# DATA STRUCTURES AND ALGORITHMS

## LECTURE 14

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# In Lecture 13...

- Binary Search Trees
- AVL Trees

# Today

1. AVL Trees
2. Final Exam

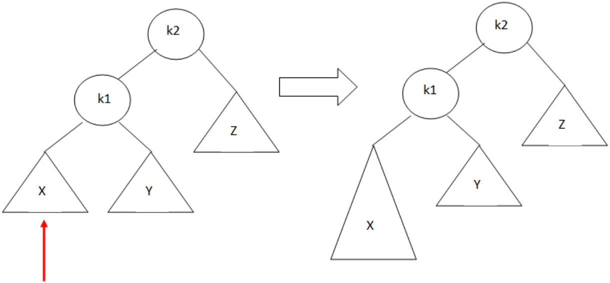
# AVL Trees

- Definition: An AVL (Adelson-Velskii Landis) tree is a binary tree which satisfies the following property (AVL tree property):
  - If $x$ is a node of the AVL tree:
    - the difference between the height of the left and right subtree of $x$ is 0, 1 or -1 (balancing information)
- Observations:
- Height of an empty tree is -1
- Height of a single node is 0

# AVL Trees - rotations

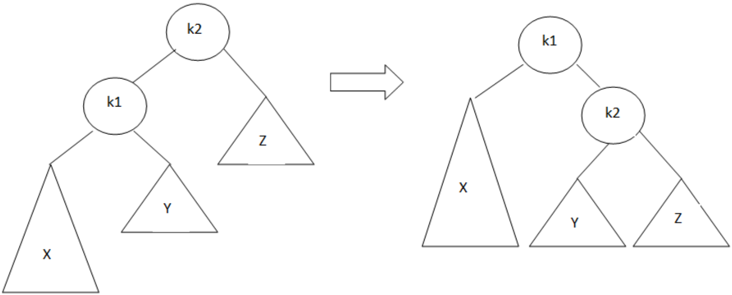
- Adding or removing a node might result in a binary tree that violates the AVL tree property.
- In such cases, the property has to be restored and only after the property holds again is the operation (add or remove) considered finished.
- The AVL tree property can be restored with operations called **rotations**.

