MATH 1060
Calculus of One Variable I

Test 2
Version A

Fall 2016
Sections 3.3 – 3.10

Student's Printed Name: ______________________ **CUID:** __________

Instructor: ______________________ **Section:** __________

Instructions: You are **not** permitted to use a calculator on any portion of this test. You are **not** allowed to use a textbook, notes, cell phone, computer, or any other technology on any portion of this test. All devices must be turned off and stored away while you are in the testing room.

During this test, **any** kind of communication with any person other than the instructor or a designated proctor is understood to be a violation of academic integrity.

No part of this test may be removed from the examination room.

Read each question carefully. To receive full credit for the free response portion of the test, you must:

1. Show legible, logical, and relevant justification which supports your final answer.
2. Use complete and correct mathematical notation.
3. Include proper units, if necessary.
4. Give answers as exact values whenever possible.

You have **90 minutes** to complete the entire test.

Do not write below this line.

Free Response Problem	Possible Points	Points Earned
1. a.	6	
1. b.	4	
2. a.	4	
2. b.	6	
2. c	2	
3.	10	
4. a.	5	
4. b.	5	
4. c.	5	

Free Response Problem	Possible Points	Points Earned
5. a.	5	
5. b.	4	
6. a.	5	
6. b.	5	
7. (Scantron)	1	
Free Response	67	
Multiple Choice	33	
Test Total	100	

red pen is precalculus material
black pen is calculus material

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Multiple Choice: There are 11 multiple choice questions. Each question is worth 3 points and has one correct answer. The multiple choice problems will be 33% of the total grade. Circle your choice on your test paper and bubble the corresponding answer on your Scantron. Any questions involving inverse trigonometric functions should be answered based on the domain restrictions for trigonometric functions used in Section 1.6.

1.
(3 pts.) Find the **second** derivative $\frac{d^2y}{dx^2}$ if $x - y^2 = 1$.

a) $\frac{d^2y}{dx^2} = -\frac{1}{4y^3}$ **b)** $\frac{d^2y}{dx^2} = -\frac{1}{2y}$ **c)** $\frac{d^2y}{dx^2} = \frac{1}{4y^3}$ **d)** $\frac{d^2y}{dx^2} = \frac{1}{2y}$

$1 - 2y\frac{dy}{dx} = 0$

$-2y\frac{dy}{dx} = -1$

$\frac{dy}{dx} = \frac{1}{2y} = \frac{1}{2}y^{-1}$

$\frac{d^2y}{dx^2} = -\frac{1}{2}y^{-2}\frac{dy}{dx}$

$= -\frac{1}{2}y^{-2}\left(\frac{1}{2}y^{-1}\right)$

$= -\frac{1}{4}y^{-3}$

2.
(3 pts.) Let $h(x) = f(g(3x))$. Use the table to evaluate $h'(1)$.

evaluating function composition

	$x=1$	$x=2$	$x=3$	$x=4$
$f(x)$	2	3	1	3
$f'(x)$	6	1	8	2
$g(x)$	1	4	4	3
$g'(x)$	4	-5	5	- 4

