$\sin(2\theta) = 2\sin\theta\cos\theta$ $\cos^2\theta = 1-\sin^2\theta$ $\sin^2\theta = \frac{1}{2}(1-\cos(2\theta))$ $\tan^2\theta = \sec^2\theta - 1$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$ $\sin^2\theta = 1-\cos^2\theta$ $\cos^2\theta = \frac{1}{2}(1+\cos(2\theta))$ $\sec^2\theta = \tan^2\theta + 1$

$\sin A \sin B = \frac{1}{2}(\cos(A-B)-\cos(A+B))$ $\Delta x(f(\overline{x_1})+f(\overline{x_2})+f(\overline{x_3})+\cdots+f(\overline{x_n}))$

$\sin A \cos B = \frac{1}{2}(\sin(A-B)+\sin(A+B))$ $\frac{\Delta x}{2}(f(x_0)+2f(x_1)+2f(x_2)+\cdots+f(x_n))$

$\cos A \cos B = \frac{1}{2}(\cos(A-B)+\cos(A+B))$ $\frac{\Delta x}{3}(f(x_0)+4f(x_1)+2f(x_2)+\cdots+2f(x_{n-2})+4f(x_{n-1})+f(x_n))$

100 Calc 2 Problems

(great for your final exam)

Video: https://youtu.be/Kwyk_mteyNc

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11/25/2019

$\frac{dP}{dt} = kP\left(1-\frac{P}{M}\right) \Rightarrow P(t) = \frac{M}{1+Ce^{-kt}},\ C = \frac{M-P_0}{P_0}$ $f(x) = \sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$

$\sinh x = \frac{e^x - e^{-x}}{2}$ $\cosh x = \frac{e^x + e^{-x}}{2}$ $\tanh x = \frac{\sinh x}{\cosh x}$ $\ln(1+x) = \sum_{n=0}^{\infty}\frac{(-1)^n}{n+1}x^{n+1}$

$\frac{d}{dx}(\sinh x) = \cosh x$ $\frac{d}{dx}(\cosh x) = \sinh x$ $\frac{d}{dx}(\tanh x) = \mathrm{sech}^2 x$ $\tan^{-1}x = \sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}x^{2n+1}$

(Q1.) Which of the following infinite series diverges by the **Test for Divergence**?

(A) $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$

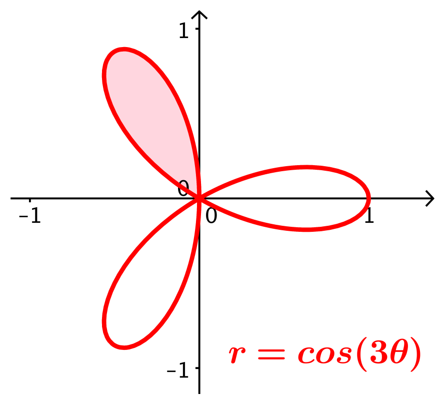
(B) $\sum_{n=1}^{\infty} \frac{n^2}{n^3+2n-1}$

(C) $\sum_{n=3}^{\infty} \frac{1}{\sqrt{n}\ln(n)}$

(D) $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+2}\right)$

(Q2.) What is the **area** of the shaded region?

(A) 0.2618
(B) 1.3231
(C) 0.5236
(D) 1.5708
(E) 0.7854
(F) 0.1309



(Q3.) Set up an integral for the **surface area** obtained by rotating the arc defined by $x = t + e^t$, $y = \cos t$ from $t = 0$ to $t = 1$ **about the y-axis**

(A) $\int_0^1 2\pi(t+e^t)\sqrt{(1+e^t)^2 + (-\sin t)^2}\,dt$

(B) $\int_0^1 2\pi \cos t\sqrt{(1+e^t)^2 + (-\sin t)^2}\,dt$

(C) $\int_0^1 2\pi(-\sin t)\sqrt{(t+e^t)^2 + (\cos t)^2}\,dt$

(D) $\int_0^1 2\pi(1+e^t)\sqrt{(t+e^t)^2 + (\cos t)^2}\,dt$

(E) $\int_0^1 2\pi(t+e^t)(\cos t)\sqrt{(1+e^t)^2 + (-\sin t)^2}\,dt$

(Q4.) Consider a sequence defined recursively by $a_1 = 5$, $a_n = 8 - a_{n-1}$ for $n \geq 2$. Which of the following statement about a_n is **true**?

