BABEŞ-BOLYAI UNIVERSITY
Faculty of Mathematics and Computer Science

Fundamentals of Programming

Lecture 9 – Search and sorting algorithms

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Course content

Programming in the large

- Introduction in the software development process
- Procedural programming
- Modular programming
- Abstract data types
- Software development principles
- Testing and debugging

Programming in the small

- Recursion
- Complexity of algorithms
- **Search and sorting algorithms**
- Backtracking and other problem solving methods
- Recap

Last time

- Recursion
    - Basic concept
    - Mechanism
    - Recursive functions

- Computational complexity
    - Analyzing the efficiency of a program
    - Run time complexity
    - Classes of complexity

Today

- Search
  - Objective and problem specification
  - Types
    - Sequential seach
    - Binary search
- Sort
  - Objective and problem specification
  - Types
    - Selection sort
    - Insert sort
    - Bubble sort
    - Quick sort

Search methods: Objective

- For a set of data stored in memory as a list of elements (el1, el2, ..., eln)
  - The list may contain elements in any order
  - The list contains elements ordered by some criteria
- Look for
  - A certain element
  - Elements that satisfy different criteria
- Return
  - True or False – if the element(s) exist in the list
  - The index of the element found

Search methods: problem specification

- Unorderd list of elements
  - Input data:
    - elem, n, list = ($list_i$) i=0,1, 2,...,n-1 (n – natural   number)
  - Results:
    - p, where $0 \leq p \leq n-1$, if elem = list[p] or -1, if elem is not in the list
- Ordered list of elements
  - Input data:
    - elem, n, list = ($list_i$), list[0]<list[1]<...<list[n-1], i=0,1, 2,...,n-1 (n – natural   number)
  - Results:
    - p, where $0 \leq p \leq n-1$, if elem = list[p] or -1, if elem is not in the list

Search methods: implementation

- **Sequential search**
  - Basic idea: the elements of the list are examined one by one (the list can be ordered or not)
  - Versions: simple and improved
- **Binary search**
  - Basic idea: the problem is divided in two similar but smaller subproblems (the list has to be ordered)

- Python
  - Functions index and find

Sequential search: implementation *Unordered list*

```
def searchSeq(el, l):
    '''
    Descr: search for an element in a list
    Data: an element and a list
    Res: the position of element in list or -1 if the elemnt is not in the list
    '''
    pos = -1
    for i in range(0, len(l)):
        if (el == l[i]):
            pos = i
    return pos

def test_searchSeq():
    assert searchSeq(2, [3,2,4]) == 1
    assert searchSeq(2, [3,5,7,2]) == 3
    assert searchSeq(2, [2,5,4]) == 0
    assert searchSeq(2, [3,7,4]) == -1
    assert searchSeq(2, [3,2,4,2,7]) == 3

test_searchSeq()
```

Case	**T(n)**
Best case	$\sum_{i=1}^{n} 1 = \mathrm{n}$
Worst case	$\sum_{i=1}^{n} 1 = \mathrm{n}$
Average case	$\sum_{i=1}^{n} 1 = \mathrm{n}$

O(n)

Sequential search: implementation
Unordered list – Improved version

```python
def searchSeq_v2(el, l):
    '''
    Descr: search for an element in a list
    Data: an element and a list
    Res: the position of element in list or
    -1 if the elemnt is not in the list
    '''
    i = 0
    while ((i < len(l)) and (l[i] != el)):
        i = i + 1
    if (i < len(l)):
        return i
    else:
        return -1

def test_searchSeq_v2():
    assert searchSeq_v2(2, [3,2,4]) == 1
    assert searchSeq_v2(2, [3,5,7,2]) == 3
    assert searchSeq_v2(2, [2,5,4]) == 0
    assert searchSeq_v2(2, [3,7,4]) == -1
    assert searchSeq_v2(2, [3,2,4,2,7]) == 1
test_searchSeq_v2()
```

Case	**T(n)**
Best case	1
Worst case	$\sum_{i=1}^{n} 1 = \text{n}$
Average case	(0+1+2+…+n-1)/n

Sequential search: implementation *Ordered list*

```python
def searchSeqOrder(el, l):
    '''
    Descr: search for an element in a list
    Data: an element and a list of ordered elements
    Res: the position of element in list or the position where the element can be inserted
    '''
    if (len(l) == 0): #l==[]
        return 0
    pos = -1
    for i in range(len(l) - 1, -1, -1):
        if (el <= l[i]):
            pos = i
    if (pos == -1):
        return len(l)
    return pos

def test_searchSeqOrder():
    assert searchSeqOrder(2, [2,3,4]) == 0
    assert searchSeqOrder(4, [2,3,4,5]) == 2
    assert searchSeqOrder(2, [1,3,5,7]) == 1
    assert searchSeqOrder(9, [1,2,3]) == 3

test_searchSeqOrder()
```

Case	T(n)
Best case	$\sum_{i=1}^{n} 1 = \mathrm{n}$
Worst case	$\sum_{i=1}^{n} 1 = \mathrm{n}$
Average case	$\sum_{i=1}^{n} 1 = \mathrm{n}$