# AVL Trees - rotations

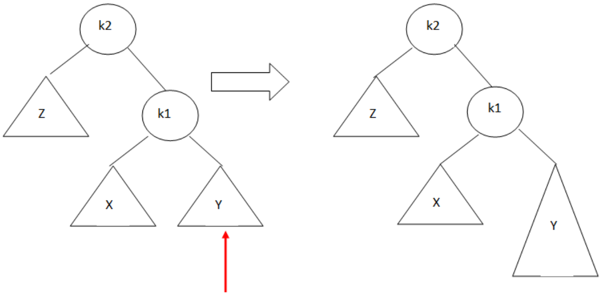



- Solution: **single rotation to right**

# AVL Trees - rotation - Single Rotation to Right

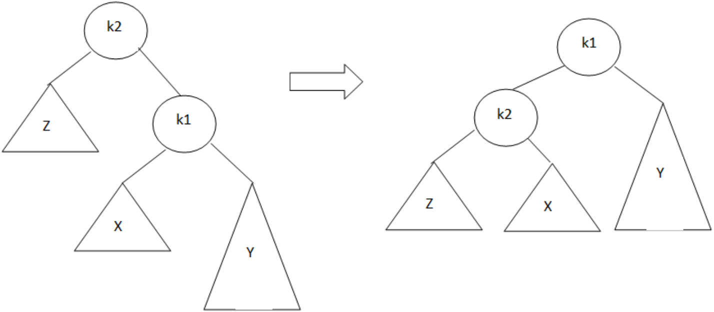


# AVL Trees - rotations

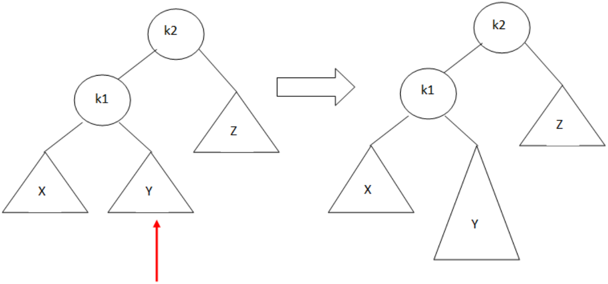



- Solution: **single rotation to left**

# AVL Trees - rotation - Single Rotation to Left

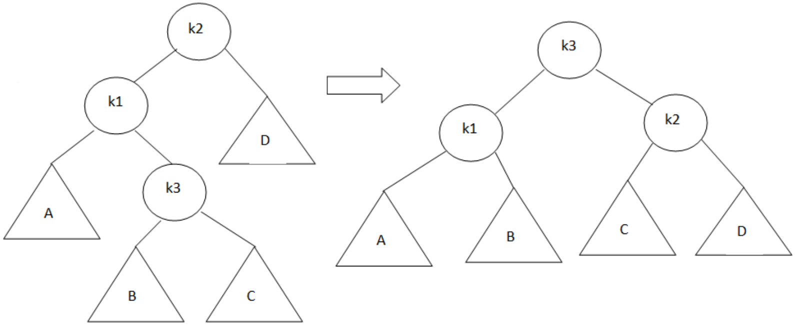


# AVL Trees - rotations - case 2

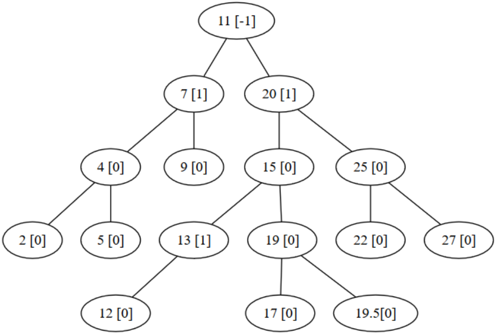



- Solution: **Double rotation to right**

# AVL Trees - rotation - Double Rotation to Right

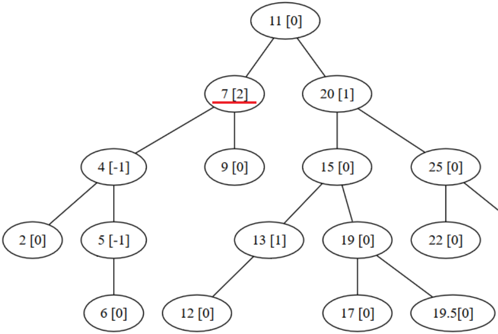


# AVL Trees - rotations - case 2 example



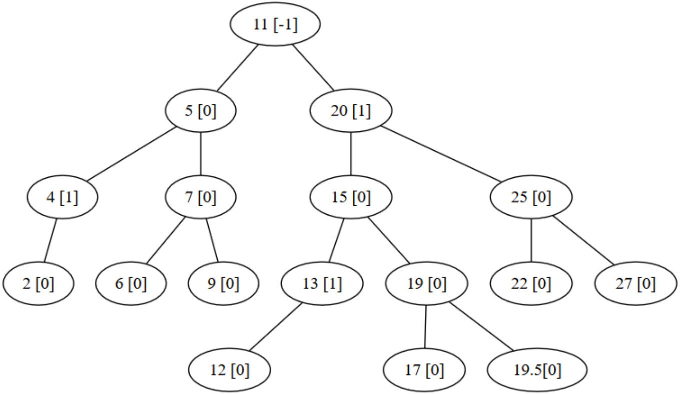

- Insert value 6

# AVL Trees - rotations - case 2 example



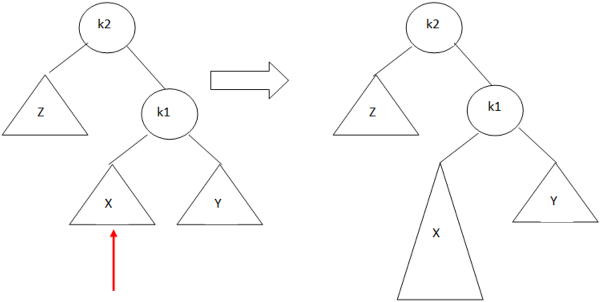

- Node 7 is imbalanced, because we inserted a new node (6) to the right subtree of the left child
- Solution: **double rotation to right**

# AVL Trees - rotation - case 2 example



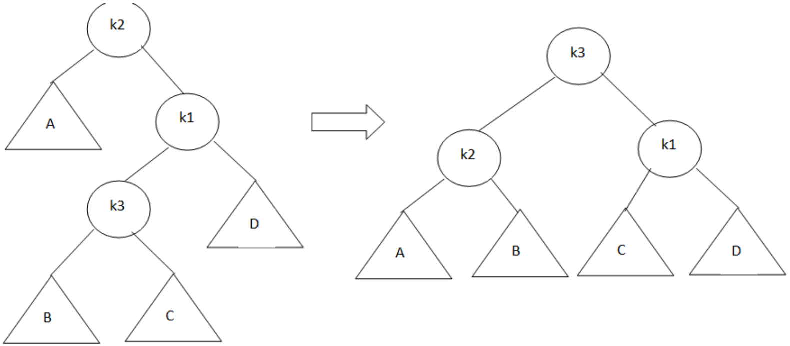

- After the rotation

# AVL Trees - rotations - case 3

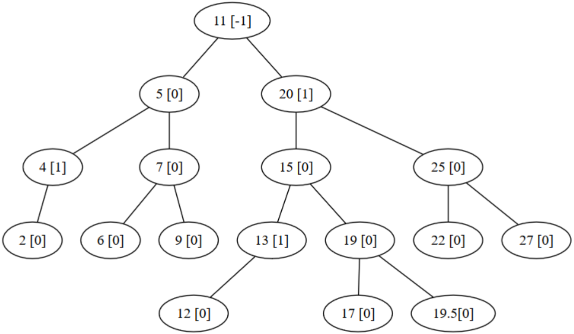



- Solution: **Double rotation to left**

# AVL Trees - rotation - Double Rotation to Left

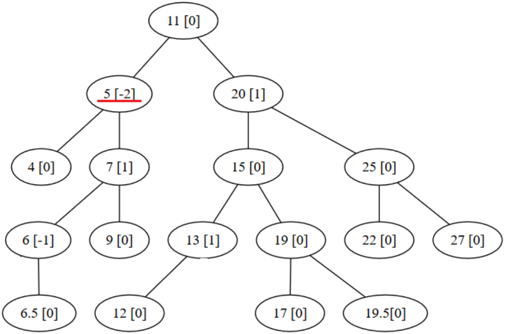


# AVL Trees - rotations - case 3 example



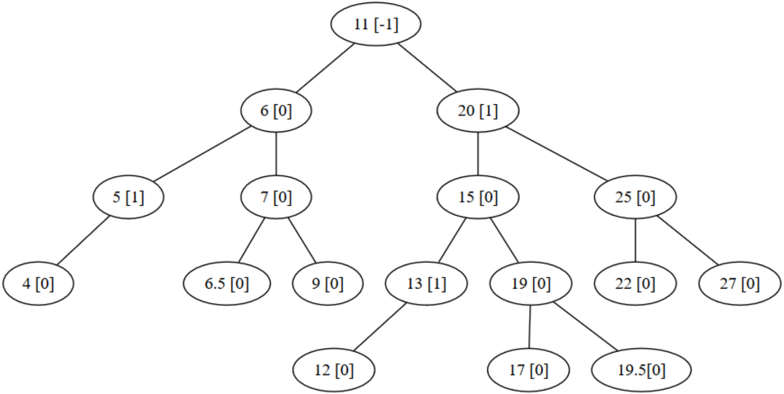

- Remove node with value 2 and insert value 6.5

# AVL Trees - rotations - case 3 example




- Node 5 is imbalanced, because we inserted a new node (6.5) to the left subtree of the right child
- Solution: **double rotation to left**

# AVL Trees - rotation - case 3 example




- After the rotation

# AVL rotations example I

- Start with an empty AVL tree
- Insert 2

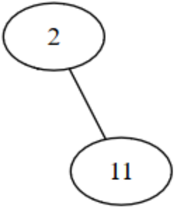
# AVL rotations example II


- Do we need a rotation?
- If yes, on which node and what type of rotation?

