HEDGE FUND PORTFOLIO OPTIMIZATION: A SEMI-PARAMETRIC APPROACH

by

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Abstract

Hedge funds, with their complex investment strategies and distinct return distributions, pose significant challenges for traditional parametric portfolio optimization models. This research presents a semi-parametric approach to hedge fund portfolio optimization, to better capture the unconventional distributional characteristics of hedge fund returns. The semi-parametric methodology combines the strengths of both parametric and non-parametric approaches, addressing the limitations inherent in each. Traditional parametric models often fall short in capturing the higher moments and tail behaviors of hedge fund returns, while non-parametric methods, although flexible, can suffer from overfitting and computational inefficiency. We use hedge fund strategy indices from the HFR database to test our semi-parametric approach with over 30 years of data. The findings reveal that the proposed model significantly improves risk-adjusted performance compared to traditional benchmarks, offering a robust framework for hedge fund portfolio optimization that aligns with real-world investment needs.

Keywords: Hedge Funds, Portfolio Optimization, Semi-Parametric Approach, Synthetic Returns, Cost Minimization, Correlation Constraints

JEL Classification: G11, G23, G32, C14, C32

Les fonds spéculatifs, avec leurs stratégies d'investissement complexes et leurs distributions de rendements distinctes, posent des défis significatifs aux modèles traditionnels d'optimisation de portefeuille paramétriques. Cette recherche présente une approche semi-paramétrique de l'optimisation de portefeuille de fonds spéculatifs, afin de mieux capturer les caractéristiques de distribution non conventionnelles des rendements de ces fonds. La méthodologie semi-paramétrique combine les forces des approches paramétriques et non paramétriques, en abordant les limitations inhérentes à chacune. Les modèles paramétriques traditionnels échouent souvent à capturer les moments d'ordre supérieur et les comportements des queues de distribution des rendements des fonds spéculatifs, tandis que les méthodes non paramétriques, bien que flexibles, peuvent souffrir de surajustement et d'inefficacité computationnelle. Nous utilisons les indices de stratégies de fonds spéculatifs de la base de données HFR pour tester notre approche semi-paramétrique avec plus de 30 ans de données. Les résultats révèlent que le modèle proposé améliore significativement la performance ajustée au risque par rapport aux benchmarks traditionnels, offrant un cadre robuste pour l'optimisation de portefeuille de fonds spéculatifs qui s'aligne sur les besoins d'investissement réels.

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Executive Summary

In hedge fund optimization, we aim to address several challenges associated with hedge fund return distributions: negative skewness (asymmetry), excess kurtosis (fat tails), autocorrelation (illiquidity), and heteroskedasticity (volatility clustering). These dynamics make traditional parametric mean-variance optimization ineffective, often leading to unintended allocations. Therefore, we employ three alternative risk measures—Conditional Value at Risk (CVaR), Conditional Drawdown at Risk (CDaR), and the Omega ratio—to mitigate these biases. While these measures are typically implemented non-parametrically, they do not easily account for autocorrelation and volatility clustering. This study seeks to bridge the gap by refining existing models to better address the specifics of hedge fund returns while introducing practical considerations such as minimizing transaction costs and adding constraints on portfolio correlations with other asset classes, specifically bonds, and stocks.

We utilize hedge fund strategy indices from the HFRI database, spanning from 1990 onwards, and sub-strategy data available from the mid-2000s. This provides us with over 30 years of data, offering increased diversification and adaptability. We identify potential biases commonly found in hedge fund databases, including survivorship, selection, backfill, and illiquidity biases. Statistical tests, such as Jarque-Bera, Ljung-Box Q, and Engle LM, are used to assess normality, autocorrelation, and heteroskedasticity. Performance measures are computed to thoroughly evaluate the hedge fund indices, prior to integrating into portfolios.

Our core methodology employs AR(1)-EGARCH(1,1) standardized residuals in a semi-parametric approach. This involves parametric estimation of return distribution tails using extreme value theory and a generalized Pareto distribution, combined with non-parametric smoothing of the distribution center via a Gaussian kernel. Dependence modeling is achieved using a multivariate Student's t distribution, generating synthetic returns from the semi-parametric residuals. Volatility, estimated from the EGARCH model, is reintroduced to the synthetic returns. Portfolios are constructed out-of-sample using these synthetic returns, with optimization measures including CVaR, CDaR, and the Omega ratio. Cost minimization is introduced to the objective function, resulting in more practical portfolios. Additionally, the model incorporates correlation constraints with bonds and stocks. We propose two portfolio styles: minimum risk and optimal, catering to different investment objectives.

We benchmark performance using the HFR Fund of Fund index (HFRIFOF), equally weighted, Minimum Volatility (MVP), and Maximum Sharpe Ratio (MSR) portfolios. Performance measures are evaluated for all portfolios, considering synthetic returns, cost minimization, and correlation constraints with stocks and bonds. Our results reveal that portfolios optimized with cost minimization and correlation constraints demonstrate superior risk-adjusted performance. Specifically, portfolios incorporating these constraints show a significant reduction in transaction costs and improved alignment with practical investment scenarios. The alternative optimization measures (CVaR, CDaR, and the Omega ratio) effectively capture the complex risk-return profiles of hedge fund investments, outperforming traditional optimization methods.

This study makes a significant contribution to the field of hedge fund portfolio optimization by addressing the unique characteristics of hedge fund returns and practical implementation challenges. By integrating cost minimization and correlation constraints into a semi-parametric model with alternative optimization measures, we provide a framework that enhances risk-adjusted performance while ensuring practical applicability. Our findings underscore the importance of advanced risk measures and realistic constraints in optimizing hedge fund portfolios, ultimately leading to better investment decisions and improved portfolio management.

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Chapter 1. Introduction

Hedge funds have emerged as significant players in the financial markets, offering sophisticated investment strategies designed to achieve high returns while managing risk. Characterized by their flexible investment strategies and diverse approaches, hedge funds seek to generate high returns and manage risk through a variety of sophisticated techniques. Unlike traditional investment vehicles, hedge funds often employ complex trading tactics, leverage, and derivatives to maximize performance, making their return distributions markedly different from traditional asset classes. This complexity poses unique challenges for portfolio optimization, as traditional parametric models, which assume normally distributed returns are often inadequate for hedge fund portfolios. These traditional models can underestimate the likelihood and impact of extreme events, leading to suboptimal investment decisions. Hedge fund returns often exhibit illiquidity and autocorrelation, complicating risk assessment and portfolio optimization. Traditional mean-variance optimization techniques fall short of accurately modeling hedge fund returns characterized by skewness, kurtosis, and fat tails. The limitations of traditional parametric approaches highlight the necessity for advanced portfolio optimization methods that can better account for the complexities inherent in hedge fund returns. Hedge funds exhibit unconventional distributional characteristics that traditional mean-variance optimization fails to capture. This shortcoming underscores the need for a semi-parametric approach that integrates both parametric and non-parametric methods, allowing for more accurate modeling of return distributions and risk assessment. This thesis aims to address a critical gap in the existing literature by proposing a semi-parametric model for hedge fund portfolio optimization. The integration of the proposed semi-parametric model using synthetic hedge fund returns offers a practical solution to achieve higher risk-adjusted returns. By considering cost minimization and correlation constraints, the model provides a more practical solution for real-world implementation. This research seeks to enhance the effectiveness of portfolio optimization, leading to better risk-adjusted returns and improved risk management for hedge fund investments. In this chapter, we dive into the hedge fund industry, first by examining hedge funds characteristics, operations, and roles before defining the goals of this research.

1.1. Hedge Funds: Definition, History, and Purpose

The Security Exchange Commission (S.E.C.) defines a hedge fund as a privately offered investment vehicle that pools the contributions of its investors to invest in a variety of asset classes, specifically a privately organized fund, administered by professional money managers and catering to sophisticated investors. Hedge funds stand apart from mutual funds by not being subject to the same regulations designed to protect the general public as well as relying on an asymmetric compensation system based on performance rather than a simple fixed percentage of assets. According to Fung and Hsieh (1999), the term was first introduced in 1949 to characterize a fund managed by Alfred Winslow Jones combining both long and short equity positions to “hedge” the portfolio’s exposure to market movements. Mr. Jones was the first manager to create a market-neutral fund seeking to profit from his stock-picking ability and not on the direction of the market. Shortly after, Jones created the first hedge fund product in 1952 by converting his fund into a limited partnership and introducing both a performance fee of twenty percent along a leveraged strategy. Hedge funds later got more attention in 1966, following an article from Fortune magazine describing Jones’ fund outperformance of the best mutual funds by 44% over five years net of fees. A rapid expansion of the industry followed for a few years until the recession of 1969-1970 along with the bear markets of 1973-1974 when many funds suffered large losses and withdrawals, resulting in hedge fund closures. This period of industry struggles against market risk saw the creation of the Quantum Fund, managed by the polarizing George Soros as well as Bridgewater Associates, founded by Ray Dalio in 1975 who would later on launch Pure Alpha and All Weather funds in the 1990s. Hedge funds went in the spotlight again in 1986 when an article in Institutional Investor highlighted Julian Robertson’s Tiger fund, founded in 1980, with a net annual return of 43%. Tiger Fund was created with $8 million and would reach $22 billion before closing in March 2000, leaving a permanent mark on the industry, and creating a legacy that would launch some of the best-known hedge funds, nicknamed “Tiger Cubs”. The 1990s are defined by Preqin as the real hedge fund boom with the emergence of superstar managers and a wave of new strategies spanning a large group of asset classes and investment styles. The fall of Long-Term Capital Management (LTCM), founded in 1994 by a group of prominent financiers, including Nobel laureates Myron Scholes and Robert C. Merton, authors of the Black-Scholes option pricing model, is certainly one of the most notable events of the 1990s, considering it mandated an intervention, orchestrated by financial regulators and Wall Street to prevent a systemic collapse. After the dot-com bubble burst, hedge funds became more popular with institutional investors such as pension funds, insurance companies, and sovereign wealth funds. The Great Financial Crisis of 2007 as well as large scandals such as Bernie Madoff's Ponzi scheme pushed for major changes in the industry regulations. The hedge fund industry today is still far less mature than the traditional asset management industry and keeps responding to regulation and an evolving investor client base. Nowadays, the total asset under management is \$3.61tn globally according to Preqin compared to \$100 trillion and \$150 trillion for both publicly traded equity and public bond market, as of the 2020 peak. Some of the largest funds are Citadel, Bridgewater, Renaissance Technologies, AQR, Viking, and Brevan Howard to name only a few.

Hedge funds' investment strategies differ considerably hence providing a single definition can be challenging. Historically, largely unregulated, they are regarded as private investments for wealthy individuals or institutional investors. They can invest in an unconstrained manner across a wide range of asset classes and instruments using a vast array of portfolio construction and risk management techniques. Previously reserved to accredited investors, the definition of which has been updated by the SEC in August 2020 to not only include net worth but also accreditation and knowledge of the industry, hedge fund products are now more accessible for smaller and retail clients. Hedge funds are characterized by more flexible investment strategies than mutual funds that seek to profit in all kinds of markets. They are allowed to use leverage and short selling, unlike mutual funds, since they are not subjected to the same regulations designed to protect investors. Usually organized as limited partnerships or limited liability companies, the structure of funds is increasingly available as managed accounts or via platforms. Hedge funds have been criticized for their high fee 2/20 structure corresponding to a 2% management fee and a 20% performance fee. This asymmetric fee can essentially be interpreted as an embedded put option that requires the manager to invest a significant portion of personal wealth to align his objectives with investors.

Hedge funds are often associated with commodity trading pools operated by commodity trading advisors (CTAs) which are registered and regulated by the Commodity Futures Trading Commission (CFTC) and limited to trading primarily futures contracts, usually transacting in over-the-counter (OTC) securities market. Hence, over the years the distinction between hedge funds and CTA funds has been blurred, for instance, LTCM was registered as a commodity pool operator. Fung and Hsieh (1999) agree that the distinction lies in the performance characteristics of the funds themselves and not their licenses.

Hedge fund investing starts with an open investment at the beginning which can be withdrawn or increased periodically, with motives for investing in such funds varying over time. In the 1980s, private investors were mainly attracted to high absolute returns. As institutional investors started allocating to hedge funds, objectives shifted as they started to look for volatility reduction, a reduced correlation to markets, and improvements on a risk-adjusted basis. Hedge funds are often praised for their low correlation to both stocks and bonds, however, Mirabile (2020) defines clear risks and benefits: Hedge funds can reduce volatility and increase returns when added to a traditional portfolio, protect principal, and enhance portfolio returns in environments not favorable to traditional stocks and bonds. Hedge funds also allow investors to participate in a variety of new financial products not available to traditional investors. Their strategies are diverse and exhibit low or uncorrelated returns to traditional portfolios, with equity long and short funds exhibiting the highest correlation to equities and macro funds the lowest. The vast majority of hedge fund strategies offer a middle ground in terms of volatility with a proposition lower than equities but higher than bonds. Moreover, consistent performing managers have low or negative exposure to factors that usually drive traditional returns. Alternatively, many strategies exhibit non-normal distributions sometimes displaying extreme skew and kurtosis, exposing investors to significant tail risk. Hedge funds often use leverage and derivatives which can create a mismatch between fund assets and liabilities. Another downside is the different biases that hedge fund databases suffer from, such as survivorship bias misrepresenting actual historical fund performance.

The industry has historically largely been unregulated with an exemption allowing funds to operate without registering with the SEC or CFTC under the “private advisor” exemption. However, in the US, the Dodd-Frank Wall Street Reform and Consumer Protection Act, implemented in 2011, made significant changes to the registration, reporting, and recordkeeping of funds, largely unchanged since the Investment Advisers Act of 1940. This reform came into effect in direct response to the great financial crisis and allows authorities to collect data from registered investment advisors about their funds to assess systemic risk by the Financial Stability Oversight Council. In Europe, a similar response was adopted, called the Alternative Investment Fund Manager Directive (AIFMD) to force hedge funds to upgrade compliance and operational frameworks. This inflow of new regulation in the industry led to new barriers to entry and an increase in competition toward a more professionalized and institutionalized industry. Alternatively, due to the nature of hedge funds, managers tend to be averse to regulation mandating a high level of transparency to preserve their “hedge” or winning strategy.

1.2. Fees

As we touched upon earlier, hedge funds usually charge both a fixed fee and a performance fee. The fixed fee is a management fee calculated as a percentage of a fund's net asset value; it can vary from 1% to 2%. The incentive fee is based on the performance of the fund to reward managers for generating positive returns; it is typically set at around 20%. The fund might also have a hurdle rate which is the minimum rate of return that a manager should generate before charging a performance fee, it can either be a fixed rate or most often a benchmark interest rate. Another feature that can be integrated into the fund contract is a high-water mark. The manager is only allowed to charge a performance fee if the fund value exceeds the high-water mark, denoting the highest value a fund has ever reached. Therefore, performance fees are only paid on new profits instead of positive performance simply offsetting previous losses. Management fees are intended to cover the fund's operating expenses and the incentive fee aligns the objectives of the manager with those of the investor by rewarding the fund owner and paying employee bonuses for good performance. Getmansky et al. (2015) explains that option-like fees charged by hedge funds exist because of the limited capacity of investment strategies, hence performance cannot be rewarded with linearly scaled fees. Furthermore, this paper suggests that fees are relevant to investors by impacting direct returns as well as manager behavior and that a high watermark can help alleviate inefficiencies created by asymmetric information. Finally, it concludes that the fees and costs of investing in hedge funds are very significant and should be monitored closely, with a net present value of fees reaching as high as 33% of the amount invested. Nowadays, recent hedge fund scandals and the crisis of 2007, which consequently pushed for more regulation of the industry, are creating higher barriers to entry for funds by inducing new costs. The post-Great Financial Crisis industry must comply with new regulations, legal, compliance, state-of-the-art hardware, and best practices, with independent audits tracking assets, as well as a clientele pushing for lower fees with more due diligence, forcing many funds to close.

1.3. Leverage

The SEC describes a hedge fund as an investment fund with the following features: it charges a performance fee, is highly leveraged, and makes use of short selling. While some funds might not use leverage, it certainly is a common feature across the industry and a significant difference with mutual funds. Leverage is typically described as the use of debt to invest to increase potential returns. It magnifies both potential gains and potential losses of an investment, therefore increasing returns as well as the risk of an investment. Hence, leverage can have multiple uses depending on the strategy of the fund. It can enhance returns while increasing the risk, amplify low-risk strategy returns, reduce volatility, or improve liquidity and lower transaction costs. The investment strategy will determine the appropriate type of leverage, for instance, a market-neutral manager might choose to enhance returns since his strategy is hedged by design to market risk and bears low volatility, whereas a long/short fund will reduce long exposure by shorting, while the liquidity benefits are experienced in the commodities market. According to Getmansky et al. (2015), leverage levels in hedge funds vary from zero to large multiples of assets under management. Economy-wide factors tend to predict changes in hedge fund leverage, for instance, a decrease in funding costs and fund returns as well as an increase in forecasted market values lead to higher leverage. It provides evidence that hedge funds are able to adjust their portfolio market exposure as market liquidity changes with the top liquidity timing funds outperforming the bottom ones by upwards of 5%. Mirabile (2020) describes the golden age of hedge funds as the period when hedge funds had access simultaneously to low-cost loans and derivatives with high implied leverage. The industry lived through a very lucrative stage, generating billions in fees before the financial crisis until major losses hit unexpected magnitudes in 2008. Large positions with heavily leveraged losses also generally lead to margin calls, defined as the broker requiring funding the position back to the initial margin, which can procure the death blow to a fund already coping with large losses and withdrawals.

One of the most common leverage methods is through shorting. By shorting a security, a manager borrows the desired asset from a lender to sell it on the market with the obligation to buy back the shorted security at a later time to return it to the lender. As a result of the sale, the manager receives cash that can be used to buy more securities than the fund originally could. A manager may decide to leverage the fund with implied leverage through the use of derivatives such as options, futures, forwards, and swaps. However, the fund is exposed to a risk of appreciation of the security potentially generating a future loss. Additionally, excess leverage in a fund can be an early warning sign of fund failures. Other warning signs include but are not restricted to, the excess size of the fund, a lack of transparency, a funding mismatch, and manager hubris. However, hedge fund failures usually result from several compounded bad decisions spanning from fraud and the many forms it can take, accounting fraud, valuation fraud, misappropriation of funds, to excessive leverage, unexpected tail events, unanticipated withdrawals, liquidity traps, short squeeze and even lack of supervision or lack of compliance.

1.4. Fund terms

Hedge funds have more peculiarities that set them aside from mutual funds, one of them being limiting opportunities to redeem shares, typically redemption periods span from monthly to quarterly or annually. In addition, funds often include a lock-up period of one year or more on new investors or even on new deposits during which investors cannot withdraw their funds. On top of these restrictions, investors must hand in an advance notice, typically 30 days but up to one year, prior to making any withdrawals. Furthermore, funds may charge a redemption fee before an investor is able to withdraw.

A manager also has the ability to impose gates, a temporary restriction to prevent or limit capital redemption, to protect the fund against any fire sale liquidations that would generate extreme losses for the fund and remaining investors. Furthermore, the fund is subject to subscription frequency limits which restrict the number of investments an investor can make within a given time, usually subscription opportunities are at least if not more frequent than redemption ones. A manager may also decide to close a fund because it has reached the capacity constraints of a strategy. It should be noted that funds that are gated, usually during a financial crisis, can be traded on the secondary market often with large discounts.

1.5. The industry's role in the Great Financial Crisis

In the post-financial crisis world of 2007 and 2008, hedge funds' responsibility and role in the crisis has long been questioned. During the first half of 2007, the public remained largely unaware of shifts in expectation of future economic conditions. Indeed, equity markets remained mainly unaffected excluding stocks in the financial sector. During July 2007, active money managers saw a performance downtrend in factor-driven portfolios based on Fama-French factors. Following the negative performance of factors the month before, some of the most successful equity hedge funds of the industry reported record losses by the first week of August 2007, affecting quantitatively managed equity market-neutral funds. Khandani and Lo (2008) described the most likely unfolding of events that would come to be dubbed the Quant Meltdown of 2007. Market-neutral hedge funds suffered losses in early August 2007 due to the forced liquidation of one or more large equity market-neutral portfolios either to raise cash or reduce leverage. These first losses instigated other funds to deleverage their portfolios resulting in additional price impacts and generating further losses and more deleveraging, sparking a deadly feedback loop of forced liquidation that would bring down many quantitative equity market neutral managers. Throughout August, the pace of liquidation increased, causing more and more dislocation of the quantitative portfolios due to liquidity providers decreasing their capital.

Litterman (2013) provides incredible insights on how the event unfolded as a partner at Goldman Sachs in 2007. As a head of quantitative research, Robert Litterman was in charge of the Global Equities' Opportunity Fund, a highly leveraged market-neutral fund. He recalls the losses of July, followed by unprecedented losses in August and sudden liquidation that got the fund in trouble. As it was, according to Litterman, too late to do anything, the losses kept getting worse and Goldman Sachs organized a bailout of $3B in the span of 48 hours. He explains the financial meltdown that followed was caused by a fundamental risk management error, a mispricing of systematic risk embedded in mortgages. Systemic risk was not recognized and priced accordingly in the mortgages until it was too late. The systemic risk was the risk of mortgage defaulting, causing economic disruption and stalling, creating a positive feedback loop and ultimately a meltdown of the financial system. He further describes the issue that not pricing this risk led to too much mortgage risk being created and when the risk was suddenly recognized, the market values of these mortgages melted away which caused the disaster. It is the emergence of incorrect incentives due to this mispricing of risk that led to the creation of the conditions for the financial crisis.

The Quant Meltdown highlighted additional risks, namely tail risk, from occasional liquidation and deleveraging, induced by events completely unrelated to equity markets which depicts the endogenous nature of liquidity and the degree of interconnectedness between markets. Khandani and Lo (2008) admits that industry participants directly involved in the Quant Meltdown may not have been aware of the broader milieu which corroborates with Litterman's version of events inside Goldman Sachs at the time. However, since the topic involves hedge funds, proprietary trading desks, their prime brokers, and credit counterparties, primary sources are virtually impossible to access, and sources are not at liberty to disclose any information. Henceforth, indirect means are the only option to gain insight into these events and we can only formulate hypotheses as to what happened.

The fall of Long-Term Capital Management in 1998 and the bailout by the Fed it necessitated, shows how hedge funds have become inextricably linked to systemic risk. Right before the crisis, hedge funds were one of the largest purchasers of the riskiest tranches of Collateralized Debt Obligations (CDOs). According to Getmanksy et al. (2015), more than half of hedge funds would engage in correlation trades, buying equity tranches and shorting other tranches simultaneously. Moreover, many funds also used credit default swaps (CDSs) to bet on different tranches of the same CDO security. Therefore, hedge funds would stand to profit from a value appreciation of CDOs and they would profit even more if the market crashed, this implies that not only do they benefit from financial instability, but they also have financial incentives to increase systemic risk. The great financial crisis highlighted the need for regulators to obtain greater transparency to be able to gauge systemic risk and keep track of crowded trades, justifying the need for the SEC to collect confidential hedge fund data. The authors agree that unconstrained and dynamic investment strategies can facilitate the buildup and propagation of systemic risk and shocks. Hedge funds are often criticized for being part of the opaque shadow banking system, however, hedge funds supported the economy in 2008 when eight investment funds, with the help of the US Treasury, created a plan to buy back troubled assets. They, with their investors, are a vital part of the financial ecosystem in providing capital. Hedge funds' contribution to systemic risk might be an unavoidable consequence of providing market liquidity and their drawbacks should be carefully supervised by the regulators.

1.6. Illiquidity and autocorrelation

A defining characteristic of hedge fund investing is illiquidity, a criterion responsible for major financial crises including the Quant Meltdown of 2007, as well as the downfall of LTCM in 1998, propping an intervention by the regulators to maintain financial stability. Understanding illiquidity in hedge funds is crucial to generating long-term wealth and evading its short-term pitfalls for an unwary investor caught in a selling stampede. Sadly, market illiquidity is a complex notion, lacking a universally agreed-upon definition, however, liquidity can be evaluated on three characteristics: the speed at which an asset can be sold, the quantity able to be sold, and the price impact generated from selling a large amount. Moreover, liquidity has proven to be time-varying, as observed during the crisis cited above, funds' assets can suddenly, depending on the context and market dynamics, switch from liquid to highly illiquid at a determining moment, putting the fund at risk if not correctly anticipated.

Several theoretical and empirical measures of illiquidity have already been proposed such as percentage bid/offer spreads, price impact, and volume-based statistics, however, these measures are flawed since they assume frequent price discovery and trading through a centralized exchange when hedge funds are largely structured as privately placed securities.

Getmansky et al. (2015) propose using autocorrelation ρ_k as a proxy for illiquidity. Efficient markets should produce serially uncorrelated returns $\rho_k = 0$ for all k > 0, implying that past returns contain little information about future returns otherwise, it could be exploited, pushing return autocorrelation of returns close to zero again. This work states that two reasons can hinder exploiting autocorrelation therefore preventing its reduction to zero and observing $\rho_k > 0$: if the autocorrelation is due to a time-varying equilibrium of returns and if trading friction such as illiquidity prevents exploiting autocorrelation as a predictor of returns. However, it also deducts that the time-varying equilibrium of returns is less likely to explain autocorrelation for shorter periods based on their definition, hence the autocorrelation of returns may be explained by illiquidity which, in practice, implies that investors cannot exploit information about these assets as they cannot be traded quickly or in large sizes or without a significant price impact. Using a proprietary model created to compute autocorrelation in hedge funds, the authors corroborate that funds with higher autocorrelation tend to be more illiquid funds. Other research work has examined return autocorrelation, and short or long redemption notice periods, observing higher autocorrelation in less liquid portfolios.

1.7. Fallouts of mean-variance optimization in hedge funds

Getmansky et al. (2015) elaborates on the implications of illiquidity in hedge funds in the context of the Modern Portfolio Theory (MPT) of Markovitz (1952). Indeed, risk in the MPT is attributed to the standard deviation of returns and is not appropriate to capture downside risk such as market disruptions caused by illiquidity. These disruptions or events such as fire sales and temporary market failures are a type of risk that should reward investors carrying it. On the basis that a traditional mean-variance optimization would determine the best return one could obtain for a certain level of risk, the authors tested mean-variance optimization with hedge funds to compute the efficient frontier while constraining autocorrelation. They observed they would limit the capture of the illiquidity premium and estimated an illiquidity premium of 2%. In other words, to earn higher returns an investor must bear the risk of market fluctuations as well as the risk of market disruptions carried by illiquidity. Investors must determine their tolerance for illiquidity on top of traditional market risk to better allocate their capital in strategies meeting these criteria. Highly illiquid hedge fund strategies might provide better opportunities and improved returns at the risk of more exposure to market failures.

Chapter 2. Literature review

During the Great Financial Crisis, the hedge fund industry suffered large trading losses and withdrawals, decimating the number of hedge funds as well as assets under management. However, as soon as April 2011 the number of funds had recovered to previous levels. Harris and Mazibas (2012) describes the role of portfolio optimization as a key factor for this recovery, enabling both the launch of investable hedge fund indexes for small investors as well as making funds of hedge funds readily available.

Hedge fund returns are notably autocorrelated due to illiquidity and exhibit significant negative skewness and excess kurtosis, influenced by the strategies employed by fund managers. Consequently, traditional parametric mean-variance optimization often yields suboptimal portfolio weights. Harris and Mazibas further demonstrate that hedge fund returns display volatility clustering and that incorporating multivariate conditional volatility models can enhance portfolio performance. This study aims to further develop the use of a semi-parametric approach to address the limitations of both parametric and non-parametric methods.

2.1. Parametric approach

Parametric estimation methods operate by first assuming a distribution function, usually a Gaussian distribution, then estimating the parameters of the distribution by using the data. Even though this model is relatively easy to implement it requires the data to accommodate the distribution of the model. Therefore, Gaussian distribution-based models require the data to conform to the same distribution function otherwise the models will lead to inaccuracies in estimations and suboptimal portfolios. Indeed, parametric models offer less flexibility in modeling various data types but can work well with smaller sample sizes, pending the assumptions hold. Kassberger and Kiesel (2006) proposes a fully parametric model capable of adequately describing both univariate and multivariate return properties. They find that the Normal distribution is inadequate as a model for hedge fund returns which are typically skewed and fat-tailed. Their parametric model based on the multivariate extension of the Normal Inverse Gaussian distribution is shown capable of capturing the characteristic distributional features of hedge fund returns. Furthermore, they also compared the VaR and CVaR based on their parametric model to risk measures based on a normal distribution, demonstrated the superiority of their model, and discussed its application to portfolio optimization. Morton et al. (2005) uses a normal-to-anything method to model and simulate non-normal hedge fund returns with prescribed marginal distributions and correlation structure. Additionally, they use their portfolio optimization model to construct a fund using hedge fund indices. Their approach is able to recognize and use the information embedded in the unusual return distributions of hedge funds to actively manage a portfolio of hedge funds with the goal of systematically maintaining performance above a given benchmark.

2.2. Non-parametric approach

On the contrary, non-parametric models make fewer assumptions than parametric ones by not relying on a specific distribution function of the data, rather empirically estimating cumulative distribution functions without imposing a structure, therefore allowing for more flexibility. However, they come with several significant drawbacks, notably the curse of dimensionality. As the number of dimensions increases, the volume of the data space grows exponentially, leading to data sparsity. This sparsity makes it challenging for non-parametric methods to find sufficient nearby data points to make reliable estimates, which can severely impact their accuracy and performance. Additionally, the computational complexity of non-parametric methods typically increases with the number of dimensions, making them computationally intensive and time-consuming in high-dimensional spaces. Jurczenko et al. (2006) proposes a nonparametric optimization criterion for the static portfolio selection problem in the mean-variance–skewness–kurtosis space. It proposes an empirical application on funds of hedge funds to show a multi-moment efficient portfolio set, thereby encompassing hedge funds' unconventional return distribution. Almeida et al. (2019) explores performance measures for hedge funds based on identifiable stochastic discount factors. By applying this methodology to a panel of individual hedge funds, their analysis reveals that fewer funds have a statistically significant positive alpha compared to Jensen's alpha obtained by the traditional linear regression approach. This discrepancy in alpha between their nonparametric methodology and the traditional Jensen's alpha also reveals considerable variation in hedge fund rankings.

2.3. A semi-parametric approach

By using a semi-parametric approach, we intend to address the shortcomings of parametric and non-parametric approaches in the context of hedge fund optimization. It consists of a hybrid approach using a parametric approach to capture the global characteristics of hedge fund returns while using non-parametric models to account for higher moments and capture the full behavior of their distribution. Fabozzi and Subbiah (2016) presents non-parametric methodologies to capture the time-varying alpha in Asian hedge fund returns while capturing the beta component through an OLS regression using Fung and Hsieh (2004) factors as predictors. They find that a combination of the OLS regression with the Fung and Hsieh factors combined with their portfolio construction produces the best information ratio. The authors also report the drawback of their non-parametric regression suffering the curse of dimensionality, from a lack of sufficiently long history to validate their methodology. Del Brio et al. (2013) compares the performance of risk measures based on parametric distribution and semi-nonparametric methodologies. Their findings show that parametric methodologies fail to forecast hedge fund VaR, whilst their semi-non-parametric and extreme value theory approaches accurately succeed in it. Furthermore, their results extend to the multivariate framework and demonstrate that their semi-nonparametric multivariate approach accurately captures portfolio risk and outperforms VaR estimates obtained through a multivariate parametric approach. Harris and Mazibas (2012) overcomes data limitations of the non-parametric approach by using extreme value theory while preserving the properties of the times series of returns to provide better forecasts of the tails of the return distribution. This approach accounts for both linear and non-linear risks while being able to capture the dynamics of volatility clustering and autocorrelation. They show significant risk-adjusted portfolio performance improvements when comparing CVaR, CDaR, and Omega to parametric mean-variance optimization models. Additionally, semi-parametric estimation of the CVaR, CDaR, and Omega risk measures offers a very significant performance increase over non-parametric estimation.

Despite their advantages, current hedge fund optimization methodologies have limitations in practical applications. The first consideration is to integrate turnover minimization into optimization. A high turnover effectively means that a high percentage of assets is sold and/or bought, each time incurring brokerage fees, bid-ask spreads, and market impact costs. Garleanu and Pedersen (2013) offers a perspective on an optimal trading strategy that weighs the expected benefit of trading against its transaction costs and risks. Instead of trading all the way to the optimal portfolio and incurring high transaction costs, they offer a smoother, more conservative portfolio that moves in the direction of the aim portfolio while limiting turnover. They conclude that their net of trading costs strategy performs significantly better since it incurs far lower trading costs while still capturing much of the return predictability and diversification benefits. Portfolio turnover is therefore a critical variable to control for, which should also be minimized in the optimization process. Moreover, optimization models failing to implement turnover also create other undesirable portfolio characteristics such as larger market impact costs while being generally more difficult to implement due to higher liquidity of assets needed, and regulatory or market constraints. Furthermore, another consideration for practical application is the ability to consider the correlation of the portfolio to other asset classes such as stocks and bonds. A portfolio with a high correlation to stocks or bonds is unlikely to be implemented by hedge fund managers for several reasons. First, from a risk exposure perspective, portfolios can become overexposed to systemic risk, especially during economic downturns when the correlation of assets shoots up, increasing the risk of the portfolio and possibly leading to larger losses than first expected. During extreme market events, a portfolio with unconstrained correlation experiences a larger tail risk, carrying an unexpected downside risk usually underestimated before the event. Second, hedge fund clients are already invested in other asset classes therefore when considering their allocation to the hedge fund asset class, one of the determining factors is the added diversification benefits that it proposes. However, diversification benefits are inversely proportional to the correlation between assets, hence a highly correlated portfolio might not align with investors' expectations regarding their allocation to hedge funds.

We follow and extend beyond the implementation performed in Harris and Mazibas (2012), considering transaction costs and especially minimizing portfolio turnover, resulting in portfolios aligning more closely with real-world implementation. Additionally, we directly constrain the portfolio correlation to a stock and bond index in the optimization model, controlling unexpected downside risk and diversification benefits.

Chapter 3. Methodology

In hedge fund optimization, several issues arise from the unconventional distribution of hedge fund returns: negative skewness (asymmetry), excess kurtosis (fat tails), autocorrelation (indicative of illiquidity), and heteroskedasticity (volatility clustering). Our proposed methodology addresses these challenges through a multi-step process designed to refine hedge fund optimization models, taking into account these unique return characteristics and the practical implementation of such models.

We implement a semi-parametric approach using the standardized residuals from an AR(1)-EGARCH(1,1) volatility model. This combines a parametric tail estimation using extreme value theory (EVT) and a Generalized Pareto Distribution (GPD) with a non-parametrically smooth center achieved through a Gaussian Kernel. To model dependence, we utilize a Student's t copula, generating synthetic returns from the semi-parametric residuals. The EGARCH model's volatility estimates are then reintroduced to the synthetic returns.