(A) a_n diverges
(B) a_n converges to 3
(C) a_n converges to 5
(D) a_n is increasing
(E) a_n is decreasing

(Q5.) If $x = te^t$ and $y = 3e^t$, then $\dfrac{d^2y}{dx^2} = ?$

(A) $\dfrac{-3}{e^t(t+1)^3}$

(B) $\dfrac{3}{e^t(t+1)^3}$

(C) $\dfrac{-3}{(t+1)^3}$

(D) $\dfrac{1}{3e^t(t+1)^3}$

(Q6.) If $f(x) = \tanh^{-1}(\sin x)$, then $f'(x) = ?$

(A) $\sec x$

(B) $\csc^2 x$

(C) $\sec^2 x$

(D) $\sec x \cos x$

(Q7.) Given $\displaystyle\int_2^\infty \frac{1}{x\sqrt{x^2-1}}\,dx$ converges. Which of the following also converges by the comparison theorem with $\displaystyle\int_2^\infty \frac{1}{x\sqrt{x^2-1}}\,dx$?

(A) $\displaystyle\int_2^\infty \frac{1}{x^2\sqrt{x^2-1}}\,dx$

(B) $\displaystyle\int_2^\infty \frac{x}{\sqrt{x^2-1}}\,dx$

(C) $\displaystyle\int_2^\infty \frac{1}{\sqrt{x^2-1}}\,dx$

(D) $\displaystyle\int_2^\infty \frac{1}{\sqrt{x^2+1}}\,dx$

(Q8.) Solve $\dfrac{dy}{dx} = xy^2$ and $y(0) = 4$

(A) $y = \dfrac{4}{1-2x^2}$

(B) $y = \dfrac{2}{1-2x^2}$

(C) $y = \dfrac{2}{1-2x^2}$

(D) $y = \dfrac{4}{1+4x^2}$

(Q9.) Integrate $\int \frac{3x^2-5x-4}{x^2-2x-3}\,dx$

(A) $3x+2\ln|x-3|-\ln|x+1|+C$

(B) $3x+\ln|x+3|-2\ln|x-1|+C$

(C) $\frac{5}{2}\ln|x-3|-\frac{1}{2}\ln|x+1|+C$

(D) $2\ln|x-3|-\ln|x+1|+C$

(E) $3x+\ln|x-2|-2\ln|x-3|+C$

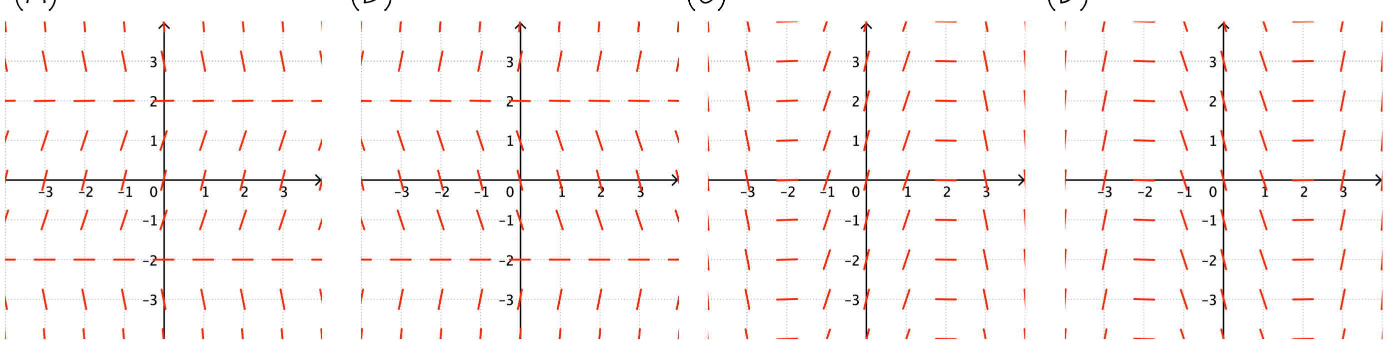
(Q10.) Evaluate $\lim\limits_{x\to\infty} \frac{\ln(2x)}{\ln(x^3+1)}$