a) 30 **b)** 10 **c)** 40 **d)** 3

$h'(x) = f'(g(3x))\,g'(3x)\cdot 3$

$h'(1) = f'(g(3))\,g'(3)\cdot 3$

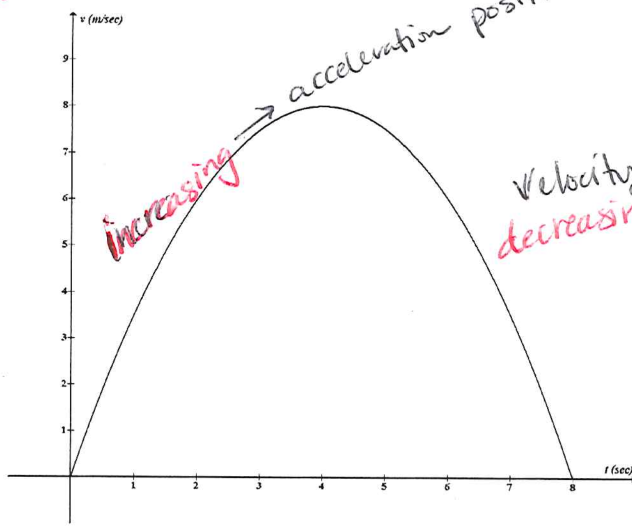
$= f'(4)(5)(3)$

$= 2(5)(3)$

$= 30$

3. **(3 pts.)** The figure shows the **velocity,** $v(t)$, of an object moving in a straight line as a function of time t, where $0 \le t \le 8$. Which statement about the velocity and acceleration of the object is true at $t = 6$?

describing graph of function



a) $v(6) < 0$ and $a(6) < 0$

b) $v(6) > 0$ and $a(6) > 0$

c) $v(6) > 0$ and $a(6) < 0$

d) $v(6) < 0$ and $a(6) > 0$

4. **(3 pts.)** Find $\frac{dy}{dx}$ if $y = 4\sqrt{x+\sqrt{x}}$.

$= 4(x + x^{1/2})^{1/2}$

$y' = 4\left[\frac{1}{2}(x + x^{1/2})^{-1/2}\left(1 + \frac{1}{2}x^{-1/2}\right)\right]$

a) $\frac{dy}{dx} = \frac{2 + \frac{1}{\sqrt{x}}}{\sqrt{x+\sqrt{x}}}$

b) $\frac{dy}{dx} = \frac{\frac{1}{2\sqrt{x}}}{2\sqrt{x+\sqrt{x}}}$

c) $\frac{dy}{dx} = \frac{\frac{1}{\sqrt{x}}}{2\sqrt{x+\sqrt{x}}}$

d) $\frac{dy}{dx} = \frac{1 + \frac{1}{2\sqrt{x}}}{2\sqrt{x+\sqrt{x}}}$

$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$

$1 + \cot^2 x = \csc^2 x$

5.
(3 pts.) Find the derivative of $y = \dfrac{5 + 5\cot^2 x}{3\csc x}$. Hint: Simplify before finding the derivative.

trig identity

a) $y' = -\dfrac{5}{3}\csc x \cot x$

b) $y' = -\dfrac{1}{3}\csc x \cot x$

c) $y' = \dfrac{-10\csc^2 x \cot x}{3\sec x \tan x}$

d) $y' = \dfrac{5}{3}\sec x \tan x$

$y = \dfrac{5(1+\cot^2 x)}{3\csc x} = \dfrac{5\csc^2 x}{3\csc x} = \dfrac{5}{3}\csc x$

$y' = \dfrac{5}{3}(-\csc x \cot x)$

6.
(3 pts.) The edge of a cube is measured to be 5 centimeters, with a maximum possible error of 0.2 centimeters. Use a differential to estimate the maximum error in calculating the **volume** of the cube.

a) 75 cm³

b) 15 cm³

c) 25 cm³

d) 1 cm³

$V = x^3$ volume of cube

$\dfrac{dv}{dx} = 3x^2 \Rightarrow dv = 3x^2\,dx$

$= 3(5)^2(0.2)$

$= 75(0.2)$

$= 75\left(\frac{2}{10}\right)$

$= 75\left(\frac{1}{5}\right)$

$= 15 \text{ cm}^3$

7. Let $f(x) = \sqrt[3]{x}$. Use the linearization of f at $x = 8$ to estimate $\sqrt[3]{9}$.
(3 pts.)

equation of line

a) $\sqrt[3]{9} \approx 2$

b) $\sqrt[3]{9} \approx 3$

c) $\sqrt[3]{9} \approx \frac{25}{12}$

d) $\sqrt[3]{9} \approx \frac{23}{12}$

$f'(x) = \frac{1}{3}x^{-2/3}$ $\quad f'(8) = \frac{1}{3}(8)^{-2/3} = \frac{1}{3}\left(\frac{1}{4}\right) = \frac{1}{12}$

