$\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$

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1. $\int \sin^3 x \cos x^{1/2}\, dx$

$= \int \sin^2 x \cos x^{1/2} \sin x\, dx$

$= \int (1-\cos^2 x) \cos x^{1/2} \sin x\, dx$

$\frac{u^{3/2}}{3/2}$

$= \int (\cos x^{1/2} - \cos x^{5/2}) \sin x\, dx$

$\frac{u^{5/2}}{5/2}$

$u = \cos x$ $du - \sin x\, dx$

$-du = \sin x\, dx$

$- = \int u^{1/2} - u^{5/2}\, du$

$= -\left(\frac{2\sqrt{u^3}}{3} - \frac{2\sqrt{u^5}}{5} \right)$

$= -\frac{2\sqrt{u^3}}{3} + \frac{2\sqrt{u^5}}{5}$ $\boxed{= -\frac{2\sqrt{\cos^3 x}}{3} + \frac{2\sqrt{\cos^5 x}}{5}}$

$x = 4\tan\theta$ $dx = 4\sec^2\theta\, d\theta$

$\int \frac{1}{x\sqrt{x^2+16}} dx = \int \frac{4\sec^2\theta}{4\tan\sqrt{16\tan^2+16}} d\theta$

$\frac{1}{\cos}$

$= \int \frac{4\sec^2\theta}{4\tan\, 4\sec\theta} d\theta = \int \frac{\sec\theta}{4\tan\theta} d\theta = \frac{1}{4}\int \frac{\sec\theta}{\tan\theta} d\theta$

$= \frac{1}{4}\int \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} d\theta = \frac{1}{4}\int \frac{1}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{\sin\theta} d\theta$

$= \frac{1}{4}\int \csc\theta\, d\theta$

Continued →

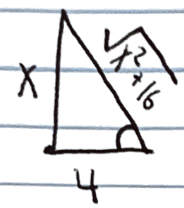
$$\arctan \frac{x}{4} = \theta$$

$$x = 4\tan\theta$$

$$= \frac{1}{4}\int \csc\theta \, d\theta \qquad = \frac{1}{4}\ln|\csc\theta - \cot\theta| + C$$

$$\boxed{= \frac{1}{4}\ln\left|\csc\left(\frac{\sqrt{x^2+16}}{x}\right) - \cot\frac{4}{x}\right| + C}$$

$\frac{O}{A}$



Sorry For the Mess!

$\frac{1}{\sin}$ $\frac{H}{O}$ $\cot\theta = \frac{A}{O}$

•Note: I couldn't see the signs of PDF

3. $$\int \frac{5x+9}{x(x^2+9)}\,dx$$

$$= \left(\frac{A}{x} + \frac{Bx+C}{x^2+9}\right)\left(x(x^2+9)\right)$$

$$5x+9 = A(x^2+9) + (Bx+C)(x)$$

$$0:\ 9 = A(9) + \cancel{(B(0)+C)}(0)$$

$\boxed{A=1}$ $\quad Ax^2 + 9A + Bx^2 + Cx$

$9 \hat{=} 9A \qquad 0 = A+B \qquad \boxed{5 = C}$

$\boxed{-B = A}$

$\boxed{-1 = B}$

A. $$\boxed{\frac{1}{x} + \frac{(-x+5)}{x^2+9}}$$

$$\int \frac{-x}{x^2+9}\,dx + \int \frac{5}{x^2+9}\,dx$$

$u = x^2+9$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$-\frac{1}{2}\int \frac{1}{u}\,du$$

$$-\frac{1}{2}\ln|x^2+9| + C$$

$$\frac{5}{9}\int \frac{1}{\left(\frac{1}{3}x\right)^2+1}\,dx \qquad \frac{15}{9}\int \frac{1}{u^2+1}\,du$$

$u = \frac{1}{3}x$

$3du = dx$

$$\frac{15}{9}\arctan u$$

B. $$\int \frac{1}{x} + \frac{(-x+5)}{x^2+9}\,dx = \boxed{\ln|x| - \frac{1}{2}\ln|x^2+9| + \frac{5}{3}\arctan\left(\frac{1}{3}x\right) + C}$$