



Calculus 1 Formulas

Limits & Continuity

Limits

Suppose $f(x)$ is defined when x is near the number a (f is defined on some open interval that contains a , except possibly at a itself). Then we write

$$\lim_{x \rightarrow a} f(x) = L$$

and say “the limit of $f(x)$ as x approaches a equals L ” if we can make the values of $f(x)$ arbitrarily close to L (as close to L as we like) by taking x to be sufficiently close to a (on either side of a) but not equal to a .

One-sided limits

Left-hand limit (the limit as x approaches a from the left)

$$\lim_{x \rightarrow a^-} f(x) = L$$

Right-hand limit (the limits as x approaches a from the right)

$$\lim_{x \rightarrow a^+} f(x) = L$$

The general limit exists if and only if the left- and right-hand limits both exist and are equal to each other

$$\lim_{x \rightarrow a} f(x) = L \quad \text{only if} \quad \lim_{x \rightarrow a^-} f(x) = L \quad \text{and} \quad \lim_{x \rightarrow a^+} f(x) = L$$



Limit laws

$$\lim_{x \rightarrow a} c = c$$

$$\lim_{x \rightarrow a} [cf(x)] = c \lim_{x \rightarrow a} f(x)$$

$$\lim_{x \rightarrow a} x = a$$

$$\lim_{x \rightarrow a} x^n = a^n$$

$$\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$$

$$\lim_{x \rightarrow a} [f(x) - g(x)] = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$$

$$\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$$

$$\lim_{x \rightarrow a} \sqrt[n]{x} = \sqrt[n]{a}$$

$$\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)}$$

$$\lim_{x \rightarrow a} [f(x)]^n = \left[\lim_{x \rightarrow a} f(x) \right]^n$$

Trigonometric limits

$$\lim_{\theta \rightarrow 0} \sin \theta = 0$$

$$\lim_{\theta \rightarrow 0} \cos \theta = 1$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$



Solving limits with substitution

$$\lim_{x \rightarrow a} f(x) = f(a)$$

Squeeze theorem

If $f(x) \leq g(x) \leq h(x)$ when x is near a (except possibly at a itself) and if

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = L \text{ then } \lim_{x \rightarrow a} g(x) = L$$

Precise definition of the limit

Let f be a function defined on some open interval that contains the number a , except possibly at a itself. Then we say that the limit of $f(x)$ as x approaches a is L , and we write

$$\lim_{x \rightarrow a} f(x) = L$$

if for every $\epsilon > 0$ there is a $\delta > 0$ such that

$$\text{if } 0 < |x - a| < \delta \quad \text{then} \quad |f(x) - L| < \epsilon$$



Continuity

A function f is continuous at a number a if

$$\lim_{x \rightarrow a} f(x) = f(a)$$

One-sided continuity

A function f is continuous from the right at a number a if $\lim_{x \rightarrow a^+} f(x) = f(a)$

A function f is continuous from the left at a number a if $\lim_{x \rightarrow a^-} f(x) = f(a)$

Continuity of elementary functions

The following functions are continuous everywhere in their domain:

Polynomials

Rational functions

Trigonometric functions

Exponential functions

Root functions

Inverse trigonometric functions

Logarithmic functions



Continuity of composites

If g is continuous at a and f is continuous at $g(a)$, then the composite function $f \circ g$ given by $(f \circ g)(x) = f(g(x))$ is continuous at a .

Intermediate value theorem

Suppose that f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in (a, b) such that $f(c) = N$.



Derivatives

Definition of the derivative

The definition of the derivative, also called the difference quotient, is the slope of the tangent line, which is given by

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} \quad \text{or} \quad f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

Differentiability

- A function is differentiable at a if $f'(a)$ exists
- A function is differentiable on an open interval (a, b) if it is differentiable at every number in the interval
- If f is differentiable at a , then f is continuous at a

Derivative rules

Power rule	$\frac{d}{dx} (x^n) = nx^{n-1}$
Product rule	$\frac{d}{dx} [f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$
Quotient rule	$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$



