Duke Kunshan University

MATH 302, Fall 2025

Prof. Georg May

Assignment 2

Due date for take-home problems: September 9, 2025, 11pm

Take-Home Assignments

Note:

- As usual, you can use the provided example codes as a starting point whenever you want.
- Unless otherwise instructed, use the double datatype for floating point numbers with C++.

Problem 1 - *10 Points*

Textbook Exercise Set 10.2, Problem 7 (b),(d). Plot the convergence curves (logarithmic y-axis).

Problem 2 - *12 Points*

Textbook Exercise Set 10.2, Problem 10. Convert the units to metric units before you do the exercise. Report the solution for parts (a) and (b), and plot the convergence curves (logarithmic y-axis).

Problem 3 - *12 Points*

Textbook Exercise Set 10.3, Problem 8. Make sure you plot the convergence curve (logarithmic y-axis).

Problem 4 - *14 Points*

Textbook, Exercise Set 6.1, Problem 10.

Problem 5 - *8 Points*

Bring the 3×3 Hilbert matrix to triangular form by using Gaussian Elimination (By hand, no pivoting!). Show the final result and the intermediate result, obtained after eliminating the non-zero entries below the diagonal in the first column.

Problem 6 - *14 Points*

Implement Gaussian elimination without pivoting in C++. Print out the triangular matrix you obtain for the 3×3 Hilbert matrix. Compare to the result from the previous problem.

Hint:

- Note that the algorithms in the Textbook and those derived in class assume that array indices start at 1. (E.g., Algorithm 2 in Lecture 2.2 for Gaussian Elimination.) For Python and C++, array indices start at zero by default. Be sure to take this into account when you implement the algorithms.
- For computational efficiency, the algorithm for Gaussian Elimination does not explicitly set the elements in the lower triangle to zero. Take this into account when printing the matrices. You can either print the upper triangle only, or explicitly set the elements in the lower triangle to zero

Problem 7 - *16 Points*

Use Gaussian elimination to solve a linear system with the 8×8 Hilbert matrix and manufactured solution $x = (1, \ldots, 1)^T$. This means, compute the right-hand-side $\mathbf{b} = H\mathbf{x}$, and then solve the linear system with that right-hand-side. Use

(a) single precision (i.e., the float datatype)

(b) double precision (i.e., the double datatype)

Only one implementation is necessary if you use templates, as discussed in the Lab. Measure the relative error

$$\frac{\|\mathbf{x} - \mathbf{x}_{exact}\|}{\|\mathbf{x}_{exact}\|} \tag{1}$$

for both cases.

Hint:

- You need to perform the elimination steps for both the matrix and the right-hand side. Here's a version of Algorithm 2 that includes the right-hand side

  for j=1,. . . , n-1 **do**
      for i=j+1,. . . ,n **do**
          $m_{ij} = \frac{a_{ij}}{a_{jj}}$
          for k=j+1,. . . , n **do**
              $a_{ik} \leftarrow a_{ik} - m_{ij} a_{jk}$
          end for
          $b_i \leftarrow b_i - m_{ij} b_1$
      end for
  end for

- You need to think of an algorithm for back substitution, i.e., for solving the triangular system, obtained after Gaussian elimination

Problem 8 - *14 Points*

A population of rodents, who live for three years has fecundity rates

Age	fecundity rate
1	0.5
2	1.7
3	0.7

The survival rates are 0.6 and 0.5 for 1 and 2-year old species, respectively. In year 2, the population of rodents aged $(1, 2, 3)$ is $(N_1, N_2, N_3) = (75, 18, 10)$.

(a) What was the population in year one?

(b) What will the population be in year three?

Hint: new species of age 1 are born at the fecundity rate for each age group. For example, the species of age 2 (N_2) contribute to new species of age 1 in the next year via $F_2 \times N_2$.

Total Points: 100