# AVL rotations example III

- No rotation is needed
- Insert 11

# AVL rotations example IV

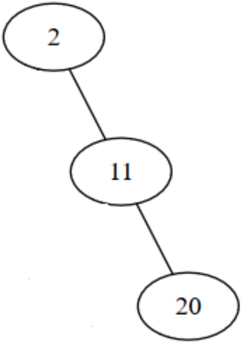



- Do we need a rotation?
- If yes, on which node and what type of rotation?

# AVL rotations example V

- No rotation is needed
- Insert 20

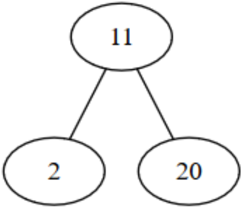
# AVL rotations example VI




- Do we need a rotation?
- If yes, on which node and what type of rotation?

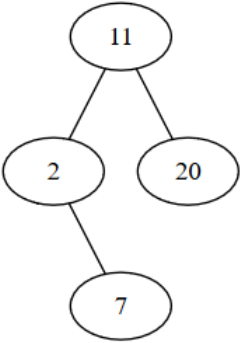
# AVL rotations example VII

- Yes, we need a single left rotation on node 2
- After the rotation:




- Insert 7

# AVL rotations example VIII

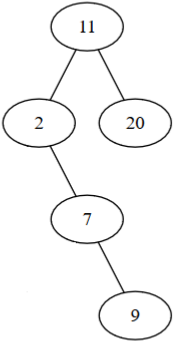



- Do we need a rotation?
- If yes, on which node and what type of rotation?

# AVL rotations example IX

- No rotation is needed
- Insert 9

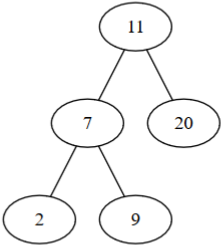
# AVL rotations example X




- Do we need a rotation?
- If yes, on which node and what type of rotation?

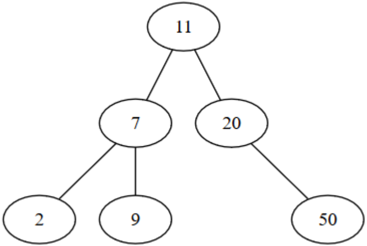
# AVL rotations example XI

- Yes, we need a single left rotation on node 2
- After the rotation:




- Insert 50

# AVL rotations example XII

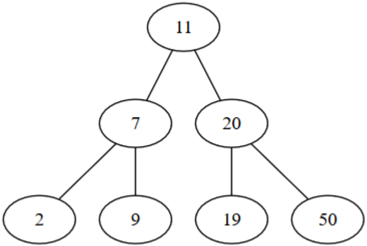



- Do we need a rotation?
- If yes, on which node and what type of rotation?

# AVL rotations example XIII

- No rotation is needed
- Insert 19

# AVL rotations example XIV

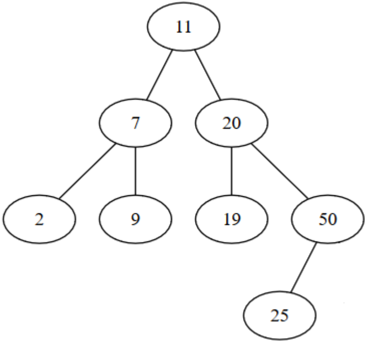



- Do we need a rotation?
- If yes, on which node and what type of rotation?

# AVL rotations example XV

- No rotation is needed
- Insert 25

# AVL rotations example XVI

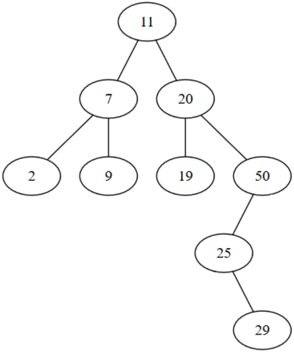



- Do we need a rotation?
- If yes, on which node and what type of rotation?

# AVL rotations example XVII

- No rotation is needed
- Insert 29

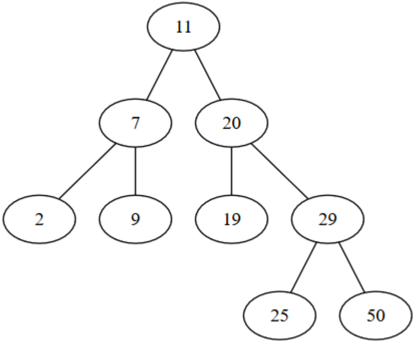
# AVL rotations example XVIII




- Do we need a rotation?
- If yes, on which node and what type of rotation?

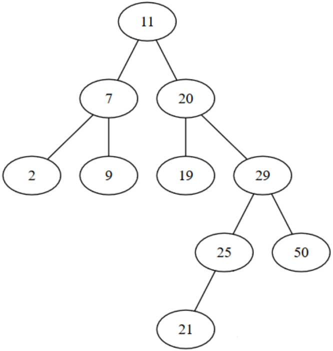
# AVL rotations example XIX

- Yes, we need a double right rotation on node 50
- After the rotation




- Insert 21

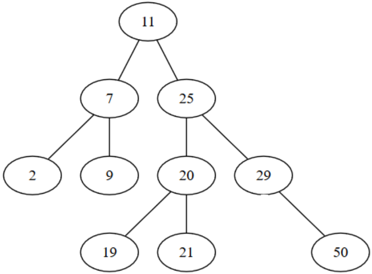
# AVL rotations example XX




- Do we need a rotation?
- If yes, on which node and what type of rotation?

