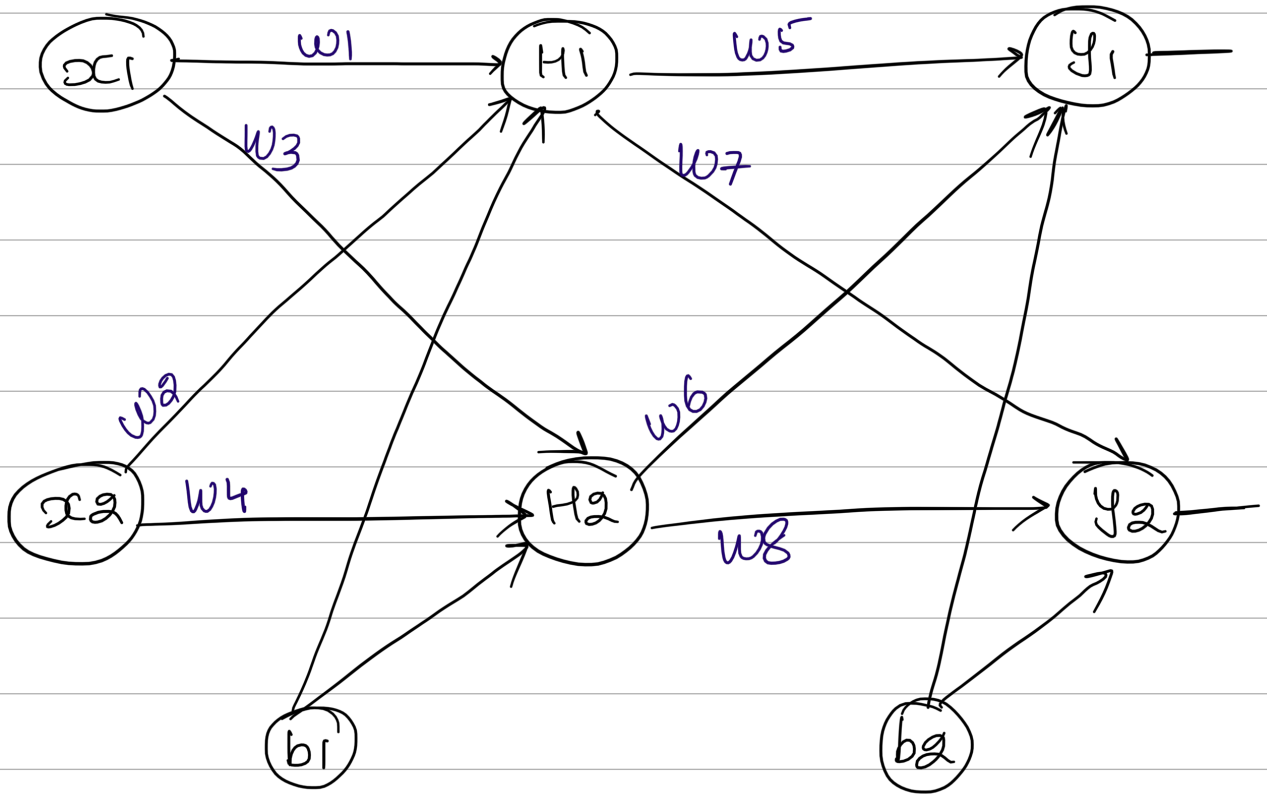
Backpropagation Example :-



$$H1 = x1w1 + x2w2 + b1$$

Activation function → sigmoid : $\frac{1}{1+e^{-x}}$

$$\text{At } H1 = \frac{1}{1+e^{-H1}} = \frac{1}{1+e^{-(x1w1+x2w2+b1)}}$$

Example :-

$x1 = 0.05$; $b1 = 0.35$

$x2 = 0.10$; $b2 = 0.60$

Initial weights :-

$w1 = 0.15$	$w5 = 0.40$	Target values
$w2 = 0.20$	$w6 = 0.45$	$T1 = 0.01$
$w3 = 0.25$	$w7 = 0.50$	$T2 = 0.99$
$w4 = 0.30$	$w8 = 0.55$	

