DATA STRUCTURES AND ALGORITHMS

LECTURE 12

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In Lecture 11...

- Hash tables
- Trees

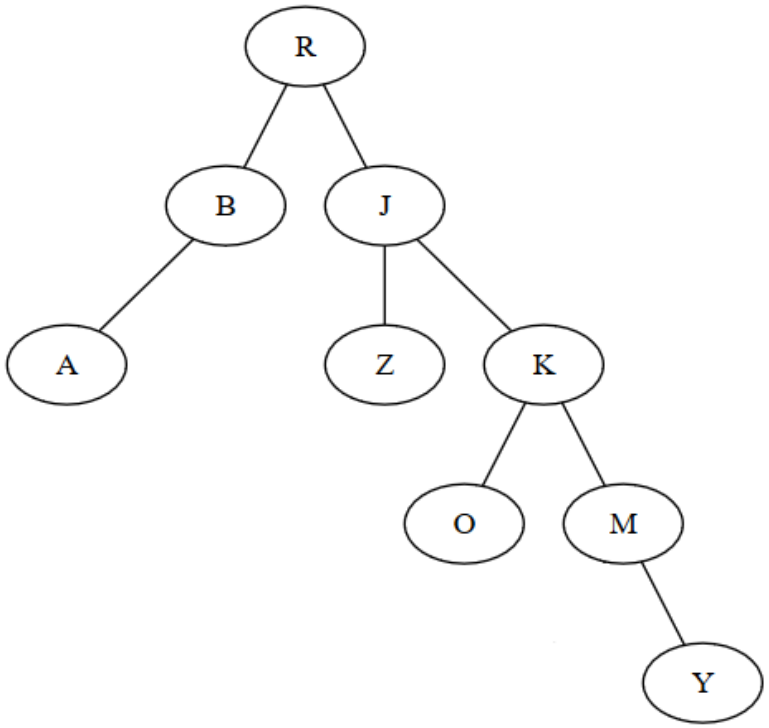
Today

1. Binary Trees
2. Huffman coding
3. Binary Search Trees

Binary trees

- An ordered tree in which each node has at most two children is called *binary tree*.
- In a binary tree we call the children of a node the *left child* and *right child*.
- Even if a node has only one child, we still have to know whether that is the left or the right one.

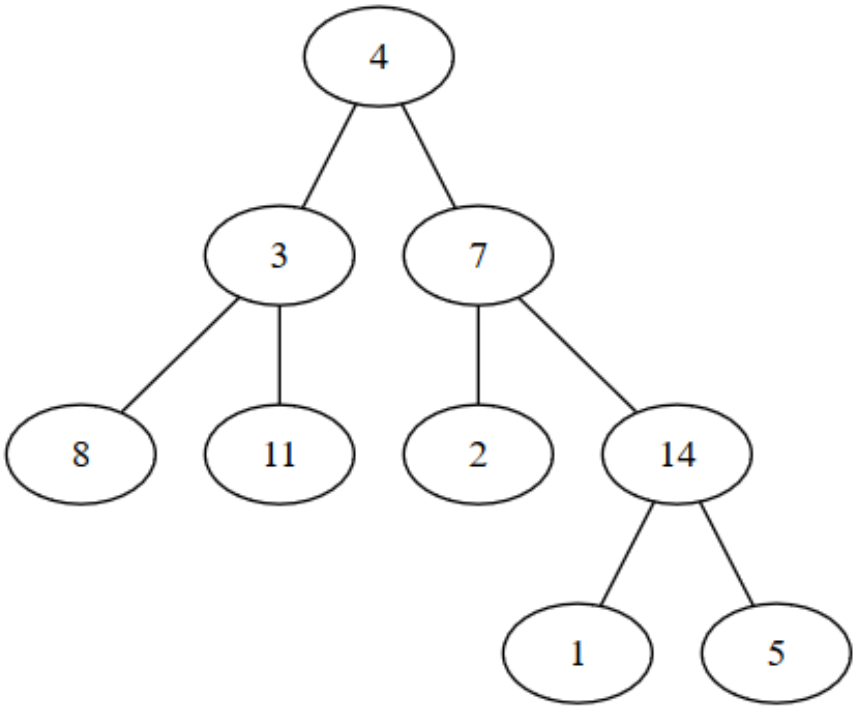
Binary tree - example



- A is the left child of B
- Y is the right child of M

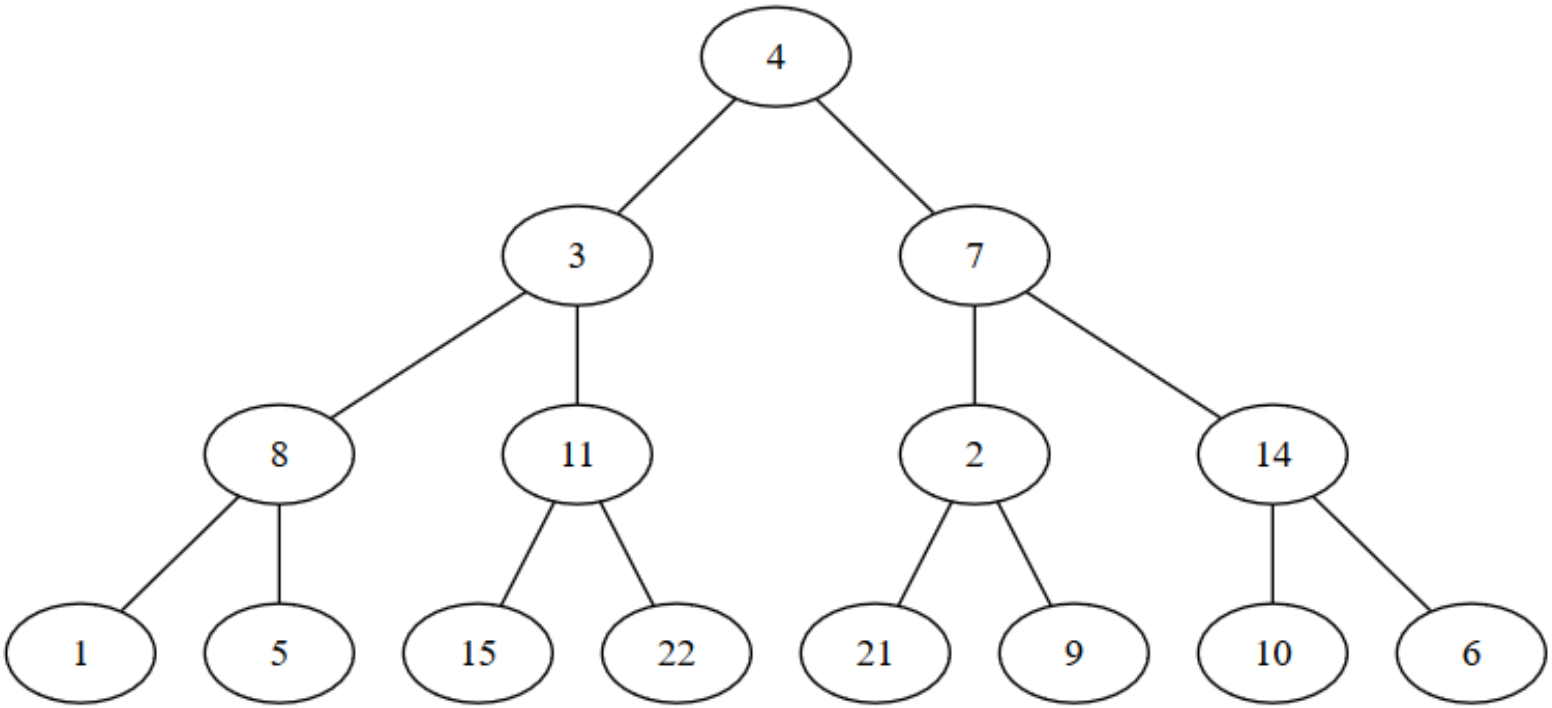
Binary tree - Terminology I

- A binary tree is called *full* if every internal node has exactly two children.



