MiE 12.07.2018 Restanta Trif Tiberiu Amaliza 2

Reexamination in Calculus 2 part 2 - Integrability on R^n

Exercise 1: Solve the double integral

$$\int \int_{A} \left(\sqrt{x^2 + 1} + y \right) dx dy$$

where

$$A = \{(x, y) \in \mathbb{R}^2 : 0 \le x \le 1, \quad 0 \le y \le 2x\}.$$

Exercise 2: Solve the triple integral

$$\int \int \int_A x^2 dx dy dz,$$

where

$$A = \{(x, y, z) \in \mathbb{R}^3 : 0 \le x^2 + y^2 + z^2 \le 9\}.$$

Exercise 3: Find the length of the parametrized path $\gamma:[0,\pi]\to\mathbb{R}^3$, defined by

$$\gamma(t) = \left(t - \sin t, \quad 1 - \cos t, \quad 4\cos\left(\frac{t}{2}\right)\right).$$

Exercise 4: Let $f:[a,b] \to \mathbb{R}^n$ be a function of the class C^1 , and let $g:[a,b] \to \mathbb{R}$ be the function defined by

$$g(x) = V_a^x(f)$$

(the total variation of f on [a, x]). Prove that g has a finite left derivative at each point of (a, b].