

June 6 and 7, 2024



Brandenburgische
Technische Universität
Cottbus - Senftenberg

<https://www.b-tu.de/fg-algorithmische-mathematik/geometrietag-2024>

About

The Geometrietag is an annual geometry conference in Germany. Past editions were held in Chemnitz (2011, 2016), Leipzig (2012, 2023), Magdeburg (2007, 2013, 2017), Dresden (2009, 2014, 2018), and Jena (2008, 2010, 2015, 2019).

In 2024, the conference is held at the Zentralkampus of BTU Cottbus–Senftenberg on the occasion of Professor Horst Martini's 70th birthday.

Organizing committee

Gennadiy Averkov (BTU Cottbus–Senftenberg)

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General information

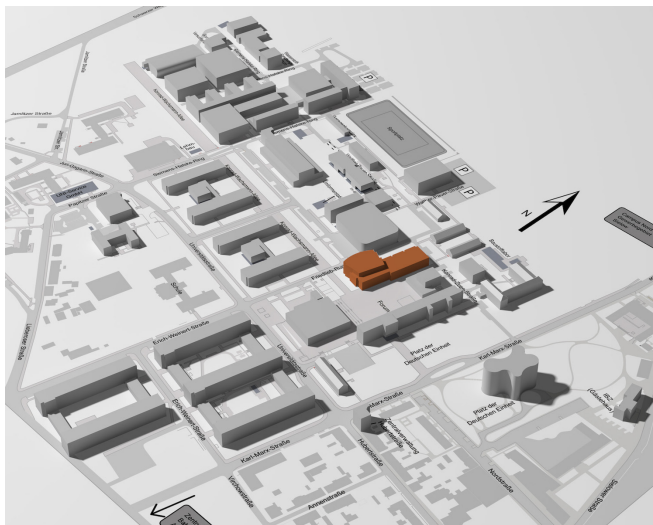
Talks will be held at the **Central Lecture Hall Building** (Zentrales Hörsaalgebäude, ZHG). It is situated at Konrad-Wachsmann-Allee 3, 03046 Cottbus.

Wi-Fi will be available during the conference. BTU also provides access to eduroam.

How to get to the BTU?

Berlin Brandenburg Airport (BER) is the best option to reach Cottbus by plane. From that airport you can reach Cottbus train station in at most 1.5 hours by train. Other airports in the nearby area are Airport Leipzig/Halle (LEJ) and Airport Dresden (DRS). From these airports the train connections take longer and are more expensive than from Berlin.

The university campus can be reached easily via public transport in less than 15 minutes.



Schedule

Thursday, June 6

8:30	Registration
8:50	Welcome remarks
9:00	Convex polytopes: examples and counterexamples, problems and conjectures Günter M. Ziegler
10:05	Looking back as a geometer Horst Martini
	Coffee
11:30	Geometric algebra for sets with betweenness relations Walter Wenzel
12:00	Optimal containment in sparse approximation and information-based complexity Thomas Jahn
	Lunch
14:30	Highly symmetric fundamental domains for point lattices Dirk Frettlöh
15:00	Self-affinity of discs under glass-cut dissections Christian Richter
15:30	Minkowski measurability for self-similar fractals Uta Freiberg
	Coffee
16:30	From the question how to properly define the diameter in generalized Minkowski spaces to relating means of convex bodies Bernardo González Merino
17:25	Mean inequalities for symmetrizations of convex sets Katherina von Dichter
17:55	General properties of (r, D, R)-Blaschke-Santaló diagrams in arbitrary Minkowski spaces Mia Runge
19:30	Conference Dinner

Friday, June 7

9:00	Totally separable packings of translates of a convex body Konrad Swanepoel
	Coffee
10:30	The geometry of the Vandermonde map Sebastian Debus
11:00	Two numerical approaches to the Blaschke-Lebesgue problem in three dimensions Gerd Wachsmuth
11:30	A 4-dimensional peabody of constant width Luis Montejano
12:00	Conclusion

List of Abstracts

Thursday, June 6

Convex polytopes: examples and counterexamples, problems and conjectures

Günter M. Ziegler

Freie Universität Berlin, Germany

My plan is to talk about some of my favourite (open and solved) polytope problems and on this occasion discuss

- beautiful conjectures (e.g. by Gil Kalai)
- the value of examples (among them the 720-cell and the Klee-Minty cubes)
- our limited tool-box of construction methods
- the value of counter-examples (e.g. to conjectures by Gil Kalai)
- my class of counter-examples (“to nearly everything”) in polytope theory.