(A) $\frac{1}{3}$
(B) $\frac{2}{3}$
(C) 1
(D) 0
(E) the limit does not exist

(Q11.) Evaluate $16-4+1-\frac{1}{4}+\frac{1}{16}-\cdots$
(A) $\frac{64}{5}$
(B) $\frac{32}{5}$
(C) $\frac{16}{5}$
(D) $\frac{32}{9}$
(E) $\frac{64}{9}$

(Q12.) Which of the following is the **slope field** for $\frac{dy}{dx}=4-y^2$

(A) (B) (C) (D)



(Q13.) Determine $\int \frac{\tan^{-1}(x^2)}{x^2}\,dx$ as a power series

(A) $C+\sum_{n=0}^{\infty}\frac{(-1)^n}{(4n+1)(2n+1)}x^{4n+1}$

(B) $C+\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)(4n+3)}x^{4n+3}$

(C) $C+\sum_{n=0}^{\infty}\frac{1}{(4n+1)(2n+1)!}x^{4n+1}$

(D) $C+\sum_{n=0}^{\infty}\frac{(-1)^n}{(4n+1)(2n+1)!}x^{4n+1}$

(Q14.) Which of the following is an example of a_n that $\lim_{n\to\infty} a_n = 0$ but $\sum_{n=1}^{\infty} a_n$ diverges?

(A) $a_n=\frac{1}{\sqrt{n}}$

(B) $a_n=\frac{1}{n!}$

(C) $a_n=\frac{1}{n^2}$

(D) $a_n=e^{-n}$

(E) $a_n=\frac{1}{\tan^{-1} n}$

(Q15.) $\int \frac{2\sin x}{\sin(2x)}\,dx = ?$

(A) $\ln|\sec x+\tan x|+C$

(B) $\ln|\sin x+\cos x|+C$

(C) $\sec x\tan x+C$

(D) $\sec x+\tan x+C$

(E) $\ln|1+\sec x\tan x|+C$

(Q16.) Evaluate $\sum_{n=1}^{\infty}\frac{1}{n^2+5n+4}$

(A) $\frac{13}{36}$

(B) $\frac{5}{4}$

(C) $\frac{1}{8}$

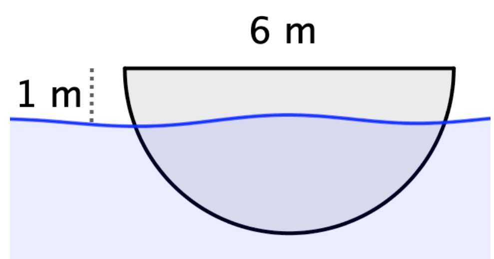
(D) $\frac{12}{5}$

(E) $\frac{3}{5}$

(Q17.) Find the **arc length** on the curve $y=\sinh x$ from $x=1$ to $x=4$?

(A) 26.437
(B) 26.115
(C) 23.639
(D) 3.655
(E) 3.639

(Q18.) The vertical plate is partially submerged in water and has the indicated shapes. Find the **hydrostatic forces** against one side of the plate



(A) 66980.42 N
(B) 63985.43 N
(C) 38980.24 N
(D) 35505.55 N
(E) 33490.28 N

(Q19.) Convert the Cartesian equation of a line $y=mx+b$ to a **polar** equation

(A) $r=\dfrac{b}{\sin\theta-m\cos\theta}$
(B) $r=\dfrac{m}{\sin\theta+b\cos\theta}$
(C) $r=\dfrac{\sin\theta}{m\sin\theta+b\cos\theta}$
(D) $r=\dfrac{b\sin\theta}{b\sin\theta-\cos\theta}$
(E) $r=\dfrac{\cos\theta}{b\sin\theta-m\cos\theta}$

(Q20.) Determine the **Maclaurin series** of $x^3\sin x$ (i.e. Taylor series at $a=0$)

(A) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{2n+4}$
(B) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{6n+4}$
(C) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^{n-1}}{(2n+1)!}x^{6n+3}$
(D) $\displaystyle\sum_{n=0}^{\infty}\frac{1}{(2n+3)!}x^{2n+3}$

(Q22.) Pikachu thinks $\int_0^{\frac{\pi}{2}} \tan x \, dx$ diverges. What do you think?

