* = exercise from another row

Examen Algebră 2 Septimiu Crivei

2018 MIE sem II 29.06.2018

III

1 Define and give example of a: subgroup, cyclic group, subring, (group homomorphism).

2 State and prove the theorem regarding the form of subgroups of a cyclic group. (All subgroups of cyclic gr. are cyclic)

* first isomorphism theorem for groups

3 ~~Let G, G' be group. We define G×G'~~ Let $(G,\cdot)$, $(G',\cdot)$ groups with identity elements e and e'. Define „·" on $G \times G'$ $(g_1, g_1') \cdot (g_2, g_2') = (g_1 \cdot g_2, g_1' \cdot g_2')$. Prove $(G\times G', \cdot)$ is a group.

4 $(\mathbb{Z}_8, +)$ determine order of each element, its subgroups and factor groups.

5 $\mathbb{Q}(\sqrt{3}) = \{a + b\sqrt{3} \mid a, b \in \mathbb{Q}\}$. Prove $\mathbb{Q}(\sqrt{3})$ its a subfield of $(\mathbb{R}, +, \cdot)$

6 Let $M = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \middle| \, a, b \in \mathbb{R} \right\} \leq M_2(\mathbb{R})$. Show $(M, +, \cdot)$ is a field and that $(M, +, \cdot)$ is isomorphic to $(\mathbb{C}, +, \cdot)$. ⇔ $(M, +, \cdot) \simeq (\mathbb{C}, +, \cdot)$

1.5 points each exercise 100 minutes without 4, 3

150 minutes all