THE MEASUREMENT OF COMET POSITIONS

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ABSTRACT

The procedure for measuring the position of a comet from a photograph is described. In view of a recent shortage of good position measurements, it is hoped that this article will encourage observers to undertake some measurements.

RÉSUMÉ

On décrit ci-dessous le procédé pour mesurer la position d'une comète à partir d'un cliché photographique. Étant donné un manque récent des bonnes mesures de position, on espère que cet article encouragera quelques observateurs à entreprendre de telles mesures.

In a recent IAU Circular, Marsden (1980) described the dearth of accurate observations of comet positions as having reached an "extremely precarious situation". This is a pity because many amateurs are perfectly able to measure good positions, given the right equipment and the know-how. Although the techniques have been well-known and routine for a long time and are described in several textbooks (e.g. Smart 1931), I thought it might be useful to describe how to do it for those who are new to it.

The items required are: a suitable photographic telescope; a measuring microscope; a good star atlas and catalogue; a programmable calculator or a computer. Not everyone will possess all of these, but many will have some of them, and the enterprising observer will be able to beg, borrow or steal the rest. If you can demonstrate seriousness of purpose, a local observatory or university might well allow access to materials you need.

The highest-precision astrometry is a task for the specialist with a large professional telescope designed for the purpose. With comets, however, the very highest precision is usually not possible, partly because comets are fuzzy objects, and partly because the brightest part is not necessarily the centre of mass. Therefore the precision that is possible comes within reach of the careful observer who is not a specialist in precise astrometry. This is not an invitation to sloppiness, of course. Anything less precise than about two seconds of arc is probably not useful except for rough positions in the first few nights following discovery of a new comet. With suitable equipment and care, a precision better than one second of arc should be aimed for.

This article will mainly discuss the calculations that are needed for reducing the measurements, but a few practical details are also mentioned.

The Telescope and Photograph. A sturdy, equatorially-mounted telescope, with a good sidereal drive and proper alignment of the polar axis, is essential. The field should be flat and free from distortion, and the images free from coma. (They should also be free from astigmatism – a fault usually due to incorrect collimation of the primary mirror, which can easily be corrected.)

The focal length is an important consideration, since this affects the *plate scale* and the size of the field. By *plate scale* is meant the number of minutes of arc on the sky that corresponds to 1 mm on the photograph. If the focal length of the telescope is f mm, then 1 mm on the photograph will correspond, roughly, to $3438/f$ minutes of arc. The use of the word "plate" suggests that the photograph is being made on a glass plate. This indeed should be the case for the highest-precision astrometry. However, I myself use film for comet work, and I seem to be able to get away with it. If you *can* use plate, it *is* better, but it seems not to be essential.

If you have a long focal length, the plate scale is large, and measurements can be made, in principle, with great precision. However, the fuzziness of a cometary image will limit the usefulness of a large plate scale, and there is the very likely possibility that, with a long focal length, your photograph may not include the very minimum of three identifiable comparison stars that are needed in the reduction. If you have a short focal length, there will be no difficulty in finding enough comparison stars, but the small plate scale will limit the precision of your measurement.

The telescope I use has a focal length of 500 mm and plate scale of 1 mm = $6\overset{'}{.}9$. It is a Schmidt, and the images are very sharp over a field of more than 5°. Consequently I virtually never have difficulty finding sufficient comparison stars. On the other hand the plate scale is very small, and 1 μm (0.001 mm) corresponds to $0\overset{''}{.}4$, which is the very limit of precision I could achieve under favorable circumstances. If I had a choice, I would prefer a rather longer focal length at the expense of a smaller field.

For pictorial photography intended to show a beautiful comet with a long tail streaming away from the sun, a long-exposure photograph in which the telescope is guided to follow the motion of the comet is required. This is precisely what is *not* wanted for astrometry. The shortest possible exposure in which the comet is just visible as a tiny image is what is needed.

Should the telescope be guided so as to follow the motion of the comet so that the stars will be trails on the photograph? Or should the telescope be held at sidereal rate, so that the stars are points and the comet is a short trail? If the comet is a very faint one, the question answers itself, for unless the telescope is guided so as to follow the motion of the comet you probably will not record the comet at all! On the other hand, if the comet is bright, only a very short exposure is necessary, and the telescope can be kept at sidereal rate. The trail of the comet then will be very short, possibly not detectable.