FORWARD PROPAGATION :-

$$H1 = x1w1 + x2w2 + b1$$
$$= 0.05 * 0.15 + 0.10 * 0.20 + 0.35$$
$$= 0.3775$$

$$\text{out\_H1} = \frac{1}{1+e^{-H1}} = \frac{1}{1+e^{-0.3775}} = 0.593269992$$

(output of H1)

Similarly :- $\text{out\_H2} = \frac{1}{1+e^{-H2}}$

$$H2 = x1w3 + x2w4 + b1$$
$$H2 = 0.15 * 0.25 + 0.20 * 0.30 + 0.35$$
$$H2 = 0.4475$$
$$\text{Out\_H2} = 0.596884378$$

Now for Calculating y1

out_H1 is output of node H1 ; out_H2 is output of node H2

$y_1 = w5 * out\_H1 + out\_H2 * w6 + b2$

$y_1 = 0.4 * 0.593269992 + 0.596884378 * 0.45 + 0.60$

$y_1 = 1.105905967$

$out\_y_1 = \frac{1}{1+e^{-y_1}} = \frac{1}{1+e^{-(1.105905967)}} =$

0.75136507

Similarly :-

$out\_y_2 = 0.772928465$

$y_2 = w7 * out\_H1 + out\_H2 * w8 + b2$

$y_2 = 0.50 * 0.593269992 + 0.596884378 * 0.55 + 0.6$

$y_2 = 1.2249$

$out\_y_2 = \frac{1}{1+e^{-y_2}} = \frac{1}{1+e^{-(1.2249)}}$

CALCULATING TOTAL ERROR :-

Given $T1 = 0.01$

$T2 = 0.99$

$out\_y_1 = 0.75136507 \simeq 0.75$

$out\_y_2 = 0.772928465 \simeq 0.77$

Target value T1 does not match with out_y1

Target value T2 does not match with out_y2

Hence we calculate error:-

$$E_{Total} = \sum \frac{1}{2}(target\_value - output)^2$$

$$= \frac{1}{2}\left(\underbrace{[0.01 - 0.75]^2}_{E1} + \underbrace{[0.99 - 0.77]^2}_{E2}\right)$$

$$\left[\frac{1}{2}(T1 - out\_y1)^2 + \frac{1}{2}(T2 - out\_y2)^2\right] = E_{Total}$$

