1 I figured out how to make Latex pretty

1. Prove $\sum_{i=0}^{n} i2^i = (n-1)2^{n+1} + 2$

We can prove this by induction
Base Case: when $n = 0$ the left side is $0 * 2^0$ which is equivalent to 0 When we plug 0 into the right side we get $(0-1)2^{0+1} + 2$ which is also equivalent to 0 so our base case holds

Induction Step: Now we want to show it holds for $n+1$. Assume it holds for n

$\sum_{i=0}^{n+1} i2^i = (\sum_{i=0}^{n} i2^i) + (n+1)2^{n+1}$
$= (n-1)2^{n+1} + 2 + (n+1)2^{n+1}$

$= 2^{n+1}(n+1+n-1) + 2$

$= 2^{n+1}(2n) + 2$

$= 2^{n+2}(n) + 2$

Thus by the principle of induction the proof it is true for all n

2. Prove that if n^2 is even then n is even

We can prove this through the contradiction method
Taking the NOT of the first expression we get: n is not even, meaning n is odd

This means there exists an integer k such that $n = 2k + 1$

Squaring both sides leads to $n^2 = (2k+1)^2 = 4k^2 + 4k + 1$

which can be rewritten as $n^2 = 2(2k^2 + 2k) + 1$

This statement says $n^2 = 2x + 1$ where $x = 2k^2 + 2k$ proving n^2 is odd. The contradiction to the hypothesis proves that n^2 is even (3). The Diagrams are inside a Separate PDF. It made it 1000x easier. The PDF is called automata.pdf (4). Find the closed form solution

(a). $T(n) = 4T\left(\frac{n}{2}\right) + n$

$a = 4, b = 2$ and $f(n) = n$

$n^{log_b a} = n^{log_4 2}$

$f(n) = n = O(n^{2-\epsilon})$ letting $\epsilon = .5$ This satisfies the first condition of the masters method

thus $T(n) = \Theta(n^{\log_b a}) = \Theta(n^2)$

(b). $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

$a = 4, b = 2$ and $f(n) = n^2$

$f(n) = \Theta(n^{log_b a})$

then by the second condition of the masters theorem

$T(N) = n^{\log_b a} \log n = n^2 \log n$

(c). $4T\left(\frac{n}{2}\right) + n^2 \log n$

Master theorem doesn't work here :(So lets expand the recursion a little bit.

$= 16T(n/2^2) + 4(\frac{n}{2})^2 \log \frac{n}{2} + n^2 \log n$

We can simplify this statement a little bit. Also the base of the log is 2

$$= 16T(n/4) + 4(\frac{n^2}{4}) \log \frac{n}{2} + n^2 \log n$$
$$= 4^{\log n} + (n)^2 \log \frac{n}{2} + n^2 \log n$$
$$= 4^{\log n} + (n^2)(\log n + \log \frac{n}{2} + ... + \log \frac{n}{2^{\log n}})$$

By the property of logs this statement can be rewritten as (I'm not sure this is correct...)

$$= 4^{\log n} + (n^2) \log \left(n + \frac{n}{2} + \frac{n}{2^{logn}}\right)$$

Which can be simplified to

$$4^{\log n} + (n^2) \log \left(\frac{n^{\log n + 1}}{2^{\frac{\log n + 1}{2}}}\right)$$

$$= n^2 + n^2 \log(n^{\frac{\log n + 1}{2}})$$

$$= n^2 \cdot n^2 \frac{\log n}{2} \cdot \log n$$

Therefore $T(n) = \Theta(n^2 \log^2 n)$ I spent an outrageous amount of time on this

I have no idea if this is correct but this confused the day lights out of me.

(5). Use generating functions to find the closed form solution of $\sum_{i=1}^{n} a^i$
I think we can

$\sum_{i=1}^{n} a^i - x \sum_{i=1}^{n} a^i$

$= (1 + x + x^2 ...) - (x + x^2 + x^3)$

which should $= 1$?

So this implies

$(1 - x) \sum_{i=1}^{n} a^i = 1$

So the closed form solution is $\dfrac{1}{1 - x}$