Student's Printed Name: ______________________________ **XID: C**______________

Instructor: ______________________________________ **Section:** ____________

No questions will be answered during this exam.

If you consider a question to be ambiguous, state your assumptions in the margin and do the best you can to provide the correct answer.

Instructions: You are **not** permitted to use a calculator on any portion of this test. You are **not** allowed to use a textbook, notes, cell phone, computer, or any other technology on any portion of this test. All devices must be turned off and stored away while you are in the testing room.

During this test, **any** kind of communication with any person other than the instructor or a designated proctor is understood to be a violation of academic integrity.

No part of this test may be removed from the examination room.

Read each question carefully. To receive full credit for the free response portion of the test, you must:

1. Show legible, logical, and relevant justification which supports your final answer.
2. Use complete and correct mathematical notation.
3. Include proper units wherever appropriate.
4. Give answers as exact values whenever possible.

You have **90 minutes** to complete the entire test.

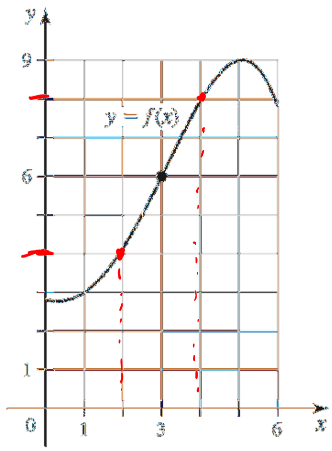
Do not write below this line.

Free Response Problem	**Possible Points**	**Points Earned**	**Free Response Problem**	**Possible Points**	**Points Earned**
1.	8		5.	9	
2.a.	5		6.a.	8	
2.b.	5		6.b.	4	
3.a.	6		7. (Scantron)	1	
3.b.	6		**Free Response**	64	
4.a.	4		**Multiple Choice**	36	
4.b.	4		**Test Total**	100	
4.c.	4				

Precalculus material in blue
calculus material in green

Multiple Choice: There are 12 multiple choice questions. Each question is worth 3 points and has one correct answer. The multiple choice problems will be 36% of the total grade. Circle your choice on your test paper. Any questions involving inverse trigonometric functions should be answered based on the domain restrictions for trigonometric functions used in Section 1.4.

1. The function f in the figure satisfies $\lim_{x\to 3} f(x) = 6$. Determine the largest value of $\delta > 0$ satisfying the statement, "If $0 < |x - 3| < \delta$, then $|f(x) - 6| < 2$."



$\delta = 1$

A) $\delta = 2$

B) $\delta = 0$

C) $\delta = 1$

D) $\delta = 3$

2. Assume the function g satisfies the inequality $a\cos(x-a) \le g(x) \le (x-a)^2 + a$ for all values of x near a where a is a positive constant. Find $\lim_{x\to a} g(x)$.

A) $\lim_{x\to a} g(x) = 0$

B) $\lim_{x\to a} g(x) = 1$

C) $\lim_{x\to a} g(x) = \infty$

D) $\lim_{x\to a} g(x) = a$

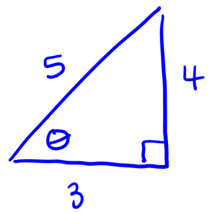
$\lim_{x\to a} a\cos(x-a) \le \lim_{x\to a} g(x) \le \lim_{x\to a} (x-a)^2 + a$

$a\cos(0) \le \lim_{x\to a} g(x) \le (0)^2 + a$

$a(1) \le \lim_{x\to a} g(x) \le a$

therefore $\lim_{x\to a} g(x) = a$ by squeeze theorem

3. Given $\sec\theta = \dfrac{5}{3}$ and $0 < \theta < \dfrac{\pi}{2}$, evaluate $\cos\theta$ and $\tan\theta$.



← found from Pythagorean theorem

we know everything is positive because $0 < \theta < \frac{\pi}{2}$

A) $\cos\theta = \dfrac{3}{5}$ and $\tan\theta = \dfrac{4}{3}$

B) $\cos\theta = \dfrac{3}{5}$ and $\tan\theta = \dfrac{3}{4}$

C) $\cos\theta = \dfrac{1}{5}$ and $\tan\theta = \dfrac{4}{5}$

D) $\cos\theta = \dfrac{4}{5}$ and $\tan\theta = \dfrac{3}{4}$

$\cos\theta = \frac{3}{5}$ $\quad$ $\tan\theta = \frac{4}{3}$

4. Given the following limits, determine the vertical asymptotes of f.

meaning of this is from precalculus

$$\lim_{x\to -3^-} f(x) = -\infty \qquad \lim_{x\to -3^+} f(x) = \infty \qquad \lim_{x\to e} f(x) = \infty$$

$$\lim_{x\to 2^-} f(x) = 1 \qquad \lim_{x\to 2^+} f(x) = 4$$

relationship between asymptotes & limits is Calculus but you'll see some limits in our class

