MIE decembrie 2017
Analiza 1, Anca Grad

Partial ~~AG~~ Analiză

Ex 1

a) $A = [\sqrt{9}, 9) \cap \mathbb{N}$; $(-\infty, 10] \cup (20, 30)$; $\mathbb{R} \setminus \mathbb{Q}$

$\min A$, $\max A$, $\min A$, $\max A$, $\inf A$, $\sup A$

b) $A = [\sqrt{9}, 9) \cap \mathbb{N}$; $(-\infty, 10] \cup (20, 30)$; $\mathbb{Q}$

$\operatorname{int} A$, $\operatorname{ext} A$, $\operatorname{bd} A$ (border), ~~$\operatorname{Izo} A$~~, $\operatorname{cl} A$, A'

c) prove $\operatorname{ext} A$ of $(-\infty, 10] \cup (20, 30)$

d) $A = (-1; 8]$; $(-2, 2)$; $[-3, 10]$; $\mathbb{R} \setminus \{-1\}$; $\mathbb{R} \setminus \mathbb{N}$; $\mathbb{R}$

$A \in v(\infty)$?; is closed?

e) prove $\mathbb{R} \setminus \mathbb{N} \in v(\infty)$?, closed?

Ex 2 a) theorem concerning bounded and convergent sequence

b) + proof

c) $\lim\limits_{n \to \infty} \dfrac{1^4 + 2^5 + \ldots + n^4}{n^5}$

Ex 3 a) Cauchy's Condensation criterion

b) necessity of a) "$\Longrightarrow$"

c) $\sum\limits_{n \geq 3} \dfrac{3^{n-2} + (-4)^{n+3}}{5^{n+1}}$ și $\sum\limits_{n \geq 2} \dfrac{\sqrt{n} - \sqrt{n+2}}{\sqrt{n(n+1)(n+2)}}$

d) study nature (c. or d.)

$\sum\limits_{n \geq 1} \left(\dfrac{n^2 + n + 1}{n^2} \cdot a \right)^n$, $a > 0$ și $\sum\limits_{n \geq 1} \dfrac{n!}{2^n}$

Ex 4 a) characterisation theorem with ε and δ of continuity

b) sufficiency of a) "$\Leftarrow$"

c) study continuity $f: (-\infty, 1] \cup \{\pi\} \cup [2\pi, \infty) \to \mathbb{R}$

$$f(x) = \begin{cases} x \sin\left(x + \frac{\pi}{2}\right) & : x \in (-\infty, 0) \\ 1 & : x \in [0, 1] \cup \{\pi\} \\ \cos\left(x + \frac{1}{2}\right) & : x \in [2\pi, \infty) \cap \mathbb{Q} \\ -1 & : x \in [2\pi, \infty) \cap (\mathbb{R} \setminus \mathbb{Q}) \end{cases}$$