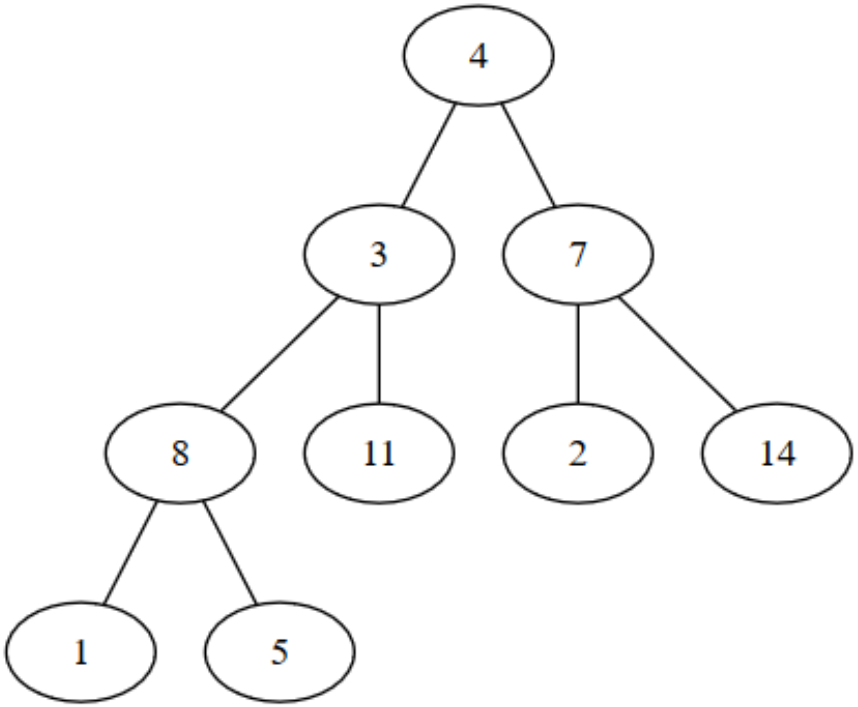
Binary tree - Terminology II

- A binary tree is called *complete* if all leaves are one the same level and all internal nodes have exactly 2 children.



Binary tree - Terminology III

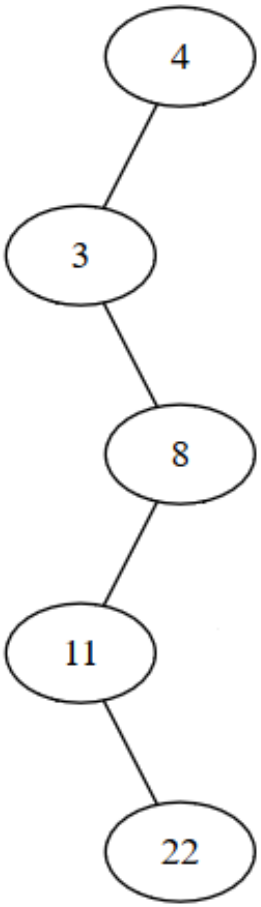
- A binary tree is called *almost complete* if it is a *complete* binary tree except for the last level, where nodes are completed from left to right (binary heap - structure).



Binary tree - Terminology IV

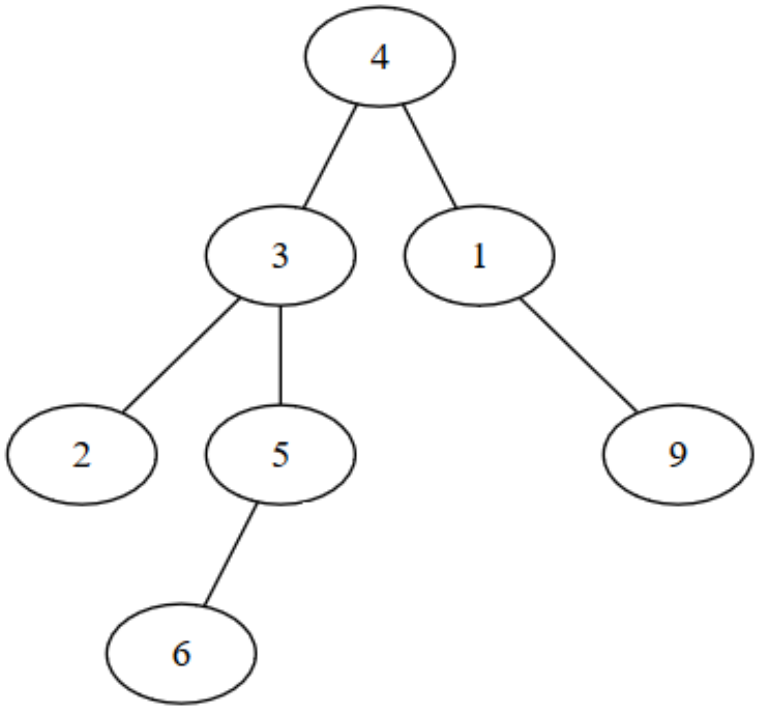
- A binary tree is called *degenerate* if every internal node has exactly one child (it is actually a chain of nodes).

Binary tree - Terminology V



Binary tree - Terminology VI

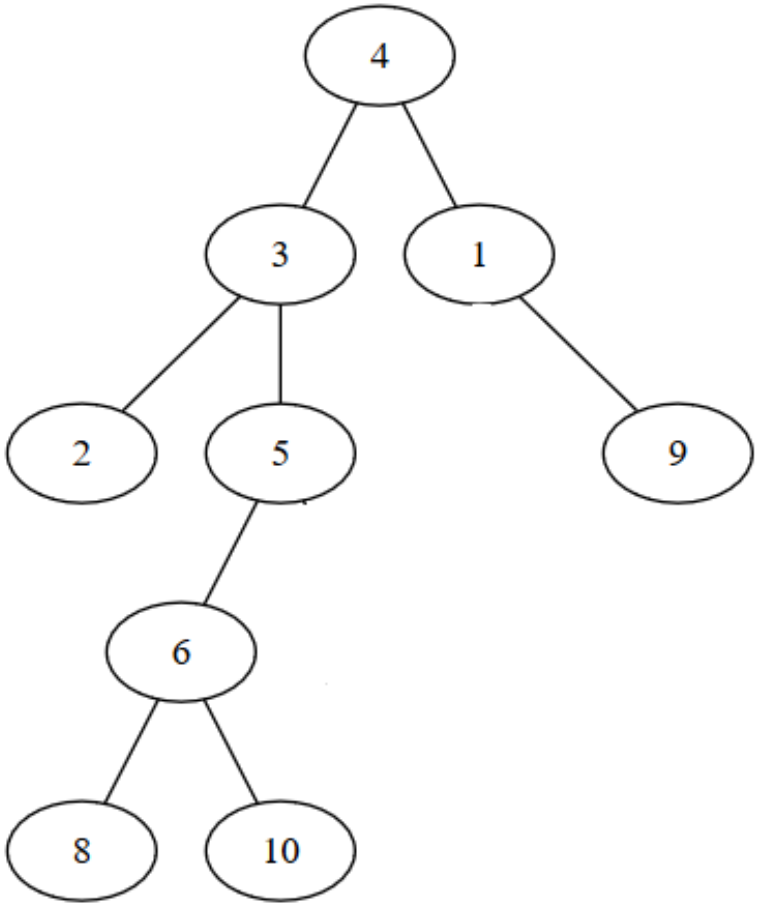
- A binary tree is called *balanced* if the difference between the height of the left and right subtrees is at most 1 (for every node from the tree).