(A) Pikachu is correct.

(B) Pikachu is wrong. $\int_0^{\frac{\pi}{2}} \tan x \, dx$ converges to 2

(C) Pikachu is wrong. $\int_0^{\frac{\pi}{2}} \tan x \, dx$ converges to 1

(D) Pikachu is wrong. $\int_0^{\frac{\pi}{2}} \tan x \, dx$ diverges $\frac{1}{\sqrt{2}}$

(Q23.) Given a polar equation $r = \sec\theta \tan\theta$. Determine $\frac{dy}{dx}$ in terms of θ

(A) $2\tan\theta$
(B) $\tan^2\theta$
(C) $\csc\theta\sec\theta$
(D) $\sec^2\theta$
(E) $3\sec^3\theta - \tan^2\theta$
(F) $\sec^3\theta + 2\tan^3\theta$

(Q24.) Determine the **interval of convergence** of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n3^n}(x-2)^n$ Hint: R=3

(A) $(-1,5]$
(B) $[-1,5)$
(C) $(-1,5)$
(D) $(-3,3]$
(E) $[-3,3]$

(Q25.) A rectangular swimming pool, with length 20 m, width 12 m and depth 2 m, is filled with water to the 1.7 m mark. How much **work** is required to pump all the water out over the side?

(A) 4598160 J
(B) 4245020 J
(C) 5192205 J
(D) 5014250 J
(E) 4910350 J

(Q26.) Compute M_3 for $\int_1^4 \frac{2^x}{1+x} dx$

(A) 5.262
(B) 5.014
(C) 4.934
(D) 4.341
(E) 3.809

(Q27.) Which of the following series **converges absolutely**?

(A) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

(B) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+2}}$

(C) $\sum_{n=1}^{\infty} \frac{(-1)^n n^3}{n^3+1}$

(D) $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^3+1}$

(Q28.) Biologists stocked a lake with 600 fish and estimated the carrying capacity to be 15000. The number of fish tripled after two years. If the size of the fish population satisfies the **logistic equation**, Find the number of fish after another two years. Round your answer to the nearest whole number.

(A) 4628

(B) 4723

(C) 5201

(D) 5400

(E) 5821

(Q29.) Evaluate $\int_0^1 x \ln x \, dx$ if it converges.

(A) $\frac{-1}{4}$

(B) $\frac{-1}{2}$

(C) 1

(D) -2

(E) $\frac{-2}{3}$

(Q30.) Evaluate $\sum_{n=0}^{\infty} \frac{1}{2^n n!}$

(A) $\sqrt{e}$

(B) $\frac{1}{\sqrt{e}}$

(C) $\frac{e}{2}$

(D) e^2

(E) diverges

(F) $\frac{1}{2e}$

(Q31.) A tank with a capacity of 400 L is full of a mixture of water and chlorine with a concentration of 0.22 g of chlorine per liter. In order to reduce the concentration of chlorine, fresh water is pumped into the tank at a rate of 12 L/min. The mixture is kept stirred and is pumped out at the same rate. Let $A(t)$ be the amount of chlorine in the tank after t minutes. What is the value of $A(30)$?

