2. AR Model (2) casual

$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + W_t, \quad t \in \mathbb{Z}$

$\phi_1 = -0.5 \quad \phi_2 = 0.3$

1) Autocovariance function γ_{t-h}

$E[X_t X_{t-h}]$

$E[\phi_1 X_{t-1} X_{t-h}] + E[\phi_2 X_{t-2} X_{t-h}] + E[X_{t-h} W_t]$

$= \phi_1 E[X_{t-1} X_{t-h}] + \phi_2 E[X_{t-2} X_{t-h}] + E[X_t$

$0 = \phi_1 \gamma(h-1) + \phi_2 \gamma(h-2)$

So $E[X_{t-h} W_t]$ is 0 by identity

auto correlation:

$$\rho(h) = -0.5\rho(h-1) + 0.3\rho(h-2)$$

d.t eq: $\rho(h) = C_1 z^{-h} + C_2 z^{-h}$

difference equation

$$\rho(h) = \left(\frac{1}{2} - \frac{23}{35}\right)(z)^{-h} + \left(\frac{1}{2} - \frac{19}{35}\right)(z)^{-h}$$

2) AR2 determine PACF

PACF cuts off after $p=2$ $h \geq 3$, lecture week 5

$$\frac{\gamma_0(1)}{\gamma_0(0)} x_1 = \rho(1) x_1$$

in R

$$PACF = \begin{cases} -0.1 & h=1 \\ 0.3 & h=2 \\ 0 & h \geq 3 \end{cases}$$

3) Simulation; fit with 10

4)

5) in R