Binary tree - Terminology VII

- Obviously, there are many binary trees that are none of the above categories, for example:

Binary tree - Terminology VIII



Binary tree - properties

- A binary tree with n nodes has exactly $n-1$ edges (this is true for every tree, not just binary trees)
- The number of nodes in a complete binary tree of height N is $2^{N+1}-1$ (it is $1+2+4+8+...+2^N$)
- The maximum number of nodes in a binary tree of height N is $2^{N+1}-1$ - if the tree is complete.
- The minimum number of nodes in a binary tree of height N is N - if the tree is degenerate.
- A binary tree with N nodes has a height between $log_2 N$ and N.

ADT Binary Tree I

- Domain of ADT Binary Tree:

$$\mathcal{BT} = \{bt \mid bt \text{ binary tree with nodes containing information of type TElem}\}$$

ADT Binary Tree II

- init(bt)
  - **descr:** creates a new, empty binary tree
  - **pre:** true
  - **post:** $bt \in \mathcal{BT}$, bt is an empty binary tree

ADT Binary Tree III

- initLeaf(bt, e)
  - **descr:** creates a new binary tree, having only the root with a given value
  - **pre:** $e \in TElem$
  - **post:** $bt \in \mathcal{BT}$, bt is a binary tree with only one node (its root) which contains the value e

ADT Binary Tree IV

- initTree(bt, left, e, right)
  - **descr:** creates a new binary tree, having a given information in the root and two given binary trees as children
  - **pre:** $left, right \in \mathcal{BT}$, $e \in TElem$
  - **post:** $bt \in \mathcal{BT}$, bt is a binary tree with left child equal to $left$, right child equal to $right$ and the information from the root is e

ADT Binary Tree V

- insertLeftSubtree(bt, left)
  - **descr:** sets the left subtree of a binary tree to a given value (if the tree had a left subtree, it will be changed)
  - **pre:** $bt, left \in \mathcal{BT}$
  - **post:** $bt' \in \mathcal{BT}$, the left subtree of bt' is equal to $left$

ADT Binary Tree VI

- insertRightSubtree(bt, right)
  - **descr:** sets the right subtree of a binary tree to a given value (if the tree had a right subtree, it will be changed)
  - **pre:** $bt, right \in \mathcal{BT}$
  - **post:** $bt' \in \mathcal{BT}$, the right subtree of bt' is equal to *right*

ADT Binary Tree VII

- root(bt)
  - **descr:** returns the information from the root of a binary tree
  - **pre:** $bt \in \mathcal{BT}$, $bt \neq \Phi$
  - **post:** $root \leftarrow e$, $e \in TElem$, e is the information from the root of bt
  - **throws:** an exception if bt is empty

ADT Binary Tree VIII

- left(bt)
  - **descr:** returns the left subtree of a binary tree
  - **pre:** $bt \in \mathcal{BT}$, $bt \neq \Phi$
  - **post:** $left \leftarrow l$, $l \in \mathcal{BT}$, l is the left subtree of bt
  - **throws:** an exception if bt is empty

ADT Binary Tree IX

- right(bt)
  - **descr:** returns the right subtree of a binary tree
  - **pre:** $bt \in \mathcal{BT}$, $bt \neq \Phi$
  - **post:** $right \leftarrow r$, $r \in \mathcal{BT}$, r is the right subtree of bt
  - **throws:** an exception if bt is empty

ADT Binary Tree X

- isEmpty(bt)
  - **descr:** checks if a binary tree is empty
  - **pre:** $bt \in \mathcal{BT}$
  - **post:**

$$empty \leftarrow \begin{cases} True, & \text{if } bt = \Phi \\ False, & \text{otherwise} \end{cases}$$

ADT Binary Tree XI

- iterator (bt, traversal, i)
  - **descr:** returns an iterator for a binary tree
  - **pre:** $bt \in \mathcal{BT}$, *traversal* represents the order in which the tree has to be traversed
  - **post:** $i \in \mathcal{I}$, i is an iterator over bt that iterates in the order given by *traversal*

ADT Binary Tree XII

- destroy(bt)
  - **descr:** destorys a binary tree
  - **pre:** $bt \in \mathcal{BT}$
  - **post:** bt was destroyed

ADT Binary Tree XIII

- Other possible operations:
  - change the information from the root of a binary tree
  - remove a subtree (left or right) of a binary tree
  - search for an element in a binary tree
  - return the number of elements from a binary tree

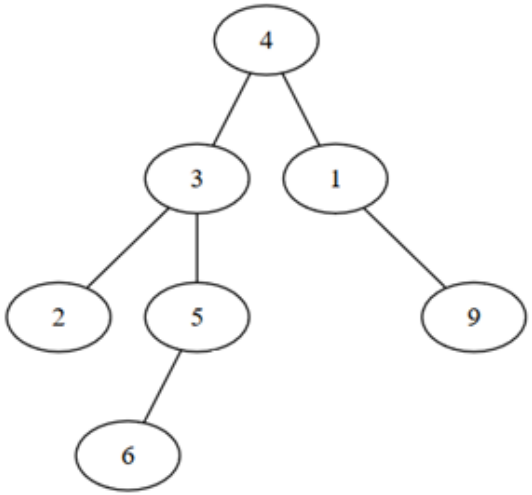
Possible representations

- If we want to implement a binary tree, what representation can we use?
- We have several options:
  - Representation using an array (similar to a binary heap)
  - Linked representation
    - with dynamic allocation
    - on an array

Possible representations I

- Representation using an array
  - Store the elements in an array
  - First position from the array is the root of the tree
  - Left child of node from position i is at position $2 * i$, right child is at position $2 * i + 1$.
  - Some special value is needed to denote the place where no element is.

Possible representations II



Pos	Elem
1	4
2	3
3	1
4	2
5	5
6	-1
7	9
8	-1
9	-1
10	6
11	-1
12	-1
13	-1
...	...

Possible representations III

- Disadvantage: depending on the form of the tree, we might waste a lot of space.

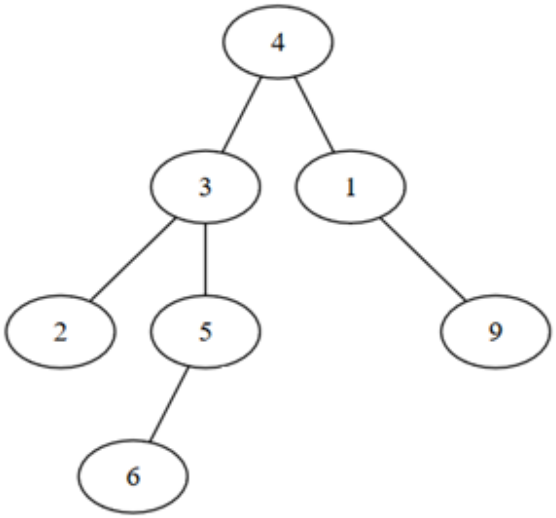
Possible representations I

- Linked representation with dynamic allocation
  - We have a structure to represent a node, containing the information, the address of the left child and the address of the right child (possibly the address of the parent as well).
  - An empty tree is denoted by the value NIL for the root.
  - We have one node for every element of the tree.

