CPSC 3120 Design and Analysis of Algorithms Homework 1

Due date: March 15, 2021 by end of the day Please use latex or word to answer all assignment questions.

1. Prove that $\sum_{i=0}^{n} i2^i = (n-1)2^{n+1} + 2$.
2. Prove that if n^2 is even, n is even.
3. Provide a deterministic or a nondeterministic finite automata that accept the following languages over the alphabet $\Sigma = \{0, 1\}$:
   (a) The set of all strings in Σ^* ending in 00.
   (b) The set of all strings in Σ^* containing 001.
   (c) The set of all strings in Σ^* such that the fifth symbol from the end of the string is a 0.
   (d) The set of all strings in Σ^* such that every substring of length four contains at least three 1s.
4. For each of the following recurrences, find the closed-form solution. Feel free to use the Master Method, but before applying it, you should check to make sure that the conditions of the Master Theorem are satisfied.
   (a) $T(n) = 4T(\frac{n}{2}) + n$
   (b) $T(n) = 4T(\frac{n}{2}) + n^2$
   (c) $T(n) = 4T(\frac{n}{2}) + n^2 \lg n$
5. Use generating functions to find closed forms for$\sum_{i=1}^{n} a^i$.