A) $x = -3,\ x = e,\ x = 2$

B) $x = -3,\ x = e$

C) $x = 2$

D) $x = e$

5. Evaluate $\sin^{-1}\left(\dfrac{-1}{2}\right)$.

$\sin^{-1}\left(-\frac{1}{2}\right) = \theta$

$\sin\theta = -\frac{1}{2}$

so $\theta = -\frac{\pi}{6}$

this is a unit circle problem & we need to know

$-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$

A) $\sin^{-1}\left(\dfrac{-1}{2}\right) = \dfrac{-\pi}{6}$

B) $\sin^{-1}\left(\dfrac{-1}{2}\right) = \dfrac{-\pi}{3}$

C) $\sin^{-1}\left(\dfrac{-1}{2}\right) = \dfrac{4\pi}{3}$

D) $\sin^{-1}\left(\dfrac{-1}{2}\right) = \dfrac{7\pi}{6}$

6. Simplify the following expression, $\log\left(\frac{\sqrt[3]{x}}{7}\right)$.

A) $\frac{1}{3}\log(x)+\log(7)$

B) $3\log(x)-\log(7)$

C) $\frac{1}{3}\log(x)-\log(7)$

D) $\frac{\log(x)}{3\log(7)}$

$\log\frac{\sqrt[3]{x}}{7} = \log\sqrt[3]{x} - \log 7$

$= \frac{1}{3}\log x - \log 7$

7. Analyze the end behavior of $f(x)=\frac{2x}{\sqrt{x^2+2}}$. State any horizontal asymptotes.

A) f has no horizontal asymptote.

B) f has horizontal asymptotes: $y=-2,\ y=2$.

C) f has horizontal asymptote: $y=2$.

D) f has horizontal asymptotes: $y=\frac{-2}{\sqrt{3}},\ y=\frac{2}{\sqrt{3}}$.

$\lim_{x\to\infty}\frac{2x}{\sqrt{x^2+2}} = \lim_{x\to\infty}\frac{\frac{2x}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{2}{x^2}}}$

$= \lim_{x\to\infty}\frac{2}{\sqrt{1+\frac{2}{x^2}}}$

$= \frac{2}{\sqrt{1}}$

$= 2$

repeat process for $\lim_{x\to-\infty}\frac{2x}{\sqrt{x^2+2}}$

8. Solve the given equation for x.

$$8^{(x^2-4)}=1$$

A) $x=-2,\ 2$

B) $x=2$

C) $x=-\sqrt{4\ln(8)},\ \sqrt{4\ln(8)}$

D) $x=-2$

$\ln 8^{x^2-4} = \ln 1$

$(x^2-4)\ln 8 = 0$

$x^2 = 4$

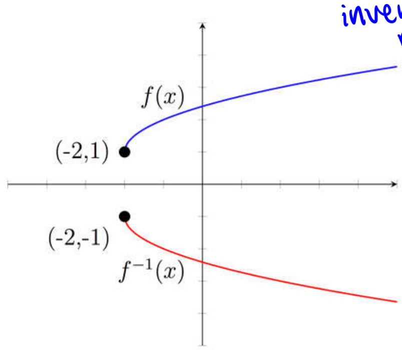
$x = \pm 2$

9. Choose the graph that shows the inverse of $f(x)=\sqrt{x+2}+1$.

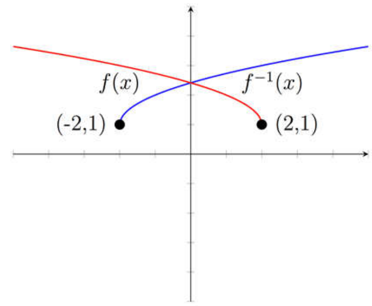
inverse will reflect across line y=x

You can also find the inverse & graph but it will take several steps

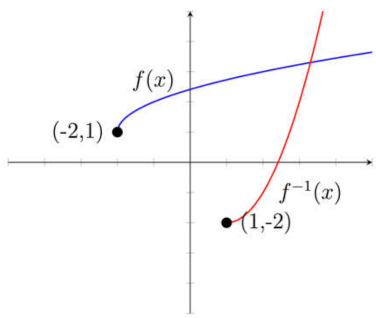
A)



