In this document, we'll review the major areas of trig we need to know. We won't cover all of trig as there's a good bit of information, but knowing what's presented here will provide you a solid foundation. Both Objectives 10 and 11 are represented here.

Angles

Be sure you know how to sketch angles as this skill will be useful for other topics. It's also important for us to know **coterminal** and **reference angles**. Coterminal angles are angles which share the same terminal side, so they are exactly a multiple of a full circle apart. Why? Knowing that, given an angle how would you calculate an angle that's coterminal with it?
Reference angles are acute angles with the horizontal, so to find these, sketch the angle and identify where the acute angle with the horizontal is. Then determine how to obtain that angle.

1. Find an angle between $0°$ and $360°$ that is coterminal with $-204°$

$-204°$ is too small, so add $360°$

$-204 + 360 = 156°$

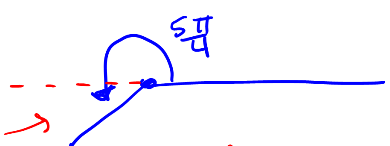
2. Find an angle between 0 and 2π that is coterminal with $\frac{33\pi}{4}$

$\frac{33\pi}{4}$ is too big, so keep subtracting 2π until it's small enough

$\frac{33\pi}{4} - 2\pi = \frac{33\pi}{4} - \frac{8\pi}{4} = \frac{25\pi}{4}$ still too big ... $\frac{25\pi}{4} - \frac{8\pi}{4} = \frac{17\pi}{4}$ still too big

$\frac{17\pi}{4} - \frac{8\pi}{4} = \frac{9\pi}{4}$ $\quad \frac{9\pi}{4} - \frac{8\pi}{4} = \frac{\pi}{4}$

3. Find the reference angle for $\frac{5\pi}{4}$



reference angle is there & that's $\frac{\pi}{4}$

Amplitude, Period, Phase Shift

Let $f(x)$ be one of the trig functions (we only need to know this for sine and cosine). We can capture some important information about its graph by considering the general case

$$y = Af(Bx + C) + D,$$

where $|A|$ is the **amplitude** (the vertical distance from the center of the graph to the top/bottom), $\frac{P}{|B|}$ is the new **period** (the horizontal distance until the graph repeats) when P is the original period of $f(x)$, $-\frac{C}{B}$ is the **phase shift** (horizontal shift).