The time of mid-exposure should be recorded to the nearest second in Coordinated Univeral Time, and this is the time for which your observation will ultimately be reported. It is, of course, essential to check your clock against a time-signal.

The Measuring Microscope. Ideally, this should be a two-coordinate measuring-engine, but if you *can* find one you will have done better than I. The most likely type of measuring microscope that you will find will have a precision-screw for measuring motion in one direction only and it will probably have been intended for measuring spectra. If this is so, you will need to adapt the microscope so that it, or more likely the photograph, can be *moved* by means of a semi-precise screw at right angles to the precision-screw, even if this motion cannot be precisely *measured*.

I purchased a rotatable microscope stage with x- and y-motions as well as rotation, of the sort that can be seen in well-equipped biology departments. This has proved to be so extremely useful that I now cannot imagine measuring a photograph without it. I dare not even mention the price, however. It is probably beyond the purses of most amateurs, but it is worth mentioning in case you have any friendly contacts in biology. I should add that nothing on this rotatable microscope stage is actually used for *measurement*, but only to move from place to place on the photograph. All measurement is done with the single precision-screw on the main body of the instrument, to a precision of 0.001 mm.

It is sometimes thought to be a good procedure to make an enlarged print of the photograph and make measurements on the print. This is an absolute no-no. The measurements are made by microscope on the original negative photograph, which is illuminated from below by some system of substage illumination. A faint, fuzzy image of a comet on a photograph can be very difficult to see under the high magnification of the microscope. The visibility can be vastly improved by quite minor changes in the method of substage illumination, and it is well worthwhile to experiment with various methods.

The Star Atlas and Catalogue. The requirements of the atlas and catalogue are such that you should be able to identify several comparison stars with ease, and that the precise right ascensions, declinations *and proper motions* of the stars should be given. Depending on your plate scale the Bečvář or SAO Atlases (see reference list) are very suitable, while the SAO Catalog (see reference list) is ideal. The SAO Catalog gives the right ascensions and declinations of the stars, as they were at 1950.0, in 1950.0 coordinates. All your measurements, as well as your final reported position, will be in 1950.0 coordinates. The only occasion when you might have to worry about precession is when you are setting the circles of your telescope; but since you are photographing a fairly wide field this is probably not important.

Choosing the Comparison Stars. A minimum of three is essential, though more than three are better. I never use more than twelve, and it would be silly to do so. Typically I use between five and ten.

The comparison stars should be chosen so that they are symmetrically distributed around the comet in all directions, perhaps within one or two degrees of the comet and of each other. Do not use two stars that are extremely close to each other, or a star which is extremely close to the comet. Thus the two components of a double star are totally unsuitable.

If the field is small and sparsely populated it may be difficult to find even three stars that surround the comet symmetrically. Nevertheless the procedure of surrounding the comet symmetrically by comparison stars is not done merely for aesthetic purposes, but it is very important for the precision of your final position. Therefore every attempt should be made to choose the most suitable stars possible.

Bright stars are easy to identify, but they are very difficult to measure and may have large proper motions. Therefore choose faint stars with small images.

The Measurement. I clamp my photograph (which is on film) between two sheets of glass. One of these is a photographic plate on which has been photographed, much reduced, two fine black lines at right angles to each other. I try to align these roughly north-south and east-west, but this is not done exactly. The emulsion side of the star photograph goes right up against the two lines, which I shall refer to as the coordinate axes. It is convenient to take the positive y direction roughly north, and the positive x direction roughly east (increasing right ascension).

The requirements of the coordinate axes are that they should be (a) exceedingly thin, (b) very straight, and (c) exactly perpendicular to each other. Figure 1 shows the appearance of the field of view.

For each star and the comet measure the distance x, in mm to the nearest 0.001 mm, between the y axis and the star. The semi-precise motion in y mentioned earlier is necessary to move from one star to the next, but is not to be used for precise measurement unless a two-coordinate engine is being used. For each star a setting must be made on the y axis as well as on the star. All settings *must* be made from the same direction, to avoid backlash. There must be no parallax between the stars and the crosshairs in the eyepiece of the microscope. The motion of the precision-screw must be accurately parallel with the x-axis. This is one place where the rotatable microscope stage is very valuable.

Then, turn the photograph through exactly 90° (more use for the rotatable microscope stage!) and measure the y coordinates.

