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1st Semester, 2018-2019

Geometry 1 (Analytic Geometry)

Exercise Sheet 3

Exercise 1. Express, in rectangular and spherical coordinates, the following equations, given in cylindrical coordinates:

(a) $r^2 + z^2 = 1$;

(b) $\theta = \frac{\pi}{4}$;

(c) $r^2 \cos(2\theta) = z$.

Exercise 2. The equations below are given in spherical coordinates. Express them in rectangular coordinates:

(a) $\rho \sin(\phi) = 2\cos(\theta)$;

(b) $\rho - 2\sin(\phi)\cos(\theta) = 0$.

Exercise 3. Let M and N be the midpoints of two opposite sides of a quadrilateral $ABCD$ and let P be the midpoint of $[MN]$. Prove that

$$\overline{PA} + \overline{PB} + \overline{PC} + \overline{PD} = \overline{0}$$

Exercise 4. In a circle of center O, let M be the intersection point of two perpendicular chords $[AB]$ and $[CD]$. Show that

$$\overline{OA} + \overline{OB} + \overline{OC} + \overline{OD} = 2\overline{OM}$$

Exercise 5. Consider, in the 3-dimensional space, the parallelograms $A_1A_2A_3A_4$ and $B_1B_2B_3B_4$. Prove that the midpoints of the segments $[A_1B_1]$, $[A_2B_2]$, $[A_3B_3]$ and $[A_4B_4]$ are the vertices of a new parallelogram.

Exercise 6. Let ABC be a triangle and a, b, c the lengths of its sides, respectively. If A_1 is the intersection point of the internal bisector of the angle $\hat{A}$ and BC and M is an arbitrary point, then

$$\overline{MA_1} = \frac{b}{b+c}\overline{MB} + \frac{c}{b+c}\overline{MC}$$

Exercise 7. If G is the centroid (center of mass) of a triangle ABC in the plane and O is a given point, then

$$\overline{OG} = \frac{\overline{OA} + \overline{OB} + \overline{OC}}{3}$$

Exercise 8. Let ABC be a triangle, H its orthocenter, O the circumcenter (the center of its circumscribed cercle), G the centroid of the triangle and A' the point on the circumcenter diametrically opposed to A. Then:

1. $\overline{OA} + \overline{OB} + \overline{OC} = \overline{OH}$; (**Sylvester's formula**)
2. $\overline{HB} + \overline{HC} = \overline{HA'}$;
3. $\overline{HA} + \overline{HB} + \overline{HC} = 2\overline{HO}$;
4. $\overline{HA} + \overline{HB} + \overline{HC} = 3\overline{HG}$;
5. the points H, G, O are collinear and $2GO = HG$. (the **Euler line**)

Exercise 9. Let $ABCD$ be a quadrilateral with $AB \cap CD = \{E\}$, $AD \cap BC = \{F\}$ and the points M, N, P the midpoints of $[BD]$, $[AC]$ and $[EF]$, respectively. Then M, N, P are collinear. (the **Newton-Gauss line**)