MATH 1060: Exam 2 Review Sheet

3.3: Rules of Differentiation

- $\frac{d}{dx}(c) = 0$
- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(x) = 1$
- $\frac{d}{dx}(cf(x)) = cf'(x)$
- $\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$

3.3: Exponential Differentiation

- $\frac{d}{dx}(e^x) = e^x$
- $\frac{d}{dx}(e^{g(x)}) = e^{g(x)}g'(x)$

3.4: The Product Rule

If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = \left[\frac{d}{dx}f(x)\right]g(x) + f(x)\left[\frac{d}{dx}g(x)\right]$$

or

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

3.4: The Quotient Rule

If f and g are both differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\left[\frac{d}{dx}f(x)\right]g(x) - f(x)\left[\frac{d}{dx}g(x)\right]}{[g(x)]^2}$$

or

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

3.7: The Chain Rule

- $(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$
  1. Take the derivative of the outside function and leave the inside unchanged
  2. Multiply by the derivative of the inside function

3.6: Marginal Cost

- $C(x)$ is the total cost to produce x units
- **Average cost:** $\frac{C(x)}{x}$
- **additional cost:** $\Delta C = C(x_2) - C(x_1)$
- **marginal cost:** $C'(n) \approx C(n+1) - C(n)$

3.6: Rates of Change

instantaneous rate of change: the derivative; also the limit of the average rates of change
elapsed time: Δt
displacement: $\Delta s = f(a + \Delta t) - f(a)$
average velocity: $\frac{\Delta s}{\Delta t}$
velocity: the derivative of position with respect to time

$$v(t) = s'(t)$$

Note: The sign of velocity indicates direction
speed: the magnitude of the velocity

$$\text{speed} = |v(t)|$$

acceleration: the derivative of velocity with respect to time

$$a(t) = v'(t) = s''(t)$$

Note:

- Speeding up: velocity and acceleration have the same sign
- Slowing down: velocity and acceleration have opposite signs

Note: To find when an object is at rest, set velocity equal to 0 and solve for t.
Free Fall:

- The object reaches its maximum height when velocity is 0.
  1. set $v(t) = 0$
  2. solve for t. This is the time the object reaches its maximum height
  3. plug in the time you found into $s(t)$ to get the maximum height
- The object hits the ground when the height=0.
  1. Find the time the object hits the ground by solving $s(t) = 0$
  2. Find the velocity with which the object hits the ground by plugging in time to velocity function.

3.5: Special Trig Limits

- $\lim_{x \to 0} \frac{\sin x}{x} = 1$
- $\lim_{x \to 0} \frac{\cos x - 1}{x} = 0$

Note: You need to make sure the correct multipliers in front of x are in place to use these special limits.

3.5: Trig Derivatives

- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\cot x) = -\csc^2 x$
- $\frac{d}{dx}(\sec x) = \sec x \tan x$
- $\frac{d}{dx}(\csc x) = -\csc x \cot x$

3.10: Inverse Trig Derivatives

- $\frac{d}{dx}\left[\sin^{-1} x\right] = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}\left[\cos^{-1} x\right] = \frac{-1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}\left[\tan^{-1} x\right] = \frac{1}{1+x^2}$
- $\frac{d}{dx}\left[\csc^{-1} x\right] = \frac{-1}{|x|\sqrt{x^2-1}}$
- $\frac{d}{dx}\left[\sec^{-1} x\right] = \frac{1}{|x|\sqrt{x^2-1}}$
- $\frac{d}{dx}\left[\cot^{-1} x\right] = \frac{-1}{1+x^2}$

Note: Do not forget CHAIN RULE!! You must multiply by the derivative of the inside!
i.e.

$$\frac{d}{dx}\left[\sin^{-1}(g(x))\right] = \frac{1}{\sqrt{1-g(x)^2}} \cdot g'(x)$$

3.8: Implicit Differentiation

implicit form: an equation that is not solved for one variable

Implicit Differentiation Method:

1. Differentiate both sides with respect to x
2. Solve for $\frac{dy}{dx}$ (or y')

Note: You need to apply the chain rule for terms involving y!