Sequential search: implementation *Ordered list – Improved version*

```
def searchSeqOrder_v2(el, l):
    '''
    Descr: search for an element in a list
    Data: an element and a list of ordered elements
    Res: the position of element in list or
    the position where the element can be inserted
    '''
    if (len(l) == 0): #l==[]
        return 0
    if (el <= l[0]):
        return 0
    if (el > l[len(l)-1]):
        return len(l)
    i = 0
    while ((i < len(l)) and (l[i] < el)):
        i = i + 1
    return i
def test_searchSeqOrder_v2():
    assert searchSeqOrder_v2(2, [2,3,4]) == 0
    assert searchSeqOrder_v2(4, [2,3,4,5]) == 2
    assert searchSeqOrder_v2(2, [1,3,5,7]) == 1
    assert searchSeqOrder_v2(9, [1,2,3]) == 3
test_searchSeqOrder_v2()
```

Case	**T(n)**
Best case	1
Worst case	$\sum_{i=1}^{n} 1 = n$
Average case	(0+1+2+...+n-1)/n

Binary search: implementation
Ordered list – recursive version

```
def binarySearch(el, l, start, end):
    if (start > end):
        return -1
    middle = (start + end) // 2
    if (el < l[middle]):
        return binarySearch(el, l, start, middle)
    elif (el > l[middle]):
        return binarySearch(el, l, middle + 1, end)
    else: #el == l[middle]
        return middle

def binarySearchRec(el, l):
    #Descr: search for an element in a list
    #Data: an element and a list
    #Res: the position of element in list or
    # -1 if the element is not in the list
    if (len(l) == 0):
        return -1
    elif (el < l[0]) or (el > l[len(l) -1]):
        return -1
    else:
        return binarySearch(el, l, 0, len(l)-1)
```

```
def test_binarySearchRec():
    assert binarySearchRec(4, [3,4,5]) == 1
    assert binarySearchRec(2, [-3,0,1,2]) == 3
    assert binarySearchRec(2, [2,5,6]) == 0
    assert binarySearchRec(2, [3,7,9]) == -1
    assert binarySearchRec(2, [1,2,2,5,6]) == 2

test_binarySearchRec()
```

Case	**T(n)**
Best case	1
Worst case	$\log_2 n$
Average case	$\log_2 n$

Binary search: implementation
Ordered list – iterative version

```
def binarySearchIter(el, l):
    '''
    Descr: search for an element in a list
    Data: an element and a list
    Res: the position of element in list or
    -1 if the elemnt is not in the list
    '''
    if (len(l) == 0):
        return -1
    elif (el < l[0]) or (el > l[len(l) -1]):
        return -1
    else:
        start = 0
        end = len(l) - 1
        while (start <= end):
            middle = (start + end) // 2
            if (el < l[middle]):
                end = middle
            elif (el > l[middle]):
                start = middle + 1
            else:
                return middle
```

```
def test_binarySearchIter():
    assert binarySearchIter(4, [3,4,5]) == 1
    assert binarySearchIter(2, [-3,0,1,2]) == 3
    assert binarySearchIter(2, [2,5,6]) == 0
    assert binarySearchIter(2, [3,7,9]) == -1
    assert binarySearchIter(2, [1,2,2,5,6]) == 2
test_binarySearchIter()
```

$$T(n) = \begin{cases} 1, if\ n = 1 \\ T(n/2) + 1, otherwise \end{cases}$$

$$if\ n = 2^k \rightarrow T\left(2^k\right) = T\left(2^{k-1}\right) + 1$$

$$T\left(2^k\right) = T\left(2^{k-1}\right) + 1$$

$$T\left(2^{k-1}\right) = T\left(2^{k-2}\right) + 1$$

$$\dots$$

$$T(2^1) = T(2^0) + 1$$

$$T\left(2^k\right) = k + 1 \qquad k = \log_2 n \rightarrow T(n) = \log_2 n + 1$$

Search methods: Python functions

- list.index(element)
  - Returns the index of the element in the list
  - If the element does not exist in the list, throws an exception
- list.count(element)
  - Returns the number of times the element appears in the list (if it exists)
  - Returns 0 if the element is not in the list

```
def test_index():
    l = [7,2,13,4,1]
    assert l.index(2) == 1
    assert l.index(1) == 4
    try:
        l.index(3)
        assert False
    except ValueError as ex:
        print("elem not found")
        assert True

test_index()
```

```
def test_count():
    l = [7,2,13,4,1]
    assert l.count(2) == 1
    assert l.count(1) == 1
    assert l.count(3) == 0

test_count()
```

Sorting methods

- Objective
  - Rearrange the elements of a container such that they are in a certain relation of order

- Problem specification
  - Input data:
    - n, list = ($list_i$) i=0,1, 2,...,n-1 (n – natural number)
  - Results:
    - n, list’ = ($list_i$’) i=0,1, 2,...,n-1, orderRelation($list_i$’, $list_{i+1}$’)=True for any i=0,1,...,n-2

Sorting methods: taxonomy

- Place where the elements are stored
  - **Internal sort** – data to be sorted is available in the internal memory
  - **External sort** – data available from files (external memory)
- The order relation
  - **Ascending sort**
  - **Descending sort**
- Keeping the initial order of the elements
  - **Stable sort –** keep the initial order of equal elements
  - **Instable sort** – the initial order of equal elements is not kept
- Space complexity
  - **In-place sort** – the additional space (to that needed for the container) is small
  - **Not-in-place / Out-of-place sort** – large additional space
- Mechanism
  - **Selection sort**
  - **Insert sort**
  - **Bubble sort**
  - **Quick sort**

Selection Sort

- Basic idea
  - Determine the smallest element from the collection and place it in first position (swap the smallest element with the first one)
  - Repeat the first step for all elements different from the smallest element
- Algorithm
- Complexity
  - Time

    $T(n) = \sum_{i=0}^{n-2}\left(\sum_{j=i}^{n-1} 1 + 3\right) \approx n(n-1)/2$ -> ***O(n²)***
  - Space

