Student's Printed Name: ______________________________ **XID: C**_______________

Instructor: ____ **Section:** _____________

No questions will be answered during this exam.

If you consider a question to be ambiguous, state your assumptions in the margin and do the best you can to provide the correct answer.

Instructions: You are **not** permitted to use a calculator on any portion of this test. You are **not** allowed to use a textbook, notes, cell phone, computer, or any other technology on any portion of this test. All devices must be turned off and stored away while you are in the testing room.

During this test, **any** kind of communication with any person other than the instructor or a designated proctor is understood to be a violation of academic integrity.

No part of this test may be removed from the examination room.

Read each question carefully. To receive full credit for the free response portion of the test, you must:

1. Show legible, logical, and relevant justification which supports your final answer.
2. Use complete and correct mathematical notation.
3. Include proper units wherever appropriate.
4. Give answers as exact values whenever possible.

You have **90 minutes** to complete the entire test.

Do not write below this line.

Free Response Problem	Possible Points	Points Earned	Free Response Problem	Possible Points	Points Earned
1.a.	6		4.	10	
1.b.	6		5.a.	4	
2.a.	8		5.b.	2	
2.b.	4		5.c.	6	
3.a.	4		6. (Scantron)	1	
3.b.	4		**Free Response**	58	
3.c.	3		**Multiple Choice**	42	
			Test Total	100	

Multiple Choice: There are 14 multiple choice questions. Each question is worth 3 points and has one correct answer. The multiple choice problems will be 42% of the total grade. Circle your choice on your test paper and bubble the corresponding answer on your Scantron. Any questions involving inverse trigonometric functions should be answered based on the domain restrictions for trigonometric functions used in Section 1.4.

1. Consider the cost function $C(x) = 500 + 0.2x,\ 0 \le x \le 4000$ ($C(x)$ is \$ when x items produced). Determine the average cost and marginal cost when $x = 1000$ items are produced.

A) Average cost is \$0.20 per item and marginal cost is \$0.10 per item.

B) Average cost is \$0.60 per item and marginal cost is \$0.10 per item.

C) Average cost is \$0.70 per item and marginal cost is \$0.20 per item.

D) Average cost is \$5.20 per item and marginal cost is \$2.00 per item.

2. Find the derivative of $f(x) = \sqrt{x}\cos x$.

A) $f'(x) = \dfrac{-\sin x}{2\sqrt{x}}$

B) $f'(x) = \dfrac{-\cos x}{2\sqrt{x}} + \sqrt{x}\sin x$

C) $f'(x) = -2\sqrt{x}\sin x$

D) $f'(x) = \dfrac{\cos x}{2\sqrt{x}} - \sqrt{x}\sin x$

3. Use the table to evaluate $\left.\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]\right|_{x=1}$.

	x = 1	**x = 3**	**x = 5**	**x = 7**	**x = 9**
$f(x)$	3	1	9	7	5
$f'(x)$	1	9	5	1	3
$g(x)$	9	7	5	3	1
$g'(x)$	5	9	3	1	7

