

Exam 3

Question 1

A. The cost of the minimum spanning tree is 19. This is calculated from just adding the edges up from the minimum spanning tree = $1 + 2 + 1 + 3 + 4 + 5 + 3 = 19$

B. Finding how many minimum spanning trees are present can be found through a few steps.

First we need to construct the adjacency matrix for the graph

Let's call this A

	A	B	C	D	E	F	G	H
A	0	1	0	0	1	0	0	0
B	1	0	1	0	1	1	0	0
C	0	1	0	1	0	1	1	0
D	0	0	1	0	0	0	1	1
E	1	1	0	0	0	1	0	0
F	0	1	1	0	1	0	1	0
G	0	0	1	1	0	1	0	1
H	0	0	0	1	0	0	1	0

We then construct the degree matrix.

Let's call this B

	A	B	C	D	E	F	G	H
A	2	0	0	0	0	0	0	0
B	0	4	0	0	0	0	0	0
C	0	0	4	0	0	0	0	0
D	0	0	0	3	0	0	0	0
E	0	0	0	0	3	0	0	0
F	0	0	0	0	0	4	0	0
G	0	0	0	0	0	0	4	0
H	0	0	0	0	0	0	0	2

So we do B - A
Which gives us

	A	B	C	D	E	F	G	H
A	2	-1	0	0	-1	0	0	0
B	-1	4	-1	0	-1	-1	0	0
C	0	-1	4	-1	0	-1	-1	0
D	0	0	-1	3	0	0	-1	-1
E	-1	-1	0	0	3	-1	0	0
F	0	-1	-1	0	-1	4	-1	0
G	0	0	-1	-1	0	-1	4	-1
H	0	0	0	-1	0	0	-1	2

Calculate the positive cofactors.
= 377

C. Minimum Spanning Tree

Edge Cost:

A-E 1
E-F 1
E-B 2
B-F 2
F-G 3
G-H 3
G-C 4
F-C 5
B-C 5
G-D 5
C-D 6
D-H 7

Text that is highlighted red is omitted edges because the adding the resulting edge would form a cycle which is not allowed for spanning trees. The resulting MST has 7 edges and 8 vertices which satisfies the requirements for a spanning tree

Question 2

Constructed min heap look like the following

[3, 10, 4, 12, 34, 78, 31, 89, 18, 46]

Performing Delete Min The first time changes our heap to

[4, 10, 12, 18, 78, 31, 89, 34, 46]

Performing Delete Min the second time changes our heap to

[10, 12, 18, 46, 31, 89, 34, 78]

Finally insert(15) changes our heap to

[10, 12, 18, 15, 31, 89, 34, 78, 46]