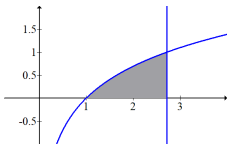
Math 1080: Calculus of One Variable II

Learning Activity 4

Area and Volumes

KEY

1. (7 points) The region R is the region bounded by $y = \ln x$ and the x-axis from $x = 1$ to $x = e$.



   (a) Set up the integral(s) with respect to x AND set up the integral(s) with respect to y needed to find the area of R.

   (b) Using the **disk/washer method**, set up the integral(s) needed to find the volume when the region R is revolved about the:
      (i) x-axis
      (ii) y-axis
      (iii) the line $x = e$

   (c) Using the **shell method**, set up the integral(s) needed to find the volume when the region R is revolved about the:
      (i) x-axis
      (ii) y-axis
      (iii) the line $x = e$

Solution: Note the graph $y = \ln x$ can also be written as $x = e^y$.

(a) $A = \int_1^e (\ln x)\, dx$

$A = \int_0^1 (e - e^y)\, dy.$

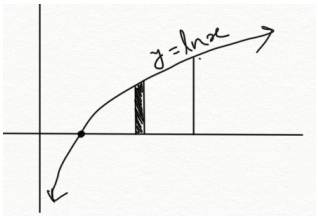


FIGURE 1. along x-axis

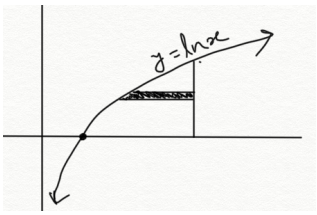


FIGURE 2. along y-axis

(b) (i) about the x-axis, disk/washer method: $V = \int_1^e \pi(\ln x)^2\,dx$

(ii) about the y-axis, disk/washer method:$V = \int_0^1 \pi\left[e^2 - (e^y)^2\right]\,dy = \int_0^1 \pi\left[e^2 - e^{2y}\right]\,dy$

(iii) about $x = e$, disk/washer method:$V = \int_0^1 \pi\left[(e - e^y)^2\right]\,dy$

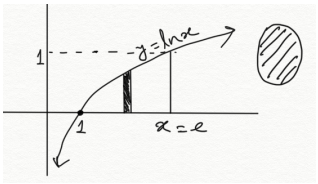


FIGURE 3. about x-axis

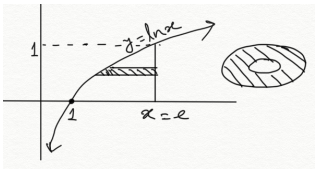


FIGURE 4. about y-axis

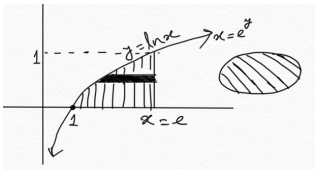


FIGURE 5. x=e

(c) (i) about the x-axis, shell method: $V = \int_0^1 2\pi y(e - e^y)\,dy.$

(ii) about the y-axis, shell method: $V = \int_1^e 2\pi x(\ln x)\,dx.$

(iii) about $x = e$, shell method: $V = \int_1^e 2\pi y(e - x)\ln x\,dx.$

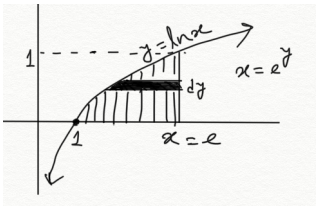


FIGURE 6. about x-axis

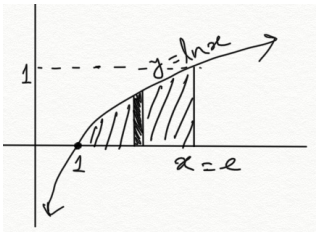


FIGURE 7. about y-axis

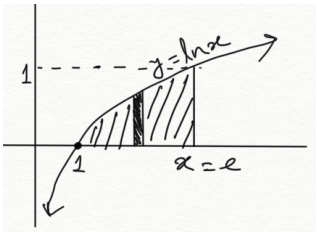
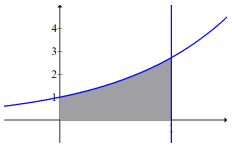


FIGURE 8. about x=e

2. (7 points) The region R is the region bounded by $y = e^x$ and the x-axis from $x = 0$ to $x = 1$.