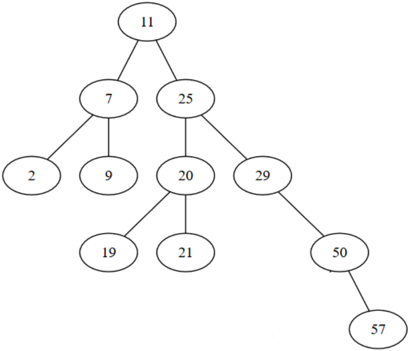
# AVL rotations example XXI

- Yes, we need a double left rotation on node 20
- After the rotation




- Insert 57

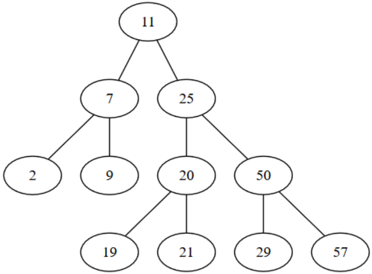
# AVL rotations example XXII




- Do we need a rotation?
- If yes, on which node and what type of rotation?

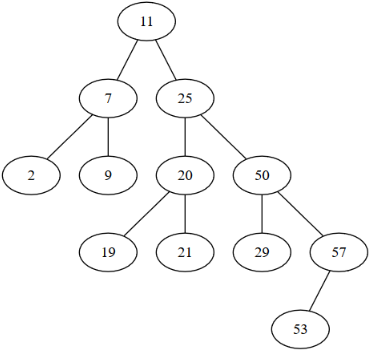
# AVL rotations example XXIII

- Yes, we need a single left rotation on node 50
- After the rotation




- Insert 53

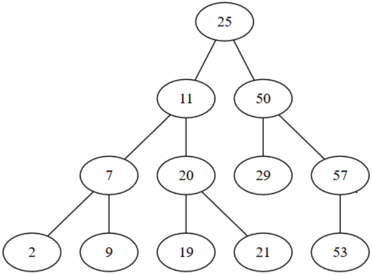
# AVL rotations example XXIV




- Do we need a rotation?
- If yes, on which node and what type of rotation?

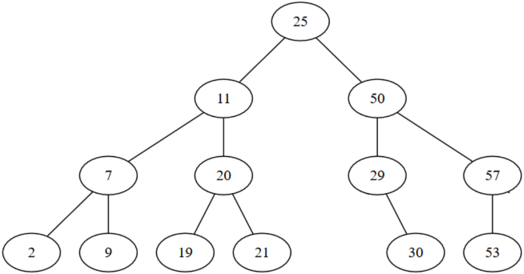
# AVL rotations example XXV

- Yes, we need a single left rotation on node 11
- After the rotation




- Insert 30

# AVL rotations example XXVI

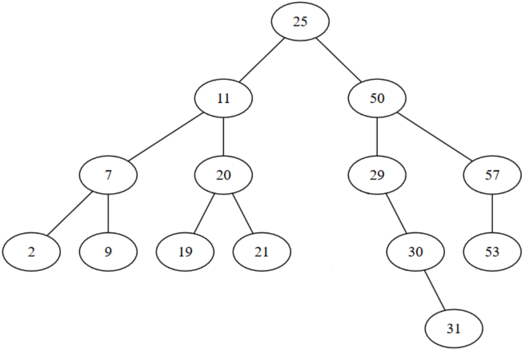



- Do we need a rotation?
- If yes, on which node and what type of rotation?

# AVL rotations example XXVII

- No rotation is needed
- Insert 31

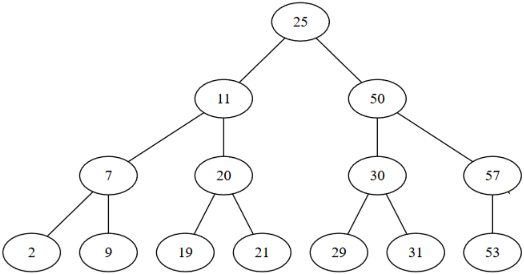
# AVL rotations example XXVIII




- Do we need a rotation?
- If yes, on which node and what type of rotation?

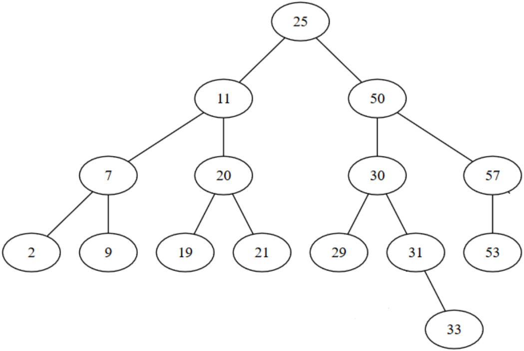
# AVL rotations example XXIX

- Yes, we need a single left rotation on node 29
- After the rotation




- Insert 33

# AVL rotations example XXX

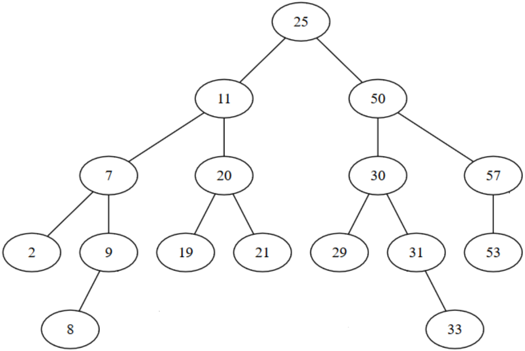



- Do we need a rotation?
- If yes, on which node and what type of rotation?

