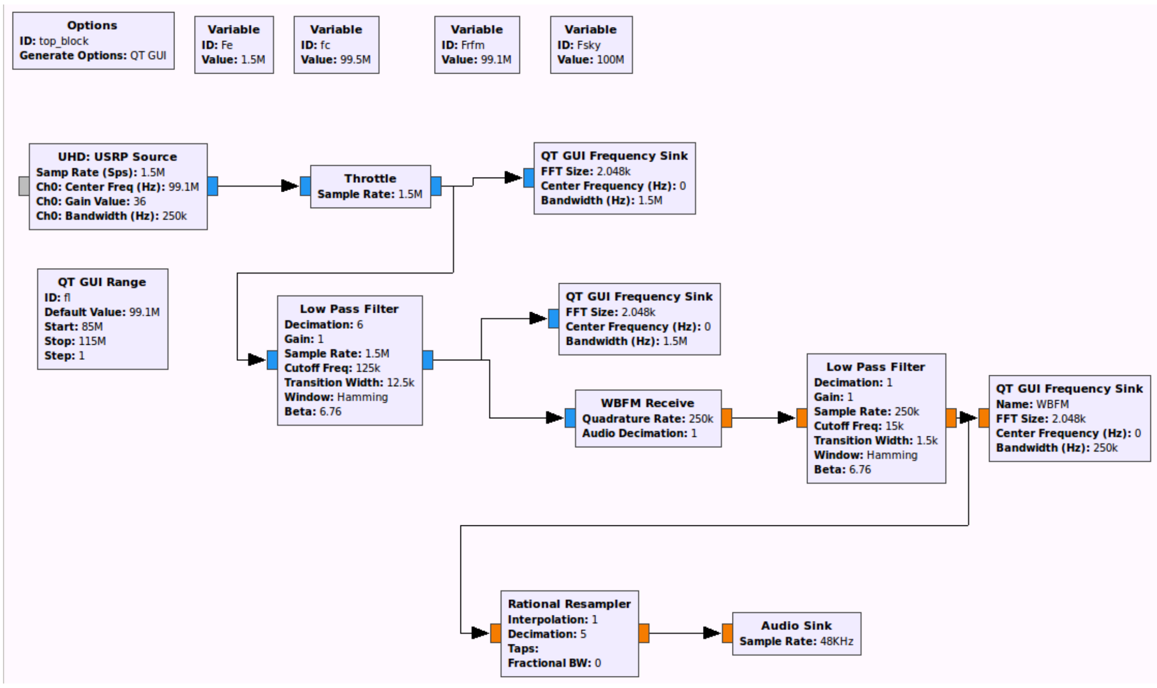

Figure 12: Signal processing to demodulate the signal received using the USRP