4. Identify the amplitude, period, and phase shift for $y = 3\cos(2x + \pi) - 2$.

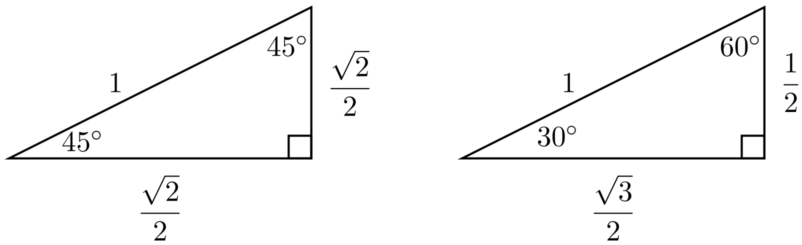
amplitude : 3

period $= \frac{2\pi}{2} = \pi$

phase shift $= -\frac{\pi}{2}$

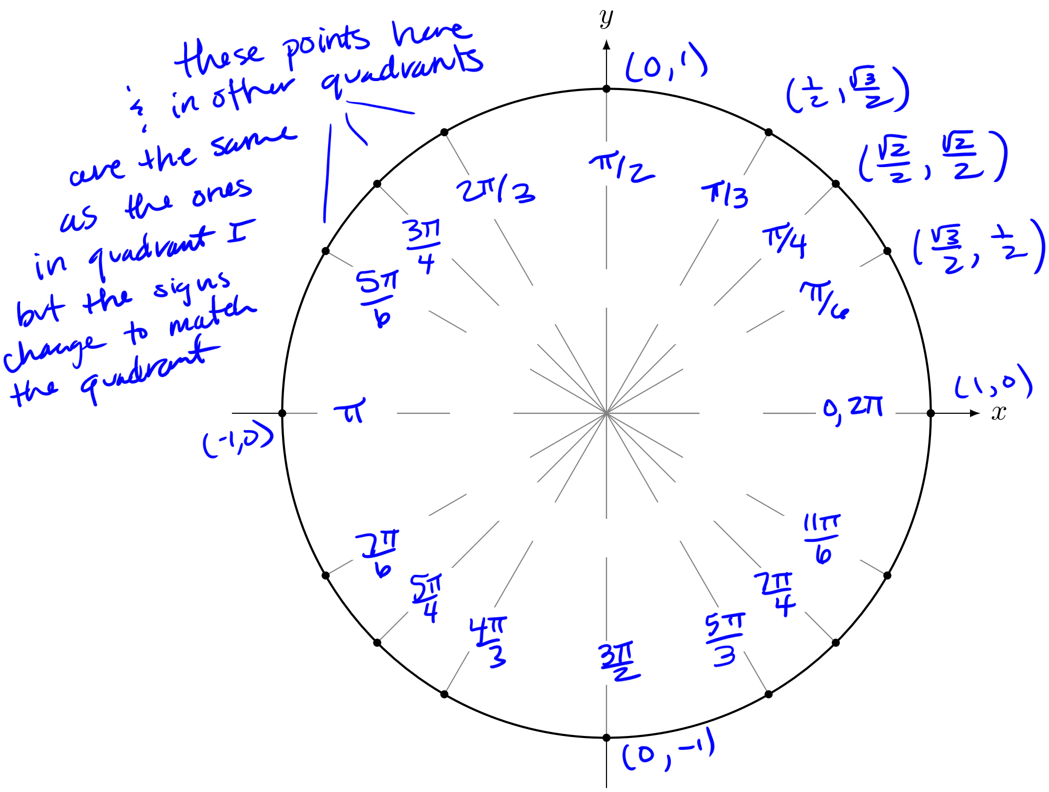
Special Right Triangles

The 30-60-90 and 45-45-90 triangles are two special right triangles we need to know. These triangles are actually part of the unit circle. If you want to know the origin for the values of the sides and hypotenuse, ask and we'll walk through the steps together.



Unit Circle

The unit circle is one of the most important items to know about trig as it allows us to calculate trig values without a calculator. This helps with graphing, solving equations, finding inverse values, etc. To get the unit circle, we need to know two different sets of values. The numbers inside the circle are the *angles*, and these are obtained by going around the circle and counting by either $\frac{\pi}{6}$ or $\frac{\pi}{4}$.



The circle is called the *unit circle* because it's a circle of radius 1. Using that information, label the coordinates for the points on the x and y axes. Then use the special right triangles to get the coordinates for the other points. The sine (cosine) of an angle on the unit circle corresponds to the y-value (x-value) of the associated point.

Here are some practice problems involving the special right triangles and the unit circle.

5. Find the terminal point on the unit circle determined by $\frac{4\pi}{3}$ radians.

from unit circle, $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

6. Find the exact value of $\sin \frac{5\pi}{6}$.

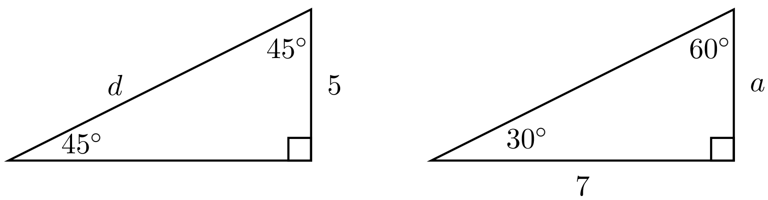
from unit circle, $\sin \frac{5\pi}{6} = \frac{1}{2}$

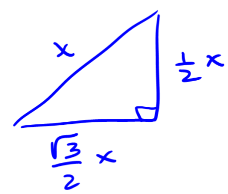
7. Find the exact value of $\csc(-\frac{8\pi}{3})$.

using coterminal angles

$\csc\left(-\frac{8\pi}{3}\right) = \csc\left(\frac{4\pi}{3}\right) = \frac{1}{\sin\left(\frac{4\pi}{3}\right)} = \frac{1}{-\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}}$

