Test 1
Version A

Student's Printed Name: ______________________________ **CUID:** ______________

Instructor: ______________________________________ **Section:** ____________

Instructions: You are **not** permitted to use a calculator on any portion of this test. You are **not** allowed to use a textbook, notes, cell phone, computer, or any other technology on any portion of this test. All devices must be turned off and stored away while you are in the testing room.

During this test, any kind of communication with any person other than the instructor or a designated proctor is understood to be a violation of academic integrity.

No part of this test may be removed from the examination room.

Read each question carefully. To receive full credit for the free response portion of the test, you must:

1. Show legible, logical, and relevant justification which supports your final answer.
2. Use complete and correct mathematical notation.
3. Include proper units, if necessary.
4. Give answers as exact values whenever possible.

You have **90 minutes** to complete the entire test.

On my honor, I have neither given nor received inappropriate or unauthorized information at any time before or during this test.

Student's Signature: ____

Do not write below this line.

Free Response Problem	Possible Points	Points Earned	Free Response Problem	Possible Points	Points Earned
1.	8		6. a.	4	
2.	5		6. b.	3	
3.	5		7.	6	
4. a.	6		8.	6	
4. b.	6		9. (Scantron)	1	
4. c.	6		**Free Response**	70	
5. a.	4		**Multiple Choice**	30	
5. b.	5		**Test Total**	100	
5. c.	5				

red pen is precalculus material (every problem requires algebra so I didn't write it for each)
black pen is calculus material

Multiple Choice: There are 10 multiple choice questions. Each question is worth 3 points and has one correct answer. The multiple choice problems will be 30% of the total grade. Circle your choice on your test paper. Any questions involving inverse trigonometric functions should be answered based on the domain restrictions for trigonometric functions used in Section 1.6.

log properties
domain of log
quadratic formula
inverse of natural log

1. **(3 pts.)** Solve the equation. Hint: Don't forget the domain for the natural log function.

$$\ln(x)+\ln(x-1)=0$$

a) No real solutions **b)** $x=1$

c) $\dfrac{1\pm\sqrt{5}}{2}$ **d)** $x=\dfrac{1+\sqrt{5}}{2}$

$\ln(x)+\ln(x-1)=\ln(x^2-x)=0$

$e^{\ln(x^2-x)}=e^0$

$x^2-x=1$

$x^2-x-1=0$ $\quad x=\dfrac{-(-1)\pm\sqrt{1-4(-1)}}{2}=\dfrac{1\pm\sqrt{5}}{2}$

but $\dfrac{1-\sqrt{5}}{2}<0$ is outside domain of $\ln x$

slope of a line = average rate of change

2. **(3 pts.)** Suppose the population P (in billions) of bacteria in a culture is given by the function $P(t)=5t-t^2$, where t is measured in hours. Find the **average** rate of change of $P(t)$ over the interval $1\le t\le 3$.

a) 2 billion/hour **b)** 1 billion/hour **c)** 3 billion/hour **d)** 6 billion/hour

average rate of change = slope of line between two given points:

$\dfrac{P(3)-P(1)}{3-1}=\dfrac{5(3)-3^2-[5-1]}{2}=\dfrac{15-9-4}{2}=1$

3. **(3 pts.)** Evaluate $\displaystyle\lim_{x\to 3}\frac{\ln(x-2)}{x+3}$.

a) ∞ **b)** $-\infty$ **c)** Does Not Exist **d)** 0

$\displaystyle\lim_{x\to\infty}\frac{\ln(x-2)}{x+3}=\frac{\ln(3-2)}{3+3}=\frac{\ln 1}{6}=\frac{0}{6}=0$

4.
(3 pts.) Evaluate $\lim_{x\to\infty} \frac{x - x\sqrt{x}}{3x\sqrt{x}+2x-7}$.