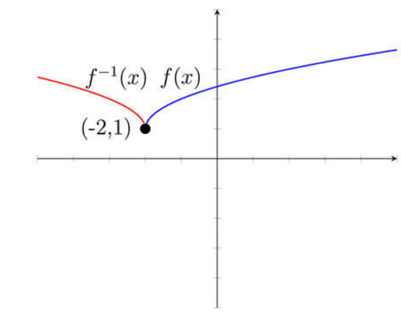
B)



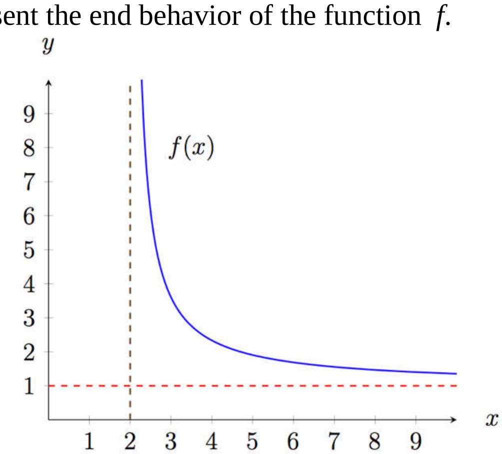
C)



D)



10. State the limits that represent the end behavior of the function f.



A) $\lim\limits_{x\to 1} f(x)=1,\ \lim\limits_{x\to 2^+} f(x)=2$

B) $\lim\limits_{x\to 1} f(x)=\infty,\ \lim\limits_{x\to 2^+} f(x)=\infty$

C) $\lim\limits_{x\to \infty} f(x)=1, \lim\limits_{x\to 2^+} f(x)=2$

D) $\lim\limits_{x\to \infty} f(x)=1,\ \lim\limits_{x\to 2^+} f(x)=\infty$

$\left.\begin{array}{l}\lim\limits_{x\to\infty} f(x)=1 \\ \lim\limits_{x\to 2^+} f(x)=\infty\end{array}\right\}$ limit stuff is calculus

but reading graphs is precalculus

11. The conditions for the Intermediate Value Theorem are:

- f is continuous on interval $[a, b]$
- L is a number strictly between $f(a)$ and $f(b)$

State the conclusion of the Intermediate Value Theorem.

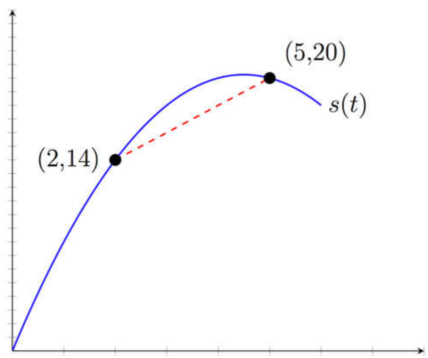
A) There exists at least one number c satisfying $f(c) = f(L)$.

B) There exists at least one number c in (a, b) satisfying $f(c) = L$.

C) There exists exactly one number c in (a, b) satisfying $f(c) = L$.

D) There exists exactly one number c in (a, b) satisfying $f(c) = 0$.

12. The graph gives the position function $s(t) = -t^2 + 9t$ of an object moving along a line at time t, where s is measured in feet and t is measured in seconds. Find the average velocity of the object over the time interval $[2,5]$.



