An Executable Operational Semantics for Rust with the Formalization of Ownership and Borrowing

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Rust is an emergent systems programming language highlighting memory safety by its Ownership and Borrowing System (OBS). The existing formal semantics for Rust only covers limited subsets of the major language features of Rust. Moreover, they formalize OBS as type systems at the language-level, which can only be used to conservatively analyze programs against the OBS invariants at compile-time. That is, they are not executable, and thus cannot be used for automated verification of runtime behavior.

In this paper, we propose RustSEM, a new executable operational semantics for Rust. RustSEM covers a much larger subset of the major language features than existing semantics. Moreover, RustSEM provides an operational semantics for OBS at the memory-level, which can be used to verify the runtime behavior of Rust programs against the OBS invariants. We have implemented RustSEM in the executable semantics modeling tool K-Framework. We have evaluated the semantics correctness of RustSEM wrt. the Rust compiler using around 700 tests. In particular, we have proposed a new technique for testing semantic consistency to ensure the absence of semantic ambiguities on all possible execution selections. We have also evaluated the potential applications of RustSEM in automated runtime and formal verification for both functional and memory properties. Experimental results show that RustSEM can enhance the memory safety mechanism of Rust, as it is more powerful than OBS in detecting memory errors.

Additional Key Words and Phrases: Rust, Operational Semantics, Ownership, Borrowing

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1 INTRODUCTION

Developing a formal semantics for a programming language could provide a mathematical foundation for the language. The semantics can be used as a reference model, and more importantly, used for proving language-level properties and constructing automated verification tools.

Rust [Rust-Team 2016] is an emergent systems programming language aiming at providing memory safety guarantees with its Ownership and Borrowing System (OBS). One of the most important guarantees maintained by the OBS invariants is the exclusive mutation capability for memory locations, which can avoid a variety of memory errors such as dangling pointers and data races.

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Several formal semantics for Rust has been developed. Unfortunately, they only cover limited subsets of the major language features of Rust, and most of them formalize either a variant or an outdated version of Rust. For instance, Patina [Reed 2015] only captures the key features related to memory safety, e.g., unique pointers and references, in an early version of Rust. RustBelt [Jung et al. 2018a] formalizes a variant λ_{Rust} of Rust in Coq, to prove in Coq that the type system of Rust can guarantee the memory and thread safe of λ_{Rust} programs.

Furthermore, most of the existing semantics for Rust is not executable, and thus cannot be used for automated verification of runtime behavior. For instance, Patina [Reed 2015] and RustBelt [Jung et al. 2018a] formalize OBS as type systems at the language-level, which can only be used to conservatively analyze programs against the OBS invariants at compile-time. But this may reject some execution traces satisfying the OBS invariants. As a result, automated verification tools for Rust have to translate programs into the input languages of general-purpose verifiers, which may hurt expressiveness and make potential verification optimizations impossible. For instance, Prusti [Astrauskas et al. 2019] translates a subset of safe constructs into the input language of Viper [Müller et al. 2016]. To support direct verification of Rust programs, a new executable semantics should be developed for automated verification.

In this paper, we propose RustSEM, a new executable operational semantics for Rust. RustSEM consists of three levels:

- (1) A sequential consistency memory model, which defines the memory layout of stacks and heaps, and a collection memory operations. More importantly, we formalize an *operational semantics of OBS* in the memory model for dynamically checking the OBS invariants. Unlike the existing semantics of OBS, which is formalized as type systems at the language-level and bound to a specific language, our semantics is language-independent and thus can be reused for other languages since most languages have similar memory operations. Moreover, we also propose a high-level abstraction of OBS and prove the refinement relation between the abstraction and the memory model, ensuring the semantics correctness of the OBS refinement.
- (2) An operational semantics for the Core Language (CL), an intermediate representation (IR) for reducing redundant semantics definitions. Recall that Rust also has an IR, namely Mid-level Intermediate Representation (MIR) [Matsakis 2016]. But the gap between Rust and CL is much smaller than MIR, facilitating the translation semantics from Rust to CL.
- (3) A translation semantics from Rust to CL, which automatically translates Rust programs into the corresponding CL programs.

To execute a Rust program, RustSEM first translates the program into a CL program, and then executes the CL program wrt. the CL semantics, in which all memory accesses are carried out by invoking the interfaces of the memory model.

RustSEM distinguishes from the existing semantics in the following aspects.

Firstly, RustSEM directly formalizes Rust, instead of a variant, and covers a much larger subset of the major language features than existing semantics. For instance, RustSEM supports both safe and unsafe constructs, concurrency, dynamic OBS, closures, pattern matching and polymorphism.

Secondly, RustSEM provides an operational semantics for OBS at the memory-level, whilst other existing semantics formalizes OBS as type systems at the language-level. The operational semantics of OBS is useful in two aspects. (1) It can be used to verify the runtime behavior of Rust programs against the OBS invariants, i.e., reject the programs violating the OBS invariants. Compared with the type systems, it provides a more flexible OBS invariant checking that accepts a larger set of correct programs. (2) It can be used to detect undefined behavior of the programs mixing safe and unsafe operations, by noting that unsafe constructs can escape from OBS and make programs prone to bugs [Davidoff 2018].

Thirdly, RustSEM is an executable semantics, which means that RustSEM can execute a Rust program with respect to its semantics. We have implemented RustSEM in the executable semantics modeling tool K-Framework (\mathbb{K}) [Roşu and Şerbănuţă 2010]. \mathbb{K} is based on rewriting-logic and has a builtin parser that enables the semantics being defined on the abstract syntax tree. \mathbb{K} has been successfully applied in formalizing the semantics of real-world programming languages, such as Java [Bogdănaş and Roşu 2015] and C [Ellison and Rosu 2012; Hathhorn et al. 2015]. \mathbb{K} 's execution and verification engines enable testing RustSEM and constructing automated verifiers.

We have evaluated the semantics correctness of RustSEM wrt. the Rust compiler using around 700 tests for the three levels, which mainly come from the Rust benchmarks [Rust-Benchmark 2020], the Rust libraries and the Rust text book [Rust-Team 2018]. Note that two semantics rules sometimes can be applied to the same construct at the same time, causing semantic ambiguities. Although one of them should be avoided, the execution engine of $\mathbb K$ always selects one rule to execute, leaving such ambiguities undetected. Thus, we have proposed a new technique for testing semantic consistency to ensure the absence of semantic ambiguities on all possible execution selections, by exploiting the verification engine of $\mathbb K$. Our new technique has discovered more than 36 semantic ambiguities. We also created tests for investigating what kinds of programs are rejected by the Rust compiler but accepted by RustSEM due to the compiler's conservation, and the reverse due to mixed safe and unsafe operations.

We have also evaluated the potential applications of RustSEM in automated runtime and formal verification for both functional and memory properties. Runtime verification with RustSEM is evaluated on 118 programs for detecting memory errors. Formal verification with RustSEM is evaluated on a collection of benchmarks, including Vec_Deque in the Rust library implementing a ring buffer. Experimental results show that RustSEM can enhance the memory safety mechanism of Rust, as it is more powerful than OBS in detecting memory errors.

In summary, we make the following contributions:

- (1) We propose a new executable operational semantics for Rust that supports a *larger* subset of the major language features, compared with all existing works.
- (2) We propose a high-level abstraction of OBS and an operational semantics of OBS in the memory model. We also prove the refinement relation between them.
- (3) We formalize the semantics of unsafe pointers and the semantics of the executions mixing both safe and unsafe pointers.
- (4) We propose a novel testing technique based on \mathbb{K} 's verification engine to detect semantic ambiguities.
- (5) We show that RustSEM can be applied to both runtime and formal verification against both functional and memory properties.

This paper is organized as follows: Section 2 recalls the OBS of Rust. Section 3 presents a high-level abstraction of OBS. Section 4 defines the operational semantics of the memory model, while Section 5 presents the basic idea of the semantics of CL and the translation semantics for Rust. Section 6 evaluates the proposed semantics. Section 7 compares related work. Section 8 concludes.

2 OWNERSHIP AND BORROWING SYSTEM

In this section, we recall the OBS of Rust and the related OBS invariants.

2.1 Ownership

A variable can declare the unique *ownership* of a memory block using a binding or an assignment. If the ownership is declared as mutable with the mut keyword, then the owner can be used to both read and write the block, otherwise it is read-only. A read-only (resp. mutable) owner is called

a shared (resp. mutable) alias of the block. We denote by $x \to_o B$ that variable x is the owner of block B, i.e., x owns B. For instance, in Listing 1, the binding "let mut v = vec![1,2]" at Line 1.1 first allocates a block B in the memory to store the vector [1,2], and then the owner v obtains the ownership of B, denoted by $v \to_o B$.

```
Listing 1.

1.1 let mut v = vec![1,2];
1.2 {

1.3 let v1 = v;
1.4 let t = v1[0];
1.5 v1[1] = 3;

1.6 }
```

An ownership can be *moved* from one variable to another. Moving an ownership from variable x to another variable y means that the ownership now belongs to y and x no longer owns it. For instance, in Listing 1, the binding "let v1 = v" at Line 1.3 moves the ownership of the vector from v to v1, i.e., v1 becomes the new owner of the vector and v can no longer be used to access it. Indeed, the vector is read and written through v1 at Lines 1.4 and 1.5, respectively.

The *lifetime* of an owner begins from the timestamp at which it obtains the ownership and ends at the timestamp at which it loses the ownership, e.g., when the ownership is moved or it goes out of the program scope (i.e., curly braces). Timestamps are a way to distinguish the execution order of program statements. In this section, we use line numbers as timestamps. The block is deallocated through its owner when the owner goes out of scope. For instance, in Listing 1, the lifetime of owner v begins at Line 1.1 and ends at Line 1.3, whilst the lifetime of owner v1 begins at Line 1.3 and ends at Line 1.6. The vector is deallocated through v1 at Line 1.6 as v1 goes out of scope.

2.2 Borrowing and Reborrowing

Borrowing is a way to create references to the owner of a memory block. There are two kinds of references: shared references (read-only, created by &) and mutable references (readable and writable, created by &mut). A shared (resp. mutable) reference is also called a shared (resp. mutable) alias of the block. A borrowing creating a shared (resp. mutable) reference is called a shared (resp. mutable) borrowing. We denote by $x \to_s y$ (resp. $x \to_m y$) that x is a shared (resp. mutable) reference to y, i.e., x borrows y. For instance, in Listing 2, the borrowing "let b1 = &v1" at Line 2.3 creates a shared reference b1 to v1, denoted by b1 \to_s v1, where b1 can only be used to read the block owned by v1. The borrowing "let b2 = &mut v2" at Line 2.4 creates a mutable reference, denoted by b2 \to_m v2, where b2 can be used to both read and write the block owned by v2.

```
Listing 2. Listing 3. 

2.1 let v1 = vec![1,2]; 3.1 let mut v = vec![1,2]; 3.2 let b1=&mut v; 3.3 let b1=&mut v; 3.3 let b2=&(*b1); 3.4 let b2=&mut v2; 2.5 let t1=(*b1)[0]; b1 \rightarrow_s v1 

2.6 let t2=(*b1)[1]; b2 \rightarrow_m v2 Listing 3. 

b1 \rightarrow_s v1 b2 \rightarrow_m v2 \rightarrow_s b1 \rightarrow_s v1 b2 \rightarrow_s v1 b2 \rightarrow_s b1 \rightarrow_s v1 b2 \rightarrow_s b1 b2 \rightarrow_s b1
```

Reborrowing is a way to create references from another reference, instead of an owner. For instance, in Listing 3, b1 borrows the owner v at Line 3.2 and b2 reborrows b1 with the referent *b1 at Line 3.3, denoted by the link b2 \rightarrow_s b1 \rightarrow_m v. To access the first element of the vector, we can use v[0], (*b1)[0] or (*b2)[0], where v, *b1 and *b2 are three paths to access the vector.

Each path uses an alias as the entry to access the block. For instance, the paths v, *b1 and *b2 use the alias entries v, b1 and b2, respectively.

As borrowing and reborrowing are similar, we define unified relations for them.

