DATA STRUCTURES AND ALGORITHMS
LECTURE 6

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In Lecture 5...

- Sorted Lists
- Circular Lists
- Linked Lists on Arrays

Today

1. Linked Lists on Arrays
2. Skip Lists
3. ADT Set
4. ADT Map
5. Iterator

Linked Lists on Arrays

elems		78	11	6	59	19		44		
next	7	6	5	-1	8	4	9	2	10	-1

head = 3

firstEmpty = 1

Linked Lists on Arrays

- In a more formal way, we can simulate a singly linked list on an array with the following:
  - an array in which we will store the elements.
  - an array in which we will store the links (indexes to the next elements).
  - the capacity of the arrays (the two arrays have the same capacity, so we need only one value).
  - an index to tell where the *head* of the list is.
  - an index to tell where the first empty position in the array is.

SLL on Array - Representation

- The representation of a singly linked list on an array is the following:

SLLA:
elems: TElem[]
next: Integer[]
cap: Integer
head: Integer
firstEmpty: Integer

SLLA - InsertFirst

subalgoritm insertFirst(slla, elem) **is:**
//pre: slla is an SLLA, elem is a TElem
//post: the element elem is added at the beginning of slla
  if slla.firstEmpty = -1 **then**
    newElems ← @an array with slla.cap * 2 positions
    newNext ← @an array with slla.cap * 2 positions
    for i ← 1, slla.cap **execute**
      newElems[i] ← slla.elems[i]
      newNext[i] ← slla.next[i]
    end-for
    for i ← slla.cap + 1, slla.cap*2 - 1 **execute**
      newNext[i] ← i + 1
    end-for
    newNext[slla.cap*2] ← -1
//continued on the next slide...

SLLA - InsertFirst

```
    //free slla.elems and slla.next if necessary
    slla.elems ← newElems
    slla.next ← newNext
    slla.firstEmpty ← slla.cap+1
    slla.cap ← slla.cap * 2
  end-if
  newPosition ← slla.firstEmpty
  slla.elems[newPosition] ← elem
  slla.firstEmpty ← slla.next[slla.firstEmpty]
  slla.next[newPosition] ← slla.head
  slla.head ← newPosition
end-subalgorithm
```

- Complexity: $\Theta(1)$ amortized

SLLA -InsertPosition

subalgorithm insertPosition(slla, elem, poz) **is:**
//pre: slla is an SLLA, elem is a TElem, poz is an integer number
//post: the element elem is inserted into slla at position pos
  if (pos $<$ 1) **then**
    @error, invalid position
  end-if
  if slla.firstEmpty $=$ -1 **then**
    @resize
  end-if
  if poz $=$ 1 **then**
    insertFirst(slla, elem)
  else
    pozCurrent $\leftarrow$ 1
    nodCurrent $\leftarrow$ slla.head
//continued on the next slide...

SLLA - InsertPosition

while nodCurrent $\neq$ -1 **and** pozCurrent $<$ poz - 1 **execute**
  pozCurrent $\leftarrow$ pozCurrent + 1
  nodCurrent $\leftarrow$ slla.next[nodCurrent]
end-while
if nodCurrent $\neq$ -1 **atunci**
  newElem $\leftarrow$ slla.firstEmpty
  slla.firstEmpty $\leftarrow$ slla.next[slla.firstEmpty]
  slla.elems[newElem] $\leftarrow$ elem
  slla.next[newElem] $\leftarrow$ slla.next[nodCurrent]
  slla.next[nodCurrent] $\leftarrow$ newElem
else
//continued on the next slide...

SLLA - InsertPosition

```
            @error, invalid position
        end-if
    end-if
end-subalgorithm
```

- Complexity: $O(n)$

SLLA - InsertPosition

- Observations regarding the *insertPosition* subalgorithm
  - The *resize* operation is done in the exact same way as for the *insertFirst*.
  - Similar to the SLL, we iterate through the list until we find the element *after* which we insert (denoted in the code by *nodCurrent* - which is an index in the array).
  - We treat as a special case the situation when we insert at the first position (no node to insert after).