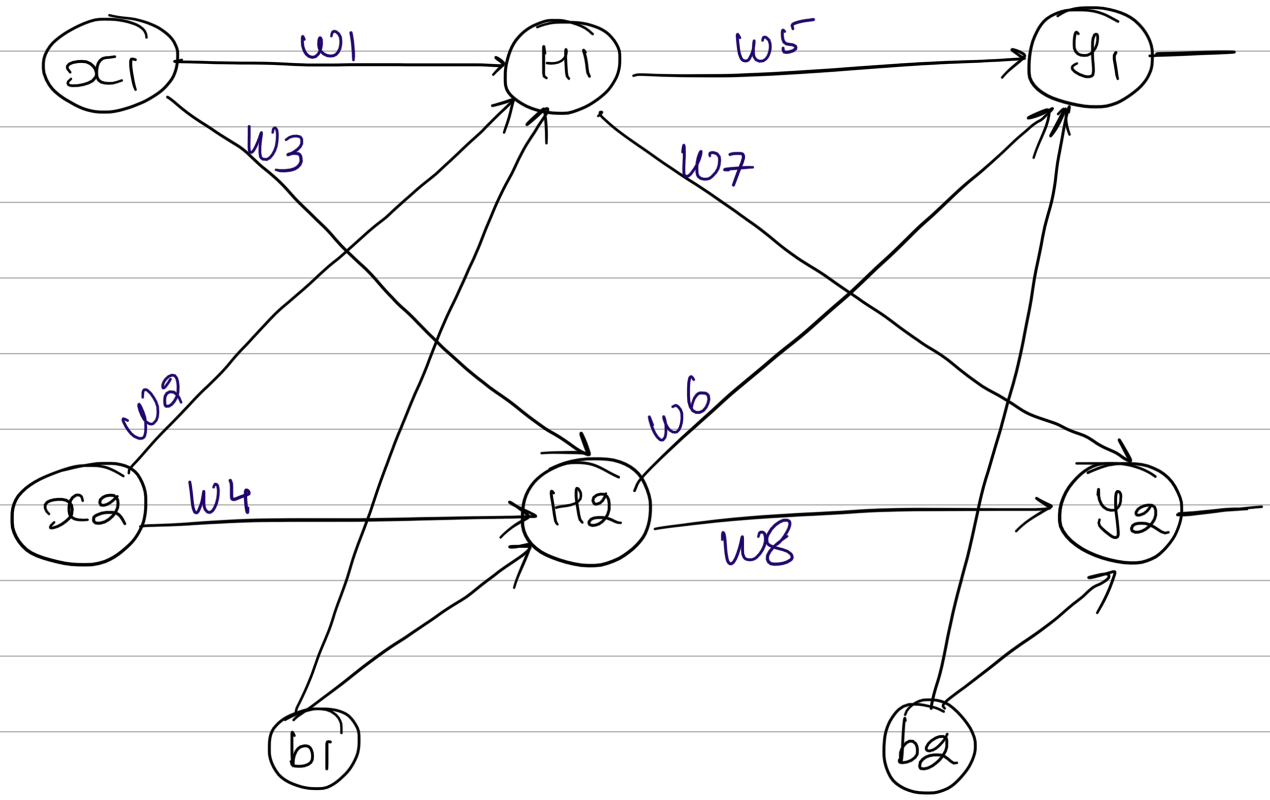
$$= 0.2748 + 0.0235 = 0.2983$$

$$E_{Total} = 0.2983$$

$$E1 = \frac{1}{2}(T1 - out\_y1)^2$$

$$E2 = \frac{1}{2}(T2 - out\_y2)^2$$

Backward Propagation



↳ Backpropagation is done to update weights

Consider $w5$

Error at $w5 = \frac{\partial E_{total}}{\partial w_5}$

$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial out_{y1}} \times \frac{\partial out_{y1}}{\partial y_1} \times \frac{\partial y_1}{\partial w_5}$$

w.k.t $E_{total} = \frac{1}{2}(T_1 - out\_y1)^2 + \frac{1}{2}(T2 - out\_y2)^2$

To solve $\frac{\partial E_{total}}{}$

$$\frac{\partial}{\partial w_5}$$

$$\frac{\partial E_{total}}{\partial out\_y1} = \frac{\partial}{\partial out\_y1}\left[\frac{1}{2}(T_1 - out\_y1)^2 + \frac{1}{2}(T2 - out\_y2)^2\right]$$

$$= \frac{1}{2}\left[2(T1 - out\_y1)(-1) + 0\right]$$

$$= -\left[(T_1 - out\_y1)\right] = -(0.01 - 0.75)$$
$$= 0.7413$$

$$\frac{\partial E_{total}}{\partial out\_y1} = 0.7413$$

$$out\_y1 = \frac{1}{1+e^{-y1}} \quad \text{as mentioned above}$$

$$\frac{\partial out\_y1}{\partial y1} = \frac{\partial}{\partial y1}\left[\frac{1}{1+e^{-y1}}\right]$$

