Engr 5011: Homework #1

Series solutions and Wronskians

Please scan the entire homework submission as a single pdf, with this page first, and the problems attached in order.

Name (print): ____________________ **TUID:** | | | | | | | | | |

1. Problem 1 score/comments:
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2. Problem 2 score/comments
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3. Problem 3 score/comments.
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4. Problem 4 score/comments
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Engr-5011: Homework #1 Problem #1

Assemble submission for this problem in the following order:
(1) this page on top, followed by
(2) your handwritten solution, followed by
(3) listing of your matlab scripts/functions,
(4) printout of matlab plotted results

Name (print): ______________________

Collaborants (print): ______________________

Documents or **resources used** (print citations here): ______________________

Solve :

$$\boxed{y'' + 10y = 0; \quad y(1) = 0; \quad y'(5) = 10}$$

using:

- the exact (closed form) solution, and,
- either the nested-series-solution method shown in class, or the matlab-specific method using `polyval`

Write both the closed-form solution and your summary of the series solution in the space below (derivations on attached pages).

Then use matlab to compare to the closed-form solution and the series solution, both plotted on $1 \leq x \leq 5$:

For the series-solution answer you will need to work out both the two basic solutions (the even and the odd functions) and their derivative functions as a series.
Create recursive nested algorithms (as demonstrated in class) or use `polyval` to compute each of these four functions.
You will need the series-solution derivative functions in a subsequent problem, and its easier to work out the concept on a simpler problem like this.

	Criteria	**Score range**	**(score)**
Technical:	Exact solution correct	[0,1]	
	Series solution for even, odd solutions correct	[0,1]	
	Computation of series solutions correct	[0,3]	
	derivatives of the series solution for even, odd solutions correct	[0,4]	
	Computation of the derivatives correct	[0,4]	
	Good agreement between the closed form and series solution	[0,4]	
Documentation:	Could be understood by a third party who is not in this course	[0,3]	
	Effective use of notation	[0,1]	
	Collaborants and/or resources properly acknowledged	[-5,0]	

Engr-5011: Homework #1 Problem #2

Assemble submission for this problem in the following order: (1) this page on top, followed by (2) your handwritten solutions

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Name (print): ______________________

Collaborants (print): ______________________

Documents or resources used (print citations here): ______________________

Derive the general solution (with two free constants) using the Wronskian 2^{nd}-order recipe.
Leave your solutions in integral form!

Write your result below and attach your written deriviations behind this page.

1. $\boxed{\left(3+2z-z^2\right)y''+(1-z)\,y'+n^2y=z^3;\ \ (n \text{ an integer})}$

2. $\boxed{y''+(e^x-1)y=e^{\frac{3x}{2}};\ \ (\text{Hint: try letting } \mu=e^{\frac{x}{2}})}$

	Criteria	Score range	(score)
Technical:	Problem 1 correct	[0,5]	
	Problem 2 correct	[0,5]	
Documentation:	Could be understood by a third party who is not in this course	[0,3]	
	Effective use of notation	[0,2]	
	Collaborants and/or resources properly acknowledged	[-5,0]	

Engr-5011: Homework #1 Problem #3

Assemble submission for this problem in the following order:
(1) this page on top, followed by
(2) your handwritten solution, followed by
(3) listing of your matlab scripts/functions,
(4) printout of matlab plotted results

Name (print): ______________________

Collaborants (print): ______________________

Documents or **resources used** (print citations here): ______________________

Solve the following two homogeneous differential equations by using

- the "bessel trick" solution, and
- the "series solution" method.

Write the summary of the bessel-trick solution and the series solution below, with derivations attached.

The formula for the derivative of any bessel function of order ν (you will need this for problem1) is:

$$\boxed{\frac{d}{dx}\left(Z_{\boldsymbol{\nu}}(x)\right) = \tfrac{1}{2}(Z_{\boldsymbol{\nu-1}}(x) - Z_{\boldsymbol{\nu+1}}(x))}$$

Plot both the bessel-trick solution and the series solution on the same plot on the interval $x \subseteq [1, 5]$

1. $\boxed{y'' + x^3 y = 0; \quad y(1) = 0; \quad y'(5) = -10 \quad \text{(this one is comparatively uncomplicated)}}$

2. $\boxed{xy'' + 3y' + 4xy = 0; \quad y(1) = -10; \quad y(5) = 4 \quad \text{(this one took me three attempts)}}$

	Criteria	**Score range**	**(score)**
Technical:	Closed-form solution problem 1 correct	[0,5]	
	Series solution problem 1 correct	[0,5]	
	Closed-form solution problem 2 correct	[0,5]	
	Series solution problem 2 correct	[0,20]	
Documentation:	Could be understood by a third party who is not in this course	[0,5]	
	Effective use of notation	[0,5]	
	Collaborants and/or resources properly acknowledged	[-10,0]	

Engr-5011: Homework #1 Problem #4

Assemble submission for this problem in the following order:
(1) this page on top, followed by
(2) your handwritten solution, followed by
(3) listing of your matlab scripts/functions,
(4) printout of matlab plotted results

Name (print): ______________________

Collaborants (print): ______________________

Documents or **resources used** (print citations here): ______________________

Solve the following two homogeneous differential equations by using the series solution method.
Note that the only difference in the two problems is the sign of the third term
(I recommend that you expand the series in $\mu = (1+x)$; explain why this in a good idea for this problem).

Write the solution summary here and attach your derivations to this sheet.
Follow your derivations with your matlab code for computing and plotting the solution on the interval $x \subseteq [0,5]$
For each problem report that maximum value of $y(x)$, and the minimum value of $y(x)$, on the interval $x \subseteq [0,5]$

1. $\boxed{(1+x)^2\, y'' + (x^2-1)y' - 12x^2 y = 0; \quad y(0) = 4; \quad y(5) = -15}$

2. $\boxed{(1+x)^2\, y'' + (x^2-1)y' + 12x^2 y = 0; \quad y(0) = 4; \quad y(5) = -15}$

	Criteria	Score range	(score)
Technical:	Problem 1 solution correct	[0,10]	
	Problem 2 solution correct	[0,10]	
	Max/min of two solution reported	[0,5]	
	Intersections of two solutions reported	[0,5]	
	Solutions plotted on $x \subseteq [0,5]$	[0,5]	
Documentation:	Could be understood by a third party who is not in this course	[0,5]	
	Effective use of notation	[0,5]	
	Collaborants and/or resources properly acknowledged	[-10,0]	