The best and most experienced measurers recommend that, in order to avoid

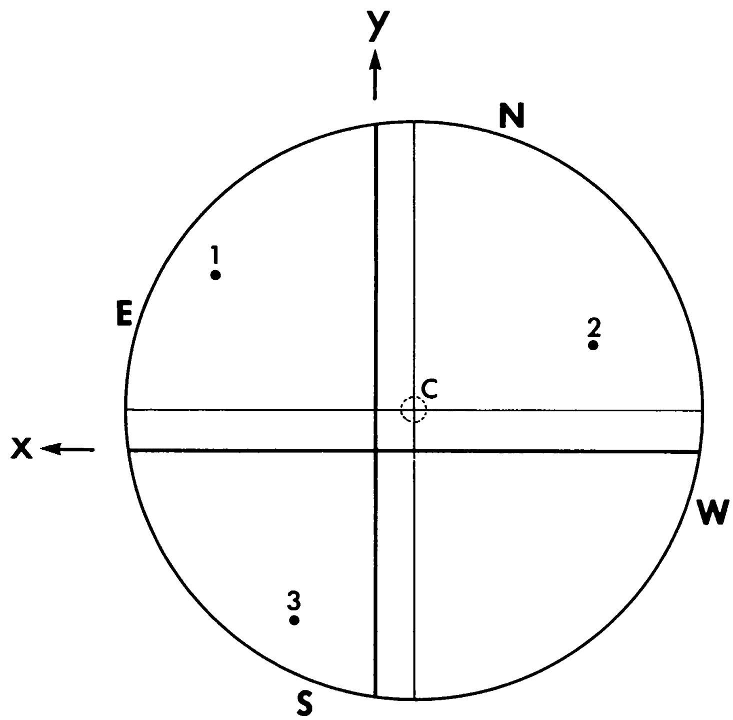

FIG. 1—Appearance of the field of view. The heavy lines are the x and y axes, which are inscribed on a sheet of glass or photographic plate as described in the text. The thin lines are the crosshairs in the microscope eyepiece; they always intersect at the centre of the field of view. Also seen are three comparison stars (1, 2, 3) and the comet (c). As illustrated, the microscope is set to measure the position of the comet, with the crosshairs bisecting the comet's image. During measurement the direction to north is not known precisely; the illustration shows the north-south direction to be not exactly parallel with the y-axis.

bias to one side in making a setting, the plate should be rotated a further 90° and the x-values re-measured in the opposite direction to before; and then a further 90° to re-measure the y-values.

The measurement is now complete. There is one other thing to do, however, before beginning the calculation. Estimate approximately from the star atlas (a few minutes of arc will do) the right ascension A and declination D of the origin of the coordinate axes.

Gathering the Data Together. You will need the following data.

1. The (A, D) of the origin of coordinates just mentioned.
2. For each star: (a) right ascension and declination (α, δ).
   (b) proper motions in α and δ (μ_α, μ_δ).
   (c) x and y coordinates.
3. For the comet: x and y coordinates.

4. Time of mid-exposure to nearest second.
5. Local sidereal time of the observation.
6. Your latitude.

You are now ready to calculate.

Proper Motion Corrections. The SAO catalogue gives the α and δ of each star as they were at the beginning of 1950. It also gives the annual proper motions. To find the α and δ at the time of your photograph, use

$$\alpha = \alpha_{1950.0} + \mu_\alpha(t - 1950.0) \quad (1)$$

$$\delta = \delta_{1950.0} + \mu_\delta(t - 1950.0). \quad (2)$$

Here t is the year (and decimal of a year if you wish) of your photograph; μ_α is the annual proper motion in α in seconds (s) per year; μ_δ is the annual proper motion in δ in seconds of arc ($''$) per year; $\alpha_{1950.0}$, $\delta_{1950.0}$ are the coordinates of the star as it was in 1950; α, δ are the required coordinates at the time of your photograph.

Determination of the Focal Length. The focal length must be determined for each photograph, since it will vary slightly with temperature, and indeed a focus adjustment must always be made to ensure a sharp photograph.

