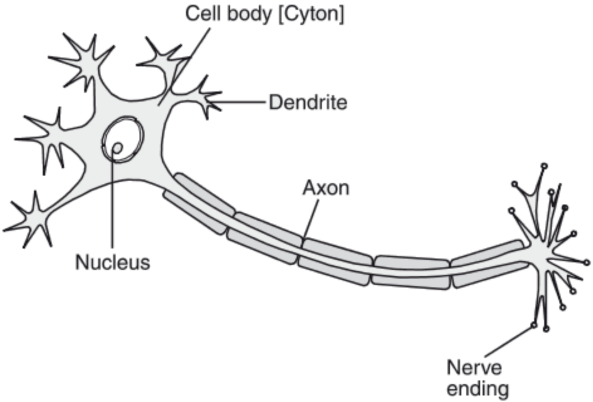
Introduction to Neural Networks

Gautom Das, Liam DeVoe, & Natnael Kelkay

Montgomery Blair High School

March 6, 2019

Introduction

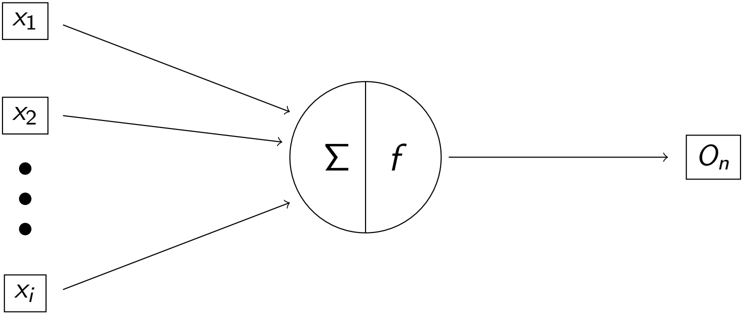


Theory

Let's start with a single neuron from the brain. Dendrites feed information into it, then there is the body of the neuron that performs some 'biology' operation on the inputs then outputs this information with its axons.

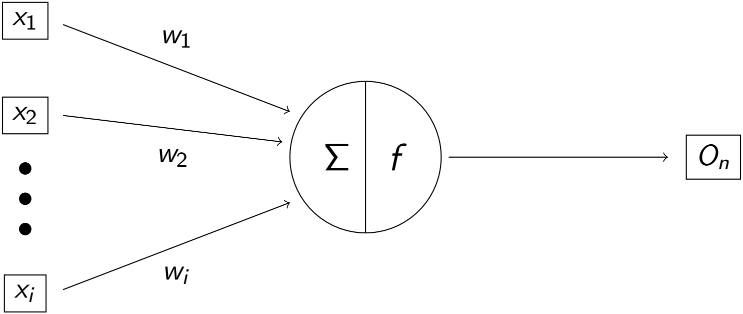
Single Neuron Construction

With this is in mind, let's try to model a single neuron mathematically:



Our input denoted by x_i have our inputs and then flow into the 'nucleus' of the neuron. There, a function is performed on all of the inputs and that information is then passed to another neuron or neurons.

Single Neuron Construction

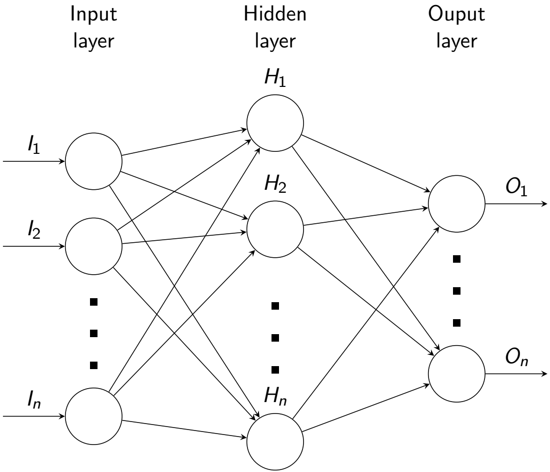


Neuron Formula

For a given neuron we can get the following formula:

$$O_n = \sigma\left(\sum_{i=1}^{n} x_i w_i + b\right)$$

Full Neural Network



Readable Mathematics

Let $X_1, X_2, \ldots, X_n$ be a sequence of independent and identically distributed random variables with $\mathrm{E}[X_i] = \mu$ and $\mathrm{Var}[X_i] = \sigma^2 < \infty$, and let

$$S_n = \frac{X_1 + X_2 + \cdots + X_n}{n} = \frac{1}{n}\sum_{i}^{n} X_i$$

denote their mean. Then as n approaches infinity, the random variables $\sqrt{n}(S_n - \mu)$ converge in distribution to a normal $\mathcal{N}(0, \sigma^2)$.