    S(n)=n+1+1

```
def selectionSort(l):
    '''
    descr: sorts the leemnts of a list
    data: a list of elements
    res: the ordered list
    '''
    for i in range(0, len(l) - 1):
        min_pos = i
        for j in range(i + 1, len(l)):
            if (l[j] < l[min_pos]):
                min_pos = j
        if (i < min_pos):
            aux = l[i]
            l[i] = l[min_pos]
            l[min_pos] = aux
    return l
```

```
l[i], l[min_pos] = l[min_pos], l[i]
```

```
def test_selectionSort():
    assert selectionSort([1,2,3]) == [1,2,3]
    assert selectionSort([3,2,1]) == [1,2,3]
    assert selectionSort([1,2,1]) == [1,1,2]

test_selectionSort()
```

Insertion Sort

- Basic idea
  - Traverse the elements of the container and insert each element at the correct position in the sub-container with the elements already sorted
  - At the end of the algorithm, the sub-container will have all the initial elements sorted
- Algorithm
- Complexity
  - Time

    $T(n) = \sum_{i=1}^{n-1}\left(1 + \sum_{j=0}^{i-1} 2 + 1\right) \approx n^2 + n - 2$ -> ***O(n²)***
  - Space

    S(n)=n+1+1+1

```
def insertSort(l):
    '''
    descr: sorts the elemnts of a list
    data: a list of elements
    res: the ordered list
    '''
    for i in range(1, len(l)):
        noOfAlreadySort = i - 1
        crtElem = l[i]
        # insert crtElem in the right position
        # (<= noOfAlreadySort)
        j = noOfAlreadySort
        while ((j >= 0) and (crtElem < l[j])):
            l[j + 1] = l[j]
            j = j - 1
        l[j + 1] = crtElem
    return l

def test_insertSort():
    assert insertSort([1,2,3]) == [1,2,3]
    assert insertSort([3,2,1]) == [1,2,3]
    assert insertSort([1,2,1]) == [1,1,2]

test_insertSort()
```

Bubble Sort

- Basic idea
  - Compare any 2 consecutive elements
    - If they are not in correct order, swap them
  - Until any 2 consecutive elements are in the correct order
- Algorithm
- Complexity
  - Time

    $T(n)$ -> ***O(n²)***
  - Space

    S(n)=n+1+1+1

```python
def bubbleSort(l):
    '''
    descr: sorts the elemnts of a list
    data: a list of elements
    res: the ordered list
    '''
    isSort = False
    while (not isSort):
        isSort = True
        for i in range(0, len(l) - 1):
            if (l[i] > l[i + 1]):
                aux = l[i]
                l[i] = l[i + 1]
                l[i + 1] = aux
                isSort = False
    return l

def test_bubbleSort():
    assert bubbleSort([1,2,3]) == [1,2,3]
    assert bubbleSort([3,2,1]) == [1,2,3]
    assert bubbleSort([1,2,1]) == [1,1,2]

test_bubbleSort()
```

Quick Sort

- Basic idea
  - Divide and conquer technique
  1. **Divide:** divide the container in 2 parts such that any element in the first sub-container ≤ any element in the second sub-container
  2. **Conquer:** sort the two sub-containers (recursively)
- Algorithm

```
def test_quickSort():
    assert quickSort([1,2,3]) == [1,2,3]
    assert quickSort([3,2,1]) == [1,2,3]
    assert quickSort([1,2,1]) == [1,1,2]

test_quickSort()
```

```
def partition(l, start, end):
    pivot = l[start]
    i = start
    j = end
    while (i != j):
        while ((pivot <= l[j]) and (i < j)):
            j = j - 1
        l[i] = l[j]
        while ((l[i] <= pivot) and (i < j)):
            i = i + 1
        l[j] = l[i]
        l[i] = pivot
    return i

def quickSortRec(l, start, end):
    pivotPos = partition(l, start, end)
    if (start < pivotPos - 1):
        quickSortRec(l, start, pivotPos - 1)
    if (pivotPos + 1 < end):
        quickSortRec(l, pivotPos + 1, end)

def quickSort(l):
    '''
    descr: sorts the elemnts of a list
    data: a list of elements
    res: the ordered list
    '''
    quickSortRec(l, 0, len(l) - 1)
    return l
```

Quick Sort: complexity

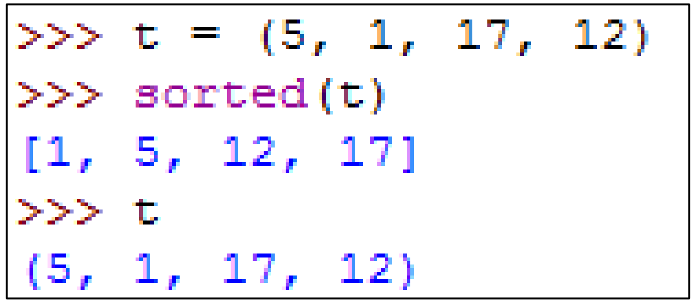
- The run time of quick-sort depends on the distribution of splits
  - The partitioning function requires linear time
  - Best case is when the partitioning function splits the array evenly