The focal length f in mm can be found from two stars by means of the formula

$$f = S/\theta, \quad (3)$$

where S is the distance in mm between the two stars on the photograph, and θ is the angular distance in radians between the two stars in the sky. The quantities S and θ are calculated from

$$S^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2, \quad (4)$$

$$\cos\theta = \sin\delta_1 \sin\delta_2 + \cos\delta_1 \cos\delta_2 \cos(\alpha_1 - \alpha_2). \quad (5)$$

In general I shall only state, but not prove, all formulas. Proofs can be found in standard texts such as that of Smart. The subscripts 1 and 2 refer to the two stars. Of course α is usually given in hours, minutes and seconds of time. It must be converted to degrees or radians, according to the calculator used. Conversion factors, of course, are

$$\pi \text{ radians} = 180° = 12^{\mathrm{h}}. \quad (6)$$

At this stage I usually calculate the focal length from each pair of stars taken in turn. Thus, if there are five stars, there are 10 (=4 + 3 + 2 + 1) determinations of focal length, and I can take the average. With a non-programmable calculator this is too tedious, but with a programmable calculator or a computer it is well worth while not so much because of increased precision but because I can immediately identify any "bad" stars ("bad" = misidentified, poor measurement, wrote down right ascension or declination wrongly, used two stars very close together, etc.), so it serves as an excellent method of checking for errors.

Now that the focal length f has been found, divide all the x and y measurements by f, so that from this point x and y are expressed not in millimetres but in units of the focal length.

Standard Coordinates. If the origin of the coordinate axes were precisely on the optic axis of the telescope, and if the coordinate axes were precisely parallel with right ascension and declination, we should be able to calculate the distances of each star from the two axes from their right ascensions and declinations (α, δ) and the right ascension and declination of the origin of coordinates (A, D). The geometry is a bit complicated, since a spherical sky is projected on a plane photograph, but the coordinates so calculated are called the standard coordinates (ξ, η) of each star. In units of the focal length they are given by

$$\xi = \frac{\sin(\alpha - A)}{\sin D \tan\delta + \cos D \cos(\alpha - A)}, \tag{7}$$

$$\eta = \frac{\tan\delta - \tan D \cos(\alpha - A)}{\tan D \tan\delta + \cos(\alpha - A)}. \tag{8}$$

These standard coordinates must be calculated for each star.

Plate Constants. The standard coordinates (ξ, η) of course are not the same as the measured coordinates (x, y) because neither of the conditions described at the beginning of the previous paragraph is satisfied. However, since (ξ, η) differ from (x, y) by only a translation and a rotation, they are linearly related. That is, the difference between ξ and x is given by

$$\xi - x = ax + by + c. \tag{9}$$

A similar relation exists for η and y:

$$\eta - y = dx + ey + f. \tag{10}$$

The constants a, b, c, d, e, f are the *plate constants* and they must now be determined. This is done by writing down equation (9) for each of the comparison stars. Let us assume for the present that there are just three comparison stars, so that we have the following three equations:

$$x_1 a + y_1 b + c = \xi_1 - x_1 \tag{9a}$$

$$x_2 a + y_2 b + c = \xi_2 - x_2 \tag{9b}$$

$$x_3 a + y_3 b + c = \xi_3 = x_3. \tag{9c}$$

These can be solved for a, b and c.

Similarly, d, e and f can be found from

$$x_1 d + y_1 e + f = \eta_1 - y_1 \tag{10a}$$

$$x_2 d + y_2 e + f = \eta_2 - y_2 \tag{10b}$$

$$x_3 d + y_3 e + f = \eta_3 - y_3. \tag{10c}$$

The Coordinates of the Comet. We already know the measured coordinates of the comet, and, now that we know the plate constants we can calculate from equations (9) and (10) its standard coordinates (ξ_0, η_0). The right ascension α_0 and declination δ_0 of the comet can now be calculated from

$$\tan(\alpha_0 - A) = \frac{\xi_0}{\cos D - \eta_0 \sin D} \tag{11}$$

$$\tan \delta_0 = \frac{(\eta_0 \cos D + \sin D)\sin(\alpha_0 - A)}{\xi_0}. \tag{12}$$

The basic calculation is now complete. Without a programmable calculator it may be rather tedious, but the possessor of one should be able to make short of it, and also make some of the refinements that follow.

More Than Three Comparison Stars. The use of more than three comparison stars not only improves the precision, but also serves as a most important check against mistakes. With only three comparison stars there is very little protection against mistakes.

