

$$(\sin^2 - 1)(\sin^2 - 1)$$

$$\sin^4 - 2\sin^2 + 1$$

$$\frac{x^2 - 4x - 77}{(x^2 - 4x + 4) - 73}$$

$$\int \frac{\cos^3 \theta}{\sin \theta} d\theta$$

$$x^2 - 4x + 5$$

$$\int \frac{\cos(\sin^2 - 1)^2}{\sin \theta} d\theta$$

$$x^2 - 4x + 4$$

$$(x-2)^2 + 9$$

$$\int \frac{\cos(\sin^4 - 2\sin^2 + 1)}{\sin \theta} d\theta$$

$$u = \sin \theta \quad du = \cos \theta d\theta$$

$$\int \frac{u^4 - 2u^2 + 1}{u} du$$

$$\int u^3 - 2u + \frac{1}{u} du = \frac{1}{4}u^4 - u^2 + \ln|u| + C$$

1.

$$= \frac{1}{4}(\sin \theta)^4 - \sin^2 \theta + \ln|\sin \theta| + C$$

2.

$$= \int \tan^5 x \sec^3 x dx$$

$$= \int \sec^3 x - 2\sec^4 x + \sec^2 x (\sec x \tan x)$$

$$u = \sec x \quad du = \sec x \tan x dx$$

$$= \int \tan^4 x \sec^2 x (\sec x \tan x) dx$$

$$\int u^6 - 2u^4 + u^2 du$$

$$= \int (\tan^2 x)^2 \sec^2 x (\sec x \tan x) dx$$

$$= \frac{1}{7}u^7 - \frac{2}{5}u^5 + \frac{1}{3}u^3 + C$$

$$= \int (\sec^2 x - 1)^2 \cdot \sec^2 x (\sec x \tan x) dx$$

$$\frac{1}{7}\sec^7 x - \frac{2}{5}\sec^5 x + \frac{1}{3}\sec^3 x + C$$

$$= \int (\sec^4 x - 2\sec^2 x + 1) \sec^2 x (\sec x \tan x) dx$$

$$\int \frac{dx}{x^3 \sqrt{x^2-1}} \quad x = \sec \theta \quad \text{then } dx = \sec \theta \tan \theta d\theta$$

$$= \int \frac{\sec \theta \tan \theta}{(\sec^3 \theta) \sqrt{\sec^2 \theta - 1}} d\theta = \int \frac{\sec \theta \tan \theta}{\sec^3 \theta \tan \theta} d\theta$$

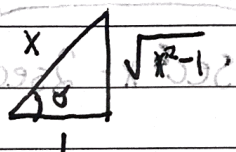
$$= \int \frac{1}{\sec^2 \theta} d\theta = \int \cos^2 \theta d\theta$$

$$= \int \frac{1 + \cos 2\theta}{2} d\theta = \int \frac{1}{2} + \frac{\cos 2\theta}{2} d\theta$$

$$= \frac{1}{2} \theta + \frac{\sin 2\theta}{4} + C$$

$$\theta = \operatorname{arcsec}\left(\frac{x}{1}\right)$$

$$x = \sec \theta$$



$$\frac{1}{2} \operatorname{arcsec}\left(\frac{x}{1}\right) + \frac{\sqrt{x^2-1}}{4x^2}$$

$$r^2 + b^2 = x^2 \quad \sqrt{x^2-1}$$

$$\boxed{\frac{1}{2} \operatorname{arcsec} x + \frac{\sqrt{x^2-1}}{2x^2} + C}$$

4.

$$\int_1^2 \frac{\sqrt{4-x^2}}{x^2} dx$$

$$x = 2 \sin \theta$$

$$dx = 2 \cos \theta d\theta$$

$$2 = 2 \sin \theta$$

$$1 = \sin \theta = \frac{\pi}{2}$$

$$1 = 2 \sin \theta$$

$$\frac{1}{2} = \sin \theta = \frac{\pi}{6}$$

$$\int_{\pi/6}^{\pi/2} \frac{\sqrt{4-4\sin^2\theta}}{4\sin^2\theta} \cdot 2\cos\theta d\theta$$

$$= \int_{\pi/6}^{\pi/2} \frac{\cos^2\theta d\theta}{\sin^2\theta} = \int_{\pi/6}^{\pi/2} \frac{\cos^2\theta}{\sin^2\theta} d\theta$$

$$= \int_{\pi/6}^{\pi/2} \frac{1-\sin^2\theta}{\sin^2\theta} d\theta = \int_{\pi/6}^{\pi/2} \left(\frac{1}{\sin^2\theta} - 1 \right) d\theta$$

$$\int_{\pi/6}^{\pi/2} \frac{1+\cos 2\theta}{2} d\theta = \int_{\pi/6}^{\pi/2} \frac{1+\cos 2\theta}{2} \cdot \frac{2}{1-\cos 2\theta} d\theta$$

$$\int_{\pi/6}^{\pi/2} \frac{1+\cos 2\theta}{1-\cos 2\theta} d\theta$$

$$\int_{\pi/6}^{\pi/2} \frac{\cos^2\theta d\theta}{1-\cos 2\theta} = \int_{\pi/6}^{\pi/2} \frac{2\cos^2\theta d\theta}{1-\cos 2\theta}$$

$$\frac{\cos}{\sin} \quad \frac{\sqrt{3}/2}{1/2}$$

$$\int_{\pi/6}^{\pi/2} \cot^2\theta d\theta = \int_{\pi/6}^{\pi/2} (\csc^2 - 1) d\theta$$

$$-\cot\theta - \theta \Big|_{\pi/6}^{\pi/2} = \left(-\frac{\pi}{2} - (-\sqrt{3} - \frac{\pi}{6}) \right)$$

$$\sqrt{3} + \frac{\pi}{6} - \frac{\pi}{2}$$