Possible representations II

- Linked representation on an array
  - Information from the nodes is placed in an array. The *address* of the left and right child is the *index* where the corresponding elements can be found in the array.
  - We can have a separate array for the parent as well.

Possible representations III



Pos	1	2	3	4	5	6	7	8
Info	4	3	2	5	6	1	9	
Left	2	3	-1	5	-1	-1	-1	
Right	6	4	-1	-1	-1	7	-1	
Parent	-1	1	2	2	4	1	6	

- We need to know that the root is at position 1 (could be any position).
- If the array is full, we can allocate a larger one.

Possible representations IV

- We can keep a linked list of empty positions to make adding a new node easier.

Binary Tree Traversal

- A node of a (binary) tree is visited when the program control arrives at the node, usually with the purpose of performing some operation on the node (printing it, checking the value from the node, etc.).
- *Traversing* a (binary) tree means visiting all of its nodes.
- For a binary tree there are 4 possible traversals:
  - Preorder
  - Inorder
  - Postorder
  - Level order (breadth first) - the same as in case of a (non-binary) tree

Binary tree representation

- In the following, for the implementation of the traversal algorithms, we are going to use the following representation for a binary tree:

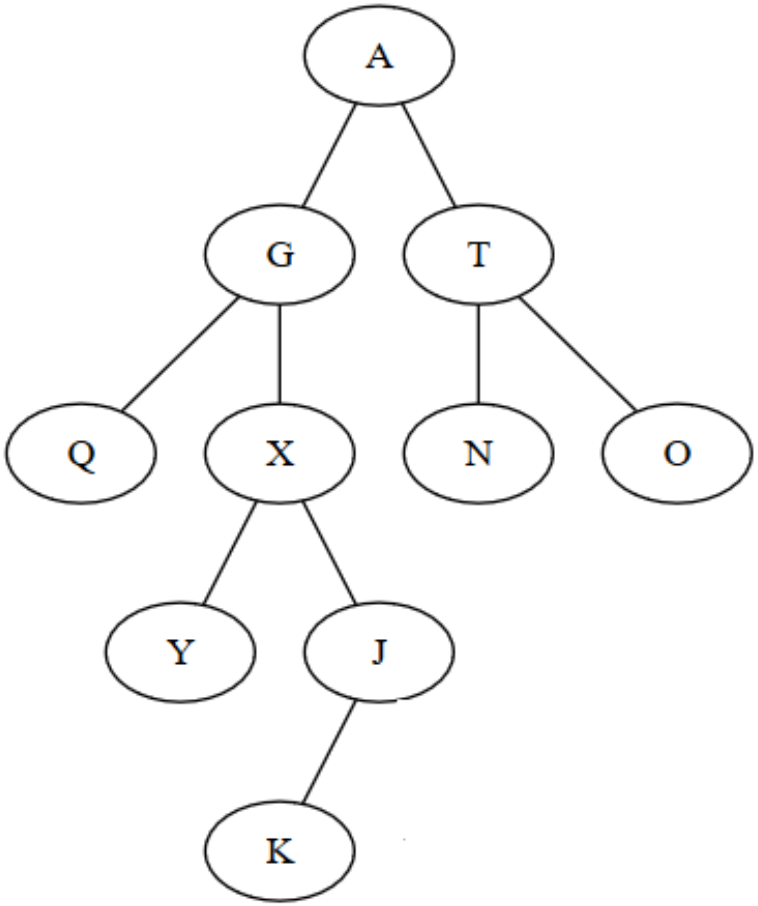
BTNode:
  info: TElem
  left: ↑ BTNode
  right: ↑ BTNode

BinaryTree:
  root: ↑ BTNode

Preorder traversal

- In case of a preorder traversal:
  - Visit the *root* of the tree
  - Traverse the left subtree - if exists
  - Traverse the right subtree - if exists
- When traversing the subtrees (left or right) the same preorder traversal is applied (so, from the left subtree we visit the root first and then traverse the left subtree and then the right subtree).

Preorder traversal example



- Preorder traversal: A, G, Q, X, Y, J, K, T, N, O

Preorder traversal - recursive implementation

- The simplest implementation for preorder traversal is with a recursive algorithm.

```
subalgorithm preorder_recursive(node) is:
//pre: node is a ↑ BTNode
    if node ≠ NIL then
        @visit [node].info
        preorder_recursive([node].left)
        preorder_recursive([node].right)
    end-if
end-subalgorithm
```

Preorder traversal - recursive implementation

- The *preorder_recursive* subalgorithm receives as parameter a pointer to a node, so we need a wrapper subalgorithm, one that receives a *BinaryTree* and calls the function for the root of the tree.

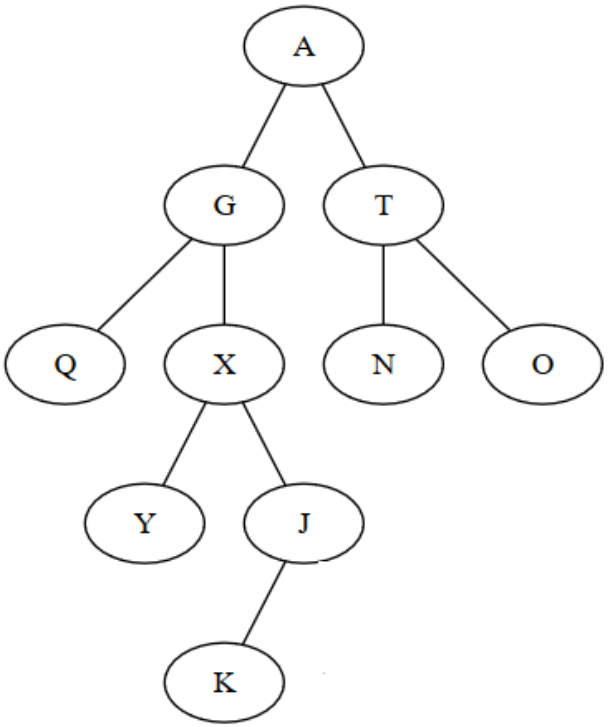
subalgorithm preorderRec(tree) **is:**
//pre: tree is a BinaryTree
  preorder_recursive(tree.root)
end-subalgorithm

- Assuming that visiting a node takes constant time (print the info from the node, for example), the whole traversal takes $\Theta(n)$ time for a tree with n nodes.

