

Seminar 1 – Complexity (Algorithm Analysis)

1. TRUE or FALSE?

- $n^2 \in O(n^3)$
- $n^3 \in O(n^2)$
- $2^{n+1} \in \Theta(2^n)$
- $2^{2n} \in \Theta(2^n)$
- $n^2 \in \Theta(n^3)$
- $2^n \in O(n!)$
- $\log_{10} n \in \Theta(\log_2 n)$
- $O(n) + \Theta(n^2) = \Theta(n^2)$
- $\Theta(n) + O(n^2) = O(n^2)$
- $O(n) + O(n^2) = O(n^2)$
- $O(f) + O(g) = O(\max\{f, g\})$
- $O(n) + \Theta(n) = O(n)$
- $(n + m)^2 \in O(n^2 + m^2)$
- $3^n \in O(2^n)$
- $\log_2 3^n \in O(\log_2 2^n)$

2. Complexity of search and sorting algorithms

Algorithm	Time Complexity				Extra Space Complexity
	Best C.	Worst C.	Average C.	Total	
Linear Search	$\Theta(1)$	$\Theta(n)$	$\Theta(n)$	$O(n)$	$\Theta(1)$
Binary Search	$\Theta(1)$	$\Theta(\log_2 n)$	$\Theta(\log_2 n)$	$O(\log_2 n)$	$\Theta(1)$
Selection Sort	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(1)$ – in place
Insertion Sort	$\Theta(n)$	$\Theta(n^2)$	$\Theta(n^2)$	$O(n^2)$	$\Theta(1)$ – in place
Bubble Sort	$\Theta(n)$	$\Theta(n^2)$	$\Theta(n^2)$	$O(n^2)$	$\Theta(1)$ – in place
Quick Sort	$\Theta(n \log_2 n)$	$\Theta(n^2)$	$\Theta(n \log_2 n)$	$O(n^2)$	$\Theta(1)$ – in place
Merge Sort	$\Theta(n \log_2 n)$	$\Theta(n \log_2 n)$	$\Theta(n \log_2 n)$	$\Theta(n \log_2 n)$	$\Theta(n)$ – out of place

3. Analyze the time complexity of the following two subalgorithms:

subalgorithm s1(n) is:

```

for i ← 1, n execute
    j ← n
    while j ≠ 0 execute
        j ← ⌊ $\frac{j}{2}$ ⌋
    end-while
end-for

```

end-subalgorithm

```
subalgorithm s2(n) is:
  for i ← 1, n execute
    j ← i
    while j ≠ 0 execute
      j ← ⌊ $\frac{j}{2}$ ⌋
    end-while
  end-for
end-subalgorithm
```

4. Analyze the time complexity of the following two subalgorithms:

```
subalgorithm s3(x, n, a) is:
  found ← false
  for i ← 1, n execute
    if  $x_i = a$  then
      found ← true
    end-if
  end-for
end-subalgorithm
```

```
subalgorithm s4(x, n, a) is:
  found ← false
  while found = false and i < n execute
    if  $x_i = a$  then
      found ← true
    end-if
    i ← i + 1
  end-while
end-subalgorithm
```

5. Analyze the time complexity of the following algorithm (x is an array, with elements $x_i \leq n$):

```
Subalgorithm s5(x, n) is:
  k ← 0
  for i ← 1, n execute
    for j ← 1,  $x_i$  execute
      k ← k +  $x_j$ 
    end-for
  end-for
end-subalgorithm
```

- a. if every $x_i > 0$
- b. if x_i can be 0

- Does the complexity change if we allow values of 0 in the array?
6. Consider the following problems and find an algorithm (having the required time complexity) to solve them :
- a. Given an arbitrary array with numbers $x_1 \dots x_n$, determine whether there are 2 equal elements in the array. Show that this can be done with $\Theta(n \log_2 n)$ time complexity.
 - b. Given an arbitrary array with numbers $x_1 \dots x_n$, determine whether there are two numbers whose sum is k (for some given k). Show that this can be done with $\Theta(n \log_2 n)$ time complexity. What happens if k is even and $k/2$ is in the array (once or multiple times)?
 - c. Given an array of distinct integers $x_1 \dots x_n$, ordered ascending, determine whether there is a position such that $A[i] = i$. Show that this can be done with $O(\log_2 n)$ complexity.
7. Analyze the time complexity of the following algorithm:

```

subalgorithm s6(n) is:
  for i ← 1, n execute
    @elementary operation
  end-for
  i ← 1
  k ← true
  while i ≤ n - 1 and k execute
    j ← i
    k1 ← true
    while j ≤ n and k1 execute
      @ elementary operation (k1 can be modified)
      j ← j + 1
    end-while
    i ← i + 1
    @elementary operation (k can be modified)
  end-while
end-subalgorithm

```

8. Analyze the time complexity of the following algorithm:

```

subalgorithm p(x,s,d) is:
  if s < d then
    m ← [(s+d)/2]
    for i ← s, d-1, execute
      @elementary operation
    end-for
    for i ← 1, 2 execute
      p(x, s, m)
    end-for
  end-if
end-subalgorithm

```

Initial call for the subalgorithm: $p(x, 1, n)$

9. Analyze the time complexity of the following algorithm:

```
Subalgorithm s7(n) is:  
   $s \leftarrow 0$   
  for  $i \leftarrow 1, n^2$  execute  
     $j \leftarrow i$   
    while  $j \neq 0$  execute  
       $s \leftarrow s + j$   
       $j \leftarrow j - 1$   
    end-while  
  end-for  
end-subalgorithm
```

10. Analyze the time complexity of the following algorithm:

```
Subalgorithm s8(n) is:  
   $s \leftarrow 0$   
  for  $i \leftarrow 1, n^2$  execute  
     $j \leftarrow i$   
    while  $j \neq 0$  execute  
       $s \leftarrow s + j - 10 * \lfloor j/10 \rfloor$   
       $j \leftarrow \lfloor j/10 \rfloor$   
    end-while  
  end-for  
end-subalgorithm
```

11. Analyze the time complexity of the following algorithm:

```
subalgorithm operation(n, i) is  
  if  $n > 1$  then  
     $i \leftarrow 2 * i$   
     $m \leftarrow \lfloor n/2 \rfloor$   
    operation(m, i-2)  
    operation(m, i-1)  
    operation(m, i+2)  
    operation(m, i+1)  
  else  
    write i  
  end-if  
end-subalgorithm
```