Partial 2 Geometrie afină
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$\boxed{P_1}$ True or false?

1) An affine morphism $\varphi: X \to X$ is injective if and only if its surjective

2) Affine morphism $\varphi: \mathbb{R}^3 \to \mathbb{R}$ and ℓ not parallel to $\pi = \varphi^{-1}(0)$

$$Pr_{\pi,\ell}(P) = P - 2\frac{\varphi(P)}{(\lim\varphi)(v)} v \text{ for } 0 \neq v \in D(\ell)$$

3) For all transformations of $\mathbb{R}^m$ the linear part operator $\lim: AGL(\mathbb{R}^m) \to GL(\mathbb{R}^m)$ is a morphism of groups.

4) An isometry of $\mathbb{E}^3$ is an affine transformation.

5) A displacement is an affine transformation φ with $[\lim\varphi] \in O(n)$

6) A real line in $\mathbb{C} \simeq \mathbb{E}^2$ has an equation of the form

$$\alpha \bar{z} + \bar{\alpha} z + \beta = 0 \text{ with } 0 \neq \alpha \in \mathbb{C} \text{ and } \beta \in \mathbb{R}$$

7) If ℓ' is obtained from a line ℓ by a homothety of $\mathbb{E}^2$, then ℓ' is ~~obtained~~ a line parallel to ℓ.

8) $\mathbb{R}$ and $\mathbb{C}$ are subfields of $\mathbb{H}$

9) the pace of helical displacement φ measures the movement of φ in the direction parallel to the rotation axis of φ

10) the norm of a purely imaginary quaternion q is $\sqrt{-q^2}$

$\boxed{P_2}$ Give the algebraic expression of the orthogonal reflection in the plane π: $3x - 4z = -1$

$\boxed{P_3}$ ... planar rotation with $45°$ ... same center as the rotation $\begin{bmatrix} x \\ y \end{bmatrix} \to \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\boxed{P_4}$ Show that the lines invariant under homothesis $H_{z_0,k}: \mathbb{C} \to \mathbb{C}$ ($z_0 \in \mathbb{C}, 0 \neq k \in \mathbb{R}$) are lines passing through z_0.

$\boxed{P_5}$ Let $g = \cos\frac{\theta}{2} + \sin\frac{\theta}{2}\,i \in \mathbb{H}$ and the map
$x \to g x g^{-1} + i + j$ restricted to $\mathbb{E}^3 \simeq \mathrm{Im}\,\mathbb{H}$

1) Why its helical displacement?
2) Find its pace and Chasles decomposition for this transformation

$\boxed{P_6}$ Let $g = \cos(2\alpha) + \sin(2\alpha)k \in \mathbb{H}$. Calculate the matrix (in standard basis) of the isometry obtained by:

1) left, right multiplication with g
2) conjugation with g, i.e. $x \to g x g^{-1}$