Preorder traversal - non-recursive implementation

- We can implement the preorder traversal algorithm without recursion, using an auxiliary *stack* to store the nodes.
  - We start with an empty stack
  - Push the root of the tree to the stack
  - While the stack is not empty:
    - Pop a node and visit it
    - Push the node's right child to the stack
    - Push the node's left child to the stack

Preorder traversal - non-recursive implementation example



- Stack: A
- Visit A, push children (Stack: T G)
- Visit G, push children (Stack: T X Q)
- Visit Q, push nothing (Stack: T X)
- Visit X, push children (Stack: T J Y)
- Visit Y, push nothing (Stack: T J)
- Visit J, push child (Stack: T K)
- Visit K, push nothing (Stack: T)
- Visit T, push children (Stack: O N)
- Visit N, push nothing (Stack: O)
- Visit O, push nothing (Stack:)
- Stack is empty, traversal is complete

Preorder traversal - non-recursive implementation

```
subalgorithm preorder(tree) is:
//pre: tree is a binary tree
    s: Stack //s is an auxiliary stack
    if tree.root ≠ NIL then
        push(s, tree.root)
    end-if
    while not isEmpty(s) execute
        currentNode ← pop(s)
        @visit currentNode
        if [currentNode].right ≠ NIL then
            push(s, [currentNode].right)
        end-if
        if [currentNode].left ≠ NIL then
            push(s, [currentNode].left)
        end-if
    end-while
end-subalgorithm
```

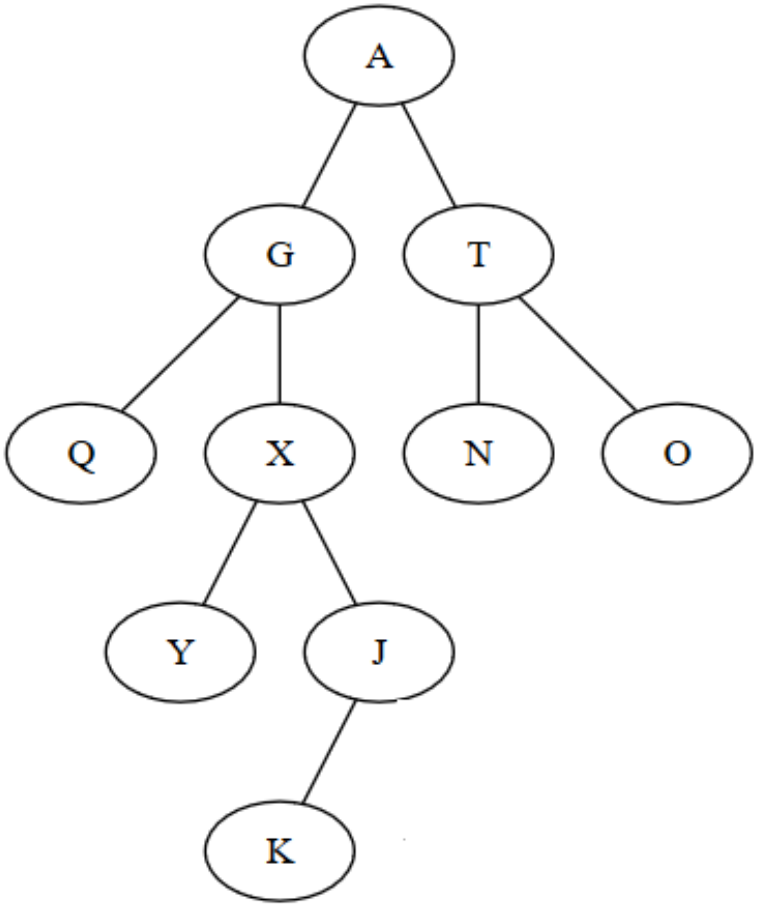
Preorder traversal - non - recursive implementation

- Time complexity of the non-recursive traversal is $\Theta(n)$, and we also need $O(n)$ extra space (the stack)

Inorder traversal

- In case of *inorder* traversal:
  - Traverse the left subtree - if exists
  - Visit the *root* of the tree
  - Traverse the right subtree - if exists
- When traversing the subtrees (left or right) the same inorder traversal is applied (so, from the left subtree we traverse the left subtree, then we visit the root and then traverse the right subtree).

Inorder traversal example



- Inorder traversal: Q, G, Y, X, K, J, A, N, T, O

Inorder traversal - recursive implementation

- The simplest implementation for inorder traversal is with a recursive algorithm.

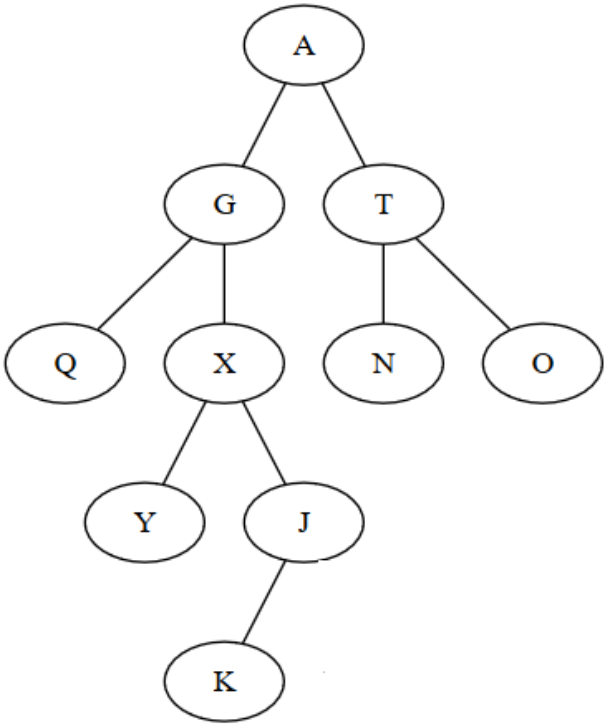
```
subalgorithm inorder_recursive(node) is:
//pre: node is a ↑ BTNode
    if node ≠ NIL then
        inorder_recursive([node].left)
        @visit [node].info
        inorder_recursive([node].right)
    end-if
end-subalgorithm
```

- We need again a wrapper subalgorithm to perform the first call to *inorder_recursive* with the root of the tree as parameter.
- The traversal takes $\Theta(n)$ time for a tree with n nodes.

Inorder traversal - non-recursive implementation

- We can implement the inorder traversal algorithm without recursion, using an auxiliary stack to store the nodes.
  - We start with an empty stack and a current node set to the root
  - While current node is not NIL, push it to the stack and set it to its left child
  - While stack not empty
    - Pop a node and visit it
    - Set current node to the right child of the popped node
    - While current node is not NIL, push it to the stack and set it to its left child

Inorder traversal - non-recursive implementation example



- CurrentNode: A (Stack:)
- CurrentNode: NIL (Stack: A G Q)
- Visit Q, currentNode NIL (Stack: A G)
- Visit G, currentNode X (Stack: A)
- CurrentNode: NIL (Stack: A X Y)
- Visit Y, currentNode NIL (Stack: A X)
- Visit X, currentNode J (Stack: A)
- CurrentNode: NIL (Stack: A J K)
- Visit K, currentNode NIL (Stack: A J)
- Visit J, currentNode NIL (Stack: A)
- Visit A, currentNode T (Stack:)
- CurrentNode: NIL (Stack: T N)
- ...

