

$$\lim_{k \rightarrow \infty} \frac{\tan^{-1} k}{1+3^k} \approx \frac{\pi/2}{3^k}$$

$$\sum_{k=1}^{\infty} \frac{\tan^{-1} k}{1+3^k} \leq \sum_{k=1}^{\infty} \frac{\pi/2}{3^k} = \frac{\pi}{2} \sum_{k=1}^{\infty} \frac{1}{3^k}$$

$$1. \sum_{k=1}^{\infty} \frac{\tan^{-1} k}{1+3^k} \leq \sum_{k=1}^{\infty} \frac{\pi/2}{3^k} = \lim_{k \rightarrow \infty} \frac{\tan^{-1} k \cdot 3^k}{1+3^k}$$

$$\frac{\pi}{2} \sum_{k=1}^{\infty} \frac{1}{3^k} = \frac{\pi}{2} \sum_{k=1}^{\infty} \left(\frac{1}{3}\right)^k$$

Geometric Series

$$r = 1/3$$

• Series converges by comparison test

$$2. |R_n| \leq A_{k+1} \quad A_k = \frac{1}{n^2+1}$$

$$n=4 \quad R_n \leq \frac{1}{(5)^2+1} = \frac{1}{26} ?$$

$$3. \sqrt{\lim_{k \rightarrow \infty} \frac{k+1}{k^2+8}} = 0$$

$$f'(x) = \frac{1(k^2+8) - (k+1)(2k)}{(k^2+8)^2} = \frac{k^2+8 - 2k^2+2k}{(k^2+8)^2}$$

$$f'(x) = -\frac{k^2+2k}{(k^2+8)}$$

• Non microimp

• Converges by the A.S.T

$$4. \sum_{k=1}^{\infty} \frac{k+1}{k^2+8} \approx \sum_{k=1}^{\infty} \frac{1}{k} \quad \frac{1}{k^1} \quad p=1, \text{ Diverges by R.S.T - P series Test}$$

$$\lim_{k \rightarrow \infty} \frac{k+1}{k^2+8} \cdot \frac{k}{1} = \lim_{k \rightarrow \infty} \frac{k^2+k}{k^2+8} = 1$$

Diverges by L.C.T

5. ~~Converges~~ Converges conditionally - because

$$\sum_{k=1}^{\infty} \left| (-1)^k \frac{k+1}{k^2+8} \right|$$

causes the series to diverge when it otherwise ~~diverged~~ converged