DATA STRUCTURES AND ALGORITHMS
LECTURE 3

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In Lecture 2...

- Algorithm Analysis
- Dynamic Array
- Iterator

Today

1. Iterators
2. Binary Heap
3. Linked Lists
   - Singly Linked Lists

Iterator

- An *iterator* is a structure that is used to iterate through the elements of a container.

- Containers can be represented in different ways, using different data structures. Iterators are used to offer a common and generic way of moving through all the elements of a container, independently of the representation of the container.

- Every container that can be iterated, has to contain in the interface an operation called *iterator* that will create and return an iterator over the container.

Iterator

- An iterator usually contains:
  - a reference to the container it iterates over
  - a reference to a *current element* from the container
- Iterating through the elements of the container means actually moving this *current element* from one element to another until the iterator becomes *invalid*
- The exact way of representing the *current element* from the iterator depends on the data structure used for the implementation of the container. If the representation/ implementation of the container changes, we need to change the representation/ implementation of the iterator as well.

Iterator - Interface I

- **Domain** of an Iterator

$\mathcal{I} = \{$**it**$|$it is an iterator over a container with elements of type TElem $\}$

Iterator - Interface II

- **Interface** of an Iterator:

Iterator - Interface III

- init(it, c)
  - **description:** creates a new iterator for a container
  - **pre:** c is a container
  - **post:** $it \in \mathcal{I}$ and it points to the first element in c if c is not empty or it is not valid

Iterator - Interface IV

- getCurrent(it, e)
  - **description:** returns the current element from the iterator
  - **pre:** $it \in \mathcal{I}$, it is valid
  - **post:** $e \in TElem$, e is the current element from it

Iterator - Interface V

- next(it)
  - **description:** moves the current element from the container to the next element or makes the iterator invalid if no elements are left
  - **pre:** $it \in \mathcal{I}$, it is valid
  - **post:** the current element from *it* points to the next element from the container

Iterator - Interface VI

- valid(it)
  - **description:** verifies if the iterator is valid
  - **pre:** $it \in \mathcal{I}$
  - **post:**

$$valid \leftarrow \begin{cases} True, & \text{if it points to a valid element from the container} \\ False & \text{otherwise} \end{cases}$$

Types of iterators I

- The interface presented above describes the simplest iterator: *unidirectional* and *read-only*
- A *unidirectional* iterator can be used to iterate through a container in one direction only (usually *forward*, but we can define a *reverse* iterator as well).
- A *bidirectional* iterator can be used to iterate in both directions. Besides the *next* operation it has an operation called *previous*.

Types of iterators II

- A *random access* iterator can be used to move multiple steps (not just one step forward or one step backward).
- A *read-only* iterator can be used to iterate through the container, but cannot be used to change it.
- A *read-write* iterator can be used to add/delete elements to/from the container.

Using the iterator

- Since the interface of an iterator is the same, independently of the exact container or its representation, the following subalgorithm can be used to print the elements of any container.

```
subalgorithm printContainer(c) is:
//pre: c is a container
//post: the elements of c were printed
//we create an iterator using the iterator method of the container
   iterator(c, it)
   while valid(it) execute
      //get the current element from the iterator
      getCurrent(it, elem)
      print elem
      //go to the next element
      next(it)
   end-while
end-subalgorithm
```

Iterator for a Dynamic Array

- How can we define an iterator for a Dynamic Array?
- How can we represent that *current element* from the iterator?

Iterator for a Dynamic Array

- How can we define an iterator for a Dynamic Array?
- How can we represent that *current element* from the iterator?
- In case of a Dynamic Array, the simplest way to represent a *current element* is to retain the position of the *current element*.

IteratorDA:
  da: DynamicArray
  current: Integer

- Let's see how the operations of the iterator can be implemented.

Iterator for a Dynamic Array - init

- What do we need to do in the *init* operation?

