Prof. Dr. Dorin Andrica

Asist. Drd. Tudor Micu

1st Semester, 2018-2019

Geometry 1 (Analytic Geometry)

Exercise Sheet 8

Exercise 1. Let $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ be distinct points in space. Prove that the equation of the plane containing P_1 and P_2 that is parallel to a vector $\overline{a} = (l, m, n)$ is $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l & m & n \end{vmatrix}$

Exercise 2. Find the equation for each of the following planes:

(a) containing $P(2, 1, -1)$ and perpendicular to the vector $\overline{n} = (1, -2, 3)$;

(b) determined by $O(0, 0, 0)$, $P_1(3, -1, 2)$ and $P_2(4, -2, -1)$;

(c) containing $P(3, 4, -5)$ and parallel to both $\overline{a_1}(1, -2, 4)$ and $\overline{a_2}(2, 1, 1)$;

(d) containing the points $P_1(2, -1, -3)$ and $P_2(3, 1, 2)$ and parallel to the vector $\overline{a}(3, -1, -4)$.

Exercise 3. Find the equation of the plane passing through $P(7, -5, 1)$ which determines on the positive half-axes three segments of the same length.

Exercise 4. Find the equation of the plane containing the perpendicular lines through $P(-2, 3, 5)$ on the planes $\pi_1 : 4x + y - 3z + 13 = 0$ and $\pi_2 : x - 2y + z - 11 = 0$.

Exercise 5. Find the equation of the plane containing the point P and having the normal vector $\overline{n}$, where:

(a) $P(2, 6, 1)$, $\overline{n}(1, 4, 2)$;

(b) $P(1, 0, 0)$, $\overline{n}(0, 1, 1)$.

Exercise 6. Find the equation of the plane passing through the points A, B and C, where:

(a) $A(-2, 1, 1)$, $B(0, 2, 3)$ and $C(1, 0, -1)$;

(b) $A(3, 2, 1)$, $B(2, 1, -1)$ and $C(-1, 3, 2)$.

Exercise 7. Show that the points $A(1, 0, -1)$, $B(0, 2, 3)$, $C(-2, 1, 1)$ and $D(4, 2, 3)$ are coplanar.

Exercise 8. Let d_1 and d_2 be two lines in $\mathcal{E}_3$, given by
$d_1 : \dfrac{x-1}{2} = \dfrac{y+1}{-1} = \dfrac{z-5}{6}$
$d_2 : \dfrac{x-1}{1} = \dfrac{y+1}{1} = \dfrac{z-5}{-3}$

(a) Find the parametric equations of d_1 and d_2;

(b) Prove that they intersect and find the coordinates of their intersection point;

(c) Find the equation of the plane determined by d_1 and d_2.

Exercise 9. Given two lines
$d_1 : x = 1 + t, y = 1 + 2t, z = 3 + t, t \in \mathbb{R}$
and
$d_2 : x = 3 + s, y = 2s, z = -2 + s, s \in \mathbb{R}$,
prove that $d_1 \parallel d_2$ and find the equation of the plane determined by d_1 and d_2.

Exercise 10. Find the parametric equations of the line

$$\begin{cases} -2x + 3y + 7z + 2 = 0 \\ x + 2y - 3z + 5 = 0 \end{cases}$$

Exercise 11. Find the parametric equations of the line passing through P_1 and P_2, where:

(a) $P_1(5, -2, 1)$ and $P_2(2, 4, 2)$;

(b) $P_1(4, 0, 7)$ and $P_2(-1, -1, 2)$.

Exercise 12. Find the parametric equations of the line passing through $(-1, 2, 4)$ and parallel to $\overline{v}(3, -4, 1)$.

Exercise 13. Find the equations of the line passing through the origin and parallel to the line given by the parametric equations: $\begin{cases} x = t \\ y = -1 + t \\ z = 2 \end{cases}$