Definition 2.1 (Unified borrowing relation). Let x and y be two variables, where y can be either an owner or a reference. x borrows or reborrows y is denoted by $x \to_b y$. The borrowing relation \to_b contains two sub-relations: *shared* borrowing \to_s and *mutable* borrowing \to_m , i.e., $x \to_b y \Leftrightarrow (x \to_s y) \lor (x \to_m y)$.

The *lifetime* of a reference begins from the timestamp of its creation and ends at the last timestamp at which it is used (read, written, or borrowed), according to the definition of Non-Lexical Lifetimes (NLL) [Rust-Team 2018]. Unlike the lifetime of owner, which can be easily decided by the program scope, the lifetime of reference is decided by the last use of it. For instance, in Listing 2, the lifetime of b1 is from Line 2.3 to 2.6, while the lifetime of b2 is from Line 2.4 to 2.7.

2.3 The OBS Invariants

Because a memory block can be accessed by multiple aliases, OBS should fulfill the following OBS guarantees in order to avoid memory errors.

- Each alias only accesses (reads or writes) one valid block. For owners, they should own the block. For references, their owners should be in their lifetimes to ensure validity.
- At any time in an execution, each block can be accessed by either multiple shared aliases (but no mutable alias) or exclusively accessed by a unique mutable alias.

In order to meet these two guarantees, OBS maintains the following invariants. We will prove that these invariants exactly ensure the guarantees in Section 3.

Definition 2.2 (The OBS invariants). The following invariants must be satisfied:

- (1) Unique owner invariant: Each block has a unique owner.
- (2) *Lifetime inclusion invariant*: If $x \to_b y$ then the lifetime of x should always be within the lifetime of y in order to avoid dangling pointers.
- (3) *Lifetime disjoint invariant*: There are *no* two references to the same referent such that their lifetimes intersect and one of them is a mutable reference.
- (4) *Writing permission invariant*: If $x \to_s y$ then the writing permission of y should be disabled until the end of x's lifetime.
- (5) Reading and writing permission invariant: If $x \to_m y$ then both the reading and writing permissions of y should be disabled until the end of x's lifetime.

We now illustrate the above invariants by examples. The unique owner invariant is obvious. The move operation preserves the invariant. Listing 4 violates Invariant (2). The lifetime of owner ν is from Line 4.3 to 4.6 and the lifetime of reference b is from Line 4.4 to 4.7. Therefore the reference's lifetime is not within the owner's lifetime.

```
Listing 4. Listing 5.

4.1 let mut b; struct PT\{x:i32,y:i32\}
4.2 {

4.3 let mut v = vec![1,2];
4.4 b = \& mut v;
4.5 let t = (*b)[1];
4.6 }

b \rightarrow_s v

Listing 5.

struct PT\{x:i32,y:i32\}

5.1 let mut v = PT\{x:1,y:2\};

5.2 let b1 = \&v;

5.3 let b2 = \&mut(v.x);

5.4 let t = (*b1).x;

5.5 (*b2) = 2;

b \rightarrow_s v

b \rightarrow_s v
```

Listing 5 violates Invariant (3). It is a more interesting example as b2 borrows the field x of v instead of v. Rust also enables fine-grained borrowings for *struct* types. Let ST be a *struct* type with fields d_1, \ldots, d_n and x be the owner of a block of the type ST. We can create borrowings for both x (e.g. $y \rightarrow_b x$) and the fields of x (e.g. $y \rightarrow_b x.d_i$). The relation between them is: $y \rightarrow_b x \Rightarrow y \rightarrow_b x.d_1 \wedge \ldots \wedge y \rightarrow_b x.d_n$. This means that if x is borrowed then all its fields are borrowed. It also means that an alias can not only be a variable but also be fields of variables. Moreover, a block could be divided into "sub-blocks" corresponding fields. The lifetime of b1 $\rightarrow_b v$ is from Line 5.2 to 5.4. We can also infer that b1 $\rightarrow_s v$. x from b1 $\rightarrow_s v$. X from b1 $\rightarrow_s v$. x and b2 $\rightarrow_m v$. x is from Line 5.3 to 5.5. The intersection of the lifetimes of b1 $\rightarrow_s v$. x and b2 $\rightarrow_m v$. x is not empty and one of them is a mutable reference. Listing 3 violates Invariant (4). The lifetime of shared reference b2 is from Line 3.3 to 3.6. Since b2 reborrows b1, b1's writing permission should be disabled within the lifetime of b2. Thus, the writing through b1 at Line 3.5 is illegal.

Both owners and references are safe pointers. But Rust also supports raw pointers, which are unsafe pointers that can escape from OBS checking. We formalize raw pointers in the memory model in Section 4. Besides raw pointers, Rust also has other unsafe features including unsafe scopes and unsafe functions, which are statement scopes and functions wherein raw pointers are read or written. We model these unsafe features in CL and the translation semantics in Section 5.

3 HIGH-LEVEL ABSTRACTION OF OBS

This section presents a high-level formalization of OBS by graphs, in which we only focus on the ownership and borrowing relations among aliases and blocks. At the end of this section, we prove that the OBS invariants exactly ensure the OBS guarantees.

Definition 3.1 (Lifetimes). Let Tim be an infinitely countable set of timestamps. (Tim, \leq) is a totally ordered set. A strict order < over Tim is defined as t < t' iff $t \leq t' \land t \neq t'$. The function $Su: Tim \to Tim$ is the successor function of timestamps, such that Su(t) = t' iff $t < t' \land \nexists t'' . t < t'' < t'$. A lifetime $t_1 \sim t_2$ (where $t_1, t_2 \in Tim$), defined as $\{t \mid t_1 \leq t \leq t_2\}$, is a subset of Tim.

Let LT denotes the set of all lifetimes. Because a lifetime is a set of timestamps, we can use set operations on lifetimes.

Blocks, aliases, and their relations constitute an OBS graph, defined as follows:

Definition 3.2 (OBS graph of a block). An OBS graph is defined as $G = (B, V, E, \mathcal{F})$, where

- *B* is a memory block,
- *V* is a set of aliases of *B*, e.g., owners and references,
- $E: V \to \{o, s, m\} \times (\{B\} \cup V)$ is a set of edges, i.e., a total mapping from aliases to the product of $\{o, m, s\}$ and $\{B\} \cup V$. E(a) = (o, B) iff $a \to_o B$, i.e. a owns B. E(a) = (s, a') iff $a \to_s a'$, i.e., a is a shared reference to a'. E(a) = (m, a') iff $a \to_m a'$, i.e., a is a mutable reference to a'.
- $\mathcal{F}: E \to LT$ is the lifetimes of edges.

Note that E is a total mapping as each alias holds exactly one value. In the sequel, E(a) = (*, a') and $E(a) \to (a') \to (a')$

We first explain the OBS graph for B. In the graph, an edge is labeled by the mutability of the borrowing and its lifetime. For instance, the edge from b2 to b1 is labeled by s[3-4], it denotes $b2 \rightarrow_s b1$ and $\mathcal{F}(b2 \rightarrow_s b1) = \{3,4\}$. The notation [3-4] denotes the range from 3 to 4. The second

struct PT {x:i32, y:i32}

1 let mut v = PT{x:1, y:2};

2 let b1 = & mut v;

3 let b2 = & (* b1);
4 let z = (*b2).x;

b1 b1
$$\rightarrow_m$$
 v

b2 $s[3-4]$ b1 $m[2-5]$ v $o[1-7]$ B

b2 $s[3-4]$ b1 $m[2-5]$ v

b3 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b2 $m[2-5]$ b3 $m[2-5]$ b3 $m[2-5]$ b1 $m[2-5]$ b3 $m[2-5]$ b4 $m[2-5]$ b3 $m[2-5]$ b3 $m[2-5]$ b4 $m[2-5]$ b5 $m[2-5]$ b6 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b2 $m[2-5]$ b3 $m[2-5]$ b3 $m[2-5]$ b3 $m[2-5]$ b4 $m[2-5]$ b3 $m[2-5]$ b4 $m[2-5]$ b5 $m[2-5]$ b6 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b3 $m[2-5]$ b3 $m[2-5]$ b4 $m[2-5]$ b5 $m[2-5]$ b6 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b3 $m[2-5]$ b3 $m[2-5]$ b4 $m[2-5]$ b5 $m[2-5]$ b6 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b2 $m[2-5]$ b3 $m[2-5]$ b4 $m[2-5]$ b5 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b2 $m[2-5]$ b3 $m[2-5]$ b4 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b2 $m[2-5]$ b3 $m[2-5]$ b1 $m[2-5]$ b3 $m[2-5]$ b4 $m[2-5]$ b5 $m[2-5]$ b6 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b2 $m[2-5]$ b3 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b2 $m[2-5]$ b3 $m[2-5]$ b1 $m[2-5]$ b2 $m[2-5]$ b3 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b2 $m[2-5]$ b3 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b2 $m[2-5]$ b1 $m[2-5]$ b2 $m[2-5]$ b3 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b2 $m[2-5]$ b1 $m[2-5]$ b2 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b2 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b2 $m[2-5]$ b1 $m[2-5]$ b2 $m[2-5]$ b2 $m[2-5]$ b1 $m[2-5]$ b2 $m[2-5]$ b3 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b2 $m[2-5]$ b2 $m[2-5]$ b1 $m[2-5]$ b2 $m[2-5]$ b3 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b1 $m[2-5]$ b2 $m[2-5]$ b1 $m[2-5]$

Fig. 1. An example of OBG graphs

OBS graph of B.0 is for the field x, since b3 directly borrows v.x. The dashed arrow in the OBS graph is inferred from the borrowing $b_1 \rightarrow_m v$.

To formalize the OBS invariants (1), (2) and (3), we define well-formedness for OBS graphs.

Definition 3.3 (Well-formed OBS graphs). Let $G = (V, B, E, \mathcal{F})$ be an OBS graph. It is well-formed if and only if it satisfies

- (1) There is a unique $a \in V$ such that E(a) = (o, B).
- (2) $\forall a, a', a'' \in V \cup \{B\}$. $E(a) = (*, a') \land E(a') = (*, a'') \implies \mathcal{F}(a \to_* a') \subset \mathcal{F}(a' \to_* a'')$. (3) $\forall a, a', a'' \in V$. $E(a') = (b, a) \land E(a'') = (m, a) \implies \mathcal{F}(a' \to_b a) \cap \mathcal{F}(a'' \to_m a) = \emptyset$.

In Fig. 1, the two OBS graphs are well-formed. For instance, the block B.0 is uniquely pointed by v.x. The lifetime of b2 \rightarrow_s b1 is within the lifetime of b1 \rightarrow_m v.x. The lifetimes of b₁ \rightarrow_m v.x and $b_3 \rightarrow_m v.x$ do not intersect. One important property for the well-formed OBS graphs is acyclic. The proof is in Appendix A.

LEMMA 3.4. A well-formed OBS graph is acyclic.

To formalize the OBS invariants (4) and (5), we define reading and writing permission functions. An alias is permitted to read or write at a specific timestamp if the reading or writing permission function returns true, respectively.

Definition 3.5 (Permission functions). Let $G = (B, V, E, \mathcal{F})$ be an OBS graph. The reading permission function $R_G: V \times Tim \rightarrow \mathbb{B}$ is defined as

$$R_G(a,t) = \begin{cases} true & if (\exists a'.a \to_* a' \in E \land t \in \mathcal{F}(a \to_* a')) \land (\nexists a''.t \in \mathcal{F}(a'' \to_m a)) \\ false & otherwise \end{cases}$$

where the condition $t \in \mathcal{F}(a \to_* a')$ denotes that t is in the lifetime of the relation. Note that the condition $\nexists a'' . t \in \mathcal{F}(a'' \to_m a)$ ensures that a's reading permission is not disabled at t.

The writing permission function $W_G: V \times Tim \to \mathbb{B}$ is defined as

$$W_G(a,t) = \begin{cases} true & if (\exists a'.a \rightarrow_{\{o,m\}} a' \in E \land t \in \mathcal{F}(a \rightarrow_{\{o,m\}} a')) \land (\nexists a''.t \in \mathcal{F}(a'' \rightarrow_b a)) \\ false & otherwise \end{cases}$$

where $a \rightarrow_{\{o,m\}} a'$ denotes $a \rightarrow_o a'$ or $a \rightarrow_m a'$.

