Student's Printed Name: ________________________________ **XID: C**_______________

Instructor: ____ **Section:** ______________

No questions will be answered during this exam.

If you consider a question to be ambiguous, state your assumptions in the margin and do the best you can to provide the correct answer.

Instructions: You are **not** permitted to use a calculator on any portion of this test. You are **not** allowed to use a textbook, notes, cell phone, computer, or any other technology on any portion of this test. All devices must be turned off and stored away while you are in the testing room.

During this test, **any** kind of communication with any person other than the instructor or a designated proctor is understood to be a violation of academic integrity.

No part of this test may be removed from the examination room.

Read each question carefully. To receive full credit for the free response portion of the test, you must:

1. Show legible, logical, and relevant justification which supports your final answer.
2. Use complete and correct mathematical notation.
3. Include proper units wherever appropriate.
4. Give answers as exact values whenever possible.

You have **90 minutes** to complete the entire test.

Do not write below this line.

Free Response Problem	Possible Points	Points Earned	Free Response Problem	Possible Points	Points Earned
1.a.	6		4.	10	
1.b.	6		5.a.	4	
2.a.	8		5.b.	2	
2.b.	4		5.c.	6	
3.a.	4		6. (Scantron)	1	
3.b.	4		**Free Response**	58	
3.c.	3		**Multiple Choice**	42	
			Test Total	100	

precalculus will be in blue
calculus will be in green

Multiple Choice: There are 14 multiple choice questions. Each question is worth 3 points and has one correct answer. The multiple choice problems will be 42% of the total grade. Circle your choice on your test paper and bubble the corresponding answer on your Scantron. Any questions involving inverse trigonometric functions should be answered based on the domain restrictions for trigonometric functions used in Section 1.4. that's precalculus

1. Consider the cost function $C(x) = 500 + 0.2x,\ 0 \le x \le 4000$ ($C(x)$ is \$ when x items produced). Determine the average cost and marginal cost when $x = 1000$ items are produced.

   A) Average cost is \$0.20 per item and marginal cost is \$0.10 per item.

   B) Average cost is \$0.60 per item and marginal cost is \$0.10 per item.

   C) Average cost is \$0.70 per item and marginal cost is \$0.20 per item.

   D) Average cost is \$5.20 per item and marginal cost is \$2.00 per item.

   average cost $= \frac{C(1000)}{1000} = \frac{500+200}{1000} = \frac{700}{1000} = 0.7$

   marginal cost $= C'(1000) = 0.2$

2. Find the derivative of $f(x) = \sqrt{x}\cos x$. product rule

   $f'(x) = \sqrt{x}(-\sin x) + \frac{1}{2}x^{-1/2}\cos x$

   A) $f'(x) = \dfrac{-\sin x}{2\sqrt{x}}$

   B) $f'(x) = \dfrac{-\cos x}{2\sqrt{x}} + \sqrt{x}\sin x$

   C) $f'(x) = -2\sqrt{x}\sin x$

   D) $f'(x) = \dfrac{\cos x}{2\sqrt{x}} - \sqrt{x}\sin x$

3. Use the table to evaluate $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]\bigg|_{x=1}$.

	x = 1	**x = 3**	**x = 5**	**x = 7**	**x = 9**
$f(x)$	3	1	9	7	5
$f'(x)$	1	9	5	1	3
$g(x)$	9	7	5	3	1
$g'(x)$	5	9	3	1	7