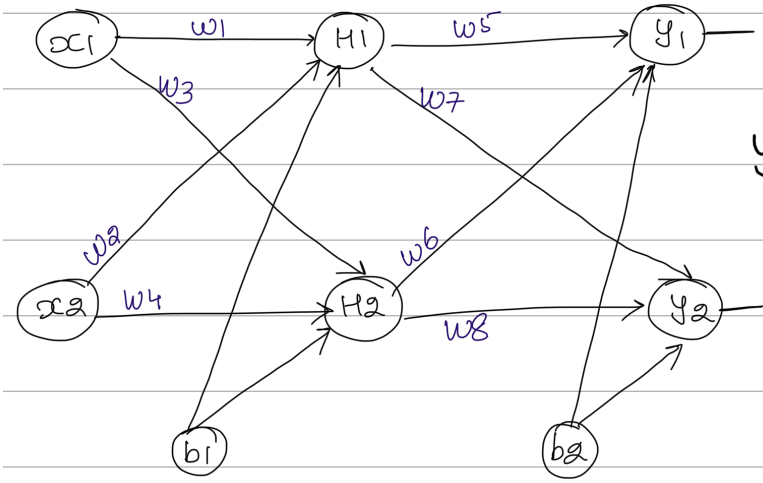
$$= \left[\frac{1}{1+e^{-y1}}\right]\left[1 - \frac{1}{1+e^{-y1}}\right]$$

$$= (out\_y1)(1 - out\_y1)$$

$$= 0.75\,(1 - 0.75) = 0.1868$$

$$\frac{\partial out\_y1}{\partial y_1} = 0.1868$$

$$\frac{\partial y_1}{\partial w_5} = \frac{\partial}{\partial w_5}\left[out\_H1 * w5 + out\_H2 * w6 + b2\right]$$



W.K.T

$y1 = out\_H1 * w5 + out\_H2 * w6 + b2$

mentioned earlier...

$$\frac{\partial y_1}{\partial w_5} = out\_H1(1) + 0 + 0 = out\_H1$$
$$= 0.593269992$$
$$\simeq 0.5932$$

$$\frac{\partial y_1}{\partial w_5} = 0.5932$$

Hence $$\frac{\partial E_{total}}{\partial w_5} = 0.7413 * 0.1868 * 0.5932$$
$$= 0.082167041$$
$$= 0.0821$$

Change in $w_5 = 0.0821$ where $0 < \eta < 1$

Updating w_5 :-

$$w_{5_{new}} = w_{5_{old}} - \eta \frac{\partial E_{total}}{\partial w_5}$$

$\eta \rightarrow$ learning rate $= 0.5$ (here)