We present our initial set of portfolios and subsequently implement turnover minimization and correlation constraints. Portfolios are constructed out-of-sample using synthetic returns, employing the following risk measures: CVaR, CDaR, and Omega ratio. Finally, we propose two styles of portfolios to cater to different investor expectations: minimum risk portfolios to minimize risk and optimal portfolios to maximize performance per unit of risk.

3.1. Volatility modeling

We begin with a volatility model, specifically the AR(1)-EGARCH(1,1) model, first introduced by Engle (1982) and refined by Nelson (1991), to obtain independent and identically distributed standardized returns. The autoregressive component (AR) of the model captures serial correlation and the flexible volatility structure (EGARCH) effectively handles asymmetry in volatility (heteroskedasticity) and volatility clustering. It also accounts for leverage effects, where negative shocks have a larger impact on volatility than positive shocks. A Student's t distribution for residuals addresses the fat tails observed in hedge fund returns.

The return process is modeled as an AR(1) process:

$$r_{i,t} = \mu_i + \theta_i r_{i,t-1} + \varepsilon_{i,t} \text{ and } u_{i,t} = \frac{\varepsilon_{i,t}}{\sigma_{i,t}}, \qquad u_{i,t} \sim t_d(\nu, 0,1)$$

where $r_{i,t}$ is the return at time t, μ_i is the unconditional mean, θ_i is the autoregressive coefficient, $\varepsilon_{i,t}$ are the residuals, and $u_{i,t}$ are the standardized residuals following a Student's t distribution with ν degrees of freedom.

The volatility process is modeled as an EGARCH (1,1):

$$\log(\sigma_{i,t}^2) = \omega_i + \alpha_i \left(|u_{i,t-1}| - \sqrt{\frac{2}{\pi}} \right) + \gamma_i u_{i,t-1} + \beta_i \cdot \log(\sigma_{t-1}^2)$$

where $\sigma_{i,t}^2$ is the conditional variance at time t, ω_i is the intercept term, α_i is the asymmetry parameter, β_i is the lag coefficient, γ_i is the leverage parameter.

We fit this model to each hedge fund index return using Ordinary Least Squares (OLS) to estimate parameters, then standardize the residuals for our semi-parametric approach, retaining the conditional volatility for future use.

3.2 Extreme Value Theory

We use a semi-parametric approach on standardized residuals to estimate the tails parametrically and smooth the center non-parametrically. This is done for each index return series.

3.2.1. Parametrical tails

We apply the Peaks Over Threshold (POT) method to model the tails of the distribution. Upper and lower thresholds are set at the 95th and 5th percentiles, respectively, creating 10% tails and a 90% center.

For the upper tail of the distribution, residuals exceeding the 95th percentile are modeled using a Generalized Pareto Distribution (GPD) with Maximum Likelihood Estimation (MLE). The GPD is a probability distribution allowing for heavy tails, well suited to model extreme events. From Hosking and Wallis (1987) the distribution function of a GPD is defined by:

$$F(x) = \begin{cases} 1-(1 - kx/\alpha)^{1/k}, & \text{if } k \neq 0 \\ 1 - e^{-x/\alpha}, & \text{if } k = 0 \end{cases}$$

as well as the density function:

$$f(x) = \begin{cases} \alpha^{-1}(1 - kx/\alpha)^{1/k-1}, & \text{if } k \neq 0 \\ \alpha^{-1}e^{-x/\alpha}, & \text{if } k = 0 \end{cases}$$

where α is the scale parameter, and k is the shape parameter. The range of x is $0 \leq x \leq \infty$ for $k \leq 0$, and $0 \leq x \leq \alpha/k$ for $k > 0$.

Subsequently, we estimate extremes from the GPD, using the fitted distribution parameters, and replace the upper tail. This parametric extrapolation of the tail allows us to estimate exceedances that lie outside the range of historical data. Similarly, we use the lower threshold defined at the 5th percentile to fit a GPD to the lower tail, draw extreme events, and replace the upper tail accordingly.

3.2.2. Non-Parametrical center

The center of the distribution is smoothed non-parametrically using a Gaussian kernel. From Chung (2012), the transformation is:

$$Y(t) = \int K(t,s)\,X(s)ds$$

where K is the kernel of the integral with X and Y are the input and output signals respectively. The Gaussian kernel K is defined as:

$$K(t) = \frac{1}{\sqrt{2\pi}}e^{t^2/2}$$

The Gaussian Kernel is scaled by σ.

$$K_\sigma(t) = \frac{1}{\sigma}K\left(\frac{t}{\sigma}\right)$$

Smoothing reduces short-term fluctuations or noise that do not represent any significant underlying trend while highlighting longer-term patterns. With the parametric estimation of tails and non-parametric smoothing of the center completed for all return indices, we proceed to model dependence.

3.3. Copula

To model the dependency structure of the semi-parametric residuals, we use a Student's t copula function. This copula is particularly useful because it allows for tail dependence, meaning extreme events in one time series can be associated with extreme events in another. This is an essential feature when modeling financial returns.

A copula of a random vector $\boldsymbol{X} = (X_1, \dots, X_d)$ is defined as:

$$\boldsymbol{C}(\boldsymbol{u}) \coloneqq C(u_1, \dots, u_d) = F\left(F_1^{-1}(u_1), \dots, F_d^{-1}(u_d)\right)$$

where $F_i^{-1}(u_i)$ are the quantile functions, also known as the inverse CDFs, of the marginal distributions. As demonstrated by Demarta and McNeil (2005), a copula remains invariant under the standardization of the marginal distributions. Hence, a copula of a $t_d(\nu, \boldsymbol{\mu}, \Sigma)$ distribution is identical to a $t_d(\nu, \mathbf{0}, \boldsymbol{P})$ distribution, where $\boldsymbol{P}$ is the correlation matrix derived from the dispersion matrix Σ.

For a random vector $\boldsymbol{X} \sim t_d(\nu, \mathbf{0}, \boldsymbol{P})$ following a multivariate t-distribution, and the return vector $\boldsymbol{U} = \left(t_\nu(X_1), \dots, t_\nu(X_d)\right)'$, where ν is the degree of freedom of a standard univariate t-distribution, the t copula can be expressed as:

$$c_{\nu,P}^{t}(u) = \frac{f_{\nu,P}\left(t_\nu^{-1}(u_1), \dots, t_\nu^{-1}(u_d)\right)}{\prod_{i=1}^{d} f_\nu\left(t_\nu^{-1}(u_i)\right)}$$

where $f_{\nu,P}$ is the joint density of a $t_d(\nu, \mathbf{0}, \boldsymbol{P})$ distributed random vector and f_ν is the density of the univariate standard t-distribution with ν degrees of freedom.

Using the correlation matrix from the semi-parametric residuals, we set the degrees of freedom at 15 for the copula. From the Student's t copula, we draw dependent uniform marginal cumulative distribution functions (CDFs). These are then transformed back into standardized residuals by applying the inverse CDF to each time series. Finally, we reinject the heteroskedasticity and autocorrelation, as estimated earlier in the EGARCH model, into the standardized residuals to generate the hedge fund returns for each index. These simulated returns, known as synthetic returns, will be used in the subsequent portfolio optimization process.

3.4. Risk measures

We use the following risk measures to construct our portfolios:

3.4.1. Value at Risk (VaR):

The Value at Risk (VaR) at a given confidence level α, denoted by VaR_α, is the α-quantile of the loss distribution. It is defined as the smallest loss x such that the cumulative distribution function (CDF) of losses F_X is at least α. Mathematically, it can be expressed as:

$$\mathrm{VaR}_\alpha = \inf\{x: F_X(x) \geq \alpha\}$$

3.4.2. Conditional Value at Risk (CVaR):

The Conditional Value at Risk (CVaR), at a given confidence level α, denoted by CVaR_α, is the expected loss given that the loss exceeds the VaR_α. CVaR captures tail risk more effectively than VaR. It is defined as:

$$\mathrm{CVaR}_\alpha(X) = \mathrm{E}[\,X \mid X \geq \mathrm{VaR}_\alpha(X)\,] \quad \text{with} \quad \mathrm{VaR}_\alpha = \inf\{x: F_X(x) \geq \alpha\}$$

where X represents the loss and F_X is CDF of X.

3.4.3. Drawdown at Risk (DaR):

The Drawdown at Risk (DaR) at a confidence level α, denoted by DaR_α, is the α-quantile of the drawdown distribution. Drawdowns are peak-to-trough declines of portfolio values during a specific period. For a time series of portfolio values $X(t)$, the drawdown at time t is defined as:

$$\mathrm{DD_t}(X) = \frac{\max_{s \le t} X(s) - X(t)}{\max_{s \le t} X(s)}$$

DaR is the smallest drawdown x such that the CDF of drawdowns $\mathrm{F_{DD}}(x)$ is at least α:

$$\mathrm{DaR}_\alpha = \inf\{x: F_{DD}(x) \ge \alpha\}$$

3.4.4 Conditional Drawdown at Risk (CDaR):

The Conditional Drawdown at Risk (CDaR) at a given confidence level α, denoted by CDaR_α, is the expected value of drawdowns exceeding DaR_α. CDaR is particularly useful for understanding the tail risk of portfolio drawdowns. It is defined as:

$$\mathrm{CDaR}_\alpha = \mathrm{E}[\,\mathrm{DD}(X) \mid \mathrm{DD}(X) \ge \mathrm{DaR}_\alpha(X)\,] \quad with\ \mathrm{DaR}_\alpha = \inf\{x: F_{DD}(x) \ge \alpha\}$$

3.4.5. Omega Ratio:

The Omega ratio is a performance measure that captures the probability-weighted ratio of gains versus losses for a given threshold return level τ. It can be expressed as:

$$\Omega(r_i) = \frac{\int_\tau^\infty [1 - F(r_i)]dr}{\int_{-\infty}^\tau F(r_i)dr}$$

where r_i indicates the returns of an asset i and $F(r_i)$ is the corresponding CDF.

3.5. Starting portfolios

To construct our portfolios, we adopt two distinct investment styles to cater to investors with different objectives and risk tolerances. Minimum-risk portfolios aim to minimize risk, making them suitable for investors who prioritize stability, capital preservation, and reduced exposure to volatility. Optimal risk portfolios aim to maximize returns per unit of risk, targeting investors willing to accept higher levels of risk for potential capital appreciation and higher gains.

The optimization processes are performed out-of-sample with monthly rebalancing to more accurately represent real-world performance. Out-of-sample methods enhance the robustness of the model, increasing confidence in its application to other datasets. We use a rolling window of length $\delta = 48$ months to generate one-month-ahead allocations. The first rolling window uses observations from $t = 1$ to δ to generate portfolio allocations for $t = \delta + 1$. The window then slides forward by one month to compute the next allocation, iterating until the end of the investment horizon $t = T$. The length of the rolling window δ involves trade-offs. A longer window captures more market conditions and economic cycles, offering more stability but reducing the back-testing period. Conversely, a shorter window allows the model to adapt quickly to recent market changes but may lack stability.

We consider n hedge fund indices and start our optimization at time $t = \delta + 1$, repeating for each rolling window until the end of the investment horizon at time $t = T$. A portfolio is characterized by a vector of weights $\boldsymbol{x} \in \mathbb{R}^n$, such that individual weights sum to 1 under the constraint $\boldsymbol{x}'\mathbf{1}_{\boldsymbol{n}} = 1$, where $\mathbf{1}_{\boldsymbol{n}}$ is an n-vector of ones. Portfolios weights $x_i \in [0,0.5]$ are constrained to a maximum of 50% to ensure minimum diversification, requiring the portfolio to be invested in at least two hedge fund indices. Furthermore, let $\boldsymbol{r} \in \mathbb{R}^n$ define the vector of returns of hedge fund indices, therefore the return of a portfolio is $r_x = \boldsymbol{x}'\boldsymbol{r}$.

For a minimum risk portfolio, the portfolio optimization problem is given by:

$$\min_x \Phi(\boldsymbol{x}) \quad \text{such that} \quad \boldsymbol{x}'\mathbf{1}_{\boldsymbol{n}} = 1,\ x_i \in [0,0.5]$$

For an optimal portfolio, the portfolio optimization problem is given by:

$$\max_x \frac{\boldsymbol{x}'\mathrm{E}(\boldsymbol{r}) - r_f}{\Phi(\boldsymbol{x})} \quad \text{such that} \quad \boldsymbol{x}'\mathbf{1}_{\boldsymbol{n}} = 1,\ x_i \in [0,0.5]$$

where $\Phi(\boldsymbol{x})$ is the portfolio risk function to be minimized, defined differently for each portfolio type:

- For CVaR portfolios: $\Phi(\boldsymbol{x}) = \mathrm{CVaR}_\alpha(\boldsymbol{x})$
- For CDaR portfolios: $\Phi(\boldsymbol{x}) = \mathrm{CDaR}_\alpha(\boldsymbol{x})$
- For Omega portfolios: $\Phi(\boldsymbol{x}) = \mathrm{E}[(\tau - \boldsymbol{x}'\boldsymbol{r})^+]$ with τ set to the risk-free rate r_f.

In the case of Omega portfolios, the minimum risk portfolio corresponds to the denominator of the Omega ratio and the optimal portfolio is the Omega ratio itself.

To benchmark, we compute a minimum variance portfolio (MVP) as well as a maximum Sharpe ratio portfolio (MSR) using historical returns, respectively defined by:

$$\min_x \left(\boldsymbol{x}' \sum \boldsymbol{x}\right)^{\mathbf{1/2}}, \qquad \text{such that } \boldsymbol{x}'\mathbf{1}_{\boldsymbol{n}} = 1,\ x_i \in [0,0.5]$$

$$\min_x \frac{\boldsymbol{x}'E(\mathbf{r}) - r_f}{(\boldsymbol{x}' \sum \boldsymbol{x})^{\mathbf{1/2}}}, \qquad \text{such that } \boldsymbol{x}'\mathbf{1}_{\boldsymbol{n}} = 1,\ x_i \in [0,0.5]$$

where $\sum$ is the $n \times n$ covariance matrix of returns.

3.6. Cost minimization

To mitigate transaction costs, we incorporate portfolio turnover PT_t in the objective function. Portfolio turnover from $t-1$ to t is defined as:

$$PT_t(\boldsymbol{x}) = \sum_{i=1}^{m} \left| x_{t,i} - x_{t-1,i} \right|$$

For a minimum risk portfolio, the new portfolio optimization problem is given by:

$$\min_x \Phi(\boldsymbol{x}) + \gamma_i PT(\boldsymbol{x}) \quad \text{such that } \boldsymbol{x}'\mathbf{1}_{\boldsymbol{n}} = 1,\ x_i \in [0,0.5]$$

For an optimal portfolio, the new portfolio optimization problem is given by:

$$\max_x \frac{\boldsymbol{x}'\mathrm{E}(\boldsymbol{r}) - r_f}{\Phi(\boldsymbol{x})} - \gamma_{\mathrm{i}} PT(\boldsymbol{x}) \quad \text{such that } \boldsymbol{x}'\mathbf{1}_{\boldsymbol{n}} = 1,\ x_i \in [0,0.5]$$

where, γ_{i} is the turnover penalty for each portfolio.

By adding the turnover in the minimization, we effectively reduce the amount of reallocation performed monthly thereby reducing transaction costs incurred.

3.7. Correlation constraints

To limit exposure to other asset classes, we set upper bounds on portfolio correlation to stock and bond indexes respectively ρ_s^{max}, ρ_b^{max}. Let the correlation between individual assets to the stock and bond index be $\rho_{s,i}$ and $\rho_{b,i}$.

Therefore, portfolio correlation with the stock index is:

$$\rho_{s,x}(\boldsymbol{x}) = \sum_{i=1}^{m} x_i \rho_{s,i}$$

and with the bond index:

$$\rho_{b,x}(\boldsymbol{x}) = \sum_{i=1}^{m} x_i \rho_{b,i}$$

For a minimum risk portfolio, the portfolio optimization problem is given by:

$$\min_x \Phi(\boldsymbol{x}) \quad \text{such that} \begin{cases} \boldsymbol{x}'\boldsymbol{1_n} = 1 \\ x_i \in [0,0.5] \\ \rho_{s,x}(\boldsymbol{x}) \leq \rho_s^{max} \\ \rho_{b,x}(\boldsymbol{x}) \leq \rho_b^{max} \end{cases}$$

For an optimal portfolio, the portfolio optimization problem is given by:

$$\max_x \frac{\boldsymbol{x}'\mathrm{E}(\boldsymbol{r}) - r_f}{\Phi(\boldsymbol{x})} \quad \text{such that} \begin{cases} \boldsymbol{x}'\boldsymbol{1_n} = 1 \\ x_i \in [0,0.5] \\ \rho_{s,x}(\boldsymbol{x}) \leq \rho_s^{max} \\ \rho_{b,x}(\boldsymbol{x}) \leq \rho_b^{max} \end{cases}$$

3.8. Final portfolios

Finally, we combine turnover minimization and correlation constraints to create portfolios using CVaR, CDaR, and Omega ratio with controlled transaction costs and a defined threshold for correlation to other asset classes. The resulting portfolios are as follows:

For a minimum risk portfolio, the final portfolio optimization problem is given by:

$$\min_x \Phi(\boldsymbol{x}) + \gamma_i PT(\boldsymbol{x}) \quad \text{such that} \begin{cases} \boldsymbol{x}'\boldsymbol{1_n} = 1 \\ x_i \in [0,0.5] \\ \rho_{s,x}(\boldsymbol{x}) \leq \rho_s^{max} \\ \rho_{b,x}(\boldsymbol{x}) \leq \rho_b^{max} \end{cases}$$

For an optimal portfolio, the final portfolio optimization problem is given by:

$$\max_x \frac{\boldsymbol{x}'\mathrm{E}(\boldsymbol{r}) - r_f}{\Phi(\boldsymbol{x})} - \gamma_{\mathrm{i}} PT(\boldsymbol{x}) \quad \text{such that} \begin{cases} \boldsymbol{x}'\boldsymbol{1_n} = 1 \\ x_i \in [0,0.5] \\ \rho_{s,x}(\boldsymbol{x}) \leq \rho_s^{max} \\ \rho_{b,x}(\boldsymbol{x}) \leq \rho_b^{max} \end{cases}$$

These final portfolios integrate constraints on turnover and correlations to ensure balanced risk management and cost efficiency while achieving the desired risk-return trade-off.

Chapter 4. Data

Before delving into our results, we begin with an exploration of different hedge fund categories, including global macro funds, equity long-short funds, fixed-income relative value funds, event-driven funds, and multi-strategy funds. Secondly, we discuss the origin of our data, recognizing the inherent biases in hedge fund databases. Finally, we present the statistical properties of our hedge fund indices as well as perform statistical tests to help identify distributional characteristics.

4.1. Global Macro funds

Global macro funds generally focus on using changes in macroeconomic factors to try to predict the price movements of different securities. It first appeared in the 1980s and received attention from the industry with the market crash of 1987. Indeed, now hedge fund superstar Paul Tudor Jones profited with short positions on S&P futures thanks to one of his earliest successes of predicting the Black Monday crash of 1987. A few years later, George Soros also helped popularize global macro funds with his infamous bet against the British pound in 1992. Global macro hedge funds can be further separated into two categories: discretionary funds versus systematic funds.

Discretionary funds use the manager's beliefs in variations of macroeconomic variables to determine the direction of the market and the relationship to the prices of financial assets. Macro strategies hedge funds aim to profit from broad market swings either from political or economic events. Several macroeconomic factors can be used by a fund as an indicator such as inflation, growth, variation in productivity, the pace of technology, regulation, and trade among nations. The manager uses its views of future states or probabilities assigned to multiple future states and builds a correlation matrix based on historical correlation to predict changes in prices, and then determines the security he will short or long. It usually starts with a trade idea based on a macroeconomic factor such as GDP growth rate, labor productivity, real interest rates, inflation, money supply, foreign direct investments, housing, employment rates, default rates, or any factor that the manager can identify to forecast price changes in securities. The manager then computes historical correlations and estimates the impact of the macroeconomic factor on a security's price. Finally, he needs to select the appropriate set of long and short to best capture profits as well as respect fund constraints of volatility target or trade size. Discretionary funds are striving to capture profits from price variations in financial assets based on manager's expectations of the economy.

Systematic funds are a type of funds that differ from discretionary by using model-driven or trend-following processes, instead of manager views, to identify future changes in financial asset prices. These funds use their quantitative models by collecting large amounts of data on financial instruments from the most liquid markets to identify patterns and create trading rules. They evaluate short- and long-term patterns to generate signals via quantitative analysis and use the model to determine their likely action. Managers in quantitative macro funds will primarily use liquid markets in futures, currency, and bond markets. A systematic fund requires significant computing power to run algorithms that evaluate price patterns to identify the type of patterns commonly used for technical analysis such as peaks, tops, bottoms, or continuation patterns. The fund then creates a set of trading rules to generate automated computer signals to buy or sell securities via an electronic platform to the exchange. A key advantage of systematic funds is the ability to back-test the set of trading rules generated using historical data.

According to Mirabile (2020), an investor might use macro funds to take advantage of its diversification benefits, as macro funds often have returns with low correlation to traditional stock and bond investments. They limit the instruments they trade to 3 common characteristics: anonymity, liquidity, and transparency. This includes futures, options on futures, currencies, government bonds, and very rarely individual stocks or corporate bonds. As a result, both systematic funds and discretionary funds offer some of the highest returns with the lowest correlation to stocks and bonds among the vast styles of hedge funds.

4.2. Equity Long-Short funds

According to Preqin (2023), equity strategies are the most widely used type of hedge fund in the industry today. Even though several types of long-short strategies exist, equities strategies can be summarized as taking a long position in stocks seen by a manager as undervalued and reciprocally a short position in stocks perceived to be overvalued. It is a strategy that might seem more approachable for a manager as it benefits from being similar to traditional long-only investing, by using the same tools for constructing portfolios, measuring performance, and evaluating risks. In the early days of hedge funds, most of the investors' attention went to global macro, continuing in the 1990s equity long-short strategies outperformed in the turmoil of the first Persian Gulf war and the Russian defaults, global macro still received most of the inflows from high-net-worth individuals seeking high returns. However, equity long-short strategies are easily understood as an extension of long only with the benefit of leverage and short selling. The strategy also saw inflows from institutional investors seeking a long-short manager's ability to generate alpha, after long-only managers failed to beat passive index returns during efficient market periods.

According to Mirabile (2020) and Preqin, equity long-short funds include a variation of funds operating under the long/short category such as long bias fund, variable bias fund, short bias, market neutral fund, and value-oriented. Long bias funds generally execute directional bets on individual stocks using leverage and relatively limited short selling to beat an index, maintaining a ratio of long superior to short positions while still preserving the possibility to transition to a variable bias fund depending on its allocation. Variable bias funds favor a more versatile long/short ratio, with the assumption of a 50% long and 50% short position allocation, they tend to lean more long than short. This type of fund performs individual bets on the success or failure of selected companies to generate alpha, reduce their correlation to the market, or lower their volatility, from a combination of long and short positions using either technical or fundamental analysis. On the other hand, short-bias funds exclusively short-sell overpriced equities by borrowing a stock and selling it today to buy it back later, hopefully at a lower price, and return it to the lender. Market-neutral funds invest in individual long and short positions in specific companies to exploit differences in stock prices with the same sector, industry, market capitalization, country, or other common factors. Equity market-neutral funds aim to produce uncorrelated absolute returns in any market environment, similar to a fixed-income instrument. These funds can be further categorized either into classic equity market neutral or quantitative market neutral with the latter using computer optimization in a portfolio to achieve net zero market value and net zero beta exposure instead of being simply managed for a specific exposure. Quantitative funds operate computer programs with mathematical models to identify trades and perform optimization to create portfolios bound by constraints of market and beta-neutral investments. Equity market neutral, however, seeks to identify patterns and themes that drive stock prices. Finally, value-oriented funds select securities perceived to be selling at a deep discount to their intrinsic value. They may use long-only positions or invest using futures or options until the true value of the security is recognized by the market. Altogether, equity long/short funds employ research processes similar to active equity investing and their performance reflects their skills as stock pickers while quantitative equity funds rely on their expertise to create models and apply them through computer programming.

4.3. Fixed Income Relative Value funds

Fixed-income relative value hedge funds seek to exploit pricing inefficiencies of fixed-income securities by executing bets on relative changes in interest rates, credit spreads, or bond prices. They use a vast set of investment strategies to exploit valuation differences and pricing anomalies between US or international government-issued bonds, corporate bonds, agency bonds, or hybrid securities. Managers exploit a complex set of derivatives to either hedge or profit from the identified mispricing such as interest rate swaps, swaptions, or credit default swaps. A mispricing is identified by seeking a security that is temporarily cheaper or more expensive than another identical or at least very highly correlated one. Macro funds already have a significant understanding of assessing trends and economic framework, hence they usually apply their views and expectations of interest rates, credit, and inflation through a fixed income relative value desk.

Fixed-income hedge funds can be further subdivided into three specific strategies: relative value, credit arbitrage, and convertible bond arbitrage. Relative value managers examine historical relationships and evaluate inefficiencies to understand if the price of the security can revert to the mean as a result of an event or at a predetermined time. These managers most generally learn to identify patterns and understand valuation models, and relationships between products, through previous bank trading desk experiences needed to grasp the complexity of fixed-income instruments. The set of tools available to a relative value fund is comprised of currency markets, liquid interest rates, or listed derivatives. Lhabitant (2006) explains that a relative value fund might for instance investigate the pricing discrepancies occurring on a newly issued 30-year US Treasury bond, considered on the run, versus the same bond issued 6 months ago. This gap in pricing, often attributed to a temporary liquidity premium, might generate an opportunity for a fixed-income fund to generate a profit on an irrational difference which at some point will converge. A fund can simply long the cheaper "off the run" treasury while shorting the more expensive "on the run" bond, securing a profit from a spread on convergence.

Credit arbitrage funds invest in debt instruments intending to profit from inefficiencies in lending by exploiting opportunities using derivatives such as credit default swaps (CDS). By implementing strategies with offset long and short in corporate bonds, equities, options, or CDSs with varying liquidity or pricing transparency, funds generate alpha when prices return to normal. Credit analysts identify catalysts that might cause the current spread to narrow or widen by comparing them to historical spreads. Credit arbitrage hedge funds are also interested in trading the spread between the cash market and CDS market or securities within the same company under different terms.

On the other hand, convertible arbitrage hedge funds focus on inefficiencies in the pricing of convertible bonds, a hybrid security with features of both debt and equity. Essentially, it is a type of corporate bond embedded with an option to convert the bond into a predetermined number of shares in the issuing company. A fund dealing with convertible bonds must understand exactly how to price those complex securities to exploit opportunities on the market. Embedded options in convertible securities are not always correctly priced resulting in an opportunity for a trader to set up a riskless position by buying the convertible bond, shorting the issuer's equity, and hedging interest rate and default risk. Therefore, a manager can obtain cheap options on equity by isolating options embedded in convertible securities. It requires a particular set of skills combining credit, interest rates, and equities to exploit mispricing on embedded options and buy cheap credit. Overall, very efficient markets may leave only meager arbitrage opportunities leaving the fund with low unleveraged returns therefore mandating an increase in leverage.

4.4. Event-driven funds

Event-driven strategies consist of exploiting pricing inefficiencies occurring from corporate events. Funds use a combination of equity and debt instruments to profit when an expected event is realized. Most strategies stand to benefit greatly from the realization of an event and stand to lose most or all its value if it does not. They rely on the realization of idiosyncratic events and are therefore difficult to hedge but offer low correlations to equities and bonds. Event-driven funds tend to be the most opportunistic category of hedge funds, implying that they can adapt their strategies as chances arise to profit from events such as initial public offerings (IPO), earning calls, bankruptcies, mergers and acquisitions, or other corporate events. However, this opportunistic behavior is justified by a fund performance constrained by the quantity and quality of deals or simply the number of companies facing those binary events.

Therefore, an event-driven hedge fund targets opportunistic returns when deals are plentiful and cash preservation when the market is unfavorable.

Event-driven funds can be further subdivided into three categories: risk/merger arbitrage, distressed debt, and activist funds. According to Lhabitant (2006), risk arbitrage or merger arbitrage is one of the oldest event-driven strategies, originating back to the 1940s with Gustave Levy at Goldman Sachs. Over time the strategy stayed the same, simultaneously purchasing shares in one company while shorting another to capture the deal premium and exit upon completion of the transaction. A fund will take a long position in the target company while short-selling the acquiring firm. Risk arbitrage funds are customarily highly concentrated with little diversification while displaying high sensitivity to financing terms and borrowing fees. It requires a tremendous amount of deal-specific due diligence for a fund to identify potential takeover targets and understand if the merger or acquisition will finalize, determining the possibility for the fund to capture the deal premium. Mirabile (2020) states that it usually requires conditions to be met for the deal to go through, such as financing, share votes, earnings tests, or regulatory approvals. The acquiring company can choose to make a stock-for-stock offer, an all-cash deal, or a combination of both. A takeover can be friendly or hostile, the latter meaning a company making an unwanted approach through the public market. The spread, that the fund is looking to capture, is a function of the perceived risk of the deal since failure to close a deal may result in large losses for the fund due to the target's stock price falling to pre-merger levels and the acquirer's stock price either unaffected or increasing, widening the spread, and generating losses on the long and short side.

Activist funds take a different approach by purchasing enough of a stake in a public stock to be able to influence or provide feedback to a corporation to improve share price or dividends by exercising their shareholder rights. Their goal is to influence the board without taking over control, some funds adopt a friendly approach and engage only with friendly management, while others choose a more hostile approach. Activists tend to have a small set of opportunities and may sit on their cash for long periods. Their approach is very opportunistic by being net long, often with no hedges or short sales and low leverage. Similarly, to merger arbitrage funds, activists' firms have a high concentration of positions with very little diversification, usually focusing on a single sector to provide the best knowledge to the target company.

Distressed strategies are another type of event-driven strategy that invests in deeply discounted equity or debt of companies under reorganization or facing bankruptcy. Funds can invest in a wide range of securities such as bonds, debt, trade claims, common stock, or preferred stock.

Most funds are willing to negotiate debt terms by giving a company more time to restructure and avoid bankruptcy while asking for a higher coupon, more collateral, or equity shares. Therefore, a distressed firm switches from low-yield and short-maturity bonds to high-yield and long maturity to avoid bankruptcy. Funds might also invest in highly illiquid trade claims trading at a deep discount which consists in declarations of rights of payments from one counterparty to another. There is an uncertainty surrounding the amount and the timing of the payment of the claim usually determined by the bankruptcy judge, which created a secondary market for trade claims. The sellers of trade claims might not wish to wait until the completion of the reorganization or be motivated by cash. Distressed funds must consider the specific risk involved with trade claims such as the recovery risk, a risk that the amount of the payment will be different than originally assumed, and the notional value risk, the risk that a claim is deemed invalid by some legal defect that impairs the gross amount due under the claim.

Event-driven funds have displayed higher returns since the 1990s than other hedge fund strategies. It is an opportunistic strategy that benefits from rising or falling markets. Due to its countercyclical properties, it has a low correlation to stocks and bonds, increasing diversification when coupled with other strategies. However, it suffers from its illiquid investments, fund terms tend to have longer lockups or higher fees as a consequence.

4.5. Multi-Strategy funds

The last type of hedge fund strategy is more general and includes funds that try to exploit opportunities across all categories. Multi-strategy funds allocate their capital to a variety of investment strategies by definition. These funds generally exhibit lower risk than equity market risk as well as a high level of diversification across assets and strategies to help smooth returns and reduce volatility. Multi-strategy funds are the result of the success of specialist hedge funds able to provide strong performance to investors. As demand grew, successful large hedge funds were faced with either turning down investors' capital or finding new ways to increase their capacity. From this dilemma, multi-strategy funds were born from managers choosing to hone their skills with new strategies or to add experienced professionals to their staff.

Multi-strategy funds have the ability to quickly assign capital to a wide variety of strategies, each benefiting from intelligence and communication from other strategies. Where single-strategy hedge funds are limited in terms of investment opportunities they can exploit, multi-strategy funds can divert capital from less attractive strategies to those that offer superior performance. As a direct drawback, such funds generally exhibit higher fees than their respective counterparty strategy-specific hedge funds. Furthermore, an investor investing through a multi-strategy hedge fund will be diversified over different strategies but exposes himself to concentration risk with a single management team. A derivation of this problem is that strategies often become highly correlated in market downturns, reducing diversification benefits, and leaving investors significantly more exposed than they anticipated. Multi-strategy funds are also able to invest in illiquid or private investments that may not be disclosed, potentially stressing the fund when investors redeem their capital and exposing them to illiquidity risks. Multi-strategy funds require a certain size to operate, they tend to be larger than most single-strategy funds and it is common for them to exceed \$1B in assets under management.

4.6. Source of Data

Acquiring data on hedge funds has always been a difficult feat due to the private nature of these funds. Obtaining reliable information or performance characteristics on individual funds remained difficult for a long time, which on top of their exclusivity is partially responsible for their negative public image. We use the data from Hedge Fund Research (HFR), a veteran of the hedge fund industry, established in 1992 and a leading supplier of hedge fund data for over 25 years.

4.6.1. Indices

We use HFRI indices for their long track record starting in 1990 which provide a comprehensive view of the hedge fund universe as well as including closed funds, providing a more unbiased measure of hedge fund performance. When considering the practical applications of this work, however, we should mention that Hedge Fund Research created HFRX investable indices reflecting real-world investable strategies which could be used with our methodology to provide practical and investable portfolios. Sadly, HFRX indices are only available from 2008 which does not provide enough historical data to grasp various market environments. Considering the scope of this work, HFRI is more appropriate to reflect the broader and more detailed viewpoint of hedge fund strategies.

We use the following HFRI index, starting from January 1990: Event-driven (HFRIEDI), Distressed/Restructuring (HFRIDSI), Merger Arbitrage (HFRIMAI), Equity Hedge (HFRIEHI), Equity Market Neutral (HFRIEMNI), Quantitative Directional (HFRIENHI), Macro (HFRIMI), Macro: Systematic Diversified (HFRIMTI), Relative Value (HFRIRVA), Fixed Income-Convertible Arbitrage (HFRICAI). We also use indices that start at a later date such as: Activist (HFRIACT), Equity Hedge: Fundamental Value (HFRIEHFV), Equity Hedge: Long/Short Directional (HFRIELD), Macro: Discretionary Thematic (HFRIMDT), Relative Value: Fixed Income-Asset Backed (HFRIFIMB). Our model can accept time series with different start points and is therefore not limited to a single set of origin periods. More flexibility means it benefits from using more than 30 years of data to back-test the model in different economic conditions while accepting newer, more niche indices to take advantage of their hedge.

4.6.2. Benchmark

As a benchmark, we chose the Fund of Fund (HFRIFOF) Index, available directly from HFRI, following multiple considerations. First, the index aggregates multiple hedge fund strategies to provide a broadly diversified benchmark, reflecting the overall performance of the hedge fund industry and representative of the quantitively constructed portfolios presented below. Second, it aims to achieve a balanced exposure to different hedge fund strategies with lower volatility than individual strategies. Furthermore, the Fund of Fund index is widely recognized and accepted in the industry, considered reliable and comprehensive, while providing extensive historical data. By diversifying across different hedge fund styles, this index aligns well with portfolios constructed from multiple hedge fund indices, reflecting a similar multi-strategy construction.