Looking back as a geometer

Horst Martini

University of Technology Chemnitz, Germany

In this lecture, a “retired geometer” will give a review on essential steps of his professional life. The partial headlines of the talk are the following ones:

- 1) Starting point: Classical curves in illumination geometry
- 2) Unfading elementary geometry
- 3) Meeting Donald Coxeter
- 4) Editor-in-Chief of a journal
- 5) Branko Grünbaum: Geometry strikes again
- 6) Convex, discrete, and Minkowski geometry
- 7) History of mathematics
- 8) Popularization of mathematics

Geometric algebra for sets with betweenness relations

Jürgen Jost, Walter Wenzel

Max Planck Institute for Mathematics in the Sciences, Leipzig, Germany

Andreas Dress and the second named author introduced the concept of the *Tutte group of a matroid* that enables an algebraic approach particularly to those matroids which are not representable over any field. This – abelian – group is defined in terms of generators and relations.

Something very similar has now been done for metric spaces – or even more general sets with a betweenness relation: Given such a betweenness relation on a set E , a certain abelian group in terms of generators and relations is investigated. – Due to the analogy to the Tutte group on the one handside and, particularly, to the work of Andreas Dress in the field of metric spaces on the other handside, we have named this group the *Dress group* – after our friend and mentor Andreas Dress. This group is in particular important for *tripod spaces*; these are metric spaces where for any three points there is a fourth one, not necessarily distinct from the others – a *median* – that lies in the interval determined by any two of the given points.

Optimal containment in sparse approximation and information-based complexity

Thomas Jahn

Catholic University of Eichstätt-Ingolstadt, Germany

A set $A \subset \mathbb{R}^d$ is said to be optimally contained in the set $B \subset \mathbb{R}^d$ if $A \subseteq B$ but for all $\varepsilon \in (0, 1)$ and $x \in \mathbb{R}^d$, we have $A \not\subseteq x + \varepsilon B$. In this talk, we discuss two optimal containment situations in approximation theory. Approximation theory problems are usually about functions: How well can functions be approximated by “sparse elements”? How well can we recover a function from a finite number of function values? By considering spaces of functions given by basis expansions with certain summability conditions on their coefficients, the problems can be translated to sequence spaces. For finite sequences, some of these questions turn into situations of optimal containment of starshaped and convex sets.

This is joint work with Tino Ullrich and Felix Voigtlaender.

Highly symmetric fundamental domains for point lattices

Dirk Frettlöh

Bielefeld University, Germany

A point lattice L in \mathbb{R}^d is the span of d linearly independent vectors in \mathbb{R}^d . The point group of L is the group of symmetries of L that fixes the origin. Considering L as a group of translations, L has a fundamental domain (not necessarily unique). Trivially, each point lattice has a fundamental domain whose symmetry group equals the point group of the lattice. We show that all lattices in \mathbb{R}^2 (and most lattices in \mathbb{R}^3) have a fundamental domain with symmetry group strictly larger than their point group.

Self-affinity of discs under glass-cut dissections

Christian Richter

Friedrich Schiller University Jena, Germany

A topological disc is called n -self-affine if it has a dissection into n affine images of itself. It is called n -gc-self-affine if the dissection is obtained by successive glass-cuts, which are cuts along segments splitting one disc into two. For every $n \geq 2$, we characterize all n -gc-self-affine discs. All such discs turn out to be either triangles or convex quadrangles. All triangles and trapezoids are n -gc-self-affine for every n . Non-trapezoidal quadrangles are not n -gc-self-affine for even n . They are n -gc-self-affine for every odd $n \geq 7$, and they are n -gc-self-affine for $n = 5$ if they aren't affine kites. Only four one-parameter families of quadrangles turn out to be 3-gc-self-affine.

Minkowski measurability of self similar fractals

Uta Freiberg

Technische Universität Chemnitz, Germany

With the help of renewal theory one can show that self-similar fractals (satisfying the OSC) are Minkowski measurable in the so-called non-lattice case. The converse is still open for fractals embedded into Euclidean spaces with dimension greater than one. We sketch the history of this problem and give a family of examples supporting the conjecture.

This is a joint ongoing project with Jonas Lippold (TU Chemnitz) and Steffen Winter (KIT).

From the question how to properly define the diameter in generalized Minkowski spaces to relating means of convex bodies

Bernardo González Merino

Universidad de Murcia, Spain

The aim of this talk is to explain to the audience the path we followed from a question asked to us by, amongst others, Horst and Thomas in 2015: Why do we insist on using the notion of diameter in generalized Minkowski spaces given by Danzer, Grünbaum, and Klee (1963) and not the standard one.

This question led us to study relations between different means of convex bodies. To do so, we made use of the Asymmetry measure of Minkowski, i.e. the smallest rescaling factor of K such that it contains a translation of $-K$, allowing us to do a meaningful comparison between those means.