A) $\frac{8}{27}$

B) $\frac{1}{3}$

C) $\frac{-2}{27}$

D) $\frac{1}{5}$

4. Find an equation of the line tangent to $g(x) = 3x^2 - x + 1$ at $x = 1$.

A) $y = 6x - 17$

B) $y = 5x - 2$

C) $y = 6x - 1$

D) $y = 5x - 14$

5. Find $\dfrac{d^2y}{dx^2}$ given $xy = 1$.

A) $\dfrac{d^2y}{dx^2} = 0$

B) $\dfrac{d^2y}{dx^2} = \dfrac{y^2+y}{x^2}$

C) $\dfrac{d^2y}{dx^2} = \dfrac{2y}{x^2}$

D) $\dfrac{d^2y}{dx^2} = \dfrac{y+xy}{x^3}$

6. Find the derivative of $g(x) = \sec\left(x^2+3\right)$.

A) $g'(x) = 2x\tan^2\left(x^2+3\right)$

B) $g'(x) = \sec\left(2x\right)\tan\left(2x\right)$

C) $g'(x) = 2x\sec\left(x^2+3\right)\tan\left(x^2+3\right)$

D) $g'(x) = 4x^2\sec\left(x^2+3\right)\tan\left(x^2+3\right)$

7. Find the derivative of $y = \tan\left(e^{3x}\right)$.

A) $\dfrac{dy}{dx} = 3e^{3x}\sec^2\left(e^{3x}\right)$

B) $\dfrac{dy}{dx} = e^{3x}\sec^2 x + 3e^{3x}\tan x$

C) $\dfrac{dy}{dx} = \sec^2\left(3e^{3x}\right)$

D) $\dfrac{dy}{dx} = e^{3x}\sec^2\left(e^{3x}\right)$

8. Find the derivative of $y = \ln\left(\dfrac{3x}{\sec x}\right)$.

A) $\dfrac{dy}{dx'} = \dfrac{1 + x\cot x}{x}$

B) $\dfrac{dy}{dx} = \dfrac{9x\sec x - 9x^2 \sec x \tan x}{\sec^3 x}$

C) $\dfrac{dy}{dx} = \dfrac{\sec x}{3x}$

D) $\dfrac{dy}{dx} = \dfrac{1 - x\tan x}{x}$

9. Find the derivative of $y = \sin^{-1}(2x^3)$.

A) $y' = \dfrac{6x^2}{1 + 4x^6}$

B) $y' = \dfrac{1}{\sqrt{1 - 4x^9}}$

C) $y' = \dfrac{6x^2}{\sqrt{1 - 4x^6}}$

D) $y' = \dfrac{6x^2}{\sqrt{4x^6 + 1}}$

10. Find y''' (the third derivative) given $y = x^e - e^x$.

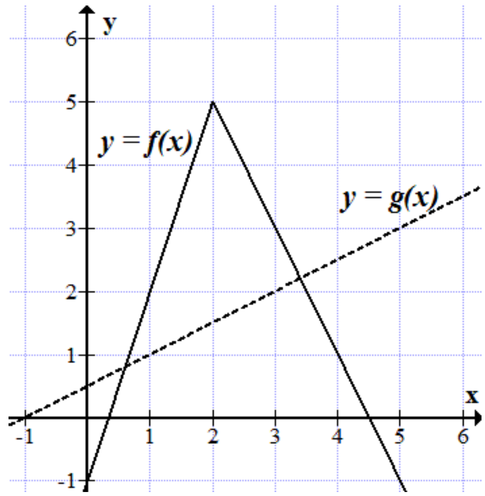
A) $y''' = e(e-1)(e-2)x^{e-3} - e^x$

B) $y''' = e^3 x^{e-3} - e^x$

C) $y''' = e(e-1)(e-2)x^{e-3} - x(x-1)(x-2)e^{x-3}$

D) $y''' = e^3 x^{e-3} - x^3 e^{x-3}$

11. Use the given graphs of f and g to evaluate $\left.\dfrac{d}{dx}\left(5f(x)+3g(x)\right)\right|_{x=4}$.



A) 32

B) $\dfrac{-17}{2}$

C) $\dfrac{23}{2}$

D) $\dfrac{25}{2}$

12. Differentiate $f(x) = 5^{\sin x}$.

A) $f'(x) = 5^{\sin x}(\cos x)$

B) $f'(x) = (\ln 5)5^{\sin x}$

C) $f'(x) = 5^{\sin x}\ln(\sin x)$

D) $f'(x) = (\ln 5)5^{\sin x}(\cos x)$

13. Evaluate $\lim_{x\to 0} \dfrac{k \sin kx}{x}$ where k is a positive constant.

A) 0

B) 1

C) k

D) k^2

14. A spherical balloon is inflated with helium at a rate of 10 ft ^{3}/min. How fast is the radius of the balloon increasing at the instant the radius is 2 feet?

Recall that the volume of a sphere is $V = \dfrac{4}{3}\pi r^3$.

A) $\dfrac{dr}{dt} = \dfrac{5}{8\pi}$ ft/min

B) $\dfrac{dr}{dt} = \dfrac{5}{4\pi}$ ft/min

C) $\dfrac{dr}{dt} = 160\pi$ ft/min

D) $\dfrac{dr}{dt} = 16\pi$ ft/min

Free Response: The Free Response questions will be 58% of the total grade. Read each question carefully. To receive full credit, you must show legible, logical, and relevant justification which supports your final answer. Give answers as exact values. Questions involving inverse trigonometric functions should be answered based on the domain restrictions in Section 1.4.