(Q32.) Find a **parametrization** (with time interval) of the full circle with radius 2, centered at (1, 2), the starting point at (1, 4), traveling… **(a)** counterclockwise **(b)** clockwise

(Q33.) $\int \sin^5 x \cos^2 x \, dx$

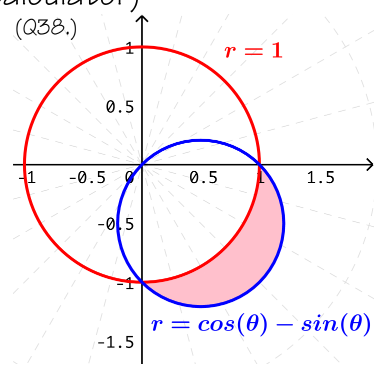
(Q34.) Determine $\int \ln(1+x^3)\, dx$ as a power series

(Q35.) Find the **centroid** of the region bounded by $y=\frac{1}{x}$, $y=0$, $x=1$ and $x=4$

(Q36.) $\int \tan^3 x \, dx$

(Q37.) Determine the power series expansion for $f(x)=\dfrac{x^5}{4+x^2}$ using **sigma notation**, at $a=0$ and state the **radius** and the **interval of convergence**

(Q38.) Find the **area** of the shade region (evaluate the integrals on your calculator)



(Q39.) Does $\sum_{n=1}^{\infty} \sin^2\left(\frac{1}{n}\right)$ converge? Justify your answer

(Q40.) $\int \sin^{-1} x \, dx$

(Q41.) $\int \dfrac{\sqrt{x^2-9}}{x^3}\, dx$

(Q42.) Write an **equation of the line tangent to** the curve defined by

$$x=t^2-6t \text{ and } y=\sqrt{t+7} \text{ at } t=2$$

(Q43.) Prove the **volume** of the cone with height h and base radius r is $V=\frac{1}{3}\pi r^2 h$

(Q44.) Determine $\int e^{x^3}\, dx$ as a **power series**. State the radius of convergence

(Q45.) How **large should n be** to guarantee that the approximation T_n to the integral $\int_1^4 e^{-x^2}\, dx$ is accurate to within 0.0005?

(Q46.) Evaluate $\lim_{t\to 0^+}\left(\dfrac{1}{t}-\dfrac{1}{e^t-1}\right)$

(Q47.) $\int x^5 e^{x^3}\, dx$

(Q48.) $\int \dfrac{1}{\sqrt{(1-x^2)^3}}\, dx$

(Q49.) For what values of r will the function $y=e^{rx}$ satisfy $3y''+2y'-8y=0$?

(Q50.) Determine the **Maclaurin series** of $\cosh x = \dfrac{e^x + e^{-x}}{2}$ (i.e. Taylor series at $a = 0$).

Hint use the Taylor formula

(Q51.) $\displaystyle\int \frac{\ln x}{\sqrt{x}}\,dx$

(Q52.) Evaluate $\displaystyle\lim_{x\to\infty}\left(\sqrt{x^2+5x+2}-\sqrt{x^2-x+5}\right)$

(Q53.) Given the sequence $\dfrac{1}{2}, \dfrac{-1}{3}, \dfrac{2}{9}, \dfrac{-4}{27}, \cdots, a_n, \cdots$

(a) does a_n converge? If so, find the value.

(b) does $\displaystyle\sum_{n=1}^{\infty} a_n$ converge? If so, find the value.

(Q54.) Find a **parametrization** (with time interval) of the line segment from (1, 2) to (5, 4).

(Q55.) Evaluate $\displaystyle\int_0^{\infty} \frac{1}{1+e^x}\,dx$ if it converges.

(Q56.) Solve $\dfrac{dy}{dx} = 8x^3 e^{-2y}$ and $y(1) = 0$

(Q57.) Find the **exactly value of surface area** obtained by

rotating the arc $\begin{cases} x = e^t - t \\ y = 4e^{\frac{t}{2}} \end{cases}$, $0 \le t \le 1$ about the x-axis

(Q58.) $\displaystyle\int x^2 \sin(3x)\,dx$

(Q59.) Does $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n\sqrt{\ln n}}$ converge? Justify your answer

(Q60.) $\displaystyle\int \frac{2x+7}{(x+1)(x^2+9)}\,dx$

(Q61.) Does $\displaystyle\int_1^{\infty} \frac{1}{x+e^x}\,dx$ converge? Justify your answer

(Q62.) Find the **centroid** of the region bounded by $y = e^{2x}$, $y = 0$, $x = 0$ and $x = 1$

(Q63.) Find the **slope of the line tangent** to the polar curve $r = \cos\theta$ at $\theta = \dfrac{\pi}{6}$

(Q64.) A tank has the shape of a frustum with height 5 m, bottom radius 6 m, top radius 2 m. It is filled with water to a height of 4 m. Find the **work** required to empty the tank by pumping all the water to the top of the tank.