Finally, we will also connect this topic to some open questions in convex geometry, some of them now partly solved due to those techniques.

Joint work with René Brandenberg and Katherina von Dichter. —

Mean inequalities for symmetrizations of convex sets

Katherina von Dichter

Technical University of Munich, Germany

The arithmetic-harmonic mean inequality can be generalized for convex sets, considering the intersection, the harmonic and the arithmetic mean, as well as the convex hull of two convex sets. We study those relations of symmetrization of convex sets, i.e., dealing with the means of some convex set C and $-C$. We determine the dilatation factors, depending on the asymmetry of C , to reverse the containments between any of those symmetrizations, and tighten the relations proven by Firey and show a stability result concerning those factors near the simplex.

General properties of (r, D, R) -Blaschke-Santaló diagrams in arbitrary Minkowski spaces

Mia Runge

Technical University Munich, Germany

We study Blaschke-Santaló diagrams for the inradius, circumradius and diameter in general Minkowski spaces. Independent of the gauge, they can be described by at most five parts of boundaries. We analyse which bodies fill these bounds and for which gauges they become redundant. Furthermore, we give a complete description of the union over all these diagrams with respect to planar symmetric gauges (solving an open problem stated by Brandenburg and González Merino in a recent paper) by providing a new inequality that tightens Bohnenblust's bound. This union is equal to the union over all diagrams with respect to intersections of triangles with their origin reflection.

Joint work with René Brandenburg and Bernardo González Merino.

Friday, June 7

Totally separable packings of translates of a convex body

Konrad Swanepoel

London School of Economics and Political Science, United Kingdom

A packing of translates of a convex body is called totally separable if any two translates can be separated by a hyperplane that does not intersect the interior of any translate of the packing. This notion was introduced by Gábor Fejes Tóth and László Fejes Tóth in 1973, and studied mostly by considering the density of such packings. More recently, Károly Bezdek and others considered the combinatorial properties of the contact graphs of totally separable packings. In a contact graph of a packing, the vertices are the translates, and two vertices are joined when the two translates touch. We will discuss the maximum degree (Hadwiger number), minimum degree, and total number of edges (contact number) of these graphs. In particular, we completely settle the problem of determining the maximum number of edges of a contact graph of a totally separable packing of n translates of a convex disc in the plane, for all convex discs. An important tool in this work is the notion of angular measure in a normed plane.

This is joint work with Márton Naszódi.

The geometry of the Vandermonde map

Sebastian Debus

University of Technology Chemnitz, Germany

In this talk we, consider the Vandermonde cell, i.e. the image of the power sum map restricted to the probability simplex. The image arises naturally in various contexts. We show that the Vandermonde cell has the combinatorial structure of a cyclic polytope. We can understand how the image behaves by letting the number of variables go to infinity. We also consider the convex hull of the image and show some connection to sums of squares and nonnegative polynomials.

This is based on joint work with J. Acevedo, G. Blekherman and C. Riener.

Two numerical approaches to the Blaschke–Lebesgue problem in three dimensions

Gerd Wachsmuth

Brandenburgische Technische Universität Cottbus-Senftenberg, Germany

The Blaschke–Lebesgue problem in three dimensions asks for the body of minimal volume given the minimal width. This problem is currently unsolved and the common belief is that the minimizers are the two Meissner bodies. In the talk, we present two numerical approaches to attack the problem. In the first approach, we approximate the body as an intersection of halfspaces. In the second approach, we minimize the volume among Reuleaux polyhedra of a fixed combinatorial structure. In both approaches, our solutions “converge” towards the Meissner bodies.

A 4-dimensional peabody of constant width

Luis Montejano

Universidad Nacional Autónoma de México, Mexico

The purpose of this talk is to describe a 4-dimensional body of constant width, obtained from the 5-simplex Reuleaux body by replacing a small neighborhood of its 2-skeleton with sections of a 4-ball envelope.

As far as we have news, besides the obvious rotational bodies of constant width, this is the only concrete body of constant width that is known in dimension 4. Behind its construction lies the classical notion of focal quadrics discussed, for example, by Hilbert and the notion of Steiner chain of balls.

List of Participants

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Sebastian Debus	TU Chemnitz, Germany
Katherina von Dichter	TU Munich, Germany
Uta Freiberg	TU Chemnitz, Germany
Dirk Frettlöh	Bielefeld University, Germany
Bernardo González Merino	Universidad de Murcia, Spain
Kurt Klement Gottwald	TU Chemnitz, Germany
Christoph Helmberg	TU Chemnitz, Germany
Martin Henk	TU Berlin, Germany
Thomas Jahn	KU Eichstätt–Ingolstadt, Germany
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