4.6.3. Eligibility

Unless specified, HFRI indices are constructed using equal weights with annual rebalancing and follow these criteria for inclusion in an index: A hedge fund must report monthly returns, report net of all fees returns, report in USD, be open to new investment, be available in a fund structure and meet the minimum AUM eligibility requirements: having at least $50M AUM on the last reported month prior to annual rebalance or; having at least $10M AUM on the last reported month prior to annual rebalance and have been actively trading for at least twelve months.[1]

4.7. Database biases

One of the many peculiarities of hedge funds is that they are prohibited from advertising to potential investors by the S.E.C. Thus, hedge fund managers release their proprietary information to commercial databases to increase their visibility and stay on the radar of potential investors. Unfortunately, this dynamic leads to biases that can be hard to identify and correct. Lhabitant (2006) provides a comprehensive description of the five biases affecting hedge fund databases: self-selection bias, sample selection bias, survivorship bias, backfill bias, and infrequent pricing bias.

4.7.1. Self-selection bias:

The first and most obvious bias in hedge fund databases is the self-selection bias. Since hedge funds are not required to disclose their performance to the public, managers choose to voluntarily report the fund's information to data providers. Hence the sample of funds available from databases may not be representative of the entire universe of funds, leading to a skewed sample. The incentive from managers to not report their funds can be multiple. On one hand, a fund with sub-par performance does not want to compare badly against better-performing peers while a very successful fund may not need to report because it has reached its capacity. On the other hand, some funds do not wish to see their performance included in an index that they use as a benchmark, mechanically reducing their apparent hedge in the industry. Finally, a small strong performing fund has a strong incentive to report to increase their visibility. The bias may be positive or negative and its impact is not possible to identify as long as the non-reporting fund remains unobservable.

[1] HFRI criteria for inclusion in an index at: https://www.hfr.com/hfri-index-methodology

4.7.2. Sample selection bias:

Sample selection bias occurs when the dataset used for analysis is not representative of the entire population, sadly every existing database is incomplete. Most databases follow different criteria for a fund to be eligible, from a minimum lifespan to excluding particular styles. Furthermore, managers can report to multiple databases but rarely to all and the method of collection often differs. Hence, performance statistics and characteristics of the hedge fund universe often vary between databases.

4.7.3. Survivorship bias:

Another bias is the survivorship bias which results from the tendency of some funds to be excluded from databases because they no longer exist. Survivorship bias is not exclusive to hedge funds however in this particular case it may lead to a large overestimation of historical returns due to the high attrition rate of hedge funds. Therefore, when considering data from the 1990s, historical performance is conditional on the survival of the fund. Indeed, a significant number of funds stop their activity each year[2] thus survivorship bias is amplified compared to other asset classes. Lhabitant (2006) covers the different situations from which a survivorship bias can arise: a fund is liquidated, closed, merged with another, or stops reporting. Sadly, survivorship bias is tricky to correct since it requires comparing the performance of surviving funds to the performance of the entire unobservable universe of funds. The author estimates it ranges from 3% to 4% per annum and may impact higher moments of the distribution of returns. Most HFR indices are free from survivorship bias after 1994.

4.7.4. Backfill bias:

Backfill bias arises when a hedge fund joins a database and voluntarily chooses to include historical performance upon integration. Instead of reporting performance in real-time, funds may backfill which retroactively includes past performance even if they were not part of the database previously. Since it is in a fund's best interest to show strong performance, the ability to backfill is equivalent to an option for the fund manager to decide when to be included in the database with part or all of the fund's track record. Backfill bias is likely to overestimate past

[2] Brooks and Kat (2001) estimate that around 30% of newly established funds do not survive the first three years.

performance. As a result, some databases prohibit backfill, but it remains a problem in the industry as a whole. Lhabitant (2006) estimates the backfill bias to be between 1.2% to 1.4%.

4.7.5. Infrequent pricing bias:

Lastly, infrequent pricing bias in hedge fund databases refers to a bias generated from hedge funds' infrequent reporting of their net asset values. It is particularly a problem when funds hold illiquid securities in which case the manager is asked to assign a price if the security has no available market price. Moreover, this bias can also be associated with the tendency for managers to smooth their returns which often overstate risk-adjusted returns. Funds with infrequent pricing may introduce inaccuracies in their performance calculations, making it impossible to identify the true performance of these funds.

4.7. Data Analysis:

Having established the potential biases of hedge fund databases, we now turn to an analysis of the distributional characteristics and dependencies within our data. In this section, we examine the distribution properties of our data and employ the Jarque-Bera test to assess normality, the Ljung-Box Q test to investigate autocorrelation, and the Engle LM test to detect conditional heteroscedasticity.

4.7.1. Statistical properties

From Table 1, we observe that most indices suffer from a slight negative skew in their distribution and that the kurtosis value differs from a mesokurtic distribution. The deviation of the skewness and kurtosis from 0 and 3 respectively indicate at first sight that hedge fund indices diverge from a normal distribution. Additionally, we observe autocorrelation for the first lag except for the macro discretionary index, while indices such as equity market neutral, convertible arbitrage, and event-driven: distressed/ restructuring display autocorrelation up to the 4^{th} lag. We use the skewness, kurtosis, and autocorrelation values we computed to perform the following statistical test for normality, autocorrelation, and heteroscedasticity.

Table 1. Summary of descriptive measures of HFR indices

	Mean	Median	Volatility	Semi Volatility	Min. monthly	Max. monthly	Skewness	Kurtosis	ACF(1)	ACF(2)	ACF(3)	ACF(4)	ACF(5)
Event-Driven (Total)	9.67%	12.75%	6.89%	6.46%	-12.40%	7.04%	-1.44	3.77	0.3068	0.1363	0.1025	0.0500	0.0584
ED: Activist	4.35%	8.42%	14.43%	11.61%	-20.33%	11.51%	-0.94	1.31	0.1548	-0.0179	0.1531	0.0307	-0.0299
ED: Distressed/Restructuring	9.84%	11.27%	6.60%	6.07%	-11.05%	7.06%	-1.13	3.23	0.4530	0.2517	0.1586	0.1308	0.0752
ED: Merger Arbitrage	7.31%	8.93%	4.32%	4.87%	-9.58%	4.84%	-2.28	11.83	0.1400	0.1209	0.0853	0.0195	0.0923
Equity Hedge (Total)	10.79%	12.63%	9.00%	6.29%	-10.89%	10.88%	-0.31	-0.74	0.1946	0.1214	0.0782	0.0362	-0.0018
EH: Fundamental Value	5.45%	9.04%	10.48%	8.24%	-13.73%	9.99%	-0.69	-0.07	0.1372	0.0633	0.0743	0.0147	0.0236
EH: Long/Short	4.54%	7.12%	10.24%	8.12%	-11.77%	8.89%	-0.72	-0.49	0.1840	0.0999	0.0780	0.0348	0.0131
EH: Equity Market Neutral	5.67%	5.41%	3.03%	2.11%	-2.87%	3.59%	-0.17	-1.27	0.1699	0.1611	0.2167	0.1793	0.0830
EH: Quantitative	10.80%	13.95%	11.51%	7.92%	-13.34%	10.74%	-0.34	-1.69	0.1705	0.0029	-0.0403	-0.0335	-0.0260
Macro (Total)	9.24%	5.91%	6.85%	3.36%	-6.40%	7.88%	0.70	-1.43	0.1747	0.0557	0.0532	0.1451	0.1847
Macro: Systematic	8.34%	8.52%	7.52%	3.89%	-6.36%	6.49%	0.09	-3.08	0.0068	0.0376	-0.0300	0.0641	0.0751
Macro: Discretionary	1.92%	1.17%	4.70%	2.96%	-4.50%	5.07%	0.14	-1.43	0.1737	0.1079	0.0635	0.0829	-0.0609
Relative Value (Total)	8.37%	8.89%	4.42%	5.65%	-9.77%	5.72%	-2.64	15.82	0.3663	0.1765	0.1009	0.0823	0.0511
RV: Asset Backed	8.22%	9.24%	4.36%	8.10%	-13.47%	3.42%	-5.61	50.74	0.2933	0.1648	0.0978	0.1036	0.0509
RV: Convertible Arbitrage	7.61%	9.67%	6.00%	7.64%	-16.01%	9.74%	-2.94	26.57	0.5130	0.2553	0.1362	0.0887	-0.0007

4.7.2. Statistical tests

We present the results of statistical tests on our dataset in the table below.

Table 2. Summary table of statistical tests.

	JB	p value	Normality	LB-Q	p value	Auto Correlation	ARCH	p value	Heteroscedasticity
Event-Driven (Total)	14.059	0.001	reject	54.571	0.000	auto correlation	10.482	0.033	ARCH effect
ED: Activist	3.278	0.194	normality	14.750	0.141	reject	14.999	0.005	ARCH effect
ED: Distressed/Restructuring	9.710	0.008	reject	130.157	0.000	auto correlation	21.358	0.000	ARCH effect
ED: Merger Arbitrage	100.507	0.000	reject	25.177	0.005	auto correlation	9.715	0.046	ARCH effect
Equity Hedge (Total)	0.576	0.750	normality	32.032	0.000	auto correlation	29.381	0.000	ARCH effect
EH: Fundamental Value	1.183	0.554	normality	9.612	0.475	reject	21.930	0.000	ARCH effect
EH: Long/Short	1.452	0.484	normality	13.456	0.199	reject	30.767	0.000	ARCH effect
EH: Equity Market Neutral	1.078	0.583	normality	113.174	0.000	auto correlation	20.927	0.000	ARCH effect
EH: Quantitative	2.084	0.353	normality	20.049	0.029	auto correlation	27.010	0.000	ARCH effect
Macro (Total)	2.499	0.287	normality	54.798	0.000	auto correlation	25.468	0.000	ARCH effect
Macro: Systematic	5.935	0.051	normality	9.193	0.514	reject	11.476	0.022	ARCH effect
Macro: Discretionary	1.328	0.515	normality	15.802	0.105	reject	12.818	0.012	ARCH effect
Relative Value (Total)	173.897	0.000	reject	78.793	0.000	auto correlation	17.543	0.002	ARCH effect
RV: Asset Backed	1687.933	0.000	reject	54.365	0.000	auto correlation	0.101	0.999	no effect
RV: Convertible Arbitrage	462.889	0.000	reject	155.519	0.000	auto correlation	86.557	0.000	ARCH effect

4.7.2.1 Jarque-Bera test

The Jarque-Bera test assesses whether a given distribution is normally distributed. The null hypothesis H_0 is that the data is normally distributed with a skewness of 0 and a kurtosis of 3, with the alternative H_1 being the negative. Furthermore, H_0 is rejected if test statistic JB is greater than the critical value of Chi-squared with 2 degrees of freedom, meaning 5.99 at 95% significance. From Table 2, we can conclude that only 6 indices out of 15 reject H_0, therefore rejecting the hypothesis of being normally distributed. This comes as a surprise as hedge fund data is generally supposed to be non-normally distributed. The JB test statistic is calculated using the following formula:

$$JB = \frac{n}{6}\left(S^2 + \frac{(K-3)^2}{4}\right) \quad \sim \chi^2(2)$$

where n is the sample size, $S = E\left(\frac{X-\mu}{\sigma}\right)^3$ is the skewness, and $K = E\left(\frac{X-\mu}{\sigma}\right)^4$ is the kurtosis for a random variable X with mean μ and variance σ.

4.7.2.2 Ljung-Box Q test

The Ljung-Box Q test, on the other hand, checks whether the autocorrelations of a time series at different lags are significantly different from 0. The null hypothesis H_0 is defined as the autocorrelations up to the 10^{th} lag are jointly zero while H_1 is the negative. We reject H_0 if the Q statistic is greater than the critical value of a Chi-squared with 10 degrees of freedom, specifically 18.307 at 95% significance, or a p-value smaller than 0.05, suggesting evidence of autocorrelation in the data. Unsurprisingly, in Table 2, we observe that 10 indices, or 67% show significant signs of autocorrelation at 95% which is expected in hedge fund data and also an indicator of illiquidity for the indices concerned. The Q test statistic is calculated using the following formula:

$$Q = n(n+2)\sum_{k=1}^{10} \frac{\hat{\rho}_k^2}{n-k} \quad \sim \chi^2(10)$$

where n is the sample size, h is the maximum lag tested, and $\hat{\rho}_k$ is the sample autocorrelation at lag k.

4.7.2.3 Engle LM test

The Engle LM test identifies the presence of conditional heteroscedasticity in a time series using the residual errors of an ARCH(4) model. It tests the null hypothesis H_0 that the variance of residual errors is constant over time corresponding to homoskedasticity while the alternative H_1 corresponds to non-constant variance, in other words, heteroskedasticity. We reject H_0 if the LM test statistic is greater than the critical value of a Chi-squared with 4 degrees of freedom, or 9.488 at 95% significance, or a p-value smaller than 0.05. In Table 2, every index excepted Relative Value: Convertible arbitrage has significant heteroskedasticity which corresponds to volatility clustering during economic downturns.

Chapter 5. Results

In this chapter, we present and analyze the results and performance metrics of the generated portfolios. We begin with a discussion of the performance of the benchmarks used for comparison. The results are then organized into four parts: portfolios without additional constraints, portfolios with cost minimization, portfolios with correlation constraints on asset classes, and portfolios combining cost minimization and correlation constraints.

We previously mentioned our motives to use HFRI, however following a practical approach we can estimate our costs per trade based on the transaction costs of investing using HFRX indices. HFRX indices themselves are not directly investable like an Exchange Traded Fund, instead, an investor might gain exposure to HFRX strategies via an asset management firm offering managed accounts, structured products offered by investment banks, or firms offering hedge fund replication strategies. The total cost of investing in HFRX tracking products is most likely higher than investing in ETFs. However, we focus on the transaction costs per trade to estimate the impact of turnover on portfolio performance. Transaction costs can be broken down into 4 components: the brokerage commissions, the bid-ask spread, market impact costs, and slippage. The transaction costs of a portfolio are dependent on the level of liquidity of the underlying asset, the order size as well as the portfolio rebalancing frequency. If we consider an average portfolio that has a mix of liquid and moderately liquid assets, we assume the following transaction costs: the brokerage fee at 5 bps, an average bid-ask spread of 10 bps, a market impact cost of 10 bps, and a slippage of 5 bps, totaling an average transaction cost per trade of 30 bps.

5.1. Benchmarks:

As outlined in the Data section, we chose the Fund of Fund (FoF) Index as our primary benchmark. Additionally, we included the equally weighted (EW) portfolio, the minimum variance (MVP) portfolio, and the maximum Sharpe ratio (MSR) portfolio, all calculated using historical returns from our dataset. The evaluation period spans from January 1990 to November 2023, with a transaction cost assumption of 30 basis points (bps).

The performance measures evaluated include the annualized average return, the Modigliani-Modigliani risk-adjusted return (M^2), the annualized volatility, the maximum drawdown (MDD), the monthly Conditional Value at Risk (CVaR), the monthly Conditional Drawdown at Risk (CDaR), the Sharpe Ratio, the Calmar ratio, the R^2 of the regression to the FoF index, the average monthly portfolio correlation to S&P500 (Corr. Stocks), the average monthly portfolio correlation to the bond index (Corr. Bonds), the portfolio correlation to the FoF index, and the portfolio turnover.

Table 3. Performance measures of the HFRI Fund of Fund index and benchmark portfolios.

Benchmark	**FoF**	**EW**	**MVP**	**MSR**
Return	4.93%	7.51% (7.45%)	5.94% (5.64%)	9.05% (4.28%)
M^2	4.93%	7.81% (7.74%)	7.20% (6.67%)	10.09% (4.29%)
Volatility	5.64%	5.16% (5.16%)	3.23% (3.24%)	4.64% (4.67%)
MDD	22.20%	15.23% (15.32%)	9.88% (9.95%)	10.61% (24.93%)
CVaR	7.21%	6.36% (6.37%)	5.42% (5.47%)	5.66% (6.08%)
CDaR	24.23%	16.05% (16.14%)	8.88% (9.20%)	9.77% (27.40%)
Sharpe	0.123	0.633 (0.621)	0.525 (0.430)	1.037 (0.008)
Calmar	0.031	0.215 (0.209)	0.172 (0.140)	0.453 (0.002)
R^2	1.000	0.824 (0.825)	0.493 (0.488)	0.367 (0.365)
Corr. Stocks	0.648	0.555 (0.555)	0.379 (0.379)	0.422 (0.422)
Corr. Bonds	0.361	0.256 (0.256)	0.222 (0.222)	0.234 (0.234)
Corr. FoF	1.000	0.908 (0.908)	0.702 (0.699)	0.606 (0.604)
Turnover	0.000	6.083	28.717	449.425

All returns, M², volatility, MDD, CVaR, and CDaR are expressed as percentages. Both performance measures CVaR and CDaR are evaluated at a confidence level $\alpha = 99\%$. The risk-free rate is determined by averaging the annualized interest rate of the market yield on U.S. Treasury securities at a 10-year constant maturity over the evaluation period. Performance measures calculated with transaction costs are displayed in parentheses.

From the performance table of benchmark portfolios (Table 3), several observations can be made. The equally weighted portfolio shows higher returns and similar standard deviation to the FoF index, resulting in a higher Sharpe ratio. The risk is also lower than the FoF index due to a lower maximum drawdown, CVaR, and CDaR. The MVP has the lowest risk among the benchmarks, with the lowest standard deviation at 3.24%, maximum drawdown under 10%, CVaR and CDaR. On the other hand, the MSR has the highest average return before cost with relatively low risk and a Sharpe ratio of over 1. However, the MSR portfolio suffers from high turnover, significantly reducing its Sharpe and Calmar ratios when integrating transaction costs with 0.008 and 0.002 respectively. Concerning the correlation to stock and bond indexes, the FoF index has the highest correlation to both stock and bond indexes, offering fewer diversification benefits than the MVP. Moreover, the equally weighted portfolio has a high R² and correlation to the FoF index, indicating that it closely follows the FoF, with the index explaining over 80% of its return.

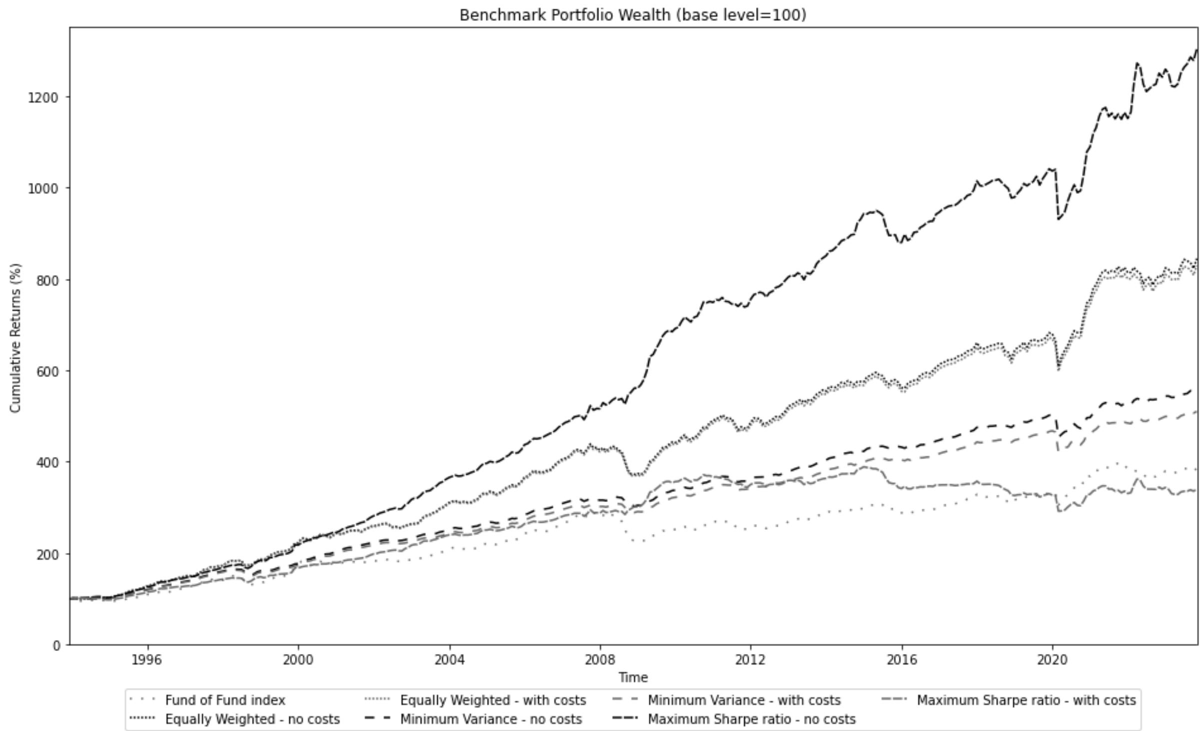

Figure 1. HFRI index and benchmark portfolios displayed with and without costs.

Figure 1 shows the performance of benchmark portfolios over 30 years, highlighting their performance through various crises. The starting level of wealth is 100, therefore cumulative returns are expressed in percentages. Total weights are equal to 1. The maximum allocation to a single index is 50%. It illustrates the cumulative returns of the benchmark portfolios over the evaluation period. The main takeaway is the impact on the MSR level when including costs and the overperformance of the equally weighted portfolio compared to the FoF index. Stacked area plots of the benchmark portfolios are available in Appendix C from Figure 5.1.1. to 5.1.3.

5.2. Part 1: Starting Portfolios

This section examines portfolios created by optimizing synthetic returns without additional constraints. These portfolios serve as a baseline for further improvements in subsequent sections. We present the results of the minimum risk CVaR (Min. CVaR), optimal CVaR (Opti. CVaR), minimum risk CDaR (Min. CDaR), optimal CDaR (Opti. CDaR), minimum risk Omega (Min. Omega), optimal Omega (Opti. Omega). Both CVaR and CDaR portfolios are built using a confidence level $\alpha = 95\%$. The threshold of the Omega ratio is the risk-free rate. Portfolio optimization is performed out of sample with a 48-month rolling window using synthetic returns.

Table 4. Performance measures of portfolios without additional constraints

P1 - Synthetic	FoF	EW	MVP	MSR	Min. CVaR	Opti. CVaR	Min. CDaR	Opti. CDaR	Min. Omega	Opti. Omega
Return	4.93%	7.51% (7.45%)	5.94% (5.64%)	9.05% (4.28%)	6.61% (6.23%)	7.29% (2.58%)	6.58% (5.96%)	7.37% (2.95%)	6.70% (5.79%)	6.72% (4.58%)
M²	4.93%	7.81% (7.74%)	7.20% (6.67%)	10.09% (4.29%)	8.08% (7.46%)	8.27% (2.04%)	8.00% (7.00%)	8.38% (2.53%)	8.14% (6.70%)	7.84% (4.74%)
Volatility	5.64%	5.16% (5.16%)	3.23% (3.24%)	4.64% (4.67%)	3.47% (3.48%)	4.27% (4.24%)	3.51% (3.51%)	4.26% (4.23%)	3.56% (3.56%)	3.88% (3.88%)
MDD	22.20%	15.23% (15.32%)	9.88% (9.95%)	10.61% (24.93%)	11.03% (11.42%)	15.26% (20.08%)	10.73% (11.39%)	14.30% (18.72%)	12.71% (13.98%)	12.14% (13.03%)
CVaR	7.21%	6.36% (6.37%)	5.42% (5.47%)	5.66% (6.08%)	4.54% (4.65%)	4.97% (5.25%)	4.13% (4.13%)	5.27% (5.51%)	4.96% (5.06%)	5.55% (5.69%)
CDaR	24.23%	16.05% (16.14%)	8.88% (9.20%)	9.77% (27.40%)	11.22% (11.73%)	16.07% (21.77%)	10.66% (11.45%)	14.94% (20.05%)	12.92% (14.49%)	11.78% (13.51%)
Sharpe	0.123	0.633 (0.621)	0.525 (0.430)	1.037 (0.008)	0.681 (0.571)	0.714 (-0.391)	0.666 (0.489)	0.735 (-0.304)	0.692 (0.435)	0.638 (0.088)
Calmar	0.031	0.215 (0.209)	0.172 (0.140)	0.453 (0.002)	0.214 (0.174)	0.200 (-0.083)	0.218 (0.151)	0.219 (-0.069)	0.194 (0.111)	0.204 (0.026)
R²	1.000	0.824 (0.825)	0.493 (0.488)	0.367 (0.365)	0.063 (0.059)	0.072 (0.065)	0.058 (0.057)	0.068 (0.064)	0.067 (0.062)	0.061 (0.060)
Corr. Stocks	0.648	0.555 (0.555)	0.379 (0.379)	0.422 (0.422)	0.407 (0.407)	0.452 (0.452)	0.406 (0.406)	0.444 (0.444)	0.428 (0.428)	0.409 (0.409)
Corr. Bonds	0.361	0.256 (0.256)	0.222 (0.222)	0.234 (0.234)	0.255 (0.255)	0.260 (0.260)	0.267 (0.267)	0.261 (0.261)	0.263 (0.263)	0.254 (0.254)
Corr. FoF	1.000	0.908 (0.908)	0.702 (0.699)	0.606 (0.604)	0.251 (0.243)	0.267 (0.256)	0.241 (0.238)	0.261 (0.254)	0.259 (0.249)	0.246 (0.246)
Turnover	0.000	6.083	28.717	449.425	35.677	449.339	58.537	420.883	86.131	202.249

In Table 4, without costs, we notice that optimal CVaR and optimal CDaR offer high annualized returns, with the best risk-adjusted return M^2. The portfolios offer respectable Sharpe and Calmar ratios with optimal CDaR portfolio boasting 0.735 and 0.219 respectively. However, high turnover results in poor risk-adjusted performance after accounting for transaction costs with simultaneously the lowest returns and highest risk of any portfolio, particularly in terms of maximum drawdown, and Sharpe ratios below 0. The optimal Omega portfolio offers a middle ground between other optimal and minimum-risk portfolios; however, it still suffers from high turnover, greatly decreasing its Sharpe and Calmar ratio.

Minimum risk portfolios provide lower standard deviation and maximum drawdown. Their lower turnover mitigates the impact of transaction costs, resulting in better risk-adjusted performance compared to optimal portfolios. However, their risk-adjusted M^2 is comparable to optimal portfolios thanks to their low volatility. Minimum risk portfolios with transaction costs display Sharpe ratios ranging from 0.435 to 0.571.

Unlike the benchmark, our portfolios, have very low R^2 with a maximum of 7% of return explained by the return of the FoF index, thanks to a construction with synthetic returns which also plays a role in the low correlation to the FoF index. Correlations to the stock index range from 0.406 to 0.452 and correlations to the bond index are around 0.26.

By comparing the performance of the portfolios presented to the benchmark, we immediately notice that the performance of optimal portfolios without costs is superior to the benchmark portfolios, except for the MSR. However, including transaction costs lowers the risk-adjusted performance of optimal portfolios, the optimal Omega portfolio being less impacted due to its lower turnover. Minimum risk portfolios show, even with costs, lower risk, and better risk-adjusted performance than FoF, MVP, and MSR benchmarks.

Figure 2 shows the performance of these portfolios with transaction costs. Benchmark portfolios are displayed in grey, and portfolio performance is displayed with transaction costs. The high turnover of optimal CDaR and CVaR portfolios significantly impacts their long-term value. Stacked area plots of these portfolios, showing the allocation of the portfolios over time into the different indices, are available in Appendix C (Figures 5.2.2 to 5.2.7).

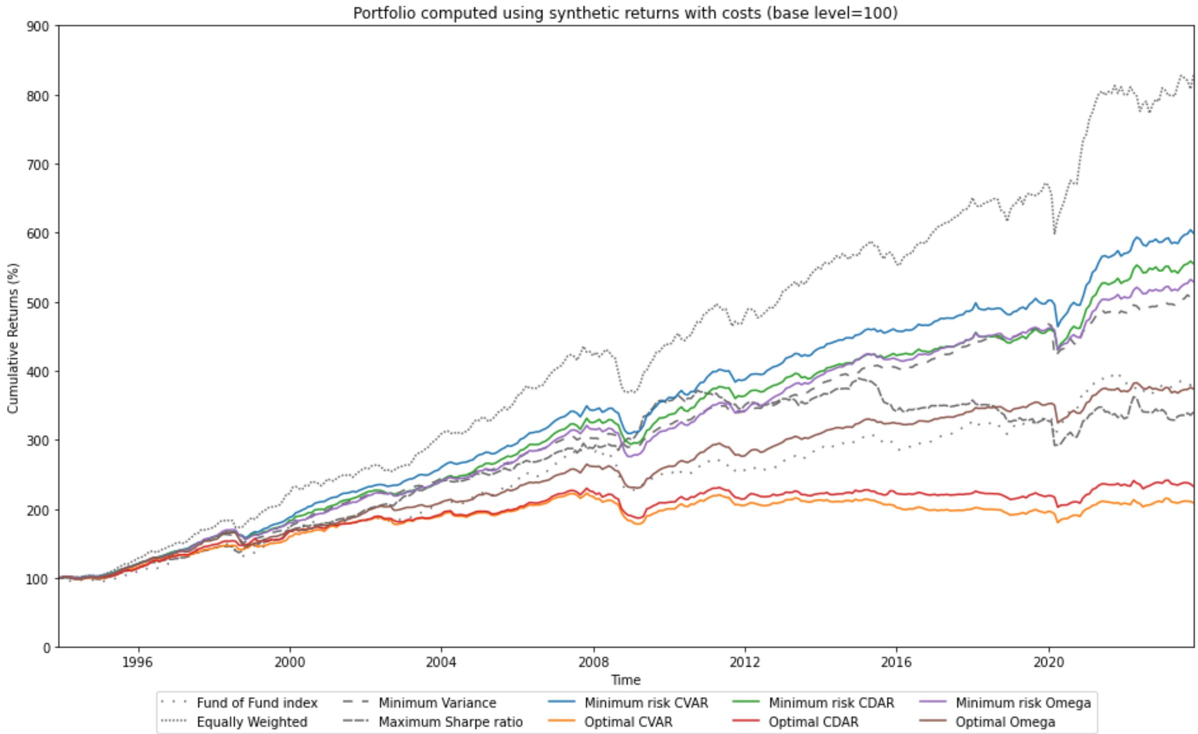

Figure 2. Portfolio without additional constraints, displayed with costs.

In Part 1, the analysis of portfolios optimized without additional constraints established a baseline for further enhancements. Optimal portfolios, despite their higher returns, suffer significantly from high turnover, leading to reduced risk-adjusted performance after accounting for transaction costs. Minimum-risk portfolios exhibit lower volatility and better stability, showcasing the importance of balancing return optimization with risk minimization.

5.3. Part 2: Portfolios with cost minimization

The proposed portfolios covered above display very promising returns at relatively low risk, however when accounting for transaction costs the risk-adjusted performance diminishes due to high portfolio turnover. In this section, we introduce turnover minimization to address the high transaction costs observed in Part 1. The turnover penalty is fixed for minimum-risk portfolios, while optimal portfolios have a dynamic turnover penalty that increases when turnover exceeds the upper turnover threshold and decreases when under the lower threshold.

Table 5. Performance measures of portfolios with cost minimization

P2 - Synthetic	FoF	EW	MVP	MSR	Min. CVaR	Opti. CVaR	Min. CDaR	Opti. CDaR	Min. Omega	Opti. Omega
Return	4.93%	7.51% (7.45%)	5.94% (5.64%)	9.05% (4.28%)	6.72% (6.42%)	7.20% (5.32%)	6.80% (6.52%)	7.28% (5.28%)	6.92% (6.25%)	7.72% (6.03%)
M²	4.93%	7.81% (7.74%)	7.20% (6.67%)	10.09% (4.29%)	8.38% (7.89%)	7.81% (5.54%)	8.17% (7.76%)	7.80% (5.45%)	8.36% (7.33%)	8.73% (6.56%)
Volatility	5.64%	5.16% (5.16%)	3.23% (3.24%)	4.64% (4.67%)	3.38% (3.37%)	4.68% (4.68%)	3.67% (3.66%)	4.81% (4.84%)	3.67% (3.66%)	4.36% (4.35%)
MDD	22.20%	15.23% (15.32%)	9.88% (9.95%)	10.61% (24.93%)	12.00% (12.38%)	14.00% (16.72%)	12.13% (12.38%)	14.58% (16.58%)	12.71% (12.99%)	11.85% (13.44%)
CVaR	7.21%	6.36% (6.37%)	5.42% (5.47%)	5.66% (6.08%)	4.13% (4.15%)	5.55% (5.78%)	4.85% (4.86%)	6.12% (6.25%)	5.00% (5.07%)	5.26% (5.31%)
CDaR	24.23%	16.05% (16.14%)	8.88% (9.20%)	9.77% (27.40%)	12.21% (12.66%)	14.58% (17.57%)	12.28% (12.60%)	15.21% (17.38%)	12.97% (13.30%)	12.20% (14.14%)
Sharpe	0.123	0.633 (0.621)	0.525 (0.430)	1.037 (0.008)	0.734 (0.647)	0.632 (0.230)	0.697 (0.624)	0.631 (0.214)	0.730 (0.548)	0.797 (0.410)
Calmar	0.031	0.215 (0.209)	0.172 (0.140)	0.453 (0.002)	0.207 (0.176)	0.212 (0.064)	0.211 (0.184)	0.208 (0.063)	0.211 (0.154)	0.293 (0.133)
R²	1.000	0.824 (0.825)	0.493 (0.488)	0.367 (0.365)	0.074 (0.072)	0.077 (0.073)	0.065 (0.064)	0.084 (0.081)	0.071 (0.070)	0.086 (0.085)
Corr. Stocks	0.648	0.555 (0.555)	0.379 (0.379)	0.422 (0.422)	0.418 (0.418)	0.512 (0.512)	0.429 (0.429)	0.508 (0.508)	0.445 (0.445)	0.483 (0.483)
Corr. Bonds	0.361	0.256 (0.256)	0.222 (0.222)	0.234 (0.234)	0.254 (0.254)	0.258 (0.258)	0.256 (0.256)	0.263 (0.263)	0.263 (0.263)	0.257 (0.257)
Corr. FoF	1.000	0.908 (0.908)	0.702 (0.699)	0.606 (0.604)	0.272 (0.268)	0.278 (0.270)	0.255 (0.254)	0.289 (0.285)	0.267 (0.264)	0.292 (0.292)
Turnover	0.000	6.083	28.717	449.425	28.070	178.004	25.899	188.374	63.424	158.437

The results in Table 5 show that the optimal omega portfolio without costs displays the best risk-adjusted performance with the highest return, M², a Sharpe ratio of 0.797, and a Calmar ratio of 0.293. Moreover, optimal portfolios have slightly higher returns and higher standard deviation than minimum-risk portfolios, but the lower volatility of the latter confers them higher risk-adjusted returns. However, when taking costs into account, the overperformance of optimal portfolios is erased by their difference in portfolio turnover compared to minimum-risk ones. Indeed, both optimal CVaR and CDaR see the biggest drop in risk-adjusted performance with lower return and higher risk. After costs, the minimum risk CVaR offers the best Sharpe Ratio and lowest risk.