Inorder traversal - non-recursive implementation

```
subalgorithm inorder(tree) is:
//pre: tree is a BinaryTree
    s: Stack //s is an auxiliary stack
    currentNode ← tree.root
    while currentNode ≠ NIL execute
        push(s, currentNode)
        currentNode ← [currentNode].left
    end-while
    while not isEmpty(s) execute
        currentNode ← pop(s)
        @visit currentNode
        currentNode ← [currentNode].right
        while currentNode ≠ NIL execute
            push(s, currentNode)
            currentNode ← [currentNode].left
        end-while
    end-while
end-subalgorithm
```

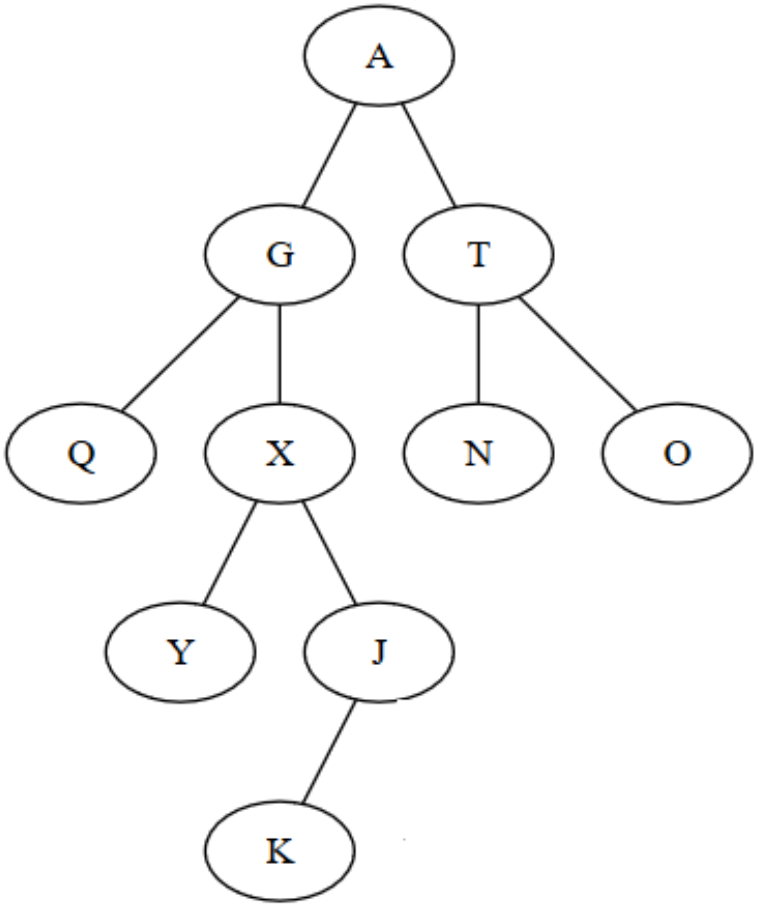
Inorder traversal - non-recursive implementation

- Time complexity $\Theta(n)$, extra space complexity $O(n)$

Postorder traversal

- In case of *postorder* traversal:
  - Traverse the left subtree - if exists
  - Traverse the right subtree - if exists
  - Visit the *root* of the tree
- When traversing the subtrees (left or right) the same postorder traversal is applied (so, from the left subtree we traverse the left subtree, then traverse the right subtree and then visit the root).

Postorder traversal example



- Postorder traversal: Q, Y, K, J, X, G, N, O, T, A

Postorder traversal - recursive implementation

- The simplest implementation for postorder traversal is with a recursive algorithm.

```
subalgorithm postorder_recursive(node) is:
//pre: node is a ↑ BTNode
    if node ≠ NIL then
        postorder_recursive([node].left)
        postorder_recursive([node].right)
        @visit [node].info
    end-if
end-subalgorithm
```

- We need again a wrapper subalgorithm to perform the first call to *postorder_recursive* with the root of the tree as parameter.
- The traversal takes $\Theta(n)$ time for a tree with n nodes.

Postorder traversal - non-recursive implementation

- We can implement the postorder traversal without recursion, but it is slightly more complicated than preorder and inorder traversals.
- We can have an implementation that uses two stacks and there is also an implementation that uses one stack.

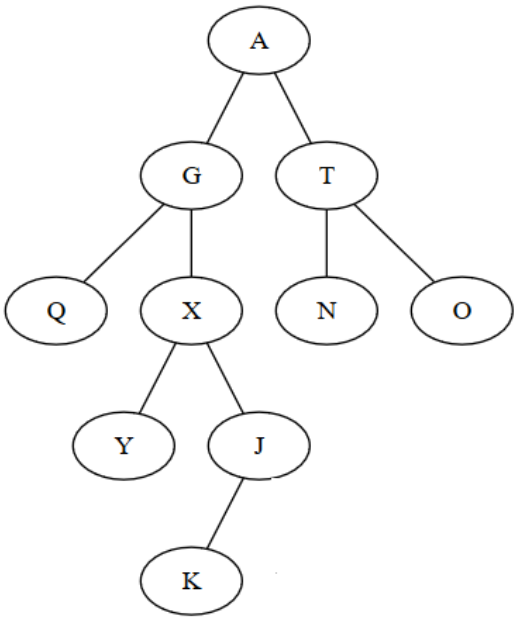
Postorder traversal with two stacks

- The main idea of postorder traversal with two stacks is to build the reverse of postorder traversal in one stack. If we have this, popping the elements from the stack until it becomes empty will give us postorder traversal.
- Building the reverse of postorder traversal is similar to building preorder traversal, except that we need to traverse the right subtree first (not the left one). The other stack will be used for this.
- The algorithm is similar to *preorder* traversal, with two modifications:
  - When a node is removed from the stack, it is added to the second stack (instead of being visited)
  - For a node taken from the stack we first push the left child and then the right child to the stack.

Postorder traversal with one stack

- We start with an empty stack and a current node set to the root of the tree
- While the current node is not NIL, push to the stack the right child of the current node (if exists) and the current node and then set the current node to its left child.
- While the stack is not empty
  - Pop a node from the stack (call it current node)
  - If the current node has a right child, the stack is not empty and contains the right child on top of it, pop the right child, push the current node, and set current node to the right child.
  - Otherwise, visit the current node and set it to NIL
  - While the current node is not NIL, push to the stack the right child of the current node (if exists) and the current node and then set the current node to its left child.

Postorder traversal - non-recursive implementation example



- Node: A (Stack:)
- Node: NIL (Stack: T A X G Q)
- Visit Q, Node NIL (Stack: T A X G)
- Node: X (Stack: T A G)
- Node: NIL (Stack: T A G J X Y)
- Visit Y, Node: NIL (Stack: T A G J X)
- Node: J (Stack: T A G X)
- Node: NIL (Stack: T A G X J K)
- Visit K, Node: NIL (Stack: T A G X J)
- Visit J, Node: NIL (Stack: T A G X)
- Visit X, Node: NIL (Stack: T A G)
- Visit G, Node: NIL (Stack: T A)
- Node: T (Stack: A)
- Node: NIL (Stack: A O T N)
- ...

Postorder traversal - non-recursive implementation

```
subalgorithm postorder(tree) is:
//pre: tree is a BinaryTree
    s: Stack //s is an auxiliary stack
    node ← tree.root
    while node ≠ NIL execute
        if [node].right ≠ NIL then
            push(s, [node].right)
        end-if
        push(s, node)
        node ← [node].left
    end-while
    while not isEmpty(s) execute
        node ← pop(s)
        if [node].right ≠ NIL and (not isEmpty(s)) and [node].right = top(s) th
            pop(s)
            push(s, node)
            node ← [node].right
//continued on the next slide
```

Postorder traversal - non-recursive implementation

```
        else
            @visit node
            node ← NIL
        end-if
        while node ≠ NIL execute
            if [node].right ≠ NIL then
                push(s, [node].right)
            end-if
            push(s, node)
            node ← [node].left
        end-while
    end-while
end-subalgorithm
```