In Fig. 1, at timestamp 3, $W_G(b1,3) = false$ as there exists b2 such that $3 \in \mathcal{F}(b2 \to sb1)$. At timestamp 7, $R_G(b3,7) = true$ as $7 \in \mathcal{F}(b3 \to_m v.x)$ and $\nexists a'.a' \to_b b3 \in E$.

In the following theorem, we show that a well-formed OBS graph satisfies exclusive mutation guarantee with respect to the permission functions. The proof is in Appendix B.

THEOREM 3.6. Let $G = (V, B, E, \mathcal{F})$ be an OBS graph and t be a timestamp. If G is well-formed then we have either (1) $\forall a \in V$, $W_G(a, t) = false$ or (2) $\exists ! a \in V$, $W_G(a, t) = true \land (\forall a'. a \neq a' \Rightarrow R_G(a', t) = W_G(a', t) = false$). The notation $\exists !$ denotes unique existential quantification.

Theorem 3.6 shows that for a block or a sub-block, if all readings or writings by the aliases in its OBG graph are executed at the timestamps that the corresponding permission functions return true then the readings and writings respect exclusive mutation guarantee.

Compared with the OBS of Rust compiler, our formalization relaxes some restrictions. For instance, in Rust compiler, an alias a could write a block owned by a' if all the borrowings in the path from a to a' in the OBS graph are all mutable borrowings and a' is declared as mutable. In the high-level abstraction, we have no such restriction. But in the memory model (Section 4) of RustSEM, we still respect this restriction.

In Rust, there is another definition of the lifetimes for references, called two-phase borrowing [Matsakis 2017], in which the lifetime of a mutable reference is from the timestamp that we start to use it instead of the timestamp that it is created. Our high-level abstraction is also compatible with it. We only need to reconsider the lifetimes of references, For instance, in Fig. 1 $\mathcal{F}(b3 \to_m v.x)$ is $\{7\}$ now. Definition 3.3 and 3.5 keep unchanged and Theorem 3.6 is still true.

4 MEMORY MODEL

In this section, we introduce our memory model, which is the core of RustSEM that formalizes OBS. The memory model supports the dynamic checking of OBS invariants. It can be viewed as an implementation or a refinement of the high-level abstraction of OBS. Moreover, it also supports sequential consistency checking for concurrent access.

We first illustrate our basic idea, especially the *dynamic lifetime extension*, with a motivation example in Fig. 2. There are 7 lines of code in the figure. Line 1 creates a block B for vec![1,2], whose address is b. The symbol $v\mapsto own(b)$ means that the variable v starts to own the block b. It denotes $v\to_o B$. The value own(b) indicates that the ownership of B belongs to v. Line 2 creates a borrowing relation $b1\to_m v$. We introduce $mut(2\sim 2,v)$ as a value to denote a *reference value* to v in the memory model, where v=v=v is a timestamp span from Line 2 to 2 in which v=v is used, since we only scan the code to Line 2 until now. When we scan Line 3, v=v is used to write the vector, the timestamp span of the reference v=v is a should be extended since it is used here. The new span is v=v is v=v. This treatment for lifetimes is called the *dynamic lifetime extension*. Line 4 writes the vector by its owner v=v. Note that v=v introduce the value v=v is lifetime is not extended to Line 4. Line 5 creates a relation v=v is v=v. We introduce the value v=v is denote a shared reference value to v=v whose lifetime is from Line 5 to 5 at the moment. Line 6 writes the vector by its owner, this is allowed at the moment since the existing two references cannot disabled it until now. But at Line 7, since v=v is used to read, the lifetime of the reference v=v should be extended to 7. Now, an error arises, i.e, the writing at Line 6 is within the lifetime of the shared reference, thus it is illegal.

From the motivation example, in order to support dynamic OBS invariant checking without looking backward, we select to store lifetimes for references. Moreover, we also need to record at which timestamps an alias is used to read or write. For instance, in Fig. 2 the owner v is recorded to be used to write at Line 4 and 6. Here we use

```
 \begin{array}{lll} 1 & | \mbox{let mut v=vec!} [1,2]; & \mbox{$v\mapsto own(b)$} \\ \hline 2 & | \mbox{let b1} = \& \mbox{mut $v$}; & \mbox{$b1\mapsto mut(2\sim2,\,v)$} \\ \hline 3 & | \mbox{$(^*b1)[0]$} = 2; & \mbox{$b1\mapsto mut(2\sim3,\,v)$, $b1$ is used for writing at 3} \\ \hline 4 & | \mbox{$v[0]$} = 1; & \mbox{$v$ is used for writing at 4} \\ \hline 5 & | \mbox{let b2} = \& \mbox{$v$}; & \mbox{$b2\mapsto shr(5\sim5,\,v)$} \\ \hline 6 & | \mbox{$v[1]$} = 2; & \mbox{$v$ is used for writing at 6} \\ \hline 7 & | \mbox{let $t=(^*b2)[1]$}; & \mbox{$b2\mapsto shr(5\sim7,\,v)$} \\ \hline \end{array}
```

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A error is detected until line 7, since b2 should disable v during its lifetime 5 ~ 7, but v is used to write at 6

```
Natural number
                           \mathsf{n},\mathsf{m}\in\mathbb{N}
Types
                             t \in \mathcal{T}
Time stamps
                          ts \in Tim
Local Lifetimes
                            lt \in LT ::= ts_1 \sim ts_2
Stack locations
                                               (L_s, \leq_s) is a partially ordered set
                             s \in L_s
Block locations
                             b \in L_b
                                               (L_b, \leq_b) is a totally ordered set and L_s \cap L_b = \emptyset
                                               L_h \triangleq L_h \times \mathbb{N}
Heap locations
                        (b, n) \in L_h
Mem locations
                            l \in L_m ::= s \mid (b, n)
Paths
                                   p ::= s | *p | p.n
Alias
                             a \in \mathcal{A} ::= s \mid a.n
Pointer Values
                                        ::= \text{own}(b) | \text{shr}(lt, p) | \text{shr}(p)
                                  pv
                                         mut(lt, p) \mid mut(p) \mid raw(l)
Primitive Values
                           sv \in SV
                                        ::= i \in Int \mid str \in Str \mid c \in Char
                                               f \in \mathsf{Float} \mid bv \in \mathsf{Bool}
                                         Values
                           v \in \mathsf{Val}
                                        := \perp |pv| sv
Mem Operations
                                тор
                                             alloc(n,t) | free(b)
                                        ::=
                                               lv(p) \mid read(p) \mid write(p, v)
                                               rawRead(l) \mid rawRead'(l)
                                               rawWrite(l, v) \mid rawWrite'(l, v)
                                               aRead(l) \mid aWrite(l, v)
```

Fig. 3. The grammar of the memory model

lines to denote timestamps for illustration. In the memory model, timestamps are generated by semantics rules. Fig. 3 illustrates the grammar of the memory model. We detail them in the following subsections.

4.1 Memory Configuration

A memory configuration is defined as a 4-tuple $mem = (S, H, \mathcal{P}, ms)$, where S is for stacks, H is for a heap, \mathcal{P} stores the latest timestamps at which an alias is used to read or write as we explained in the motivation example, ms is used for sequential consistency checking of concurrent access. We elaborate them as follows.

The stacks are modeled as a finite partial map $S:L_s \xrightarrow{fin} (Val \times T)$ from stack locations in the set L_s ("Stack locations" in Fig. 3) to typed values, denoted as v:t, where $v\in Val$ is a value and $t\in T$ is its type. Different stacks have disjoint location spaces in L_s to ensure that the stacks are local to their corresponding threads. L_s is partial order under \leq_s since only stack locations in the same thread are ordered to denote the creation order of local variables. Various strategies could be selected to implement L_s . For instance, a stack location could be a pair (tid, n), where tid is a thread id and n is a natural number to denote a local stack location of tid.

The heap is defined as a set of blocks $H = \{B_1, \dots, B_n\}$. A block B_i is 4-tuple (b_i, n_i, m_i, t_i) , where $b_i \in L_b, L_b$ is a set of block locations ("Block locations" in Fig. 3) and each block has a unique block location, $n_i \in \mathbb{N}$ is the size of the block, $m_i : [0, n_i - 1] \to \text{Val} \times \mathcal{T}$ is a map from the offsets in

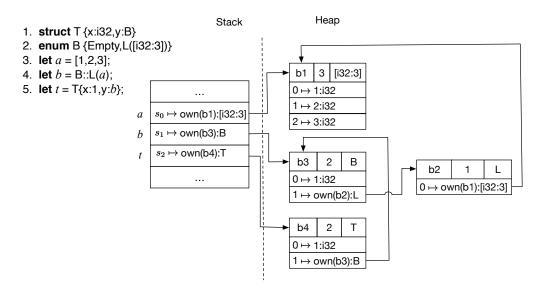


Fig. 4. An example of the stacks and heap

the range $[0, n_i - 1]$ to the corresponding typed values ($[n_1, n_2]$ denotes the set of natural numbers $n_1, n_1 + 1, \ldots, n_2$), and $t_i \in \mathcal{T}$ is the block's type. A heap location is defined as a pair (b, m) ("Heap locations" in Fig. 3), where b is a block location and m is the offset within the block b.

The definitions of stacks and heaps are capable of storing values of various types, such as primitive types (integers, boolean values, among others), pointers, arrays, product types, and sum types. Heap blocks storing values of primitive types or pointers have size 1. Blocks storing arrays or values of product types have size greater or equal than 1, according to the number of elements the types are composed of, and two elements in the case of sum types: one for the value itself and another to indicate the constructor that the sum type selects to construct the value. Fig. 4 illustrates the stacks and heap created by the program on the left in the figure. Three variables a, b, t created at lines 3, 4, 5 have the corresponding stack locations s_0 , s_1 , s_2 , respectively. Line 1 and 2 define a struct type and an enum type, respectively. Line 3 creates a block to store an array of the type [i32:3] (a 32 bits integer array of the size 3). The location, size, and type of the block are b1, 3, and [i32:3] respectively. The offsets 0, 1, 2 store the values 1, 2, 3 of type i32, respectively. Line 4 creates two blocks b2 and b3. The block b2 stores the value of the type L defined by the constructor L in the enumeration type B. The symbol own(b1) denotes a value storing the address of the block b1. The block b3 stores the value of the type B, which has two constructors: Empty and L. The value "1" at the offset 0 in the block b3 indicates the constructor L is selected, otherwise if it is 0 then the constructor Empty is selected. Line 5 creates the block b4 storing the value of type T.

In order to access memory locations, paths and aliases are introduced (Paths, Aliases in Fig. 3). A path could be a stack location, a dereference, or a field. It can be used to access a memory block with an alias that refers to the block. Consider the following code "struct $P\{x:i32,y:i32\}$ let $v = P\{x:1,y:2\}$; let z = & v; ", if we want to access the field x of v with the alias v then the path is v and v assume the stack location of v is and the field v corresponds to the offset 0. The path in the memory model is v and v has an alias, which is defined

as: $alias(p) \triangleq \begin{cases} p & p = s \text{ or } p = s.n_1...n_m \\ alias(p') & p = *p' \text{ or } p = (*p').n \end{cases}$. Roughly speaking, the alias of a path is the

term innermost the path without dereferences. An alias of a block itself is also a memory location.

For instance, $s_1.1$ corresponds to the heap location (b3, 1) in Fig. 4, which refers to the block b2. We define a predicate des(a', a) for two aliases as $des(a', a) \triangleq \exists m \geq 0.a' = a.n_1....n_m$. It means that a' is a field or nested field of a. For instance, in Fig. 4, $s_2.1$ is a field of s_2 , $s_2.1.1$ if a field of $s_2.1$ and the nested field of $s_2.1$ Note that, des(a, a) = true. This is introduced for fine-grained borrowings. For a borrowing relation $a \rightarrow_b a'$, if des(a'', a') then $a \rightarrow_b a''$.

The third element in the configuration mem is a finite partial map $\mathcal{P}: \mathcal{A} \xrightarrow{fin} Tim \times Tim$ from aliases to pairs of timestamps, which stores the latest timestamps at which an alias was used to read and write, respectively. For instance, $s \mapsto (ts_1, ts_2) \in \mathcal{P}$ means that the latest reading and writing using s happened at ts_1 and ts_2 , respectively. This information is used to check whether a reading or writing is disabled by other references.