$f(8) = \sqrt[3]{8} = 2$

$y - 2 = \frac{1}{12}(x-8)$

thus linearization is

$L(x) = 2 + \frac{1}{12}(x-8)$

$L(9) = 2 + \frac{1}{12}(9-8) = 2 + \frac{1}{12} = \frac{25}{12}$

8. Evaluate $\lim_{x\to 0} \frac{3\sin(ax)}{bx}$ (constants a and b, where $b \neq 0$)
(3 pts.)

a) $\frac{3}{b}$

b) $\frac{3a}{b}$

c) $\frac{3}{ab}$

d) $\frac{3}{a}$

$$\lim_{x\to 0} \frac{3\sin(ax)}{bx} = \lim_{x\to 0} \frac{\sin ax}{ax} \cdot \frac{3a}{b}$$

$$= 1 \cdot \frac{3a}{b}$$

$$= \frac{3a}{b}$$

equation of line
evaluating trig function

9. (3 pts.) Find the equation of the line tangent to the graph of $y = 2x - \cos(2x)$ at $x = \frac{\pi}{2}$.

a) $y = 2x + 1$ b) $y = -x$

c) $y = -x + 2\pi$ d) $y = -x + \pi$

$y' = 2 + \sin 2x \cdot 2 = 2 + 2\sin 2x$

$y'(\frac{\pi}{2}) = 2 + 2\sin\pi = 2$

$y(\frac{\pi}{2}) = \pi - \cos\pi = \pi + 1$

$y - (\pi + 1) = 2(x - \frac{\pi}{2})$

$y - \pi - 1 = 2x - \pi$ $y = 2x + 1$

Circumference of a circle

10. (3 pts.) The circumference of a circle is increasing at 2 centimeters per second. At what rate is the radius of the circle increasing when the radius is exactly 20 centimeters?
Note: The circumference C of a circle of radius r is $C = 2\pi r$.

a) $\frac{dr}{dt} = 2\ cm/s$ b) $\frac{dr}{dt} = \frac{1}{20\pi}\ cm/s$

c) $\frac{dr}{dt} = \frac{20}{\pi}\ cm/s$ d) $\frac{dr}{dt} = \frac{1}{\pi}\ cm/s$

$C = 2\pi r$

$\frac{dc}{dt} = 2\pi \frac{dr}{dt}$ ← calculus

$\frac{dr}{dt} = \frac{dc}{dt} \cdot \frac{1}{2\pi} = 2 \cdot \frac{1}{2\pi} = \frac{1}{\pi}$

11.
(3 pts.) Assume g is a differentiable function, where $y = \sin^{-1}(e^{g(x)})$. Find $\frac{dy}{dx}$.

a) $\frac{dy}{dx} = \frac{e^{g(x)} g'(x)}{\sqrt{1+(e^{g(x)})^2}}$

b) $\frac{dy}{dx} = \frac{e^{g(x)}}{\sqrt{1-(e^{g(x)})^2}}$

c) $\frac{dy}{dx} = \frac{e^{g(x)} g'(x)}{\sqrt{1-(e^{g(x)})^2}}$

d) $\frac{dy}{dx} = \frac{e^{g(x)}}{\sqrt{1+(e^{g(x)})^2}}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(e^{g(x)})^2}} \cdot e^{g(x)} g'(x)$$

Free Response: The Free Response questions will be 67% of the total grade. Read each question carefully. To receive full credit, you must show legible, logical, and relevant justification which supports your final answer. Give answers as exact values. Any questions involving inverse trigonometric functions should be answered based on the domain restrictions for trigonometric functions used in Section 1.6.