SLLA - DeleteElement

subalgorithm deleteElement(slla, elem) **is:**
//pre: slla is a SLLA; elem is a TElem
//post: the element elem is deleted from SLLA
  nodC ← slla.head
  prevNode ← -1
  while nodC ≠ -1 **and** slla.elems[nodC] ≠ elem **execute**
    prevNode ← nodC
    nodC ← slla.next[nodC]
  end-while
  if nodC ≠ -1 **then**
    if nodC = slla.head **then**
      slla.head ← slla.next[slla.head]
    else
      slla.next[prevNode] ← slla.next[nodC]
    end-if
//continued on the next slide...

SLLA - DeleteElement

//add the nodC position to the list of empty spaces
    slla.next[nodC] ← slla.firstEmpty
    slla.firstEmpty ← nodC
  else
    @the element does not exist
  end-if
end-subalgorithm

- Complexity: $O(n)$

SLLA - Iterator

- Iterator for a SSLA is a combination of an iterator for an array and of an iterator for a singly linked list:
- Since the elements are stored in an array, the *currentElement* will be an index from the array.
- But since we have a linked list, going to the next element will not be done by incrementing the *currentElement* by one; we have to follow the *next* links.

DLLA

- Obviously, we can define a doubly linked list as well without pointers, using arrays.
- For the DLLA we will see another way of representing a linked list on arrays:
    - The main idea is the same, we will use array indexes as links between elements
    - We are using the same information, but we are going to structure it differently
    - However, we can make it look more similar to linked lists with dynamic allocation

DLLA - Node

- Linked Lists with dynamic allocation are made of nodes. We can define a structure to represent a node, even if we are working with arrays.
- A node (for a doubly linked list) contains the information and links towards the previous and the next nodes:

<u>DLLANode:</u>
  info: TElem
  next: Integer
  prev: Integer

DLLA

- Having defined the *DLLANode* structure, we only need one array, which will contain *DLLANodes*.
- Since it is a doubly linked list, we keep both the head and the tail of the list.

DLLA:
    nodes: DLLANode[]
    cap: Integer
    head: Integer
    tail: Integer
    firstEmpty: Integer

DLLA - Allocate and free

- To make the representation and implementation even more similar to a dynamically allocated linked list, we can define the *allocate* and *free* functions as well.

```
function allocate(dlla) is:
//pre: dlla is a DLLA
//post: a new element will be allocated and its position returned
   newElem ← dlla.firstEmpty
   if newElem ≠ -1 then
      dlla.firstEmpty ← dlla.nodes[dlla.firstEmpty].next
      dlla.nodes[dlla.firstEmpty].prev ← -1
      dlla.nodes[newElem].next ← -1
      dlla.nodes[newElem].prev ← -1
   end-if
   allocate ← newElem
end-function
```

DLLA - Allocate and free

subalgorithm free (dlla, poz) **is:**
//pre: dlla is a DLLA, poz iss an integer number
//post: the pozition poz was freed
dlla.nodes[poz].next ← dlla.firstEmpty
dlla.nodes[poz].prev ← -1
dlla.nodes[dlla.firstEmpty].prev ← poz
dlla.firstEmpty ← poz
end-subalgorithm

DLLA - InsertPosition

subalgorithm insertPosition(dlla, elem, poz) **is:**
//pre: dlla is a DLLA, elem is a TElem, poz is an integer number
//we assume that poz is a valid position
//post: the element elem is inserted in dlla at position poz

DLLA - InsertPosition

```
subalgorithm insertPosition(dlla, elem, poz) is:
//pre: dlla is a DLLA, elem is a TElem, poz is an integer number
//we assume that poz is a valid position
//post: the element elem is inserted in dlla at position poz
   newElem ← alocate(dlla)
   if newElem = -1 then
      @resize
      newElem ← alocate(dlla)
   end-if
   dlla.nodes[newElem].info ← elem
   if poz = 1 then
      if dlla.head = -1 then
         dlla.head ← newElem
         dlla.tail ← newElem
      else
//continued on the next slide...
```

DLLA - InsertPosition

```
            dlla.nodes[newElem].next ← dlla.head
            dlla.nodes[dlla.head].prev ← newElem
            dlla.head ← newElem
        end-if
    else
        nodC ← dlla.head
        pozC ← 1
        while nodC ≠ -1 and pozC < poz - 1 execute
            nodC ← dlla.nodes[nodC].next
            pozC ← pozC + 1
        end-while
        if nodC ≠ -1 then
            nodNext ← dlla.nodes[nodC].next
            dlla.nodes[newElem].next ← nodNext
            dlla.nodes[newElem].prev ← nodC
            dlla.nodes[nodC].next ← newElem
//continued on the next slide...
```

DLLA - InsertPosition

```
            if nodNext = -1 then
                dlla.tail ← newElem
            else
                dlla.nodes[nodNext].prev ← newElem
            end-if
        end-if
    end-if
end-subalgorithm
```

- Complexity: $O(n)$

DLLA - Iterator

- The iterator for a DLLA contains as *current element* the index of the current node from the array.