# AVL rotations example XXXI

- No rotation is needed
- Insert 8

# AVL rotations example XXXII

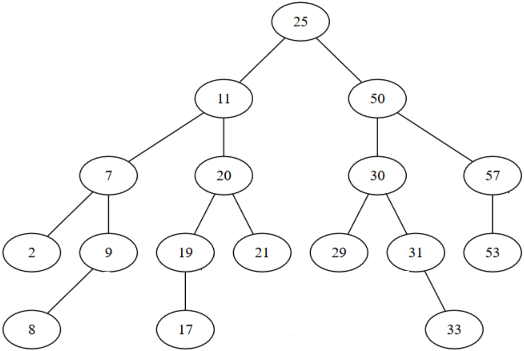



- Do we need a rotation?
- If yes, on which node and what type of rotation?

# AVL rotations example XXXIII

- No rotation is needed
- Insert 17

# AVL rotations example XXXIV

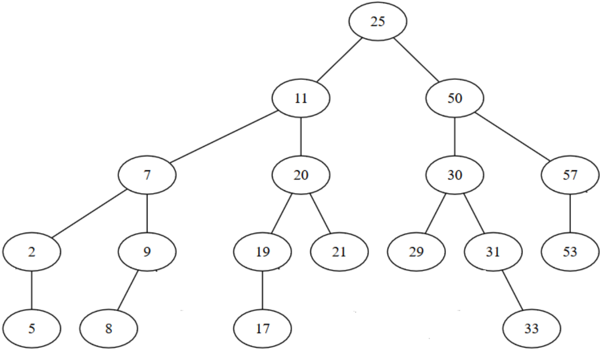



- Do we need a rotation?
- If yes, on which node and what type of rotation?

# AVL rotations example XXXV

- No rotation is needed
- Insert 5

# AVL rotations example XXXVI

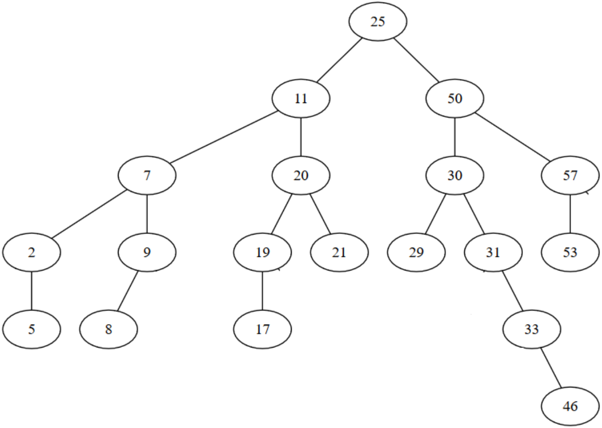



- Do we need a rotation?
- If yes, on which node and what type of rotation?

# AVL rotations example XXXVII

- No rotation is needed
- Insert 46

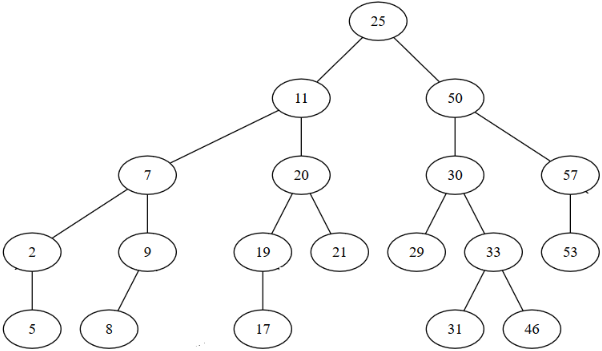
# AVL rotations example XXXVIII




- Do we need a rotation?
- If yes, on which node and what type of rotation?

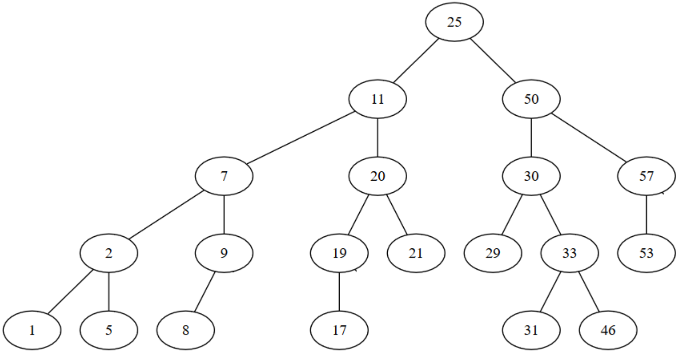
# AVL rotations example XXXIX

- Yes, we need a single left rotation on node 31
- After the rotation




- Insert 1

# AVL rotations example XL

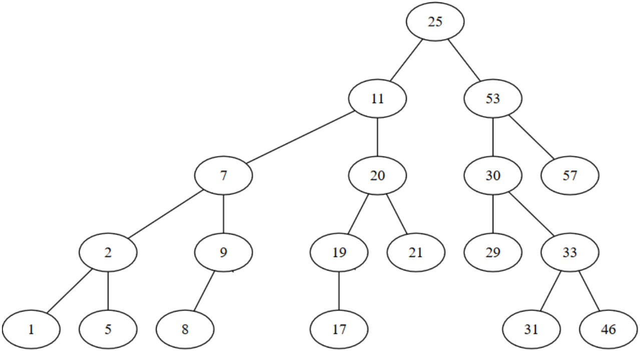



- Do we need a rotation?
- If yes, on which node and what type of rotation?