To further this analysis, it is valuable to look at the evolution of the portfolios between Part 1 and Part 2 to quantify the effects of adding turnover minimization on performance. If we first compare both tables without costs, we notice slightly higher returns for minimum-risk portfolios at the costs of higher standard deviation. On the other hand, optimal portfolios CDaR and CVaR show slightly lower returns as well as higher standard deviation and lower Sharpe ratios. The optimal omega portfolio displays higher risk-adjusted performance than the original with similar characteristics except for a slightly higher standard deviation from 3.88% to 4.36%.

By comparing both tables with transaction costs, we immediately observe the effect of the turnover reduction, especially for optimal portfolios with previously very high turnover. For instance, the turnover of the optimal CVaR has been reduced to 178.004 from 449.339 originally, and the optimal CDaR from 420.883 to 188.374. Other portfolios also benefit from the minimization of turnover albeit to a smaller extent, such as the optimal Omega with 158.437 turnover from 202.249 or the minimum risk CDaR with 25.899 from 58.537 previously. This reduction in portfolio turnover has a considerable effect on performance, manifested by an after-cost increase return, risk-adjusted M^2, Sharpe and Calmar ratios. The Sharpe ratios of both optimal CVaR and CDaR changed the most dramatically from below 0 to 0.230 and 0.214 respectively. The portfolio with the best risk-adjusted performance is still the minimum risk CVaR which also benefits from an increase in Sharpe ratio from 0.571 previously to 0.647 currently. Furthermore, the Calmar ratio of every portfolio increased after turnover minimization. In conclusion, including turnover minimization in the optimization reduced the costs of implementing the portfolios, therefore increasing average returns and risk-adjusted portfolio metrics.

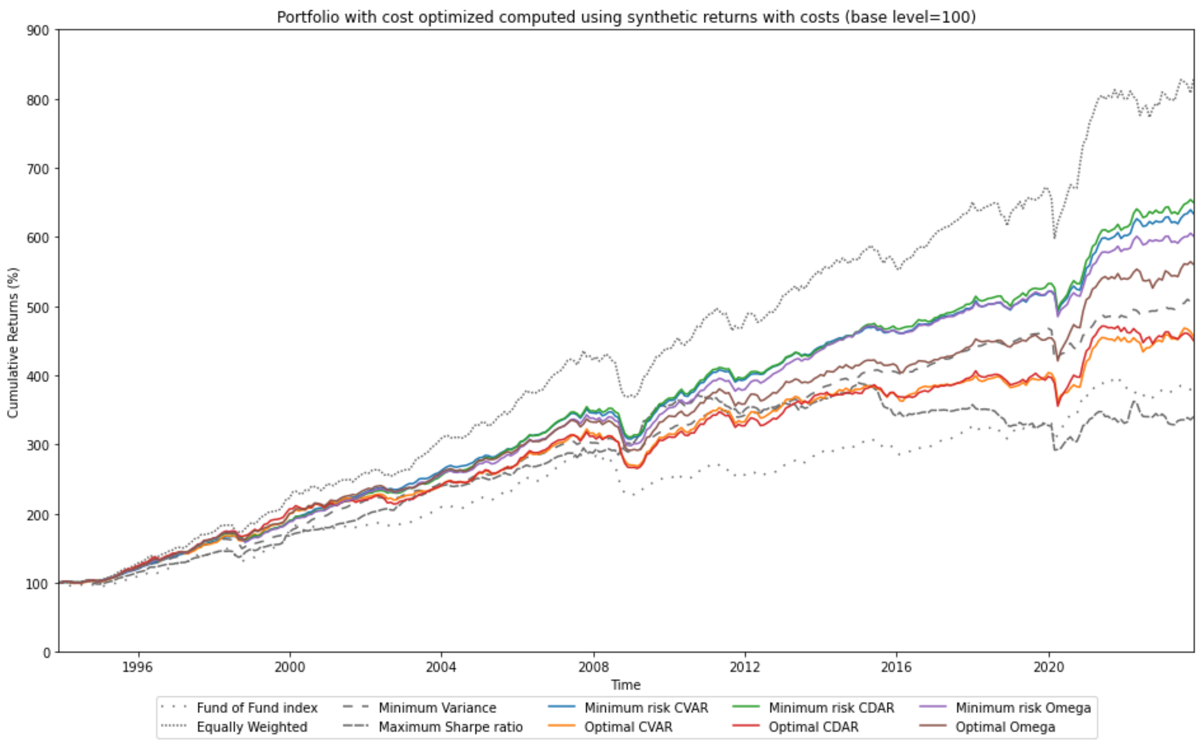

Figure 3. Portfolio with cost minimization, displayed with costs.

Figure 3 illustrates the improvement in the end value of portfolios due to turnover minimization, especially with optimal portfolios. Stacked area plots and turnover comparison plots in Appendix C (Figures 5.3.2 to 5.3.13) further demonstrate the benefits of turnover minimization. For instance, the stacked area plot and turnover comparison plots of the optimal synthetic CVaR show a reduction in portfolio turnover which coincides with a less unstable weight allocation.

In Part 2, introducing turnover minimization significantly improved the performance of portfolios by reducing transaction costs. Optimal portfolios see a dramatic reduction in turnover, translating to higher returns after costs, and improved Sharpe and Calmar ratios. This enhancement in net performance highlights the practical necessity of incorporating cost considerations in portfolio optimization.

5.4. Part 3: Portfolios with correlation constraints

This section adds maximum correlation constraints to the portfolios to further reduce risk while offering greater diversification to stocks and bonds. Portfolios are constrained to a maximum correlation of 0.35 to the S&P500 and 0.25 to a bond index: ICE BofA US Corporate Index.

Table 6. Performance measures of portfolios with correlation constraints

P3 - Synthetic	**FoF**	**EW**	**MVP**	**MSR**	**Min. CVaR**	**Opti. CVaR**	**Min. CDaR**	**Opti. CDaR**	**Min. Omega**	**Opti. Omega**
Return	4.93%	7.51% (7.45%)	5.94% (5.64%)	9.05% (4.28%)	6.90% (6.00%)	7.23% (3.78%)	6.95% (5.98%)	7.27% (3.93%)	7.09% (6.02%)	6.77% (4.99%)
M^2	4.93%	7.81% (7.74%)	7.20% (6.67%)	10.09% (4.29%)	8.64% (7.14%)	8.62% (3.56%)	8.59% (7.01%)	8.68% (3.79%)	8.98% (7.18%)	8.30% (5.44%)
Volatility	5.64%	5.16% (5.16%)	3.23% (3.24%)	4.64% (4.67%)	3.41% (3.43%)	3.85% (3.88%)	3.52% (3.53%)	3.84% (3.88%)	3.38% (3.40%)	3.52% (3.54%)
MDD	22.20%	15.23% (15.32%)	9.88% (9.95%)	10.61% (24.93%)	5.86% (6.69%)	5.68% (8.85%)	5.26% (5.80%)	5.64% (7.89%)	5.38% (5.95%)	7.26% (8.10%)
CVaR	7.21%	6.36% (6.37%)	5.42% (5.47%)	5.66% (6.08%)	2.55% (2.64%)	3.02% (3.32%)	2.76% (2.89%)	3.12% (3.38%)	2.73% (2.84%)	3.48% (3.59%)
CDaR	24.23%	16.05% (16.14%)	8.88% (9.20%)	9.77% (27.40%)	5.71% (6.69%)	5.70% (8.71%)	4.98% (5.59%)	5.39% (7.66%)	4.62% (5.34%)	6.26% (7.29%)
Sharpe	0.123	0.633 (0.621)	0.525 (0.430)	1.037 (0.008)	0.779 (0.513)	0.776 (-0.120)	0.770 (0.491)	0.788 (-0.081)	0.840 (0.521)	0.720 (0.212)
Calmar	0.031	0.215 (0.209)	0.172 (0.140)	0.453 (0.002)	0.454 (0.263)	0.526 (-0.053)	0.516 (0.299)	0.536 (-0.040)	0.529 (0.298)	0.348 (0.093)
R^2	1.000	0.824 (0.825)	0.493 (0.488)	0.367 (0.365)	0.035 (0.037)	0.027 (0.025)	0.029 (0.030)	0.024 (0.023)	0.027 (0.027)	0.027 (0.028)
Corr. Stocks	0.648	0.555 (0.555)	0.379 (0.379)	0.422 (0.422)	0.309 (0.309)	0.325 (0.325)	0.312 (0.312)	0.324 (0.324)	0.319 (0.319)	0.313 (0.313)
Corr. Bonds	0.361	0.256 (0.256)	0.222 (0.222)	0.234 (0.234)	0.199 (0.199)	0.197 (0.197)	0.202 (0.202)	0.197 (0.197)	0.198 (0.198)	0.199 (0.199)
Corr. FoF	1.000	0.908 (0.908)	0.702 (0.699)	0.606 (0.604)	0.186 (0.192)	0.164 (0.158)	0.170 (0.172)	0.154 (0.150)	0.163 (0.164)	0.164 (0.169)
Turnover	0.000	6.083	28.717	449.425	84.486	327.957	92.015	316.647	100.607	168.489

If we directly compare Table 6 to Table 4 from Part 1, correlation constraints reduced the portfolios' correlations to stocks, bonds, and the FoF index. Indeed, the average correlation of portfolios to the stock index is around 0.31 from 0.42 before and 0.19 for the bond correlation to 0.26 previously. The correlation of the portfolios to the FoF index declined significantly as well from an average of 0.25 to 0.16 under the constraints. Without costs, portfolios exhibit significantly lower maximum drawdown, CVaR, and CDaR values, indicating reduced risk. For instance, the optimal CVaR portfolio previously had a maximum drawdown of 15.26% which is reduced to 5.68% with correlation constraints. This considerable reduction in maximum drawdown benefits particularly the Calmar ratio, on average more than doubling it, the most striking improvement is 0.536 from 0.219 originally, in the case of the Optimal CDaR. Moreover, we notice higher Sharpe ratios, noticeably from the minimum risk Omega portfolio boasting 0.84, an increase from the previous 0.692. Subsequently, the performance of portfolios with costs carries much less risk as well, the risk-adjusted returns are higher than in Part 1, despite high portfolio turnover. Correlation constraints on portfolios offer a risk reduction as well as an improvement in diversification benefits of portfolios while not significantly impacting portfolio performance.

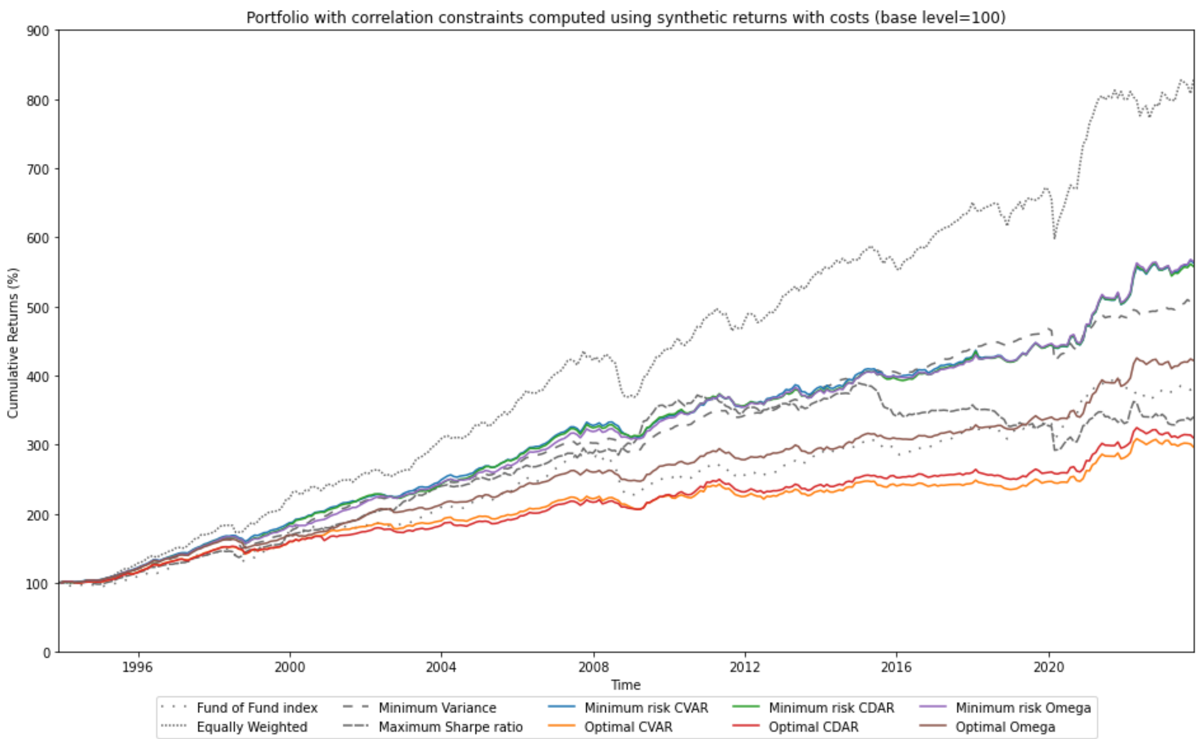

Figure 4. Portfolio with correlation constraints, displayed with costs.

Figure 4 shows the performance of these portfolios, which have higher end-period values due to lower risk and improved diversification.

Stacked area plots and correlation constraint plots in Appendix C (Figures 5.4.1 to 5.4.13) illustrate the impact of these constraints. Correlation plots show the correlation of a portfolio to the bond and stock index at a specific time and compare the correlation of the constrained portfolios to the unconstrained ones. Regarding the minimum risk CVaR (Figure 5.4.3.), the stock correlation constraint is binding from 1998 to 2003 and again from 2009 to 2020. Moreover, it also shows that in 1999 and 2021 to 2022 the correlation constraint of the portfolios to the stock index cannot be respected and the effective portfolio correlation is higher than its constrained level.

In Part 3, introducing correlation constraints to portfolios effectively reduced risk and enhanced diversification benefits by limiting exposure to stock and bond indices. The constraints lead to lower maximum drawdowns, CVaR, and CDaR values, indicating better resilience to extreme market events. These superior risk-adjusted returns achieved while maintaining a lower correlation to traditional assets highlight the efficacy of correlation constraints.

5.5. Part 4: Final portfolios

Finally, the last set of portfolios is a combination of the previous cost minimization and correlation constraints. The goal of the following portfolios is to supply an easily implementable solution that provides an investment vehicle offering the benefits of both approaches. The benefits of turnover minimization can be summarized as lower transaction costs such as brokerage fees and market impact costs, greater stability of investments as in disclosing fewer changes in holding, and higher tax efficiency by exhibiting fewer taxable events. The benefits of constraining the maximum correlation of the portfolio to other asset classes can be summarized as limiting portfolio exposure to market movements to be more resilient to economic downturns and enhancing diversification benefits to an investor. Finally, these portfolios can be tailored to meet investors' risk tolerance and investment objectives, including regulation requirements on limitations of turnover or correlation to a benchmark.

Portfolios in this section combine the construction of Part 2 and Part 3, therefore, portfolios are subject to a maximum correlation of 0.35 to the S&P500 and a maximum correlation of 0.25 to the ICE BofA US Corporate Index and costs are minimized with the added term in the objective function.

Table 7. Performance measures of portfolios with costs minimization and correlation constraints

P4 - Synthetic	FoF	EW	MVP	MSR	Min. CVaR	Opti. CVaR	Min. CDaR	Opti. CDaR	Min. Omega	Opti. Omega
Return	4.93%	7.51% (7.45%)	5.94% (5.64%)	9.05% (4.28%)	6.97% (6.24%)	6.95% (5.40%)	7.04% (6.31%)	6.93% (5.43%)	6.89% (5.99%)	7.10% (5.55%)
M²	4.93%	7.81% (7.74%)	7.20% (6.67%)	10.09% (4.29%)	8.76% (7.53%)	8.05% (5.86%)	8.66% (7.50%)	8.02% (5.90%)	8.57% (7.08%)	8.28% (6.08%)
Volatility	5.64%	5.16% (5.16%)	3.23% (3.24%)	4.64% (4.67%)	3.41% (3.43%)	4.00% (4.02%)	3.57% (3.59%)	4.00% (4.02%)	3.45% (3.47%)	3.99% (4.01%)
MDD	22.20%	15.23% (15.32%)	9.88% (9.95%)	10.61% (24.93%)	5.62% (6.19%)	5.83% (6.69%)	5.23% (5.72%)	5.75% (6.55%)	5.30% (5.90%)	7.71% (9.16%)
CVaR	7.21%	6.36% (6.37%)	5.42% (5.47%)	5.66% (6.08%)	2.55% (2.64%)	3.30% (3.45%)	2.89% (2.98%)	3.27% (3.41%)	2.73% (2.84%)	3.61% (3.77%)
CDaR	24.23%	16.05% (16.14%)	8.88% (9.20%)	9.77% (27.40%)	5.33% (6.00%)	5.70% (6.72%)	4.62% (5.09%)	5.66% (6.61%)	4.94% (5.65%)	6.34% (8.10%)
Sharpe	0.123	0.633 (0.621)	0.525 (0.430)	1.037 (0.008)	0.801 (0.583)	0.676 (0.288)	0.784 (0.577)	0.670 (0.294)	0.768 (0.503)	0.716 (0.327)
Calmar	0.031	0.215 (0.209)	0.172 (0.140)	0.453 (0.002)	0.486 (0.323)	0.464 (0.173)	0.535 (0.362)	0.466 (0.181)	0.501 (0.296)	0.371 (0.143)
R²	1.000	0.824 (0.825)	0.493 (0.488)	0.367 (0.365)	0.035 (0.038)	0.031 (0.032)	0.032 (0.033)	0.031 (0.033)	0.030 (0.032)	0.035 (0.038)
Corr. Stocks	0.648	0.555 (0.555)	0.379 (0.379)	0.422 (0.422)	0.314 (0.314)	0.343 (0.343)	0.322 (0.322)	0.342 (0.342)	0.328 (0.328)	0.367 (0.367)
Corr. Bonds	0.361	0.256 (0.256)	0.222 (0.222)	0.234 (0.234)	0.199 (0.199)	0.201 (0.201)	0.202 (0.202)	0.201 (0.201)	0.197 (0.197)	0.202 (0.202)
Corr. FoF	1.000	0.908 (0.908)	0.702 (0.699)	0.606 (0.604)	0.188 (0.194)	0.177 (0.178)	0.178 (0.182)	0.177 (0.182)	0.174 (0.178)	0.188 (0.196)
Turnover	0.000	6.083	28.717	449.425	69.220	146.217	68.717	141.610	85.257	145.917

To further understand the effects of the turnover minimization and correlation constraints it is valuable to compare the portfolios to their counterparts from Part 1. First, we notice that the correlations to stocks and bonds of the new portfolios are greatly reduced. The optimal CVaR portfolio previously had a correlation of 0.452 to the S&P500 which decreased to 0.343. Similarly, the minimum risk CDaR saw its correlation to the bond index drop from 0.260 to 0.202. Additionally, the correlation of the portfolios to the FoF index is around 0.18 which is a reduction compared to the original 0.25 from Part 1. Similarly, the R^2 of the regression to the FoF index halved from the first set of portfolios to around 0.035, differentiating them even more from the benchmark.

As observed in Part 3, correlation constraints allow for further reduction of risk particularly observable through alternative risk measures such as maximum drawdown, CVaR, and CDaR which purport much lower levels of risk than the benchmarks. For instance, the maximum drawdown of the MSR is at 24.93% or 15.32% for the EW, compared to around 6% for the portfolios, except for the optimal Omega which is slightly riskier sitting at 9.16%. Furthermore, we observe a strong reduction in portfolio turnover on optimal portfolios from 449 to 146 for the optimal CVaR, 421 to 141 for the optimal CDaR, and 202 to 146 for optimal Omega. However, on minimum-risk portfolios, portfolio turnover is slightly higher than in Part 1 due to the correlation constraints. This increase in turnover is minimal, for instance, minimum risk CVaR jumps from 36 originally to 69, which overall has very little effect on performance. By cumulating the effect of the correlation constraints and the turnover minimization, the portfolios benefit from lower correlation to other asset classes, lower risk, less investment instability, and lower transaction costs which reduces their negative effect on net portfolio performance.

The portfolios proposed here vary significantly from those first introduced in Part 1. They display very low maximum drawdowns, CvaR, and CDaR, with a volatility of a maximum of 4%, combined with risk-adjusted returns of 7.53% in the case of the minimum risk CVaR. As a result, we get very good Sharpe and Calmar ratios, the Calmar ratio being more representative of risk-adjusted performance based on other risk metrics than volatility. For instance, in the case of the minimum risk CDaR, the 99% CVaR is 2.98% implying an expected loss of 2.98% for 1% of monthly returns while providing a Sharpe ratio of 0.577 with a risk-free rate of 4.24% as per the average annualized 10-year T-bill rate.

The reduction of turnover solved the significant underperformance problems after costs, particularly in the case of optimal portfolios, which mechanically increased Sharpe ratios. Simultaneously, the correlation constraints significantly reduce the correlation of the portfolios to other asset classes while greatly reducing maximum drawdown, CVaR, and CDaR values. Portfolios from Part 4 offer the best risk-adjusted performance of all previous iterations and benchmarks while displaying relatively low correlations to other asset classes for greater diversification from market downturns as well as providing a solution for investors with a correlation budget.

For completeness, identical portfolios were also computed using historical returns (Appendix C, Figure 5.5.20). Therefore, we can directly observe the benefits of synthetic returns over historical ones to create these portfolios. First, the average return of portfolios is lower with slightly higher volatility. Other risk measures such as maximum drawdown, CVaR, and CDaR are also worse with historical returns portfolios. As a result, Sharpe and Calmar ratios are less attractive. Finally, portfolios constructed with historical returns have much higher R^2 around 0.42 and 0.546 for optimal Omega, implying that the returns of these portfolios are partly explained by the returns of the FoF benchmark, a stark difference from the 0.035 R^2 of synthetic

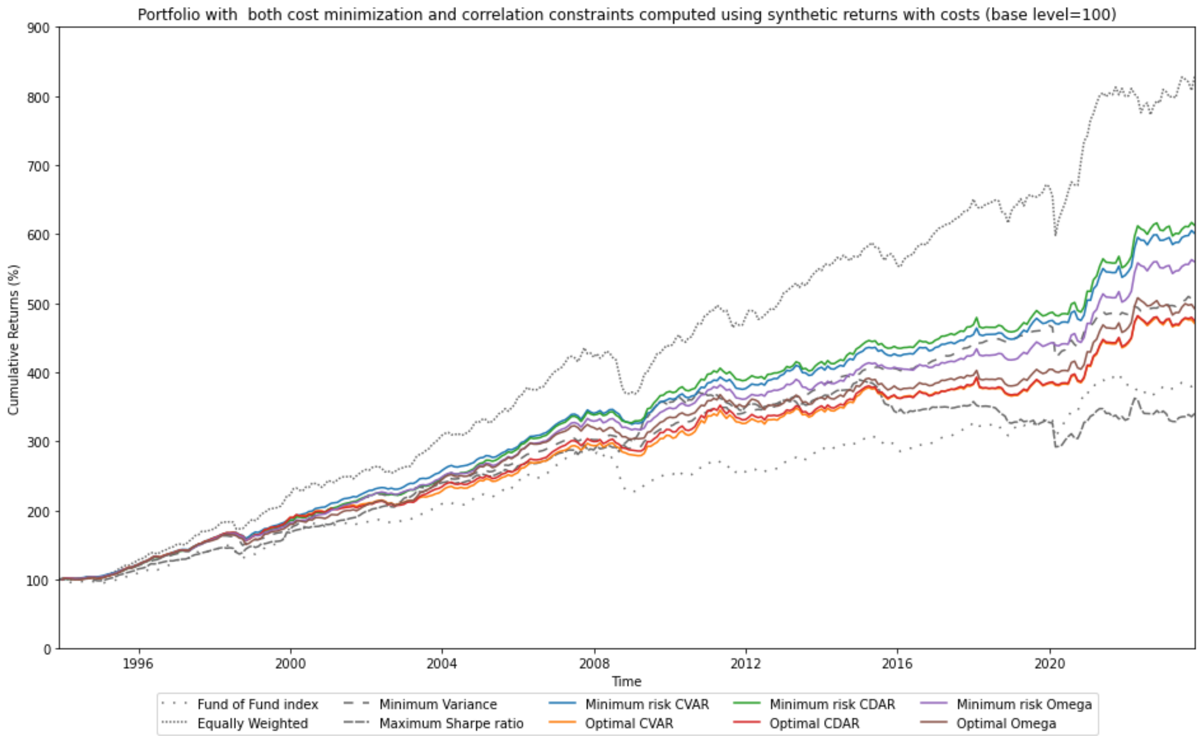

Figure 5. Portfolio with cost minimization and correlation constraint, displayed with costs.

return portfolios. Unsurprisingly, the correlation to the benchmark index is also much higher. Portfolios constructed with historical returns not only deliver lower risk-adjusted performance but also exhibit much higher R^2 values and correlation to the benchmark. Therefore, portfolios constructed using synthetic returns versus historical returns reveal the advantages of synthetic returns in achieving lower risk, better diversification, and greater independence.

In Figure 5, the portfolios do not boast the highest end-of-period value, however as mentioned above they offer the best risk-adjusted performance. When compared to Figure 2 from Part 1, the performance is now much more consistent across the different portfolios. The correlation constraints as well as the turnover minimization reduced risk while increasing returns.

Figures in Appendix C (Figures 5.5.1 to 5.5.19) provide detailed graphical representations, showing the reduction of portfolio turnover and the stability of allocations due to the combined constraints. Indeed, the reduction of portfolio turnover consequently allocates longer-lasting investments in a particular index, unlike the unconstrained portfolio which displayed almost complete monthly reallocation to other indices. Graphically, we also notice that the maximum weight of 50% constraint, set for minimum diversification, is a lot less binding than in Part 1. Concerning correlation graphs, we still observe the effect of the constraints reducing portfolio correlation to the bond and stock index to the maximum level set, occasionally exceeding this threshold in cases where the optimization simply cannot produce a portfolio under those constraints. Moreover, turnover plots compare the portfolio turnover to those in Part 1. We observe a strong reduction in portfolio turnover in optimal portfolios with an emphasis on CVaR and CDaR which previously displayed extreme turnover. However, the turnover on minimum-risk portfolios is higher than its previous counterparts due to correlation constraints, simultaneously increasing turnover from the original very low turnover.

In Part 4, the results highlight the importance of incorporating cost minimization and correlation constraints in portfolio optimization to achieve superior risk-adjusted performance and better diversification. These portfolios achieve the lowest maximum drawdowns and the highest risk-adjusted returns, with significantly reduced correlations to other asset classes. The combined approach proves to be the most effective, providing a practical and resilient investment strategy that balances performance with risk management.

Chapter 6. Conclusion

In this research, we developed a semi-parametric approach to hedge fund portfolio optimization, addressing the limitations of traditional parametric models. By integrating parametric models with non-parametric estimation techniques, we were able to capture the unique distributional characteristics of hedge fund returns, including skewness and leptokurtosis. This hybrid approach allowed us to incorporate higher moments and the full distribution behavior of hedge fund returns.

We generated synthetic returns using a combination of an AR(1)-EGARCH(1,1) volatility model, extreme value theory, and a Student's t copula. These synthetic returns, when used in portfolio construction, effectively captured the dynamic nature of hedge fund returns, enhancing out-of-sample performance. Furthermore, by integrating alternative risk measures such as Conditional Value at Risk (CVaR) and Conditional Drawdown at Risk (CDaR), our model provided a more comprehensive risk assessment framework, significantly enhancing the risk-adjusted performance of hedge fund portfolios.

To ensure practical applicability, we addressed constraints such as portfolio turnover and correlation with traditional asset classes. Minimizing turnover in the optimization process reduced transaction costs, making the proposed model more cost-effective and feasible in real-world applications.

Our methodology offers a new perspective on handling non-normal return distributions, bridging the gap between parametric and non-parametric methods in portfolio optimization. This provides hedge fund managers with a more effective semi-parametric model for better risk management and higher risk-adjusted performance. Additionally, our optimization process incorporates risk measures like CVaR, CDaR, and Omega, as well as practical considerations such as portfolio turnover minimization and correlation constraints with other asset classes like stocks and bonds. This enhances the hedge fund's ability to manage extreme risks through a practically implementable model aligned with real-world investment strategies.

While our findings are promising, we acknowledge several limitations. The accuracy of the model depends on the quality and availability of data, which, in the context of hedge funds, can be hard to obtain and often expensive. The computation of synthetic returns and portfolio optimization can be computationally intensive, especially as the number of constituents increases, leading to exponentially longer computation times.

In conclusion, this research presents a robust, flexible, and practical semi-parametric approach to hedge fund portfolio optimization. By addressing the unique challenges posed by hedge fund return distributions, we offer significant improvements in portfolio performance and risk management. The incorporation of alternative risk measures and the use of synthetic returns for model evaluation demonstrate the model's potential for improving risk-adjusted performance. Despite its limitations, this research contributes significantly to both academic literature and practical applications in hedge fund management. Future research can build on these findings by exploring applications of this model to a dataset of individual hedge funds, potentially providing a quantitative investing model for multi-strategy hedge funds or funds of funds.

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Appendix

Appendix A - Methodology

The proprietary algorithm created for this research runs on Python (Version 3.9.12). The following Python libraries were used:

- pandas (Version 1.4.1)
- numpy (Version 1.21.5)
- matplotlib (Version 3.5.1)
- scipy (Version 1.7.3)
- statsmodels (Version 0.13.2)
- arch (Version 6.1.0)
- seaborn (Version 0.11.2)
- tqdm (Version 4.64.1)
- scikit-learn (Version 1.2.2)

Appendix B - Data

A description of every HFRI index used in this paper can be found directly on HFR's website available at: https://www.hfr.com/family-indices/hfri/#index-descriptions

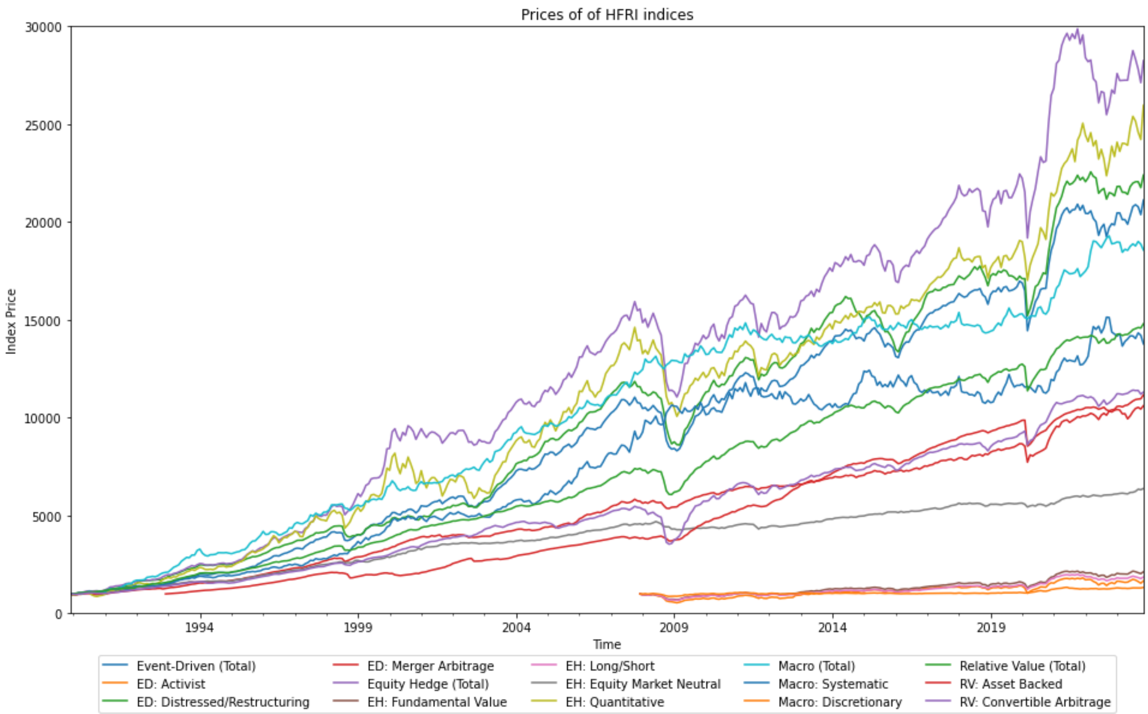

Figure 4.1.1: Evolution of price levels of HFRI indices. The starting level is 1000.

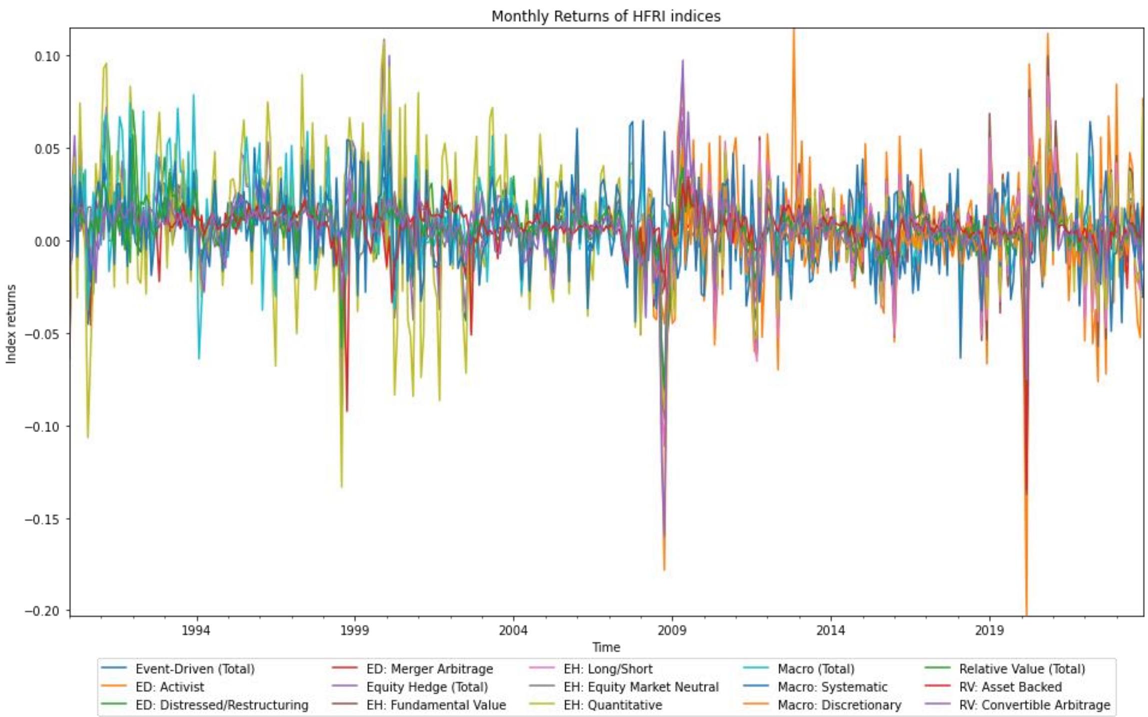

Figure 4.1.2: Evolution of monthly returns of HFRI indices.