1. **(10 pts.)** Consider the curve defined by the equation $\cos(y^2)+x=e^y$.

a) **(6 pts.)** Find $\frac{dy}{dx}$.

$$-\sin(y^2)\cdot 2y\frac{dy}{dx}+1=e^y\frac{dy}{dx}$$

$$1=e^y\frac{dy}{dx}+\sin(y^2)\cdot 2y\frac{dy}{dx}$$

$$1=\frac{dy}{dx}\left(e^y+\sin(y^2)2y\right)$$

$$\frac{dy}{dx}=\frac{1}{e^y+\sin(y^2)\cdot 2y}$$

b) **(4 pts.)** Find the equation of the line **normal** to the curve at the point $(0,0)$.

equation of normal line

$$\left.\frac{dy}{dx}\right|_{(0,0)}=\frac{1}{e^0+\sin(0)(0)}=1 \quad \text{so slope} = -1$$

$$y-0=-1(x-0)$$

$$y=-x$$

2. **(12 pts.)** A feather is dropped on the moon from a height of 40 meters. Its height s (in meters) above the surface of the moon t seconds after it is dropped is given by $s(t) = 40 - 0.8t^2$.

a) **(4 pts.)** At what time will the feather be 20 meters above the surface? purely precalculus problem

$s(t) = 20$

$40 - 0.8t^2 = 20$

$-0.8t^2 = -20$

$t^2 = 25$

$t = \pm 5$ but $t \neq -5$ since time is positive

$t = 5$ seconds

b) **(6 pts.)** Find the velocity of the feather the moment it strikes the surface.

$s(t) = 0$

$40 - 0.8t^2 = 0$

$0.8t^2 = 40$

$t^2 = 50$

$t = \pm 5\sqrt{2}$ but $t \neq -5\sqrt{2}$

$s'(t) = -1.6t$

$s'(5\sqrt{2}) = -1.6(5\sqrt{2})$

$= -8\sqrt{2} \frac{m}{s}$

c) **(2 pts.)** What is the acceleration due to gravity on the moon?

$s''(t) = -1.6 \, m/s^2$

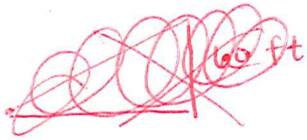
3. **(10 pts.)** A hot-air balloon is rising vertically at a constant rate of 20 feet per second. A motorcycle traveling in a straight line on flat ground passes directly beneath the balloon at the moment the balloon is 60 feet above the ground. If the motorcycle is traveling at a constant rate of 50 feet per second, at what rate is the distance between the balloon and the motorcycle changing two seconds after the motorcycle passes beneath the balloon?

parsing word problem

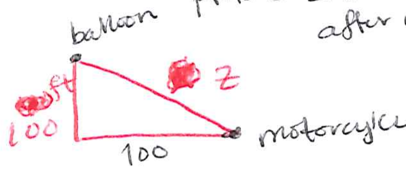
Pythagorean theorem

20 ft/s $\uparrow$

$\rightarrow 50$ ft/s



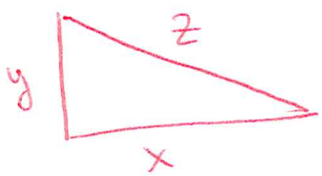
picture 2 seconds after motorcycle passed balloon



z : distance between balloon & motorcycle

$$z^2 = 100^2 + 100^2 = \underset{10000}{} + 10000 = \overset{10^4}{} 20000 = 100\sqrt{2}$$

so at any time,



$$z^2 = x^2 + y^2$$

$$\frac{dz}{dt} = ? \qquad 2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$$

$$\left.\frac{dz}{dt}\right|_{t=2} = \frac{100(50) + 100(20)}{100\sqrt{2}}$$

$$= \frac{70}{\sqrt{2}} \frac{\text{ft}}{\text{sec}}$$

4. **(15 pts.)** Find the indicated derivatives. Assume $g(x)$ is a differentiable function wherever it appears. **DO NOT TRY TO SIMPLIFY YOUR ANSWERS.**

a. **(5 pts.)** Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left(g(e^{3x})\right)$