If there are, for example, six comparison stars, there will be six equations of the type (9a)–(9c), and another six of the type (10a)–(10b). How do we handle six equations like (9a)–(9c) if there are only three unknowns, a, b and c? We shall not be able to find values of a, b and c that satisfy all six equations exactly, because of tiny errors in our measurements. What we have to do is to find values of a, b, c that best satisfy all six equations according to a criterion known as the principle of least squares. I shall stick to my policy in this paper of not supplying proofs, and go straight to the solution.

In brief, if there are n comparison stars, we have to calculate the following quantities, which are easily calculated on most modern scientific calculators:

$$A = \Sigma x_i^2 \qquad i = 1, 2, \ldots, n \tag{13}$$

$$B = \Sigma y_i^2 \tag{14}$$

$$C = \Sigma x_i y_i \tag{15}$$

$$D = \Sigma x_i \tag{16}$$

$$E = \Sigma y_i \tag{17}$$

$$L = \Sigma x_i \xi_i \tag{18}$$

$$M = \Sigma y_i \xi_i \tag{19}$$

$$N = \Sigma \xi_i \tag{20}$$

$$P = \Sigma x_i \eta_i \tag{21}$$

$$Q = \Sigma y_i \eta_i \tag{22}$$

$$R = \Sigma \eta_i \tag{23}$$

We then set up the following three equations (the "normal equations"):

$$A(1 + a) + Cb + Dc = L \tag{24}$$

$$C(1 + a) + Bb + Ec = M \tag{25}$$

$$D(1 + a) + Eb + nc = N \tag{26}$$

and solve them for a, b and c.

Likewise we set up the equations

$$Ad + C(1 + e) + Df = P \tag{27}$$

$$Cd + B(1 + e) + Ef = Q \tag{28}$$

$$Dd + E(1 + e) + nf = R \tag{29}$$

and solve them for d, e and f. We then proceed as before to determine the coordinates of the comet.

I have to admit that, for myself, I have programmed all of this for a computer, so that it just takes a few seconds. I must therefore be a little careful about telling an amateur armed only with a hand calculator how easy it all is. Nevertheless there is one step I always take, and which I regard as being of the utmost importance, even though it does double the amount of calculation. From the n comparison stars I choose one that lies inside the polygon formed by the remaining $n - 1$ and I pretend it is the "unknown". I then use the $n - 1$ stars to calculate the α and δ of the "unknown" star, and I compare these with the catalogue α and δ (duly corrected, of course, for proper motion). If this is correct within the likely errors of measurement (i.e. a couple of micrometres or so in my case) it is certain that no mistake has been made in the data concerning any of the comparison stars, or in measurement or in calculation. If it is not correct, it is equally certain that there *is* a mistake somewhere. The computer programme also includes some tests to determine where the mistake is if there is one. The easiest way is to look at the calculations of the focal length mentioned earlier. A "bad" star will show up there.

Differential Refraction. For the very highest-precision astrometric measurements there are a number of further refinements that could be made, but for a fuzzy object like a comet the game may not be worth the candle. The correction for differential refraction, however, should be made, especially as comets are often photographed at rather low altitudes, where refraction is rather severe. The refraction itself is not the trouble; rather it is the *differential* refraction – the fact that the refraction near the bottom of the photograph is greater than the refraction near the top.

The exact amount of refraction by the earth's atmosphere of course depends on atmospheric conditions and it is fairly complicated to determine. However, to the precision with which most cometary images can be measured, the difference in refraction between the top and bottom of the photograph can usually be calculated in a relatively simple matter. The way I do it is as follows.

First I calculate the "true" zenith distance z and the azimuth A for each star – that is, the zenith distance and azimuth the star would have if the earth had no atmosphere. These are calculated from

$$\cos z = \sin \phi \sin \delta + \cos \phi \cos \delta \cos H \tag{30}$$

$$\tan A = \frac{\sin H}{\cos \phi \tan \delta - \sin \phi \cos H}. \tag{31}$$

In these equations ϕ is the observer's latitude, and H is the hour angle of the star, which is the local sidereal time minus the right ascension.

Next, I calculate the "apparent" zenith distance ζ of each star as it appears in the sky after refraction by the earth's atmosphere. This is calculated from

$$\zeta = z - 58''.2 \tan \zeta. \tag{32}$$

(Since ζ appears on the right hand side, the equation is solved by iteration.)