Appendix C - Results

5.1. Benchmarks

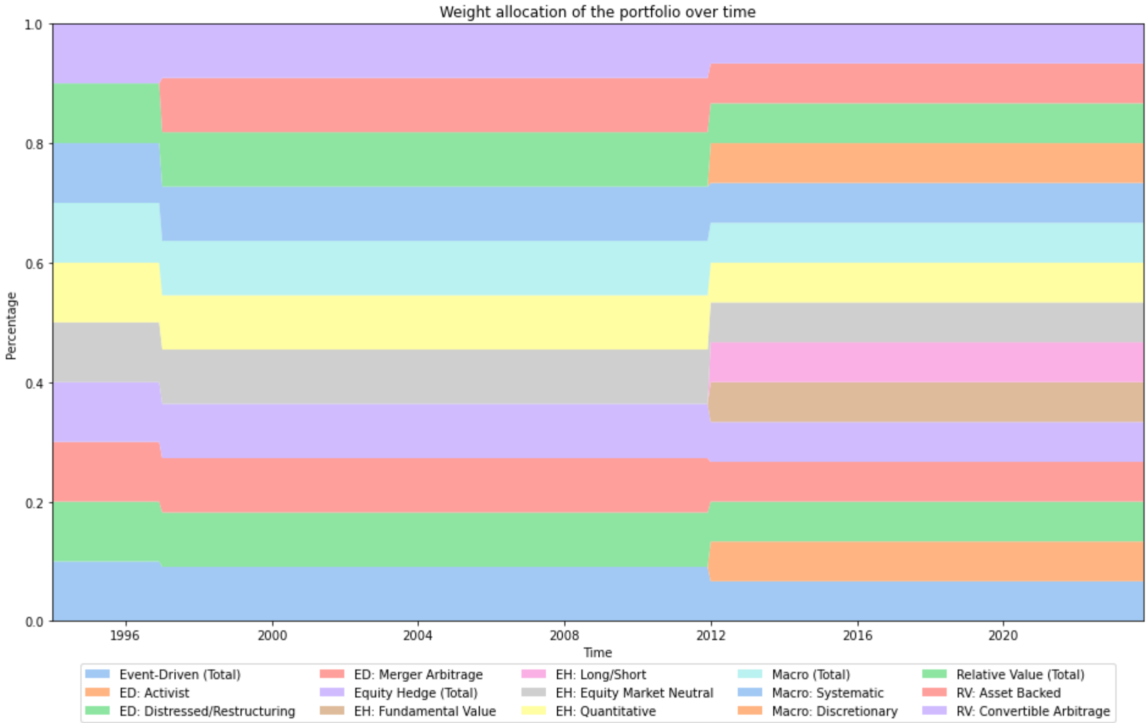

Figure 5.1.1. Stacked area plot of the **equally weighted** benchmark (EW) portfolio.

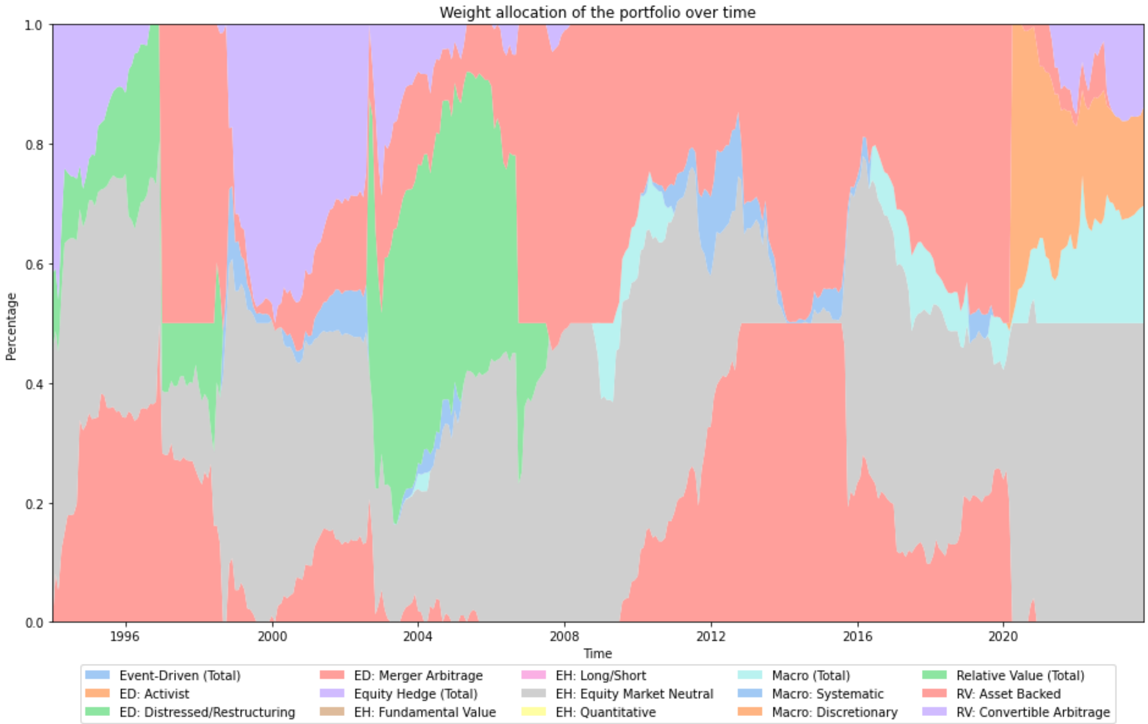

Figure 5.1.2. Stacked area plot of the **Minimum Variance Portfolio** benchmark (MVP) portfolio.

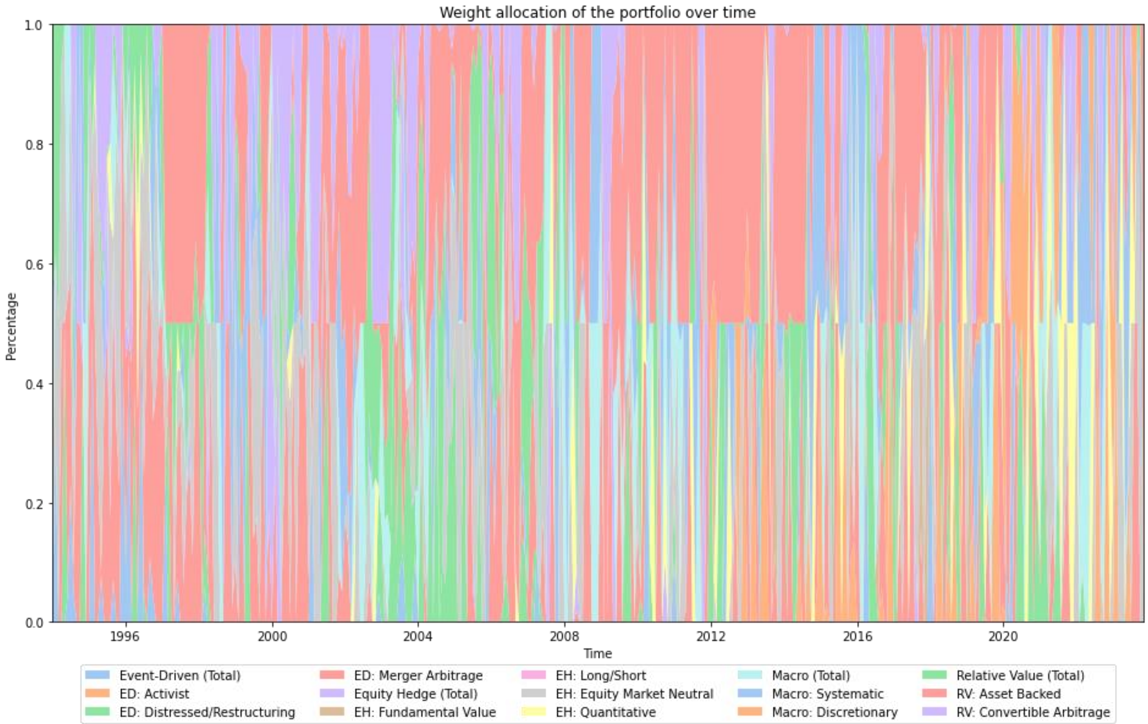

Figure 5.1.3. Stacked area plot of the **Maximum Sharpe Ratio** benchmark (MSR) portfolio.

5.2. Part 1: Starting Portfolios

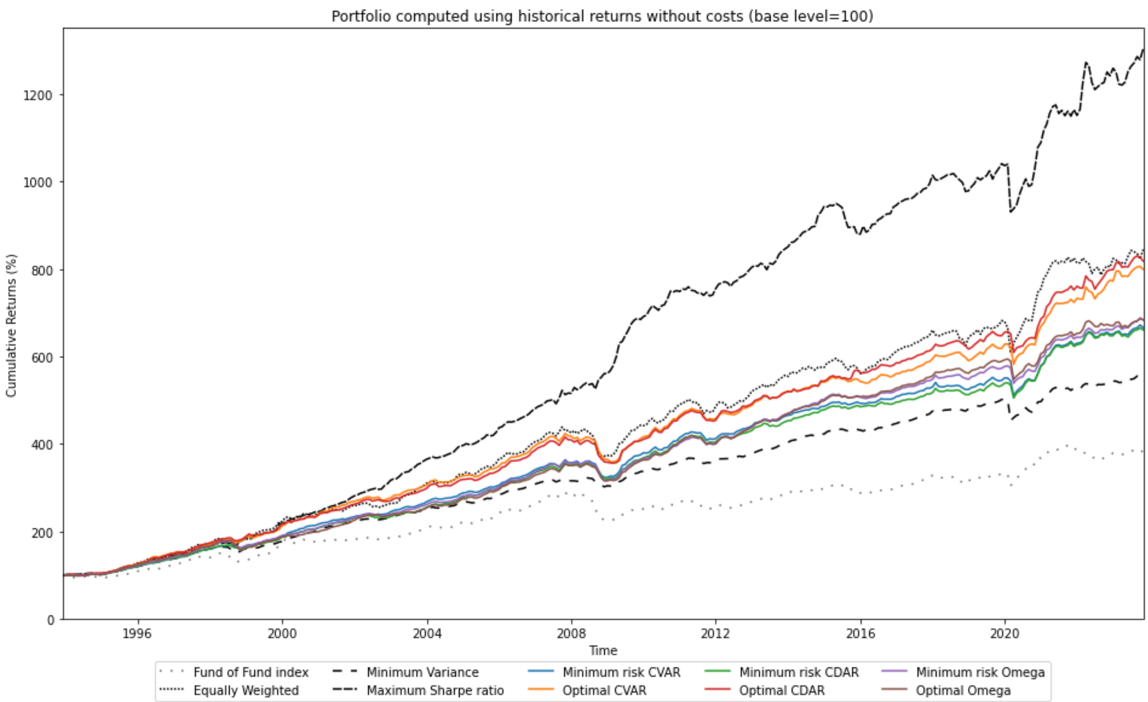

Figure 5.2.1. Starting portfolios displayed without costs.

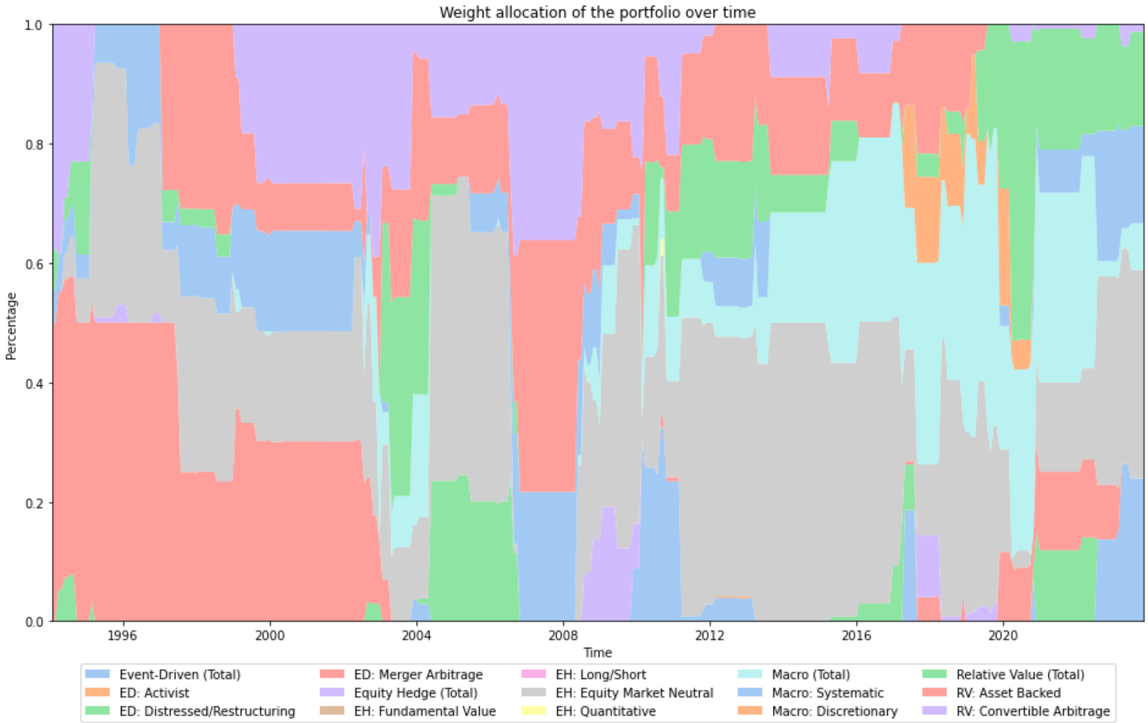

Figure 5.2.2. Stacked area plot of the **Minimum risk synthetic CVaR** portfolio without additional constraints.

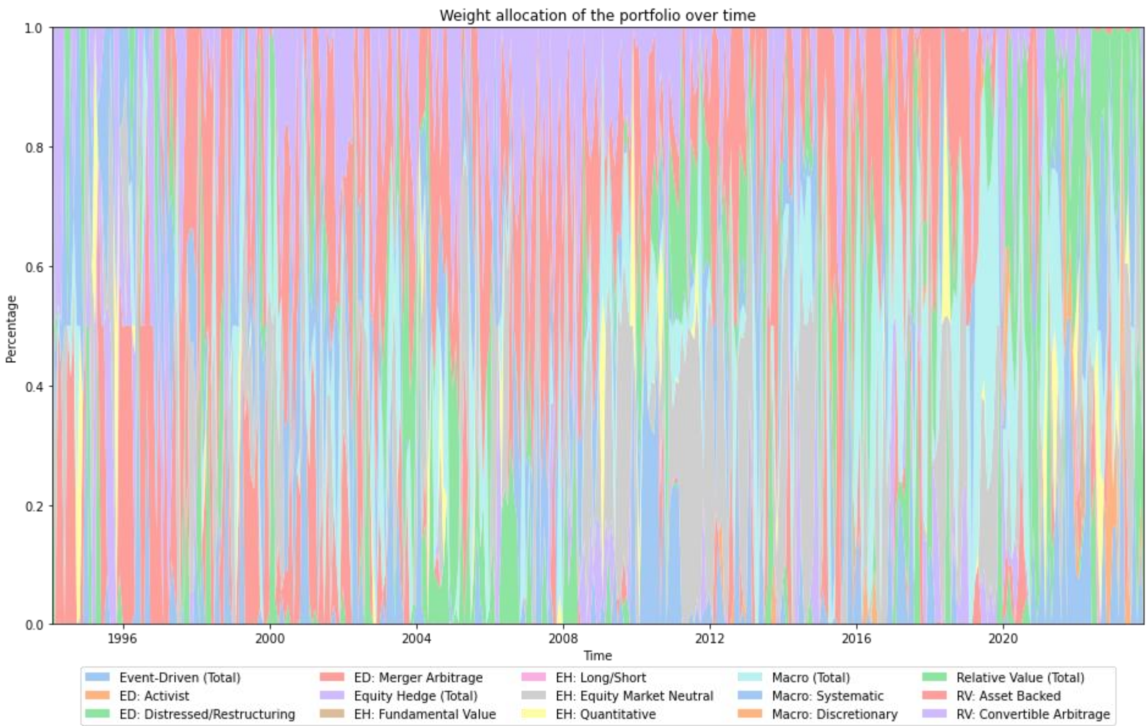

Figure 5.2.3. Stacked area plot of the **Optimal synthetic CVaR** portfolio without additional constraints.

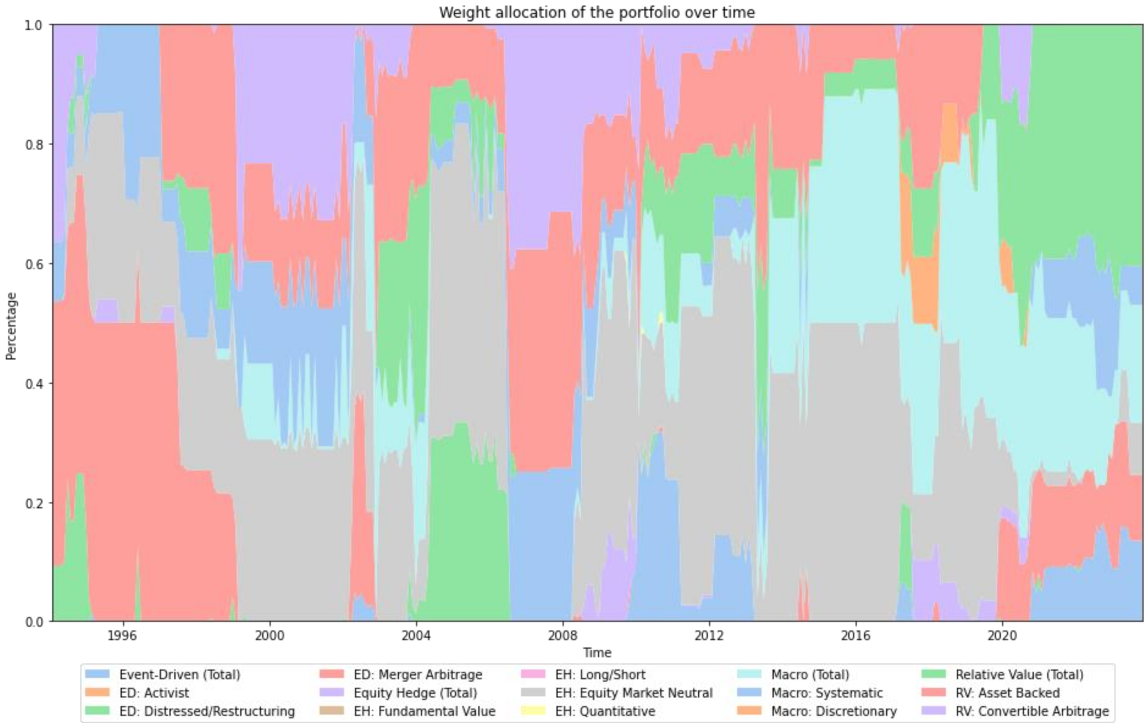

Figure 5.2.4. Stacked area plot of the **Minimum risk synthetic CDaR** portfolio without additional constraints.

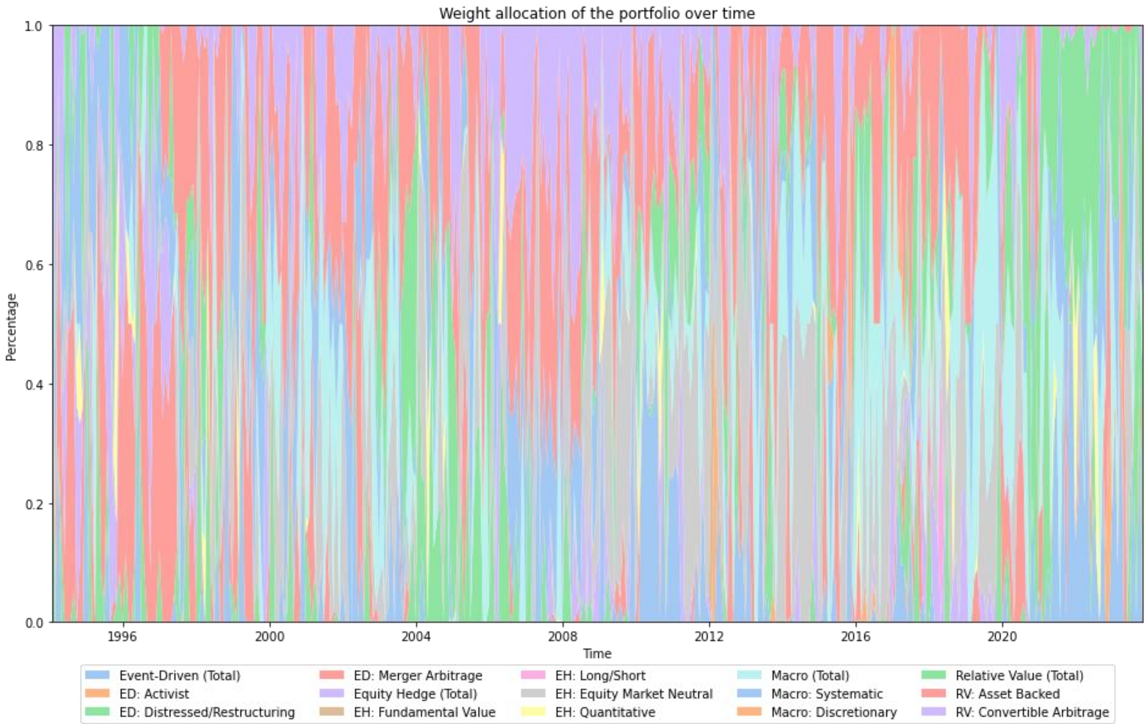

Figure 5.2.5. Stacked area plot of the **Optimal synthetic CDaR** portfolio without additional constraints.

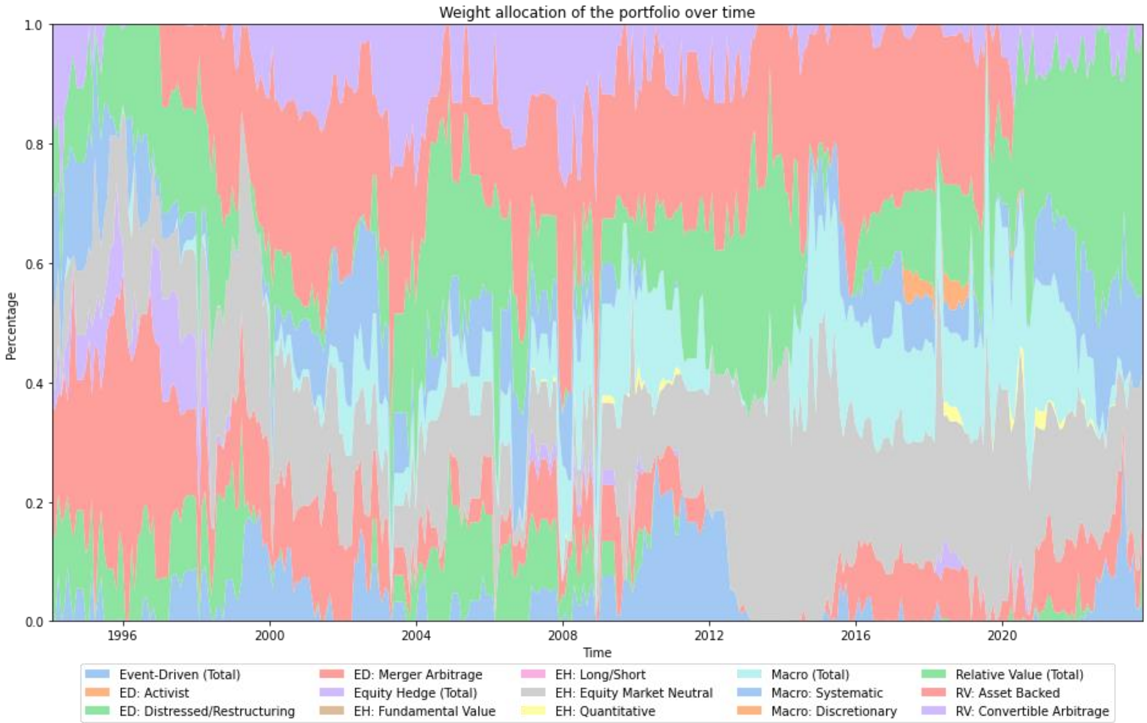

Figure 5.2.6. Stacked area plot of the **Minimum risk synthetic Omega** portfolio without additional constraints.

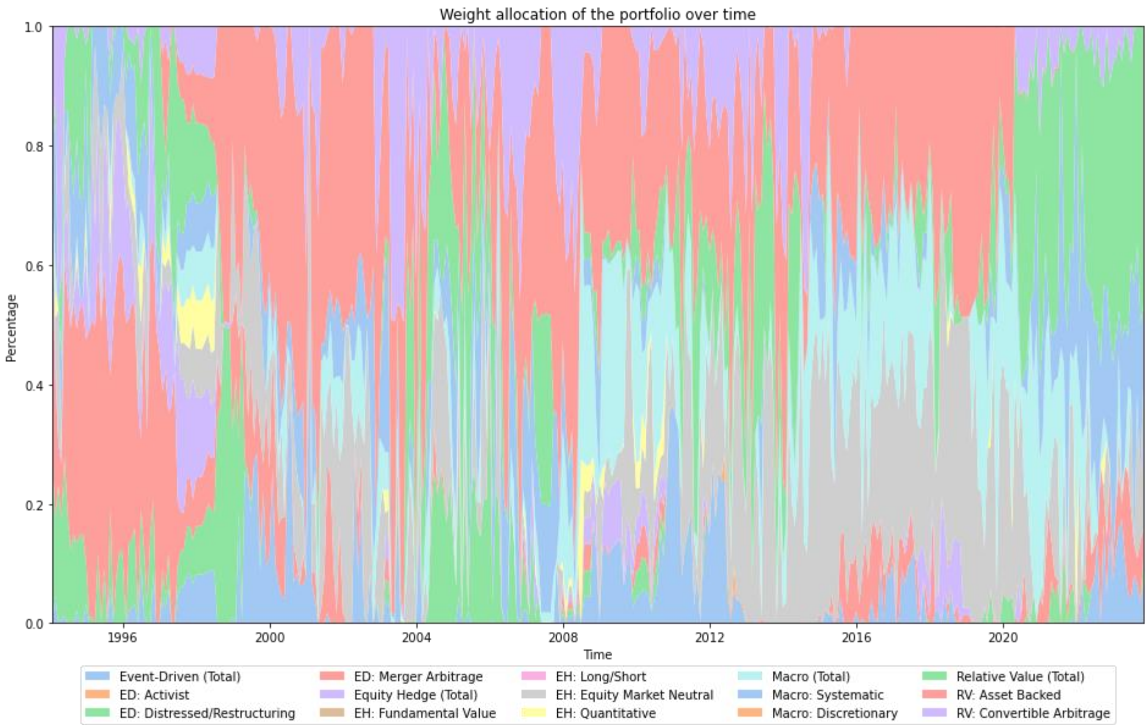

Figure 5.2.7. Stacked area plot of the **Optimal synthetic Omega** portfolio without additional constraints.

5.3. Part 2: Portfolios with cost minimization

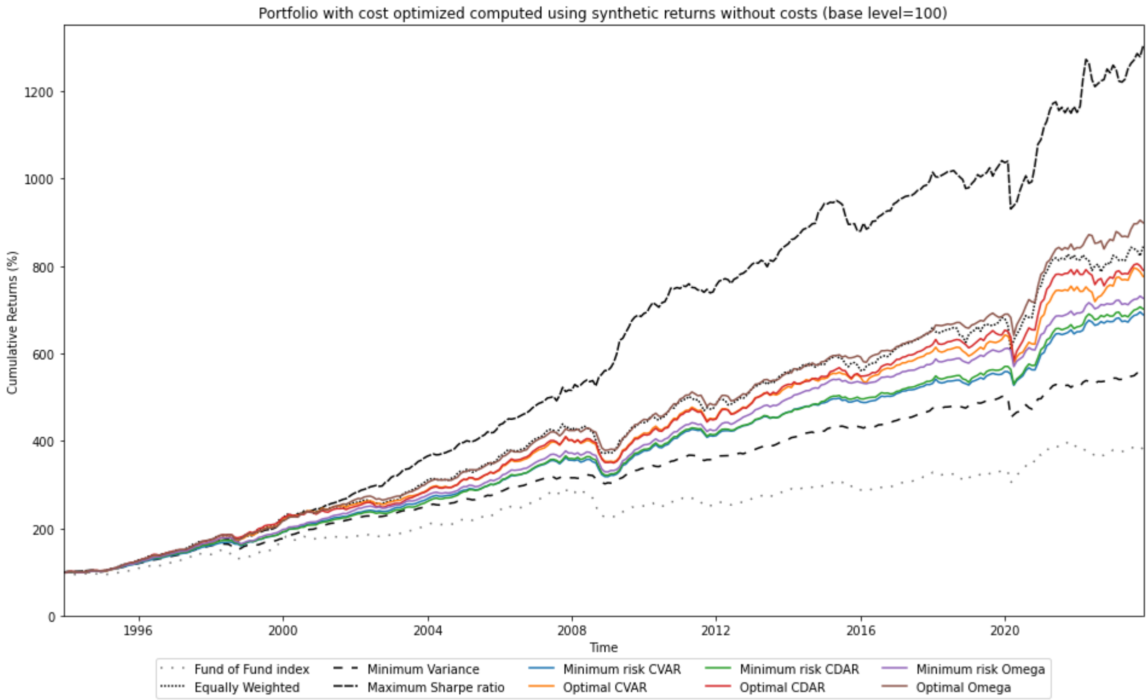

Figure 5.3.1. Portfolios with cost minimization, displayed without costs.

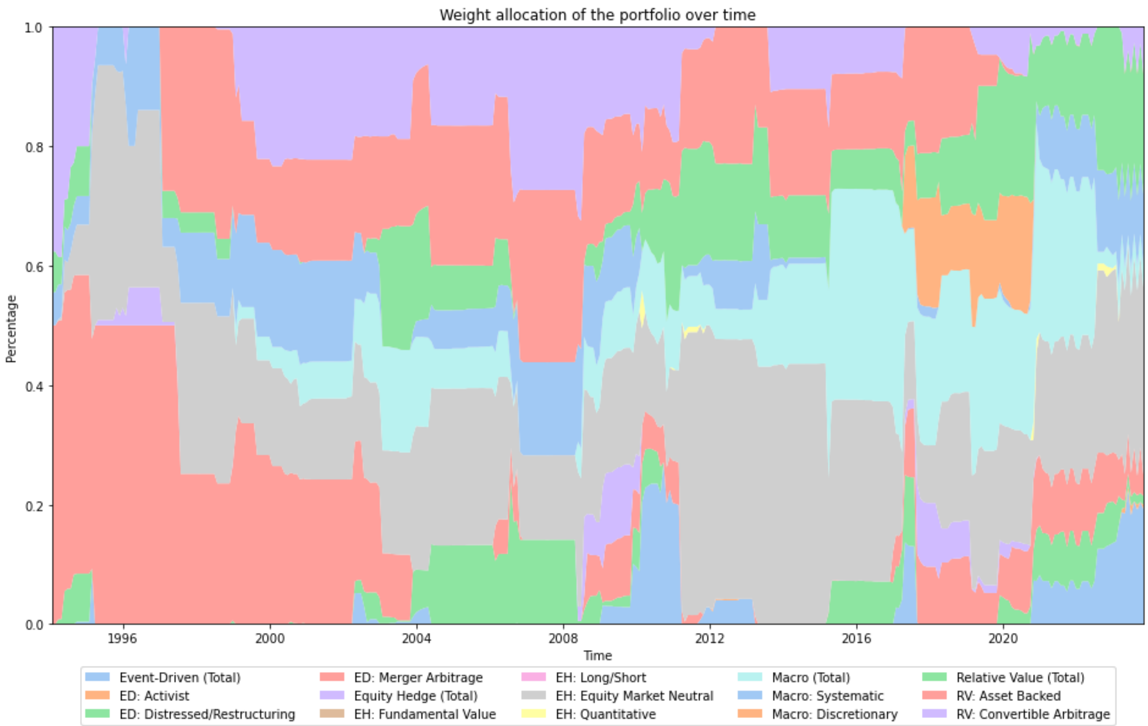

Figure 5.3.2. Stacked area plot of the **Minimum risk synthetic CVaR** portfolio with cost minimization.

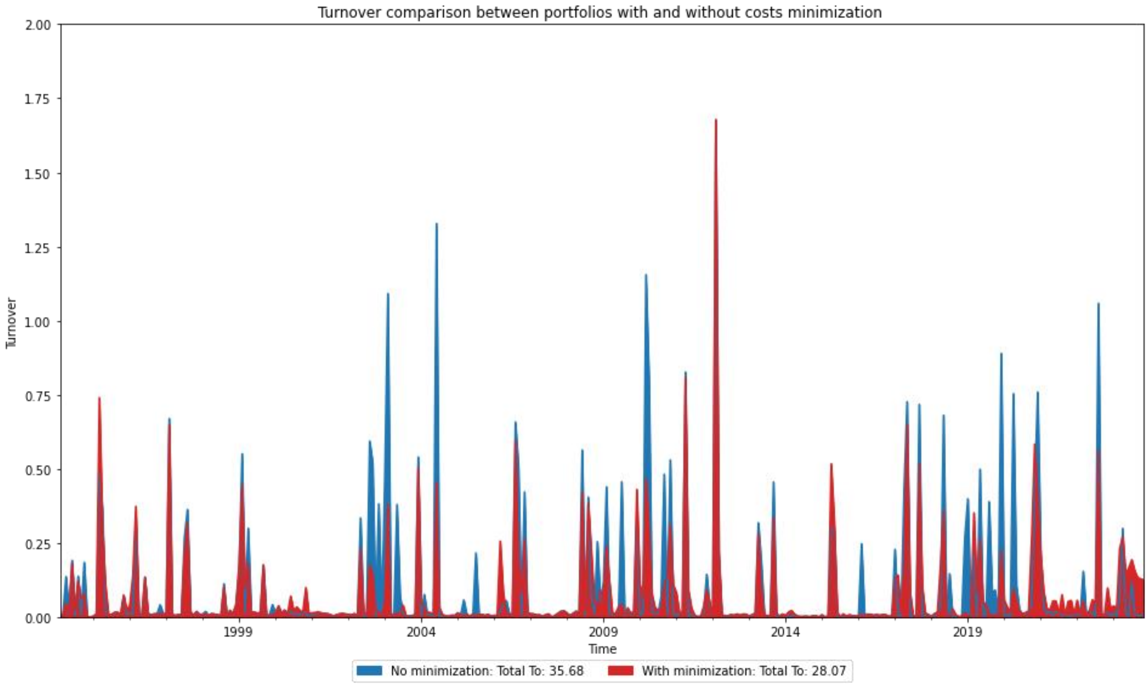

Figure 5.3.3. Portfolio turnover comparison of the **Minimum risk synthetic CVaR** portfolio with cost minimization.