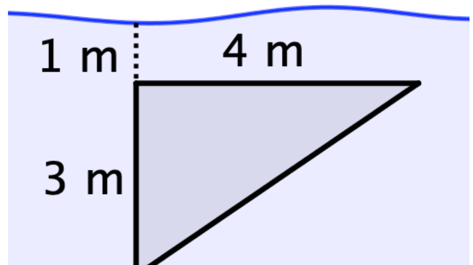
(Q65.) $\displaystyle\int \frac{1}{x^2-4x+7}\,dx$

(Q66.) Does $\displaystyle\sum_{n=1}^{\infty} \frac{4^n n!}{n^n}$ converge? Justify your answer

(Q67.) Compute S_4 for $\int_0^8 \sqrt{1+x^3}\,dx$

(Q68.) Let $y(x)$ be the solution to the differential equation $\frac{dy}{dx} = xy-2$ and $y(1)=3$. Use **Euler's Method**, with step size 0.05, to estimate $y(1.2)$

(Q69.) The vertical plate is submerged in water and has the indicated shapes. Find the **hydrostatic forces** against one side of the plat



(Q69.)

(Q70.) **True/False** $\frac{d}{dx}(\cosh x) = -\sinh x$

(Q71.) **True/False** The sum of infinitely many rational numbers has to be rational

(Q72.) **True/False** $y = x^3$ is a solution to $x^2y'' + 6xy' + 6y = 0$

(Q73.) **True/False** The polar equation $r = \theta^2$ represents a parabola

(Q74.) **True/False** $\frac{d^2y}{dx^2} = \frac{\frac{d^2y}{dt^2}}{\frac{d^2x}{dt^2}}$

(Q75.) **True/False** $\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$ converges conditionally

(Q76.) **True/False** We can use the ratio test to show that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

(Q77.) **True/False** $\lim_{x\to 0^+} \sin(x)\ln(x) = 1$

(Q78.) **True/False** $\int \sinh^3 x\,dx = \frac{1}{4}\sinh^4 x + C$

(Q79.) **True/False** If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} (a_n)^2$ has to converge

(Q80.) Prove $\sinh^{-1} x = \ln\left(x + \sqrt{1+x^2}\right)$ for any real number x

(Q81.) Prove $\frac{d}{dx}\left(\sinh^{-1} x\right) = \frac{1}{\sqrt{1+x^2}}$

(Q82.) Consider the sequence $a_n = \frac{1}{(n+2)n!}$

(a) Find a formula for the n^th partial sum

(b) Evaluate $\sum_{n=1}^{\infty} a_n$ if it converges.

(Q83.) Prove the **area** under one arch of the cycloid $\begin{cases} x = r(t-\sin t) \\ y = r(1-\cos t) \end{cases}$ is $3\pi r^2$.

(Q84.) Solve $\frac{dy}{dx} = x\sqrt{1-y^2}$ and $y(0) = \frac{1}{2}$

(Q85.) Evaluate $\lim_{x\to\infty}\frac{(\ln x)^2}{x}$

(Q86.) Does $\sum_{n=3}^{\infty}\frac{1}{2^n \ln n}$ converge? Justify your answer

(Q87.) Convert the polar equation $r=6\sin\theta-4\cos\theta$ to **Cartesian** and describe the shape

(Q88.) $\int \sin^4 x\,dx$

(Q89.) At what **point(s)** on the curve $\begin{cases} x=3t^2+1 \\ y=t^3-1 \end{cases}$ does the tangent line have slope $\frac{1}{4}$?