Chain rule

$$\frac{d}{dx} [f(g(x))] = f'(g(x))g'(x)$$

Derivative of exponential functions

$$\frac{d}{dx} (e^x) = e^x \ln e = e^x(1) = e^x$$

$$\frac{d}{dx} (a^x) = a^x \ln a$$

Derivatives of logarithmic functions

$$\frac{d}{dx} (a \log_b x) = \frac{a}{x \ln b}$$

$$\frac{d}{dx} (a \ln x) = \frac{a}{x}$$

Derivatives of trig functions

$$\frac{d}{dx} [a \sin (f(x))] = a \cos (f(x)) \cdot f'(x)$$

$$\frac{d}{dx} [a \csc (f(x))] = -a \csc (f(x)) \cot (f(x)) \cdot f'(x)$$

$$\frac{d}{dx} [a \cos (f(x))] = -a \sin (f(x)) \cdot f'(x)$$

$$\frac{d}{dx} [a \sec (f(x))] = a \sec (f(x)) \tan (f(x)) \cdot f'(x)$$

$$\frac{d}{dx} [a \tan (f(x))] = a \sec^2 (f(x)) \cdot f'(x)$$

$$\frac{d}{dx} [a \cot (f(x))] = -a \csc^2 (f(x)) \cdot f'(x)$$

Derivatives of inverse trig functions

$$\frac{d}{dx} [a \sin^{-1} (f(x))] = \frac{a \cdot f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$\frac{d}{dx} [a \csc^{-1} (f(x))] = -\frac{a \cdot f'(x)}{|f(x)| \sqrt{[f(x)]^2 - 1}}$$



$$\frac{d}{dx} \left[a \cos^{-1} (f(x)) \right] = - \frac{a \cdot f'(x)}{\sqrt{1 - [f(x)]^2}} \quad \frac{d}{dx} \left[a \sec^{-1} (f(x)) \right] = \frac{a \cdot f'(x)}{|f(x)| \sqrt{[f(x)]^2 - 1}}$$

$$\frac{d}{dx} \left[a \tan^{-1} (f(x)) \right] = \frac{a \cdot f'(x)}{1 + [f(x)]^2} \quad \frac{d}{dx} \left[a \cot^{-1} (f(x)) \right] = - \frac{a \cdot f'(x)}{1 + [f(x)]^2}$$

Derivatives of hyperbolic trig functions

$$\frac{d}{dx} \left[a \sinh (f(x)) \right] = a \cosh (f(x)) \cdot f'(x) \quad \frac{d}{dx} \left[a \operatorname{csch}(f(x)) \right] = - a \operatorname{csch}(f(x)) \coth(f(x)) \cdot f'(x)$$

$$\frac{d}{dx} \left[a \cosh (f(x)) \right] = a \sinh (f(x)) \cdot f'(x) \quad \frac{d}{dx} \left[a \operatorname{sech}(f(x)) \right] = - a \tanh(f(x)) \operatorname{sech}(f(x)) \cdot f'(x)$$

$$\frac{d}{dx} \left[a \tanh (f(x)) \right] = a \operatorname{sech}^2(f(x)) \cdot f'(x) \quad \frac{d}{dx} \left[a \coth (f(x)) \right] = - a \operatorname{csch}^2(f(x)) \cdot f'(x)$$

Derivatives of inverse hyperbolic trig functions

$$y = \sinh^{-1} [g(x)] \quad y' = \frac{g'(x)}{\sqrt{[g(x)]^2 + 1}}$$

$$y = \cosh^{-1} [g(x)] \quad y' = \frac{g'(x)}{\sqrt{[g(x)]^2 - 1}} \quad g(x) > 1$$

$$y = \tanh^{-1} [g(x)] \quad y' = \frac{g'(x)}{1 - [g(x)]^2} \quad |g(x)| < 1$$



$$y = \coth^{-1} [g(x)] \qquad y' = \frac{g'(x)}{1 - [g(x)]^2} \qquad |g(x)| > 1$$

$$y = \operatorname{sech}^{-1} [g(x)] \qquad y' = -\frac{g'(x)}{g(x)\sqrt{1 - [g(x)]^2}} \qquad 0 < g(x) < 1$$

$$y = \operatorname{csch}^{-1} [g(x)] \qquad y' = -\frac{g'(x)}{|g(x)|\sqrt{[g(x)]^2 + 1}} \qquad g(x) \neq 0$$