A) $\frac{8}{27}$

B) $\frac{1}{3}$

C) $\frac{-2}{27}$

D) $\frac{1}{5}$

quotient rule

$\frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

$\frac{g(1)f'(1) - f(1)g'(1)}{(g(1))^2}$

$= \frac{9(1) - (3)(5)}{(9)^2}$

$= \frac{9-15}{81}$

$= \frac{-6}{81}$

$= \frac{-2}{27}$

4. Find an equation of the line tangent to $g(x) = 3x^2 - x + 1$ at $x = 1$.

A) $y = 6x - 17$

B) $y = 5x - 2$

C) $y = 6x - 1$

D) $y = 5x - 14$

$g'(x) = 6x - 1$ $g'(1) = 5$ $g(1) = 3 - 1 + 1 = 3$

$y - 3 = 5(x - 1)$

$y = 5x - 5 + 3 = 5x - 2$

5. Find $\dfrac{d^2y}{dx^2}$ given $xy = 1$.

A) $\dfrac{d^2y}{dx^2} = 0$

B) $\dfrac{d^2y}{dx^2} = \dfrac{y^2 + y}{x^2}$

C) $\dfrac{d^2y}{dx^2} = \dfrac{2y}{x^2}$

D) $\dfrac{d^2y}{dx^2} = \dfrac{y + xy}{x^3}$

$xy = 1$

$x\dfrac{dy}{dx} + y = 0$

$\dfrac{dy}{dx} = -\dfrac{y}{x} = -yx^{-1}$

$\dfrac{d^2y}{dx^2} = -y(-x^{-2}) - \dfrac{dy}{dx}x^{-1}$

$= yx^{-2} + yx^{-2}$

$= 2yx^{-2}$

6. Find the derivative of $g(x) = \sec\left(x^2 + 3\right)$.

A) $g'(x) = 2x\tan^2\left(x^2 + 3\right)$

B) $g'(x) = \sec(2x)\tan(2x)$

C) $g'(x) = 2x\sec\left(x^2 + 3\right)\tan\left(x^2 + 3\right)$

D) $g'(x) = 4x^2\sec\left(x^2 + 3\right)\tan\left(x^2 + 3\right)$

$g'(x) = \sec(x^2+3)\tan(x^2+3)\,(2x)$

7. Find the derivative of $y = \tan\left(e^{3x}\right)$.

A) $\dfrac{dy}{dx} = 3e^{3x}\sec^2\left(e^{3x}\right)$

B) $\dfrac{dy}{dx} = e^{3x}\sec^2 x + 3e^{3x}\tan x$

C) $\dfrac{dy}{dx} = \sec^2\left(3e^{3x}\right)$

D) $\dfrac{dy}{dx} = e^{3x}\sec^2\left(e^{3x}\right)$

$y' = \sec^2(e^{3x})\,e^{3x}\,(3)$

8. Find the derivative of $y = \ln\left(\dfrac{3x}{\sec x}\right)$.

A) $\dfrac{dy}{dx} = \dfrac{1 + x\cot x}{x}$

B) $\dfrac{dy}{dx} = \dfrac{9x\sec x - 9x^2\sec x\tan x}{\sec^3 x}$

C) $\dfrac{dy}{dx} = \dfrac{\sec x}{3x}$

D) $\dfrac{dy}{dx} = \dfrac{1 - x\tan x}{x}$

$$y' = \frac{1}{\frac{3x}{\sec x}} \cdot \frac{\sec x(3) - 3x\sec x\tan x}{\sec^2 x}$$

$$= \frac{\sec x}{3x} \cdot \frac{3 - 3x\tan x}{\sec x}$$

$$= \frac{1 - x\tan x}{x}$$

9. Find the derivative of $y = \sin^{-1}(2x^3)$.

A) $y' = \dfrac{6x^2}{1 + 4x^6}$

B) $y' = \dfrac{1}{\sqrt{1 - 4x^9}}$

C) $y' = \dfrac{6x^2}{\sqrt{1 - 4x^6}}$

D) $y' = \dfrac{6x^2}{\sqrt{4x^6 + 1}}$

$$y' = \frac{1}{\sqrt{1 - (2x^3)^2}}(6x^2)$$

10. Find y''' (the third derivative) given $y = x^e - e^x$.

A) $y''' = e(e-1)(e-2)x^{e-3} - e^x$

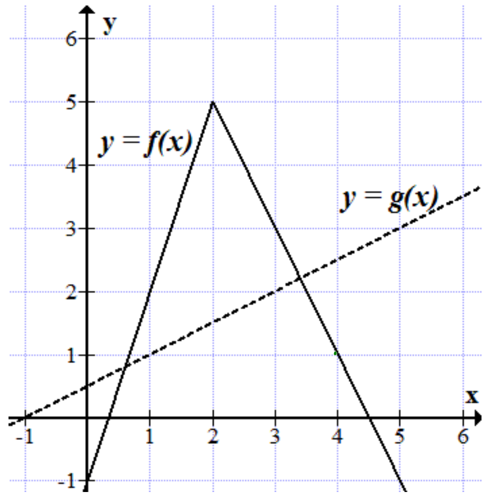
B) $y''' = e^3 x^{e-3} - e^x$

C) $y''' = e(e-1)(e-2)x^{e-3} - x(x-1)(x-2)e^{x-3}$

D) $y''' = e^3 x^{e-3} - x^3 e^{x-3}$

$y' = ex^{e-1} - e^x$
$y'' = e(e-1)x^{e-2} - e^x$
$y''' = e(e-1)(e-2)x^{e-3} - e^x$

