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Geometry 1 (Analytic Geometry)

Exercise Sheet 12

Exercise 1. Find the equation of the circle

(a) of diameter $[AB]$, where $A(1,2)$ and $B(-3,-1)$;

(b) of center $I(2,-3)$ and radius $R=7$;

(c) of center $I(-1,2)$ and which passes through $A(2,6)$;

(d) centered at the origin and tangent to $d : 3x - 4y + 20 = 0$;

(e) passing through $A(3,1)$ and $B(-1,3)$ and having the center on the line $d : 3x - y - 2 = 0$;

(f) determined by $A(1,1)$, $B(1,-1)$ and $C(2,0)$;

(g) tangent to both $d_1 : 2x + y - 5 = 0$ and $d_2 : 2x + y + 15 = 0$, if the tangency point with d_1 is $M(3,1)$.

Exercise 2. (a) Determine the position of the point $A(1,-2)$ relative to the circle $C : x^2 + y^2 - 8x - 4y - 5 = 0$;

(b) Find the intersection between the line $d : 7x - y + 12 = 0$ and the circle $C : (x-2)^2 + (y-1)^2 - 25 = 0$;

(c) Determine the position of the line $d : 2x - y - 3 = 0$ relative to the circle $C : x^2 + y^2 - 3x + 2y - 3 = 0$.

Exercise 3. Find the equation of

(a) the tangent line to $C : x^2 + y^2 - 5 = 0$ at the point $A(-1, 2)$;

(b) the tangent lines to $C : x^2 + y^2 + 10x - 2y + 6 = 0$, parallel to $d : 2x + y - 7 = 0$;

(c) the tangent lines to $C : x^2 + y^2 - 2x + 4y = 0$, orthogonal on $d : x - 2y + 9 = 0$.

Exercise 4. Let $C_\lambda : x^2 + y^2 + \lambda x + (2\lambda + 3)y = 0,\ \lambda \in \mathbb{R}$, be a family of circles. Prove that the circles from the family have two fixed points.

Exercise 5. Find the geometric locus of the points in the plane for which the sum of the squares of the distances to the sides of an equilateral triangle is constant.

Exercise 6. Let P and Q be two fixed points and d a line, orthogonal on PQ. Two variable orthogonal lines, passing through P, cut d at A, respectively B. Find the geometric locus of the orthogonal projection of the point A on the line BQ.

Exercise 7. Two circles of centers O, respectively O', intersect each other at A and B. A variable line passing through A cuts the two circles at C, respectively C'. Find the geometric locus of the intersection point between the lines OC and $O'C'$.