<u>DLLAIterator:</u>
  list: DLLA
  currentElement: Integer

DLLAIterator - init

subalgorithm init(it, dlla) **is:**
//pre: dlla is a DLLA
//post: it is a DLLAIterator for dlla
  it.list ← dlla
  it.currentElement ← dlla.head
end-subalgorithm

- For a (dynamic) array, currentElement is set to 0 when an iterator is created. For a DLLA we need to set it to the head of the list (which might be position 0, but it might be a different position as well).

DLLAIterator - getCurrent

subalgorithm getCurrent(it, e) **is:**
//pre: it is a DLLAIterator, it is valid
//post: e is a TElem, e is the current element from it
  e ← it.list.nodes[it.currentElement].info
end-subalgorithm

DLLAIterator - next

subalgoritm next (it) **is:**
//pre: it is a DLLAIterator, it is valid
//post: the current elements from it is moved to the next element
 it.currentElement ← it.list.nodes[it.currentElement].next
end-subalgorithm

- In case a (dynamic) array, going to the next element means incrementing the *currentElement* by one. For a DLLA we need to follow the links.

DLLAIterator - valid

function valid (it) **is:**
//pre: it is a DLLAIterator
//post: valid return true is the current element is valid, false otherwise
  if it.currentElement = -1 **then**
    valid ← False
  else
    valid ← True
  end-if
end-function

Skip Lists

- Assume that we want to memorize a sequence of sorted elements. The elements can be stored in:
  - dynamic array
  - linked list
- What is the time complexity of inserting a new element into the sequence?
  - We can divide the insertion into two steps: *finding the position* and *inserting the element*.

Skip List

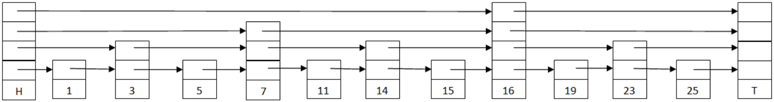
- A skip list is a data structure that allows *fast search* in an ordered sequence.

- How can we do that?

Skip List

- A skip list is a data structure that allows *fast search* in an ordered sequence.
- How can we do that?
    - Starting from an ordered linked list, we add to every second node another pointer that skips over one element.
    - We add to every fourth node another pointer that skips over 3 elements.
    - etc.

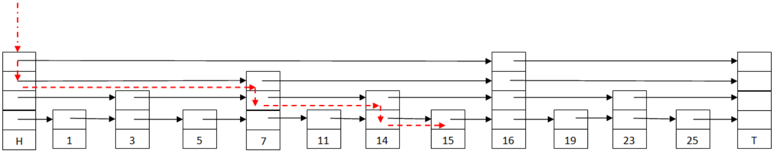
Skip List



- H and T are two special nodes, representing *head* and *tail*. They cannot be deleted, they exist even in an empty list.

Skip List - Search

- Search for element 15.



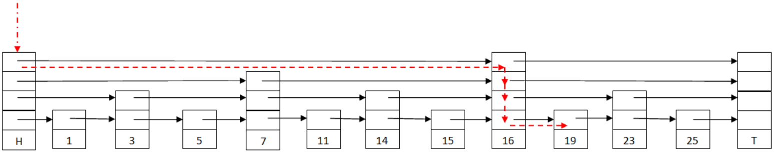
- Start from head and from highest level.
- If possible, go right.
- If cannot go right (next element is greater), go down a level.

Skip List

- Lowest level has all n elements.
- Next level has $\frac{n}{2}$ elements.
- Next level has $\frac{n}{4}$ elements.
- etc.
- $\Rightarrow$ there are approx $log_2 n$ levels.
- From each level, we check at most 2 nodes.
- Complexity of search: $O(log_2 n)$

Skip List - Insert

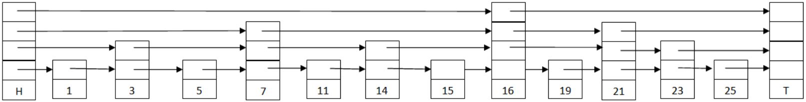
- Insert element 21.



