Prof. Dr. Dorin Andrica
Asist. Drd. Tudor Micu
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# Geometry 1 (Analytic Geometry)

## Exercise Sheet 13

**Exercise 1.** Find:

1. the foci of the ellipse $\mathcal{E} : 9x^2 + 25y^2 - 225 = 0$;
2. the foci of the hyperbola $\mathcal{H} : \dfrac{x^2}{9} - \dfrac{y^2}{4} - 1 = 0$;
3. the focus and the director line of the parabola $y^2 - 24x = 0$.

**Exercise 2.** Sketch the graph of $y = -\dfrac{3}{4}\sqrt{16 - x^2}$.

**Exercise 3.** Find the intersection points between:

1. the line $d_1 : x + 2y - 7 = 0$ and the ellipse $\mathcal{E} : x^2 + 3y^2 - 25 = 0$;
2. the line $d_2 : 2x - y - 10 = 0$ and the hyperbola $\mathcal{H} : \dfrac{x^2}{20} - \dfrac{y^2}{5} - 1 = 0$;

**Exercise 4.** Find the position of the line $d : 2x + y - 10 = 0$ relative to the ellipse $\mathcal{E} : \dfrac{x^2}{9} + \dfrac{y^2}{4} - 1 = 0$.

**Exercise 5.** Find the area of the triangle determined by the asymptotes of the hyperbola $\mathcal{H} : \dfrac{x^2}{4} - \dfrac{y^2}{9} - 1 = 0$ and the line $d : 9x + 2y - 24 = 0$.

**Exercise 6.** Find the equation of the parabola having the focus $F(-7, 0)$ and the director line $x - 7 = 0$.

**Exercise 7.** Find the equation of the tangent line(s) to:

1. the ellipse $\mathcal{E} : x^2 + 4y^2 - 20 = 0$, orthogonal on the line $d_1 : 2x - 2y - 13 = 0$;
2. the hyperbola $\mathcal{H} : \dfrac{x^2}{20} - \dfrac{y^2}{5} - 1 = 0$, orthogonal to the line $d_2 : 4x + 3y - 7 = 0$;
3. the parabola $\mathcal{P} : y^2 - 8x = 0$, parallel to $d_3 : 2x + 2y - 3 = 0$.

**Exercise 8.** Find the equations of the tangent line(s) to:

1. the ellipse $\mathcal{E} : \dfrac{x^2}{25} + \dfrac{y^2}{16} - 1 = 0$, passing through $P_1(10, -8)$;
2. the hyperbola $\mathcal{H} : \dfrac{x^2}{3} - \dfrac{y^2}{5} - 1 = 0$ passing through $P_2(1, -5)$;
3. the parabola $\mathcal{P} : y^2 - 36x = 0$, passing through $P_3(2, 9)$.

**Exercise 9.** Find the equation of the tangent line to the parabola $y^2 - 4x = 0$ at the point $P(1, 2)$.

**Exercise 10.** Let $\mathcal{P}_1 : y^2 - 2px = 0$ and $\mathcal{P}_2 : y^2 - 2qx = 0$ be two parabolas with $0 < q < p$. A mobile tangent to $\mathcal{P}_2$ intersects $\mathcal{P}_1$ at $M_1$ and $M_2$. Find the geometric locus of the midpoint of the segment $[M_1M_2]$.

**Exercise 11.** Let $A$, $B$ and $C$ be three distinct points on the parabola of equation $y^2 = 2px$. The tangent lines at $A$, $B$, respectively $C$ to the parabola determine a triangle $A'B'C'$. Prove that the line passing through the centers of gravity of the triangles $\triangle ABC$ and $\triangle A'B'C'$ is parallel to $Ox$.

## Bonus exercises

**Exercise 12.** Find the geometric locus of :

1. the orthogonal projections of a focus of an ellipse on the tangent lines to the ellipse;
2. the orthogonal projections of a focus of an hyperbola on the tangent lines to the hyperbola;
3. the orthogonal projections of the focus of a parabola on the tangent lines to the parabola.

**Exercise 13.** Let $d_1$ and $d_2$ be two variable orthogonal lines, passing through the point $A(a, 0)$ of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and let $P_1$ and $P_2$ be the intersection points of these two lines and the ellipse. Prove that the line $P_1P_2$ passes through a fixed point.

**Exercise 14.** Let $a$, $b$ and $c$ be the tangent lines at three distinct points of a parabola and $ABC$ the triangle determined by the tangents. Prove that the focus of the parabola belongs to the circumscribed circle of the triangle $ABC$.