

Luca Laura-Claudia

914

ADT Sparse Matrix

Domain:

$SM = \{ sm \mid sm \text{ is a sparse matrix with elements of type } TElem, \text{ each elements having a unique position determined by a line and a column (both integers) in } sm \}$

Operations:

- **init(sm, nrL, nrC)**
Pre: nrL \in integer, nrC \in integer
Post: sm \in SM, sm is an “empty” matrix (only contains 0_{TElem} values)
- **nrLines(sm)**
Pre: sm \in SM
Post: nrLines \leftarrow number of lines (nrL)
- **nrColumns(sm)**
Pre: sm \in SM
Post: nrColumns \leftarrow number of columns (nrC)
- **element(sm, i, j)**
Pre: sm \in SM, i \in TLine , j \in TColumn, valid(i, j)
Post: e \in TElem
element \leftarrow e \in (i, j) (the element on line i and column j)
*@throws exception **if** the pair (i, j) is not valid*
- **modify(sm, i, j, val)**
Pre: sm \in SM, i \in TLine , j \in TColumn, val \in TElem, valid(i, j)
Post: the element from position (i, j) = val
*@throws exception **if** the pair (i, j) is not valid*
- **destroy(sm)**
Pre: sm \in SM
Post: sm was destroyed (allocated memory was freed)
- **iterator(sm, i)**
Pre: sm \in SM
Post: i \in IteratorSM

Representation :

SMElement:

line: TLine

column: TColumn

value: TElem

SMNode:

Info : SMElement

Next: \uparrow SMNode

SM:

elems: \uparrow SMNode [] //an array of pointers to nodes

m: Integer

h:TFunction //the hash function

hc: TFunction //The hashCode function

nrL: Integer

nrC: Integer

Iterator SM

Operations:

- **init(it,sm):**
Pre: $sm \in SM$
Post: $it \in IteratorSM$
- **getCurrent(it,elem)**
Pre: $it \in IteratorSM$, the iterator is valid
Post: $elem \in SMElement$
- **next(it)**
Pre: $it \in IteratorSM$, the iterator is valid
Post: goes to the next element
- **valid(it)**
Pre: $it \in IteratorSM$
*Post: checks **if** the current element is valid*

Representation:

IteratorSM:

sm: Sparse Matrix

pos: integer

findMin:TFunction

ADT Sparse Matrix – Operation implementation

subalgorithm init (sm, nrL, nrC) is :

```
sm.m ← 10
@create an array empty with m positions in sm.elems
for i ← 1, m, 1 execute:
    sm.elems[i] ← NIL
end-for
sm.nrL ← nrL
sm.nrC ← nrC
```

Complexity: $\theta(m)$, is the size of the array

function nrLines(sm) is:

```
nrLines ← sm.nrL
```

Complexity: $\theta(1)$

function nrColumns(sm) is:

```
nrColumns ← sm.nrC
```

Complexity: $\theta(1)$

function element (sm, i, j) is:

```
if i < sm.nrL or i > sm.nrL or j < sm.nrC or j > sm.nrC then
    @throw exception
else
    key ← sm.hc(i, j)
    position ← sm.h(key)
    current ← sm.elems[position]
    while currentN ≠ NIL and not ([currentN].line = i and [currentN].column = j) execute
        current ← [currentN].next
    end-while
    if currentN ≠ NIL then
        element ← [currentN].info
    else
        element ← 0
    end-if
end-if
```

Complexity:

Best Case: the current element is NIL $\Rightarrow \theta(1)$

Worst Case: the element is on the last position of the SLL
 $\Rightarrow \theta(n)$, n is the number of elements in the SLL

Average Case: $O(n)$

Final Complexity: $O(n)$

subalgorithm modify(sm, i, j, val) is:

if $i < \text{sm.nrL}$ **or** $i > \text{sm.nrL}$ **or** $j < \text{sm.nrC}$ **or** $j > \text{sm.nrC}$ **then**

@throw exception

end-if

if $\text{sm.element}(i, j) = 0$ **then**

if $\text{val} \neq 0$ **then**

$\text{modifyZN}(\text{sm}, i, j, \text{val})$

end-if

else

if $\text{val} = 0$ **then**

$\text{modifyNZ}(\text{sm}, i, j, \text{val})$

end-if

end-if

Complexity: $O(n)$

subalgorithm modifyNZ(sm, l, j, val) is:

$\text{key} \leftarrow \text{sm.hc}(l, j)$

$\text{position} \leftarrow \text{sm.h}(\text{key})$

$\text{current} \leftarrow \text{sm.elems}[\text{position}]$

while $[\text{current}].\text{next} \neq \text{NIL}$ **and not**

$([[\text{current}].\text{next}].\text{info.line} = i \text{ and } [[\text{current}].\text{next}].\text{info.column} = j)$ **execute:**

