Prof. Dr. Dorin Andrica

Asist. Drd. Tudor Micu

1st Semester, 2018-2019

Geometry 1 (Analytic Geometry)

Exercise Sheet 7

Exercise 1. Given the line $d : 2x + 3y + 4 = 0$, find the equation of a line d_1 passing through the point $M_0(2, 1)$, in the following situations:

(a) d_1 is parallel with d;

(b) d_1 is orthogonal on d;

(c) the angle determined by d and d_1 is $\pi/4$.

Exercise 2. The vertices of the triangle $\triangle ABC$ are the intersection points of the lines $d_1 : 4x + 3y - 5 = 0$, $d_2 : x - 3y + 10 = 0$, $d_3 : x - 2 = 0$.

1. Find the coordinates of A, B, C.
2. Find the equations of the median lines of the triangle.
3. Find the equations of the heights of the triangle.

Exercise 3. Find the coordinates of the symmetrical of the point $P(-5, 13)$ with respect to the line $d : 2x - 3y - 3 = 0$.

Exercise 4. Find the coordinates of the point P on the line $d : 2x - y - 5 = 0$, for which the sum $AP + PB$ attains its minimum, when $A(-7, 1)$ and $B(-5, 5)$.

Exercise 5. Find the coordinates of the circumcenter (the center of the circumscribed circle) of the triangle determined by the lines $4x - y + 2 = 0$, $x - 4y - 8 = 0$ and $x + 4y - 8 = 0$.

Exercise 6. Prove that, in any triangle $\triangle ABC$, the orthocenter H, the center of gravity G and the circumcenter O are collinear.

Exercise 7. Given the bundle of lines of equations

$(1 - t)x + (2 - t)y + t - 3 = 0, t \in \mathbb{R}$

$x + y - 1 = 0$, find:

1. the coordinates of the vertex of the bundle;

2. the equation of the line in the bundle which cuts Ox and Oy in M, respectively N, such that $OM^2 \cdot ON^2 = 4(OM^2 + ON^2)$;

Exercise 8. Let B be the bundle of vertex $M_0(5, 0)$. An arbitrary line from B intersects the lines $d_1 : y - 2 = 0$ and $d_2 : y - 3 = 0$ in M_1 respectively M_2. Prove that the line passing through M_1 and parallel to OM_2 passes through a fixed point.

Exercise 9. The vertices of the quadrilateral $ABCD$ are $A(4, 3)$, $B(5, -4)$, $C(-1, -3)$ and $D(-3, -1)$.

(a) Find the coordinates of the points $E = AB \cap CD$ and $F = BC \cap AD$;

(b) Prove that the midpoints of the segments $[AC]$, $[BD]$ and $[EF]$ are collinear.

Exercise 10. Let M be a point whose coordinates satisfy

$$\frac{4x + 2y + 8}{3x - y + 1} = \frac{5}{2}$$

(a) Prove that M belongs to a fixed line;

(b) Find the minimum of $x^2 + y^2$, when $M \in d \setminus \{M_0(-1, -2)\}$.

Exercise 11. Find the geometric locus of the points whose distances to two orthogonal lines have a constant ratio.