Calculus on $\mathbb{R}$ Lect. dr. Anca Grad
Homework. 7 **Sweek7-8, 2017**

Limits of functions

We recall that:

$$\lim_{x\to\infty} q^x = \begin{cases} +\infty & : q > 1 \\ 1 & : q = 1 \\ 0: & |q| < 1 \\ \nexists & : q \leq 1 \end{cases}$$

$$\lim_{x\to\infty} q^x = q^{x_0}, \forall q \in (0,\infty) \quad si \quad x_0 \in \mathbb{R}$$

$$\lim_{x\to x_0} \log_a x = \log_a x_0, \forall a \in (0,\infty)\setminus\{1\}, x_0 > 0.$$

$$\lim_{x\to\infty} \log_a x = \begin{cases} +\infty & : a > 1 \\ -\infty & : 0 < a < 1 \end{cases}$$

$$\lim_{x\to 0} \frac{q^x - 1}{x} = \ln q, \forall q > 0 \quad si \quad \lim_{x\to 0} \frac{\ln(1+x)}{x} = 1.$$

Exercise1: Determine the limits of the following functions at the specified points:

$$a)\lim_{x\to\infty} x\cos^2\frac{x+2}{x} \quad b)\lim_{x\to 1}\frac{x}{x^2+1} \quad c)\lim_{x\to-\infty}\frac{x^2+5}{x^3} \quad d)\lim_{x\to\infty}\frac{(x+2)(2x+1)}{x^2+3x+5}$$

$$e)\lim_{x\to 1}\frac{x^2-1}{x^3-1} \quad f)\lim_{x\to 2}\left(\frac{1}{2-x}-\frac{2x}{4-x^2}\right)$$

$$g)\lim_{x\to 1}\frac{1+x+x^2+...+x^n-(n+1)}{x-1}, n\in\mathbb{N} \quad h)\lim_{x\to 1}\frac{x+x^2+...+x^n-n}{x+x^2+...+x^m-m}, \forall m,n\in\mathbb{N}.$$

$$i)\lim_{x\to 27}\frac{x-27}{\sqrt[3]{x}-3} \quad j)\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1}$$

$$k)\lim_{x\to\infty}\left(\sqrt[3]{ax^3+x^2+bx+c}-(bx+c)\right)\forall a,b,c>0.$$

Exercise 2: Determine the limits of the following functions at the specified points:

$$a)\lim_{x\to\infty}\left(\frac{1}{x}\right)^{\frac{5x+1}{2x+4}} \quad b)\lim_{x\to 0}\left(\frac{3\sin x-\tan x}{x}\right)^{\frac{\sin x+2x}{x}}$$

$$c)\lim_{x\to 0}(1+\cos x)^{\frac{1}{x^2}} \quad d)\lim_{x\to 0}(e^x-x+1)^{\frac{1}{1-\cos x}}$$

$$e)\lim_{x\to 0}(1+\sin x)^{\frac{1}{x}} \quad f)\lim_{x\to\infty}\left(\frac{x+7}{x}\right)^x$$

Exercise 3

Determine the limits of the following functions at the specified points:

$$a)\lim_{n\to\infty}\left[\lim_{x\to 0}\left(1+\sin^2 x+\sin^2 2x+...+\sin^2 nx\right)^{\frac{1}{n^3x^2}}\right]$$

$$a)\lim_{n\to\infty}\left[\lim_{x\to 0}\left(1+\ln(1+x)+\ln(1+2x)+...+\ln(1+nx)\right)^{\frac{1}{n^2x}}\right]$$

Exercise 4

Determine the limits of the following functions at the specified points:

$$a)\lim_{x\to 0}\frac{e^{2x}-1}{3x}; \quad b)\lim_{x\to 0}\frac{e^x-cos}{3x}.$$