A) $v_{av} = -2$ ft/s

$$\frac{20-14}{5-2} = \frac{6}{3} = 2$$

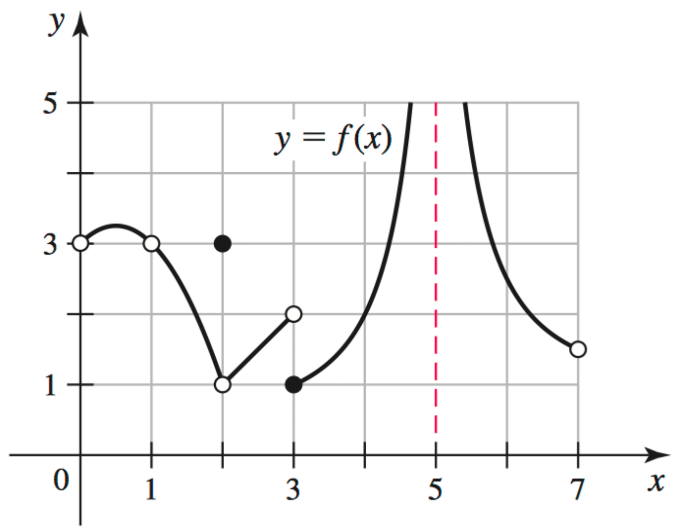
B) $v_{av} = \frac{34}{3}$ ft/s

C) $v_{av} = \frac{-34}{3}$ ft/s

D) $v_{av} = 2$ ft/s

Free Response: The Free Response questions will be 64% of the total grade. Read each question carefully. To receive full credit, you must show legible, logical, and relevant justification which supports your final answer. Give answers as exact values. Questions involving inverse trigonometric functions should be answered based on the domain restrictions in Section 1.4.

1. **(8 pts.)** Use the graph of $f(x)$ to evaluate the limits and answer the question.
Infinite limits should be answered with "$=\infty$" or "$=-\infty$", whichever is appropriate.
If the limit does not exist (and cannot be answered as ∞ or $-\infty$), state "DNE."



a. **(2 pts.)** $\lim_{x\to 2} f(x) = 1$

b. **(2 pts.)** $\lim_{x\to 3^-} f(x) = 2$

c. **(2 pts.)** $\lim_{x\to 3^+} f(x) = 1$

d. **(2 pts.)** $\lim_{x\to 5} f(x) = \infty$

2. **(10 pts.)** Find the following limits. Show all work. **Do not use L'Hopital's Rule.**
Infinite limits should be answered with "= ∞" or "= –∞", whichever is appropriate.
If the limit does not exist (and cannot be answered as ∞ or –∞), state "DNE."

a. **(5 pts.)** $\lim_{x\to 0} \frac{\ln a^x}{x^2 + ax}$ where a is a positive constant

$$\lim_{x\to 0} \frac{\ln a^x}{x^2+ax} = \lim_{x\to 0} \frac{x \ln a}{x^2+ax}$$

$$= \lim_{x\to 0} \frac{\ln a}{x+a}$$

$$= \frac{\ln a}{0+a}$$

$$= \frac{\ln a}{a}$$

b. **(5 pts.)** $\lim_{x\to\infty} \frac{x^2 - 4x + 3}{3 - x}$ **(Must show work consistent with limits at infinity.)**

$$\lim_{x\to\infty} \frac{x^2-4x+3}{3-x} = \lim_{x\to\infty} \frac{\frac{x^2}{x} - \frac{4x}{x} + \frac{3}{x}}{\frac{3}{x} - \frac{x}{x}}$$

$$= \lim_{x\to\infty} \frac{x - 4 + \frac{3}{x}}{\frac{3}{x} - 1}$$

$$= \frac{\infty - 4 + 0}{0 - 1}$$

$$= -\infty$$

OR

$$\lim_{x\to\infty} \frac{(x-3)(x-1)}{-(x-3)}$$

$$= \lim_{x\to\infty} \frac{x-1}{-1}$$

$$= -\infty$$

3. **(12 pts.)** Find the following limits. Show all work. **Do not use L'Hopital's Rule.**
Infinite limits should be answered with "= ∞" or "= –∞", whichever is appropriate.
If the limit does not exist (and cannot be answered as ∞ or –∞), state "DNE."

a. **(6 pts.)** $\lim_{x\to 3}\frac{\sqrt{2x^2+7}-5}{x-3}$

$$\lim_{x\to 3}\frac{\sqrt{2x^2+7}-5}{x-3} = \lim_{x\to 3}\frac{\sqrt{2x^2+7}-5}{x-3}\cdot\frac{\sqrt{2x^2+7}+5}{\sqrt{2x^2+7}+5}$$