$$w_{5_{new}} = 0.4 - 0.5(0.0821)$$
$$= 0.3589$$

$$w_{5_{new}} = 0.3589$$

Exercise :-

Calculate w_6, w_7, w_8 in similar fashion

check your Answers :-

$w_{6_{new}} = 0.4086$

$w_{7_{new}} = 0.5113$

$w_{8_{new}} = 0.5613$

} new updated weights

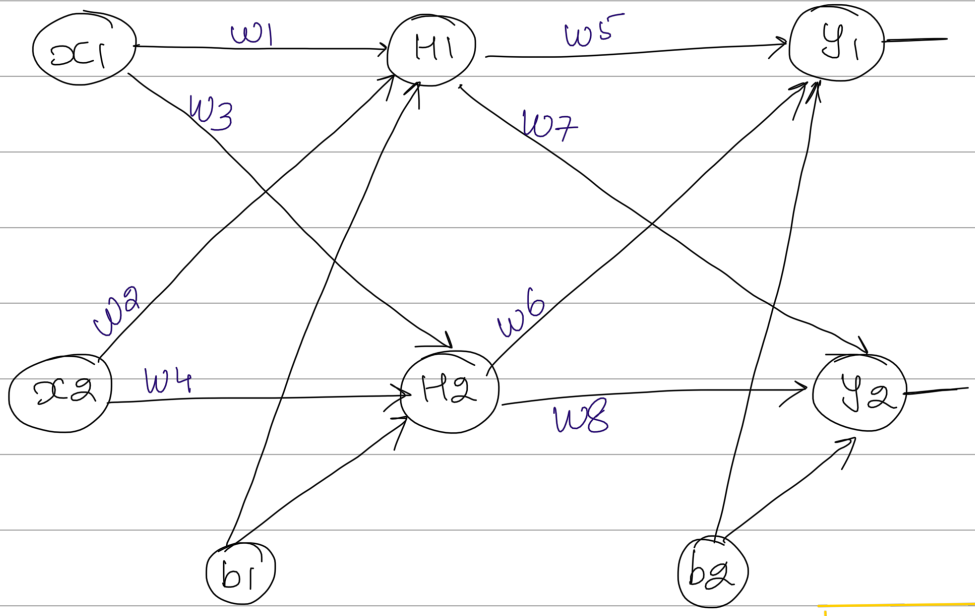
Updating w1, w2, w3, w4 weights :-

Now at hidden layer, updating w1, w2, w3, w4

$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out\_H1} * \frac{\partial out\_H1}{\partial H1} * \frac{\partial H1}{\partial w_1}$$

[since $E_{Total} = E_1 + E_2$]

$$\frac{\partial E_{total}}{\partial out\_H1} = \frac{\partial E_1}{\partial out\_H1} + \frac{\partial E_2}{\partial out\_H1} = \frac{\partial}{\partial out\_H1}[E1 + E2]$$



$$\frac{\partial E_1}{\partial out\_H1} = \frac{\partial E_1}{\partial y_1} * \frac{\partial y_1}{\partial out\_H1}$$

$$= 0.055399$$

Here

$$\frac{\partial E_1}{\partial out\_y_1} = (T_1 - out\_y_1)(-1)$$

$$= (0.01 - 0.75)(-1)$$

$$= 0.74$$

$$\frac{\partial E_1}{\partial y_1} = \frac{\partial E_1}{\partial out\_y_1} * \frac{\partial out\_y_1}{\partial y_1} = 0.7413 * 0.1868$$

$$= 0.1384$$

Target values

$T1 = 0.01$

$T2 = 0.99$

w.k.t $E_{total} = \frac{1}{2}(T_1 - out\_y1)^2 + \frac{1}{2}(T2 - out\_y2)^2$

$$\frac{\partial y_1}{\partial out\_H1} = w5(1) = 0.40$$

w.k.t:-

$y1 = w5 * out\_H1 + out\_H2 * w6 + b2$

$$\frac{\partial E2}{\partial out\_H1} = -0.0190$$

Exercise try deriving $\frac{\partial E2}{\partial out\_H1}$

$$\frac{\partial E_{total}}{\partial out\_H1} = 0.055399 + (-0.0190)$$

$$= 0.036399$$

Since:-

$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out\_H1} * \frac{\partial out\_H1}{\partial H1} * \frac{\partial H1}{\partial w_1}$$

computed above

w.k.t

$$out\_H1 = \frac{1}{1 + e^{-H1}}$$

$$\frac{\partial}{\partial H1}(out\_H1) = \frac{\partial}{\partial H1}\left[\frac{1}{1+e^{-H1}}\right] = out\_H1(1-out\_H1)$$

$$= 0.241300709$$

$$\simeq 0.2413$$

w.r.t

$$H1 = w1 * x1 + w2 * x2 + b1$$

$$\frac{\partial H1}{\partial w1} = \frac{\partial}{\partial w1}[w1x1 + w2x2 + b1] = x1$$

$$= 0.05$$

Since:-

$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out\_H1} * \frac{\partial out\_H1}{\partial H1} * \frac{\partial H1}{\partial w_1}$$

computed above | computed above | computed above

$$\frac{\partial E_{total}}{\partial w1} = 0.03635 * 0.241300 * 0.05$$

$$= 0.000043858$$

UPDATING w1

$$w1\,new = w1\,old - \eta \frac{\partial E_{total}}{\partial w1}$$

$$w1\,new = 0.15 - 0.5 * 0.000043858$$

$w_{1\,new} = 0.149780716$

$w_{1\,new} = 0.1497$

In the same way:-

$w_{2\,new} = 0.19956$

$w_{3\,new} = 0.24975$

$w_{4\,new} = 0.29950$

Exercise try to derive $w_{2\,new}$, $w_{3\,new}$, $w_{4\,new}$ and compare your answer