1. $H = \begin{bmatrix} a - 2b \\ a + b \\ 3b \end{bmatrix} \quad \vec{u} = a \underbrace{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}}_{\vec{v}_1} + b \underbrace{\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}}_{\vec{v}_2}$

$H = \text{span}\{v_1, v_2\}$
is it a basis?

$-1 \left[\begin{array}{cc|c} 1 & -2 & 0 \\ 1 & 1 & 0 \\ 0 & 3 & 0 \end{array}\right] \xrightarrow{-R_1+R_2 \to R_2} \left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 3 & 0 \\ 0 & 3 & 0 \end{array}\right] \xrightarrow{-R_2+R_3} \left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{array}\right]$

- linearly independent
- Basis

$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} \right\}$

~~$x_1 - 2x_2 = 0$~~

~~$3x_2 = 0$~~

$\boxed{\text{Dim}(H) = 2}$

2.

A. $\left[\begin{array}{cccc|c} 1 & 0 & -2 & -12 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 \end{array}\right]$

Basis if linearly independent
- It is linearly independent

~~$\left[\begin{array}{cccc|c} 1 & 0 & -2 & -12 & 1 \\ 0 & 2 & 0 & 0 & 4 \\ 0 & 0 & 4 & 0 & 7 \\ 0 & 0 & 0 & 8 & 0 \end{array}\right]$ RREF~~

B. $\left[\begin{array}{cccc|c} 1 & 0 & -2 & -12 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & \frac{7}{4} \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$

$-12R_4 + R_1$ $\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & \frac{7}{4} \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$ $2R_3 + R_1 \to R_1$ $\longrightarrow$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{9}{2} \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & \frac{7}{4} \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] = \begin{bmatrix} \frac{9}{2} \\ 2 \\ \frac{7}{4} \\ 0 \end{bmatrix}$$