Iterator for a Dynamic Array - init

- What do we need to do in the *init* operation?

subalgorithm init(it, da) *is:*
//it is an IteratorDA, da is a Dynamic Array
  it.da ← da
  it.current ← 1
end-subalgorithm

- Complexity:

Iterator for a Dynamic Array - init

- What do we need to do in the *init* operation?

subalgorithm init(it, da) *is:*
//it is an IteratorDA, da is a Dynamic Array
  it.da $\leftarrow$ da
  it.current $\leftarrow$ 1
end-subalgorithm

- Complexity: $\Theta(1)$

Iterator for a Dynamic Array - getCurrent

- What do we need to do in the *getCurrent* operation?

Iterator for a Dynamic Array - getCurrent

- What do we need to do in the *getCurrent* operation?

subalgorithm getCurrent(it, e) *is:*
  e $\leftarrow$ it.da.elems[it.current]
end-subalgorithm

- Complexity:

Iterator for a Dynamic Array - getCurrent

- What do we need to do in the *getCurrent* operation?

subalgorithm getCurrent(it, e) *is:*
  e $\leftarrow$ it.da.elems[it.current]
end-subalgorithm

- Complexity: $\Theta(1)$

Iterator for a Dynamic Array - next

- What do we need to do in the *next* operation?

Iterator for a Dynamic Array - next

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subalgorithm next(it) *is:*
    it.current ← it.current + 1
end-subalgorithm

- Complexity:

Iterator for a Dynamic Array - next

- What do we need to do in the *next* operation?

subalgorithm next(it) *is:*
  it.current $\leftarrow$ it.current + 1
end-subalgorithm

- Complexity: $\Theta(1)$

Iterator for a Dynamic Array - valid

- What do we need to do in the *valid* operation?

Iterator for a Dynamic Array - valid

- What do we need to do in the *valid* operation?

function valid(it) *is:*
  if it.current $<=$ it.da.len **then**
    valid $\leftarrow$ True
  else
    valid $\leftarrow$ False
  end-if
end-function

- Complexity:

Iterator for a Dynamic Array - valid

- What do we need to do in the *valid* operation?

function valid(it) *is:*
  if it.current <= it.da.len **then**
    valid ← True
  else
    valid ← False
  end-if
end-function

- Complexity: $\Theta(1)$

Iterator for a Dynamic Array

- We can print the content of a Dynamic Array in two ways:
  - Using an iterator (as present above for a container)
  - Using the positions (indexes) of elements

subalgorithm printDA(da) **is:**
//pre: da is a Dynamic Array
//post: the elements of da were printed
    for i $\leftarrow$ 1, size(da) **execute**
        getElement(da, i, elem)
        print elem
    end-for
end-subalgorithm

Iterator for a Dynamic Array

- In case of a Dynamic Array both printing algorithms have $\Theta(n)$ complexity
- For other data structures/containers we need iterator because
  - there are no positions in the data structure/container (for example for the Bag ADT)
  - the time complexity of iterating through all the elements is smaller

Binary Heap

- A binary heap is a data structure that can be used as an efficient representation for Priority Queues (will be discussed later).
- A binary heap is a kind of hybrid between a dynamic array and a binary tree.
- The elements of the heap are actually stored in the dynamic array, but the array is visualized as a binary tree.
- **Note:** There are several heap data structures (Binary Heap, Binominal Heap, Fibonacci Heap, etc.) in the following we are going to use the term *heap* to represent a binary heap.

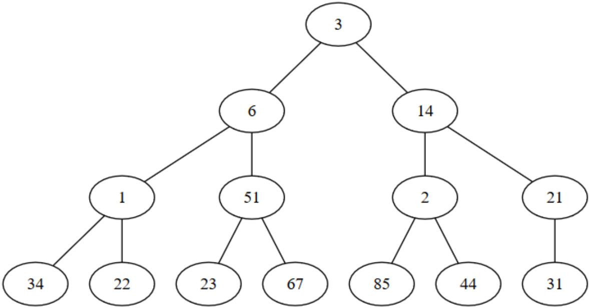
Heap

- Assume that we have the following array (upper row contains positions, lower row contains elements):

1	2	3	4	5	6	7	8	9	10	11	12	13	14
3	6	14	1	51	2	21	34	22	23	67	85	44	31

Heap

- We can visualize this array as a binary tree, in which each node has exactly 2 children, except for the last two rows, but there the children of the nodes are completed from left to right.