8. Find the exact values of the side lengths d and a.





$d = 5\sqrt{2}$

$\frac{\sqrt{3}}{2}x = 7$

$x = \frac{14}{\sqrt{3}}$

$a = \frac{1}{2} \cdot \frac{14}{\sqrt{3}} = \frac{7}{\sqrt{3}}$

Triangle Problems

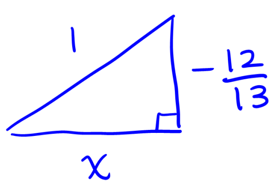
Most trig problems can be solved using the unit circle, but there are some problems that require us to draw a triangle and potentially use the following relationships:

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypothenuse}} \qquad \cos(\theta) = \frac{\text{adjacent}}{\text{hypothenuse}} \qquad \tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{\text{hypothenuse}}{\text{opposite}} \qquad \sec(\theta) = \frac{1}{\cos(\theta)} = \frac{\text{hypothenuse}}{\text{adjacent}} \qquad \cot(\theta) = \frac{1}{\tan(\theta)} = \frac{\text{adjacent}}{\text{opposite}}$$

Here are some examples of these triangle problems.

9. Suppose that $(x, -\frac{12}{13})$ is a point in quadrant IV lying on the unit circle. Find x.



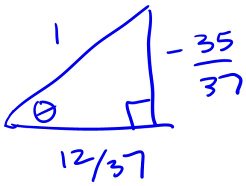
$1^2 = x^2 + \left(-\frac{12}{13}\right)^2$

$x^2 = 1 - \left(-\frac{12}{13}\right)^2 = 1 - \frac{144}{169} = \frac{25}{169}$

$x = \frac{5}{13}, \cancel{-\frac{5}{13}}$

can't be because this is quadrant IV

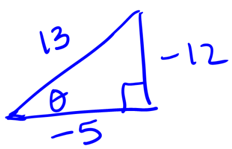
10. Suppose that θ is an angle in standard position whose terminal side intersects the unit circle at $(\frac{12}{37}, -\frac{35}{37})$. Find the exact values of $\sec\theta, \tan\theta$, and $\cos\theta$.



$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{12/37} = \frac{37}{12}$ $\cos\theta = \frac{12}{37}$

$\tan\theta = \frac{35/37}{12/37} = \frac{35}{12}$

11. Let θ be an angle in quadrant III such that $\cos\theta = -\frac{5}{13}$. Find the exact values of $\csc\theta$ and $\cos\theta$.



$-\sqrt{13^2-(-5)^2} = -\sqrt{169-25} = -\sqrt{144} = -12$

$\cos\theta = \frac{-5}{13}$ $\csc\theta = \frac{1}{\sin\theta} = \frac{1}{-12/13} = \frac{-13}{12}$

Trig Identities

Be able to use the following trig identities:

- sum/difference identities:
  $\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \sin(\beta)\cos(\alpha)$; $\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$
- double-angle identities for sine and cosine:
  $\sin(2x) = 2\sin(x)\cos(x)$; $\cos(2x) = \cos^2(x) - \sin^2(x) = 1 - 2\sin^2(x) = 2\cos^2(x) - 1$
- half-angle identities for sine and cosine: $\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos(\alpha)}{2}}$; $\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos(\alpha)}{2}}$

The Formulas Overview assignment provides a good overview of how you might use these identities.

Inverse Trig

Be able to solve inverse trig problems. When you see these problems, it may be helpful to think 'What angle would give the value in the problem?' For example, the problem could be

Find the exact value of $\sin^{-1}(-\frac{\sqrt{2}}{2})$

which means 'What angle on the unit circle has a corresponding y-coordinate (it's y-coordinate since this problem involves sine) of $-\frac{\sqrt{2}}{2}$. You should find two possible answers based on the unit circle, but be sure to apply the restrictions on inverse trig functions (talk to one of us if it's not clear why those restrictions are needed; this involves the idea of 1-to-1 functions from an earlier Objective). Once that restriction is applied, only one answer is possible.

Graphing and Solving Trig Equations

Graphing trig functions is covered in the Parent Graphs and Transformations review and solving trig equations is covered in the Solving Equations review.