$\text{current} \leftarrow [\text{current}].\text{next}$

end-while

if $[\text{current}].\text{next} = \text{NIL}$ **then**

$\text{deallocate}(\text{sm.elems}[\text{position}])$

$\text{sm.elems}[\text{position}] \leftarrow \text{NIL}$

else

$\text{node} \leftarrow [\text{current}].\text{next}$

$[\text{current}].\text{next} \leftarrow [[\text{current}].\text{next}].\text{next}$

$\text{deallocate}(\text{node})$

end-if

Complexity: $O(n)$, n is the number of elements in the SLL

subalgorithm modifyZN(sm, l, j, val) is:

$\text{key} \leftarrow \text{sm.hc}(l, j)$

$\text{position} \leftarrow \text{sm.h}(\text{key})$

@create new SMLElement in elem

$[\text{elem}].\text{info.line} \leftarrow i$; $[\text{elem}].\text{info.column} \leftarrow j$; $[\text{elem}].\text{info.val} \leftarrow \text{val}$; $[\text{elem}].\text{next} \leftarrow \text{NIL}$

if $\text{sm.elems}[\text{position}] = \text{NIL}$ **then**

$\text{sm.elems}[\text{position}] \leftarrow \text{elem}$

else

$\text{current} \leftarrow \text{sm.elems}[\text{position}]$

while $[\text{current}].\text{next} \neq \text{NIL}$ **and not** $([[\text{current}].\text{next}].\text{info.line} > [\text{elem}].\text{info.line} \text{ or } [[\text{current}].\text{next}].\text{info.line} = [\text{elem}].\text{info.line})$ **execute:**

$\text{current} \leftarrow [\text{current}].\text{next}$

end-while

if [current].next = NIL

if current=sm.elems[position]

if [current].next.info.line = i > [elem].info.line **then**

 [current].next←elem

else

if [current].next.info.line = i < [elem].info.line **then**

 [elem].next←current

 Sm.elems[position]<-elem

else

if [current].next.info.column = i < [elem].info.column **then**

 [current].next←elem

else

 [elem].next←current

 Sm.elems[position]←elem

end-if

end-if

else

if [current].info.line < [elem].info.line

 [current].next←elem

end-if

end-if

else

if [[current].next].info.line > [elem].info.line

 [elem].next←[current].next

 [elem].next←elem

else

if [[current].next].info.line = [elem].info.line **then**

if [[current].next].info.column > [elem].info.column **then**

 [elem].next←[current].next

 [elem].next←elem

else

 [elem].next←[[current].next].next

 [[current].next].next←elem

end-if

end-if

end-if

Complexity: $O(n)$, n is the number of elements in the SLL

subalgorithm destroy(sm) is:

```
for i←1,m,1 execute
    currentN←elems[i]
    while currentN≠ NIL execute:
        nextNode←[currentN].next
        free(currentN)
        currentN←nextNode
    end-while
end-for
```

Complexity: $O(m+e)$, e is the number of elements in the Sparse Matrix

subalgorithm iterator(sm,i) is:

```
i←IteratorSM(sm)
```

Complexity: $\theta(1)$

Iterator SM – Operation Implementation

subalgorithm init(sm,i) is:

```
@create a new sparse matrix in matrix with sm's attributes
for i←1,sm.m,1 execute:
    Nod←sm.elems[i]
    While nod ≠ NIL execute
        @create a new SMLElement in elem
        matrix.modify( [nod].info.line, [nod].info.column, [nod].next)
        Nod←[nod].next
    end-while
i.sm←matrix
Pos←-i.findMin()
```

Complexity: $O(m+e)$, e is the number of elements in the Sparse Matrix

function findMin() is:

```
minLin ←maxInt
minCol ←maxInt
iminPos← -1
for k←1,sm.m, execute
    nod←i.sm.elems[k]
    if nod ≠NIL
        if [nod].info.line <minLin
            minL=[nod].info.line
            minPos←k
        else
            if [nod].info.line = minLin and [nod].info.column < minCol then
```

```

                                minCol ← [nod].info.column
                                minPos ← k
                        end-if
                end-if
        end-for
    findMin ← minPos
Complexity:  $\theta(m)$ 

subalgorithm next(i) is:
    i.sm.elems[pos] ← [i.sm.elems[pos]].next
    pos ← i.findMin(i)
Complexity:  $\theta(m)$ 

function valid(i) is:
    if pod = -1
        valid ← false
    else
        valid ← true
    Complexity:  $\theta(1)$ 

subalgorithm destroy(i) is:
    i.sm.destroy()
Complexity:  $O(m+e)$ ,  $e$  is the number of elements in the Sparse Matrix

```

Problem statement:

We are given the adjacency matrix for a directed graph with n vertices and m edges, where $n \in [10, 999]$ and $m \in (0, 30]$, n and m are natural numbers. Find the outbound of a given vertex v .

