

$$\begin{cases} -x \leq 4 - \frac{2}{3}x \\ -x \leq 4 + \frac{2}{3}x \\ 1 \leq 4 - \frac{2}{3}x \\ 1 \leq 4 + \frac{2}{3}x \end{cases} \Leftrightarrow \begin{cases} -\frac{1}{3}x \leq 4 \\ -\frac{5}{3}x \leq 4 \\ \frac{2}{3}x \leq 3 \\ -\frac{2}{3}x \leq 3 \end{cases} \Leftrightarrow \begin{cases} x \geq -12 \\ x \geq -\frac{12}{5} \\ x \leq \frac{9}{2} \\ x \geq -\frac{9}{2} \\ x \leq 2 \end{cases}$$

$$x \in \left[-\frac{12}{5}, 2\right]$$

37. What are the Voronoi cells of  $\{(m, m) : m, m \in \mathbb{N}\} \subseteq \mathbb{R}^2$

### SEMINAR 7

18.04.2018.

$$a = (1 \ 0 \ 0 \ 20)$$

$$b = (0 \ 2 \ 0 \ 0 \ 1)$$

$$c = (1 \ 2 \ 0 \ 0 \ 0)$$

$$d = (0 \ 0 \ 0 \ 2 \ 1)$$

$$A = a + \langle b, c \rangle \quad \mathbb{R}^5$$

$$B = c + \langle b, d \rangle$$

$$\text{aff}(A \cap B) = A \cap B$$

$$A \ni P_A(t, s) = a + tb + sc \quad s, t \in \mathbb{R}$$

$$B \ni P_B(p, q) = c + pb + qd$$

$$P_A(t, s) = P_B(p, q)$$

$$P_A(t, 0) = P_B(t-1, 1)$$

$$\Rightarrow a + tb + sc = c + pb + qd$$

$$a + \underbrace{(t-p)}_x b + \underbrace{(s-1)}_y c - \underbrace{q}_z d = 0$$

$$\begin{cases} 1+y=0 \Rightarrow y=-1 \\ 2x+2y=0 \Rightarrow x=1 \\ 0=0 \\ 2+2z=0 \Rightarrow 2z=-2 \Rightarrow z=-1 \\ x+z=0 \Rightarrow z=-x=-1 \end{cases}$$

$$\begin{cases} t-p=1 \Rightarrow t=1+p \\ s-1=-1 \Rightarrow s=0 \\ -z=-1 \Rightarrow z=1 \end{cases}$$

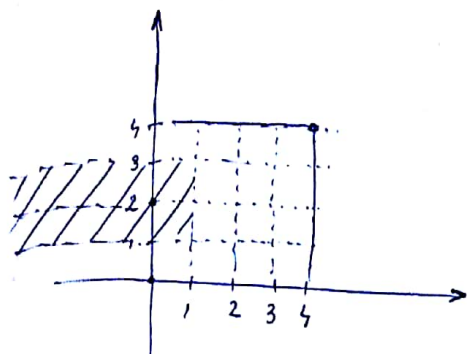
$$A \cap B = a + \langle b \rangle$$

$$= c + \{(t-1)b + d; t\}$$

$$\text{aff}(A \cup B) = a + \langle b, c, d, c-a \rangle$$

38)  $M = \{(2m, 2m) : m, m \in \mathbb{N}\} \subseteq \mathbb{R}^2$

determine the Voronoi cells of  $\begin{cases} M \\ MU(C_{i,j}) + M \end{cases}$



$f(2, 2)$

$C_{(m,m)} = [m-1, m+1] \times [m-1, m+1]$

$C_{(0,m)} = (-\infty, m+1] \times [m-1, m+1]$

$C_{(0,0)} = (-\infty, 1] \times (-\infty, 1]$

$C_{(2m, 2m)} = \begin{cases} x-y+C_1 \leq 0 \\ x-y+(-2m+1-2m) \geq 0 \\ x+y+(-2m-2m-1) \leq 0 \\ x+y+(-2m-2m+1) \geq 0 \end{cases}$

$2m+1+2m+C_1=0$

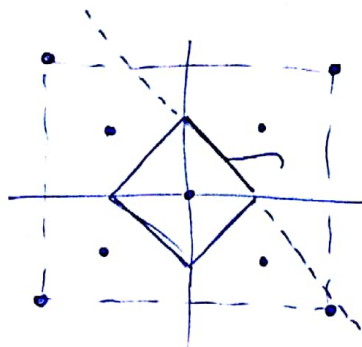
$C_1 = -2m-2m-1$

$x-y-2m+2m-1 \leq 0$

$2m-2m-2m+2m-1 \leq 0$

$-1 \leq 0$

$2m+2m-2m-2m-1 \leq 0$



39)

40)  $f: X \rightarrow X$   $X$  aff /  $k$  char  $k \neq 2$

1. If  $f$  fixes  $A$  and  $B \Rightarrow f$  fixes the line  $AB$
2.  $f$  (midpoint of segment  $S$ ) = midpoint of  $f(S)$
3.  $f$  (parallelogram) is a parallelogram
4.  $f$  (plane) is a plane
5.  $L_1 \parallel L_2 \subseteq X$   $f(L_1) \parallel f(L_2)$   
lines

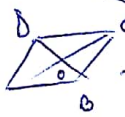
1.  $f(\text{Box}(A; B; (1-t), t)) = \text{Box}(f(A), f(B); 1-t, t)$

2.  $[A, B]$

$t = \frac{1}{2}$



3.



$f$  (line) is a line

2.  $\Rightarrow f(0)$  and  $f(1)$  and  $f(2)$  and  $f(3)$

$$1 = \mu_1 + \mu_2 + \mu_3$$

$$\text{Bar}(\underbrace{f(A), f(B), f(C)}_{\text{f trans?}}, \mu_1, \mu_2, \mu_3)$$

f trans?

so these are in general position