$$y' = \frac{1}{1+(g(e^{3x}))^2} \cdot g'(e^{3x})\, e^{3x} \cdot 3$$

b. **(5 pts.)** Find $g'(x)$ if $g(x) = \ln\left(\frac{5+\sin x}{7^x}\right)$

$$g'(x) = \frac{1}{\frac{5+\sin x}{7^x}} \left[\frac{7^x \cos x - (5+\sin x)\, 7^x \ln 7}{(7^x)^2}\right]$$

c. **(5 pts.)** Find $f'(x)$ if $f(x) = \sqrt[5]{[g(x)]^{10} - \sec(x^5)}$

$$f'(x) = \frac{1}{5}\left[(g(x))^{10} - \sec x^5\right]^{-4/5} \left[10(g(x))^9 g'(x) - \sec x^5 \tan x^5 \cdot 5x^4\right]$$

parsing word problems

inverse functions

5. **(9 pts.)** The population of wolves on an island is given by $P(t) = P_0e^{kt}$, where P is the number of wolves and t is time in years since the beginning of the year 2010, e.g, the beginning of 2016 corresponds to $t = 6$. The population is known to double every 4 years.

a) **(5 pts.)** If there were P_0 wolves present in 2010, find the population of wolves in the year 2020. Give you final answer in the form aP_0, where a is a constant.

$P(4) = 2P_0$

so $P(4) = P_0 e^{k(4)} = 2P_0$

$e^{4k} = 2$

$4k = \ln 2$

$k = \frac{\ln 2}{4}$

$P(10) = P_0 e^{\frac{\ln 2}{4}(10)}$

$= P_0 (e^{\ln 2})^{5/2}$

$= P_0 (2)^{5/2}$

b) **(4 pts.)** If there were P_0 wolves present in 2010, in what year will the population be five times that amount? Give you answer in terms of natural logarithms.

$P(t) = 5P_0$

$P_0 e^{\frac{\ln 2}{4}t} = 5P_0$

$e^{\frac{\ln 2}{4}t} = 5$

$\frac{\ln 2}{4}t = \ln 5$

$t = \frac{4\ln 5}{\ln 2}$

so in the year

$2010 + \frac{4\ln 5}{\ln 2}$

6. **(10 pts.)** Let $y = f(x)^{\tan x}$, where $f(x)$ is a differentiable function with

i) $f(0) = e$ *ii)* $\lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = 0$

a) **(5 pts.)** Find $\frac{dy}{dx}$.

$\ln y = \tan x \ln f(x)$

$$\frac{1}{y}\frac{dy}{dx} = \tan x \cdot \frac{1}{f(x)} \cdot f'(x) + \sec^2 x \ln f(x)$$

$$\frac{dy}{dx} = f(x)^{\tan x}\left[\frac{\tan x}{f(x)} \cdot f'(x) + \sec^2 x \ln f(x)\right]$$

b) **(5 pts.)** Find the equation of the line tangent to the graph of $y = f(x)^{\tan x}$ at $x = 0$.

$$\left.\frac{dy}{dx}\right|_{x=0} = e^0\left[\frac{0}{e}(0) + 1 \cdot \ln e\right] = 1$$

$y - 1 = 1 \cdot (x - 0)$ $y(0) = f(0)^{\tan 0} = e^0 = 1$

$y = x + 1$

Scantron **(1 pt.)**

Check to make sure your Scantron form meets the following criteria. If any of the items are NOT satisfied when your Scantron is handed in and/or when your Scantron is processed one point will be subtracted from your test total.

My Scantron:

- ☐ is bubbled with firm marks so that the form can be machine read;
- ☐ is not damaged and has no stray marks (the form can be machine read);
- ☐ has **11** bubbled in answers;
- ☐ has **MATH 1060** and my section number written at the top;
- ☐ has my instructor's last name written at the top;
- ☐ has Test No. **2** written at the top;
- ☐ has the correct test version written at the top **and** bubbled in below my XID;
- ☐ shows my correct XID both written and bubbled in;

  Bubble a zero for the leading C in your XID.

Please read and sign the honor pledge below.

On my honor, I have neither given nor received inappropriate or unauthorized information at any time before or during this test.

Student's Signature: ______________________________________