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Geometry 1 (Analytic Geometry)

Exercise Sheet 10

Exercise 1. Find the equations of the line passing through $P(6,4,-2)$ and parallel to the line

$$d : \frac{x}{2} = \frac{y-1}{-3} = \frac{z-5}{6}$$

Exercise 2. Given the lines

$d_1 : x = 4 - 2t, y = 1 + 2t, z = 9 + 3t$

and

$d_2 : \frac{x-1}{2} = \frac{y+2}{3} = \frac{z-4}{2}$

find the intersection points between the two lines and the coordinate planes.

Exercise 3. Let d_1 and d_2 be the lines given by

$d_1 : x = 3 + t, y = -2 + t, z = 9 + t, t \in \mathbb{R}$

and

$d_2 : x = 1 - 2s, y = 5 + s, z = -2 - 5s, s \in \mathbb{R}$

(a) Prove they are coplanar.

(b) Find the equation of the line passing through the point $P(4,1,6)$ and orthogonal on the plane determined by d_1 and d_2.

Exercise 4. Prove that the intersection lines of the planes

$\pi_1 : 2x - y + 3z - 5 = 0$

$\pi_2 : 3x + y + 2z - 1 = 0$

$\pi_3 : 4x + 3y + z + 2 = 0$

are parallel.

Exercise 5. Verify that the lines
$d_1 : \dfrac{x-3}{1} = \dfrac{y-8}{3} = \dfrac{z-3}{4}$
and
$d_2 : \dfrac{x-4}{1} = \dfrac{y-9}{2} = \dfrac{z-9}{5}$
are coplanar and find the equation of the plane determined by the two lines.

Exercise 6. Determine whether the line

$$\begin{cases} x = 3 + 8t \\ y = 4 + 5t \\ z = -3 - t \end{cases}$$

is parallel to the plane $x - 3y + 5z - 12 = 0$.

Exercise 7. Find the intersection point between the line $\begin{cases} x = 3 + 8t \\ y = 4 + 5t \\ z = -3 - t \end{cases}$ and the plane $x - 3y + 5z - 12 = 0$.

Exercise 8. Prove that the lines $d_1 : \begin{cases} x = 1 + 4t \\ y = 5 - 4t \\ z = -1 + 5t \end{cases}$ and $d_2 : \begin{cases} x = 2 + 8t \\ y = 4 - 3t \\ z = 5 + t \end{cases}$ are skew.

Exercise 9. Find the parametric equations of the line passing through $(5, 0, -2)$ and parallel to the planes $x - 4y + 2z = 0$ and $2x + 3y - z + 1 = 0$.

Exercise 10. Find the equation of the plane containing the point $P(2,0,3)$ and the line $d : \begin{cases} x = -1 + t \\ y = t \\ z = -4 + 2t \end{cases}$

Exercise 11. Show that the line $d = \begin{cases} x = 0 \\ y = t \\ z = t \end{cases}$ is contained inside the plane $6x + 4y - 4z = 0$.

Exercise 12. Let $M_1(2,1,-1)$ and $M_2(-3,0,2)$ be two points. Find:

(a) the equation of the bundle of planes passing through M_1 and M_2;

(b) the plane π from the bundle, which is orthogonal on xOy;

(c) the plane ρ from the bundle, which is orthogonal on π.

Exercise 13. Given the points $A(1, 2\alpha, \alpha)$, $B(3,2,1)$, $C(-\alpha, 0, \alpha)$ and $D(-1,3,-3)$, find the parameter α, such that the bundle of planes passing through AB has a common point with the bundle of planes passing through CD.

Exercise 14. Given the planes $\pi_1 : 2x + y - 3z - 5 = 0$
and
$\pi_2 : x + 3y + 2z + 1 = 0$,
find the equations of the bisector planes of the dihedral angle and choose which one belongs to the acute dihedral angle.