a) ∞ **b)** $-\infty$ **c)** $-\frac{1}{3}$ **d)** Does Not Exist

$$\lim_{x\to\infty} \frac{x - x\sqrt{x}}{3x\sqrt{x}+2x-7} = \lim_{x\to\infty} \frac{x - x^{3/2}}{3x^{3/2}+2x-7} = \lim_{x\to\infty} \frac{\frac{x}{x^{3/2}} - \frac{x^{3/2}}{x^{3/2}}}{\frac{3x^{3/2}}{x^{3/2}} + \frac{2x}{x^{3/2}} - \frac{7}{x^{3/2}}}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{x^{1/2}} - 1}{3 + \frac{2}{x^{1/2}} - \frac{7}{x^{3/2}}} = \frac{0-1}{3+0-0} = \frac{-1}{3}$$

5.
(3 pts.) Solve the equation.

log's exponential properties

$2^{x-4} = 3$

a) $x = \frac{\ln 3}{\ln 2} + 4$ **b)** $x = \ln\left(\frac{3}{2}\right) + 4$

c) $y = \frac{3}{4\ln 2}$ **d)** No real solutions.

$$2^{x-4} = 3$$
$$\ln 2^{x-4} = \ln 3$$
$$(x-4)\ln 2 = \ln 3$$
$$x - 4 = \frac{\ln 3}{\ln 2}$$
$$x = \frac{\ln 3}{\ln 2} + 4$$

6.
(3 pts.) Evaluate $\lim_{x\to(-2)^+} \frac{x^2}{x+2}$.

a) ∞ **b)** $\frac{1}{2}$ **c)** $-\infty$ **d)** $-\frac{1}{2}$

$$\lim_{x\to(-2)^+} \frac{x^2}{x+2} = \frac{(-2)^2}{sp} = \frac{+4}{\text{small positive}} = \infty$$

7. Let $h(x)=\dfrac{g(x)f(x)}{3}$. Use the table to evaluate $h'(4)$.
(3 pts.)

evaluating a function using a table of values

	$x=1$	$x=2$	$x=3$	$x=4$
$f(x)$	2	3	1	3
$f'(x)$	6	1	8	2
$g(x)$	1	4	4	3
$g'(x)$	4	-5	5	-4

a) $-\frac{8}{3}$ **b)** -2 **c)** -6 **d)** 6

$$h'(x) = \frac{g(x)f'(x)+g'(x)f(x)}{3}$$

$$h'(4) = \frac{g(4)f'(4)+g'(4)f(4)}{3}$$

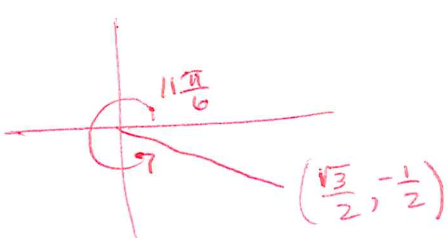
$$= \frac{3(2)+(-4)(3)}{3} = \frac{6-12}{3} = \frac{-6}{3} = -2$$

8. Evaluate $\lim\limits_{x\to 11}\left[\tan\left(\frac{x\pi}{6}\right)\right]$.
(3 pts.)

unit circle

a) -2 **b)** $-\frac{1}{2}$ **c)** $-\frac{\sqrt{3}}{2}$ **d)** $-\frac{1}{\sqrt{3}}$

$$\lim_{x\to 11}\tan\left(\frac{x\pi}{6}\right) = \tan\frac{11\pi}{6} = \frac{-1/2}{\sqrt{3}/2} = \frac{-1}{2}\cdot\frac{2}{\sqrt{3}} = \frac{-1}{\sqrt{3}}$$



9.
(3 pts.) Find $\lim_{x\to 1} f(x)$ if $f(x)=\begin{cases} -e^{\ln(5x)}+2 & \text{if } x<1 \\ 7x & \text{if } x=1 \\ 5x-8 & \text{if } x>1 \end{cases}$

inverse functions
how piecewise functions work

a) -3 **b)** 1

c) 8 **d)** Does Not Exist

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} -e^{\ln(5x)}+2 = -e^{\ln 5}+2 = -5+2=-3$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (5x-8) = 5(1)-8 = -3$$

10.
(3 pts.) Find the intervals on which the function $f(x)=\dfrac{2x+5}{e^x(x^2-6x+8)}$ is continuous.