(Q90.) Does $\frac{1}{3}-\frac{2}{5}+\frac{3}{7}-\frac{4}{9}+\cdots$ converge? Justify your answer

(Q91.) Prove the **arc length** formula for the polar curve: $L=\int_{\theta_1}^{\theta_2}\sqrt{r^2+(\frac{dr}{d\theta})^2}\,d\theta$

(Q92.) A bottle of beer at room temperate of 75 °F is placed in a refrigerator where the temperature is 36 °F. After 40 minutes the beer has cooled to 60 °F. Use the **Newton's Law of Cooling** to find the temperature of the beer after another 40 minutes.

(Q93.) $\int e^{3x}\sin(2x)\,dx$

(Q94.) Convert $\begin{cases} x=t^2-6t \\ y=2t+1 \end{cases}$ to **Cartesian**

(Q95.) Does $\sum_{n=1}^{\infty}\left(1-\frac{1}{n}\right)^{n^2}$ converge? Justify your answer

(Q96.) Consider a population modeled by the differential equation $\frac{dP}{dt}=0.02P\left(1-\frac{P}{4500}\right)$

(a) For what values of P is the population increasing?

(b) For what values of P is the population decreasing?

(c) What are the equilibrium solutions?

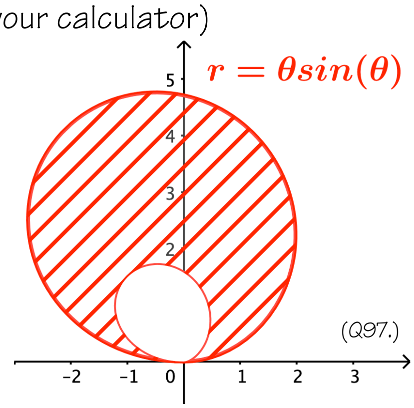
(Q97.) Find the **areas** of the shared regions (evaluate the integrals on your calculator)

(Q98.) Evaluate $\lim_{x\to 1}x^{\frac{1}{1-x}}$

(Q99.) $\int\sqrt{e^x-1}\,dx$

(Q100.) Does $\sum_{n=1}^{\infty}\frac{2n^3-n}{\sqrt{n^9+10n^3-8}}$ converge? Justify your answer

(Q101.) Find the Taylor series, in sigma notation, of the **best friend** centered at $a=3$. State the **radius** and **interval** of convergence



(Q97.)

"Best of luck on your calc 2 final!"... blackpenredpen

Errors in the video…

(Q21.) I realized I didn't include Q21 or any Quotient rule problems, so let me make it up here:

$$\frac{d}{dx}\left(\frac{\sinh x}{1+\cosh x}\right)=$$

(A) $\dfrac{1}{1+\cosh x}$ (B) $\dfrac{1}{(1+\cosh x)^2}$ (C) $\dfrac{\sinh x\cosh x}{(1+\cosh x)^2}$ (D) $\dfrac{-1}{(1+\cosh x)^2}$

(Q27.) In the video, I put $\sum_{n=1}^{\infty}\left|\frac{(-1)^n}{n!}\right|=\sum_{n=1}^{\infty}\frac{1}{n!}\le\sum_{n=1}^{\infty}\frac{1}{n^2}$. The inequality is **not correct** since $\sum_{n=1}^{\infty}\frac{1}{n!}=e-1\approx 1.718$ but $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}\approx 1.645$. What I needed to say is $\sum_{n=k}^{\infty}\frac{1}{n!}\le\sum_{n=k}^{\infty}\frac{1}{n^2}$ for some k. This is true because $n^2 << n!$ as $n\to\infty$. Alternatively, you can also use the ratio test to show $\sum_{n=k}^{\infty}\frac{1}{n!}$ converges.

(Q82.) In the video, I had $s_n=\frac{1}{2}-\frac{2}{(n+2)!}$ but it should really be $s_n=\frac{1}{2}-\frac{1}{(n+2)!}$.

Please let me know if there are more. I am sorry for the confusions these might have caused.
Thanks to everyone who watched the video and pointed out my mistakes.
11/27/2019