Restanță Algebră 2 MIE 2018 iulie

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1 Define and give example of a: semigroup, subring, ring homomorphism.

2 State and prove the first isomorphism theorem for rings.

3 Let $n \geq N$, $n \geq 2$ and $SL_n(\mathbb{R}) = \{A \in M_n(\mathbb{R}) \mid \det(A) = 1\}$, $GL_n(\mathbb{R}) = \{A \in M_n(\mathbb{R}) \mid \det(A) \neq 0\}$. Show that $SL_n(\mathbb{R})$ is a normal subgroup of the group $(GL_n(\mathbb{R}), \cdot)$

4 Let $f: G \to G'$ group homomorphism and let $x \in G$ be an element of finite order. Prove that $\operatorname{ord} f(x)$ is finite and $\operatorname{ord} f(x) \mid \operatorname{ord} x$ („|" divides)

5 Let $(R,+,\cdot)$ be a ring. Let $\mathbb{Z} \times R$ be a set with the operations „+" and „·" defined by:

$(m, a) + (n, b) = (m+n, a+b)$

$(m, a) \cdot (n, b) = (m \cdot n, a \cdot b + n \cdot a + m \cdot b)$, where $(m,a), (n,b) \in \mathbb{Z} \times R$

Prove that $(\mathbb{Z} \times R, +, \cdot)$ is a ring with identity.

6 Let $A = \left\{ \begin{pmatrix} a & b \\ 0 & b \end{pmatrix} \middle| a, b \in \mathbb{R} \right\} \subseteq M_2(\mathbb{R})$. Is A an ideal of the ring $M_2(\mathbb{R})$? Is A a subring of $M_2(\mathbb{R})$? ~~Justificate.~~ Justify.