The vertices are denoted by integer numbers from 0 to $n-1$.

Justification for using ADT Sparse Matrix:

Considering the fact that we are given a fairly large number of vertices, but a small number of edges, the adjacency matrix will contain a large number of 0 and a small number of 1. In order to save memory / space, we only memorize the positions (i, j) on which the value is 1.

Solution

function getData() is:

```
@open file
@read in nrL and nrC the number of lines and column of the matrix
init(matrix,nrL,nrC)
for i←1,nrL,1 execute:
    @read a line in string mline
    for j←1,nrC,1 execute:
        modify(matrix,i,j,mline[j])
getData←matrix
```

Complexity: $O(nrL * nrC)$

subalgorithm solution() is:

```
@create an array of 100 positions in arr
@read a number from the console in nr
matrix←getData()
post=0
lter←sm.iterator()
while [lter].valid() execute
    Elem←[lter].current
    If element.line = nr
        Pos←pos+1
        Arr[pos]←elem.column
    end-if
    [lter].next
end-while
If pos=0
    @print "No outbound"
else
    @print the content of arr
```

Complexity: $O(m * e)$, e is the number of elements in the Sparse Matrix

Tests for the ADT

```
void testSM()
{
    SM matrix(1000, 1100);
    assert(matrix.getNrL() == 1000);
    assert(matrix.getNrC() == 1100);
    assert(matrix.element(2, 2) == 0);
    matrix.modify(1, 5, 5);
    assert(matrix.element(1, 5) == 5);
    matrix.modify(2, 4, 8);
    assert(matrix.element(2, 4) == 8);
    matrix.modify(2, 2, 22);
    assert(matrix.element(2, 2) == 22);
    matrix.modify(0, 4, 80);
    assert(matrix.element(0, 4) == 80);
    matrix.modify(1, 3, 222);
    assert(matrix.element(1, 3) == 222);
    matrix.modify(2, 103, 28);
    assert(matrix.element(2, 103) == 28);
    matrix.modify(2, 3, 18);
    assert(matrix.element(2, 3) == 18);
    matrix.modify(4, 3, 28);
    assert(matrix.element(2, 103) == 28);
    matrix.modify(4, 103, 18);
    assert(matrix.element(2, 3) == 18);
    matrix.modify(1, 1, 88);
    assert(matrix.element(1, 1) == 88);
    matrix.modify(1, 1, 0);
    assert(matrix.element(1, 1) == 0);
    matrix.modify(0, 8, 55);
    matrix.modify(1, 7, 8);
    matrix.modify(7, 1, 8);
    matrix.modify(2, 6, 8);
    matrix.modify(3, 5, 333);
    matrix.modify(2, 6, 0);

    assert(matrix.element(2, 6) == 0);
    matrix.modify(3, 5, 0);
    assert(matrix.element(3, 5) == 0);
    matrix.modify(0, 8, 55);
    matrix.modify(2, 6, 8);
    matrix.modify(2, 6, 0);
    assert(matrix.element(2, 6) == 0);
    matrix.modify(7, 2, 2);
    matrix.modify(8, 1, 22);
    matrix.modify(6, 3, 555);
    matrix.modify(0, 7, 44);
    matrix.modify(0, 17, 66);
    matrix.modify(0, 87, 66);
    matrix.modify(0, 97, 66);
    matrix.modify(0, 27, 66);
    matrix.modify(0, 37, 66);

    matrix.modify(0, 5, 222);
    matrix.modify(0, 15, 222);
    matrix.modify(0, 25, 222);
    matrix.modify(0, 45, 222);
    matrix.modify(0, 35, 222);

    IteratorSM* iter = matrix.iterator();
    while (iter->valid())
    {
        iter->next();
    }
    iter->destroy();
    delete iter;
    matrix.destroy();
}
```

Tests for the Iterator

```
void testIT()
{
    SM matrix{ 1000,1000 };
    matrix.modify(1, 1, 22);
    matrix.modify(0, 2, 44);
    matrix.modify(3, 103, 9);
    matrix.modify(3, 3, 88);
    IteratorSM iter(matrix);
    while (iter.valid())
    {
        iter.next();
    }
}
```