# AVL rotations example XLI

- No rotation is needed
- Remove 50

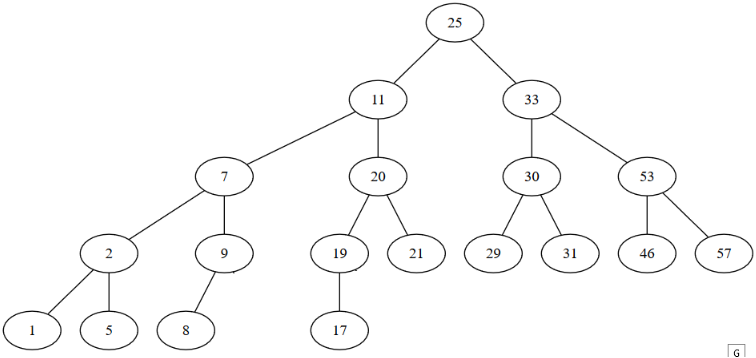
# AVL rotations example XLII




- Do we need a rotation?
- If yes, on which node and what type of rotation?

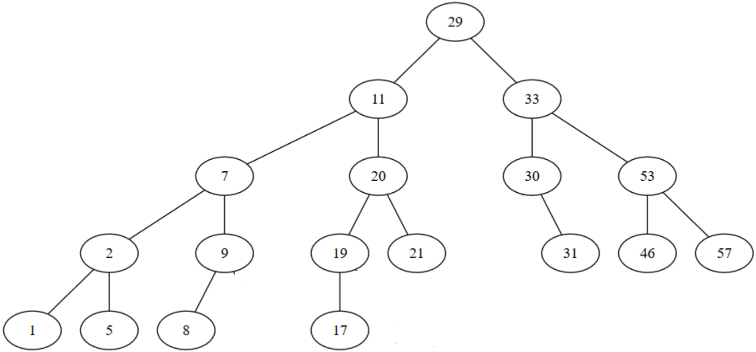
# AVL rotations example XLIII

- Yes we need double right rotation on node 53
- After the rotation




- Remove 25

# AVL rotations example XLIV

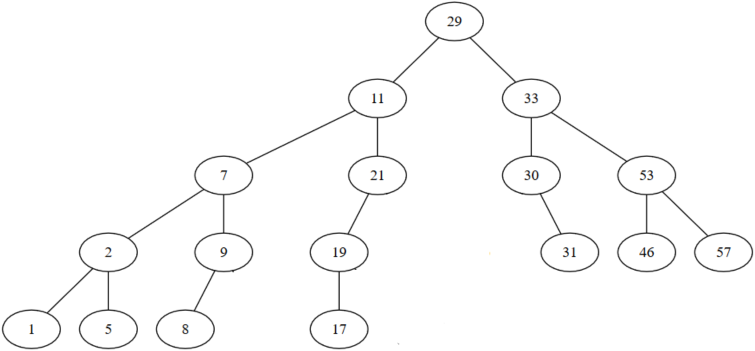



- Do we need a rotation?
- If yes, on which node and what type of rotation?

# AVL rotations example XLV

- No rotation is needed
- Remove 20

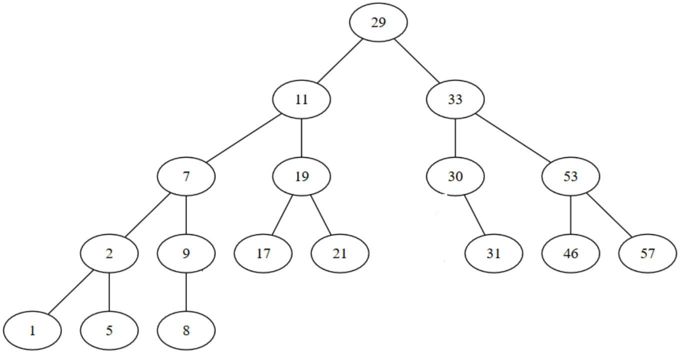
# AVL rotations example XLVI




- Do we need a rotation?
- If yes, on which node and what type of rotation?

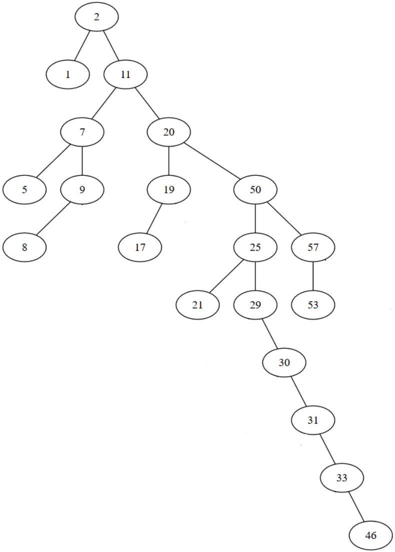
# AVL rotations example XLVII

- Yes, we need a single right rotation on node 21
- After the rotation



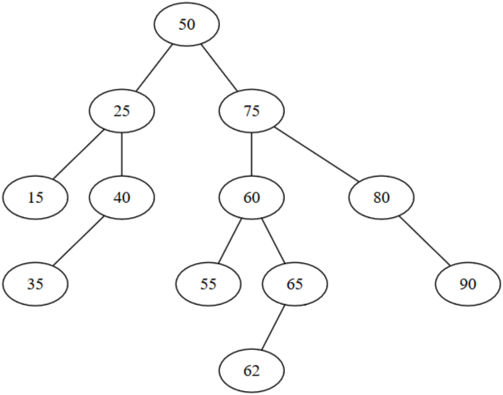
# Comparison to BST

- If, instead of using an AVL tree, we used a binary search tree, after the insertions the tree would have been:


2
1
11
7
20
5
9
19
50
8
17
25
57
21
29
53
30
31
33
46

# Rotations for remove

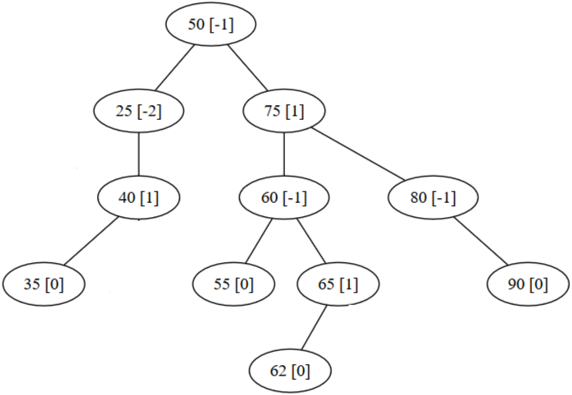
- When we remove a node, we might need more than 1 rotation:




- Remove value 15

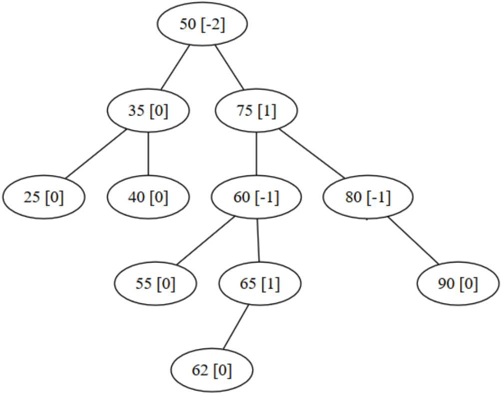
# Rotations for remove

- After remove:



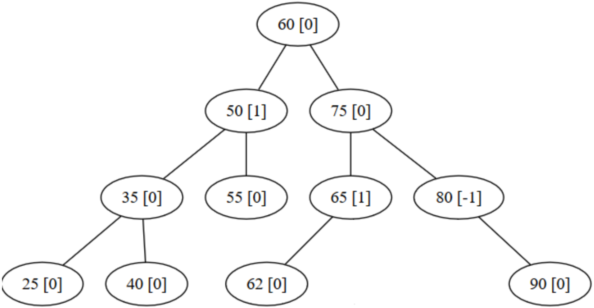
# Rotations for remove

- After the rotation



# Rotations for remove

- After the second rotation



# AVL Trees - representation

- What structures do we need for an AVL Tree?

# AVL Trees - representation

- What structures do we need for an AVL Tree?

<u>AVLNode:</u>
    info: TComp *//information from the node*
    left: $\uparrow$ AVLNode *//address of left child*
    right: $\uparrow$ AVLNode *//address of right child*
    h: Integer *//height of the node*

<u>AVLTree:</u>
    root: $\uparrow$ AVLNode *//root of the tree*

# AVL Tree - implementation

- We will implement the *insert* operation for the AVL Tree.
- We need to implement some operations to make the implementation of *insert* simpler:
    - A subalgorithm that (re)computes the height of a node
    - A subalgorithm that computes the balance factor of a node
    - Four subalgorithms for the four rotation types (we will implement only one)
- And we will assume that we have a function, *createNode* that creates and returns a node containing a given information (left and right are NIL, height is 0).

# AVL Tree - height of a node

**subalgorithm** recomputeHeight(node) **is:**
*//pre: node is an* $\uparrow$ *AVLNode. All descendants of node have their height (h) set*
*//to the correct value*
*//post: if node* $\neq$ *NIL, h of node is set*

# AVL Tree - height of a node

**subalgorithm** recomputeHeight(node) **is:**
*//pre: node is an* $\uparrow$ *AVLNode. All descendants of node have their height (h) set*
*//to the correct value*
*//post: if node* $\neq$ *NIL, h of node is set*
  **if** node $\neq$ NIL **then**
    **if** [node].left = NIL **and** [node].right = NIL **then**
      [node].h $\leftarrow$ 0
    **else if** [node].left = NIL **then**
      [node].h $\leftarrow$ [[node].right].h + 1
    **else if** [node].right = NIL **then**
      [node].h $\leftarrow$ [[node].left].h + 1
    **else**
      [node].h $\leftarrow$ max ([[node].left].h, [[node].right].h) + 1
    **end-if**
  **end-if**
**end-subalgorithm**

- Complexity: $\Theta(1)$

# AVL Tree - balance factor of a node

**function** balanceFactor(node) **is:**
*//pre: node is an* $\uparrow$ *AVLNode. All descendants of node have their height (h) set*
*//to the correct value*
*//post: returns the balance factor of the node*

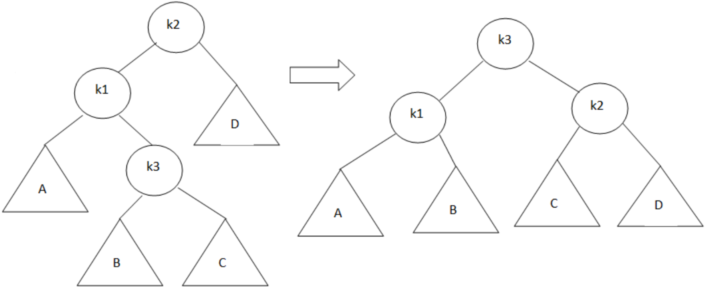
# AVL Tree - balance factor of a node

**function** balanceFactor(node) **is:**
*//pre: node is an ↑ AVLNode. All descendants of node have their height (h) set*
*//to the correct value*
*//post: returns the balance factor of the node*
  **if** [node].left = NIL **and** [node].right = NIL **then**
    balanceFactor ← 0
  **else if** [node].left = NIL **then**
    balanceFactor ← -1 - [[node].right].h *//height of empty tree is -1*
  **else if** [node].right = NIL **then**
    balanceFactor ← [[node].left].h + 1
  **else**
    balanceFactor ← [[node].left].h - [[node].right].h
  **end-if**
**end-subalgorithm**

- Complexity: $\Theta(1)$

# AVL Tree - rotations

- Out of the four rotations, we will only implement one, double right rotation (DRR).
- The other three rotations can be implemented similarly (RLR, SRR, SLR).