Examples:

- $\frac{d}{dx}[y] = \frac{dy}{dx} = y'$
- $\frac{d}{dx}[y^2] = 2y\frac{dy}{dx} = 2yy'$

Implicit Differentiation for Second Derivatives

1. First find $\frac{dy}{dx}$
2. Differentiate $\frac{dy}{dx}$
3. Solve for $\frac{d^2y}{dx^2}$
4. Substitute $\frac{dy}{dx}$ into $\frac{d^2y}{dx^2}$

3.9: Exponential Function with Base a

$$\frac{d}{dx}(a^x) = a^x \ln a$$
$$\frac{d}{dx}(a^{g(x)}) = a^{g(x)} g'(x) \ln a$$

Note: Remember you can always use logarithmic differentiation for these if you can't remember this rule.

3.9: Derivatives of Logarithmic Functions

- $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$
- $\frac{d}{dx}(\ln x) = \frac{1}{x}$
- $\frac{d}{dx}(\ln |x|) = \frac{1}{x}$
- $\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$ OR $\frac{d}{dx}[\ln(g(x))] = \frac{g'(x)}{g(x)}$

Logarithmic Differentiation:
We use Log Diff when we have a variable in our base and a variable in our exponent

1. Take ln of both sides
2. Use the properties of $\ln(x)$ to simplify
3. Use implicit differentiation to take derivative of both sides
4. Isolate y' (or $\frac{dy}{dx}$)
5. Substitute y into result.

Recall Laws of Logarithms:

- $\ln(xy) = \ln(x) + \ln(y)$ for $x > 0, y > 0$
- $\ln(\frac{x}{y}) = \ln(x) - \ln(y)$ for $x > 0, y > 0$
- $\ln(x^p) = p\ln(x)$ for $x > 0$ and $p \in \mathbb{R}$

3.11: Related Rates

Method:

1. Draw a diagram
2. introduce notation and include units
3. express the given information and the required rate in terms of derivatives
4. write an equation that relates the various quantities
5. use implicit differentiation and the chain rule to differentiate both sides of the equation with respect to time
6. substitute the given info into the result
7. solve for the unknown rate
8. write a summary sentence

Things to Remember

Find where the tangent line is horizontal:

1. Find derivative
2. Set derivative equal to 0
3. Solve for x

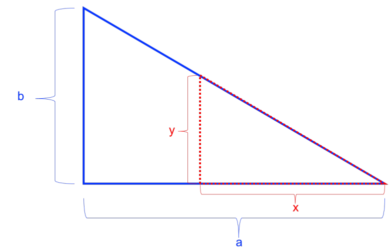
Find the equation of tangent line:

1. Find derivative.
2. If you are not given both x_1 and y_1 of the point of tangency, calculate y_1 by evaluating the ORIGINAL function at x_1.
3. Evaluate derivative at the point of tangency, (x_1, y_1). This is your slope of the tangent line, $m_{\tan}$
4. Plug $x_1, y_1, m_{\tan}$ into the point slope formula $y - y_1 = m_{\tan}(x - x_1)$ and solve for y.

Find the equation of normal line:

1. Find derivative.
2. If you are not given both x_1 and y_1 of the point of tangency, calculate y_1 by evaluating the ORIGINAL function at x_1.
3. Evaluate derivative at the point of tangency, (x_1, y_1). This is your slope of the tangent line, $m_{\tan}$. To get the slope of the normal line, take the negative reciprocal of $m_{\tan}$. $m_{\text{norm}} = \frac{-1}{m_{\tan}}$
4. Plug $x_1, y_1, m_{\text{norm}}$ into the point slope formula $y - y_1 = m_{\text{norm}}(x - x_1)$ and solve for y.

Similar Triangles



$\frac{a}{b} = \frac{x}{y}$