Heap

- If the elements of the array are: $a_1, a_2, a_3, ..., a_n$, we know that:
  - a_1 is the root of the heap
  - for an element from position i, its children are on positions $2 * i$ and $2 * i + 1$ (if $2 * i$ and $2 * i + 1$ is less than or equal to n)
  - for an element from position i (i $>$ 1), the parent of the element is on position $[i/2]$ (integer part of i/2)

Heap

- A *binary heap* is an array that can be visualized as a binary tree having a *heap structure* and a *heap property*.
  - *Heap structure:* in the binary tree every node has exactly 2 children, except for the last two levels, where children are completed from left to right.
  - *Heap property:* $a_i \geq a_{2*i}$ (if $2*i \leq n$) and $a_i \geq a_{2*i+1}$ (if $2*i+1 \leq n$)
  - The $\geq$ relation between a node and both its descendants can be generalized (other relations can be used as well).

Heap - Notes

- If we use the $\geq$ relation, we will have a *MAX-HEAP*.
- If we use the $\leq$ relation, we will have a *MIN-HEAP*.
- The height of a heap with n elements is $log_2 n$, so the operations performed on the heap have $O(log_2 n)$ complexity.

Heap - operations

- A heap can be used as representation for a Priority Queue and it has two specific operations:
    - add a new element in the heap (in such a way that we keep both the heap structure and the heap property).
    - remove (we always remove the root of the heap - no other element can be removed).

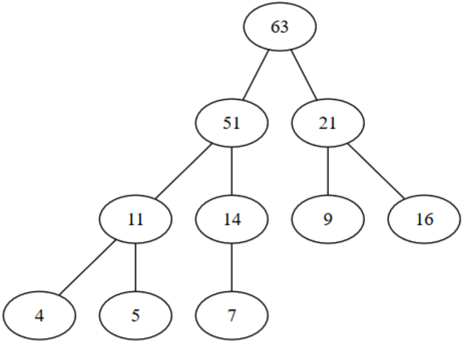
Heap - representation

Heap:
cap: Integer
len: Integer
elems: TElem[]

- For the implementation we will assume that we have a MAX-HEAP.

Heap - Add - example

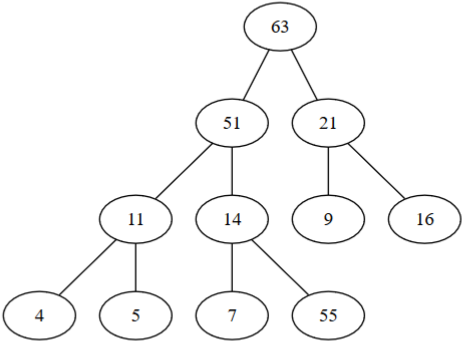
- Consider the following (MAX) heap:



- Let's add the number 55 to the heap.

Heap - Add - example

- In order to keep the *heap structure*, we will add the new node as the right child of the node 14 (and as the last element of the array in which the elements are kept).

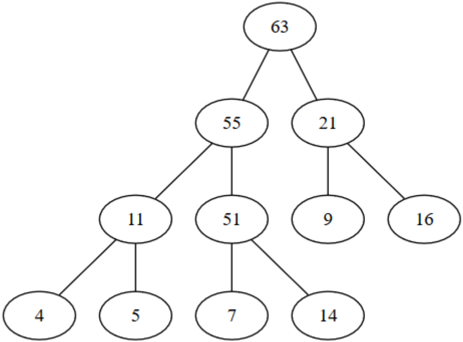


Heap - Add - example

- *Heap property* is not kept: 14 has as child node 55 (since it is a MAX-heap, each node has to be greater than or equal to its descendants).
- In order to restore the heap property, we will start a *bubble-up* process: we will keep swapping the value of the new node with the value of its parent node, until it gets to its final place. No other node from the heap is changed.