Definitions of hyperbolic trig functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\operatorname{csch} x = \frac{1}{\sinh x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\coth x = \frac{\cosh x}{\sinh x}$$

Hyperbolic trig identities

$$\sinh(-x) = -\sinh x$$

$$\cosh(-x) = \cosh x$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$$

$$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$$



Applications of Derivatives

Linear approximation and linearization

The linear approximation, or tangent line approximation, of f at a is

$$f(x) \approx f(a) + f'(a)(x - a)$$

To find the linearization of f at a , evaluate at a specific point a .

$$L(x) = f(a) + f'(a)(x - a)$$



Optimization and graph sketching

$f(x)$	$f'(x)$	$f''(x)$	
Cubic (x^3)	Parabolic (x^2)	Linear (x)	
Increasing	Positive (above the x -axis)		
Decreasing	Negative (below the x -axis)		
Concave up	Increasing	Positive (above the x -axis)	
Concave down	Decreasing	Negative (below the x -axis)	
	Concave up	Increasing	
	Concave down	Decreasing	
Critical point (extrema)	0 (x intercepts)		
Inflection point	Critical point (extrema)	0 (x intercepts)	
	Inflection point	Critical point (extrema)	
$f(x)$ is...			
	$f'(x) = +$	$f'(x) = -$	$f'(x) = 0$
$f''(x) = +$	inc/concave up	dec/concave up	min/concave up
$f''(x) = -$	inc/concave down	dec/concave down	max/concave down
$f''(x) = 0$	inc/inflection point	dec/inflection point	possible inflection point



Rolle's theorem

If f is a function that satisfies these:

1. f is continuous on the closed interval $[a, b]$.
2. f is differentiable on the open interval (a, b) .
3. $f(a) = f(b)$

Then there's a number c in the interval (a, b) where $f'(c) = 0$.

In other words, there's a point c in the interval (a, b) at which the derivative of f is 0, which means the slope of the graph is 0, the tangent line is horizontal, the point $x = c$ represents a critical point, and the graph changes direction there.

Mean value theorem

If f satisfies these:

1. f is continuous on the closed interval $[a, b]$.
2. f is differentiable on the open interval (a, b) .

Then there is a number c in the interval (a, b) where

$$f'(c) = \frac{f(b) - f(a)}{b - a} \quad \text{or} \quad f(b) - f(a) = f'(c)(b - a)$$



Newton's method

If the n th approximation is x_n and $f'(x_n) \neq 0$, then the next approximation is given by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

L'Hospital's rule

If $h(x) = \frac{f(x)}{g(x)}$,

and if $\lim_{x \rightarrow a} \frac{f(a)}{g(a)}$ gives an indeterminate form like $0/0$ or ∞/∞ ,

then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$

if the limit on the right side exists (or is ∞ or $-\infty$). If the limit on the right side does not exist, continue applying L'Hospital's rule as many times as necessary until the limit on the right side gives a real number answer or ∞ or $-\infty$.



Position, velocity and acceleration

Position $s(t)$ is the position function of an object that moves in a straight line

Velocity $v(t)$ is the velocity of the object

$$v(t) = s'(t)$$

Acceleration $a(t)$ is the acceleration of the object

$$a(t) = v'(t) = s''(t)$$

Marginal cost, revenue and profit

Cost

$C(x)$ is the cost function (the cost of producing x units)

$C'(x)$ is the marginal cost function (the derivative of $C(x)$, which is the rate of change of C with respect to x)

Revenue

$R(x)$ is the revenue function if

$$R(x) = xp(x) \text{ where}$$

x is the number of units sold and

$p(x)$ is the price per unit (the demand function, or price function, which is a decreasing function of x)



$R'(x)$ is the marginal revenue function (the derivative of $R(x)$, which is the rate of change of R with respect to x)

Profit

$P(x)$ is the profit function if

$$P(x) = R(x) - C(x) \text{ where}$$

$R(x)$ is the revenue function and

$C(x)$ is the cost function

$P'(x)$ is the marginal profit function (the derivative of $P(x)$, which is the rate of change of P with respect to x)



