

## Sequences of real numbers - 2nd part

**Exercise 1:** Determine the nature (convergence or divergence) of the following series of real numbers:

$$a) \sum_{n \geq 1} \frac{n+7}{\sqrt{n^2+7}}, \quad b) \sum_{n \geq 1} \frac{1}{\sqrt[n]{n}}, \quad c) \sum_{n \geq 1} \frac{1}{\sqrt[n]{n!}}, \quad d) \sum_{n \geq 1} \left(1 + \frac{1}{n}\right)^n.$$

**Exercise 2:** Determine the nature (convergence or divergence) of the following series of real numbers:

$$a) \sum_{n \geq 1} \frac{2^n + 3^n}{5^n}, \quad b) \sum_{n \geq 1} \frac{2^n}{3^n + 5^n}.$$

**Exercise 3:** Determine the nature (convergence or divergence) of the following series of real numbers:

$$a) \sum_{n \geq 1} \frac{1}{2n-1}, \quad b) \sum_{n \geq 1} \frac{1}{(2n-1)^2}, \quad c) \sum_{n \geq 1} \frac{1}{\sqrt{4n^2-1}}, \quad d) \sum_{n \geq 1} \frac{\sqrt{n^2+n}}{\sqrt[3]{n^5-n}}.$$

**Exercise 4:** Determine the nature (convergence or divergence) of the following series of real numbers:

$$a) \sum_{n \geq 1} \frac{100^n}{n!}, \quad b) \sum_{n \geq 1} \frac{2^n n!}{n^n}, \quad c) \sum_{n \geq 1} \frac{3^n n!}{n^n}, \quad d) \sum_{n \geq 1} \frac{(n!)^2}{2^{n^2}}, \quad e) \sum_{n \geq 1} \frac{n^2}{\left(2 + \frac{1}{n}\right)^n}.$$

**Exercise 5:** Determine the nature (convergence or divergence), by discussing the value of the parameter  $a > 0$ , of the following series of real numbers:

$$a) \sum_{n \geq 1} \frac{a^n}{n^n}, \quad b) \sum_{n \geq 1} \left( \frac{n^2 + n + 1}{n^2} a \right)^n, \quad c) \sum_{n \geq 1} \frac{3^n}{2^n + a^n}.$$

## Remark

A brief synthesis of the theory concerning the exercises will be posted on the site, on 12.11.2017.