1. **(12 pts.)** Find the derivative of each of the following functions. **SIMPLIFY YOUR ANSWERS.**

a. **(6 pts.)** $y=\left(\dfrac{x^2}{3x^2+2}\right)^3$

$$\frac{dy}{dx}=3\left(\frac{x^2}{3x^2+2}\right)^2\cdot\frac{(2x)(3x^2+2)-(x^2)(6x)}{(3x^2+2)^2}$$

$$=\frac{3x^4}{(3x^2+2)^2}\cdot\frac{6x^3+4x-6x^3}{(3x^2+2)^2}=\frac{3x^4\cdot 4x}{(3x^2+2)^4}=\frac{12x^5}{(3x^2+2)^4}$$

Work on Problem:	Points
Applies the chain rule and finds the derivative of the outside function leaving the inside unchanged	2 points
Finds the derivative of the inside function using the quotient rule	2 points
Simplifies the result	2 points

Notes:
- Subtract 4 points for applying the chain rule incorrectly by substituting the derivative inside instead of multiplying by the derivative → 2 points may be awarded if the derivative of the inside function is correct and there is some attempt to simplify

b. **(6 pts.)** $y=\tan^{-1}\left(\dfrac{x}{2}\right)$

$$\frac{dy}{dx}=\frac{1}{1+\left(\frac{x}{2}\right)^2}\cdot\frac{1}{2}$$

$$=\frac{1}{1+\frac{x^2}{4}}\cdot\frac{1}{2}=\frac{1}{\left(\frac{4+x^2}{4}\right)}\cdot\frac{1}{2}=\frac{4}{4+x^2}\cdot\frac{1}{2}=\frac{2}{4+x^2}$$

Work on Problem:	Points
Applies the chain rule and finds the derivative of the outside function leaving the inside unchanged	2 points
Finds the derivative of the inside function	2 points
Simplifies the result	2 points

Notes:
- Subtract 4 points for applying the chain rule incorrectly by substituting the derivative inside instead of multiplying by the derivative → 2 points may be awarded if the derivative of the inside function is correct and there is some attempt to simplify

2. **(12 pts.)** Consider the equation $\sin y = 6x^5 - 6$.

a. **(8 pts.)** Use **Implicit Differentiation** to find $\dfrac{dy}{dx}$.

$$\frac{d}{dx}(\sin y) = \frac{d}{dx}\left(6x^5 - 6\right)$$

$$(\cos y)\left(\frac{dy}{dx}\right) = 30x^4$$

$$\frac{dy}{dx} = \frac{30x^4}{\cos y}$$

If derivative in part a is incorrect due to multiple egregious differentiation and algebra errors, work should NOT be followed through part b.

Work on Problem:	**Points**
Explicitly or implicitly knows to take the derivative of both sides with respect to *x*	2 points
Takes the derivative of the left-hand side	2 points
Takes the derivative of the right-hand side	2 points
Solves for *dy/dx* (Prime notation okay)	2 points

Notes:
- Subtract ½ point for notation errors with a maximum deduction of 1 point

b. **(4 pts.)** Find the equation of the tangent line to the curve at the point $(1, \pi)$.

$$m_{\tan} = \left.\frac{dy}{dx}\right|_{(1,\ \pi)} = \frac{30(1)^4}{\cos \pi} = \frac{30}{-1} = -30$$

$$y - \pi = -30(x - 1)$$

$$y = -30x + 30 + \pi$$

Work on Problem:	**Points**
Substitutes given point into the derivative from part a, following work	1 point
Evaluates the derivative to find the slope of the tangent line	1 point
States any form of the equation of the tangent line, following work	2 points

Notes:
- Subtract ½ point for notation if the point is substituted into the expression for the derivative without appropriate notation for evaluating the derivative at a point
- Subtract 1 point for substituting values into opposite variables
- Subtract ½ point for incorrectly simplifying a correct equation of the tangent line

3. **(11 pts.)** Suppose a stone is thrown vertically upward with an initial velocity of 64 ft/s from a bridge 80 ft above a river. By Newton's laws of motion, the position of the stone (measured in feet above the river) after t seconds is $s(t) = -16t^2 + 64t + 80$ where $s = 0$ is the level of the river.

a. **(4 pts.)** Find the velocity and acceleration functions.

