Partial I - Geometrie afină   MIE anul I
sem. II
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$\boxed{P_1}$ Write which ones of the following are true and which are false:

a) $\varphi : X \to Y$, $y \in Y$, is $\varphi^{-1}(y)$ an affine subspace?

b) A point on the line PQ is represented with $(1+t)P - tQ$.

c) $Y \cap Z = \emptyset$ then $\dim(Y \cup Z) = \dim(D(Y) + D(Z)) + 1$

d) convex hull operator $(M \subseteq N)$ has the property that $\text{conv}(M) \supseteq \text{conv}(N)$

e) two hyperplanes intersect montrivially then have a plane in common

f) Graham's scan determines Veronoi cells.

$\boxed{P_2}$ Which ones are affine subvarieties?

$A = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : 2x_1 - x_2 + x_3 - 2 = 0\}$

$B = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : (x_1 + x_2, 2x_2 + x_3, x_3 - 2x_1) \in A\}$

$C = \{ \text{—} \| \text{—} : x_1^2 + x_2^2 + x_3^2 - 2x_1x_2 - 2x_1x_3 - 2x_2x_3 = 0\}$

$D = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1^4 + x_2 - 2x_3 + x_4 = 0\}$

$\boxed{P_3}$ $\Delta ABC \subseteq X$, $C' \in AB$, $B' \in AC$; $\overrightarrow{AC'} = \lambda \overrightarrow{BC'}$, $\overrightarrow{AB'} = \mu \overrightarrow{CB'}$, $BB' \cap CC' = M$

$O \in X$. Show that $M = \dfrac{A - \lambda B - \mu C}{1 - \lambda - \mu}$

Show that $G = \dfrac{A+B+C}{3}$, $J = \dfrac{aA + bB + cC}{a+b+c}$, $H = \dfrac{(\tan\hat{A}) \cdot A + (\tan\hat{B}) \cdot B + (\tan\hat{C}) \cdot C}{\tan\hat{A} + \tan\hat{B} + \tan\hat{C}}$

$\boxed{P_4}$ Show that the roots of P' lie in the convex hull of the roots of P.
$P \in \mathbb{C}[X]$

$\boxed{P_5}$ $a = (\ , \ , \ , \ )$, $b = (\ , \ , \ , \ )$, $c = (\ , \ , \ , \ )$, $d = (\ , \ , \ , \ )$

$A = a + \langle b, d \rangle$, $B = c + \langle b, d \rangle$. Determine $\dim \text{aff}(A \cup B)$, $\dim \text{aff}(A \cap B)$

$\boxed{P_6}$ Determine the Veronoi cells for the points $(3m, 2m) \leftarrow$ point in 2D