Project 1 (0.2 points - for the first 5 solutions)

- *Input*: non-zero natural number n
- *Output*:
  1. the number of associative operations on a set $A = \{a_1, \dots, a_n\}$
  2. the operation table of each associative operation (for $n \leq 4$)

Send solutions by e-mail: commented source code together with relevant input and output files (at least all cases $n \leq 4$ and a couple of other cases).

Example:

- *Input*: $n = 2$
- *Output*:
  1. the number of associative operations on a set $A = \{a_1, a_2\}$ is 8
  2. identifying an operation table $\begin{array}{c|cc} & a_1 & a_2 \\ \hline a_1 & x & y \\ a_2 & z & t \end{array}$ by the matrix $\begin{pmatrix} x & y \\ z & t \end{pmatrix} \in M_2(A)$, the operation tables of the associative operations on $A = \{a_1, a_2\}$ are given by the matrices:

$$\begin{pmatrix} a_1 & a_1 \\ a_1 & a_1 \end{pmatrix}, \begin{pmatrix} a_1 & a_1 \\ a_1 & a_2 \end{pmatrix}, \begin{pmatrix} a_1 & a_1 \\ a_2 & a_2 \end{pmatrix}, \begin{pmatrix} a_1 & a_2 \\ a_1 & a_2 \end{pmatrix}, \begin{pmatrix} a_1 & a_2 \\ a_2 & a_1 \end{pmatrix}, \begin{pmatrix} a_1 & a_2 \\ a_2 & a_2 \end{pmatrix}, \begin{pmatrix} a_2 & a_1 \\ a_1 & a_2 \end{pmatrix}, \begin{pmatrix} a_2 & a_2 \\ a_2 & a_2 \end{pmatrix}.$$

Project 2 (0.2 points - for the first 5 solutions)

- *Input*: non-zero natural number n
- *Output*:
  1. the number of abelian group structures which can be defined on a set $A = \{a_1, \ldots, a_n\}$
  2. the operation table of each such abelian group (for $n \leq 7$)

Send solutions by e-mail: commented source code together with relevant input and output files (at least all cases $n \leq 7$ and a couple of other cases).

Example: The operation table of a group G has the property that each element of G appears exactly once on each row and on each column. The operation table of an abelian group is symmetric with respect to the main diagonal. We may identify an operation table by a matrix.

- *Input*: $n = 4$
- *Output*:
  1. the number of abelian group structures on a set $G = \{a_1, a_2, a_3, a_4\}$ is 16
  2. the abelian group structures on G with identity element a_1 are given by the matrices:

$$\begin{pmatrix} \mathbf{a_1} & \mathbf{a_2} & \mathbf{a_3} & \mathbf{a_4} \\ \mathbf{a_2} & \boxed{a_1} & a_4 & a_3 \\ \mathbf{a_3} & a_4 & a_1 & a_2 \\ \mathbf{a_4} & a_3 & a_2 & a_1 \end{pmatrix}, \begin{pmatrix} \mathbf{a_1} & \mathbf{a_2} & \mathbf{a_3} & \mathbf{a_4} \\ \mathbf{a_2} & \boxed{a_1} & a_4 & a_3 \\ \mathbf{a_3} & a_4 & a_2 & a_1 \\ \mathbf{a_4} & a_3 & a_1 & a_2 \end{pmatrix}, \begin{pmatrix} \mathbf{a_1} & \mathbf{a_2} & \mathbf{a_3} & \mathbf{a_4} \\ \mathbf{a_2} & \boxed{a_3} & a_4 & a_1 \\ \mathbf{a_3} & a_4 & a_1 & a_2 \\ \mathbf{a_4} & a_1 & a_2 & a_3 \end{pmatrix}, \begin{pmatrix} \mathbf{a_1} & \mathbf{a_2} & \mathbf{a_3} & \mathbf{a_4} \\ \mathbf{a_2} & \boxed{a_4} & a_1 & a_3 \\ \mathbf{a_3} & a_1 & a_4 & a_2 \\ \mathbf{a_4} & a_3 & a_2 & a_1 \end{pmatrix}.$$

There are 4 similar abelian group structures for each possible identity element.

Project 3 (0.2 points - for the first 5 solutions)

- *Input*: natural numbers $m, n \geq 2$
- *Output*:
  1. the number of subgroups of the abelian group $(\mathbb{Z}_m \times \mathbb{Z}_n, +)$
  2. the subgroups of the abelian group $(\mathbb{Z}_m \times \mathbb{Z}_n, +)$ (for $2 \leq m, n \leq 10$)

Send solutions by e-mail: commented source code together with relevant input and output files (at least all cases $m, n \leq 6$ and a couple of other cases).

Example: For a finite group $(G, +)$, a non-empty subset H of G is a subgroup of $(G, +)$ if and only if H is a stable subset of $(G, +)$.

- *Input*: $m = 2$, $n = 4$
- *Output*:
  1. the number of subgroups of the abelian group $(\mathbb{Z}_2 \times \mathbb{Z}_4, +)$ is 8
  2. the subgroups of the abelian group $(\mathbb{Z}_2 \times \mathbb{Z}_4, +)$ are:

$H_1 = \{(0,0)\}$
$H_2 = \{(0,0),(1,0)\}$
$H_3 = \{(0,0),(0,2)\}$
$H_4 = \{(0,0),(1,2))\}$

$H_5 = \{(0,0),(0,1),(0,2),(0,3)\}$
$H_6 = \{(0,0),(1,0),(0,2),(1,2)\}$
$H_7 = \{(0,0),(1,1),(0,2),(1,3)\}$
$H_8 = \mathbb{Z}_2 \times \mathbb{Z}_4$