Finally, $ms: L_m \xrightarrow{fin} \mathbb{N} \times \mathbb{N}$ is a finite partial map from memory locations ("Memory locations" in Fig. 3) to pairs of natural numbers, used to detect data races with respect to sequential consistency, where $l \mapsto (\mathsf{n}_1, \mathsf{n}_2) \in ms$ represents that there are n_1 and n_2 reading and writing simultaneous accesses from/to the location l. The idea is inspired by Jung and others [Jung et al. 2018a].

At the end of this subsection, we introduce some notations for $mem = (S, H, \mathcal{P}, ms)$. For a partial map M, M(k) denotes value of the key k in the map. If k has no value in M then M(k) is undefined. The notation $M[k \leftarrow v]$ denotes a new map obtained from M by replacing the value

of
$$k$$
 with v , which is defined as: $M[k \leftarrow v](k') \triangleq \begin{cases} v & k = k' \\ M[v] & k \neq k' \end{cases}$. Let $B = (b, n, m, t)$ be a block

in H and $l = (b, n'), 0 \le n' < n$. $H(l) \triangleq m(n')$ denotes the value stored in the heap location l. $H[l \leftarrow v] \triangleq (H \setminus \{B\}) \cup \{(b, n, m[n' \leftarrow v], t)\}$ denotes a new heap by replacing the value in l with v. Moreover, we use mem(l) = v to denote $S(l) = v \vee H(l) = v$ and $mem[l \leftarrow v]$ to denote a new memory configuration in which S is replaced with $S[l \leftarrow v]$ or H is replaced with $H[l \leftarrow v]$ when l is a stack or a heap location, respectively.

4.2 Values of Memory Model

We now introduce the kinds of values used in the memory. Generally speaking, there are three kinds of values: primitive values, pointers, and \bot denoting the uninitialization of a memory location. Primitive values include integers, floating-pointer values, boolean values, characters, and strings, which are standard. Here we focus on the elaboration of pointer values ("Pointer Values" in Fig. 3). Pointer values consist of the following 3 kinds:

- Own pointers (own(b)) to indicate the ownership of a block b, an alias holds an own pointer means it owns the block.
- Shared and mutable reference values. The shared (resp. mutable) reference values have two forms shr(lt,p) and shr(p) (resp. mut(lt,p) and mut(p)). The reason is that lifetimes are transparent for users using the model, whilst in the memory, a reference should be attached a lifetime for OBS checking.
- Unsafe pointers (raw(l)), which are raw pointers to memory locations. Unsafe pointers belong to the unsafe features of Rust.

Own pointers, shared and mutable reference values are safe pointers. Rust distinguishes safe and unsafe pointers to decide whether to carry out OBS invariant checking.

The values shr(p) and mut(p) create references to alias(p). For instance, we can write a borrowing like "let x = & (*v);" in Rust. Assume the stack location of v is s then the reference value to be assigned to s is shr(*s). Here the path is s and its alias is s. For two aliases s and s, if s holds a shared (resp. mutable) reference value to s, i.e., $shr(lt,p) \land alias(p) = s$ (resp. s muts) in s mes, then we also write s and s in s mes and s mes and s mes and s mes mes in s mes mes in s mes mes mes and s mes me

and the notation lifetime(a, mem) denotes lt in the values. Moreover the notation $a_1 \to_b^* a_n \in mem$ denotes that there exists a sequence $a_1, a_2, \ldots, a_{n-1}, a_n$ for $n \ge 1$ such that $a_i \to_b a_{i+1} \in mem$, $1 \le i < n$, where \to_b denotes either \to_m or \to_s . Note that $a \to_b^* a \in mem$.

The lifetime $ts_1 \sim ts_2$ in a reference value indicates that the reference is created at the timestamp ts_1 and the last timestamp it is used to read or write is ts_2 . Note that, In Fig. 3, the meta-variable lt is called *local lifetimes*. The reason is that for a reference a, it is *in use* if it is used to read, write, or borrowed by other references. The local lifetime is the span only from its creation to the timestamp that it is used to read or write. Therefore the exact lifetime of a reference in mem, denoted as $\mathcal{L}(a, mem)$, is defined as $\mathcal{L}(a, mem) = \bigcup_{a' \in \{a'' \mid a'' \rightarrow_b^* a \in mem\}} lifetime(a', mem)$. It is the union of all local lifetimes of a' such that $a' \rightarrow_b^* a \in mem$.

4.3 Lifetime-Free Memory Operation Interfaces

The memory operation interfaces ("Memory Operations" in Fig. 3) are lifetime-free, which means that the parameters of memory operations have no lifetimes. This design aims at abstracting lifetime information from the memory interfaces to enable reusability, since other languages may not have the notion of lifetimes. Memory operations include:

- Allocation (alloc(n, t)), allocates a new memory block of size n for storing the value of the type t. Free (free(b)), deallocates the memory block b.
- Raw reading and writing (rawRead(l), rawWrite(l, v)) provide non-atomic reading and writing without integrating OBS invariant checking.
- Atomic reading and writing (aRead(l), aWrite(l, v)) provide atomic reading and writing without integrating OBS invariant checking.
- Reading and Writing (read(p), write(p, v)) provide reading and writing with the integration
 of OBS invariant checking.
- Lvalue $(1 \lor (p))$, computes the Lvalue of the path p. Lvalue of a path is the memory location identified by the path and Rvalue is the value stored in the Lvalue of the path.

Here we need to restrict that if the parameter v in either rawWrite, aWrite, or write is a reference value then it can only be shr(l) or mut(l), i.e., no lifetimes. Actually, lifetime computation is hidden in the implementation of memory operations, which will be introduced in the following.

The semantics is defined by two kinds of transition relations. The first one is $\langle mem, tm \rangle^{ts} \leadsto_m \langle mem', tm' \rangle^{ts'}$. tm and tm' are terms that can be a value, a memory operation, or "." indicating that the operation is consumed. The symbols ts and ts' are the timestamps of the pairs and satisfying ts \leq ts'. The second one is $\langle mem, tm \rangle^{ts} \leadsto_m$ stuck indicating the semantics gets stuck. Some of S, H, \mathcal{P}, ms can be omitted in rules if they are not used to make rules more concise.

4.3.1 Operational Semantics for Allocation and Free. Rule Allocation defines the semantics for alloc(n, t), where n and t are the size and type of the new block, respectively. It creates a new block with a fresh block location (fresh(H)) and adds the new block to the heap H. Moreover, it also initializes ms for the new block. initBlk(n) denotes the map $\{0 \mapsto \bot, \ldots, n-1 \mapsto \bot\}$. initMS(b,n) = $\{(b,0) \mapsto (0,0),\ldots,(b,n-1) \mapsto (0,0)\}$. Free removes a block from H. The set bLoc(H) denotes all the block locations used in H, i.e., for any $b \in bLoc(H)$, there is a block (b, n, m, t) in H. "_" matches anything. The timestamps are increased by Su(ts).

```
ALLOCATION: FREE: b = fresh(H) \quad H' = H \cup (b, n, initBlk(n), t) \\ ms' = ms \cup initMS(b, n) \\ \langle (H, ms), alloc(n, t) \rangle^{ts} \leadsto_m \langle (H', ms'), own(b) \rangle^{Su(ts)} (H, ms), free(b) \rangle^{ts} \leadsto_m \langle (H', ms'), \rangle^{Su(ts)}
```

4.3.2 Operational Semantics for Non-Atomic Raw Reading and Writing. The rules for rawWrite and rawRead are non-atomic writing and reading that are directly applied to raw pointers. The execution of a raw operation will get stuck under data races. For a location l and $l \mapsto (n_1, n_2) \in ms$, data races are defined as (1) $(n_1 + n_2 \ge 2) \land (n_2 \ge 1)$, i.e., there are at least two threads accessing the location and at least one of them writes the location. In order to simulate non-atomic operations, both raw reading and writing are decomposed into two steps that can be interleaved.

The raw writing is defined by Rule RawWrite and RawWrite'. It is a two step operation where the first step modifies ms and translates rawWrite(l,v) to rawWrite'(l,v) and ms(l) = (0,0) ensures that the writing will not cause a data race. The second step writes v in the heap location l, and resets ms. The two steps can be interrupted by other threads. The semantics of raw reading is similar to the raw writing, which is defined by Rule RawRead and RawRead'. Timestamps are not increased, since they are not for safe pointers. We still need the semantics for detecting race conditions. Rule Race-RawRead defines the semantics for the race conditions of the reading. It evolves into the stuck state. The race semantics for other operations are similar.

```
RAWWRITE:
                                                                                                                                                RAWWRITE':
          \frac{\mathit{ms}(l) = (0,0) \quad \mathit{ms'} = \mathit{ms}[l \leftarrow (0,1)]}{\langle (H,\mathit{ms}), \mathsf{rawWrite}(l,v) \rangle^{\mathsf{ts}}}
                                                                                                                                                                       ms(l) = (0,1) H' = H[l \leftarrow v]
                                                                                                                                                 ms' = ms[l \leftarrow (0,0)]
\langle (H, ms), \text{rawWrite}'(l,v) \rangle^{\text{ts}} \leadsto_m \langle (H', ms'), . \rangle^{\text{ts}}
                          \leadsto_m \langle (H, ms'), \text{rawWrite}'(l, v) \rangle^{\text{ts}}
RAWREAD:
                                                                                                                                                          RAWREAD':
ms(l) = (\mathsf{n}, 0) \quad \mathsf{n} \ge 0 \quad ms' = ms[l \leftarrow (\mathsf{n} + 1, 0)]
                                                                                                                                                                          ms(l) = (n, 0) H(l) = v v \neq \bot
                                                                                                                                                           \begin{array}{c|c} \mathbf{n} \geq 1 & ms' = ms[l \leftarrow (\mathbf{n} - 1, 0)] \\ \hline \langle (H, ms), \mathsf{rawRead'}(l) \rangle^{\mathsf{ts}} \leadsto_m \langle (H, ms'), v \rangle^{\mathsf{ts}} \\ \end{array} 
                   \langle (H, ms), rawRead(l) \rangle^{ts}
                                   \rightsquigarrow_m \langle (H, ms'), \text{rawRead}'(l, v) \rangle^{\text{ts}}
RACE-RAWREAD:
                                                                                                                                          Concur-Stuck:
                                                                                                                                         \langle mem, t_1 \rangle^{\text{ts}} \leadsto_m \text{stuck or } \langle mem, t_2 \rangle^{\text{ts}} \leadsto_m \text{stuck}
\langle mem, t_1 \mid\mid t_2 \rangle \leadsto_c \text{stuck}
mem = (S, H, \mathcal{P}, ms) ms(l) = (\mathsf{n}_1, \mathsf{n}_2) \land \mathsf{n}_2 > 0
                  \langle mem, rawRead(l) \rangle^{ts} \leadsto_m stuck

\frac{\langle mem, t_1 \rangle^{\mathsf{ts}} \leadsto_m \langle mem', t \rangle^{\mathsf{ts}}}{\langle mem, t_1 \mid t_2 \rangle \leadsto_c \langle mem', t \mid t_2 \rangle} \qquad \frac{\langle mem, t_2 \rangle^{\mathsf{ts}} \leadsto_m \langle mem', t \rangle^{\mathsf{ts}}}{\langle mem, t_1 \mid t_2 \rangle \leadsto_c \langle mem', t_1 \mid t \rangle}
```

For instance, we consider the concurrent execution of two operations rawRead(l) and rawWrite(l,v) under a concurrent semantics based on interleaving, whose grammar is $\texttt{con} := tm \mid\mid tm$, where tm can be a memory operation, a value, or "." The semantics is defined by the Rule Concur-1, Concur-2, and Concur-Stuck, with the relation \leadsto_c (The concurrent semantics is only for illustration here. RustSEM concurrent semantics is implemented in CL level.