$$= \lim_{x\to 3}\frac{2x^2+7-25}{(x-3)(\sqrt{2x^2+7}+5)}$$

$$= \lim_{x\to 3}\frac{2x^2-18}{(x-3)(\sqrt{2x^2+7}+5)}$$

$$= \lim_{x\to 3}\frac{2(x-3)(x+3)}{(x-3)(\sqrt{2x^2+7}+5)}$$

$$= \lim_{x\to 3}\frac{2(x+3)}{\sqrt{2x^2+7}+5}$$

$$= \frac{12}{10} = \frac{6}{5}$$

b. **(6 pts.)** $\lim_{x\to -\infty}\frac{17x}{\sqrt{3x^2+1}}$

$$\lim_{x\to -\infty}\frac{17x}{\sqrt{3x^2+1}} = \lim_{x\to -\infty}\frac{\frac{17x}{-x}}{\sqrt{\frac{3x^2}{x^2}+\frac{1}{x^2}}}$$

$$= \lim_{x\to -\infty}\frac{-17}{\sqrt{3+\frac{1}{x^2}}}$$

$$= \frac{-17}{\sqrt{3}}$$

4. **(12 pts.)**

Let $f(x)=\begin{cases} x^2+x & \text{if } x<1 \\ a & \text{if } x=1 \\ 3x+5 & \text{if } x>1 \end{cases}$

a. **(4 pts.)** Determine the value of a for which f is continuous from the left at 1.

$f(1)=a$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x^2+x) = 2$

So, $a=2$

b. **(4 pts.)** Determine the value of a for which f is continuous from the right at 1.

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (3x+5) = 8$

So, $a=8$

c. **(4 pts)** Is there a value of a for which f is continuous at 1? YES or NO
(Circle your answer.)

Explain.

$\lim_{x\to 1} f(x)$ DNE since $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$

5. **(9 pts.)** Use the delta-epsilon definition of a limit to prove $\lim_{x \to 2}(5x-6)=4$.

Let $f(x)=5x-6$.

$|f(x)-4|<\varepsilon$

$|5x-6-4|=|5x-10|=5|x-2|<\varepsilon$

so use $\delta=\frac{\varepsilon}{5}$

Let $\varepsilon>0$ be given.

Assume $0<|x-2|<\delta$, where $\delta=$ $\varepsilon/5$. (**Fill in the blank** with your choice of δ.)

(Complete your proof below.)

then $|f(x)-4| = |5x-6-4|$

$= |5x-10|$

$= 5|x-2|$

$< 5\delta$

$= 5\left(\frac{\varepsilon}{5}\right)$

$= \varepsilon$

therefore, $\lim_{x \to 2}(5x-6)=4$.

6. **(12 pts.)** Use the **limit definition** of the derivative at a point to find $f'(1)$ if $f(x) = 6x^2 - x + 2$
(You will receive no credit for using derivative theorems to find $f'(x)$.)

a. **(8 pts.)** Complete the limit definition of the derivative at a point.
Then, calculate the limit. Show all work.

$$f'(1) = \lim_{h\to 0}\left(\frac{f(1+h) - f(1)}{h}\right)$$

$$= \lim_{h\to 0} \frac{6(1+h)^2 - (1+h) + 2 - (6(1)^2 - (1) + 2)}{h}$$

$$= \lim_{h\to 0} \frac{6(1+2h+h^2) - 1 - h + 2 - 7}{h}$$

$$= \lim_{h\to 0} \frac{6 + 12h + 6h^2 - 1 - h + 2 - 7}{h}$$

$$= \lim_{h\to 0} \frac{12h + 6h^2 - h}{h}$$

$$= \lim_{h\to 0} (11 + 6h)$$

$$= 11$$

b. **(4 pts.)** Find the equation of the tangent line to $f(x) = 6x^2 - x + 2$ at $x = 1$.

$f'(1) = 11$

$f(1) = 6 - 1 + 2 = 7$

$y - 7 = 11(x - 1)$

Scantron **(1 pt.)**

Check to make sure your Scantron form meets the following criteria. If any of the items are NOT satisfied when your Scantron is handed in and/or when your Scantron is processed one point will be subtracted from your test total.

My Scantron:

- ◻ is bubbled with firm marks so that the form can be machine read;
- ◻ is not damaged and has no stray marks (the form can be machine read);
- ◻ has **12** bubbled in answers;
- ◻ has **MATH 1060** and my section number written at the top;
- ◻ has my instructor's last name written at the top;
- ◻ has Test No. **1** written at the top;
- ◻ has the correct test version written at the top **and** bubbled in below my XID;
- ◻ shows my correct XID both written and bubbled in;

  Bubble a zero for the leading C in your XID.

Please read and sign the honor pledge below.

On my honor, I have neither given nor received inappropriate or unauthorized information at any time before or during this test.

Student's Signature: ____