Velocity $v(t) =$ $\underline{s'(t) = -32t + 64}$ ft/s.

Acceleration $a(t) =$ $\underline{v'(t) = -32}$ ft/s^2.

Work on Problem:	Points
Differentiates the position function to find the velocity function	2 points
Differentiates the velocity function to find the acceleration function	2 points

Notes:

- Only the derivative is required here. The notation in the response is preferred, but not required.

b. **(4 pts.)** What is the highest point above the river reached by the stone?

$v(t) = 0$ when $-32t + 64 = 0$

$-32(t-2) = 0$

$t = 2$ seconds

$s(2) = -16(2)^2 + 64(2) + 80 = -64 + 128 + 80 = 144$ feet

The stone reaches its highest point when $t =$ $\underline{2}$ seconds.

The highest point reached by the stone is $\underline{144}$ feet.

Work on Problem:	Points
Sets velocity function equal to zero and finds time t when stone reaches highest point	2 points
Evaluates the position function at the determined time, following work	2 points

Notes:

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c. **(3 pts.)** When will the stone strike the river? (That is, how long is the stone in the air?)

$s(t) = 0$ when $-16t^2 + 64t + 80 = 0$

$-16(t^2 - 4t - 5) = 0$

$-16(t-5)(t+1) = 0$

$t = 5$ seconds OR ~~$t = -1$ seconds~~

The stone strikes the river after $\underline{5}$ seconds.

Work on Problem:	Points
Sets position function equal to zero and factors or uses the quadratic formula to solve	2 points
States the correct time when the stone strikes the river	1 point

Notes:

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4. **(10 pts.)** Find $f'(1)$ when $f(x) = x^{2/x}$.

1st Step. **Differentiate.** There are 2 methods for finding the derivative.
Either method will receive full credit when correct. Simplification, including solving for *dy/dx* and/or substituting the original expression for *f(x)*, is not required.

Method I. Logarithmic Differentiation

Let $y = f(x)$.

$$\ln y = \ln x^{2/x}$$

$$\ln y = \frac{2}{x}\ln x = \frac{2\ln x}{x}$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}\left(\frac{2\ln x}{x}\right)$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{\frac{2}{x}\cdot x - (2\ln x)\cdot 1}{x^2}$$

$$\frac{dy}{dx} = y\left(\frac{2-2\ln x}{x^2}\right)$$

$$f'(x) = x^{2/x}\left(\frac{2-2\ln x}{x^2}\right)$$

Work on Problem: Derivative is worth 8 points	Points
Takes the natural log (ln) of both sides	2 points
Simplifies the right-hand side using properties of logs	2 points
Finds the derivative of the left-hand side using the chain rule	2 points
Finds the derivative of the right-hand side using the quotient rule	2 points

Notes:
- No credit awarded if the natural log is not used
- Maximum of half-credit (4 points) awarded if the natural log is applied to only one side of the equation
- Subtract 2 points for taking the derivative of one side of the equation and not the other in the same step, then taking the derivative of the other side at a later step, regardless of whether the derivatives are correct or not
- Subtract 2 – 4 points for incorrect use of logarithm properties (egregious algebra errors)

Method II. Express $x^{2/x}$ as $e^{\ln x^{2/x}}$

$$f(x) = e^{\ln x^{2/x}} = e^{\frac{2}{x}\ln x} = e^{\left(\frac{2\ln x}{x}\right)}$$

$$f'(x) = \frac{d}{dx}\left(e^{\left(\frac{2\ln x}{x}\right)}\right)$$

$$= e^{\left(\frac{2\ln x}{x}\right)}\left(\frac{\frac{2}{x}\cdot x - (2\ln x)\cdot 1}{x^2}\right)$$

$$= x^{2/x}\left(\frac{2-2\ln x}{x^2}\right)$$

Work on Problem:	Points
Expresses the right-hand side using e and ln	2 points
Simplifies the exponent using properties of logs	2 points
Finds the derivative of the outside function using the chain rule	2 points
Finds the derivative of the inside function (exponent) using the quotient rule	2 points