Heap - Add - example

- When *bubble-up* ends:



Heap - add

subalgorithm add(heap, e) **is:**
//heap - a heap
//e - the element to be added
  if heap.len = heap.cap **then**
    @ resize
  end-if
  heap.elems[heap.len+1] ← e
  heap.len ← heap.len + 1
  bubble-up(heap, heap.len)
end-subalgorithm

Heap - add

```
subalgorithm bubble-up (heap, p) is:
//heap - a heap
//p - position from which we bubble the new node up
   poz ← p
   elem ← heap.elems[p]
   parent ← p / 2
   while poz > 1 and elem > heap.elems[parent] execute
      //move parent down
      heap.elems[poz] ← heap.elems[parent]
      poz ← parent
      parent ← poz / 2
   end-while
   heap.elems[poz] ← elem
end-subalgorithm
```

- Complexity:

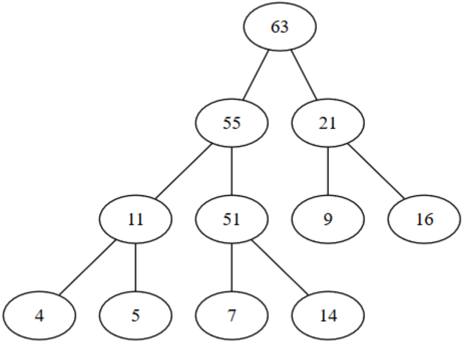
Heap - add

subalgorithm bubble-up (heap, p) **is:**
//heap - a heap
//p - position from which we bubble the new node up
    poz ← p
    elem ← heap.elems[p]
    parent ← p / 2
    while poz > 1 **and** elem > heap.elems[parent] **execute**
        //move parent down
        heap.elems[poz] ← heap.elems[parent]
        poz ← parent
        parent ← poz / 2
    end-while
    heap.elems[poz] ← elem
end-subalgorithm

- Complexity: $O(log_2 n)$
- Can you give an example when the complexity of the algorithm is less than $log_2 n$ (best case scenario)?

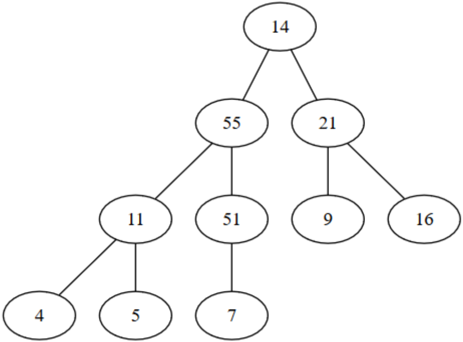
Heap - Remove - example

- From a heap we can only remove the root element.



Heap - Remove - example

- In order to keep the *heap structure*, when we remove the root, we are going to move the last element from the array to be the root.

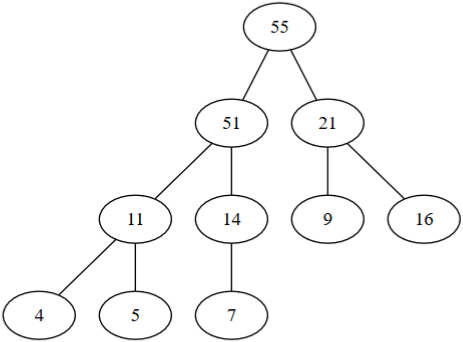


Heap - Remove - example

- *Heap property* is not kept: the root is no longer the maximum element.
- In order to restore the heap property, we will start a *bubble-down* process, where the new node will be swapped with its maximum child, until it becomes a leaf, or until it will be greater than both children.

