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$\boxed{\frac{1}{x-1} + \frac{-x+5}{x^2+4}}$

1.

A. 9 $\int \frac{6x-1}{(x-1)(x^2+4)}\,dx$ $\quad \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+4)}$ $\quad (x-1)(x^2+4)$

$u = \frac{1}{2}x \quad du = \frac{1}{2}dx$

$6x-1 = A(x^2+4) + (Bx+c)(x-1)$

$\frac{10}{4}\int \frac{1}{u^2+1} \quad 2du = dx$

$x=1:$ $5 = A(1+4) + (B(1)\cancel{+c})(0)$

$\int \frac{5}{x^2+4}$

$5 = 5A \quad \boxed{A = 1}$

$\frac{5}{4}\int \frac{1}{x^2+4}$

$x=0:$ $-1 = 4A + (B(0)+c)(0-1)$

$\frac{5}{4}\int \frac{1}{\frac{1}{4}x^2+1}$

$-1 = 4A - c$

$u = x^2+4$

$-1+4$

$-1 = -4 - c$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$-5 = -c \quad \boxed{5 = c}$

$-x^2 - 5$

$x=2:$ $11 = A(4+4) + (2B+c)(1)$

$11 = A4 + A4 + 2B + c$

$\int \frac{x}{x^2+4}\,dx = -\frac{1}{2}\int \frac{1}{u}\,du$

$11 = 8 + 2B + 5$

$11 = 13 + 2B$

$-\frac{1}{2}\ln|x^2+4| + c$

$-2 = 2B \quad \boxed{B = -1}$

B.

$= \int \frac{1}{x-1} + \left(\frac{-x+5}{x^2+4}\right)dx = \int \frac{1}{x-1} - \left(\frac{x-5}{x^2+4}\right)dx$

$$\boxed{= \ln|x-1| - \frac{1}{2}\ln|x^2+4| + \frac{5}{2}\arctan\left(\frac{1}{2}x\right) + C}$$

$\frac{\cos}{\sin}$ $\quad -\cos$

$1-\sin^2$

$\sin^2 + \cos^2 = 1$

$\frac{1}{\sin} - \sin$

$\cos^2 = 1-\sin^2\theta$ $\quad A^2 - x^2 \quad = \sin$ sub

$\csc - \sin$

2. $\displaystyle\int \frac{\sqrt{9-x^2}}{x}$ $\qquad x = 3\sin\theta \qquad dx = 3\cos\theta\, d\theta$

$$= \int \frac{\sqrt{9-(3\sin\theta)^2}\cdot 3\cos\theta\, d\theta}{3\sin\theta}$$

$$= \int \frac{\sqrt{9-9\sin^2\theta}\cdot 3\cos\theta\, d\theta}{3\sin\theta}$$

$$= \int \frac{\sqrt{9\cos^2\theta}\cdot 3\cos\theta\, d\theta}{3\sin\theta}$$

$\cos \rightleftarrows \sin$

$$= \int \frac{\cancel{3}\cos^2\theta}{\cancel{3}\sin\theta}\, d\theta$$

$\sin \to \cos$

$\cos \to -\sin$

$$= \int \cot\theta \cos\theta$$

$$= \int \cancel{\cot\theta}\ \frac{1-\sin^2\theta}{\sin\theta}\, d\theta$$

$\frac{O}{A}$

$\frac{A}{O}$

$$= \int \csc\theta - \sin\theta \ d\theta$$

$\frac{1}{\sin}$ $\quad \frac{H}{O}$

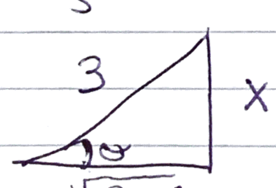
$$= \ln|\csc\theta - \cot\theta| + \cos\theta + C$$

$x = 3\sin\theta$ $\qquad A^2 + x^2 = 9$

$\frac{x}{3} = \sin\theta$ $\qquad A = 9-x^2$

$\frac{O}{H}$

$= \sqrt{9-x^2}$



$$\boxed{= \ln\left|\frac{3}{x} - \frac{\sqrt{9-x^2}}{x}\right| + \frac{\sqrt{9-x^2}}{3} + C}$$

$1 + \tan^2 = \sec^2$

3.

$\cancel{(1+\tan^2 x)(1+\tan^2 x)}$

$\cancel{1+\tan^2 x+\tan^2 x+\tan^4 x}$

$\cancel{= \int \sec^4 x \tan^8 x \, dx}$

$\cancel{= \int (1+\tan^2 x)^2 \tan^8 x \, dx}$

$\cancel{= \int (1+2\tan^2 x+\tan^4 x)\tan^8}$

3. $\int \sec^4 x \tan^8 x \, dx$

$= \int \sec^2 x \tan^8 x \sec^2 x \, dx$

$= \int (1+\tan^2 x) \tan^8 x \sec^2 x \, dx$

$= \int (\tan^8 x + \tan^{10} x) \sec^2 x \, dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$= \int u^8 + u^{10} \, du$

$= \frac{1}{9} u^9 + \frac{1}{11} u^{11} + C$

$\boxed{= \frac{1}{9} \tan^9 x + \frac{1}{11} \tan^{11} x + C}$

f(x) is continuous
f(x) is decreasing
f(x) is positive

4. $$\sum_{k=1}^{\infty} \frac{1}{(2k+3)^{\cancel{3}\,2}}$$

A. $f(x) = \frac{1}{(2x+3)^2}$

- continuous for all $x > 1$ (series starts at 1)
- positive for any value of $x > 1$ (series starts at 1)

$f'(x) = (2x+3)^{-2}$

$-2(2x+3)^{-1}$

$f'(x)\ \frac{-2}{2x+3}$ → decreasing for all $x > 1$

B. $$\lim_{b\to\infty} \int_1^b \frac{1}{(2x+3)^2}\,dx$$

u^{-2}

$-\frac{1}{u}$

$u = 2x+3$

$du = 2dx$

$\frac{1}{2}du = dx$

$$= \lim_{b\to\infty} \frac{1}{2}\int_5^{2b+3} u^{-2}$$

$$= \lim_{b\to\infty} -\frac{1}{2u}\Big|_5^{2b+3} \quad \cancel{-\frac{1}{4b+6}}^{\,0} - \left(-\frac{1}{10}\right) \quad \left(\frac{1}{10}\right)$$

$\cancel{= \frac{1}{10}}$

$\boxed{\text{The integral converges to } \frac{1}{10}}$

5.

A $\displaystyle\sum_{k=1}^{\infty} \frac{k^3+2k-1}{4+5k^3}$

• Divergence test

$$\lim_{k\to\infty} \frac{k^3+2k-1}{4+5k^3} = \boxed{\frac{1}{5}}$$

• Integral diverges by the divergence test as $\frac{1}{5}$ does not equal 0

B $\displaystyle\sum_{k=1}^{\infty} \frac{(-2)^{k+1}}{3\cdot 5^k}$

$$= \sum_{k=1}^{\infty} \frac{(-2)^k \cdot -2}{3\cdot(5)^{k}}$$

$1+1$

$$= -\frac{2}{3} \sum_{k=1}^{\infty} \left(\frac{-2}{5}\right)^k$$

$\left|\frac{2}{5}\right| < 1$

• Converges by PST

$$\frac{\frac{4}{15}}{\frac{5}{5}+\frac{2}{5}} = \frac{\frac{4}{15}}{\frac{7}{5}} = \frac{4}{15}\cdot\frac{5}{7}$$

$$\frac{4}{3}\cdot\frac{1}{7}$$

$$\boxed{=\frac{4}{21}}$$