11. Use the given graphs of f and g to evaluate $\left.\dfrac{d}{dx}\left(5f(x)+3g(x)\right)\right|_{x=4}$.



this involves calculating the slopes of those functions at $x=4$

$f'(4) = -2$

$g'(4) = \frac{1}{2}$

so $5(-2)+3\left(\frac{1}{2}\right) = -10+\frac{3}{2}$

$= -\frac{17}{2}$

A) 32

B) $\dfrac{-17}{2}$

C) $\dfrac{23}{2}$

D) $\dfrac{25}{2}$

12. Differentiate $f(x) = 5^{\sin x}$.

$f'(x) = 5^{\sin x}(\ln 5)\cos x$

A) $f'(x) = 5^{\sin x}(\cos x)$

B) $f'(x) = (\ln 5)5^{\sin x}$

C) $f'(x) = 5^{\sin x}\ln(\sin x)$

D) $f'(x) = (\ln 5)5^{\sin x}(\cos x)$

13. Evaluate $\lim_{x\to 0}\frac{k\sin kx}{x}$ where k is a positive constant.

A) 0

B) 1

C) k

D) k^2

$$\lim_{x\to 0}\frac{k\sin kx}{x} = \lim_{x\to 0} k^2\frac{\sin kx}{kx}$$

$$= k^2\lim_{x\to 0}\frac{\sin kx}{kx}$$

$$= k^2$$

14. A spherical balloon is inflated with helium at a rate of 10 ft^3/min. How fast is the radius of the balloon increasing at the instant the radius is 2 feet?

Recall that the volume of a sphere is $V=\frac{4}{3}\pi r^3$.

A) $\frac{dr}{dt}=\frac{5}{8\pi}$ ft/min

B) $\frac{dr}{dt}=\frac{5}{4\pi}$ ft/min

C) $\frac{dr}{dt}=160\pi$ ft/min

D) $\frac{dr}{dt}=16\pi$ ft/min

$$\frac{dV}{dt} = \frac{4}{3}\pi\left(3r^2\frac{dr}{dt}\right) = 4\pi r^2\frac{dr}{dt}$$

$$10 = 4\pi(2)^2\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{10}{16\pi}$$

Free Response: The Free Response questions will be 58% of the total grade. Read each question carefully. To receive full credit, you must show legible, logical, and relevant justification which supports your final answer. Give answers as exact values. Questions involving inverse trigonometric functions should be answered based on the domain restrictions in Section 1.4.

1. **(12 pts.)** Find the derivative of each of the following functions. **SIMPLIFY YOUR ANSWERS.**

a. **(6 pts.)** $y = \left(\frac{x^2}{3x^2+2}\right)^3$

$$y' = 3\left(\frac{x^2}{3x^2+2}\right)^2 \left[\frac{(3x^2+2)(2x) - x^2(6x)}{(3x^2+2)^2}\right]$$

$$= \frac{3x^4}{(3x^2+2)^2}\left[\frac{6x^3+4x-6x^3}{(3x^2+2)^2}\right]$$

$$= \frac{12x^5}{(3x^2+2)^4}$$

b. **(6 pts.)** $y = \tan^{-1}\left(\frac{x}{2}\right)$

$$y' = \frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2}$$

$$= \frac{1}{1+\frac{x^2}{4}} \cdot \frac{1}{2}$$

$$= \frac{1}{\frac{4+x^2}{4}} \cdot \frac{1}{2}$$

$$= \frac{2}{4+x^2}$$

2. **(12 pts.)** Consider the equation $\sin y = 6x^5 - 6$.

a. **(8 pts.)** Use **Implicit Differentiation** to find $\frac{dy}{dx}$.

$$\cos y \frac{dy}{dx} = 30x^4$$

$$\frac{dy}{dx} = \frac{30x^4}{\cos y}$$

b. **(4 pts.)** Find the equation of the tangent line to the curve at the point $(1, \pi)$.

$$\left.\frac{dy}{dx}\right|_{(1,\pi)} = \frac{30(1)^4}{\cos \pi} = \frac{30}{-1} = -30$$

$$y - \pi = -30(x-1)$$

3. **(11 pts.)** Suppose a stone is thrown vertically upward with an initial velocity of 64 ft/s from a bridge 80 ft above a river. By Newton's laws of motion, the position of the stone (measured in feet above the river) after t seconds is $s(t) = -16t^2 + 64t + 80$ where $s = 0$ is the level of the river.

a. **(4 pts.)** Find the velocity and acceleration functions.