- How *high* should the new node be?

Skip List - Insert

- *Height* of a new node is determined *randomly*, but in such a way that approximately half of the nodes will be on level 2, a quarter of them on level 3, etc.



- Assume we randomly generate the height 3 for the node with 21.

Skip List

- Skip Lists are *probabilistic* data structures, since we decide randomly the height of a newly inserted node.
- There might be a worst case, where every node has height 1 (so it is just a linked list).
- In practice, they function well.

ADT Set

- A *Set* is a container in which the elements are unique, and their order is not important (they do not have positions).
  - No operations based on positions.
  - We cannot make assumptions regarding the order in which elements are stored and will be iterated.
- Domain of the ADT Set:
  $\mathcal{S} = \{s | \text{s is a set with elements of the type TElem}\}$

Set - Interface I

- init (s)
  - **descr:** creates a new empty set.
  - **pre:** true
  - **post:** $s \in \mathcal{S}$, s is an empty set.

Set - Interface II

- add(s, e)
  - **descr:** adds a new element into the set.
  - **pre:** $s \in \mathcal{S}$, $e \in TElem$
  - **post:**$s' \in \mathcal{S}$, $s' = s \cup \{e\}$ (e is added only if it is not in s yet. If s contains the element e already, no change is made).
  - What happens if e is already in s?

Set - Interface III

- remove(s, e)
  - **descr:** removes an element from the set.
  - **pre:** $s \in \mathcal{S}$, $e \in TElem$
  - **post:** $s \in \mathcal{S}$, $s' = s \setminus \{e\}$ (if e is not in s, s is not changed).

Set - Interface IV

- find(s, e)
  - **descr:** verifies if an element is in the set.
  - **pre:** $s \in \mathcal{S}$, $e \in TElem$
  - **post:**

$$find \leftarrow \begin{cases} True, & \text{if } e \in s \\ False, & \text{otherwise} \end{cases}$$

Set - Interface V

- size(s)
  - **descr:** returns the number of elements from a set
  - **pre:** $s \in \mathcal{S}$
  - **post:** size $\leftarrow$ the number of elements from s

Set - Interface VI

- iterator(s, it)
  - **descr:** returns an iterator for a set
  - **pre:** $s \in \mathcal{S}$
  - **post:** $it \in \mathcal{I}$, it is an iterator over the set s

Set - Interface VII

- destroy (s)
  - **descr:** destroys a set
  - **pre:** $s \in S$
  - **post:**the set s was destroyed.

Set - Interface VIII

- Other possible operations (characteristic for sets from mathematics):
  - reunion of two sets
  - intersection of two sets
  - difference of two sets (elements that are present in the first set, but not in the second one)

Sorted Set

- We can have a Set where the elements are ordered based on a *relation* $\rightarrow$ *SortedSet*.
- The only change in the interface is for the *init* operation that will receive the *relation* as parameter.
- For a sorted set, the iterator has to iterate through the elements in the order given by the *relation*.

Set

- If we want to implement the ADT Set (or ADT SortedSet), we can use the following data structures as representation:
  - (dynamic) array
  - linked list
  - hash tables - to be discussed later
  - (balanced) binary trees - for sorted sets - to be discussed later
  - skip lists - for sorted sets

ADT Map

- A *Map* is a container where the elements are <*key, value*> pairs.
- Each *key* has one single associated *value*, and we can access the values only by using the key $\rightarrow$ no positions in a *Map*.
- Keys have to be unique in a *Map*, and each *key* has one single associated value (if a key can have multiple values we have a *MultiMap*).
- When we implement a *Map*, we should use a data structure that makes finding the *key*s easy.

Map

- Examples of using a map:
  - Bank account number (as key) and every information associated with the bank account (as value)
  - Student id (as key) and every information about the student (as value)
  - etc.
- Domain of the ADT Map:

$\mathcal{M} = \{m | \text{m is a map with elements } e = (k, v), \text{ where } k \in TKey \text{ and } v \in TValue\}$

Map - Interface I

- init(m)
  - **descr:** creates a new empty map
  - **pre:** true
  - **post:** $m \in \mathcal{M}$, m is an empty map.