Best case	$T(n)=2*T(n/2)+n \rightarrow O(n\,log_2\,n)$
Worst case	$T(n)=T(1)+T(n-1)+n=T(n-1)+n+1$ $T(n-1)=T(1)+T(n-2)+(n-1)=T(n-2)+n$ … $T(2)=T(1)+T(1)+2=4$ ____________________ $T(n)=(n+1)(n+2)/2-(1+2+3)=>O(n^2)$
Average case	$L(n)=2*U(n/2)+n$ $U(n)=L(n-1)+n$ $L(n)=2*(L(n/2-1)+n/2)+n=2L(n/2-1\_+2n=>O(n\,log_2\,n)$

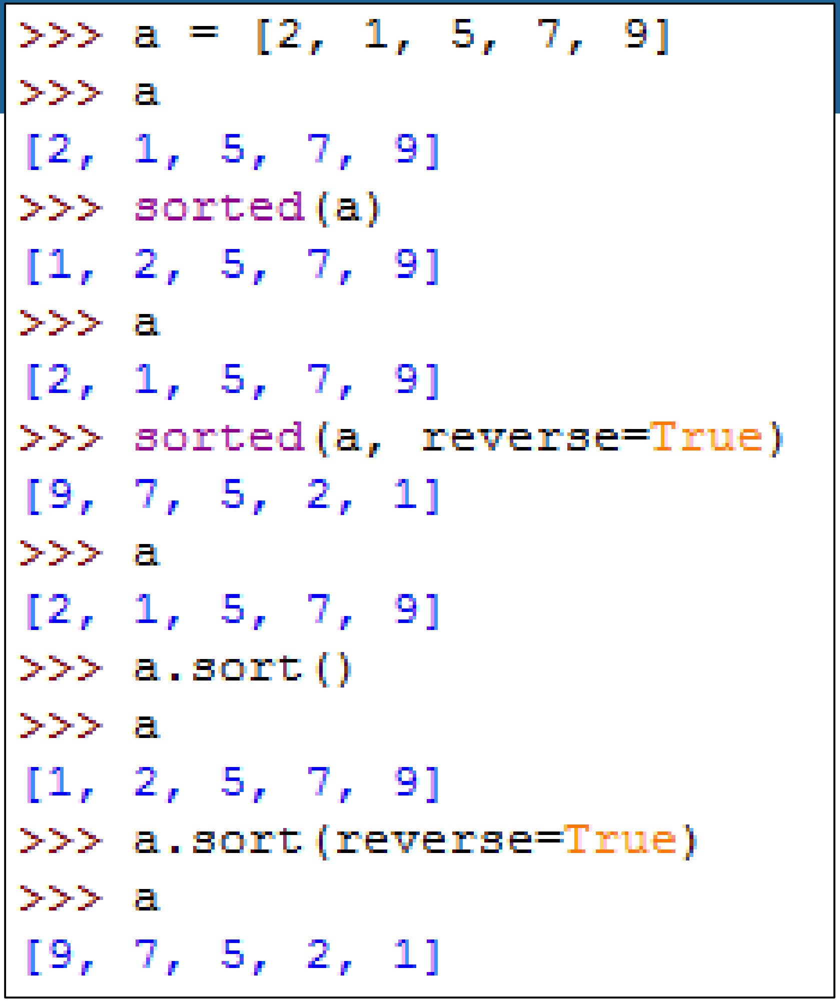
- Space complexity
  - Average: S(n) = $\log_2 n$
  - Worst: S(n) = n

Sorting in Python

- `list.sort()`
- `sorted(lista)`



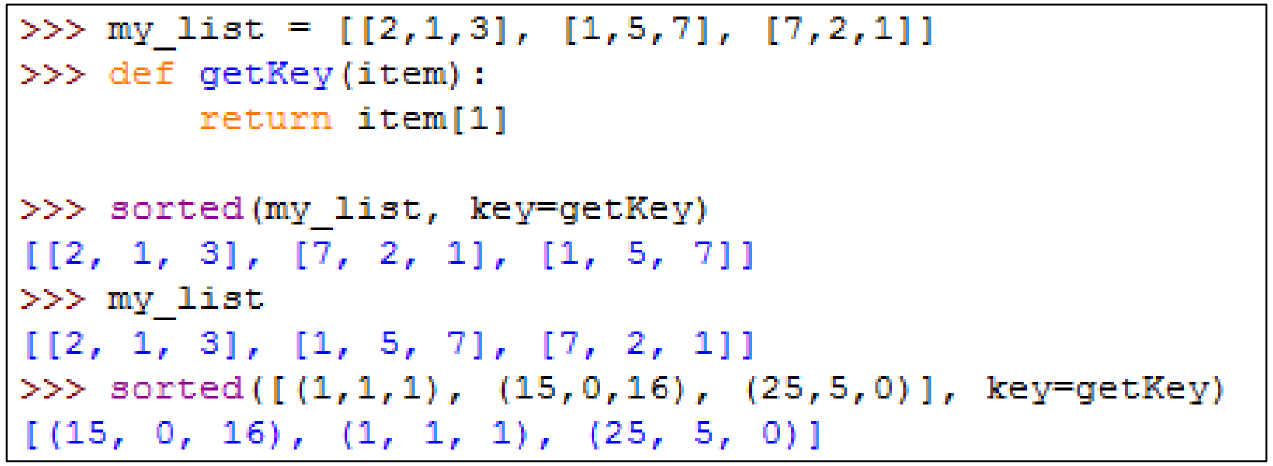
```
>>> t = (5, 1, 17, 12)
>>> sorted(t)
[1, 5, 12, 17]
>>> t
(5, 1, 17, 12)
```



```
>>> a = [2, 1, 5, 7, 9]
>>> a
[2, 1, 5, 7, 9]
>>> sorted(a)
[1, 2, 5, 7, 9]
>>> a
[2, 1, 5, 7, 9]
>>> sorted(a, reverse=True)
[9, 7, 5, 2, 1]
>>> a
[2, 1, 5, 7, 9]
>>> a.sort()
>>> a
[1, 2, 5, 7, 9]
>>> a.sort(reverse=True)
>>> a
[9, 7, 5, 2, 1]
```

