8p
1 Consider the sets $A=\{(x,y)\in\mathbb{R}^2 : 1<x^2+y^2\leq 4\}$

$B=\{\frac{1}{m} : m\in\mathbb{N}\}$

$C=\mathbb{R}\setminus\mathbb{N}$, $D=(0,1)$

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a) define int, cl, bd

b) det. int, cl, bd for $A, B, C, A\times B, A\times D$

c) specify which are either open or closed

2p
2 Let $f:\mathbb{R}^2\to\mathbb{R}$, $f(x,y)=\begin{cases}(x^2+y^2)\sin\frac{1}{\sqrt{x^2+y^2}}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$

Study the differentiability of f and write its diff. function at an arbitrary point.

2p
3 Let $A=(0,\infty)\times\mathbb{R}$, $f:A\to\mathbb{R}$, $f(x,y)=x\left(y^2+(\ln x)^2\right)$

a) Is A an open set? Prove.

b) determine the gradient of f and the norm of gradient $\|\nabla f(x,y)\|$ for all $(x,y)\in A\times\mathbb{R}$

c) Determ all the critical pts. of f

d) Determ all the extreme points. Specify their nature and the corresponding extreme values.

e) write the diff. of f at $(e,1)$; $df(e,1)$

5p
4 Let f: be a fct. twice Frechet diff. at 0_m
Prove that if 0_m is a local maximum point then $d^2f(0_m)$ is a negative semidefinite quadratic form.