Dr. Burr

Main idea: Given the rate Q' at which a quantity Q changes over time, we can use integration to calculate the net change in the quantity Q over a certain time interval and to find the value of Q at some future time. We can do this because of the Fundamental Theorem of Calculus.

Theorem (Fundamental Theorem of Calculus, part 2). If f is continuous on [a, b] and F is any antiderivative of f on [a, b], then

$$\int_{a}^{b} f(x) \ dx = \int_{a}^{b} F'(x) \ dx = F(b) - F(a)$$

Net Change and Future Value: Suppose a quantity Q changes over time at a known rate Q'.

• Then the **net change** in Q between t = a and t = b > a is

$$Q(b) - Q(a) =$$

• Given the initial value Q(0), the **future value** of Q at time $t \geq 0$ is

$$Q(t) =$$

Velocity, Position, Displacement, Distance Traveled, and Acceleration

Let s(t) be the position (relative to the origin) of an object moving along a line at time t. Then

- velocity of an object at time t is v(t) = s'(t) and speed of the object at time t is |v(t)|.
- acceleration of the object at time t is a(t) = v'(t).
- displacement of the object between t = a and t = b > a is

$$s(b) - s(a) =$$

- distance traveled by the object between t = a and t = b > a is
- position from velocity: s(t) =

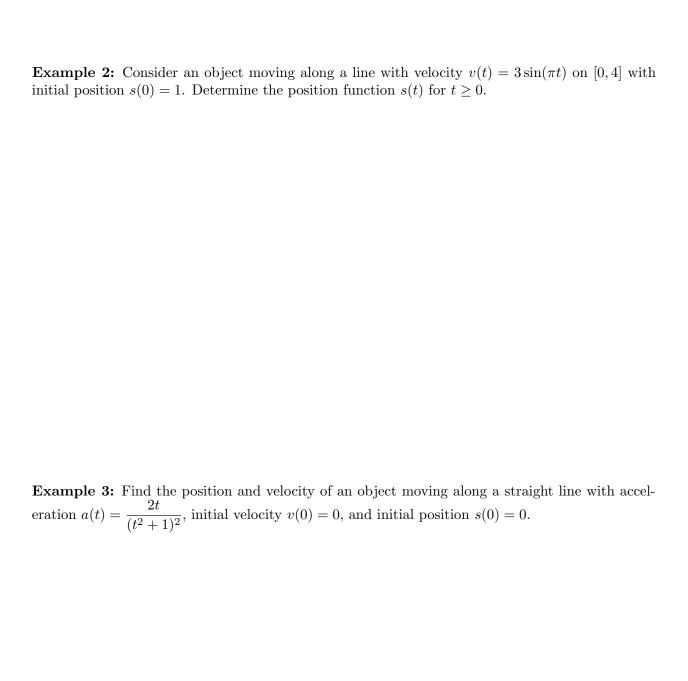
for $t \geq 0$, given v(t) and initial position s(0).

• velocity from acceleration: v(t) =

for $t \geq 0$, given a(t) and initial velocity v(0).

Example 1: Consider an object moving along a line with velocity $v(t) = 3t^2 - 6t$ on [0,3], where time t is measured in seconds and velocities have units of m/s.

- (a) Determine when the motion is in the positive direction and when it is in the negative direction.
- (b) Find the displacement over the interval [0, 3].
- (c) Find the distance traveled over the interval [0, 3].



Example 4: Water flows from the bottom of a storage tank at rate of r(t) = 200 - 4t liters per minute, where $0 \le t \le 50$. Find the amount of water that flows from the tank during the first 10 minutes.

Example 5: When records were first kept (t = 0), the population of a rural town was 250 people. During the following years, the population grew at a rate of $P'(t) = 30(1+\sqrt{t})$, where t is measured in years.

- (a) Find the population after 9 years.
- (b) Find the population P(t) at any time $t \geq 0$.

Example 6: A data collection probe is dropped from a stationary balloon, and it falls with a velocity (in m/s) given by v(t) = 9.8t, neglecting air resistance. After 10 seconds, a chute deploys and the probe immediately slows to a constant speed of 10 meters/second, which it maintains until it enters the ocean.

- (a) Graph the velocity function.
- (b) How far does the probe fall in the first 30 seconds after it is released?
- (c) If the probe was released from an altitude of 3 kilometers, when does it enter the ocean?