Notes:
- No credit awarded if the natural log is not used
- Subtract 2 – 4 points for incorrect use of logarithm properties (egregious algebra errors)

2nd Step. **Evaluate the derivative at $x = 1$.**

$$f'(1) = 1^{2/1}\left(\frac{2-2\ln 1}{1^2}\right) = 1^2\left(\frac{2-2\cdot 0}{1}\right) = 1(2-0) = 2$$

Work on Problem:	Points
Substitutes $x = 1$ into the derivative formula and evaluates, following reasonable work	2 points

Notes:
- Subtract ½ point for notation errors
- If the *y*-coordinate is needed to evaluate the derivative, then that value ($y = 1$) should be shown in the work. In this case, subtract ½ point if the work to find *y* is missing AND the *y*-value is omitted or disappears from the computation.

5. **(12 pts.)** A 12-ft ladder is leaning against a vertical wall when Trevor begins pulling the bottom of the ladder away from the wall at a rate of 0.5 ft/s. How fast is the top of the ladder sliding down the wall when the bottom of the ladder is 5 ft from the wall?

Variables: * Let x be the distance from the wall to the bottom of the ladder (in ft).
* Let y be the distance from the top of the ladder to the ground (in ft).
* Let t be time (in s).

a. **(4 pts.)** Identify the rate(s) that are given and the rate that is to be determined, using derivative notation and units. Make a sketch to organize the variables and given information.

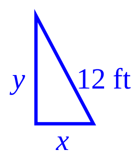
Given rate(s): $\frac{dx}{dt} = \frac{1}{2}$ ft/s

1 point

Rate to be determined: $\frac{dy}{dt} = ?$ ft/s when $x = 5$ ft

1 point

Sketch: (Include labels for variables and any constants.)



2 points
* 1 point for right triangle figure
* 1 point for labels

b. **(2 pts.)** Write an equation that expresses the basic relationship among the variables x and y.

$x^2 + y^2 = (12)^2$ by the Pythagorean Theorem

2 points for setting up equation

c. **(6 pts.)** Use your equation from part (b) to determine the rate at which the top of the ladder is sliding down the wall when the bottom of the ladder is 5 ft from the wall. State your answer in a complete sentence.

$$\frac{d}{dt}\left(x^2 + y^2\right) = \frac{d}{dt}(144)$$

When $x = 5$ ft, $y = \sqrt{144 - 25} = \sqrt{119}$ ft (by the Pythagorean Theorem)

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{-5}{\sqrt{119}} \cdot \frac{1}{2}$$

$$\frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{-5}{2\sqrt{119}} \text{ ft/sec}$$

When the bottom of the ladder is 5 feet from the wall, the top of the ladder is sliding down the wall at a rate of 5/(2sqrt(119)) ft/s.

Work on Problem:	**Points**
Takes the derivative of both sides (of part (b)) w.r.t. time t, following work (This result does not have to be solved for *dy/dt*.)	3 points
Correctly uses implicit differentiation	3 points
Substitutes the given information into the result and solves for *dy/dt* *(Due to the difference in arithmetic difficulty between test versions A and B, all six points were awarded to work up to this point, regardless of final answer and sentence.)*	

Notes:
- Subtract ½ point for notation errors
- Subtract ½ point for missing or incorrect units or sign error

Scantron **(1 pt.)**

Check to make sure your Scantron form meets the following criteria. If any of the items are NOT satisfied when your Scantron is handed in and/or when your Scantron is processed one point will be subtracted from your test total.

My Scantron:

- ◻ is bubbled with firm marks so that the form can be machine read;
- ◻ is not damaged and has no stray marks (the form can be machine read);
- ◻ has **14** bubbled in answers;
- ◻ has **MATH 1060** and my section number written at the top;
- ◻ has my instructor's last name written at the top;
- ◻ has Test No. **2** written at the top;
- ◻ has the correct test version written at the top **and** bubbled in below my XID;
- ◻ shows my correct XID both written and bubbled in;

  Bubble a zero for the leading C in your XID.

Please read and sign the honor pledge below.

On my honor, I have neither given nor received inappropriate or unauthorized information at any time before or during this test.

Student's Signature: ____