Assume the initial memory configuration satisfies ms(l) = (0,0) and $H(l) \neq \bot$, there are 4 possible execution sequences as follows:

```
(Sequence 1) \mathsf{rawRead}(l); \mathsf{rawRead}'(l); \mathsf{rawWrite}(l,v); \mathsf{rawWrite}'(l,v); (Sequence 2) \mathsf{rawWrite}(l,v); \mathsf{rawWrite}'(l,v); \mathsf{rawRead}(l); \mathsf{rawRead}(l); \mathsf{rawRead}(l); \mathsf{rawRead}(l); \mathsf{rawWrite}(l,v); \mathsf{stuck}; (Sequence 4) \mathsf{rawWrite}(l,v); \mathsf{rawRead}(l); \mathsf{stuck}.
```

Sequences (1) and (2) are safe, but Sequence (3) and (4) are unsafe. For instance, in Sequence (4), after executing rawWrite(l, v), we have that ms(l) = (0, 1), which makes rawRead(l) get stuck (Rule Race-RawRead). We will not present atomic reading and writing without OBS invariant checking as their semantics rules are trivial.

4.3.3 Operational Semantics for Reading and Writing with OBS Checking. The reading and writing operations for the safe pointers need to maintain the invariants of OBS. The invariants correspond to the well-formed memory configurations, which are defined as follows.

Definition 4.1 (Well-formed memory configurations). Let $mem = (S, H, \mathcal{P}, ms)$ be a memory configuration. It is well-formed, denoted as well-form(mem), iff it satisfies the following invariants.

- (1) $\forall b, l, l'.(mem(l) = mem(l') = own(b)) \Longrightarrow l = l'.$
- (2) $\forall a, a_1, a_2.a_1 \rightarrow_m a \in mem \land a_2 \rightarrow_b a' \in mem \land des(a, a') \Longrightarrow \mathcal{L}(a_1, mem) \cap \mathcal{L}(a_2, mem) = \emptyset.$
- $(3) \ \forall \texttt{a}, \texttt{a}'.\texttt{a} \rightarrow_{\texttt{s}} \texttt{a}' \in \textit{mem} \Longrightarrow (\forall \texttt{a}_1. \texttt{des}(\texttt{a}_1, \texttt{a}') \land \mathcal{P}(\texttt{a}_1) = (\texttt{ts}, \texttt{ts}') \Longrightarrow \texttt{ts}' \notin \mathcal{L}(\texttt{a}, \textit{mem})).$
- (4) $\forall a, a'.a \rightarrow_m a' \in mem \Longrightarrow (\forall a_1.des(a_1, a') \land \mathcal{P}(a_1) = (ts, ts') \Longrightarrow ts \notin \mathcal{L}(a, mem) \land ts' \notin \mathcal{L}(a, mem).$
- (5) $\forall l.ms(l) = (n_1, n_2) \Longrightarrow \neg((n_1 + n_2 \ge 2) \land (n_2 > 1)).$

Invariant (1) ensures that no block is owned by more than one locations. Invariant (2) ensures that an alias cannot be borrowed by two references simultaneously with one of them being mutable. Invariant (3) ensures if an alias is borrowed by a shared reference then its latest writing timestamp cannot be in the lifetime of the reference. Invariant (4) ensures if an alias is borrowed by a mutable reference then both its latest reading and writing timestamps cannot be in the lifetime of the reference. The last invariant ensures no data race.

Compared with well-formed OBS graphs, well-formed memory configurations also contain *unique owner* and *lifetime disjoint* invariants, but no *lifetime inclusion* invariant. Lifetime inclusion invariant will be maintained by the semantics rules directly. Invariants (4) and (5) of well-formed memory configurations correspond to the permission functions, but not exactly since it only disable the latest reading and writing. We will prove that it enforces memory operations to follow the permission functions in Subsection 4.4. Data race is not considered in the well-formed OBS graphs, since OBS graphs only consider about safe pointers

The semantics of Lvalue. The Ivalue of a path is the location identified by the path that is to be written or read. The lifetimes of references accessed during the computation of 1v should be extended since the references are used. The result of 1v(p) is pair (l, wp) where l is the Lvalue and wp is a boolean value indicates whether p is permitted to write. A path is permitted to write iff the borrowing links "alias $(p) \rightarrow_m a \dots a' \rightarrow_m x$ " from the alias alias(p) to the owner x are all mutable borrowings.

The first rule for Lvalue is Lv-Deref, which computes the Lvalue of *p, where p is a path. We use ref(p') (resp. ref(lt, p')) to denote a reference, which can be either a shared reference shr(p') (resp. shr(lt, p')) or a mutable reference mut(p') (resp. mut(lt, p')).

- Premise (1) computes the Lvalue of *p*, which is *l*. Lvalue semantics does not increase the timestamp as it is one of the sub-steps for reading and writing.
- Premise (2) checks whether the location is being written by a non-atomic operation. [mem₁].ms
 denotes the element ms in the memory configuration mem₁. It requires that the number of
 writings is 0 and the number of readings is greater than or equal to 0. Therefore it could be
 used to check the data races that a non-atomic writing is carrying out but interleaved by safe
 operations.
- Premise (3) requires that the location l must be a shared or mutable reference, since we can only dereference a reference. It is a shorthand for $\exists p', lt.mem_1(l) = ref(lt, p')$.
- As the reference *l* is used, its lifetime is extended to the timestamp ts (Premise (4)) by the function extLT, which is defined as follows.

$$\mathsf{extLT}(\textit{mem}, l, \mathsf{ts}) \triangleq \begin{cases} \textit{mem}[l \leftarrow \mathsf{shr}(\mathsf{ts}_1 \sim \mathsf{ts}, p')] & \textit{mem}(l) = \mathsf{shr}(\mathsf{ts}_1 \sim \mathsf{ts}_2, p') \\ \textit{mem}[l \leftarrow \mathsf{mut}(\mathsf{ts}_1 \sim \mathsf{ts}, p')] & \textit{mem}(l) = \mathsf{mut}(\mathsf{ts}_1 \sim \mathsf{ts}_2, p') \end{cases}$$

- Since the referent p' is a path, its Lyalue needs to be further computed (Premise (5)).
- Premise (6) ensures the memory configuration is still well-formed after the computation.

• Premise (7) checks whether l is a mutable reference (The notation e? v_1 , v_2 denotes that if e is true then v_1 otherwise v_2).

The resulting permission is decided by $wp \wedge wp_1 \wedge wp_2$, i.e., whether all references accessed by the semantics rule are mutable references. Rule LV-LOCATION and LV-FIELD compute the Lvalues of a location (which is itself) and a field, respectively.

LV-DEREF:

```
(1) \langle mem, 1 \lor (p) \rangle^{\text{ts}} \leadsto_m \langle mem_1, (l, wp_1) \rangle^{\text{ts}} \quad (2) \ \llbracket mem_1 \rrbracket. ms(l) = (\mathsf{n}, 0) \land \mathsf{n} \geq 0 (3) \ mem_1(l) = \mathsf{ref}(lt, p') \quad (4) \ mem_2 = \mathsf{extLT}(mem_1, l, \mathsf{ts}) \quad (5) \langle mem_2, 1 \lor (p') \rangle^{\text{ts}} \leadsto_m \langle mem_3, (l', wp_2) \rangle^{\text{ts}} (6) \ \mathsf{wellform}(mem_3) \quad (7) \ wp = (mem_1(l) = \mathsf{mut}(lt, p')?true : false) \langle mem, 1 \lor (*p) \rangle^{\text{ts}} \leadsto_m \langle mem_3, (l', wp \land wp_1 \land wp_2) \rangle^{\text{ts}}
```

```
LV-LOCATION:  \langle mem, 1 \lor (l) \rangle^{\mathsf{ts}} \leadsto_m \langle mem', (l, true) \rangle^{\mathsf{ts}}   (1) \langle mem, 1 \lor (p) \rangle^{\mathsf{ts}} \leadsto_m \langle mem_1, (l, wp) \rangle^{\mathsf{ts}}   (2) mem_1(l) = \mathsf{own}(b) \quad (3) \ b \in bLoc(mem_1)   \langle mem, 1 \lor (p, n) \rangle^{\mathsf{ts}} \leadsto_m \langle mem_1, ((b, n, ), wp) \rangle^{\mathsf{ts}}
```

For instance, assume in the memory, we have $s_1 \mapsto \text{mut}(1 \sim 2, s_2)$, $s_2 \mapsto \text{own}(b)$ and $ms(s_1) = (0,0)$, $ms(s_2) = (0,0)$. If we try to compute $lv(*s_1)$, Rule Lv-Deref works as follows.

- Premise (1) computes $\langle mem, lv(s_1) \rangle$ and gets $(s_1, true)$ by Rule Lv-Location.
- Premise (2) and (3) are true as $ms(s_1) = (0, 0)$ and s_1 holds $mut(1 \sim 2, s_2)$.
- Premise (4) extends its lifetime to the current timestamp (assume it is 4). Thus value of s_1 is updated as $mut(1 \sim 4, s_2)$ now.
- Premise (5) further computes the lvalue of $lv(s_2)$, which is $(s_2, true)$. Therefore $lv(*s_1)$ is s_2 .
- The writing permission is true as s_1 a mutable reference and the writing permission of both $lv(s_1)$ and $lv(s_2)$ are true.

The semantics of writing. The semantics of write (p, v), writes the value v to the Lvalue of p. Rule Write-Ref defines the semantics for writing a reference value ref(p') to the Lvalue of the path p. The semantics is elaborated as follows.

- Premise (1) computes the Lyalue of the path p by lv(p) and requires the permission to write.
- Premise (2) ensures no reading or writing to the location *l*,
- Premise (3) writes the value v to the location l. The reference value should be attached with the local lifetime ts \sim ts, i.e., the reference starts to have a lifetime.
- Premise (4) updates the latest writing timestamp of p's alias by the function addWrite. Assume $mem = (S, H, \mathcal{P}, ms)$, addWrite is defined as: addWrite(mem, a, ts) $\triangleq (S, H, \mathcal{P}[a \leftarrow (ts_1, ts)], ms)$ if $\mathcal{P}(a) = (ts_1, ts_2)$.
- Premise (5) ensures that the memory configuration is well-formed after the writing.

WRITE-REF:

```
(1) \langle mem, \mathbb{I} \vee (p) \rangle^{\mathsf{ts}} \leadsto_m \langle mem_1, (l, true) \rangle^{\mathsf{ts}} \quad (2) \ [\![mem_1]\!].ms(l) = (0, 0)
(3) \ mem_2 = mem_1[l \leftarrow \mathsf{ref}(\mathsf{ts} \sim \mathsf{ts}, p')] \quad (4) \ mem_3 = \mathsf{addWrite}(mem_2, \mathsf{alias}(p), \mathsf{ts}) \quad (5) \ \mathsf{wellform}(mem_3)
\langle mem, \mathsf{write}(p, \mathsf{ref}(p')) \rangle^{\mathsf{ts}} \leadsto_m \langle mem_3, . \rangle^{Su(\mathsf{ts})}
```

WRITE-OWN:

```
(1) \ \langle mem, 1v(p) \rangle^{\text{ts}} \leadsto_m \langle mem_1, (l, true) \rangle^{\text{ts}}
(2) \ \llbracket mem_1 \rrbracket .ms(l) = (0, 0)
(3) \ mem_2 = mem_1 [l \leftarrow \text{own}(b)]
(4) \ mem_3 = \text{addWrite}(mem_2, \text{alias}(p), \text{ts})
(5) \ \text{wellform}(mem_3) \quad (6) \ \text{b} \in bLoc(mem)
\langle mem, \text{write}(p, \text{own}(b)) \rangle^{\text{ts}} \leadsto_m \langle mem_3, . \rangle^{Su(\text{ts})}
```

WRITE-PRIMITIVE:

```
(1) \langle mem, 1v(p) \rangle^{\texttt{ts}} \leadsto_m \langle mem_1, (l, true) \rangle^{\texttt{ts}} \\ (2) v \in SV \quad (2) \ [mem_1] \ .ms(l) = (0, 0) \\ (3) mem_2 = mem_1[l \leftarrow v] \\ (4) mem_3 = \text{addWrite}(mem_2, \text{alias}(p), \text{ts}) \\ \langle mem, \text{write}(p, v) \rangle^{\texttt{ts}} \leadsto_m \langle mem_3, .\rangle^{Su(\texttt{ts})}
```

Rule Write-Own is the semantics for writing an owner to a memory location. Rule Write-Primitive is the semantics for writing primitive values, such as integers and boolean values.

The semantics of reading. The operation read(p) reads a value from the Lvalue of a path p. Read-Ref defines the semantics for reading a reference.