Figure 5.3.4. Stacked area plot of the **Optimal synthetic CVaR** portfolio with cost minimization.

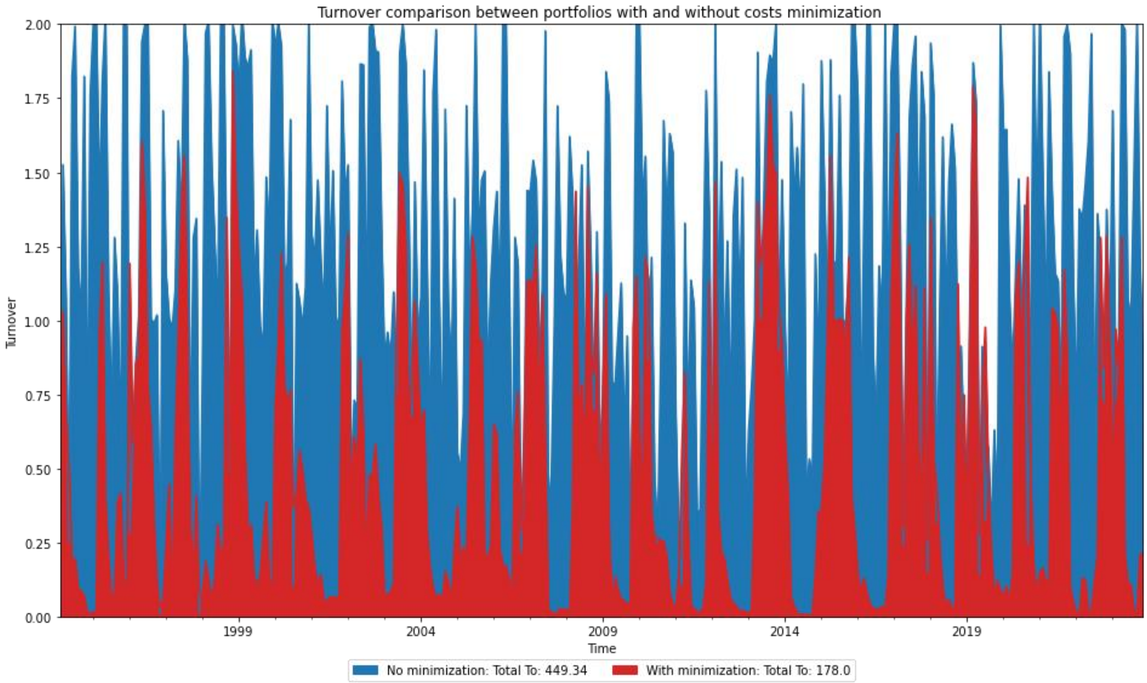

Figure 5.3.5. Portfolio turnover comparison of the **Optimal synthetic CVaR** portfolio with cost minimization.

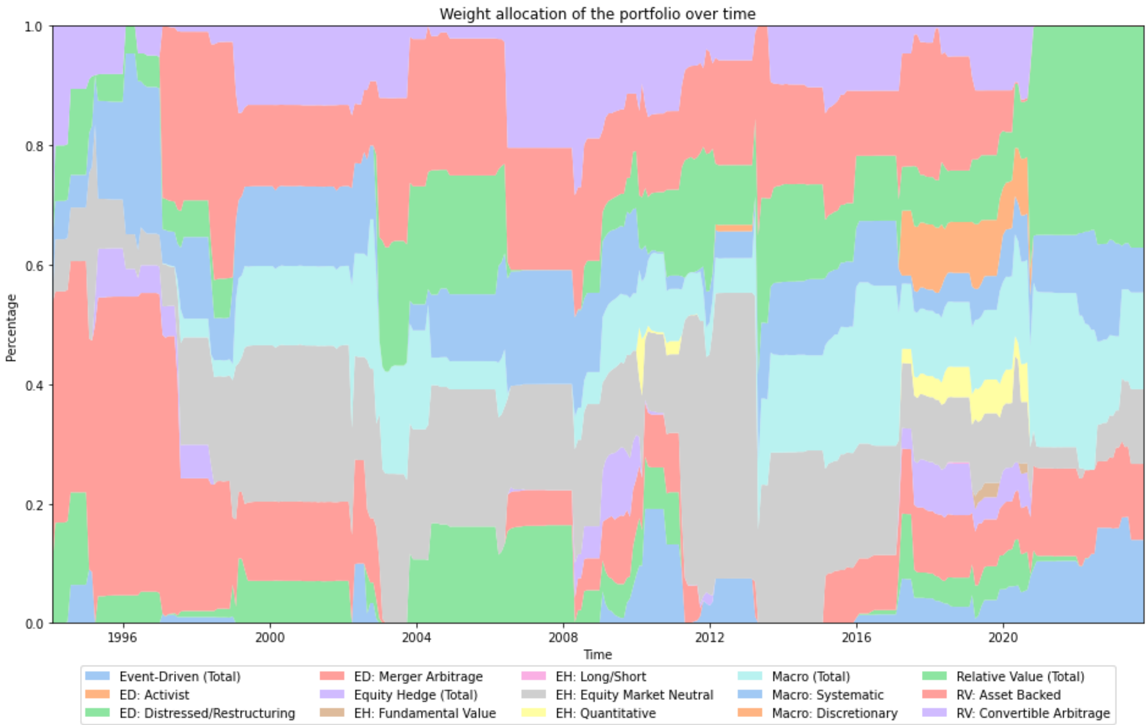

Figure 5.3.6. Stacked area plot of the **Minimum risk synthetic CDaR** portfolio with cost minimization.

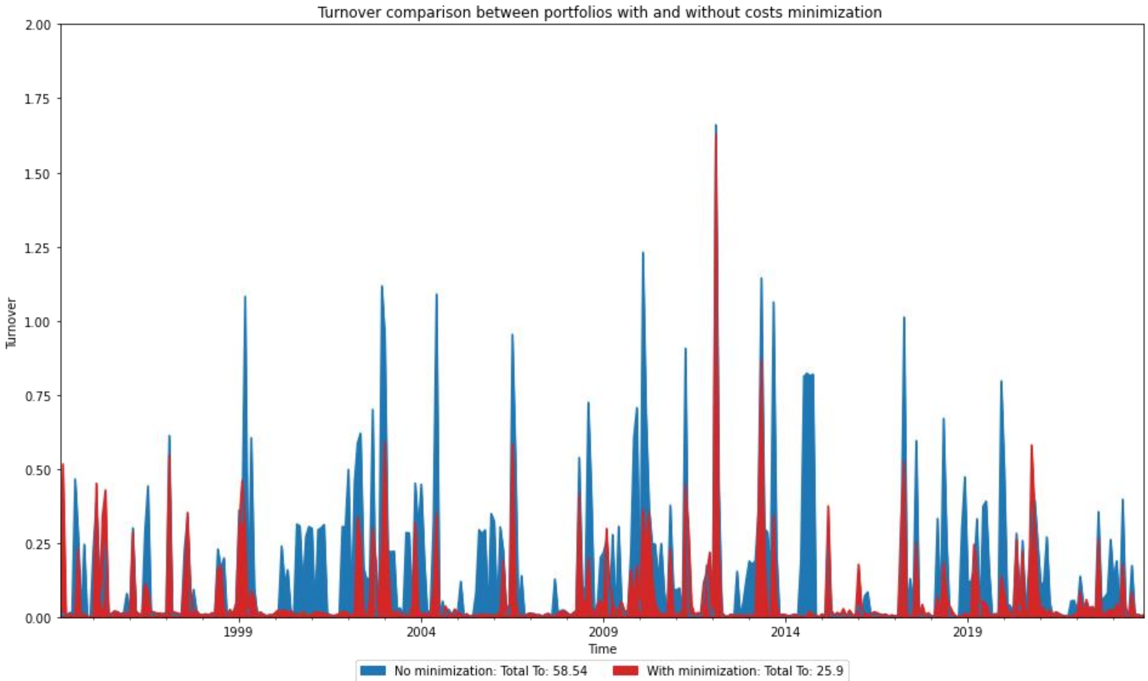

Figure 5.3.7. Portfolio turnover comparison of the **Minimum risk synthetic CDaR** portfolio with cost minimization.

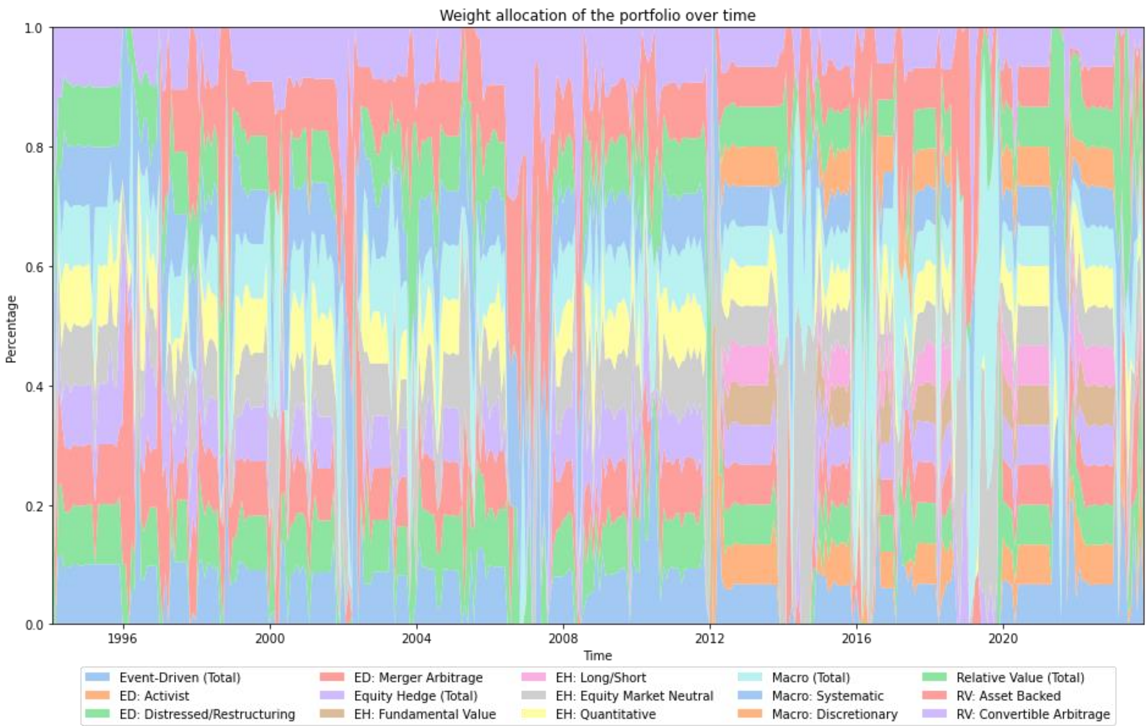

Figure 5.3.8. Stacked area plot of the **Optimal synthetic CDaR** portfolio with cost minimization.

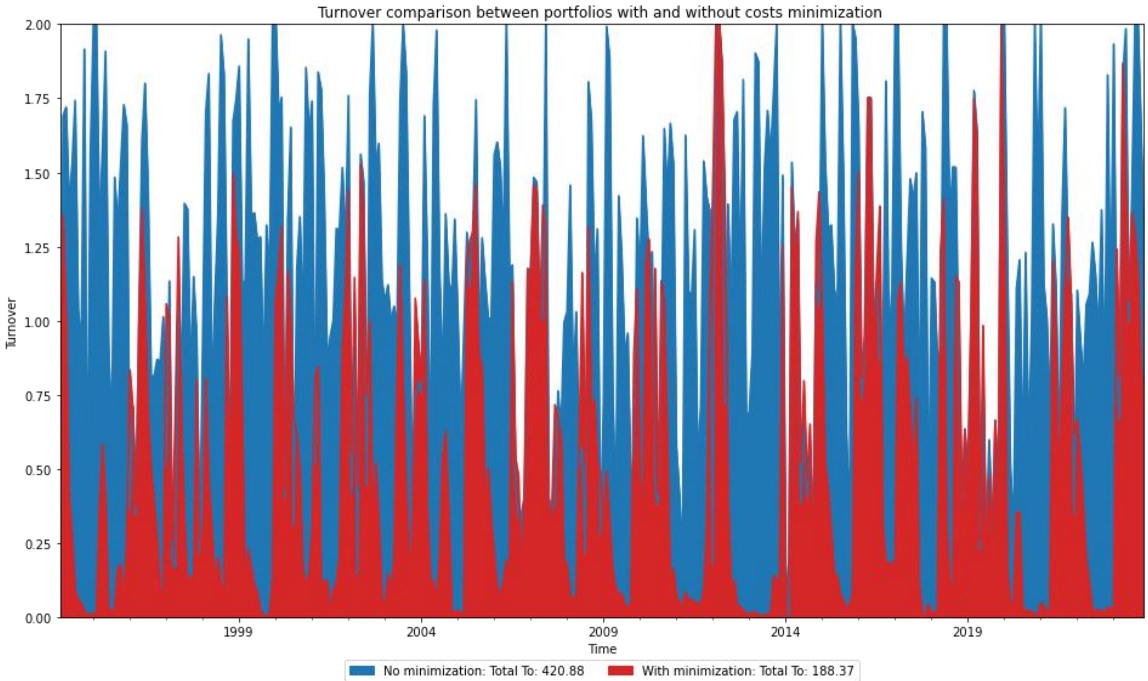

Figure 5.3.9. Portfolio turnover comparison of the **Optimal synthetic CDaR** portfolio with cost minimization.

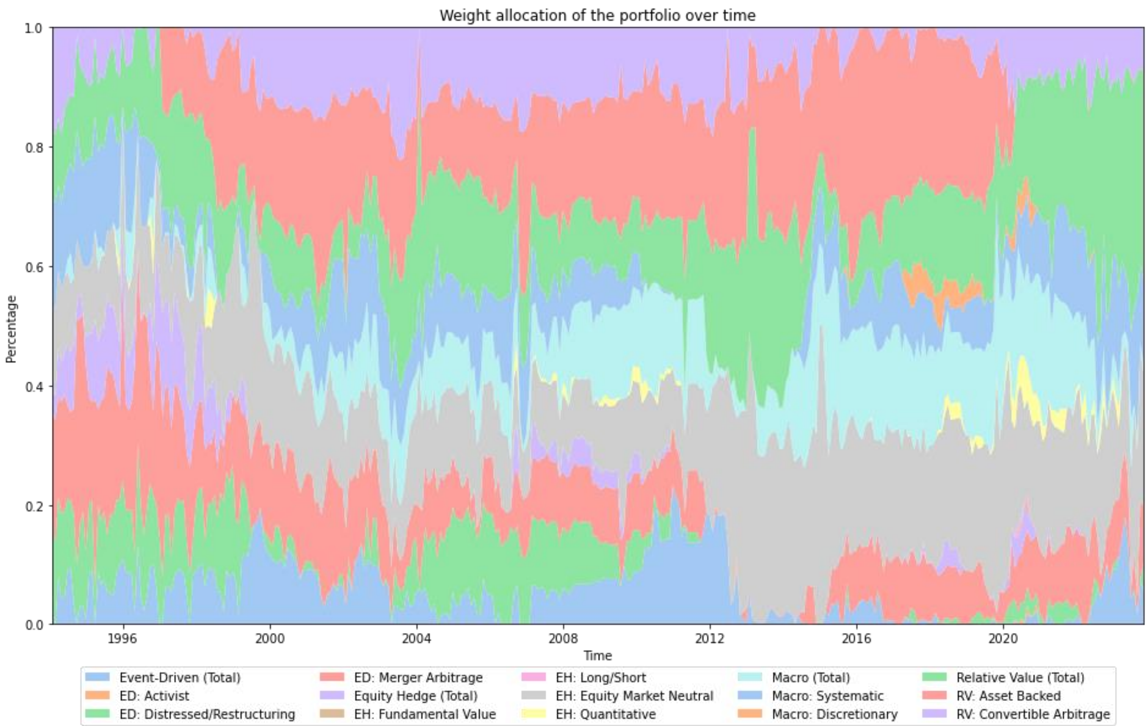

Figure 5.3.10. Stacked area plot of the **Minimum risk synthetic Omega** portfolio with cost minimization.

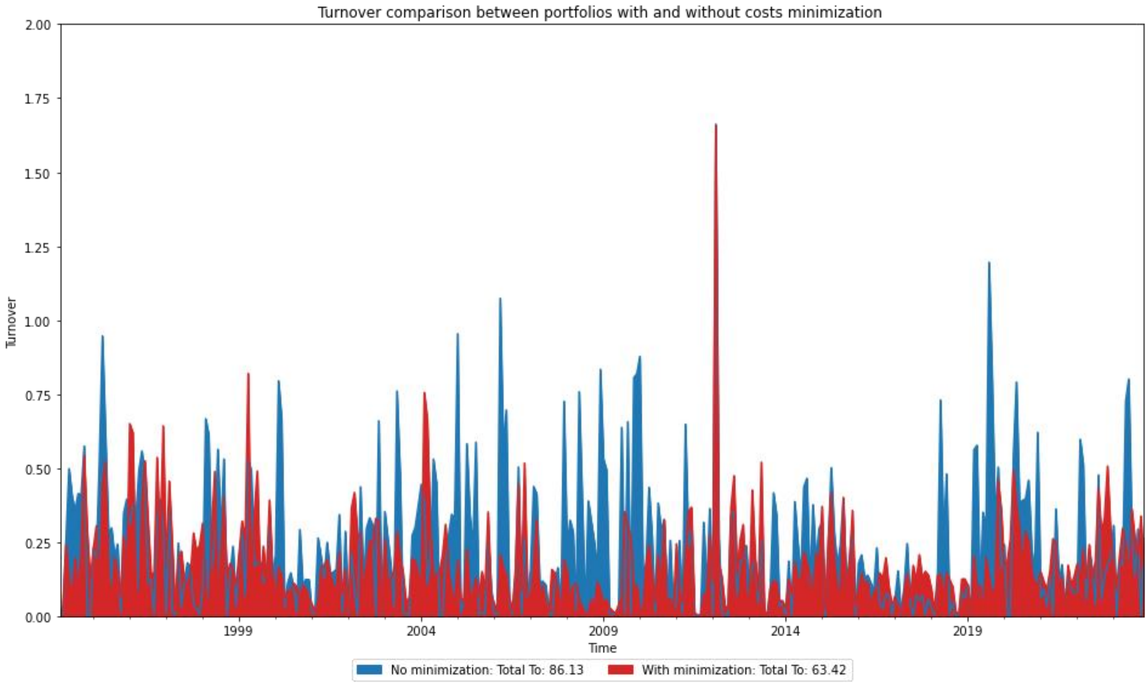

Figure 5.3.11. Portfolio turnover comparison of the **Minimum risk synthetic Omega** portfolio with cost minimization.

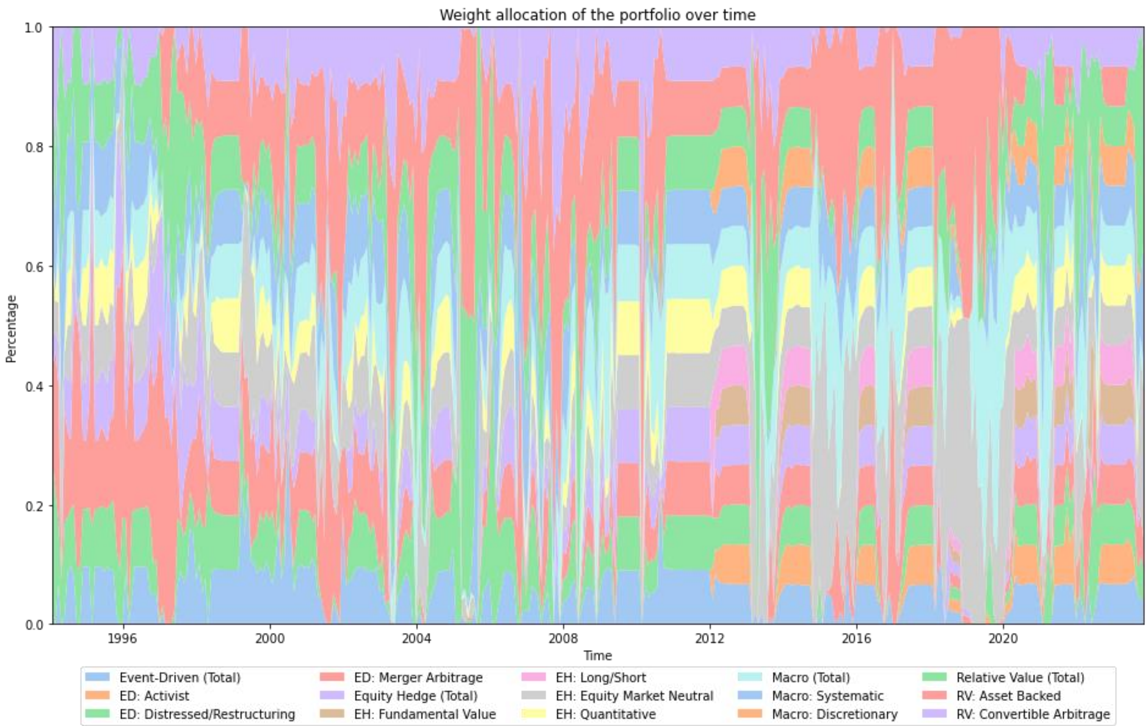

Figure 5.3.12. Stacked area plot of the **Optimal synthetic Omega** portfolio with cost minimization.

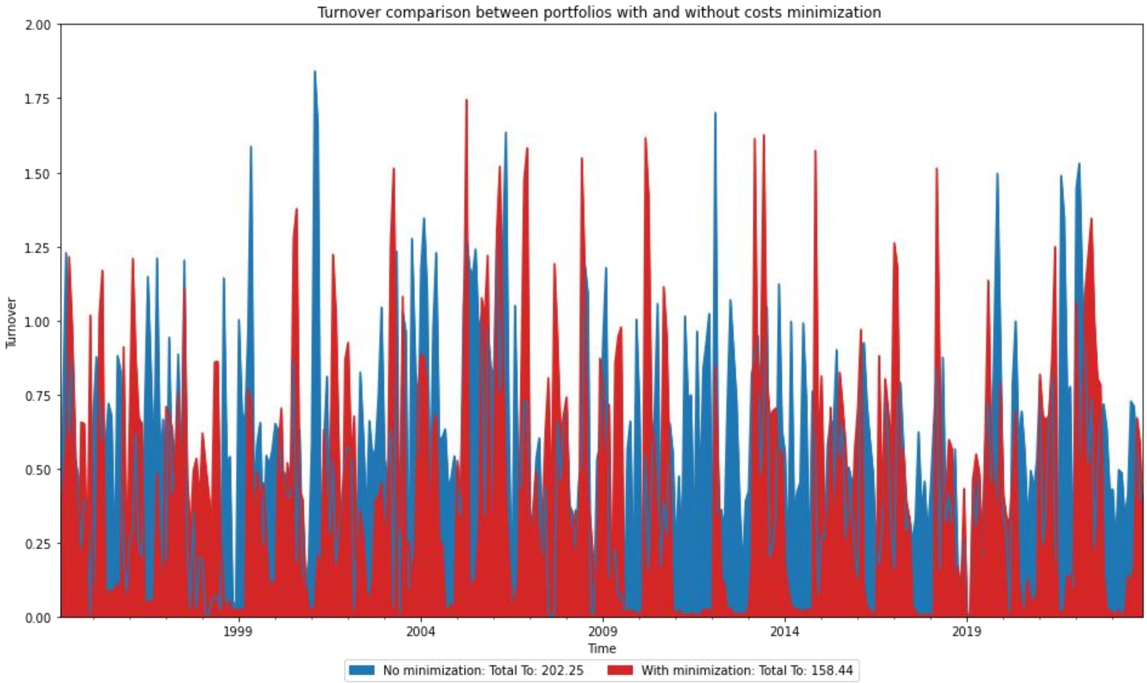

Figure 5.3.13. Portfolio turnover comparison of the **Optimal synthetic Omega** portfolio with cost minimization.

5.4. Part 3: Portfolios with correlation constraint

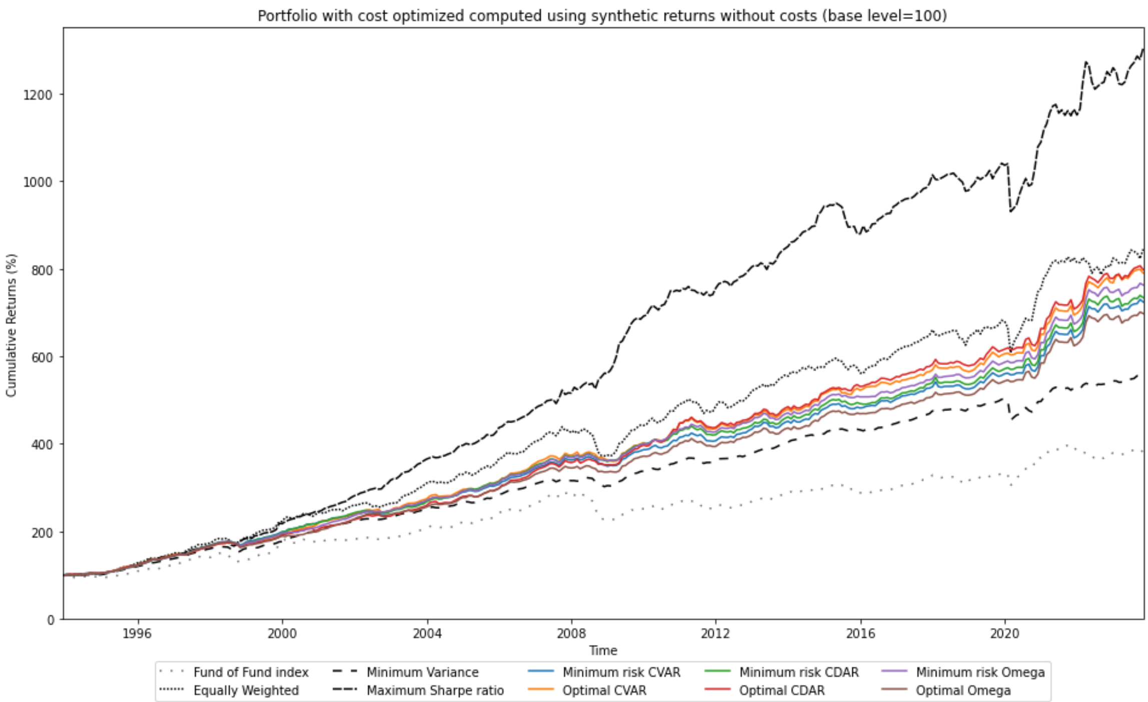

Figure 5.4.1. Portfolio with correlation constraint, displayed without costs.

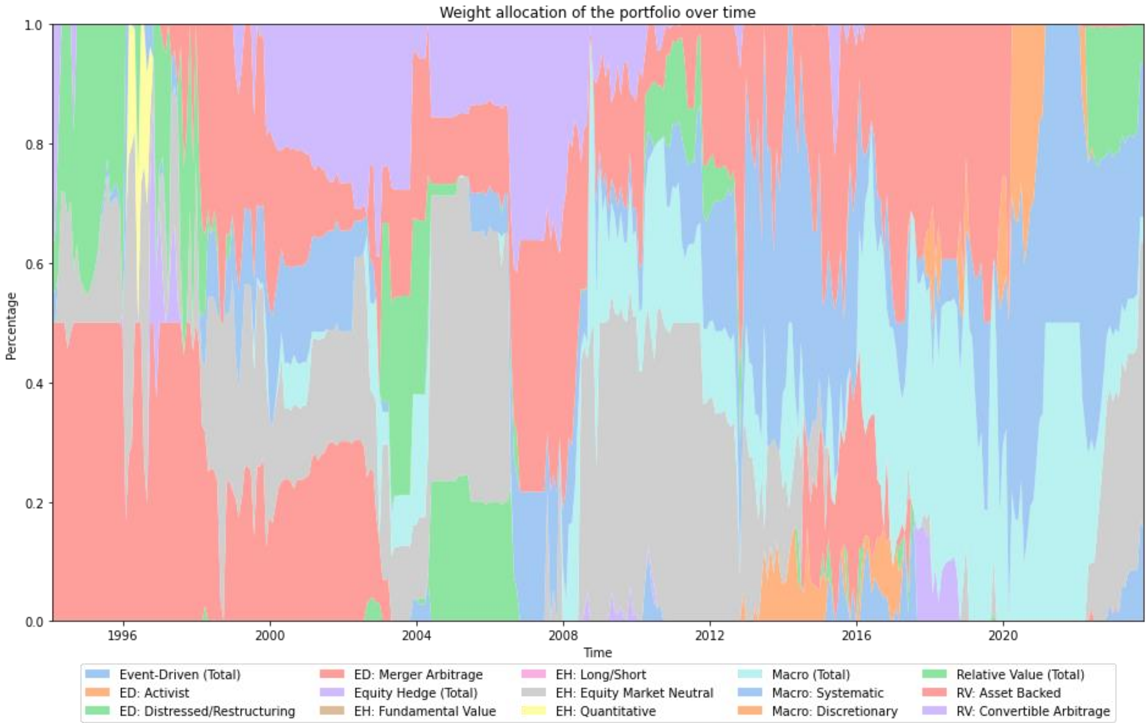

Figure 5.4.2. Stacked area plot of the **Minimum risk synthetic CVaR** portfolio with correlation constraint.

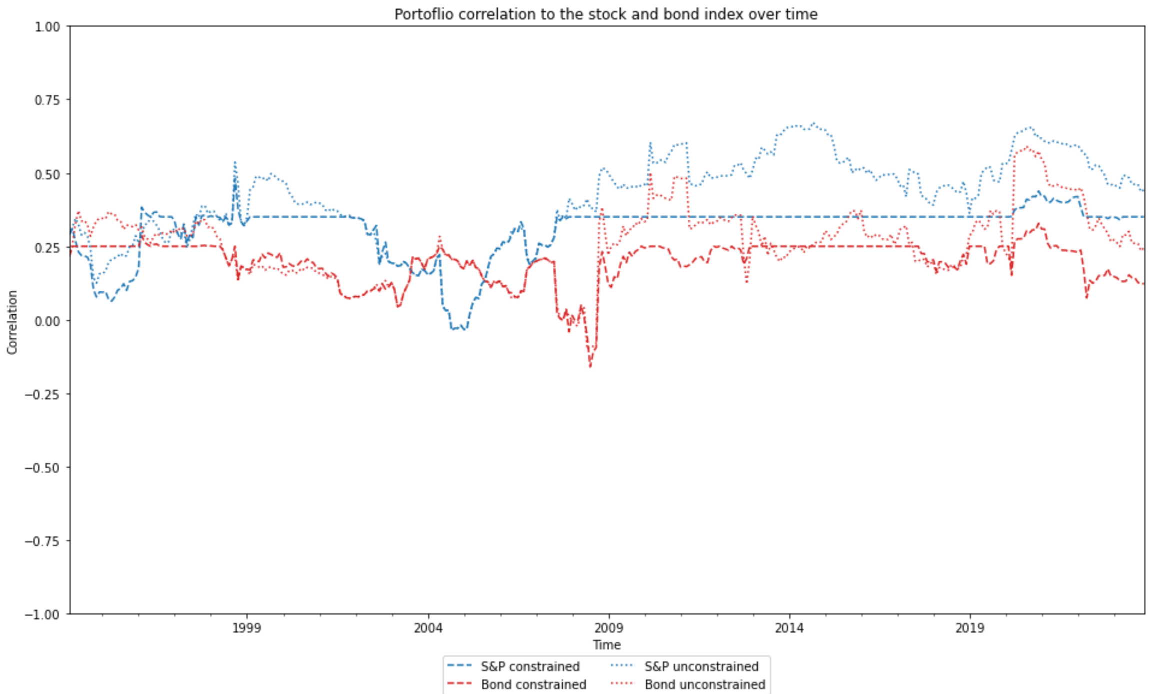

Figure 5.4.3. Portfolio correlation over time of the **Minimum risk synthetic CVaR** portfolio with correlation constraint.

Figure 5.4.4. Stacked area plot of the **Optimal synthetic CVaR** portfolio with correlation constraint.

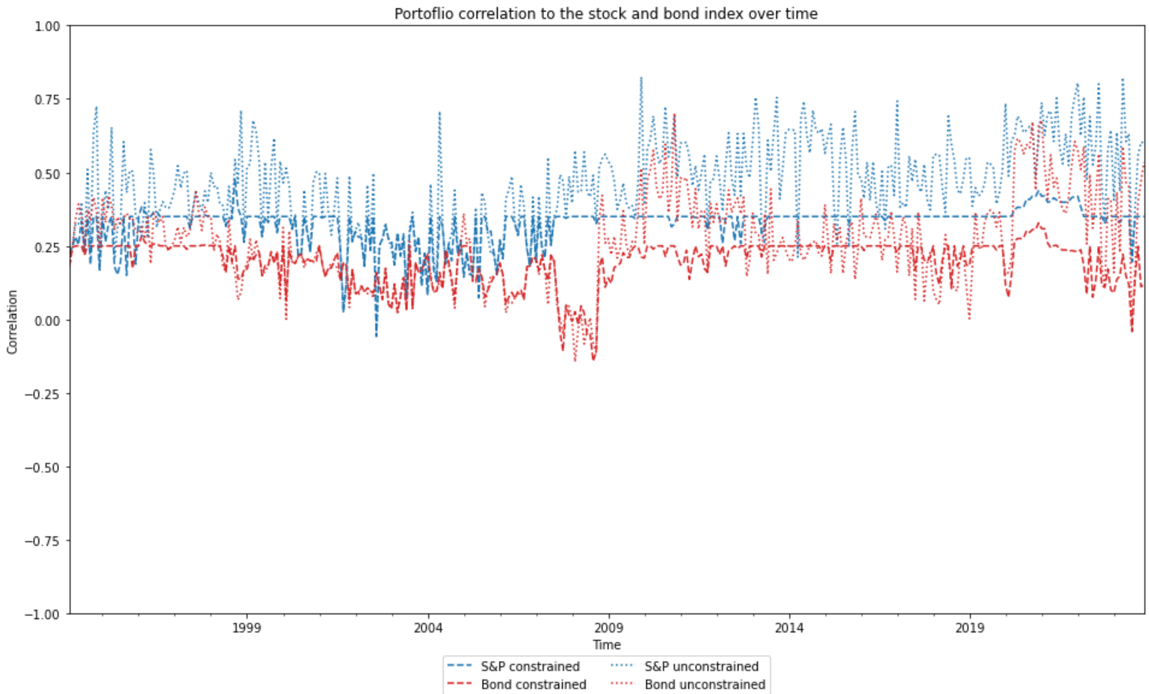

Figure 5.4.5. Portfolio correlation over time of the **Optimal synthetic CVaR** portfolio with correlation constraint.

