Differentiable Functions

Exercise 1: Determine the n-th derivative of the following functions:

a) $f : (-1, \infty) \to \mathbb{R}$ defined by $f(x) = (1 + x)^r$, with $r \in \mathbb{R}$;

b) $f : (-1, \infty) \to \mathbb{R}$ defined by$f(x) = x \cdot \ln(1 + x)$;

c) $f : (-\infty, -1) \to \mathbb{R}$ defined by $f(x) = x \cdot \ln(1 - x)$;

d) $f : (-1, 1) \to \mathbb{R}$ defined by $f(x) = \sqrt{3x + 4}$;

e) $f : (-\frac{1}{2}, \infty) \to \mathbb{R}$ defined by $f(x) = \frac{1}{\sqrt{2x+1}}$.

Exercise 2: Determine the n-th derivative of the following functions::

a) $f : \mathbb{R} \setminus \{-\frac{b}{a}\} \to \mathbb{R}$ definită prin $f(x) = \frac{1}{ax+b}$;

b) $f : \mathbb{R} \to \mathbb{R}$ definită prin $f(x) = \sin(ax + b)$;

c) $f : \mathbb{R} \to \mathbb{R}$ definită prin $f(x) = \cos(ax + b)$;

d) $f : \mathbb{R} \to \mathbb{R}$ definită prin $f(x) = e^{ax+b}$.

Exercise 3: Compute the first derivative of the functions:

a) $f : (0, \infty) \to \mathbb{R}$ definită prin $f(x) = x^x$;

b) $f : (0, \infty) \to \mathbb{R}$ definită prin $f(x) = x^{\frac{1}{x}}$;

c) $f : (0, \pi) \to \mathbb{R}$ definită prin $f(x) = \sin x^x$;

d) $f : (0, \infty) \to \mathbb{R}$ definită prin $f(x) = x^{\sin x}$;

Exercise 4: Prove that $\frac{1}{x+1} < \ln(x + 1) - \ln x < \frac{1}{x}$ forall $x > 0$.

Exercise 5:

a) Prove that for all $n \geq 2$ the following inequalities hold

$$na^{n-1} < \frac{b^n - a^n}{b - a} < nb^{n-1}$$

for all $a, b \in (0, +\infty)$ with $a < b$.

b) Prove that, for all $n \in \mathbb{N}$,

$$\left(1+\frac{1}{n}\right)^n < \left(1+\frac{1}{n+1}\right)^{n+1}, \qquad \left(1+\frac{1}{n}\right)^{n+1} > \left(1+\frac{1}{n+1}\right)^{n+2}.$$

Excercise 6:

Let the function $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = x + |x-1|$$

for all$x \in \mathbb{R}$.

a) Prove that f has first-order side derivatives at $x_0 = 1$, and compute them;

b) If the function f differentiable on the left at $x_0 = 1$? Or on the right?

c) Does the function f have a first order derivative at $x_0 = 1$?

d) Is the function f differentiable at $x_0 = 1$;