Gavin McKay

1. $\sum_{k=1}^{\infty} \frac{\sin^2 k}{k^2}$ $\sum_{k=1}^{\infty} b_k = \frac{1}{k^2}$

- All positive, $\frac{\sin^2 k}{k^2} < \frac{1}{k^2}$

Converges by Comparison Test

2. $\sum_{k=1}^{\infty} \frac{e^{1/k}}{k}$ All positive

Comparison $\sum_{k=1}^{\infty} \frac{1}{k}$

$\lim \frac{e^{1/k}}{\cancel{k}} \cdot \frac{\cancel{k}}{1} = 1$

Diverges by limit Comparison

3. $\sum_{k=1}^{\infty} \frac{(-1)^k}{(2k+1)^3}$ $B_k = \frac{1}{(2k+1)^3}$

A $B_{k+1} = \frac{1}{(2(k+1)+1)^3} \geq \frac{1}{(2k+3)^3}$

$\boxed{(2k+3)^3 \geq (2k+1)^3}$

$B_{k+1} \leq B_k$ $\lim_{k\to\infty} \frac{1}{(2k+1)^3} = 0$

- Converges by <u>AST</u>

B $\frac{1}{(2n+3)^3} < \frac{1}{10^3}$ $10^3 < (2n+3)^3$

$10 < 2n+3$ $7 < 2n$ $\frac{3}{5} < n$

$\boxed{\text{need 4 terms}}$

C. ~~R~~ $= \dfrac{1}{(2(7)+1)^3} = \dfrac{1}{(15)^3} = \dfrac{1}{3375}$

4. $\sum_{k=1}^{\infty} (-1)^k \dfrac{k}{\sqrt{k^3+2}} \approx \sum_{k=1}^{\infty} \dfrac{k}{\sqrt{k^3}} \approx \sum_{k=1}^{\infty} \dfrac{1}{\sqrt{k}}$

• Diverges by p-series ~~for~~ $p = 1/2 < 1$

$\lim_{k\to\infty} \dfrac{k}{\sqrt{k^3+2}} \cdot \dfrac{\sqrt{k}}{1}$

$= \lim_{k\to\infty} \dfrac{\sqrt{k^3}}{\sqrt{k^3+2}} = \sqrt{1} = 1$

• Diverges by L.C.T

2. $\lim_{k\to\infty} \dfrac{k}{\sqrt{k^3+2}} = \dfrac{\frac{1}{\sqrt{k}}}{\sqrt{1+\frac{2}{k^3}}} = 0$

$\boxed{\text{Converges Absolutely}}$ by A.S.T