- Time complexity $\Theta(n)$, extra space complexity $O(n)$

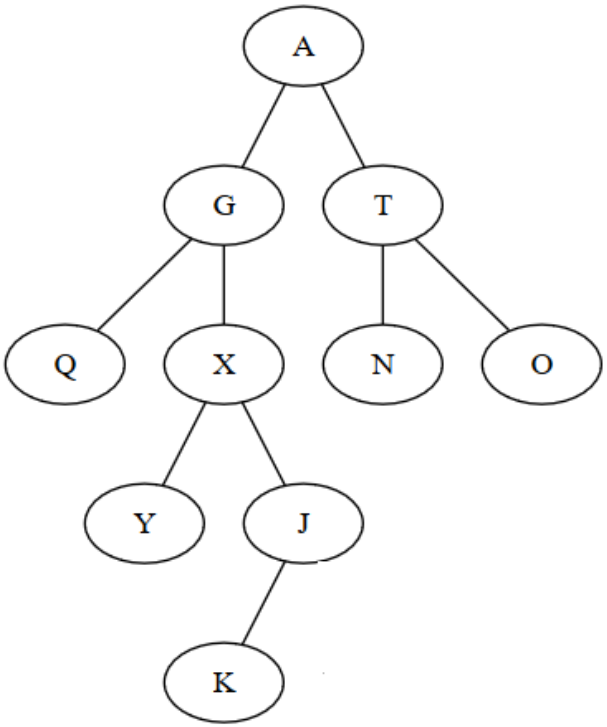
Traversal without a stack

- Preorder, postorder and inorder traversals can be implemented without an auxiliary stack if we use a representation for a node, where we keep a pointer to the parent node and a boolean flag to show whether the current node was visited or not.
- When we start the traversal we assume that all nodes have the *visited* flag set to false.
- During the traversal we set the flags to true, but when traversal is over, we have to make sure that they are set to false again (otherwise a second traversal is not possible).

Inorder traversal without a stack

- We take a current node which is set to the root of the tree
- Repeat the following until the node becomes NIL
  - If the node has a left child and it was not visited, the node becomes the left child
  - Otherwise, if the node is not visited, we will visit it
  - Otherwise, if the node has a right child and it was not visited, the node becomes the right child
  - Otherwise, set the children (left and right) of this node back to unvisited and change the node to its parent)

Inorder traversal without a stack - example



- Start with node A and go left when possible (Node: Q)
- Visit Q and go right if possible, if not, go up (Node: G)
- Visit G and go right if possible (Node: X)
- Go left as much as possible (Node: Y)
- Visit Y and go right if possible, if not, go up (Node: X)
- Visit X and go right if possible (Node: J)
- Go left as much as possible (Node: K)
- Visit K and go right if possible, if not go up (Node: J)
- Visit J and go right if possible, it not go up and set children of J to non-visited (Node: X)
- Go up and set children of X to non-visited (Node: G)
- ...

Inorder traversal without a stack - implementation

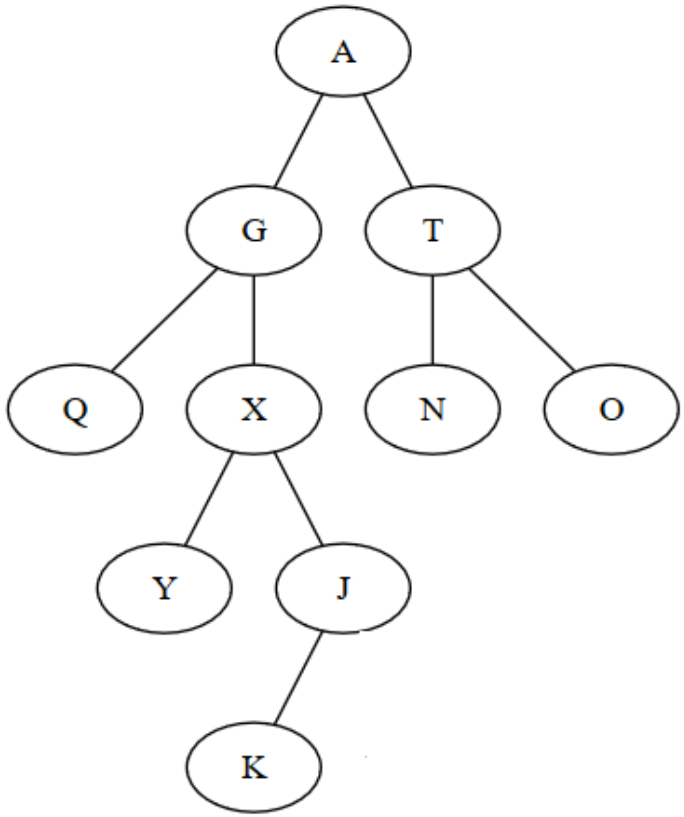
```
subalgorithm inorderNoStack(tree) is:
//pre: tree is a BinaryTree, with nodes containing pointer to parent and visited
    current ← tree.root
    while current ≠ NIL execute
        if [current].left ≠ NIL and [[current].left].visited = false then
            current ← [current].left
        else if [current].visited = false then
            @visit current
            [current].visited ← true
        else if [current].right ≠ NIL and [[current].right].visited = false then
            current ← [current].right
        else
        //we are going up, but before that reset the children to not-visited
            if [current].left ≠ NIL then
                [[current].left].visited ← false
            end-if
//continued on the next slide...
```

Inorder traversal without a stack - implementation

```
        if [current].right ≠ NIL then
            [[current].right].visited ← false
        end-if
        current ← [current].parent
    end-if
  end-while
  if tree.root ≠ NIL then
    [tree.root].visited ← false
  end-if
end-subalgorithm
```

Level order traversal

- In case of level order traversal we first visit the root, then the children of the root, then the children of the children, etc.



- Level order traversal: A, G, T, Q, X, N, O, Y, J, K

Binary tree iterator

- The interface of the binary tree contains the *iterator* operation, which should return an iterator.
- This operation receives a parameter that specifies what kind of traversal we want to do with the iterator (preorder, inorder, postorder, level order)
- The traversal algorithms discussed so far, traverse all the elements of the binary tree at once, but an iterator has to do element-by-element traversal.
- For defining an iterator, we have to divide the code into the functions of an iterator: *init*, *getCurrent*, *next*, *valid*

Inorder binary tree iterator

- Assume an implementation without a parent node.
- What fields do we need to keep in the iterator structure?

InorderIterator:
bt: BinaryTree
s: Stack
currentNode: ↑ BTNode

Inorder binary tree iterator - init

- What should the *init* operation do?

```
subalgorithm init (it, bt) is:
//pre: it - is an InorderIterator, bt is a BinaryTree
    it.bt ← bt
    init(it.s)
    node ← bt.root
    while node ≠ NIL execute
        push(it.s, node)
        node ← [node].left
    end-while
    if not isEmpty(it.s) then
        it.currentNode ← top(it.s)
    else
        it.currentNode ← NIL
    end-if
end-subalgorithm
```

Inorder binary tree iterator - getCurrent

- What should the *getCurrent* operation do?

```
function getCurrent(it) is:
    getCurrent ← [it.currentNode].info
end-function
```

Inorder binary tree iterator - valid

- What should the *valid* operation do?

```
function valid(it) is:
    if it.currentNode = NIL then
        valid ← false
    else
        valid ← true
    end-if
end-function
```

Inorder binary tree iterator - next

- What should the *next* operation do?

```
subalgorithm next(it) is:
    node ← pop(it.s)
    if [node].right ≠ NIL then
        node ← [node].right
        while node ≠ NIL execute
            push(it.s, node)
            node ← [node].left
        end-while
    end-if
    if not isEmpty(it.s) then
        it.currentNode ← top(it.s)
    else
        it.currentNode ← NIL
    end-if
end-subalgorithm
```

Preorder, Inorder, Postorder

- How to remember the difference between traversals?
  - Left subtree is always traversed before the right subtree.
  - The visiting of the root is what changes:
    - PREorder - visit the root before the left and right
    - INorder - visit the root between the left and right
    - POSTorder - visit the root after the left and right

Think about it

- Assume you have a binary tree, but you do not know how it looks like, but you have the *preorder* and *inorder* traversal of the tree. Give an algorithm for building the tree based on these two traversals.