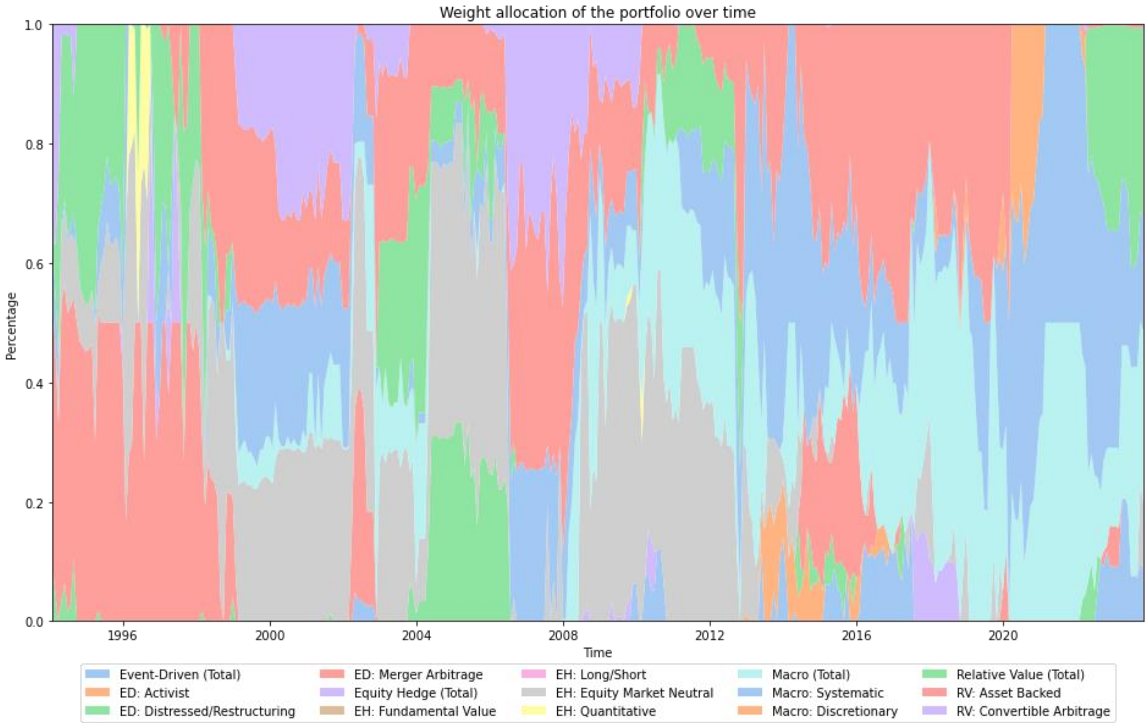

Figure 5.4.6. Stacked area plot of the **Minimum risk synthetic CDaR** portfolio with correlation constraint.

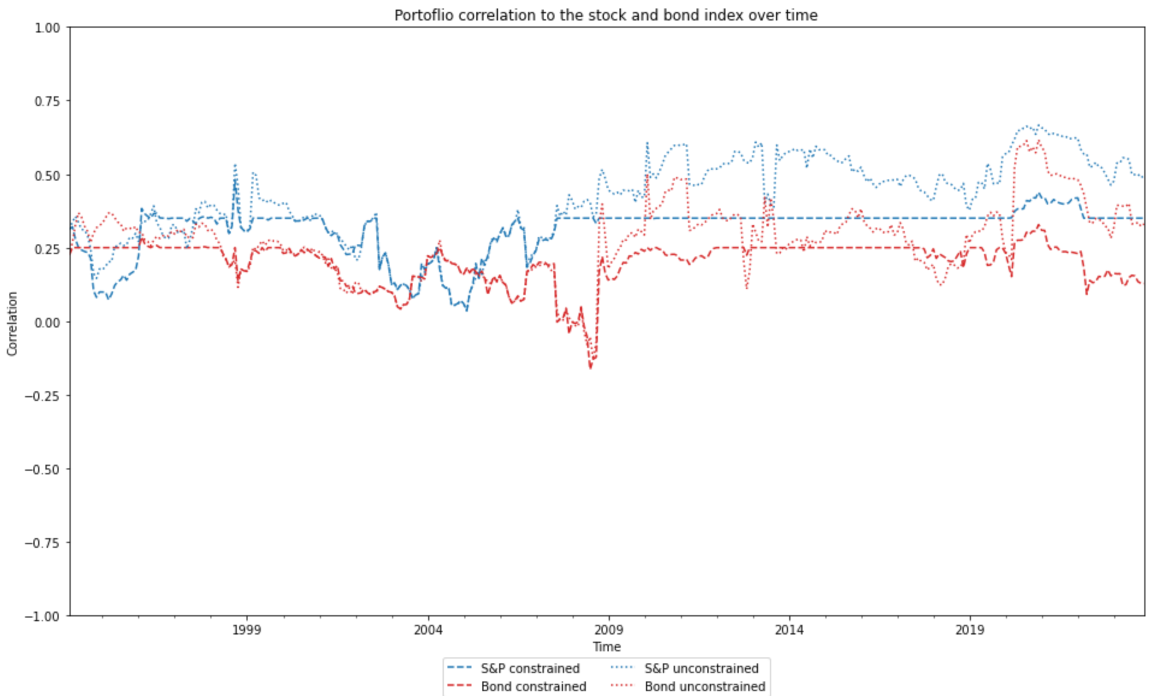

Figure 5.4.7. Portfolio correlation over time of the **Minimum risk synthetic CDaR** portfolio with correlation constraint.

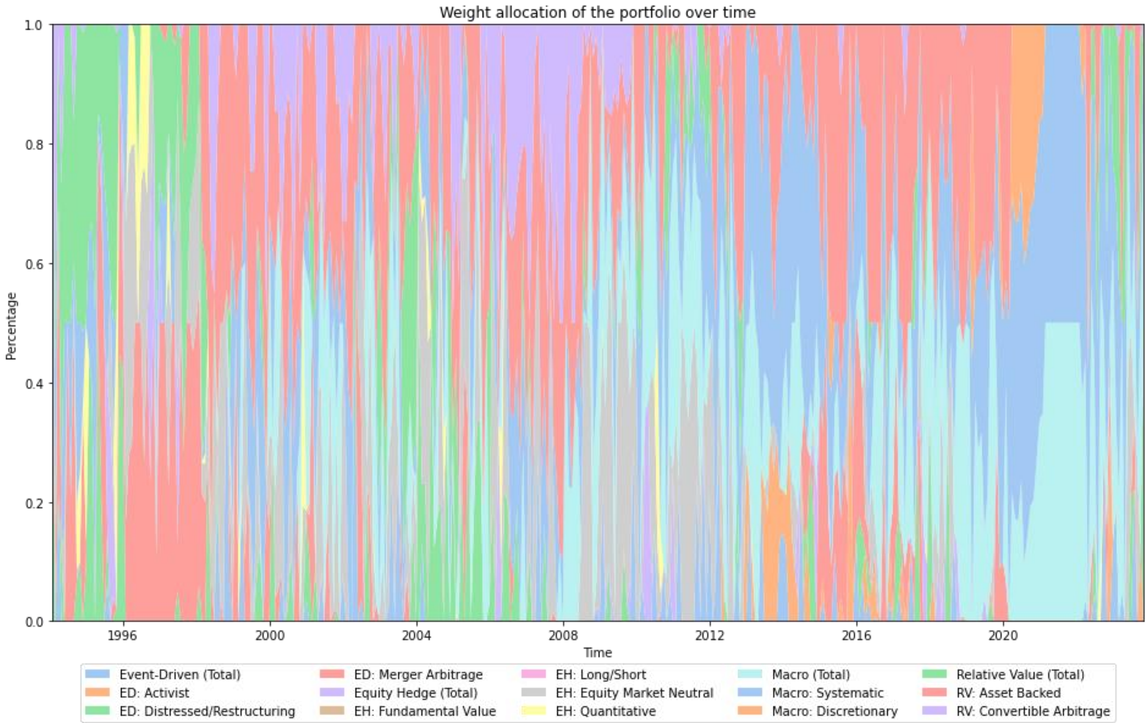

Figure 5.4.8. Stacked area plot of the **Optimal synthetic CDaR** portfolio with correlation constraint.

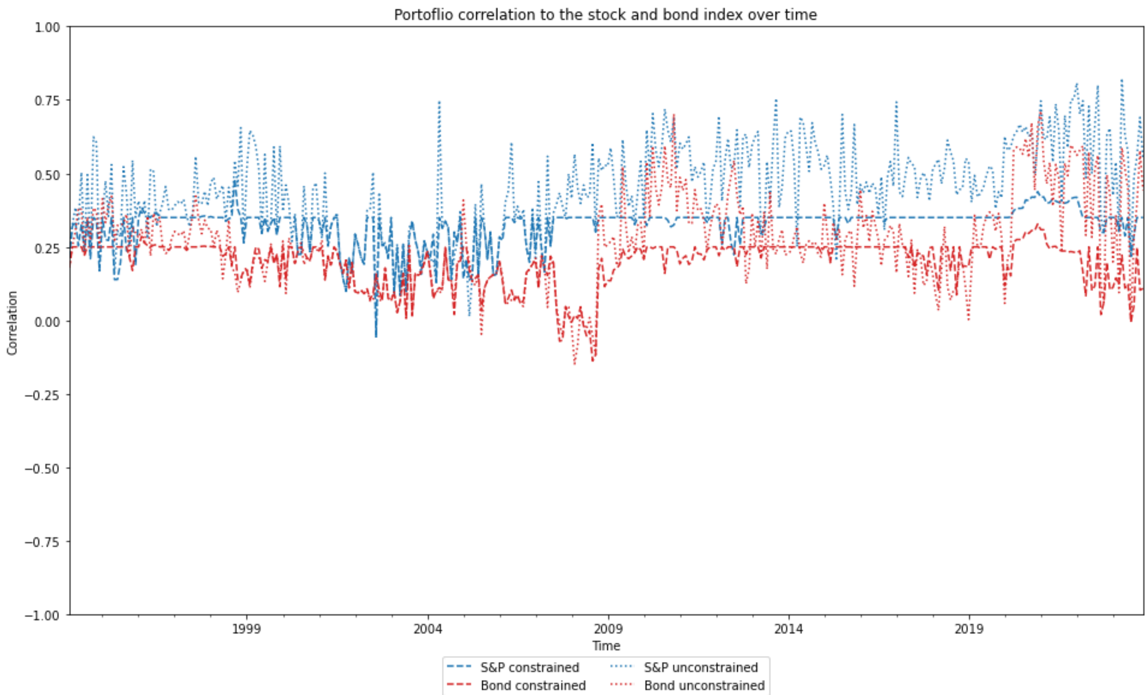

Figure 5.4.9. Portfolio correlation over time of the **Optimal synthetic CDaR** portfolio with correlation constraint.

Figure 5.4.10. Stacked area plot of the **Minimum risk synthetic Omega** portfolio with correlation constraint.

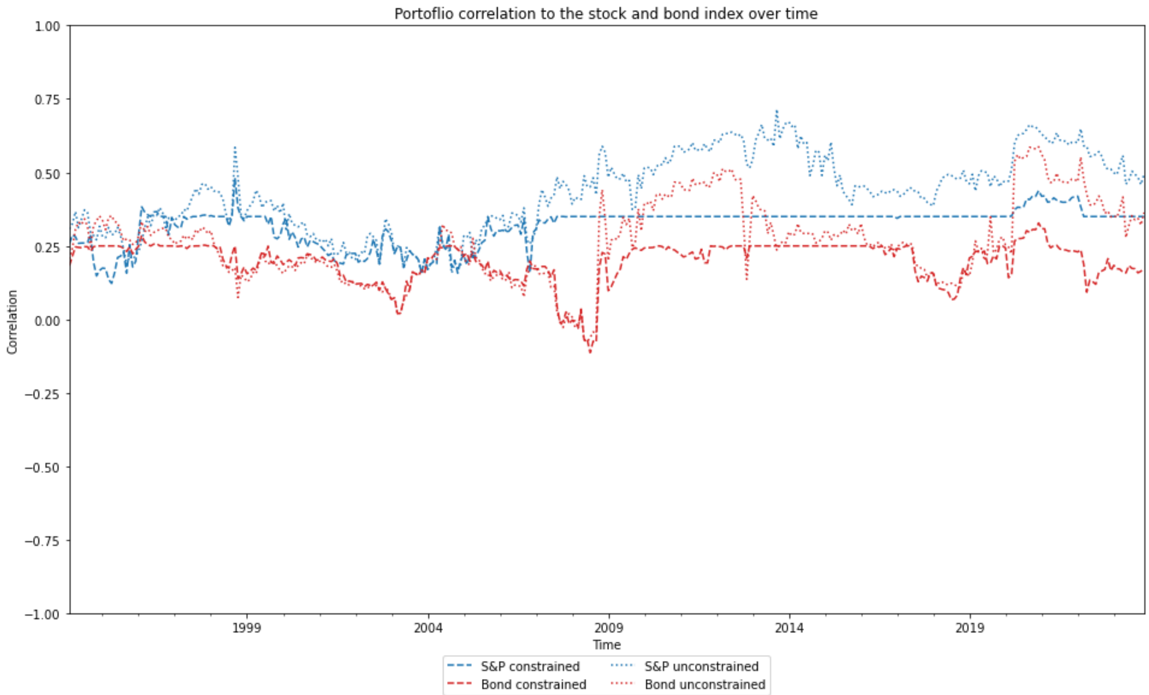

Figure 5.4.11. Portfolio correlation over time of the **Minimum risk synthetic Omega** portfolio with correlation constraint.

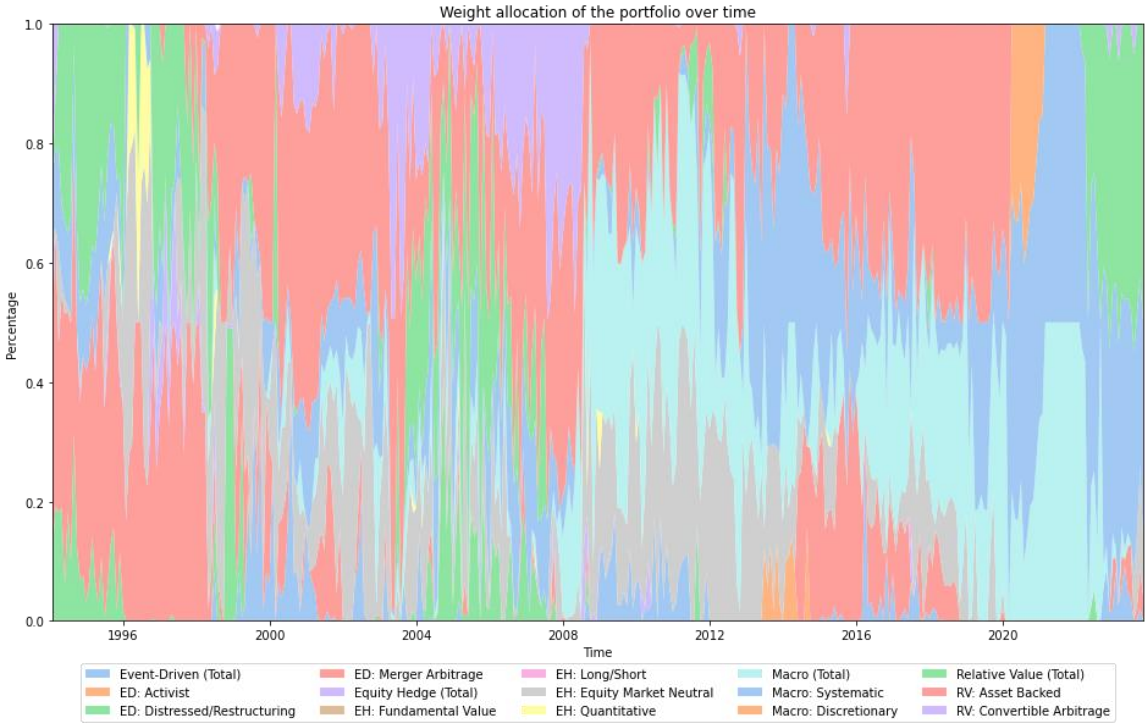

Figure 5.4.12. Stacked area plot of the **Optimal synthetic Omega** portfolio with correlation constraint.

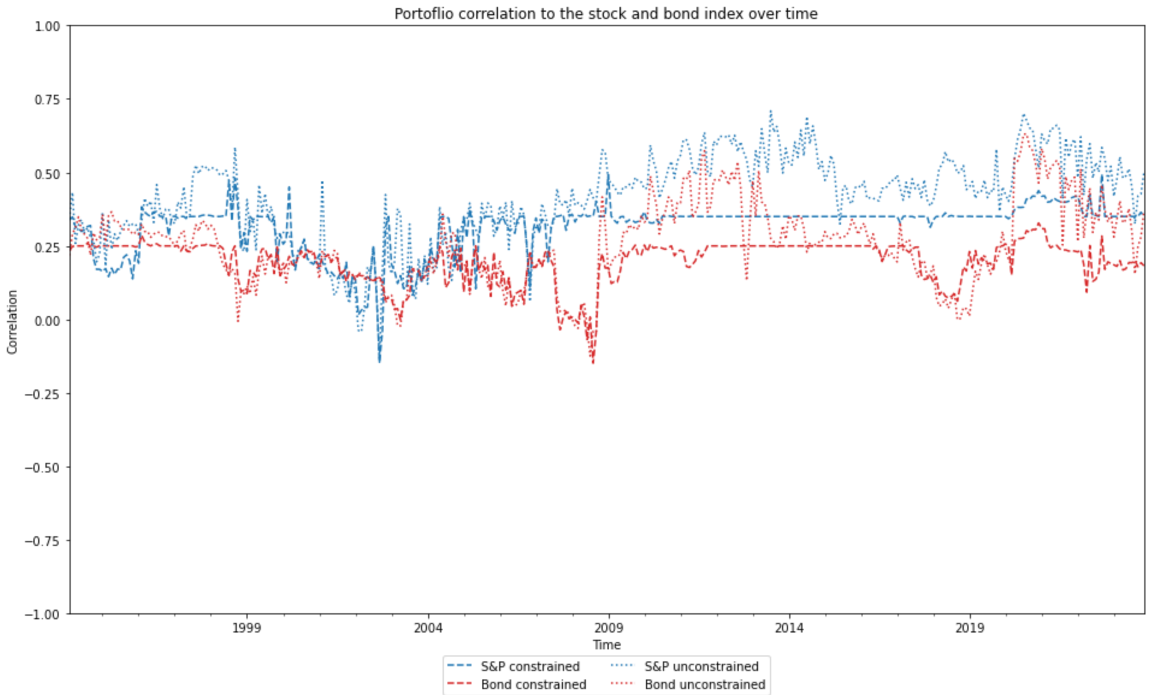

Figure 5.4.13. Portfolio correlation over time of the **Optimal synthetic Omega** portfolio with correlation constraint.

5.5. Part 4: Final portfolios

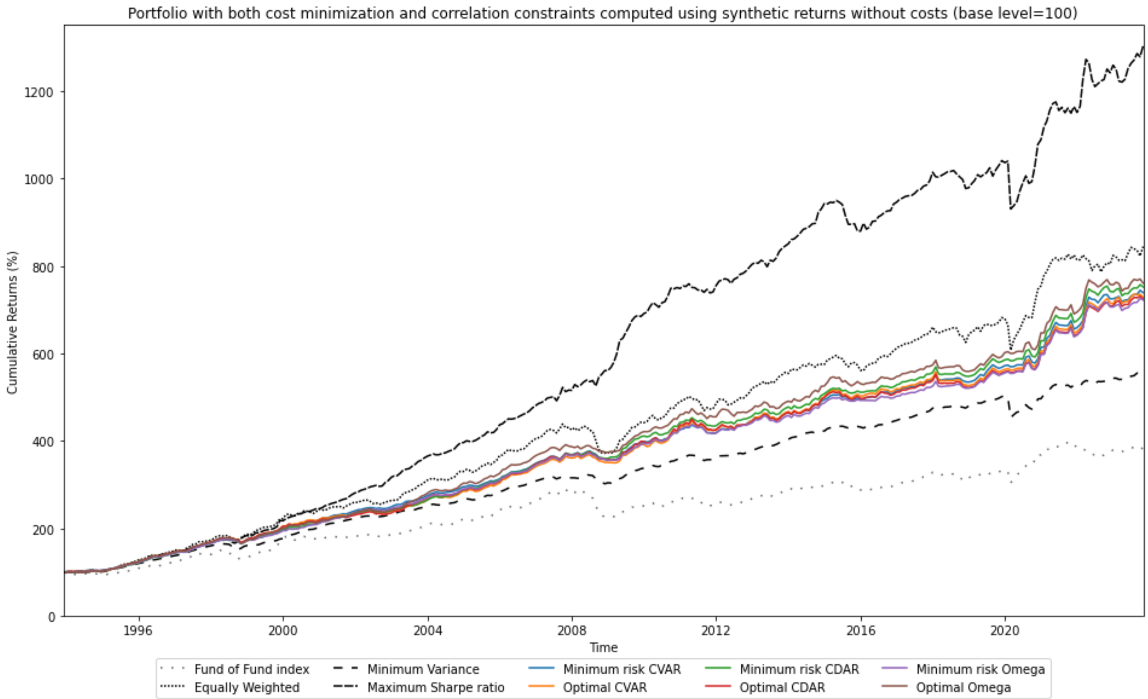

Figure 5.5.1. Portfolios with cost minimization and correlation constraint, displayed without costs.

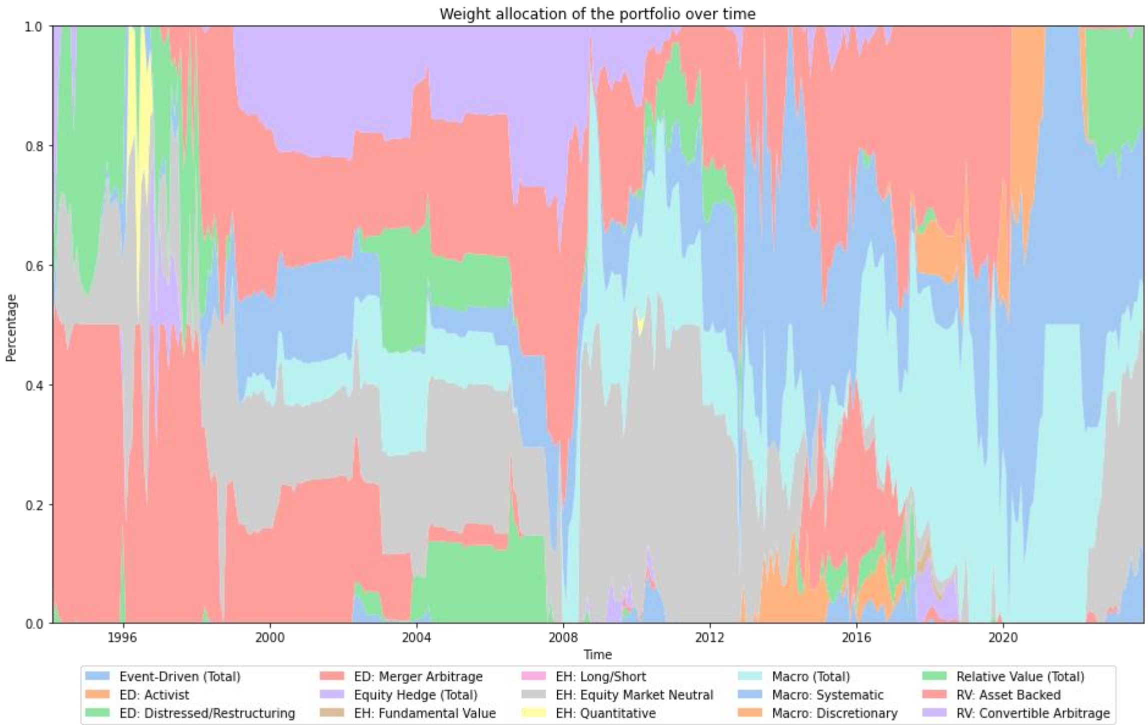

Figure 5.5.2. Stacked area plot of the **Minimum risk synthetic CVaR** portfolio with cost minimization and correlation constraint.

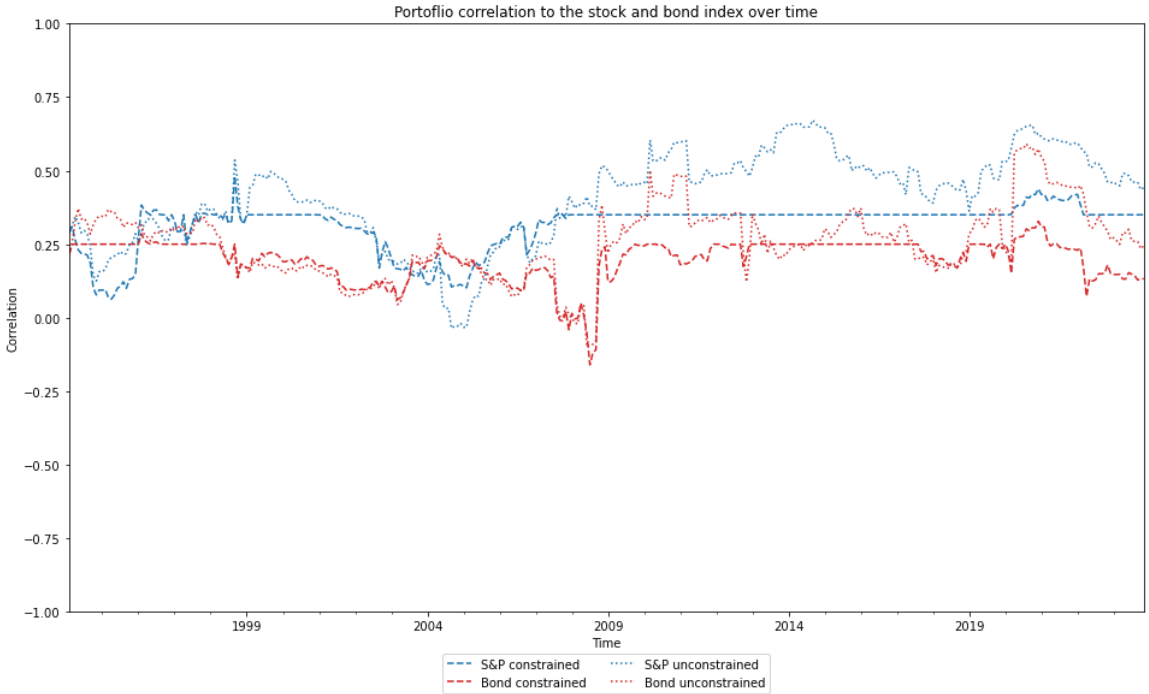

Figure 5.5.3. Portfolio correlation over time of the **Minimum risk synthetic CVaR** portfolio with cost minimization and correlation constraint.

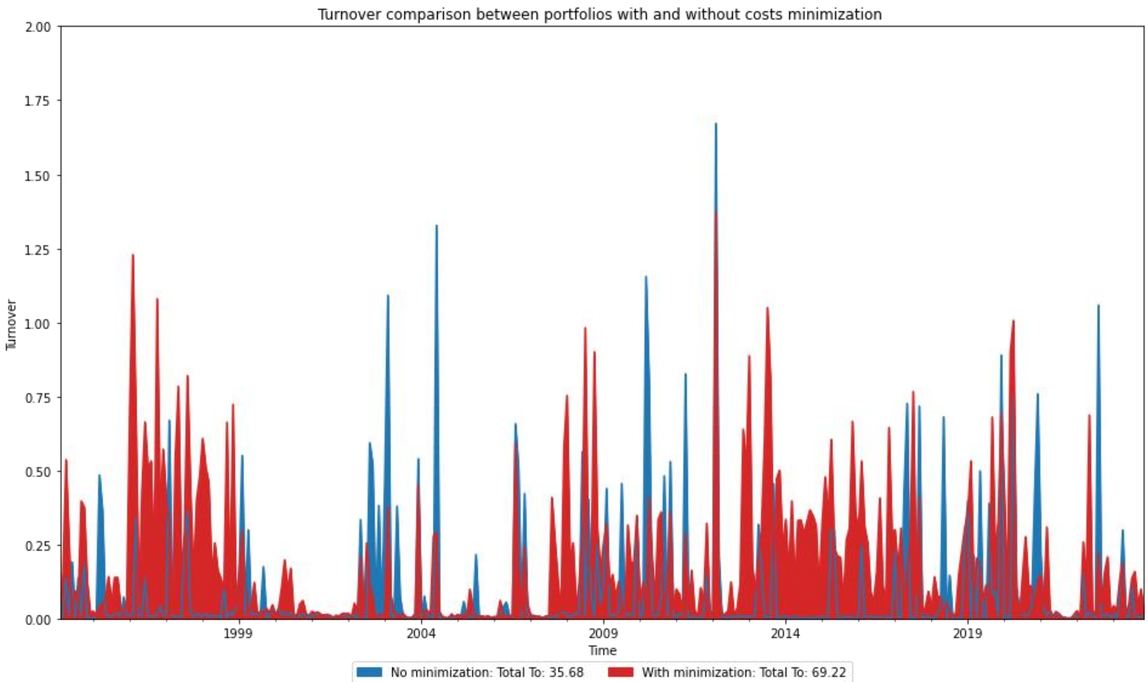

Figure 5.5.4. Portfolio turnover comparison of the **Minimum risk synthetic CVaR** portfolio with cost minimization and correlation constraint.

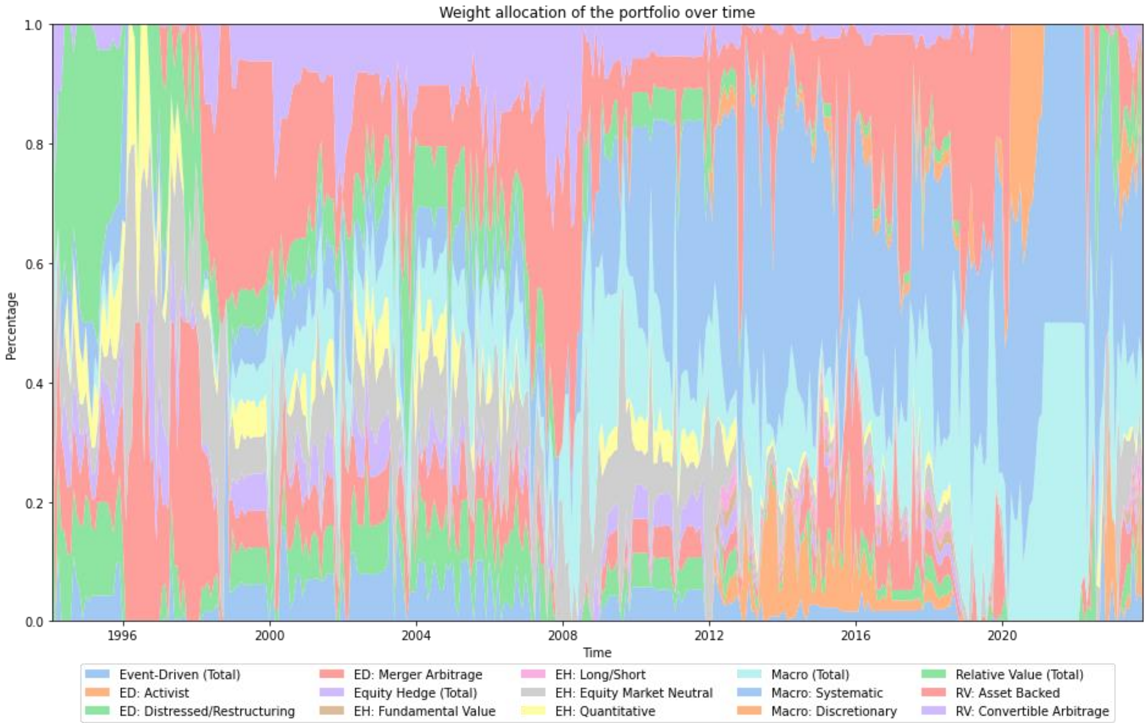

Figure 5.5.5. Stacked area plot of the **Optimal synthetic CVAR** portfolio with cost minimization and correlation constraint.

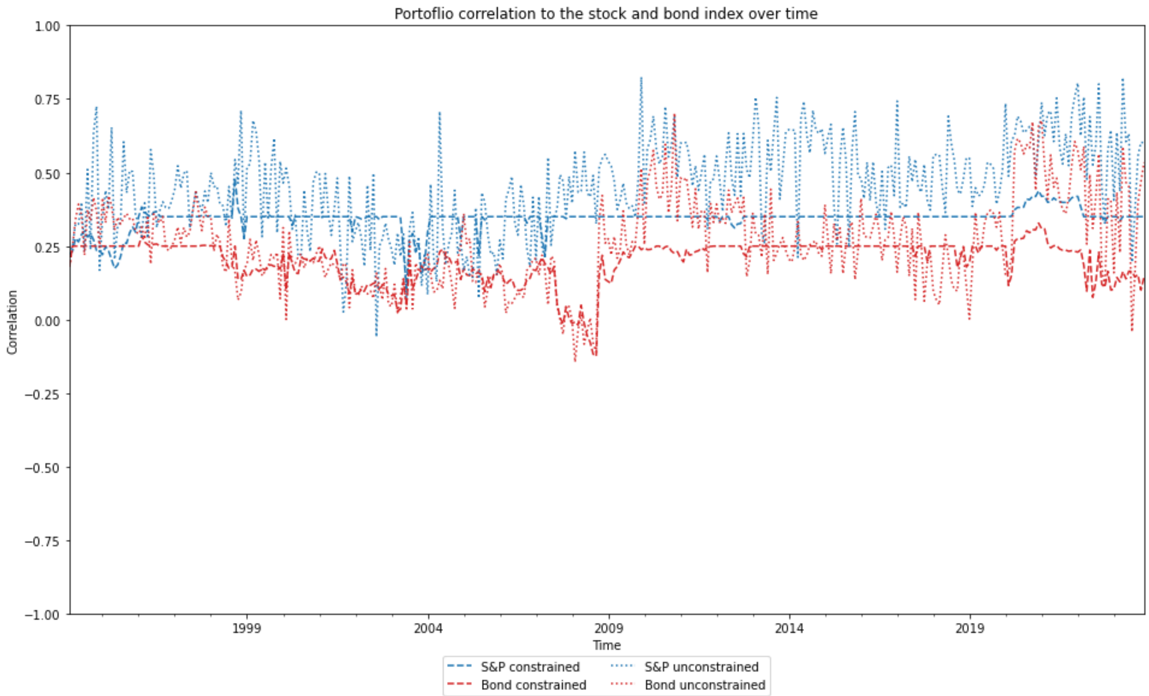

Figure 5.5.6. Portfolio correlation over time of the **Optimal synthetic CVAR** portfolio with cost minimization and correlation constraint.