Heap - Remove - example

- When the bubble-down process ends:



Heap - remove

```
function remove(heap, deletedElem) is:
//heap - is a heap
//deletedElem - is an output parameter, the removed element
   if heap.len = 0 then
      @ error - empty heap
    end-if
   deletedElem ← heap.elems[1]
   heap.elems[1] ← heap.elems[heap.len]
   heap.len ← heap.len - 1
   bubble-down(heap, 1)
   remove ← deletedElem
end-function
```

Heap - remove

```
subalgorithm bubble-down(heap, p) is:
//heap - is a heap
//p - position from which we move down the element
    poz ← p
    elem ← heap.elems[p]
    while poz ≤ heap.len execute
        maxChild ← -1
        if poz * 2 ≤ heap.len then
        //it has a left child, assume it is the maximum
            maxChild ← poz*2
        end-if
        if poz*2+1 ≤ heap.len and heap.elems[2*poz+1]>heap.elems[2*poz] th
        //it has two children and the right is greater
            maxChild ← poz*2 + 1
        end-if
//continued on the next slide...
```

Heap - remove

```
        if maxChild ≠ -1 and heap.elems[maxChild] > elem then
            heap.elems[poz] ← heap.elems[maxChild]
            poz ← maxChild
        else
            heap.elems[poz] ← elem
            poz ← heap.len + 1
            //to stop the while loop
        end-if
    end-while
end-subalgorithm
```

- Complexity:

Heap - remove

```
        if maxChild ≠ -1 and heap.elems[maxChild] > elem then
            heap.elems[poz] ← heap.elems[maxChild]
            poz ← maxChild
        else
            heap.elems[poz] ← elem
            poz ← heap.len + 1
            //to stop the while loop
        end-if
    end-while
end-subalgorithm
```

- Complexity: $O(log_2 n)$
- Can you give an example when the complexity of the algorithm is less than $log_2 n$ (best case scenario)?

Questions

- In a max-heap where can we find the:
  - maximum element of the array?

Questions

- In a max-heap where can we find the:
  - maximum element of the array?
  - minimum element of the array?

Questions

- In a max-heap where can we find the:
  - maximum element of the array?
  - minimum element of the array?
- Assume you have a MAX-HEAP and you need to add an operation that returns the minimum element of the heap. How would you implements this operation, using constant time and space? (Note: we only want to return the minimum, we do not want to be able to remove it).

Think about it

- How could you find the k^{th} smallest element from an array with n elements?
  - with complexity $O(n * log_2 k)$
- How could you find the k^{th} smallest element from a min-heap with n elements?
  - with complexity $O(k * log_2 n)$
  - with complexity $O(k * log_2 k)$ - assume you have access to the representation of the min-heap (you can access elements of the array based on their positions)

Heap-sort

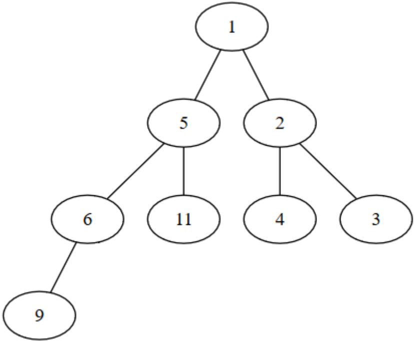
- There is a sorting algorithm, called *Heap-sort*, that is based on the use of a heap.
- In the following we are going to assume that we want to sort a sequence in ascending order.
- Let's sort the following sequence: [6, 1, 3, 9, 11, 4, 2, 5]

Heap-sort - Naive approach

- Based on what we know so far, we can guess how heap-sort works:
    - Build a min-heap adding elements one-by-one to it.
    - Start removing elements from the min-heap: they will be removed in the sorted order.

Heap-sort - Naive approach

- The heap when all the elements were added:



- When we remove the elements one-by-one we will have: 1, 2, 3, 4, 5, 6, 9, 11.

Heap-sort - Naive approach

- What is the time complexity of the heap-sort algorithm described above?

Heap-sort - Naive approach

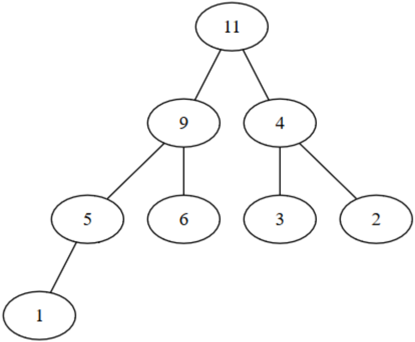
- What is the time complexity of the heap-sort algorithm described above?
- The time complexity of the algorithm is $O(nlog_2 n)$
- What is the extra space complexity of the heap-sort algorithm described above (do we need an extra array)?