- For example:
  - Preorder: A B F G H E L M
  - Inorder: B G F H A L E M

Think about it

- Can you rebuild the tree if you have the *postorder* and the *inorder* traversal?
- Can you rebuild the tree if you have the *preorder* and the *postorder* traversal?

Huffman coding

- The *Huffman coding* can be used to encode characters (from an alphabet) using variable length codes.
- In order to reduce the total number of bits needed to encode a message, characters that appear more frequently have shorter codes.
- Since we use variable length code for each character, *no code can be the prefix of any other code* (if we encode letter E with 01 and letter X with 010011, during deconding, when we find a 01, we will not know whether it is E or the beginning of X).

Huffman coding

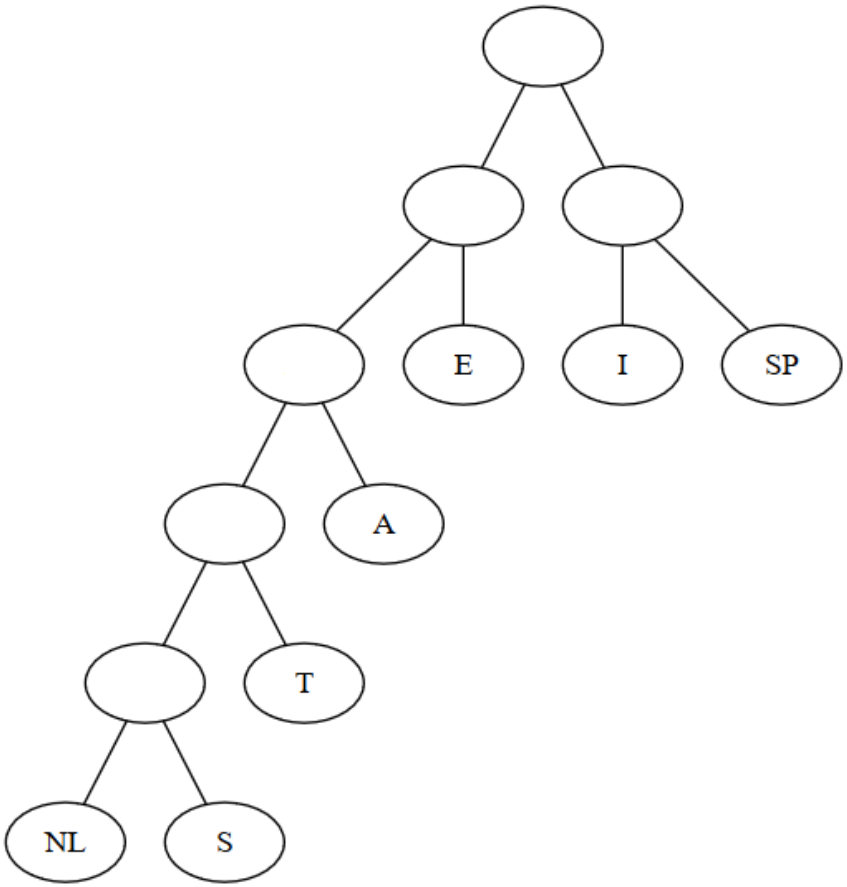
- When building the Huffman encoding for a message, we first have to compute the frequency of every character from the message, because we are going to define the codes based on the frequencies.
- Assume that we have a message with the following letters and frequencies

Character	a	e	i	s	t	space	newline
Frequency	10	15	12	3	4	13	1

Huffman coding

- For defining the Huffman code a binary tree is build in the following way:
  - Start with trees containing only a root node, one for every character. Each tree has a weight, which is frequency of the character.
  - Get the two trees with the least weight (if there is a tie, choose randomly), combine them into one tree which has as weight the sum of the two weights.
  - Repeat until we get have only one tree.

Huffman coding



Huffman coding

- Code for each character can be read from the tree in the following way: start from the root and go towards the corresponding leaf node. Every time we go left add the bit 0 to encoding and when we go right add bit 1.
- Code for the characters:
  - NL - 00000
  - S - 00001
  - T - 0001
  - A - 001
  - E - 01
  - I - 10
  - SP - 11
- In order to encode a message, just replace each character with the corresponding code

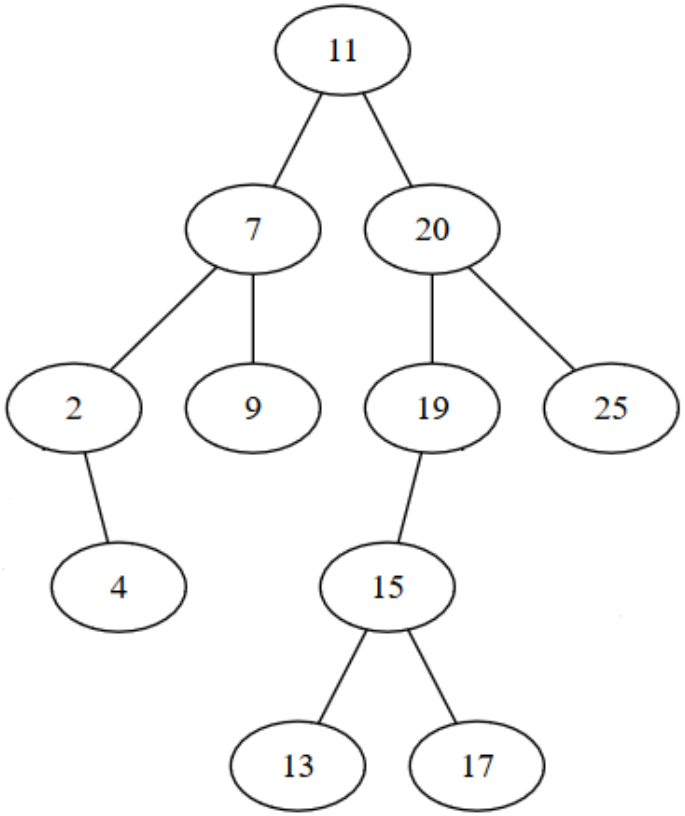
Huffman coding

- Assume we have the following code and we want to decode it: 01101100010001001110010000 0
- We do not know where the code of each character ends, but we can use the previously built tree to decode it.
- Start parsing the code and iterate through the tree in the following way:
  - Start from the root
  - If the current bit from the code is 0 go to the left child, otherwise go to the right child
  - If we are at a leaf node we have decoded a character and have to start over from the root
- The decoded message: E I SP T T A SP I E NL

Binary search trees

- A *Binary Search Tree* is a binary tree that satisfies the following property:
  - if x is a node of the binary search tree then:
    - For every node y from the left subtree of x, the information from y is less than or equal to the information from x
    - For every node y from the right subtree of x, the information from y is greater than or equal to the information from x
- In order to have a binary search tree, we need to store information in the tree that is of type *TComp*.
- Obviously, the relation used to order the nodes can be considered in an abstract way (instead of having "$\leq$" as in the definition).

Binary Search Tree Example



- If we do an inorder traversal of a binary search tree, we will get the elements in increasing order (according to the relation used).