Velocity and Net Change (6.1)

Main idea: Given the rate Q' at which a quantity Q changes over time, we can use integration to calculate the net change in the quantity Q over a certain time interval and to find the value of Q at some future time. We can do this because of the Fundamental Theorem of Calculus.

Theorem (Fundamental Theorem of Calculus, part 2)**.** If f is continuous on $[a, b]$ and F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x)\,dx = \int_a^b F'(x)\,dx = F(b) - F(a)$$

Net Change and Future Value: Suppose a quantity Q changes over time at a known rate Q'.

- Then the **net change** in Q between $t = a$ and $t = b > a$ is

$$Q(b) - Q(a) =$$

- Given the initial value $Q(0)$, the **future value** of Q at time $t \geq 0$ is

$$Q(t) =$$

Velocity, Position, Displacement, Distance Traveled, and Acceleration

Let $s(t)$ be the position (relative to the origin) of an object moving along a line at time t. Then

- **velocity** of an object at time t is $v(t) = s'(t)$ and **speed** of the object at time t is $|v(t)|$.
- **acceleration** of the object at time t is $a(t) = v'(t)$.
- **displacement** of the object between $t = a$ and $t = b > a$ is

$$s(b) - s(a) =$$

- **distance traveled** by the object between $t = a$ and $t = b > a$ is
- **position from velocity**: $s(t) =$,

  for $t \geq 0$, given $v(t)$ and initial position $s(0)$.
- **velocity from acceleration**: $v(t) =$,

  for $t \geq 0$, given $a(t)$ and initial velocity $v(0)$.

Example 1: Consider an object moving along a line with velocity $v(t) = 3t^2 - 6t$ on $[0, 3]$, where time t is measured in seconds and velocities have units of m/s.

(a) Determine when the motion is in the positive direction and when it is in the negative direction.
(b) Find the displacement over the interval $[0, 3]$.
(c) Find the distance traveled over the interval $[0, 3]$.

Example 2: Consider an object moving along a line with velocity $v(t) = 3\sin(\pi t)$ on $[0, 4]$ with initial position $s(0) = 1$. Determine the position function $s(t)$ for $t \geq 0$.

Example 3: Find the position and velocity of an object moving along a straight line with acceleration $a(t) = \dfrac{2t}{(t^2+1)^2}$, initial velocity $v(0) = 0$, and initial position $s(0) = 0$.

Example 4: Water flows from the bottom of a storage tank at rate of $r(t) = 200 - 4t$ liters per minute, where $0 \leq t \leq 50$. Find the amount of water that flows from the tank during the first 10 minutes.

Example 5: When records were first kept ($t = 0$), the population of a rural town was 250 people. During the following years, the population grew at a rate of $P'(t) = 30(1+\sqrt{t})$, where t is measured in years.

(a) Find the population after 9 years.

(b) Find the population $P(t)$ at any time $t \geq 0$.

Example 6: A data collection probe is dropped from a stationary balloon, and it falls with a velocity (in m/s) given by $v(t) = 9.8t$, neglecting air resistance. After 10 seconds, a chute deploys and the probe immediately slows to a constant speed of 10 meters/second, which it maintains until it enters the ocean.

(a) Graph the velocity function.
(b) How far does the probe fall in the first 30 seconds after it is released?
(c) If the probe was released from an altitude of 3 kilometers, when does it enter the ocean?