Map - Interface II

- destroy(m)
  - **descr:** destroys a map
  - **pre:** $m \in \mathcal{M}$
  - **post:** m was destroyed

Map - Interface III

- add(m, k, v)
  - **descr:** add a new key-value pair to the map (the operation can be called *put* as well)
  - **pre:** $m \in \mathcal{M}, k \in TKey, v \in TValue$
  - **post:** $m' \in \mathcal{M}, m' = m \cup < k, v >$

  - What happens if there is already a pair with k as key?

Map - Interface IV

- remove(m, k, v)
  - **descr:** removes a pair with a given key from the map
  - **pre:** $m \in \mathcal{M}, k \in TKey$
  - **post:** $v \in TValue$, where

$$v \leftarrow \begin{cases} v', & \text{if } \exists < k, v' > \in m \text{ and } m' \in \mathcal{M}, \\ & m' = m \setminus < k, v' > \\ 0_{TValue}, & \text{otherwise} \end{cases}$$

Map - Interface V

- search(m, k, v)
  - **descr:** searches for the value associated with a given key in the map
  - **pre:** $m \in \mathcal{M}, k \in TKey$
  - **post:** $v \in TValue$, where

$$v \leftarrow \begin{cases} v', & \text{if } \exists < k, v' > \in m \\ 0_{TValue}, & \text{otherwise} \end{cases}$$

Map - Interface VI

- iterator(m, it)
  - **descr:** returns an iterator for a map
  - **pre:** $m \in \mathcal{M}$
  - **post:** $it \in \mathcal{I}$, it is an iterator over m.

Map - Interface VII

- size(m)
  - **descr:** returns the number of pairs from the map
  - **pre:** $m \in \mathcal{M}$
  - **post:** size $\leftarrow$ the number of pairs from m

Map - Interface VIII

- keys(m, s)
  - **descr:** returns the set of keys from the map
  - **pre:** $m \in \mathcal{M}$
  - **post:** $s \in \mathcal{S}$, s is the set of all keys from m

Map - Interface IX

- values(m, b)
  - **descr:** returns a bag with all the values from the map
  - **pre:** $m \in \mathcal{M}$
  - **post:** $b \in \mathcal{B}$, b is the bag of all values from m

Map - Interface X

- pairs(m, s)
  - **descr:** returns the set of pairs from the map
  - **pre:** $m \in \mathcal{M}$
  - **post:** $s \in \mathcal{S}$, s is the set of all pairs from m

Sorted Map

- We can have a Map where we can define an order (a relation) on the set of possible keys: instead of *TKey* we will have *TComp*.
- The only change in the interface is for the *init* operation that will receive the *relation* as parameter.
- For a sorted map, the iterator has to iterate through the pairs in the order given by the *relation*, and the operations *keys* and *pairs* return SortedSets.

Map

- If we want to implement the ADT Map (or ADT SortedMap), we can use the following data structures as representation:
  - (dynamic) array
  - linked list
  - hash tables - to be discussed later
  - (balanced) binary trees - for sorted maps - to be discussed later
  - skip lists - for sorted maps

Iterator - why do we need it? I

- Most containers have iterators and for every data structure we will discuss how we can implement an iterator for a container defined on that data structure.

- Why are iterators so important?

Iterator - why do we need it? II

- They offer a uniform way of iterating through the elements of any container

subalgorithm printContainer(c) **is:**
//pre: c is a container
//post: the elements of c were printed
//we create an iterator using the iterator method of the container
  iterator(c, it)
  while valid(it) **execute**
    //get the current element from the iterator
    getCurrent(it, elem)
    print elem
    //go to the next element
    next(it)
  end-while
end-subalgorithm

Iterator - why do we need it? III

- For most containers the iterator is the only thing we have to *see* the content of the container.
    - List (will be discussed later) is the only container that has positions, for other containers we can use only the iterator.

Iterator - why do we need it? IV

- Giving up positions, we can gain performance.
    - Containers that do not have positions can be represented on data structures where some operations have good complexities, but where the notion of a position does not naturally exist and where enforcing positions is really complicated (ex. hash tables).

Iterator - why do we need it? V

- Even if we have positions, using an iterator might be faster.
  - Going through the elements of a linked list with an iterator is faster than going through every position one-by-one.