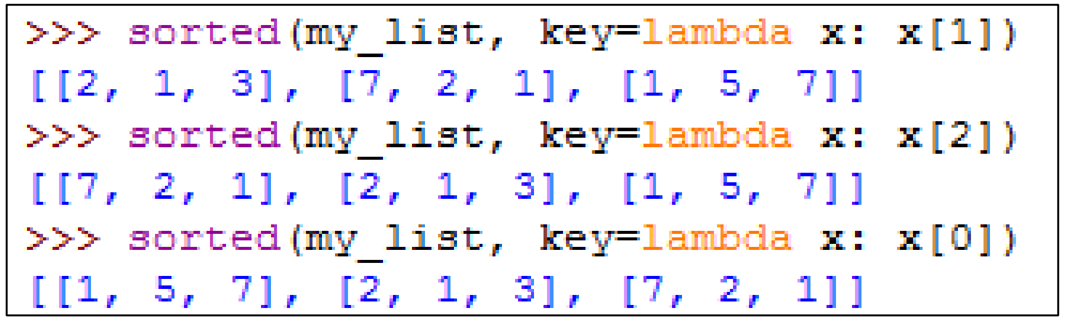
Sorting in Python: list of lists / tuples

- Use the key argument in sorted/sort



```
>>> my_list = [[2,1,3], [1,5,7], [7,2,1]]
>>> def getKey(item):
        return item[1]

>>> sorted(my_list, key=getKey)
[[2, 1, 3], [7, 2, 1], [1, 5, 7]]
>>> my_list
[[2, 1, 3], [1, 5, 7], [7, 2, 1]]
>>> sorted([(1,1,1), (15,0,16), (25,5,0)], key=getKey)
[(15, 0, 16), (1, 1, 1), (25, 5, 0)]
```

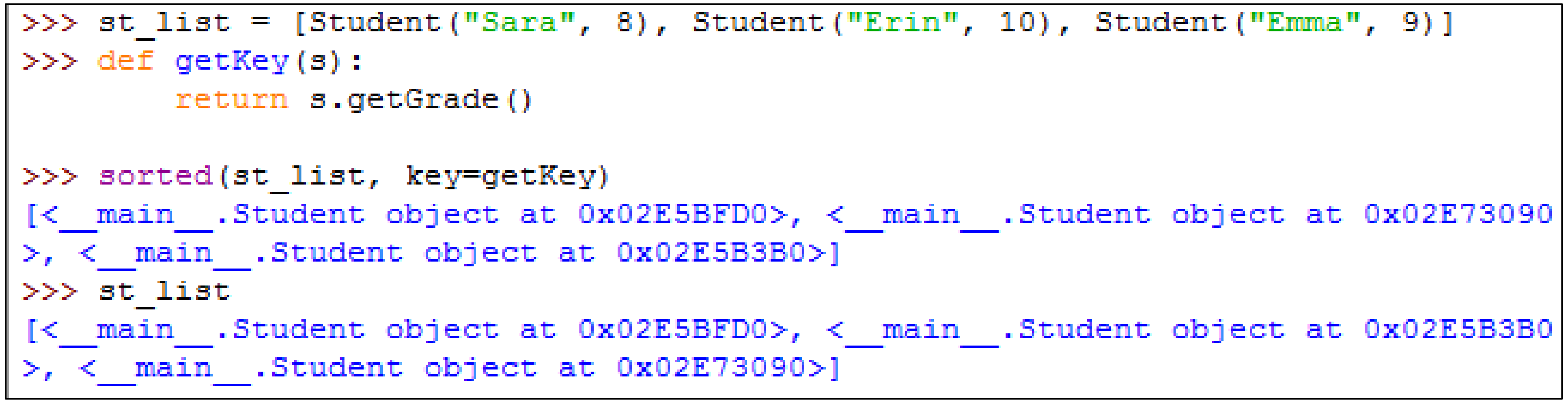
key argument &
lambda expressions



```
>>> sorted(my_list, key=lambda x: x[1])
[[2, 1, 3], [7, 2, 1], [1, 5, 7]]
>>> sorted(my_list, key=lambda x: x[2])
[[7, 2, 1], [2, 1, 3], [1, 5, 7]]
>>> sorted(my_list, key=lambda x: x[0])
[[1, 5, 7], [2, 1, 3], [7, 2, 1]]
```