# AVL Tree - DRR

**function** DRR(node) **is:** *//pre: node is an* $\uparrow$ *AVLNode on which we perform the double right rotation*
*//post: DRR returns the new root after the rotation*

k2 $\leftarrow$ node
k1 $\leftarrow$ [node].left
k3 $\leftarrow$ [k1].right
k3left $\leftarrow$ [k3].left
k3right $\leftarrow$ [k3].right

# AVL Tree - DRR

**function** DRR(node) **is:** *//pre: node is an* $\uparrow$ *AVLNode on which we perform the double right rotation*
*//post: DRR returns the new root after the rotation*

```
   k2 ← node
   k1 ← [node].left
   k3 ← [k1].right
   k3left ← [k3].left
   k3right ← [k3].right
   //reset the links
   newRoot ← k3
   [newRoot].left ← k1
   [newRoot].right ← k2
   [k1].right ← k3left
   [k2].left ← k3right
```

*//continued on the next slide*

# AVL Tree - DRR

*//recompute the heights of the modified nodes*
  recomputeHeight(k1)
  recomputeHeight(k2)
  recomputeHeight(newRoot)
  DRR $\leftarrow$ newRoot
**end-function**

- Complexity: $\Theta(1)$

# AVL Tree - insert

**function** insertRec(node, elem) **is**
*//pre: node is a ↑ AVLNode, elem is the value we insert in the (sub)tree that*
*//has node as root*
*//post: insertRec returns the new root of the (sub)tree after the insertion*
  **if** node = NIL **then**
    insertRec ← createNode(elem)
  **else if**elem ≤ [node].info **then**
    [node].left ← insertRec([node].left, elem)
  **else**
    [node].right ← insertRec([node].right, elem)
  **end-if**
*//continued on the next slide...*

# AVL Tree - insert

recomputeHeight(node)
balance $\leftarrow$ getBalanceFactor(node)
**if** balance $=$ -2 **then**

# AVL Tree - insert

```
recomputeHeight(node)
balance ← getBalanceFactor(node)
if balance = -2 then
//right subtree has larger height, we will need a rotation to the LEFT
   rightBalance ← getBalanceFactor([node].right)
   if rightBalance < 0 then
```

# AVL Tree - insert

```
recomputeHeight(node)
balance ← getBalanceFactor(node)
if balance = -2 then
//right subtree has larger height, we will need a rotation to the LEFT
    rightBalance ← getBalanceFactor([node].right)
    if rightBalance < 0 then
    //the right subtree of the right subtree has larger height, SRL
        node ← SRL(node)
    else
        node ← DRL(node)
    end-if
//continued on the next slide...
```

# AVL Tree - insert

**else if** balance $= 2$ **then**
*//left subtree has larger height, we will need a RIGHT rotation*
  leftBalance $\leftarrow$ getBalanceFactor([node].left)
  **if** leftBalance $> 0$ **then**

# AVL Tree - insert

```
else if balance = 2 then
//left subtree has larger height, we will need a RIGHT rotation
   leftBalance ← getBalanceFactor([node].left)
   if leftBalance > 0 then
   //the left subtree of the left subtree has larger height, SRR
      node ← SRR(node)
   else
      node ← DRR(node)
   end-if
end-if
insertRec ← node
end-function
```

# AVL Tree - insert

- Complexity of the *insertRec* algorithm: $O(log_2 n)$
- Since *insertRec* receives as parameter a pointer to a node, we need a wrapper function to do the first call on the root

**subalgorithm** insert(tree, elem) **is**
*//pre: tree is an AVL Tree, elem is the element to be inserted*
*//post: elem was inserted to tree*
  tree.root $\leftarrow$ insertRec(tree.root, elem)
**end-subalgorithm**

- remove subalgorithm can be implemented similarly (start from the remove from BST and add the rotation part).

# Project presentation

- Project presentation schedule is available online:
- Every student has to come to the presentation with his/her own group.
- Do not forget to bring the documentation on paper.
- Be prepared to make modifications to your project (small ones). Failure to perform the modifications will result in a failing grade for the project.

# Project presentation

- If you fail your project, the will have to redo it for the retake session.
- **No matter what your grade for the project is, you can participate in the written exam in the regular session - if you have the required number of seminar attendances**.

# Written exam

| Group | Primary date | Secondary date |
|---|---|---|
| 911 | 25.06 | 11.06 |
| 912 | 11.06 | 27.06 |
| 913 | 25.06 | 22.06 |
| 914 | 27.06 | 22.06 |
| 915 | 22.06 | 11.06 |
| 916 | 13.06 | 27.06 |
| 917 | 22.06 | 27.06 |
| 2nd, 3rd year Students | 25.06 | 27.06 |

- Rooms and the starting hours are available at the faculty's webpage.

# Written exam

- Every student has to participate in the exam on the primary date.
- In the secondary date you can only participate if you have a good reason for asking this, and if you announce me at least 48 hours in advance and have my OK.
- Exam will take 2.5 - 3 hours - results will be given as soon as we can.
- You will need a grade of at least 5 for the written exam to be able to pass this course.

# Written exam I

- Subjects for the written exam will be from everything we have covered this semester.
- You will have 5 problems:
  - Problem 1 implementation - either something on a simple data structure, or a problem where you use an ADT to solve a problem
  - Problem 2 - points a, b, c - mainly drawings, or short text answers (not code)
  - Problem 3 - "Pick the right answer from a, b, c, d and explain" - 6 questions
  - Problem 4 - "implement a given operation for a given ADT represented with a given DS" or "find the best SD to solve this problem with a given complexity and solve it"
  - Problem 5 - with binary trees

# Written exam II

- The data structures for which we did not discuss implementation (skip lists, binomial heap, etc.) will appear only at problems 2 or 3.
- "Think about it" problems are good candidates for exam problems.