- Premise (1) computes the Lvalue of p, which is l and Premise (3) reads the value from l,
- Premise (2) ensures no writing to the location. Premise (4) requires the value stored in *l* should be a shared or mutable reference.
- Premise (5) updates the latest timestamp, at which the alias of *p* is read, by the function addRead, which is similar to addWrite.
- Premise (6) extends the lifetime of *l* as it is used and Premise (7) checks well-formedness..

READ-REF:

```
(1) \langle mem, lv(p) \rangle^{ts} \leadsto_m \langle mem_1, (l, \_) \rangle^{ts} (2) [mem_1] .ms(l) = (n, 0) \land n \ge 0 (3) v = mem_1(l) (4) v = ref(lt, p') (5) mem_2 = addRead(mem_1, alias(p), ts) (6) mem_3 = extLT(mem_1, l, ts) (7) wellform(mem_3) \langle mem, read(p) \rangle^{ts} \leadsto_m \langle mem_3, v \rangle^{Su(ts)}

READ-Non-Reference:

(1) \langle mem, lv(p) \rangle^{ts} \leadsto_m \langle mem_1, (l, \bot) \rangle^{ts} (2) [mem_1] [ms(l) = (n, 0) \land n \ge 0] (3) v = mem_1(l)
```

```
(1) \langle mem, 1v(p) \rangle^{\text{ts}} \leadsto_m \langle mem_1, (l, \_) \rangle^{\text{ts}} (2) [\![mem_1]\!].ms(l) = (n, 0) \land n \ge 0 (3) v = mem_1(l) (4) v is not a reference (5) v \ne \bot (7) if v = \text{own}(b) then b \in bLoc(mem) (6) mem_2 = \text{addRead}(mem_1, \text{alias}(p), \text{ts}) \langle mem, \text{read}(p) \rangle^{\text{ts}} \leadsto_m \langle mem_2, v \rangle^{Su(\text{ts})}
```

Rule Read-Non-Reference defines the semantics of reading non-reference values.

Compared with weak memory models [Jung et al. 2018b; Podkopaev et al. 2019], RustSEM only concerns about sequential consistency and lies in higher abstraction level without bridging the gap between OBS and specific hardware architectures. We could further refine our memory model by incorporating weak memory consistency in the future to strengthen its usability.

4.4 Refinement Relation between High-Level OBS and Memory Model

In this subsection, we prove the refinement relation between high-level abstraction of OBS and the memory model. We begin with the definition of safe sequences.

Definition 4.2. Let $\pi = (mem_0, ts_0), op_1, (mem_1, ts_1), \dots, (mem_{n-1}, ts_{n-1}), op_n, (mem_n, ts_n)$ be an alternating sequence of memory configurations with timestamps and operations. For each pair (mem, ts), mem is a memory configuration and ts is the timestamp reaching mem. π is a safe sequence iff

- (1) mem_0 is an empty configuration, i.e., S, \mathcal{P} , ms are empty maps and H is an empty set,
- (2) op_i , $1 \le i \le n$, is one of the memory operations: alloc(n, t), read(p), write(p, v), free(b),
- (3) $\langle mem_i, op_{i+1} \rangle^{\mathsf{ts}_i} \leadsto_m \langle mem_{i+1}, re \rangle^{\mathsf{ts}_{i+1}}$, for all $0 \le i < n$, where re can only be a value if op_{i+1} is read or alloc, otherwise consumed ".".

The memory configurations are the *data refinement* of the high-level OBS graphs. For each memory configuration, it has an underlying OBS graph.

Definition 4.3. Let $\pi = (mem_0, ts_0), op_1, (mem_1, ts_1), \dots, (mem_{n-1}, ts_{n-1}), op_n, (mem_n, ts_n)$ be a safe sequence and B be a block in mem_i , $(0 \le i \le n)$, whose location is b. The OBS graph (V, B, E, \mathcal{F}) of mem_i is defined as:

• V is a set of aliases in mem_i that can access B,

- $\bullet \text{ for any node } a \text{ in } V, E(a) = \begin{cases} (o,B) & \text{ if } mem(a) = \text{own(b)} \\ (s,a') & \text{ if } a \rightarrow_s a'' \in mem_i \land \text{des}(a',a'') \\ (m,a') & \text{ if } a \rightarrow_m a'' \in mem_i \land \text{des}(a',a'') \end{cases}$ $\bullet \ \mathcal{F}(a \rightarrow_b a') = \mathcal{L}(a,mem_i). \ \mathcal{F}(a \rightarrow_o B) = \text{ts}_j \sim \text{ts}_i \text{ where for all } j \leq k \leq i, mem_k(a) = i \text{ the substitution of } a' \text{ the substitution of } a'$
- own(b) and $mem_{i-1}(a) \neq own(b)$.

It is now sufficient to present Theorem 4.4, which specifies the refinement relation between the high-level abstraction and the memory model.

THEOREM 4.4. Let $\pi = (mem_0, ts_0), op_1, (mem_1, ts_1), \dots, (mem_{n-1}, ts_{n-1}), op_n, (mem_n, ts_n)$ be a safe sequence and B be a block. Assume the OBS graph of B in mem_i $(0 \le i \le n)$ is $G = (V, B, E, \mathcal{F})$ then we have

- (1) (Data refinement) G is well-formed.
- (2) (Operation refinement) For any two nodes $a, a' \in V \cup \{B\}$ such that $\mathcal{F}(a \to_* a') = lt$, for any operation op_k (0 < $k \le n$), which is executed at the timestamp $ts \in lt$,
 - if op_k is an operation that reading B by the alias a then $R_G(a, ts) = true$,
 - if op_k is an operation that writing B by the alias a then $W_G(a, ts) = true$.

The proof is in Appendix C. Theorem 4.4 shows that all operations by an alias a follow the permission functions of mem_i 's OBS graph for B.

CL LANGUAGE AND TRANSLATION SEMANTICS

In this section, we present the basic idea of CL language and the translation semantics, the supported features of Rust, and the efforts of developing RustSEM. The semantics rules are not presented due to page limitation. Actually, the core of RustSEM, OBS, is already formalized in the memory model.

The design of CL is to reduce Rust grammar to a simple core in order to avoid redundant formalization. Table 1 illustrates the Rust features supported by RustSEM (Column "Rust features") and the corresponding grammars in CL (Column "CL grammar") for implementing the Rust features on the same row.

In CL, all constructs are expressions e that can be evaluated to values. The notion of variables (x), values (v), dereferences (*e), fields (e.e'), arithmetic and boolean expressions, print, skip (clskip), and assertions are standard. Shared (&e) and mutable (& mut e) references are the same as in Rust.

Functions in CL are defined as expressions that can be evaluated and assigned to variables. This is designed to represent Rust closures, which are anonymous functions with environments. There are two forms of function calls: call and pcall. The former just calls the function f with a sequence of arguments, whilst the latter is a polymorphic call $pcall(e', f, \vec{e})$ implementing dynamic dispatchs. For instance, in Rust there are expressions like e'. $f(e_1, \ldots, e_n)$, for which we only know that e' is an object implementing a trait (traits are like interfaces) containing f. Since there may be more than one type implementing the trait, the concrete function f to be invoked depends on the type of e' at runtime. As we modeled the memory as typed, i.e. each memory location stores both values and their types, the type of e' at runtime can be obtained by reading the memory. Branches in CL are defined by the case construct. Each case $e \rightarrow e'$ is a guarded action such that e' can be executed only if *e* evaluates to be true. The constructs of threads (*thread*), bindings (*bindings*), assignment (assign), and sequence (seg) are standard. The memory operations mop are also constructs in CL. It means that the memory access in CL invokes the memory operations in Section 4. As the OBS is formalized in the memory model, CL semantics does not need to concern about it.

The translation semantics translates a Rust program to a CL program. Table 1 shows that most features in Rust can be translated to corresponding CL constructs. We explain the last two rows

Rust features	CL grammar			
• primitive values • shared and mutable references	$e := v \mid x \mid *e \mid e.e' \mid \text{allocInit}(n, \vec{e}, t)$			
• casting pointers • arithmetic expressions	$ e_1 + e_2 e_1 < e_2 e_1 \& e_2 \dots$			
• print • assertion • move semantics • dereference	move(e) &e & mut e			
 field access • array access 	print(e) clskip assert(e)			
• binding • execution block	$binding ::= let x = e in \{e'\}$			
• assignment	assign := e := e'			
• function • impl block • trait definition • trait impl	$fun := \operatorname{fun} f(\vec{x})\{e\} \mid \operatorname{call}(f, \vec{e}) \mid \operatorname{pcall}(e', f, \vec{e})$			
• function call • method call • dynamic dispatch				
• polymorphism • closure				
• branch • pattern matching	$branch ::= case\{\overrightarrow{e} \rightarrow \overrightarrow{e'}\}$			
loop	$loop ::= loop e \{e'\} \mid break \mid continue$			
	seq := e; e'			
• concurrency	$thread ::= fork(e) \mid wait(e)$			
• raw pointers • intrinsic function	e ::= mop			
• struct and enum types • generic types	supported by the translation			
Box	-			

Table 1. Correspondence between Rust and CL features

in the table. For struct and enum types (user-defined types), in CL, the definitions of user-defined types are not stored. When a Rust program tries to create an object of a user-defined type, the translation generates an allocation in memory block with size of the type in CL. Translation semantics records the information of types, therefore it knows the memory block size for each type. Three data structures: Box, Array, and Vector are not hard-coded in the semantics, since they are just a collection of Rust programs.

Although RustSEM models a considerably large subset of Rust, it is not complete. Rust has no standard and evolves very fast, which makes it hard or almost impossible to some extent to build a complete semantics for it. We do not model module definitions, #[derive] attributes, among others.

Developing a formal semantics for real world languages always requires huge efforts. RustSEM is formalized in \mathbb{K} and consists of around 1100 semantics rules, taking two and a half man-years.

6 EVALUATION OF SEMANTICS CORRECTNESS AND APPLICATIONS

In this section, we evaluate the semantics correctness of RustSEM (Subsection 6.1) and its applications in verification (Subsection 6.2).

6.1 Semantics Correctness

We have evaluated RustSEM in the following three aspects: (1) Functional correctness. For each input of a program, we compare the execution output of RustSEM and the machine code generated by the Rust compiler. (2) OBS checking evaluation. For each program, we check whether RustSEM maintains the OBS invariants. It is compared with the compilation result of the Rust compiler. (3) Semantic consistency, i.e., the absence of semantic ambiguities.

We elaborate semantic ambiguities by an example. Consider the if-construct: "if e B_1 else B_2 " where e is an expression and B_i is an execution block for i=1,2. The semantics for the construct could be:

Branch:
$$\frac{(C,e) \leadsto (C',v)}{(C,\text{if e } B_1 \text{ else } B_2) \leadsto (C',\text{if v } B_1 \text{ else } B_2)}$$

TRUE:
$$V = true$$
 FALSE:
$$(C, if \lor B_1 else B_2) \leadsto (C, B_1)$$

$$(C, if \lor B_1 else B_2) \leadsto (C, B_2)$$

The semantics is a transition relation $(C, m) \sim (C', m')$, where C, C' are configurations and m, m' are terms, where a term could be a program or a value. Rule Branch evaluates the expression e to a value v under the configuration C and C evolves into a new configuration C' since the evaluation may have side effects. The if-construct is reduced to another if-construct with e replaced by its value v. Rule True is for the case that v equals true, in which the first block should be executed. Rule False is for the case that v equals false, in which the second block should be executed. Here we deliberately remove the checking v = false from the premise of Rule False, which will lead to ambiguity. Both Rule True and Rule False can match the if-construct with v being true. This is called t0 semantic t1 ambiguity, i.e., more than one semantics rules could be applied to the same pair of configuration and term. If t2 always tries to match Rule True first then the execution has no problem. But if t3 matches Rule False first then the semantics is not correct. This bug could be fixed by adding v1 adding v2 and v3 false to the premises of False.

There is another semantic ambiguity in the example. If we do not distinguish values from expressions in the semantics then the construct "if true B_1 else B_2 " can match both Rule Branch and True, since true is also an expression. Rule Branch could match it infinitely many times, which should be avoided in our semantics.

Now we formally define the semantics correctness and explain our testing methods.