(a) Set up the integral(s) with respect to x AND set up the integral(s) with respect to y needed to find the area of R.

(b) Using the **disk/washer method**, set up the integral(s) needed to find the volume when the region R is revolved about the:
   (i) x-axis
   (ii) y-axis
   (iii) the line $y = e$

(c) Using the **shell method**, set up the integral(s) needed to find the volume when the region R is revolved about the:
   (i) x-axis
   (ii) y-axis
   (iii) the line $y = e$

Solution:

(a) $A = \displaystyle\int_0^1 e^x \, dx.$

$$A = \int_0^1 1 \, dy + \int_1^e (1 - \ln y) \, dy = e - \int_1^e \ln y \, dy.$$

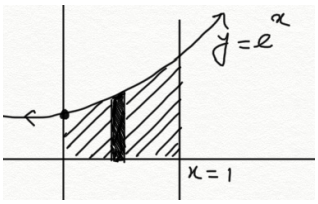


FIGURE 9. along x-axis

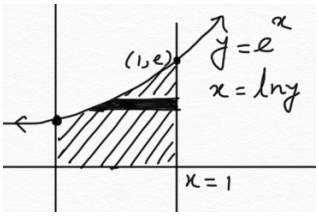


FIGURE 10. along y-axis

(b) (i) about the x-axis, disk/washer method: $V = \int_0^1 \pi(e^x)^2\,dx$.

(ii) about the y-axis, disk/washer method:$V = \int_0^1 \pi(1)^2\,dy + \int_1^e \pi\left[1^2 - (\ln y)^2\right]\,dy$.

(iii) about $y = e$, disk/washer method: $V = \int_0^1 \pi\left[e^2 - (e - e^x)^2\right]\,dx$.

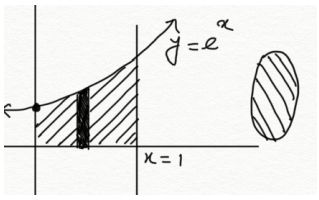


FIGURE 11. about x-axis

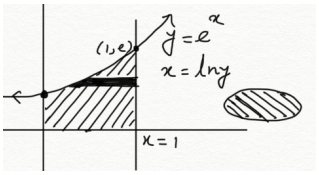


FIGURE 12. about y-axis

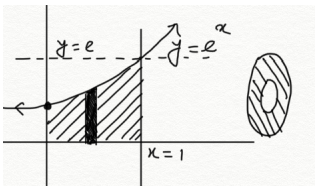


FIGURE 13. about y=e

(c) (i) about the x-axis, shell method: $V = \int_0^1 2\pi y(1)\,dy + \int_1^e 2\pi y(1 - \ln y)\,dy$.

(ii) about the y-axis, shell method: $V = \int_0^1 2\pi x e^x\,dx$.

(iii) about $y = e$, shell method: $V = \int_0^1 2\pi(e - y)(1)\,dy + \int_1^e 2\pi(e - y)(1 - \ln y)\,dy$.

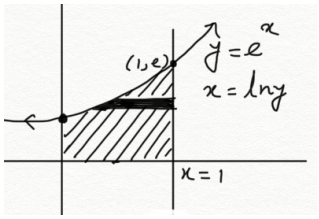


FIGURE 14. about x-axis

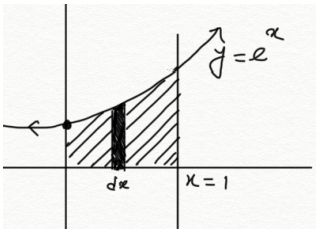


FIGURE 15. about y-axis

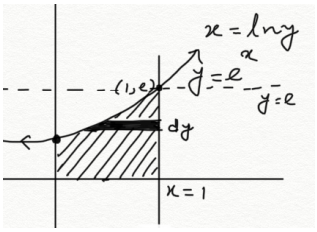
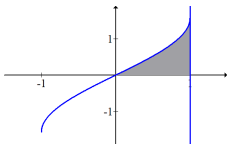


FIGURE 16. about y=e

3. (7 points) The region R is the region bounded by $y = \arcsin x$ and the x-axis from $x = 0$ to $x = 1$.