domain of a function
factoring

a) $(-\infty,\ -2),\ (-2,\ 4),\ (4,\ \infty)$ **b)** $(-\infty,\ -4),\ (-4,\ 2),\ (2,\ \infty)$

c) $(-\infty,\ 2),\ (2,\ \infty)$ **d)** $(-\infty,\ 2),\ (2,\ 4),\ (4,\ \infty)$

f(x) consists of products of functions, so continuity of f(x) is the same as the continuity of its parts; all the parts are continuous on their domains

e^x has domain $(-\infty,\infty)$ & is never zero so continuous everywhere

$2x+5$ has domain $(-\infty,\infty)$ so continuous everywhere

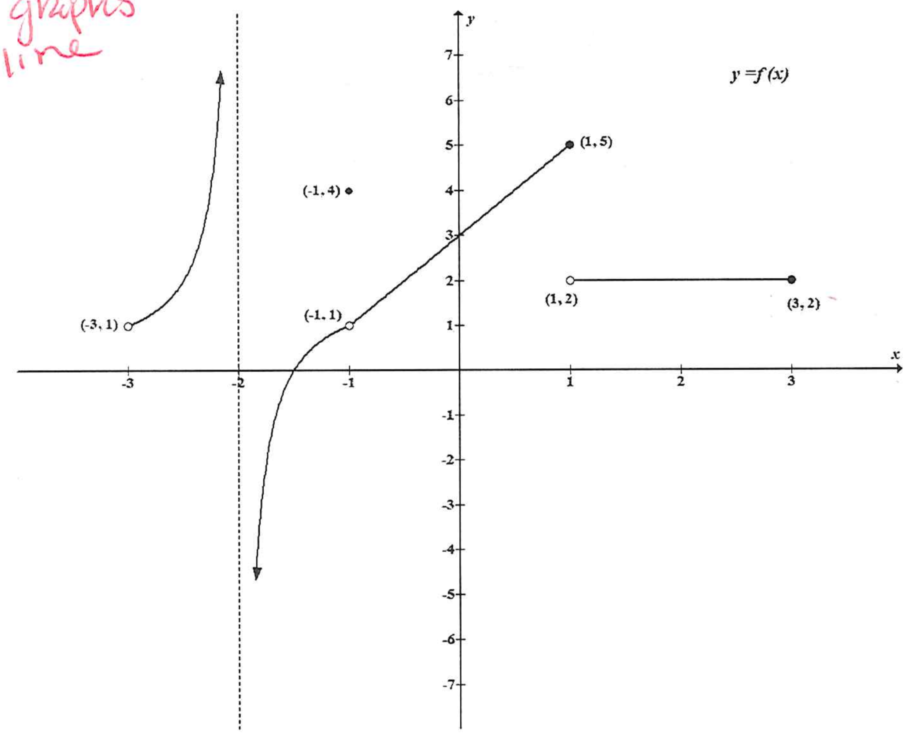
$\dfrac{1}{x^2-6x+8} = \dfrac{1}{(x-4)(x-2)}$ has domain $(-\infty,2)\cup(2,4)\cup(4,\infty)$ so continuous on that

putting everything together gives
$(-\infty,2)\cup(2,4)\cup(4,\infty)$

Free Response: The Free Response questions will be 70% of the total grade. Read each question carefully. To receive full credit, you must show legible, logical, and relevant justification which supports your final answer. Give answers as exact values. Any questions involving inverse trigonometric functions should be answered based on the domain restrictions for trigonometric functions used in Section 1.6.

1. **(8 pts.)** Use the graph of $f(x)$ to find each of the following limits, if it exists. **(1 pt. each)**
Infinite limits should be answered with "$=\infty$" or "$=-\infty$", whichever is appropriate.
If the limit does not exist (and cannot be answered as ∞ or $-\infty$), state "DNE."

reading graphs
slope of line



a. $\lim\limits_{x\to -1} f(x) = 1$

e. $\lim\limits_{x\to 2}\left(\dfrac{d}{dx}[f(x)]\right) = 0$

b. $\lim\limits_{x\to -3^+} f(x) = 1$

f. $\lim\limits_{x\to -1}(3f'(x)) =$

typo

c. $\lim\limits_{x\to 0}\left(f(x)\tan^{-1}(e^x)\right) = \dfrac{3\pi}{4}$

g. $\lim\limits_{x\to -2^+} f(x) = -\infty$

d. $\lim\limits_{x\to 1} f(x)$ DNE

h. $\lim\limits_{h\to 0}\dfrac{f(0+h)-f(0)}{h}$ } this is derivative at $x=0$ which is the same as the slope of that line