Heap-sort - Naive approach

- What is the time complexity of the heap-sort algorithm described above?
- The time complexity of the algorithm is $O(nlog_2 n)$
- What is the extra space complexity of the heap-sort algorithm described above (do we need an extra array)?
- The extra space complexity of the algorithm is $\Theta(n)$ - we need an extra array.

Heap-sort - Better approach

- If instead of building a min-heap, we build a max-heap (even if we want to do ascending sorting), we do not need the extra array.



Heap-sort - Better approach

- We can improve the time complexity of building the heap as well.

Heap-sort - Better approach

- We can improve the time complexity of building the heap as well.
  - If we have an unsorted array, we can transform it easier into a heap: the second half of the array will contain leaves, they can be left where they are.
  - Starting from the first non-leaf element (and going towards the beginning of the array), we will just call *bubble-down* for every element.
  - Time complexity of this approach: $O(n)$ (but removing the elements from the heap is still $O(nlog_2 n)$)

Linked Lists

- A *linked list* is a linear data structure, but the order of the elements is determined not by indexes, but by a pointer which is placed in each element.

- A linked list is a structure that consists of *nodes* (sometimes called *links*) and each node contains, besides the data (that we store in the linked list), a pointer to the address of the next node (and possibly a pointer to the address of the previous node).

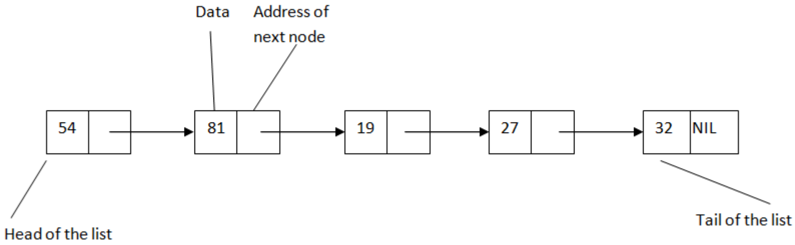
- The nodes of a linked list are not necessarily adjacent in the memory, this is why we need to keep the address of the successor in each node.

Linked Lists

- Elements from a linked list are accessed based on the pointers stored in the nodes.
- We can directly access only the first element (and maybe the last one) of the list.

Linked Lists

- Example of a linked list with 5 nodes:



Singly Linked Lists - SLL

- The linked list from the previous slide is actually a *singly linked list - SLL*.
- In a SLL each node from the list contains the data and the address of the next node.
- The first node of the list is called *head* of the list and the last node is called *tail* of the list.
- The tail of the list contains the special value *NIL* as the address of the next node (which does not exist).
- If the head of the SLL is *NIL*, the list is considered empty.

Singly Linked Lists - Representation

- For the representation of a SLL we need two structures: one structure for the node and one for the list itself.

SLLNode:
  info: TElem *//the actual information*
  next: $\uparrow$ SLLNode *//address of the next node*

Singly Linked Lists - Representation

- For the representation of a SLL we need two structures: one structure for the node and one for the list itself.

<u>SLLNode:</u>
info: TElem *//the actual information*
next: ↑ SLLNode *//address of the next node*

<u>SLL:</u>
head: ↑ SLLNode *//address of the first node*

- Usually, for a SLL, we only memorize the address of the head. However, there might be situations when we memorize the address of the tail as well (if the application requires it).

SLL - Operations

- Possible operations for a singly linked list:
  - search for an element with a given value
  - add an element (to the beginning, to the end, to a given position, after a given value)
  - delete an element (from the beginning, from the end, from a given position, with a given value)
  - get an element from a position
- These are *possible* operations; usually we need only part of them, depending on the container that we implement using a SLL.