Further Mathematics, Signals and Systems:

Curve Sketching

This week's tutorial consists of some questions involving drawing graphs by hand. The remainder of in-class time can be used to revise material from Laplace Transforms.

The appendix includes a computer-based investigation that you can conduct outside of class. I recommend the "mupad" plug-in for MATLAB, accessed by typing `mupad` in the command line, as it is a more intuitive package with the capabilities of MATLAB and displays output in a way that is useful to us.

In all cases, use radians and not degrees.

Drawing General Waveforms

By hand, plot the following trigonometric functions:

1. $y = \frac{1}{2}\cos(\pi t)$
2. $y = 15\sin\left(x+2\right)$
3. $y = -0.2\cos\left(2t-\pi\right)$
4. $y = 3\sin\left(3t+\frac{\pi}{4}\right) - 2$

Computer Investigation:

To use mupad for plotting $y = \sin(x+3)$ and $y = 2\sin(5x - \pi)$ on the same graph for $0 < x < 2\pi$:

```
[ y1:=sin(x+3)
[ y2:=2*sin(5*x-PI)
[ plotfunc2d(y1,y2,x=0..2*PI)
```

If instead you wish to use regular MATLAB, for example to plot $y = \sin(x)$, the commands used are slightly different:

```
>> x=linspace(0,4*pi);
>> y=sin(x);
>> plot(x,y);
```

This produces an approximation to the graph $y = \sin(x)$. In particular, the `linspace` command generates 100 evenly-spaced points between $[0, 4\pi]$, then the y-values at each of those points is calculated, and the plot function attempts to "join the dots". If the function was of a high-enough frequency, this could result in a misleading graph. You can use the command `hold on;` to plot multiple functions on the same image, as otherwise each new use of `plot` will replace the previous graph.

1. The goal of this excerise is to help you intuitively understand the role of each constant $(\alpha, \beta, \gamma, \delta)$ in the graph of $f(x) = \alpha \sin(\beta x + \gamma) + \delta$. For a range of values from $[0, 4\pi]$ radians, plot the graphs of:

   (a) $f(x) = \sin(x)$
   (b) $g(x) = \sin(2x)$
   (c) $b(x) = \sin(2x + \pi)$
   (d) $c(x) = \sin(2(x + \pi))$
   (e) $z(x) = 13\sin(x)$
   (f) $h(x) = \sin(x - 3)$
   (g) $u(x) = \sin(x + 3)$
   (h) $y(x) = \sin(x) + 4$
   (i) $v(x) = 3\sin(2x)$
   (j) $s(x) = \sin(x + \frac{1}{2}) - 19$
   (k) $t(x) = 7\sin(x + \frac{4}{3})$
   (l) $p(x) = 13\sin(x) + 2$
   (m) $q(x) = 13(\sin(x) + 2)$
   (n) $s(x) = \sin(\frac{x}{\pi} + 2)$
   (o) $m(x) = \sin(\frac{x}{\pi}) + 2$
   (p) $n(x) = 13\sin(\frac{x}{\pi}) + 2$

   It may help to plot some of the related functions (e.g. $z(x)$ and $p(x)$) on the same image in order to observe the effect of the change. (Using mupad's `plotfunc2d()` function will label each automatically.)

   In your own words, describe the effect (including direction of transformation when relevant) of:

   - The amplitude α.
   - The angular frequency β.
   - The phase angle γ.
   - The vertical shift δ.

2. For a range of values from $[0, 4\pi]$ radians, plot the graphs of:

   (a) $y(x) = \sin(x) + \frac{1}{3}\sin(3x)$
   (b) $y(x) = \sin(x) + \frac{1}{3}\sin(3x) + \frac{1}{5}\sin(5x)$
   (c) $y(x) = \sin(x) + \frac{1}{3}\sin(3x) + \frac{1}{5}\sin(5x) + \frac{1}{7}\sin(7x)$
   (d) $y(x) = \sin(x) + \frac{1}{3}\sin(3x) + \frac{1}{5}\sin(5x) + \frac{1}{7}\sin(7x) + \frac{1}{9}\sin(9x)$
   (e) $y(x) = \sin(x) + \frac{1}{3}\sin(3x) + \frac{1}{5}\sin(5x) + \frac{1}{7}\sin(7x) + \frac{1}{9}\sin(9x) + \frac{1}{11}\sin(11x)$

   As you can see, this series of sin terms of increasing frequency appears to be converging on a square wave. This is the fundamental idea behind Fourier series. In particular, the infinite series $\sum_{n\ odd} \frac{1}{n}\sin(nx)$ is the Fourier series expansion of the square wave with amplitude $\pi/4$ and period 2π.