Lab 2: Image Statistics

due Feb. 15, 2016

Goal:

In this lab you will implement **multivariate image statistics** in MATLAB.

Instructions:

Your assignment is to write a MATLAB program to compute the covariance and correlation matrices for the `test_ms.img` image. This image is the multispectral (multiple band) version of the image you used in the first lab. While this is the image you (and I) will use for testing purposes, you should write your code to be generic enough to work with any image with little modification.

Recall that the **covariance** is defined as:

$$\mathrm{cov}_{kl} = \frac{\sum_{1}^{n} (BV_{ik} - \mu_k)(BV_{il} - \mu_l)}{n - 1} \qquad (1),$$

where BV_{ik} is the brightness of a given pixel i (at some row and column location, say, row a, column b) in band k; BV_{il} is the brightness of that same pixel i (at row a, column b) in band l; μ_k and μ_l are the means of bands k and l, respectively; and n is total the number of pixels (i.e. n = total number of rows x total number of columns).

Also recall that the **correlation coefficient** is defined as:

$$r_{kl} = \frac{\mathrm{cov}_{kl}}{\sigma_k \sigma_l} \qquad (2),$$

where σ_k and σ_l are the standard deviations of bands k and l, respectively.

Since they are multivariate statistics, both cov_{kl} and r_{kl} are defined for <u>pairs</u> of variables, in this case the brightness value of a pixel in one band vs. the brightness value of that <u>same</u> pixel in <u>another</u> band. In the case of `test_ms.img`, you have 9 bands, and there are a number of possible, unique band pairs you can make. The formula for determining how many **combinations** of q objects you can make out of a set of n total objects is:

$$C\left(\frac{n}{q}\right) = \frac{n!}{q!(n-q)!} \qquad (3),$$

For our purposes, n = 9 and q = 2, which leads to 36 possibilities. There will then be a total of 36 unique elements in each of the two resulting matrices. *How many total elements should these matrices have, though? Is that number also 36? Why or why not?*

Here is the covariance matrix for the case of 4 bands (*yours will be larger than this*):

$$\begin{matrix} \mathrm{cov}_{1,1} & \mathrm{cov}_{1,2} & \mathrm{cov}_{1,3} & \mathrm{cov}_{1,4} \\ \mathrm{cov}_{2,1} & \mathrm{cov}_{2,2} & \mathrm{cov}_{2,3} & \mathrm{cov}_{2,4} \\ \mathrm{cov}_{3,1} & \mathrm{cov}_{3,2} & \mathrm{cov}_{3,3} & \mathrm{cov}_{3,4} \\ \mathrm{cov}_{4,1} & \mathrm{cov}_{4,2} & \mathrm{cov}_{4,3} & \mathrm{cov}_{4,4} \end{matrix}$$

Note that $\mathrm{cov}_{i,j} = \mathrm{cov}_{j,i}$ and that the diagonal elements are the **variances**, or the squares of the **standard deviations**, for each band. *Why is that?*

Here is the correlation matrix, also for the case of just 4 bands (*yours will be larger than this*):

$$\begin{matrix} 1 & r_{1,2} & r_{1,3} & r_{1,4} \\ r_{2,1} & 1 & r_{2,3} & r_{2,4} \\ r_{3,1} & r_{3,2} & 1 & r_{3,4} \\ r_{4,1} & r_{4,2} & r_{4,3} & 1 \end{matrix}$$

Note that the diagonal elements are all 1, and that $r_{i,j} = r_{j,i}$. *Why is that?*

What to Turn In and How:

Turn in the `.m` file(s) for your MATLAB program(s). Do not turn in any input or output files, including images. Be sure to put your name in a comment at the top of any and all `.m` files you create. Also be sure to adequately comment your code so that I know what your programs do and how they work. You may use the built-in MATLAB functions for mean and standard deviation, but, obviously, you should not use the built-in MATLAB functions for covariance and correlation. Turn in your code via a folder named `lab2<yourname>` that you should place in the `DROPBOX` on `yowa.geo.utep.edu`.