Sorting in Python: list of custom objects

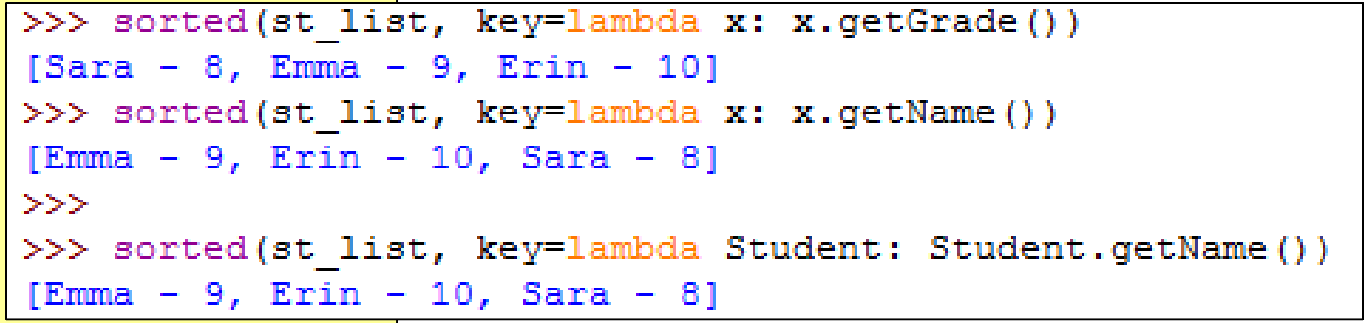
```
class Student:
    def __init__(self, name, grade):
        self.__name = name
        self.__grade = grade

    def getName(self):
        return self.__name

    def getGrade(self):
        return self.__grade
```



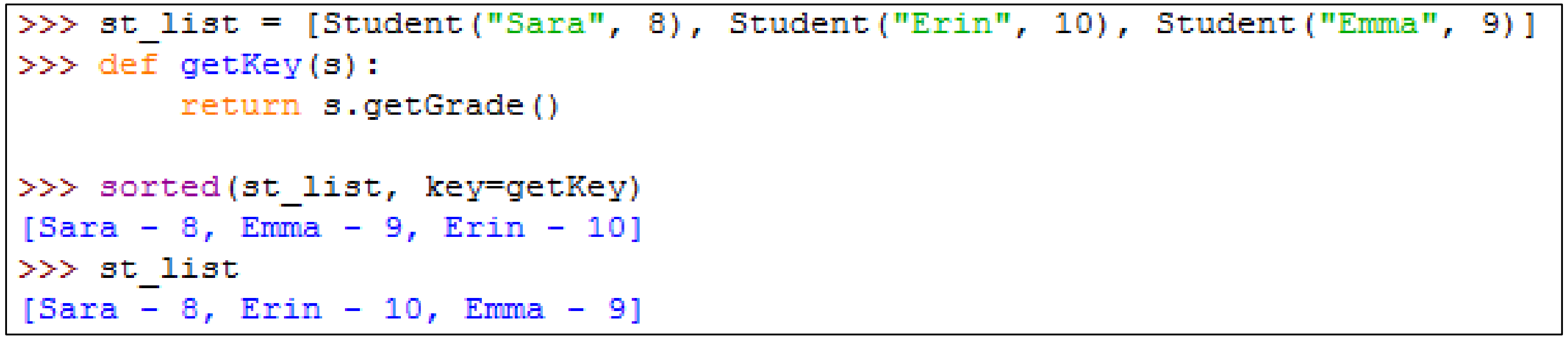
```
>>> st_list = [Student("Sara", 8), Student("Erin", 10), Student("Emma", 9)]
>>> def getKey(s):
        return s.getGrade()

>>> sorted(st_list, key=getKey)
[<__main__.Student object at 0x02E5BFD0>, <__main__.Student object at 0x02E73090
>, <__main__.Student object at 0x02E5B3B0>]
>>> st_list
[<__main__.Student object at 0x02E5BFD0>, <__main__.Student object at 0x02E5B3B0
>, <__main__.Student object at 0x02E73090>]
```

Sorting in Python: list of custom objects

```python
class Student:
    def __init__(self, name, grade):
        self.__name = name
        self.__grade = grade

    def getName(self):
        return self.__name

    def getGrade(self):
        return self.__grade

    def __repr__(self):
        return self.__name + " - " + str(self.__grade)
```



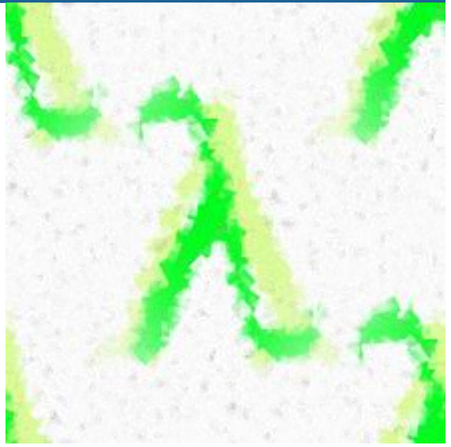
```
>>> sorted(st_list, key=lambda x: x.getGrade())
[Sara - 8, Emma - 9, Erin - 10]
>>> sorted(st_list, key=lambda x: x.getName())
[Emma - 9, Erin - 10, Sara - 8]
>>>
>>> sorted(st_list, key=lambda Student: Student.getName())
[Emma - 9, Erin - 10, Sara - 8]
```



```
>>> st_list = [Student("Sara", 8), Student("Erin", 10), Student("Emma", 9)]
>>> def getKey(s):
        return s.getGrade()

>>> sorted(st_list, key=getKey)
[Sara - 8, Emma - 9, Erin - 10]
>>> st_list
[Sara - 8, Erin - 10, Emma - 9]
```

Lambda expressions

- Small anonymous functions
- Defined and used in the same place



✓ *Syntactically restricted to a single expression*

✓ *Can reference variables from the containing scope (just like nested functions)*