Definition 6.1 (Semantics rules and semantics). Let C be a set of configurations and M be a set of terms. A semantics rule is a partial function: $Ru : C \times M \rightarrow C \times M$, i.e., mapping (or reducing) a pair (c, m) of configuration c and term m to a new pair (c', m') of configuration c' and term m'. A semantics is a set of semantics rules: $SL = \{Ru_0, \ldots, Ru_n\}$.

Definition 6.2 (Execution traces of semantics). Let SL be a semantics for a language. A finite execution trace of SL is a sequence $\pi = (c_0, m_0)(c_1, m_1) \dots (c_n, m_n)$, such that for each $0 \le i < n$, $\exists Ru \in SL.Ru(c_i, m_i) = (c_{i+1}, m_{i+1})$. The trace π is called terminated if for all rule $Ru \in SL$, $Ru((c_n, m_n))$ is undefined. An infinite or divergence trace is an infinite sequence $\pi = (c_0, m_0)(c_1, m_1) \dots$, such that for each $0 \le i$, $\exists Ru \in SL.Ru(c_i, m_i) = (c_{i+1}, m_{i+1})$.

Definition 6.3 (Semantics Correctness). Let m be a program in a language, whose semantics is SL, the subset of configurations $pre_m \subseteq C$ be the precondition of m, the relation $post_m \subseteq C \times (C \cup \{\infty\})$ be the postcondition of m, where ∞ denotes that the computation is diverge. The semantics SL is called partially correct iff for all program m and for all $c \in pre_m$, either there is a terminating trace $(c, m) \dots (c', m')$ in SL satisfying $((c, m), (c', m')) \in post_m$ or there is an infinite trace starting from (c, m) satisfying $((c, m), \infty) \in post_m$. The semantics SL is called completely correct iff for all program m and for all $c \in pre_m$, every terminating traces $(c, m) \dots (c', m')$ in SL satisfying $((c, m), (c', m')) \in post_m$ and if there is an infinite trace starting from (c, m) then $((c, m), \infty) \in post_m$.

Let SL be a semantics, c be a configuration and m be a term. We denote by $Trace_{SL}(c, m)$ the set of all terminating or infinite traces with (c, m) as the initial pair. \mathbb{K} 's execution engine selects one of the traces in $Trace_{SL}(c, m)$ for execution, whilst \mathbb{K} 's verification engine searches for all possible execution traces in $Trace_{SL}(c, m)$.

An ambiguous semantics might be *partially correct* but not *completely correct*. Thus, we need to detect semantic ambiguities to ensure *complete correctness*. Unlike Coq or Isabelle, \mathbb{K} is not a theorem prover that can be used to prove some properties of RustSEM. Instead, we can use its execution engine for testing. However, only using its execution engine is not enough for detecting semantic ambiguities if it always selects the right trace for execution. Therefore, we also use its

verification engine for testing. Note that this is still called testing because test cases are used and the input of each test case is fixed. For each test case, the following three steps are carried out.

- (1) Running the execution engine on the test case to detect bugs quickly.
- (2) Running the verification engine to detect semantic ambiguities.
- (3) Appending assert (False) to the end of each test case and run the verification engine again.

Note that Step (3) is used to detect semantic ambiguities missed by Steps (1) and (2). If a test case is not properly selected, it will be correct under all possible execution traces. Thus \mathbb{K} only outputs "sat" and some semantic ambiguities are missed. If we append assert(False) at the end of the test case then the verification engine can report all execution traces to assert(False) as the program always fails.

This testing strategy is efficient and effective. It is efficient because the test cases are small and the inputs are fixed, thus no state explosion exists. It is effective because it can detect semantic ambiguities, even with very simple test cases. For instance, the following test case with the if-construct could detect the two semantic ambiguities in Rules Branch, True, and False by running the verification engine.

```
if (true) { assert(true) } else { assert(false) }
if (false) { assert(false) } else { assert(true) }
```

We have used 400 test cases for the memory operations and CL semantics, 300 test cases for the translation semantics. These test cases mainly come from the Rust benchmark [Rust-Benchmark 2020], the Rust libraries and the Rust text book [Rust-Team 2018]. Besides, we have also created new test cases for testing our new grammars of the memory operations and CL. The execution result of each test in RustSEM is compared with the execution result and compilation result of the Rust compiler version 1.4.5. All experiments are conducted on a computer with an Intel Xeon(R) CPU E5-1650 v3 @ $3.50 \text{GHz} \times 12$ and 16 GB DDR4 RAM. The semantics of one construct in Rust is usually formalized with a collection of $\mathbb K$ rules, and thus one test case can cover a collection of rules.

The experimental results show that:

- (1) Some correct tests are accepted by RustSEM but rejected by the Rust compiler, due to the conservative analysis of the compiler.
- (2) Some incorrect tests are rejected by RustSEM but accepted by the Rust compiler, showing that RustSEM is more powerful in detecting memory errors, especially in the programs mixed with safe and unsafe operations. For example, after the ownership of a block has been moved away from a variable by a safe operation, a raw pointer reads the block through the reference to the variable.
- (3) More than 36 semantic ambiguities are detected, showing that our testing technique for ambiguities is effective.
- (4) For a collection of tests with memory errors, RustSEM can successfully detect all errors. More details are discussed in the application in runtime verification (Subsection 6.2).

6.2 Applications in Verification

Runtime Verification for Detecting Memory Errors. RustSEM provides a runtime checker for detecting memory errors, since the memory model integrates checkings for invalid memory accesses as explained in Section 4. It is based on semantics instrumentation rather than most of the existing runtime checkers, such as [Chen et al. 2019], which are based on code instrumentation. RustSEM detects: (1) accessing uninitialized locations, (2) reading dangling pointers, (3) double frees, (4) data races and deadlocks, (5) ownership and borrowing errors, (6) buffer overflows.

Table 2 shows the evaluation result for runtime verification. "TC", "IN", "Avg. Time(s)", and "Avg. Mem(KB)" denote the number of test cases, the number of inputs for each test case, average execution time and average memory consumption, respectively. For each kind of detectable memory errors, a collection of test cases is created. All the memory errors in the test cases can be detected with the inputs that

Table 2. Results for runtime verification.

Memory errors	TC	IN	Avg. Time(s)	Avg. Mem(KB)	
read uninit. values	22	10	6.16	387240	
read dangling pointers	11	10	5.89	390356	
double frees	10	10	5.73	346096	
data races	14	10	7.36	394152	
ownership & borrowing	40	10	6.64	344088	
index overflows	21	10	7.16	379788	

can trigger the errors. Data races are not always detected in an execution, since executions are non-deterministic.

Compared with other runtime checkers, RustSEM is less efficient and depends on the efficiency of $\mathbb K$ execution engine. In order to execute a program , $\mathbb K$ first constructs its abstract syntax tree and searches the corresponding semantics rules for the tree at each execution step. The advantage of RustSEM is that the memory checking is based on semantics instrumentation instead of code instrumentation. The biggest test case is around 210 LOC with the execution time of around 10 seconds, including both translation and execution time.

Formal Verification for both Memory and Functional Properties. K has a verification engine, which can be used to construct a program verifier for a language by instantiating it with an operational semantics of that language. The basic idea of the infrastructure could be explained as follows. \mathbb{K} verification engine takes an operational semantics given in \mathbb{K} and generates queries to a theorem prover (for example, Z3 [de Moura and Bjørner 2008]). The program correctness properties are given as reachability rules between matching patterns in K semantics rules. Internally, the verifier uses the operational semantics to perform symbolic execution. Also, it has an internal matching logic prover for reasoning about implication between patterns (states), which reduces to SMT reasoning (refer to K verification infrastructure [Ştefănescu et al. 2016] for more details).

For the verification, it is necessary to specify a pre- and post-condition similar to Hoare logic, and loops need to be annotated with invariants. For simplicity, we show the verified properties as a pair {Precondition}{Postcondition}, indicating that if the input of the program satisfies the precondition then the result satisfies the postcondition. Thanks to the memory access checking formalized in RustSEM, we can uniformly specify a memory property, such as data races, buffer overflows, or borrowing errors, as {True}{¬stuck}, which means that for any input, the program cannot get stuck. For other kinds of properties, such as functional properties, the program should satisfy both the memory guarantees of RustSEM and the properties being specified.

Table 3 shows the verification results of a number of programs verified in RustSEM using the \mathbb{K} verifier, where LOC, Time(s) and Mem(KB) denote the lines of code, execution time, and memory consumption, respectively. "sum(N)" is a function that computes the sum from 1 to the input N, verifying $\{N \geq 1\}\{N \times (N+1)/2\}$. "sumvec(V)" computes the sum of all elements in V, and we verify $\{[v_1, \ldots, v_n] \in$

Table 3. Results for formal verification

Prog	grams	LOC	Time(s)	Mem(KB)	
$\operatorname{sum}(N)$		15	44.65	698240	
sumvec(V)		18	63.92	1470880	
$insertion_sort(V)$		43	125.43	8736592	
polymorphism		60	42.20	678992	
closure		22	36.45	663268	
tr	trait		39.22	654672	
conci	ır-race	58	47.25	647328	
concu	r-order	57	60.82	2931760	
. LvesDkolue, 1	new		76.90	1160640	
	npush front	cle 211 P ub	lication date	: 1667140	
	push_back		140.70	2025244	
	reserve		85	1859304	

Proc. ACM Program.

vec<i32>} $\{\Sigma_{1 \leq i \leq n} v_i\}$, with the precondition being a vector of n random integers, and the postcondition the sum of its elements. "insertion_sort" implements an insertion sort algorithm, where we verify

 $\{V \in \text{vec} < \text{i} 32 > \} \{V' \in \text{vec} < \text{i} 32 > s.t. \text{ same} \text{Elem}(V, V') \land \text{order}(V') \}$. The precondition is an integer vector and the postcondition is new vector with the same elements (checked by sameElem) and the vector is ordered (checked by order). The program "concur-race" is a multi-threaded program with a data race bug where we verify $\{True\}\{\neg stuck\}$, that is the program finishes without going into a stuck state. The program has a data race, and the verification can successfully detect it, going to a stuck state thanks to the invalid access checking in the memory model, hence the property is invalid. The program "concur-order" is a multi-thread program without data races. The property is the same as for "concur-race". The verifier explores all possible interleavings in the concurrent programs to detect bugs. In RustSEM, we optimized the semantics to allow interleaving only if the rules access some shared resources.

The program "VecDeque" is a module from the Rust standard library, for which a previous version [issue $44800\ 2017$] contained a bug. We verify this old version implementation of VecDeque to rediscover the error by verifying the functions new, push_front, push_back, and reserve. The property is: $\{0 \le \text{head} < \text{cap} \land 0 \le \text{tail} < \text{cap}\}\{0 \le \text{head}' < \text{cap}' \land 0 \le \text{tail}' < \text{cap}'\}$. VecDeque has a head to indicate the memory location for push_front and and a tail to indicate the memory location for push_back. The variable cap is the capacity of the VecDeque. The property requires that the head and tail of a VecDeque have to be within the range [0, cap) before and after executing any of its methods. The primed variables denote the variables' values after executing the methods. The bug [issue $44800\ 2017$] can be rediscovered by RustSEM in the method reserve resulting in a buffer overflow. RustSEM does not support infinite heap structure specification and verification now. For the verification we use an approach similar to bounded model checking, setting a bound in the maximum capacity of vecDeque to 16, limiting the state space exploration. The benchmarks polymorphism, closure, and trait are programs using polymorphism, closures, and traits, respectively and we verify some functional properties.

Now, we discuss the limitations of formal verification. Firstly, the property specification language in $\mathbb K$ needs to specify pre- and post-conditions and loop invariants for each memory locations, which needs a lot of efforts. We only give an abstract description of the properties being verified here. More details of specification languages could be found in $\mathbb K$ verification infrastructure [Ştefănescu et al. 2016]. Secondly, we cannot specify infinite heap data structures, such as trees. $\mathbb K$ has the capability to reason about infinite heap data structures, such as binary search trees, which is shown in $\mathbb K$ verification infrastructure [Ştefănescu et al. 2016]. The challenge is that $\mathbb K$ verification infrastructure uses a memory model different from RustSEM, as RustSEM incorporates OBS. Therefore, we cannot directly inherit the framework from it. Actually, more work needs to be done to define a language that could be used to inductively define infinite data structures in RustSEM. One solution is to discover the relations between RustSEM's memory model and separation logic [Ishtiaq and O'Hearn 2001] [Reynolds 2002]. In this paper, we focus on the semantics of Rust. A more user-friendly and powerful verifier is our future work.