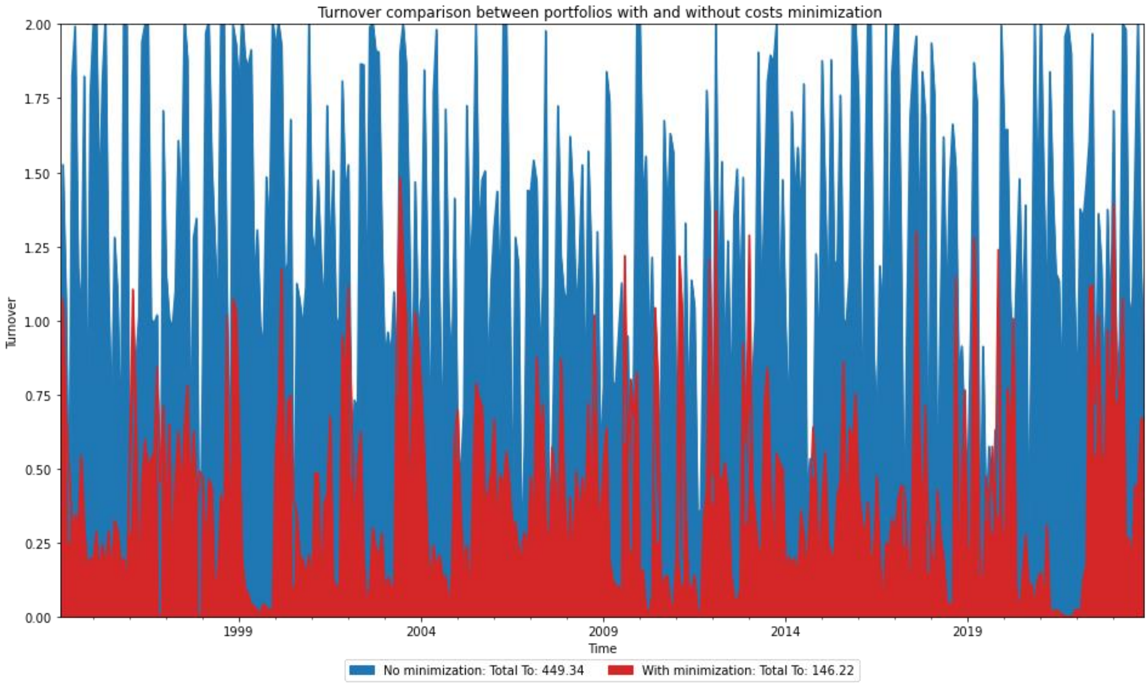

Figure 5.5.7. Portfolio turnover comparison of the **Optimal synthetic CVAR** portfolio with cost minimization and correlation constraint.

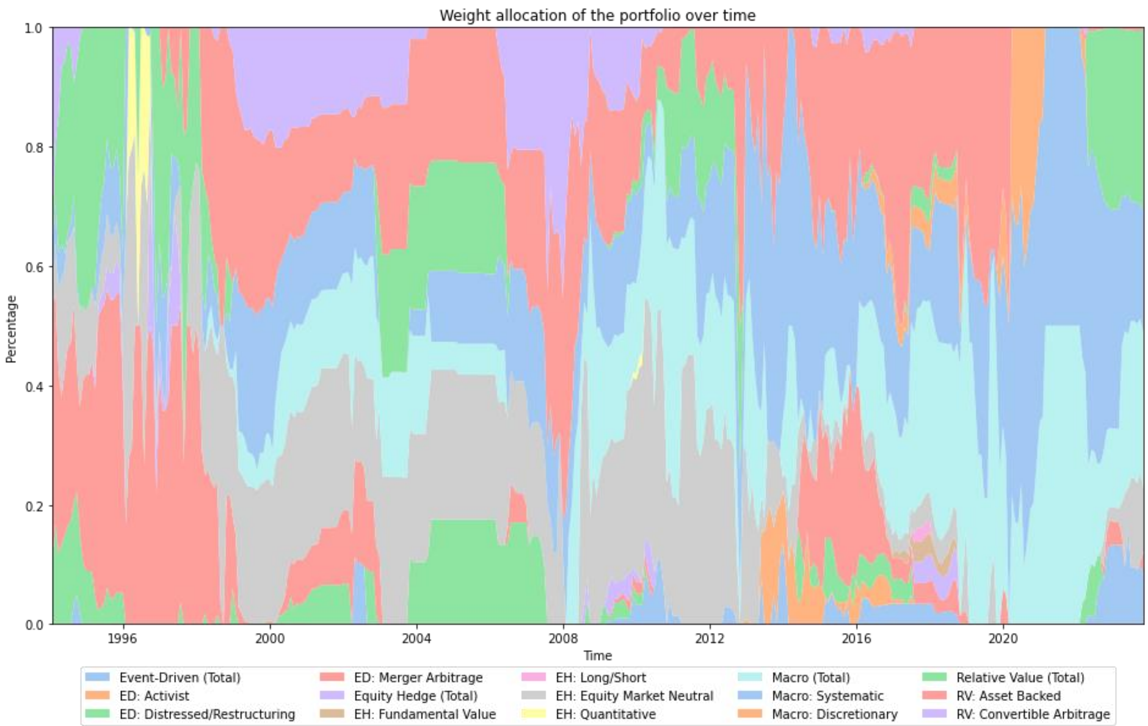

Figure 5.5.8. Stacked area plot of the **Minimum risk synthetic CDaR** portfolio with cost minimization and correlation constraint.

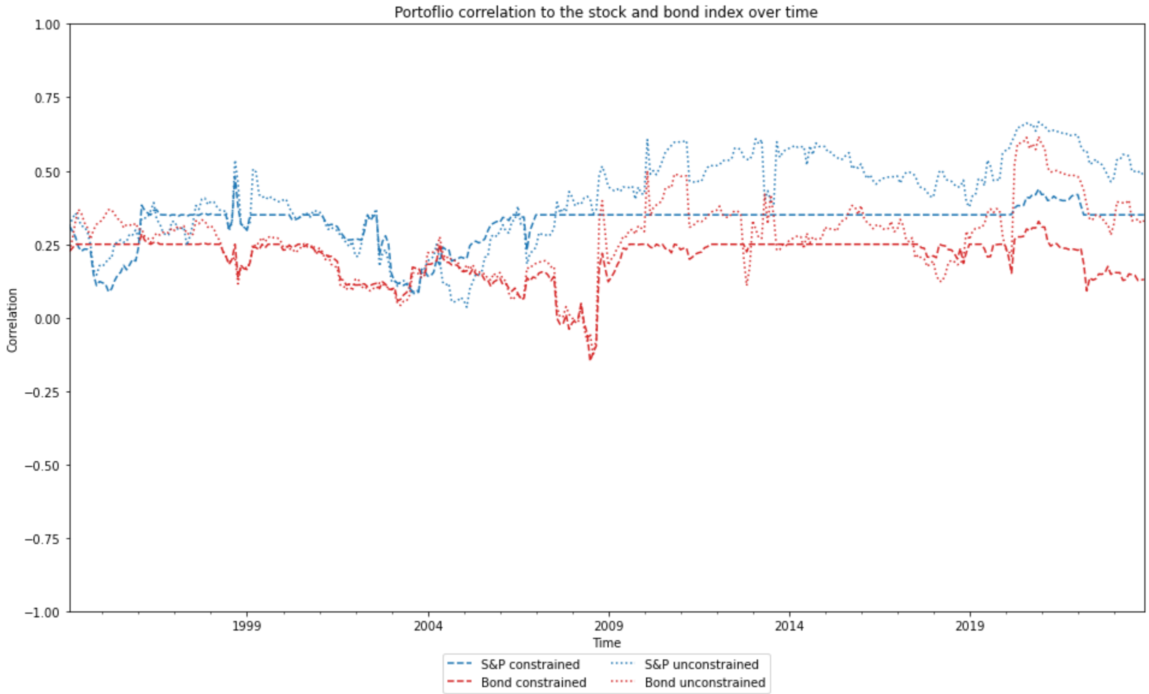

Figure 5.5.9. Portfolio correlation over time of the **Minimum risk synthetic CDaR** portfolio with cost minimization and correlation constraint.

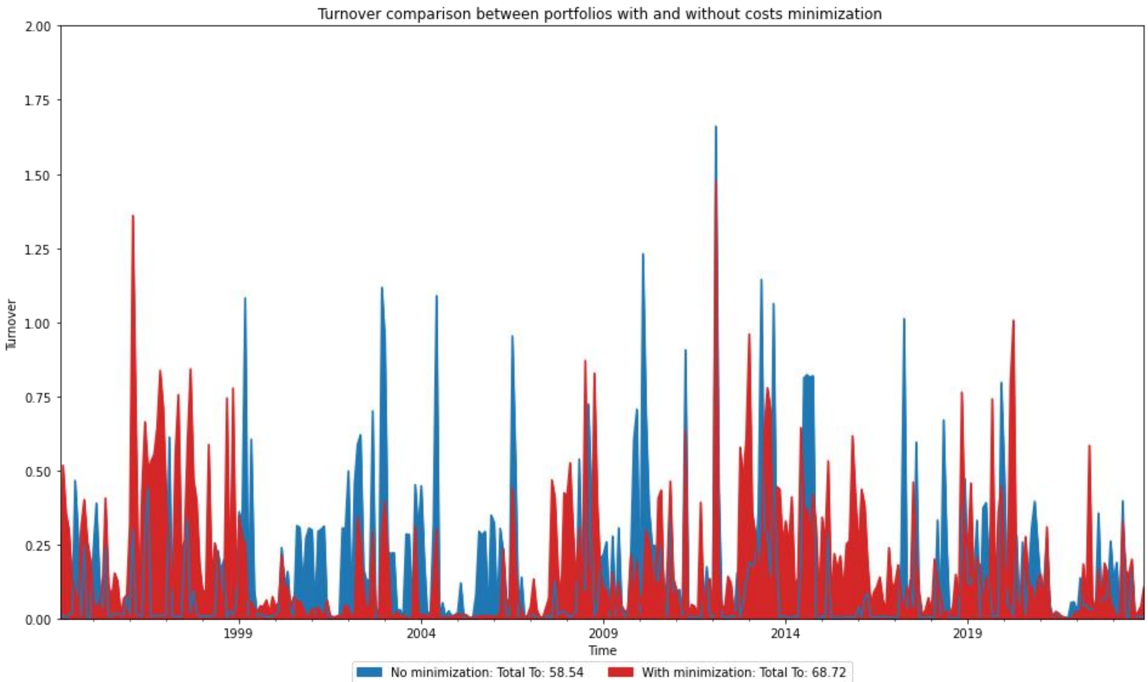

Figure 5.5.10. Portfolio turnover comparison of the **Minimum risk synthetic CDaR** portfolio with cost minimization and correlation constraint.

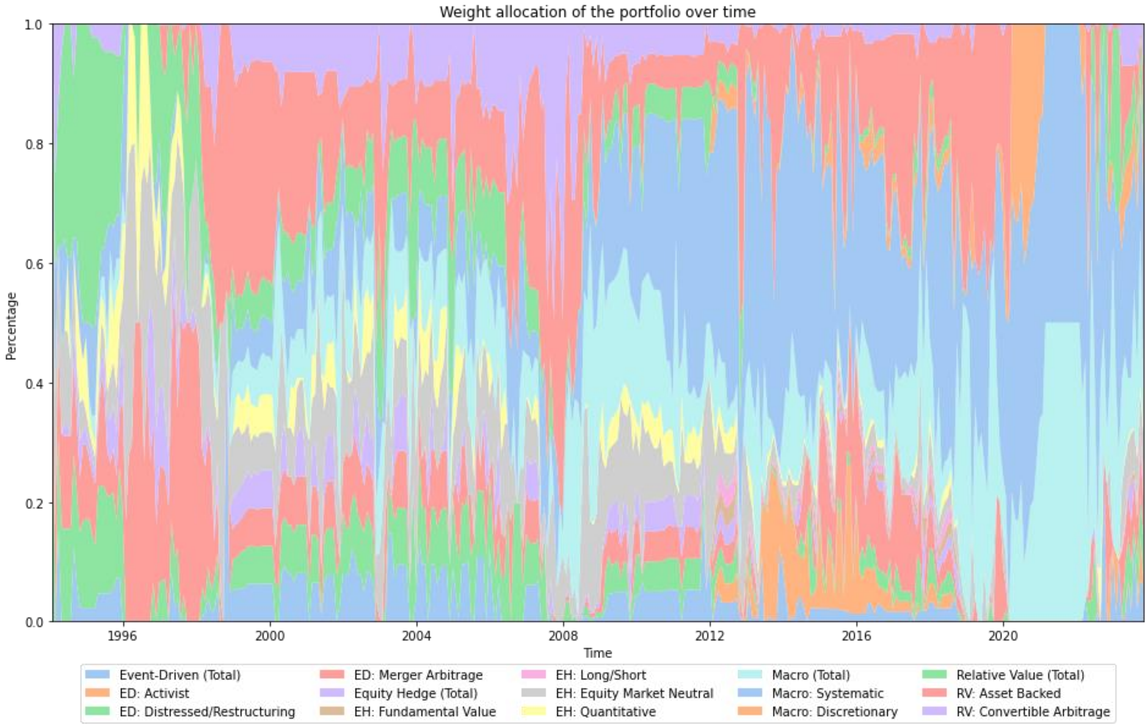

Figure 5.5.11. Stacked area plot of the **Optimal synthetic CDAR** portfolio with cost minimization and correlation constraint.

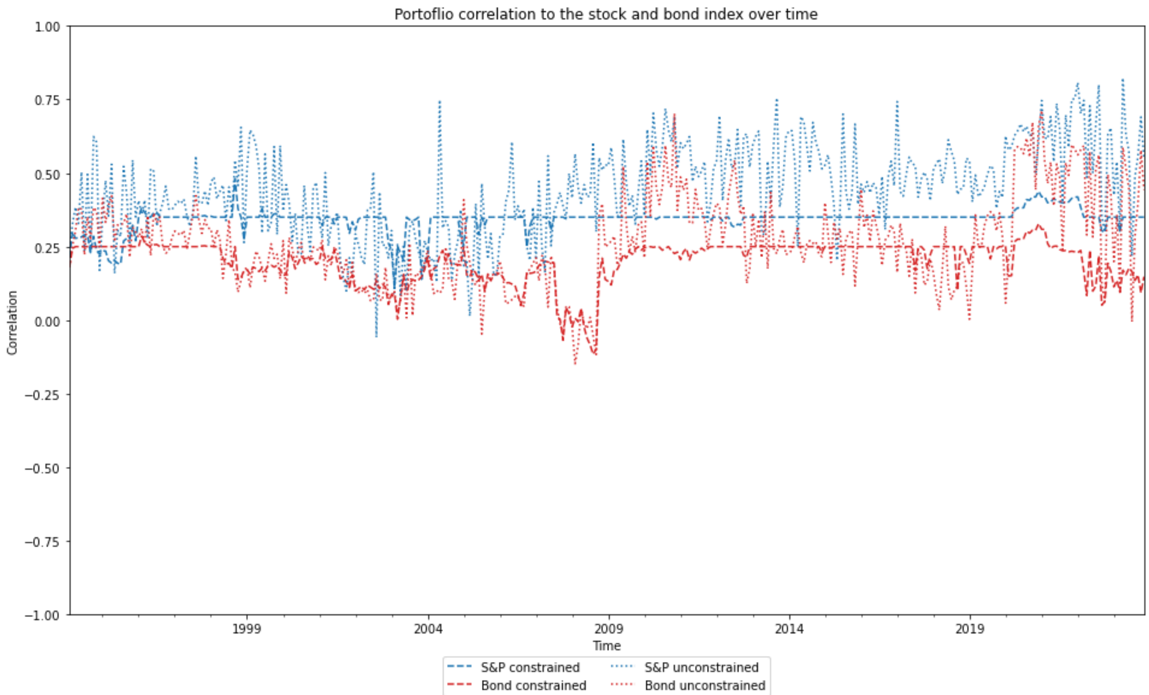

Figure 5.5.12. Portfolio correlation over time from 1990 to 2023 of the **Optimal synthetic CDAR** portfolio with cost minimization and correlation constraint.

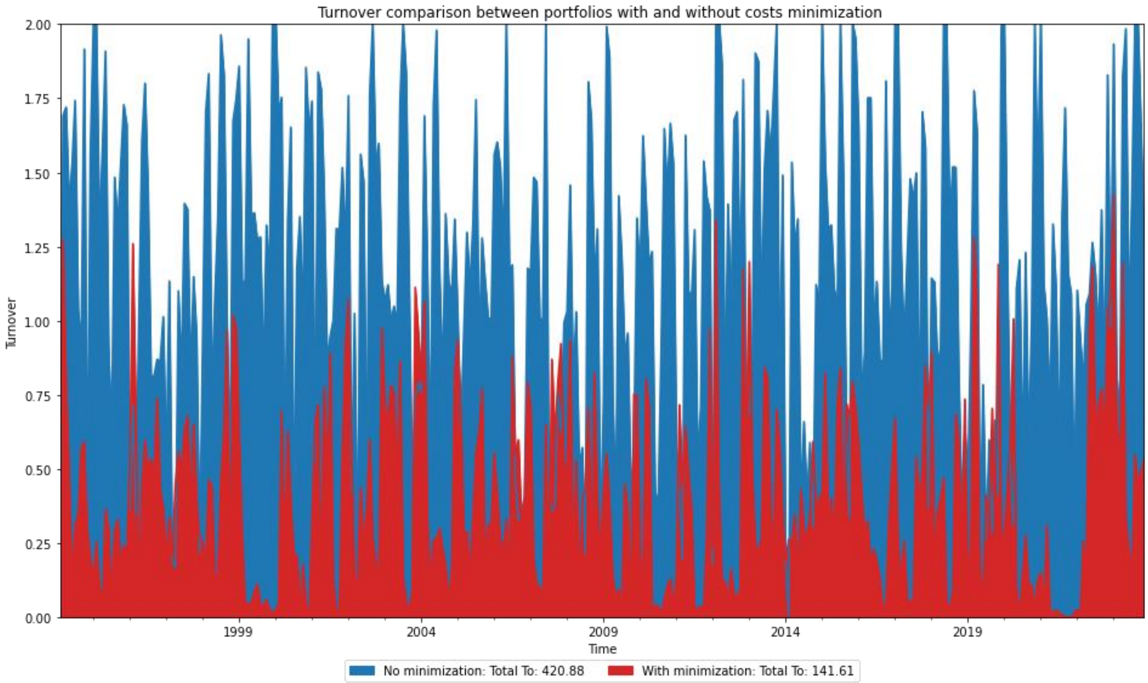

Figure 5.5.13. Portfolio turnover comparison of the **Optimal synthetic CDAR** portfolio with cost minimization and correlation constraint.

Figure 5.5.14. Stacked area plot of the **Minimum risk synthetic Omega** portfolio with cost minimization and correlation constraint.

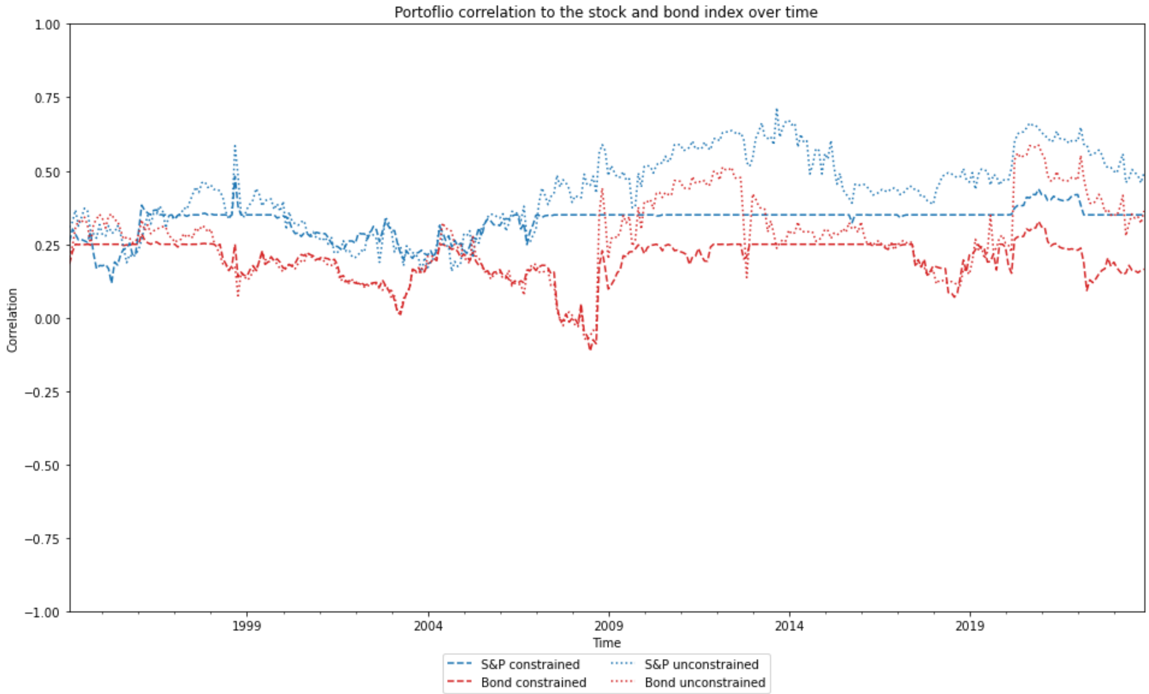

Figure 5.5.15. Portfolio correlation over time of the **Minimum risk synthetic Omega** portfolio with cost minimization and correlation constraint.

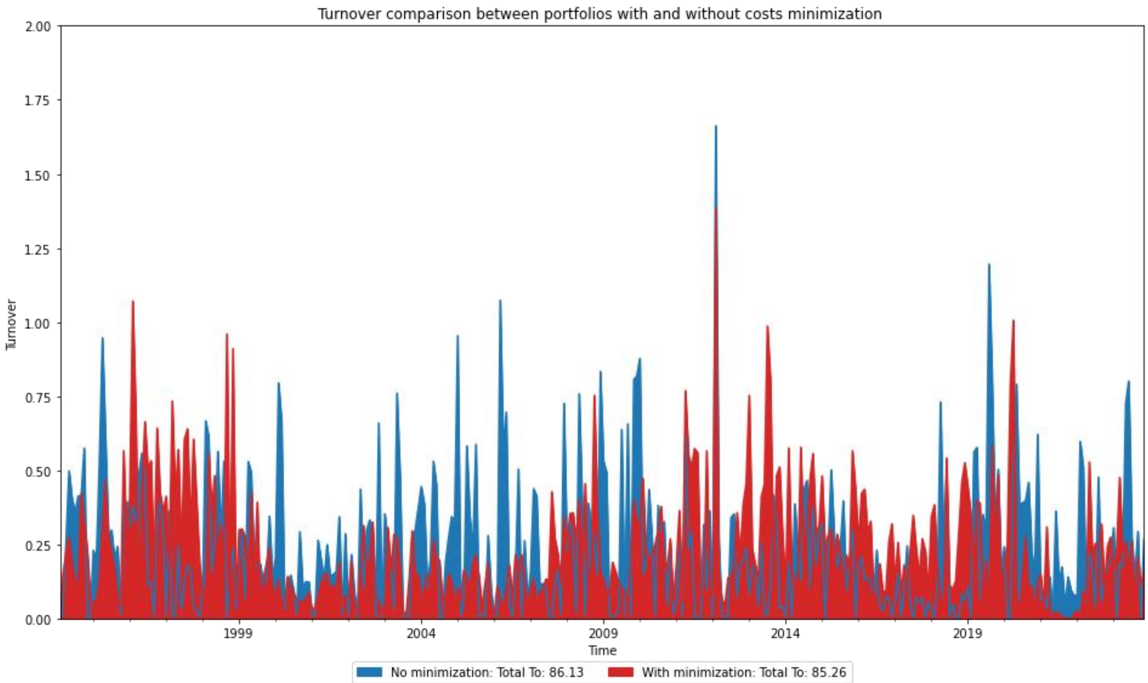

Figure 5.5.16. Portfolio turnover comparison of the **Minimum risk synthetic Omega** portfolio with cost minimization and correlation constraint.

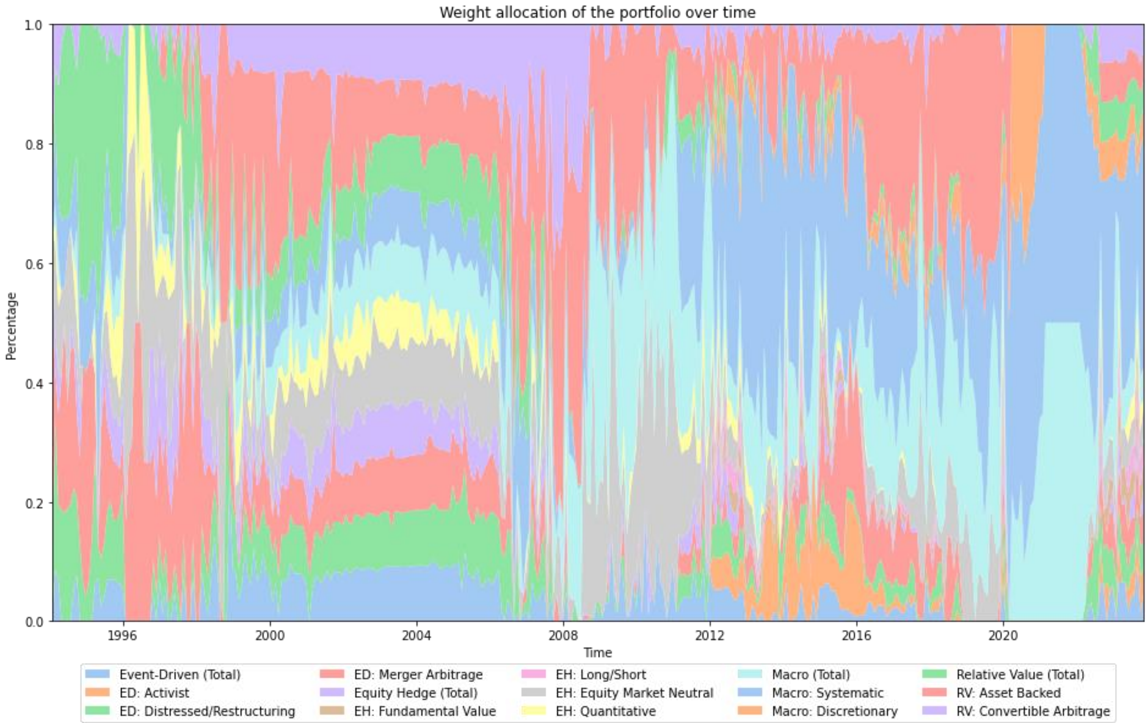

Figure 5.5.17.: Stacked area plot of the **Optimal synthetic Omega** portfolio with cost minimization and correlation constraint.

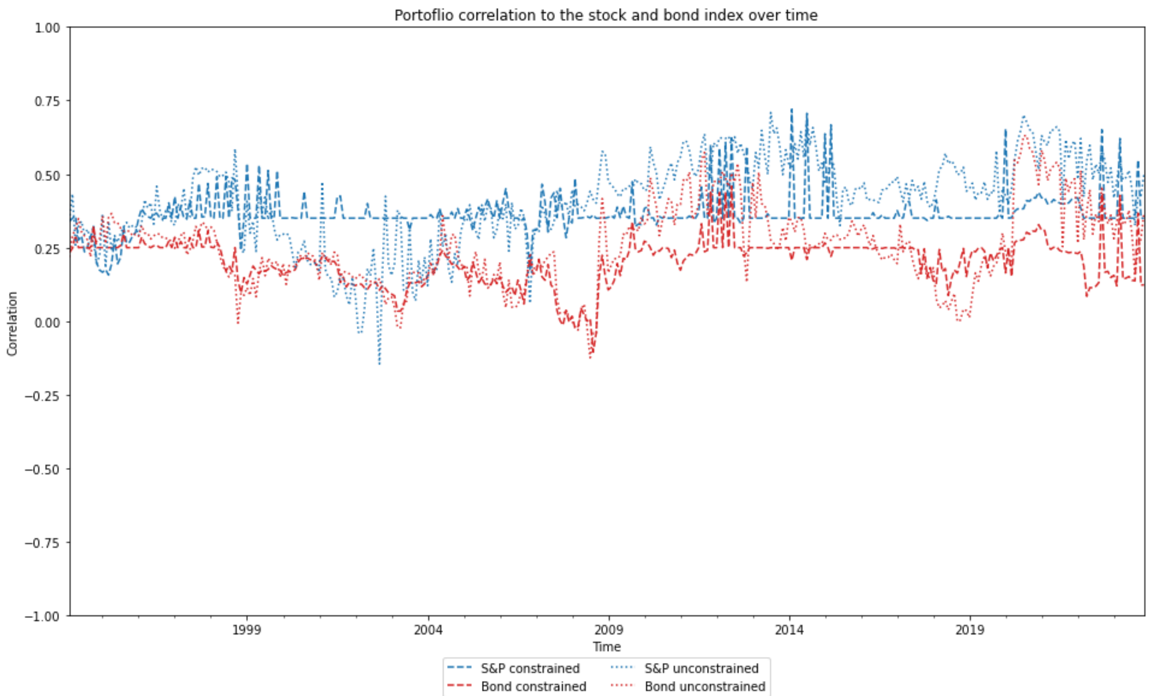

Figure 5.5.18. Portfolio correlation over time of the **Optimal synthetic Omega** portfolio with cost minimization and correlation constraint.

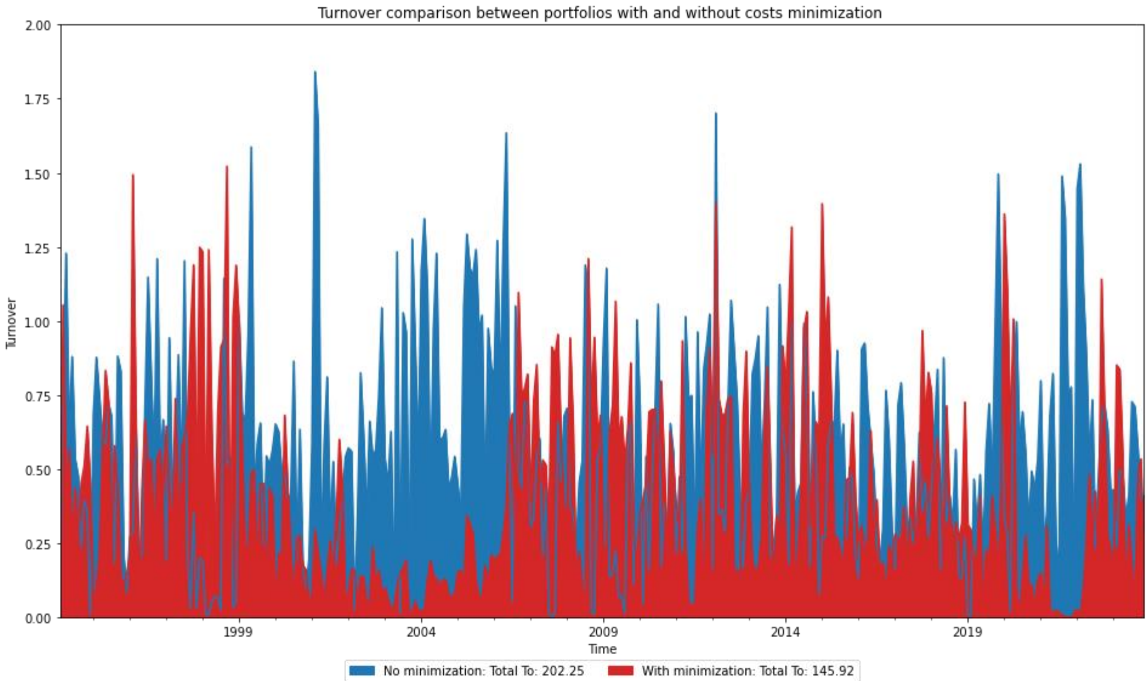

Figure 5.5.19. Portfolio turnover comparison of the **Optimal synthetic Omega** portfolio with cost minimization and correlation constraint.

P4 - Historical	FoF	EW	MVP	MSR	Min. CVaR	Opti. CVaR	Min. CDaR	Opti. CDaR	Min. Omega	Opti. Omega
Return	4.93%	7.51% (7.45%)	5.94% (5.64%)	9.05% (4.28%)	6.70% (6.02%)	6.73% (5.17%)	6.98% (6.34%)	6.69% (5.14%)	6.46% (5.54%)	7.05% (4.98%)
M²	4.93%	7.81% (7.74%)	7.20% (6.67%)	10.09% (4.29%)	7.88% (6.86%)	7.41% (5.42%)	8.19% (7.24%)	7.40% (5.39%)	7.69% (6.27%)	7.67% (5.14%)
Volatility	5.64%	5.16% (5.16%)	3.23% (3.24%)	4.64% (4.67%)	3.81% (3.84%)	4.42% (4.46%)	3.91% (3.93%)	4.38% (4.42%)	3.62% (3.62%)	4.62% (4.68%)
MDD	22.20%	15.23% (15.32%)	9.88% (9.95%)	10.61% (24.93%)	8.09% (8.91%)	9.45% (10.59%)	8.55% (9.14%)	9.87% (11.17%)	7.81% (8.48%)	10.02% (12.28%)
CVaR	7.21%	6.36% (6.37%)	5.42% (5.47%)	5.66% (6.08%)	4.80% (4.87%)	5.19% (5.37%)	4.83% (4.92%)	5.22% (5.39%)	4.80% (4.87%)	5.86% (6.04%)
CDaR	24.23%	16.05% (16.14%)	8.88% (9.20%)	9.77% (27.40%)	7.38% (7.97%)	8.02% (9.37%)	7.52% (8.04%)	7.99% (9.30%)	7.24% (7.76%)	9.23% (10.83%)
Sharpe	0.123	0.633 (0.621)	0.525 (0.430)	1.037 (0.008)	0.645 (0.464)	0.562 (0.209)	0.699 (0.532)	0.560 (0.203)	0.612 (0.359)	0.608 (0.159)
Calmar	0.031	0.215 (0.209)	0.172 (0.140)	0.453 (0.002)	0.304 (0.200)	0.263 (0.088)	0.320 (0.229)	0.248 (0.080)	0.284 (0.153)	0.280 (0.060)
R²	1.000	0.824 (0.825)	0.493 (0.488)	0.367 (0.365)	0.417 (0.422)	0.439 (0.442)	0.419 (0.423)	0.430 (0.434)	0.395 (0.397)	0.551 (0.546)
Corr. Stocks	0.648	0.555 (0.555)	0.379 (0.379)	0.422 (0.422)	0.308 (0.308)	0.328 (0.328)	0.316 (0.316)	0.326 (0.326)	0.311 (0.311)	0.397 (0.397)
Corr. Bonds	0.361	0.256 (0.256)	0.222 (0.222)	0.234 (0.234)	0.195 (0.195)	0.202 (0.202)	0.193 (0.193)	0.199 (0.199)	0.194 (0.194)	0.228 (0.228)
Corr. FoF	1.000	0.908 (0.908)	0.702 (0.699)	0.606 (0.604)	0.646 (0.650)	0.662 (0.664)	0.648 (0.651)	0.656 (0.659)	0.629 (0.630)	0.743 (0.739)
Turnover	0.000	6.083	28.717	449.425	64.252	147.607	60.710	147.370	86.623	195.804

Figure 5.5.20. Performance of historical portfolios with cost minimization and correlation constraints