~~[illegible]~~ $\dfrac{5-1}{1-(-1)} = \dfrac{4}{2} = 2$

range ~~domain~~ of cosine

2. **(5 pts.)** Find $\lim_{x\to 0}\left[x^8\cos\left(\frac{1}{x}\right)\right]$, if it exists. Show your work, citing any theorems that you use in finding the value of the limit.

$$-1 \le \cos\frac{1}{x} \le 1$$

$$-x^8 \le x^8\cos\frac{1}{x} \le x^8$$

$$\lim_{x\to 0} -x^8 \le \lim_{x\to 0} x^8\cos\frac{1}{x} \le \lim_{x\to 0} x^8$$

$$0 \le \lim_{x\to 0} x^8\cos\frac{1}{x} \le 0$$

therefore $\lim_{x\to 0} x^8\cos\frac{1}{x} = 0$ by the squeeze theorem

3. **(5 pts.)** For what value of a is $f(x)$ continuous at $x=2$? Show all supporting work, including the appropriate left and right limits.

how piecewise functions work

$$f(x)=\begin{cases} ax^2+2x, & x<2 \\ x^3-ax, & x\ge 2\end{cases}$$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(ax^2+2x) = a(4)+4$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(x^3-ax) = 8-a(2)$$

thus, $\lim_{x\to 2} f(x)$ must exist which implies $4a+4=8-2a$

$6a=4$

$a=\frac{4}{6}=\frac{2}{3}$

lastly, $f(2)=2^3-a(2)$ is satisfied/exists since $f(2)=\lim_{x\to 2^+} f(x)$

$a=\frac{2}{3}$

4. **(18 pts.)** Find the following limits. If a limit does not exist, state "does not exist" and provide a brief explanation. Show all work. **Do not use L'Hopital's Rule.**

factoring

a. **(6 pts.)** $\lim_{x\to 3}\frac{2x^3+4x^2-30x}{2x^3-4x^2-6x}$

$$= \lim_{x\to 3}\frac{2x(x^2+2x-15)}{2x(x^2-2x-3)} = \lim_{x\to 3}\frac{2x(x+5)(x-3)}{2x(x-3)(x+1)}$$

$$= \lim_{x\to 3}\frac{x+5}{x+1}$$

$$= \frac{8}{4}$$

$$= 2$$

multiply by conjugate

b. **(6 pts.)** $\lim_{x\to 5}\frac{2(x-5)}{\sqrt{5x}-5}\cdot\frac{\sqrt{5x}+5}{\sqrt{5x}+5} = \lim_{x\to 5}\frac{2(x-5)(\sqrt{5x}+5)}{5x-25}$

$$= \lim_{x\to 5}\frac{2(x-5)(\sqrt{5x}+5)}{5(x-5)}$$

$$= \lim_{x\to 5}\frac{2(\sqrt{5x}+5)}{5}$$

$$= \frac{2(\sqrt{25}+5)}{5}$$

$$= \frac{2(10)}{5}$$

$$= 4$$

c. **(6 pts.)** $\lim\limits_{x\to-\infty} \dfrac{10x^3 - 3x^2 + 8}{\sqrt{25x^6 + x^4 + 2}}$

root functions

note: $\sqrt{x^6} = -x^3$ when $x < 0$

$$\lim_{x\to-\infty} \frac{10x^3 - 3x^2 + 8}{\sqrt{25x^6 + x^4 + 2}} = \lim_{x\to-\infty} \frac{\frac{10x^3}{-x^3} - \frac{3x^2}{-x^3} + \frac{8}{-x^3}}{\sqrt{\frac{25x^6}{x^6} + \frac{x^4}{x^6} + \frac{2}{x^6}}}$$

$$= \lim_{x\to-\infty} \frac{-10 + \frac{3}{x} - \frac{8}{x^3}}{\sqrt{25 + \frac{1}{x^2} + \frac{2}{x^6}}}$$