(a) Set up the integral(s) with respect to x AND set up the integral(s) with respect to y needed to find the area of R.

(b) Using the **disk/washer method**, set up the integral(s) needed to find the volume when the region R is revolved about the:
   (i) x-axis
   (ii) y-axis
   (iii) the line $x = 1$

(c) Using the **shell method**, set up the integral(s) needed to find the volume when the region R is revolved about the:
   (i) x-axis
   (ii) y-axis
   (iii) the line $x = 1$

Solution:

(a) $A = \displaystyle\int_0^1 \arcsin x \, dx.$

$A = \displaystyle\int_0^{\frac{\pi}{2}} (1 - \sin y) \, dy.$

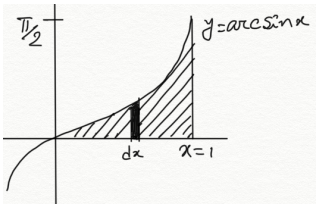


FIGURE 17. about x-axis

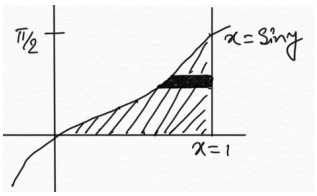


FIGURE 18. about y-axis

(b) (i) about the x-axis, disk/washer method: $V = \displaystyle\int_0^1 \pi(\arcsin x)^2\,dx$.

(ii) about the y-axis, disk/washer method: $V = \displaystyle\int_0^{\frac{\pi}{2}} \pi\left[1^2 - (\sin y)^2\right]\,dy$.

(iii) about $x = 1$, disk/washer method: $V = \displaystyle\int_0^{\frac{\pi}{2}} \pi(1 - \sin y)^2\,dy$.

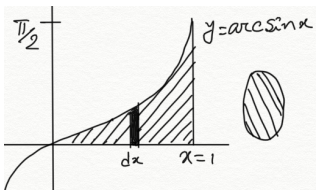


FIGURE 19. about x-axis

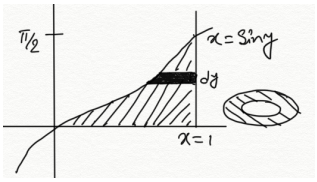


FIGURE 20. about y-axis

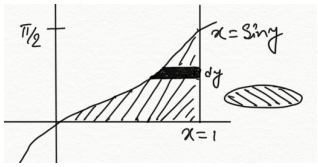


FIGURE 21. about x=1

(c) (i) about the x-axis, shell method: $V = \displaystyle\int_0^{\frac{\pi}{2}} 2\pi y(1 - \sin y)\,dy$.

(ii) about the y-axis, shell method: $V = \displaystyle\int_0^1 2\pi x \arcsin x\,dx$.

(iii) about $x = 1$, shell method: $V = \displaystyle\int_0^1 2\pi(1 - x)\arcsin x\,dx$.

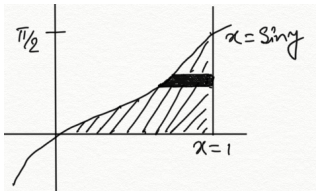


FIGURE 22. about x-axis

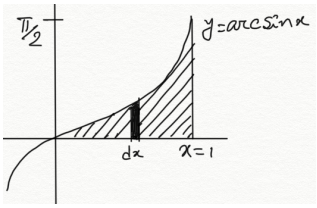


FIGURE 23. about y-axis

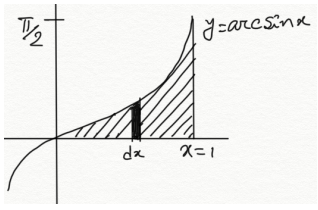


FIGURE 24. about x=1