✓ *They are syntactic sugar for a function definition*

Lambda expressions

- Syntax

```
lambda arg1, arg2, ...argN : expression using arguments
```

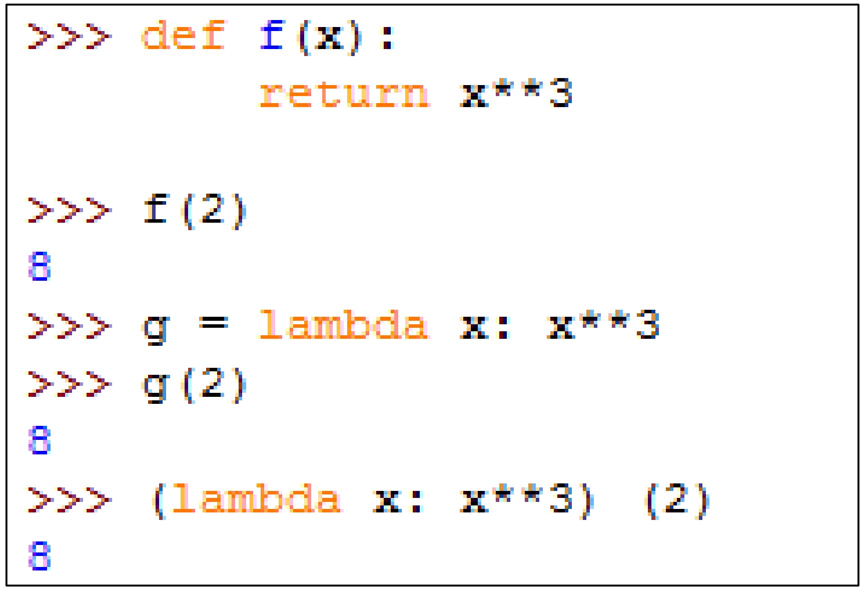
- Lambda is an expression

(def - a function with name, statement)

- Body – is simply an expression

(not a block of stataments,

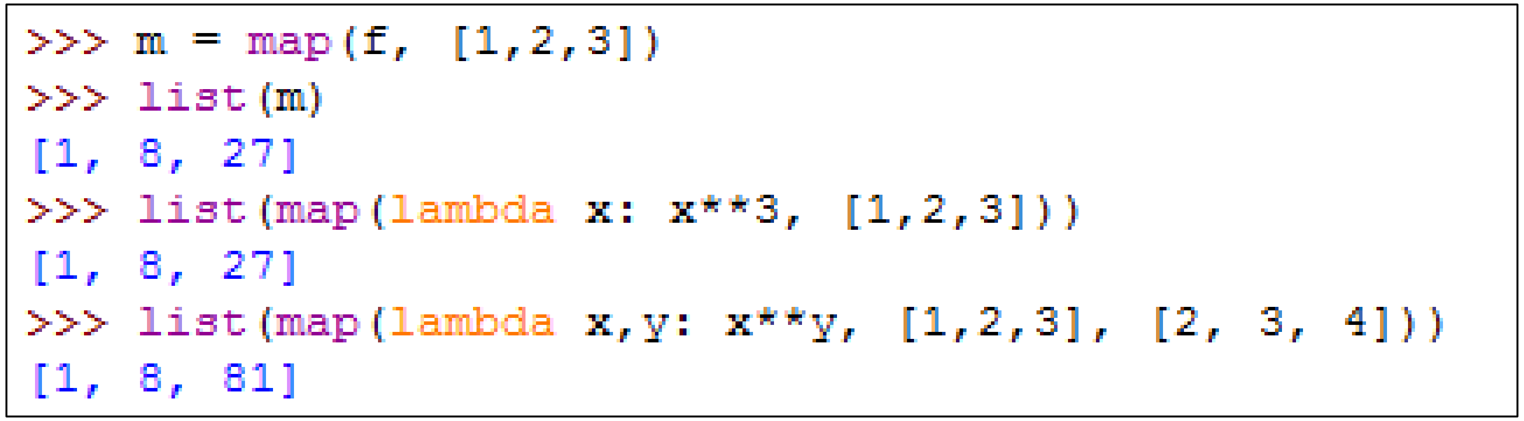
no return statement)



```
>>> def f(x):
        return x**3

>>> f(2)
8
>>> g = lambda x: x**3
>>> g(2)
8
>>> (lambda x: x**3) (2)
8
```