$$= \frac{-10 + 0 - 0}{\sqrt{25 + 0 + 0}}$$

$$= \frac{-10}{5}$$

$$= -2$$

5. **(14 pts.)** Find the derivatives of the following functions. Assume $g(x)$ is a differentiable function wherever it appears. **Do NOT simplify your answers.**

rewriting roots as powers

a) **(4 pts.)** $h(t) = \frac{\pi^5}{5}\left(\sqrt[3]{t} - e^{11} - \sqrt{48}\right)$ $= \frac{\pi^5}{5}\left(t^{1/3} - e^{11} - \sqrt{48}\right)$

$$h'(t) = \frac{\pi^5}{5}\left(\frac{1}{3}t^{-2/3}\right)$$

b) **(5 pts.)** $f(x) = \frac{xe^x}{g(x)}$

$$f'(x) = \frac{g(x)[xe^x + e^x] - xe^x g'(x)}{g^2(x)}$$

c) **(5 pts.)** $f(x) = g(x)\left(x + \frac{e}{x}\right)$ $= g(x)\left(x + ex^{-1}\right)$

$$f'(x) = g(x)\left(1 + e(-x^{-2})\right) + g'(x)\left(x + \frac{e}{x}\right)$$

6. **(6 pts.)** Consider the function $f(x)=3x-2x^2$.

a) **(4 pts.)** Find $f'(x)$ using the definition of the derivative.

FOIL
evaluating
functions

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{3(x+h)-2(x+h)^2-(3x-2x^2)}{h}$$

$$=\lim_{h\to 0}\frac{3x+3h-2(x^2+2xh+h^2)-3x+2x^2}{h}$$

$$=\lim_{h\to 0}\frac{3h-4xh-2h^2}{h}$$

$$=\lim_{h\to 0}(3-4x-2h)$$

$$= 3-4x$$

b) **(2 pts.)** Calculate $f'(2)$ using your function in part (a). Briefly describe what this value represents.

$$f'(2)=3-4(2)=-5$$

- this is slope of tangent line @ $x=2$ on $f(x)$
- rate of change of $f(x)$ at $x=2$

calculus stuff

7. **(6 pts.)** Let $f(x)=5x-4$. Use the delta-epsilon definition of a limit to prove $\lim_{x\to 2} f(x)=6$.

finding $\delta>0$,

$|f(x)-6|<\varepsilon$

$|5x-4-6|<\varepsilon$

$|5x-10|<\varepsilon$

$5|x-2|<\varepsilon$

$|x-2|<\frac{\varepsilon}{5}$

So take $\delta=\frac{\varepsilon}{5}$

proof: Given any $\varepsilon>0$, let $\delta=\frac{\varepsilon}{5}$.

If $0<|x-2|<\delta$ then

$|f(x)-6| = |5x-4-6|$

$= |5x-10|$

$= 5|x-2|$

$< 5\delta$

$= 5\cdot\frac{\varepsilon}{5}$

$= \varepsilon$

8. **(6 pts.)** Find the equation of the line tangent to the graph of $f(x)$ at the given value.

equation of line
evaluating a function

$$f(x)=\frac{2e^x}{x^2+1}, \quad x=0$$

$$f'(x) = \frac{(x^2+1)\cdot 2e^x - 2e^x(2x)}{(x^2+1)^2}$$

$$f'(0) = \frac{1\cdot 2e^0 - 2e^0(0)}{1}$$

$$= 2$$

$$f(0) = \frac{2e^0}{1} = 2$$

$$y-2 = 2(x-0)$$

Scantron **(1 pt.)**

Check to make sure your Scantron form meets the following criteria. If any of the items are NOT satisfied when your Scantron is handed in and/or when your Scantron is processed one point will be subtracted from your test total.

My Scantron:

- ☐ is bubbled with firm marks so that the form can be machine read;
- ☐ is not damaged and has no stray marks (the form can be machine read);
- ☐ has **10** bubbled in answers;
- ☐ has **MATH 1060** and my section number written at the top;
- ☐ has my instructor's last name written at the top;
- ☐ has Test No. **1** written at the top;
- ☐ has the correct test version written at the top **and** bubbled in below my XID;
- ☐ shows my correct XID both written and bubbled in;

Bubble a zero for the leading C in your XID.