7 RELATED WORK

Rust semantics. Table 4 compares the existing semantics for Rust on their target languages (lang), the support of major features including safe and unsafe constructs, concurrency (concur), dynamic OBS (D-OBS) and type system, as well as their verification capabilities, i.e., whether support automated verification (AV) and machine-checked manual verification (MV).

	long	features				Verif	Verification	
	lang	safe	unsafe	type	D-OBS	concur	AV	MV
RustBelt [Jung et al. 2018a]	λ_{Rust}	•	•	•		•		$\overline{}$
Patina [Reed 2015]	old Rust	1		•				
Oxide [Weiss et al. 2019]	Oxide	•		•				
KRust [Wang et al. 2018]	Rust	•					√	
K-Rust[Kan et al. 2018]	core-lang	•		•			\checkmark	
RustSEM	Rust	•	•		•	•	√	
nartially supported				111v cun	norted			

Table 4. The comparison with related semantics works

partially supportedfully supported

RustSEM and KRust are the only two that directly formalize the Rust language, whereas all the remaining semantics, i.e., RustBelt, Oxide, Patina and K-Rust, formalizes either a variant or an outdated version of Rust.

RustSEM covers more major features than exiting semantics. Indeed, RustSEM fully supports both safe and unsafe constructs and concurrency. In contrast, existing semantics only partially supports them. For instance, RustBelt does not formalize some advanced features such as trait objects, as it is for a variant of Rust, while Oxide, Patina, K-Rust and KRust only support a subset of safe constructs without concurrency. Furthermore, RustSEM is the only one supporting dynamic OBS. RustSEM focuses the operational semantics of OBS, thus the type system is not included.

RustSEM, K-Rust and KRust can be used for automated verification. Although RustBelt, implemented in the Coq theorem prover, can verify λ_{Rust} programs using Coq, the verification is interactive and needs expertise and manual inspection. Patina and Oxide have not yet been implemented in any tool. Thus they provide neither automated nor machine-checked verification.

Rust verification. CRust [Toman et al. 2015], SMACK-Rust [Baranowski et al. 2018] and Viperbased Prusti [Astrauskas et al. 2019] can verify Rust programs by translating them into the input languages of existing verification tools, instead of building formal semantics for Rust. For instance, SMACK-Rust compiles Rust programs into the LLVM code which can be verified by SMACK [Carter et al. 2016]. Prusti translates a subset of safe constructs to the intermediate language of Viper [Müller et al. 2016] to construct core proofs. Note that these tools cover less features than RustSEM.

Stackborrow [Jung et al. 2020] presents an alias model to regulate the use of unsafe pointers, and checks borrowings by creating a stack for each memory location, instead of explicitly using lifetimes. Moreover, Stackborrow does not support concurrency. Stackborrow is implemented in Miri [Olson 2016], which is not a formal implementation.

Ownership and Borrowing. The concept of ownership has been proposed for many years and most of related works exploit type systems to enforce various ownership disciplines. Cyclone [Jim et al. 2002] is designed to be a safe dialect of the C language by using region based memory management [Tofte and Talpin 1994]. Mezzo [Balabonski et al. 2016] and Alms [Tov and Pucella 2011] follow ML-tradition and employ substructural type systems for managing ownership. There are also many type systems associating reading and writing permissions to aliases [Boyland et al. 2001], such as Mezzo [Balabonski et al. 2016], Pony [Clebsch et al. 2015] and AEminium [Stork et al. 2014]. All these works have different technique details with Rust and exploit types systems to enforce memory safety. Our work aims at explaining the techniques of Rust from an operational aspects instead of type systems in order to provide a new insight of OBS.

8 CONCLUSION

We have proposed a high-level abstraction of OBS and an executable operational semantics for Rust. Compared with existing works, it covers a larger subset of the major language features of Rust. The core of the semantics is the memory model with the operational semantics of OBS, which can be potentially reused by other languages to improve memory safety. We have proved the refinement relation between the high-level abstraction of OBS and the operational semantics in the memory model. Moreover, we proposed a technique for testing semantic consistency in order to detect semantic ambiguities and provide completely correct semantics. We have shown that RustSEM can be applied in automated runtime and formal verification.

As future works, we are working on two directions. The first one is to extend the verification infrastructure of $\mathbb K$ to support infinite heap structure verification for the memory model of RustSEM. The second one is to reuse our memory model for the $\mathbb K$ semantics of the C programming language to improve C memory safety by checking the preservation of the OBS invariants.

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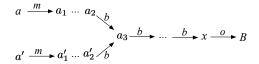


Fig. 5. The proof idea of Theorem 3.6

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A PROOF OF LEMMA 3.4

PROOF. Assume there is a cycle in a well-formed OBS graph, there must exist $a \to_b a'$ and $a' \to_b a_1 \dots a_2 \to_b a$. We can infer that $\mathcal{F}(a \to_b a') \subset \mathcal{F}(a_2 \to_b a)$ and $\mathcal{F}(a_2 \to_b a) \subset \mathcal{F}(a \to_b a')$ by the condition (2) of Definition 3.3, which cannot be true,

B PROOF OF THEOREM 3.6

PROOF. This theorem specifies that exclusive mutation guarantee are maintained in a well-formed OBS graph with respect to the permission functions.

If the owner and all references are shared aliases then the theorem is trivially proved. If there is a mutable alias enabled at the timestamp t then it is proved by two cases.

Case 1: Assume the owner is the mutable alias. We can infer that there is no reference that has the writing or reading permission to the memory location at t. Otherwise the reference should disable the owner according to the definition of the permission functions and the unique owner invariant of well-formed OBS graphs.

Case 2: Assume there is a mutable reference a to the memory block enabled at t, and the owner of the memory block is x, we have that there exists a link $a \to_m a_1 \dots x \to_o B$. The owner and all other references in the link should be disabled by a and thus their writing permissions are not enabled.

Now assume there is another reference $a' \neq a$ enabled at t. It should not be in the link from a to x. We have that there exists another link $a' \to_m a'_1 \dots x \to_o B$. Then there must exists a_2, a'_2, a_3 such that $a \to_m a_1 \dots a_2 \to_b a_3 \dots x \to_o B$ and $a' \to_m a'_1 \dots a'_2 \to_b a_3 \dots x \to_o B$ (See Fig. 5). According to Invariant (3) of the definition for well-formed OBS graphs (Definition 3.3), the lifetimes of $a_2 \to_b a_3$ and $a'_2 \to_b a_3$ should not intersect. According to Invariant (2) of Definition 3.3, the lifetime of $a \to_m a_1$ should be within the lifetime of $a_2 \to_b a_3$ and the lifetime of $a' \to_m a'_1$ should be within the lifetimes of $a \to_m a_1$ and $a' \to_m a'_1$ cannot intersect. Therefore the lifetime of $a' \to_m a'_1$ does not contain the current timestamp t and $t' \to_m t$ is not permitted to write.

C PROOF OF THEOREM 4.4

The first claim that G is a well-formed OBS graph is proved by Lemma C.1 and C.3. Lemma C.1 proves that each memory configuration in a safe sequence is a well-formed configuration and Lemma C.3 proves the lifetime inclusion variant for well-formed configurations. The second claim is proved by Lemma C.4 and C.5.

LEMMA C.1. Let $\pi = (mem_0, ts_0), op_1, (mem_1, ts_1), \dots, (mem_{n-1}, ts_{n-1}), op_n, (mem_n, ts_n)$ be a safe sequence. Then all $mem_i, 0 \le i \le n$, are well-formed.

PROOF. Since m_0 is an empty configuration, it is trivially well-formed. The allocation operation does not change the well-formedness of a memory layout. The free operation also does not change the well-formedness of a memory configuration, since it only removes a block from the heap. For reading and writing operations, the semantics rules always ensure the resulting memory configurations are well-formed.

LEMMA C.2. Let $\pi = (mem_0, ts_0), op_1, (mem_1, ts_1), \dots, (mem_{n-1}, ts_{n-1}), op_n, (mem_n, ts_n)$ be a safe sequence. Let $mem_i, 0 \le i \le n$ be a memory configuration. We have:

- (1) For any $a \to_s a' \in mem_i$, there is no writing by a' during the lifetime of a in mem_i .
- (2) For any $a \to_m a' \in mem_i$, there is no reading or writing by a' during the lifetime of a in mem_i .

PROOF. We first prove the claim (1). Assume a's lifetime is $ts_b \sim ts_e$ in mem_i (i.e., $\mathcal{L}(a, mem_i) = ts_b \sim ts_e$) and ts is a timestamp at which a' is used to write and $ts \in ts_b \sim ts_e$.

The writing by a' at ts should not be the latest writing in mem_i , that is, if $mem_i = (S, H, \mathcal{P}, ms)$ and $\mathcal{P}(a') = (_, ts')$ then $ts \neq ts'$, otherwise it should be disabled according to Definition 4.1. Therefore we have $ts' > ts_e$.

Without loss of generality, we assume there is no writing by a' during $ts \sim ts_e$. Then we have that the memory configuration mem_k in (mem_k, ts_k) with $ts_k = ts_e$ is not a well-formed memory configuration as the writing by a' at ts is the lasting writing by a' in mem_k but the lifetime of a in mem_k is $ts_b \sim ts_e$. The writing should be disabled according to Definition 4.1. This contradicts to Lemma C.2.

LEMMA C.3. Let $\pi = (mem_0, ts_0)$, $op_1, (mem_1, ts_1), \ldots, (mem_{n-1}, ts_{n-1})$, $op_n, (mem_n, ts_n)$ be a safe sequence. Let mem_i , $0 \le i \le n$ be a memory configuration and B be a block with the location b. Let the OBS graph of B in mem_i be $G = (V, B, E, \mathcal{F})$. We have that G is well-formed.

PROOF. From Lemma C.1, mem_i is well-formed, which ensures that G satisfies the *unique owner invariant* and *no intersection invariant*. We still need to prove the *lifetime inclusion invariant*.

For references, according the definition of $\mathcal{L}(a, mem)$, we know that the lifetime of a will always include the lifetime of a', where $a' \to_a^b a \in mem$.

We need to consider the owner. If an owner's writing permission is always disabled by its references then the ownership cannot be moved. In this case, its lifetime always includes it's references' lifetimes. From Lemma C.2, we know that the owner is always disabled.

LEMMA C.4. Let $\pi = (mem_0, \mathsf{ts}_0)$, $op_1, (mem_1, \mathsf{ts}_1), \ldots, (mem_{n-1}, \mathsf{ts}_{n-1})$, $op_n, (mem_n, \mathsf{ts}_n)$ be a safe sequence. Let $G = (V, B, E, \mathcal{F})$ be the OBS graph of B in $mem_i, 0 \le i \le n$, we have for any two nodes $a, a' \in V \cup \{B\}$, if a reading operation $\mathsf{read}(p)$, where $\mathsf{alias}(p) = a$, is carried out at the timestamp $\mathsf{ts} \in \mathcal{F}(a \to_* a')$ then $R_G(a, \mathsf{ts}) = true$.

PROOF. The key to prove the lemma is to ensure that during the lifetime of *a*, the reading by *a* at ts is not disabled. Actually it is proved by Lemma C.2.

LEMMA C.5. Let $\pi = (mem_0, \mathsf{ts}_0), op_1, (mem_1, \mathsf{ts}_1), \dots, (mem_{n-1}, \mathsf{ts}_{n-1}), op_n, (mem_n, \mathsf{ts}_n)$ be a safe sequence. Let $G = (V, B, E, \mathcal{F})$ be the OBS graph of B in $mem_i, 0 \le i \le n$, we have for any two nodes $a, a' \in V \cup \{B\}$, if a writing operation write(p, v), where alias(p) = a, is carried out at the timestamp $\mathsf{ts} \in \mathcal{F}(a \to_* a')$ then $W_G(a, \mathsf{ts}) = true$.

PROOF. Proof is similar to Lemma C.4.

Theorem 4.4 is proved by Lemma C.1, C.3, C.4, and C.5.