

Gavin Miley

1. $\int_1^{\infty} x e^{-x^2} dx$

$u = -x^2 \quad du = -2x dx$

$-\frac{1}{2} du = x dx$

$-\frac{1}{2} \int_1^{\infty} e^u du = \lim_{c \rightarrow \infty} -\frac{1}{2} \int_1^c e^u du$

$= \lim_{c \rightarrow \infty} -\frac{1}{2} e^{-x^2} \Big|_1^c$ ~~$-\frac{1}{2} e^{-c^2} - (-\frac{1}{2} e^{-1})$~~

$= \boxed{+\frac{1}{2e} \text{ Converges}}$

2. On the PDF I want see if its $x-2$ or $x+2$ I'm going to assume its $x-2$

$\left(= \lim_{c \rightarrow 2^-} \int_1^c \frac{1}{(x-2)^{1/3}} dx + \lim_{c \rightarrow 2^+} \int_c^5 \frac{1}{(x-2)^{1/3}} dx \right)$

~~$\frac{1}{2} \left(\frac{1}{2} \right)^{1/3} \left(\frac{1}{2} \right)^{2/3} \left(\frac{1}{2} \right)^{3/3} \left(\frac{1}{2} \right)^{4/3} \left(\frac{1}{2} \right)^{5/3}$~~

$\left\{ \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} \right\}$

B. Appears to be converging towards 0

~~$\frac{1}{2} \left(\frac{1}{2} \right)^{1/3} \left(\frac{1}{2} \right)^{2/3} \left(\frac{1}{2} \right)^{3/3} \left(\frac{1}{2} \right)^{4/3} \left(\frac{1}{2} \right)^{5/3}$~~

~~$\frac{1}{2} \left(\frac{1}{2} \right)^{1/3} \left(\frac{1}{2} \right)^{2/3} \left(\frac{1}{2} \right)^{3/3} \left(\frac{1}{2} \right)^{4/3} \left(\frac{1}{2} \right)^{5/3}$~~

Gavin Mulvey

3. $(-1)^{k+1} \left(\frac{1}{2}\right)^k$ $(-1)^3 \left(\frac{1}{2}\right)^2$

A. $(-1)^{1+1} \left(\frac{1}{2}\right)^1$

$$= \left\{ +\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16} \right\}$$

B. Appears to be converging towards zero $|r| < 1$

C. $\frac{1}{2}, \left(\frac{2}{4} - \frac{1}{4}\right), \left(\frac{2}{8} + \frac{1}{8}\right), \left(\frac{3}{8} - \frac{1}{16}\right)$

$$\left(\frac{1}{2}, \frac{1}{4}, \frac{3}{8}, \frac{5}{16} \right)$$

ILATE

4. A $\int x \sec^2 x \, dx$ • Integration by parts

$$\left(\begin{array}{l} u = x \quad du = dx \\ dv = \sec^2 x \, dx \quad v = \tan x \end{array} \right)$$

B. $\int \frac{3x^2 + 2}{x^3 + 2x} \, dx$ • Partial Fraction Decomposition

$$= \int \frac{(3x^2 + 2)}{x(x^2 + 2)} \, dx = \left(\frac{A}{x} + \frac{Bx + C}{x^2 + 2} \right)$$

C. I can't see if it's $\sqrt{16 + x^2}$ or

$\sqrt{16 - x^2}$? It doesn't show up

• Try Substitution

~~$x = 4 \sin \theta$~~ ~~$dx = 4 \cos \theta \, d\theta$~~
 ~~$x = 4 \tan \theta$~~ ~~$dx = 4 \sec^2 \theta \, d\theta$~~

- If it's $\sqrt{16 - x^2}$

$x = 4 \sin \theta \quad dx = 4 \cos \theta \, d\theta$

~~if it's $\sqrt{16 + x^2}$~~ $\left(= \int \frac{\sqrt{16 + (4 \sin \theta)^2} \cdot 4 \cos \theta \, d\theta}{(4 \sin \theta)^2} \right)$

if it's $\sqrt{16 + x^2} \quad x = 4 \tan \theta \quad dx = 4 \sec^2 \theta \, d\theta$

$$\left(= \int \frac{\sqrt{16 + (4 \tan \theta)^2} \cdot 4 \sec^2 \theta \, d\theta}{(4 \tan \theta)^2} \right)$$