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PARTIAL EXAM 1, APRIL 18 2018

			Group:			Signature:			
Points	P1 18	P2 12	P3 20	P4 10	P5 10	P6 10	P7 10		Grade
Obtained	-3	6	1	0	10	4	5	+10	3,3

+?

P1. True or false? Encircle the correct answer.

(a) (true/false) For any map $\varphi : X \to Y$ between affine spaces and $y \in Y$, $\varphi^{-1}(y)$ is an affine subspace of X.

(b) (true/false) If P and Q are two points in an affine space, a point on the line PQ has the form $(1+t)P - tQ$ for some scalar t.

(c) (true/false) For two affine subspaces Y, Z in an affine space X, if $Y \cap Z = \emptyset$ then $\dim(Y \cup Z) = \dim(D(Y) + D(Z)) + 1$.

(d) (true/false) The convex hull operator has the property that if $M \subseteq N$ are two subsets of an affine space, then $\mathrm{conv}(M) \supseteq \mathrm{conv}(N)$.

(e) (true/false) In a 4-dimensional affine space any two hyperplanes which intersect nontrivially have a plane in common.

(f) (true/false) Graham's scan is an algorithm for determining Voronoi cells.

P2. Which of the following are affine subvarieties? Justify your answer.

$$A = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : 2x_1 - x_2 + x_3 - 2 = 0\}$$

$$B = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : (x_1 + x_2, 2x_2 + x_3, x_3 - 2x_1) \in A\}$$

$$C = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1^2 + x_2^2 + x_3^2 - 2x_1x_2 - 2x_1x_3 + 2x_2x_3 = 0\}$$

$$D = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1^4 + x_2 - 2x_3 + x_4 = 0\}.$$

P3. In an affine space consider the points C' and B' on the sides AB and AC of the triangle ABC such that $\overrightarrow{AC'} = \lambda \overrightarrow{BC'}$ and $\overrightarrow{AB'} = \mu \overrightarrow{CB'}$. The lines BB' and CC' meet in the point M. Show that

$$M = \frac{1}{1-\lambda-\mu}A - \frac{\lambda}{1-\lambda-\mu}B - \frac{\mu}{1-\lambda-\mu}C.$$

Further, for the centroid G, the incenter I and the orthocenter H of the triangle ABC, show that

$$G = \frac{1}{3}A + \frac{1}{3}B + \frac{1}{3}C, \quad I = \frac{a}{p}A + \frac{b}{p}B + \frac{c}{p}C, \quad H = \frac{\tan\hat{A}}{t}A + \frac{\tan\hat{B}}{t}B + \frac{\tan\hat{C}}{t}C$$

where $a = |BC|$, $b = |CA|$, $c = |AB|$ and $p = a + b + c$, $\hat{A}, \hat{B}, \hat{C}$ are the interior angles of the triangle and $t = \tan\hat{A} + \tan\hat{B} + \tan\hat{C}$.

P4. Let $P(X) \in \mathbb{C}[X]$ be a polynomial with complex coefficients. Show that the roots of P' lie in the convex hull of the roots of P.

P5. For a set $\mathcal{S} = \{A_1, \ldots, A_p\}$, let $\mathcal{S}_i = \mathcal{S} \setminus \{A_i\}$. Show that

$$\mathrm{Bar}(S) = \mathrm{Bar}(\mathrm{Bar}(\mathcal{S}_1), \ldots, \mathrm{Bar}(\mathcal{S}_p))$$

Let $\mathcal{S}' = \{A_1, \ldots, A_q\}$ and $\mathcal{S}'' = \{A_{q+1}, \ldots, A_p\}$. Show that

$$\mathrm{Bar}(S) = \frac{q}{p}\mathrm{Bar}(S') + \frac{p-q}{p}\mathrm{Bar}(S'').$$

P6. In $\mathbb{R}^5$ consider the elements

$$\begin{aligned} a &= (1,0,0,2,0) \\ b &= (0,2,0,0,1) \\ c &= (1,2,0,0,0) \\ d &= (0,0,0,2,1) \end{aligned}$$

and the affine subspaces $A = a + \langle b, c\rangle$ and $B = c + \langle b, d\rangle$. Determine

$$\mathrm{aff}(A \cap B) \quad \text{and} \quad \mathrm{aff}(A \cup B).$$

7. Describe the Voronoi cells for the points

$$\{(2m, 3n) : n, m \in \mathbb{N}\}.$$