The apparent declination δ', apparent hour angle H' and apparent right ascension α' of each star are now calculated from

$$\sin \delta' = \sin \phi \cos \zeta + \cos \phi \sin \zeta \cos A, \tag{33}$$

$$\tan H' = \frac{\sin A \tan \zeta}{\cos \phi - \sin \phi \cos A \tan \zeta}, \tag{34}$$

the apparent right ascension α' being the local sidereal time minus H'.

I usually like to re-calculate the azimuth of each star at this stage using the formula

$$\sin A = \frac{\cos \delta' \sin H'}{\sin \zeta}. \tag{35}$$

Since refraction does not affect the azimuth of a star this should give the same result as equation (31), and it serves as a most useful check against mistakes.

Having found α' and δ' for each of the comparison stars, I then calculate the plate constants as before, and hence get the right ascension and declination of the comet. But this, of course, gives only the *apparent* right ascension α_0' and *apparent* declination δ_0'. We have to reverse the refraction calculation in order to find our final values for α_0 and δ_0.

To do this, calculate the apparent zenith distance ζ_0 and azimuth A_0 of the comet from

$$\cos \zeta_0 = \sin \phi \sin \delta_0' + \cos \phi \cos \delta_0' \cos H_0', \tag{36}$$

and

$$\tan A_0 = \frac{\sin H_0'}{\cos \phi \tan \delta_0' - \sin \phi \cos H_0'}. \tag{37}$$

Here H_0' is the apparent hour angle of the comet, which is the local sidereal time minus its apparent right ascension α_0'. Then calculate the true zenith distance of the comet, z_0:

$$z_0 = \zeta_0 + 58''.2 \tan \zeta_0. \tag{38}$$

Finally calculate its true declination δ_0:

$$\sin \delta_0 = \sin \phi \cos z_0 + \cos \phi \sin z_0 \cos A_0 \tag{39}$$

and the true hour angle H_0:

$$\tan H_0 = \frac{\sin A_0 \tan z_0}{\cos \phi - \sin \phi \cos A_0 \tan z_0}. \tag{40}$$

The true right ascension α_0 is the local sidereal time minus H_0. A check against errors is afforded by recalculating the azimuth from

$$\sin A_0 = \frac{\cos \delta_0 \sin H_0}{\sin z_0}. \tag{41}$$

This should give the same answer as equation (37).

Review. I have described the several stages of the calculation in the order in which they can be most easily understood. This is not quite the same order in which they are performed, so let us review the entire calculation.

1. Correct the star positions for proper motions (equations (1) and (2)).

2. Calculate the apparent coordinates (α', δ') of each star by applying the refraction corrections (equations (30)–(35)).

3. Determine the focal length from each pair of stars in turn (equations (3)–(6)) and express the x and y measurements in units of f.

4. Calculate the standard coordinates (ξ, η) of the comparison stars (equations (7)–(8)). Note that in equations (7) and (8) the apparent right ascensions α' and declinations δ' should be used on the right hand side. (The distinction between α,

δ and α', δ' had not been made when we first met these equations.) It is not necessary to make a refraction correction for the origin of coordinates A, D.

5. Calculate the quantities A–R (equations (13)–(23)), and solve equations (24)–(29) for the plate constants a–f.

6. Calculate the standard coordinates of the comet (equations (9)–(10)) and hence calculate its right ascension and declination (equations (11)–(12)). At this stage what has actually been calculated are the apparent α and δ of the comet, i.e. α_0', δ_0'. The primes do not appear in equations (11)–(12), since the distinction had not been made when these equations were first encountered.

7. Calculate the true right ascension α_0 and declination δ_0 of the comet (equations (39) and (40)).

8. Repeat the process using a test star rather than the comet, in order to check for mistakes. (See discussion following equation (29)).

It is worth mentioning again that your calculator will probably be working either in degrees or in radians, whereas right ascension, hour angle and local sidereal time are commonly expressed in hours of time. Suitable care must be taken throughout the calculation to use the appropriate units (equation (6)).

I hope that this account will encourage observers who may not have done so before to try their hand at measuring comet positions. But I also hope that anyone who tries this will bear in mind that no mistakes are permitted! This is why stage (8) above is most vitally important. Reliable position measurements are a valuable contribution, and much enhance the enjoyment of observing, while modern calculators make light work of the mathematics. I would suggest that those who would like to do some of this work give themselves a fair bit of practice on test stars to begin with, to iron out the early mistakes that are bound to occur and to give confidence and assurance of accuracy when it comes to the real thing.

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