Velocity $v(t) =$ $-32t + 64$ ft/s.

Acceleration $a(t) =$ -32 ft/s^2.

b. **(4 pts.)** What is the highest point above the river reached by the stone?

when $v(t) = 0$ $\quad -32t + 64 = 0$

$t = 2$

$s(2) = 144$ ft

The stone reaches its highest point when $t =$ 2 seconds.

The highest point reached by the stone is 144 feet.

c. **(3 pts.)** When will the stone strike the river? (That is, how long is the stone in the air?)

$s(t) = 0$ $\quad -16t^2 + 64t + 180 = 0$

$-16(t^2 - 4t - 5) = 0$

$t^2 - 4t - 5 = 0$

$(t - 5)(t + 1) = 0$

$t = 5, \cancel{-1}$ time can't be negative

The stone strikes the river after 5 seconds.

4. **(10 pts.)** Find $f'(1)$ when $f(x) = x^{2/x}$.

$$\ln f = \frac{2}{x}\ln x$$

$$\frac{1}{f}f' = \frac{2}{x}\cdot\frac{1}{x} + \frac{-2}{x^2}\ln x$$

$$f' = f\left[\frac{2}{x^2} - \frac{2\ln x}{x^2}\right]$$

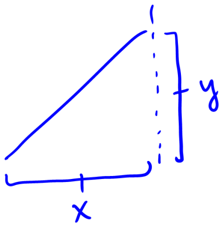
$$= x^{2/x}\left(\frac{2-2\ln x}{x^2}\right)$$

$$f'(1) = 1^2\left(\frac{2-2\ln 1}{1^2}\right)$$

$$= 1\left(\frac{2}{1}\right)$$

$$= 2$$

5. **(12 pts.)** A 12-ft ladder is leaning against a vertical wall when Trevor begins pulling the bottom of the ladder away from the wall at a rate of 0.5 ft/s. How fast is the top of the ladder sliding down the wall when the bottom of the ladder is 5 ft from the wall?



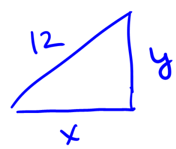
Variables: * Let x be the distance from the wall to the bottom of the ladder (in ft).
* Let y be the distance from the top of the ladder to the ground (in ft).
* Let t be time (in s).

a. **(4 pts.)** Identify the rate(s) that are given and the rate that is to be determined, using derivative notation and units. Make a sketch to organize the variables and given information.

Given rate(s): $\frac{dx}{dt}$

Rate to be determined: $\frac{dy}{dt}$

Sketch: (Include labels for variables and any constants.)



b. **(2 pts.)** Write an equation that expresses the basic relationship among the variables x and y.

right triangle so $x^2+y^2=144$
from pythagorean theorem

c. **(6 pts.)** Use your equation from part (b) to determine the rate at which the top of the ladder is sliding down the wall when the bottom of the ladder is 5 ft from the wall. State your answer in a complete sentence. when $x=5$, $y=\sqrt{119}$

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

$$\frac{dy}{dt}=-\frac{x}{y}\frac{dx}{dt}$$

so $\frac{dy}{dt}=\frac{-5}{\sqrt{119}}\left(\frac{1}{2}\right)=\frac{-5}{2\sqrt{119}}$ ft/sec

when the bottom of the ladder is 5 ft from the wall, the top of the ladder is sliding down at a rate of $\frac{5}{2\sqrt{119}}$ ft/sec

Scantron **(1 pt.)**

Check to make sure your Scantron form meets the following criteria. If any of the items are NOT satisfied when your Scantron is handed in and/or when your Scantron is processed one point will be subtracted from your test total.

My Scantron:

- ◻ is bubbled with firm marks so that the form can be machine read;
- ◻ is not damaged and has no stray marks (the form can be machine read);
- ◻ has **14** bubbled in answers;
- ◻ has **MATH 1060** and my section number written at the top;
- ◻ has my instructor's last name written at the top;
- ◻ has Test No. **2** written at the top;
- ◻ has the correct test version written at the top **and** bubbled in below my XID;
- ◻ shows my correct XID both written and bubbled in;
  Bubble a zero for the leading C in your XID.

Please read and sign the honor pledge below.

On my honor, I have neither given nor received inappropriate or unauthorized information at any time before or during this test.

Student's Signature: ____