Further Mathematics, Signals and Systems:

Supplementary Revision Questions

1. **Partial Fractions expansion:** This is a method of expanding fractions that have a polynomial on the denominator that can be factorised, and obtaining a linear combination of simpler fractions that are easier to work with. The form of the expansion depends on the nature of the fraction, but there are three main possibilities:

   - All linear terms in the denominator:
   $$\frac{7x-1}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3}$$
   - Repeated term in the denominator:
   $$\frac{2x^2+5x-13}{(x+5)(x-4)^2} = \frac{A}{x+5} + \frac{B}{x-4} + \frac{C}{(x-4)^2}$$
   - x^2 in the denominator (make sure to check for common factors on the numerator and denominator first):
   $$\frac{4x^2-7x+17}{(x-5)(x^2+7)} = \frac{A}{x-5} + \frac{Bx+C}{x^2+7}$$

   Find the partial fraction expansions of the following:

   (a)
   $$\frac{3x+5}{(x-3)(2x+1)}$$
   (b)
   $$\frac{1}{(x+4)(x-2)}$$
   (c)
   $$\frac{3x+1}{(x-1)^2(x+2)}$$
   (d)
   $$\frac{5x}{(x^2+x+1)(x-2)}$$

   Using partial fractions expansion, find:
   $$\int \frac{1}{x(x^2+1)}dx$$

2. **Integration by Parts:**

   Recall the formula for integration by parts:

$$\int u\frac{dv}{dt}dt = uv - \int v\frac{du}{dt}$$

   Using this method, integrate the following:

   (a)
$$\int x\cos(x)dx$$

   (b)
$$\int 5x\,\mathrm{e}^{2x}\,dx$$

   (c)
$$\int_0^1 x^2\,\mathrm{e}^x\,dx$$

   (d)
$$\int \mathrm{e}^x\sin(x)dx$$

3. **Computational practice:**

   Using MATLAB, EXCEL, or geogebra, plot the graphs of the following functions between $-2\pi < x < 4\pi$:

   (a)
$$f(x) = 5\sin(x)$$

   (b)
$$f(x) = 30\sin\left(5x - \frac{\pi}{4}\right)$$

   (c)
$$f(x) = \frac{x}{4}\,\mathrm{e}^{-3x}$$

   (d)
$$f(x) = \cos(2x)\,\mathrm{e}^{\frac{x}{4}}$$