Map and Lambda expressions

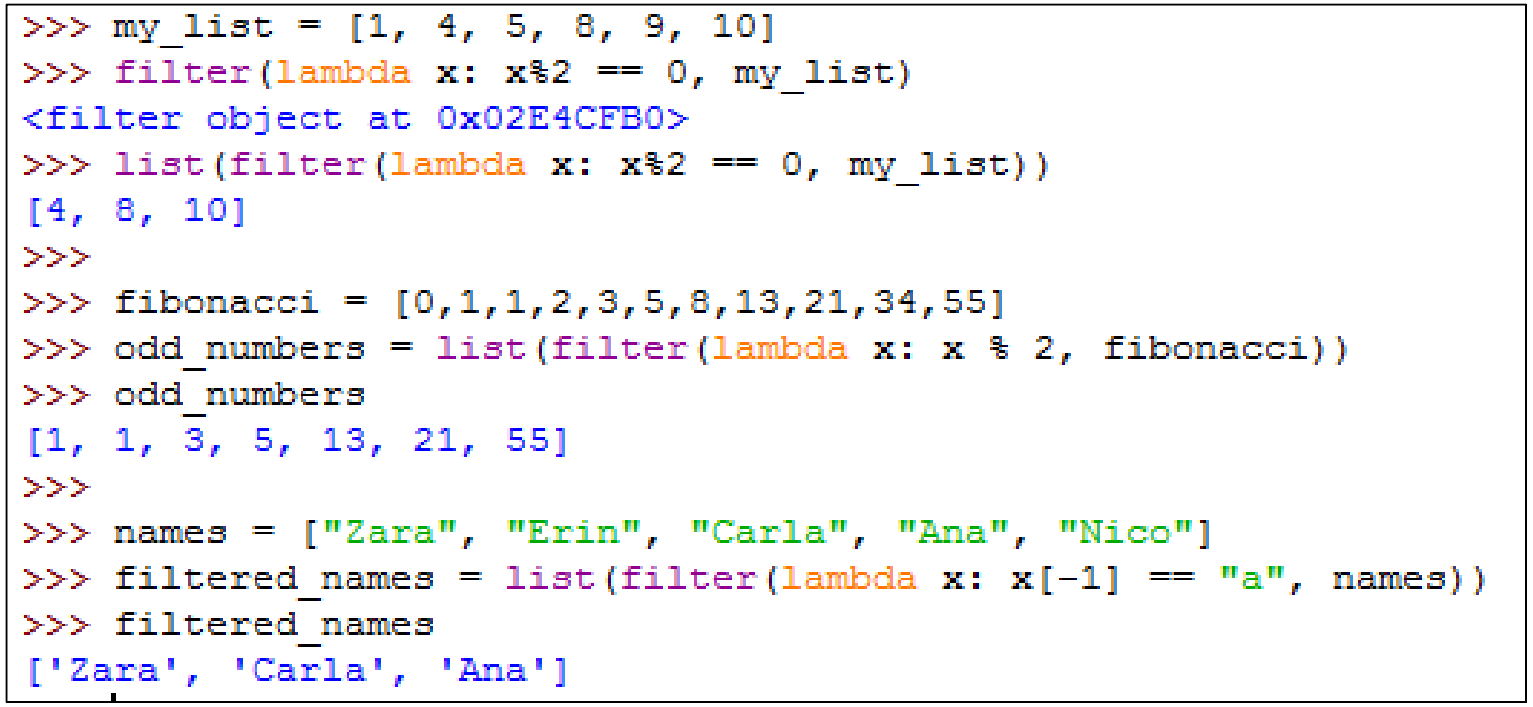
- Function map
  - r=map(function, sequence)



```
>>> m = map(f, [1,2,3])
>>> list(m)
[1, 8, 27]
>>> list(map(lambda x: x**3, [1,2,3]))
[1, 8, 27]
>>> list(map(lambda x,y: x**y, [1,2,3], [2, 3, 4]))
[1, 8, 81]
```

Filter and lambda expressions

- Function filter
  - `filter(function, sequence)`



```
>>> my_list = [1, 4, 5, 8, 9, 10]
>>> filter(lambda x: x%2 == 0, my_list)
<filter object at 0x02E4CFB0>
>>> list(filter(lambda x: x%2 == 0, my_list))
[4, 8, 10]
>>>
>>> fibonacci = [0,1,1,2,3,5,8,13,21,34,55]
>>> odd_numbers = list(filter(lambda x: x % 2, fibonacci))
>>> odd_numbers
[1, 1, 3, 5, 13, 21, 55]
>>>
>>> names = ["Zara", "Erin", "Carla", "Ana", "Nico"]
>>> filtered_names = list(filter(lambda x: x[-1] == "a", names))
>>> filtered_names
['Zara', 'Carla', 'Ana']
```

Sort and Lambda expressions

```python
class Person:
    def __init__(self, n, a):
        self.name = n
        self.age = a

    def getName(self):
        return self.name

    def getAge(self):
        return self.age

    def __repr__(self):
        return self.name + "-" + str(self.age)
```

```python
def sort_python():
    l1 = [4,2,3,1]
    l1.sort()
    print(l1)

    l1s = sorted(l1)
    print(l1s)

    p1 = Person("nnnn", 20)
    p2 = Person("eeee", 21)
    p3 = Person("ttt", 10)
    l2 = [p1, p2, p3]
    l2s = sorted(l2, key=lambda Person:Person.getName())
    print(l2s)
    l2s = sorted(l2, key=lambda Person:Person.getAge())
    print(l2s)

sort_python()
```

Recap today

- Search
  - Sequential seach
  - Binary search

- Sort
  - Selection sort
  - Insert sort
  - Bubble sort
  - Quick sort

Next time

- Algorithms
  - Backtracking
  - Divide and conquer

Reading materials and useful links

1. The Python Programming Language - https://www.python.org/
2. The Python Standard Library - https://docs.python.org/3/library/index.html
3. The Python Tutorial - https://docs.python.org/3/tutorial/
4. M. Frentiu, H.F. Pop, Fundamentals of Programming, Cluj University Press, 2006.
5. MIT OpenCourseWare, Introduction to Computer Science and Programming in Python, https://ocw.mit.edu, 2016.
6. K. Beck, Test Driven Development: By Example. Addison-Wesley Longman, 2002. http://en.wikipedia.org/wiki/Test-driven_development
7. M. Fowler, Refactoring. Improving the Design of Existing Code, Addison-Wesley, 1999. http://refactoring.com/catalog/index.html

Bibliography

The content of this course has been prepared using the reading materials from previous slide, different sources from the Internet as well as lectures on Fundamentals of Programming held in previous years by:

- Prof. Dr. Laura Dioşan - www.cs.ubbcluj.ro/~lauras
- Conf. Dr. Istvan Czibula - www.cs.ubbcluj.ro/~istvanc
- Lect. Dr. Andreea Vescan - www.cs.ubbcluj.ro/~avescan
- Lect. Dr. Arthur Molnar - www.cs.ubbcluj.ro/~arthur