BABEŞ-BOLYAI UNIVERSITY
Faculty of Mathematics and Computer Science

Fundamentals of Programming

Lecture 10 – Problem solving methods (I)

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Course content

Programming in the large

- Introduction in the software development process
- Procedural programming
- Modular programming
- Abstract data types
- Software development principles
- Testing and debugging

Programming in the small

- Recursion
- Complexity of algorithms
- Search and sorting algorithms
- **Problem solving methods**
  - **Generate and test, Backtracking**
  - **Divide et impera**
- Recap

Last time

- Search
  - Sequential seach
  - Binary search

- Sort
  - Selection sort
  - Insert sort
  - Bubble sort
  - Quick sort

Today

- Problem solving methods
  - Types

  - Techniques
    - Exact methods
    - Heuristic methods

  - Algorithms
    - Backtracking
    - Divide and conquer

Problem solving methods

- Strategies for solving difficult problems
- General algorithms that can be applied to solve certain type of problem (the problem needs to satisfy certain required criteria)

- Problem characteristics
  - Structure
  - Number of solutions
  - Search, optimization, simulation, etc

Problem types

- **By structure**
  - Problems that can be divided in sub-problems
    e.g. search for an element in a list
  - Problems that can not be divided in sub-problems
    e.g. place queens on a chessboard
- **By number of solutions**
  - Problems with a single solution
    e.g. sort a list
  - Problems with several solutions
    e.g. generate permutations
- **By solving possibilities**
  - Problems that can be deterministically solved
    e.g. compute the sin or the square root of a number
  - Problems that can be solved stochastically (heuristics)
    e.g. Real-world problems such as vehicle routing optimization
    Need to *search* for a solution

Problem types

- **By run time complexity**
  - Problems from class P – can be solved in polynomial time (n^2, n^3,...)
    e.g. sorting problems
  - Problems from class NP – can not be solved in polynomial time ($n!$, 2^n,...)
    e.g. the shortest path in a graph of cities
- **By scope**
  - Search / optimization problems
    e.g. planning, scheduling, resource allocation
  - Modeling problems
    e.g. forecasting, classification, prediction
  - Simulation
    e.g. economic game theory

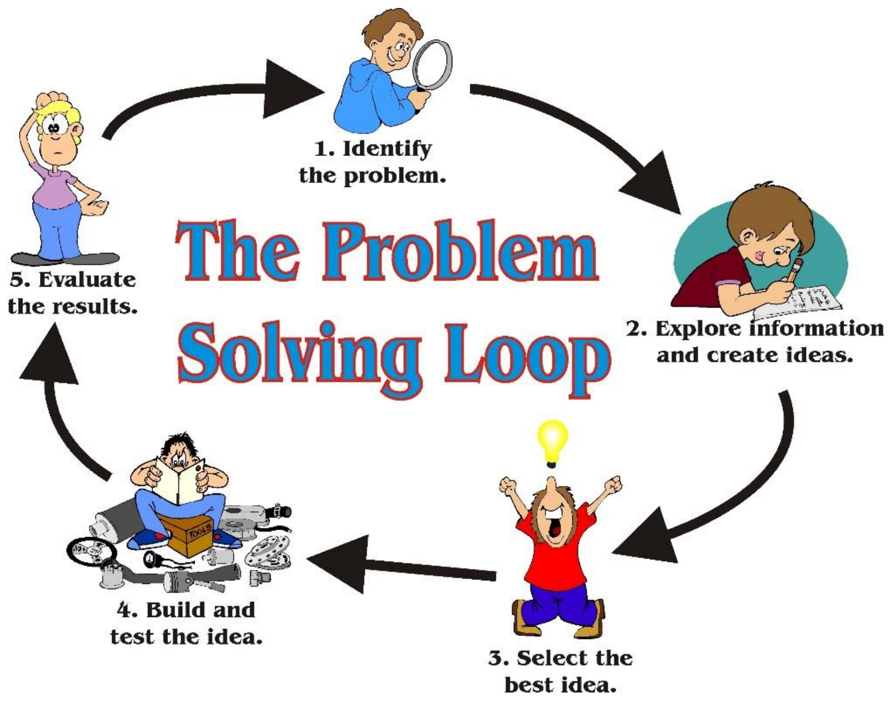
Problem solving

- Identification of a solution
  - Computer science – search process
  - Engineering and mathematics – optimization process

- How?
  - Representation of (partial) solutions – points in the search space
  - Design of search operators – transform a possible solution in a new solution

The problem solving loop

- Problem definition
- Problem analysis
- Choose problem solving technique
  - **Search**
  - Knowledge representation
  - Abstraction



Problem solving steps

- Choose a problem solving technique
  - Solve using rules (and a control strategy) to move in the search space until a path from the initial state to the final one is identified
  - Solve using search
    - Sistematically analyse states in order to identify:
      - A path from initial state to the final one
      - An optimal state
      - Search space – all possible states and the operators that allow moving from a state to another
    - How to choose the search strategy?
      - Computational complexity (run time and space)
      - Completeness – the algorithm always ends and finds a solution if one exists
      - Optimality – the algorithm finds the optimal solution

Problem solving by search

- Many search strategies – how to choose one?
  - Computational complexity
    - Performance depends on:
      - Time needed to run the algorithm
      - Space (memory) needed for the run
      - Size of the input data
      - Computer speed
      - Processor quality

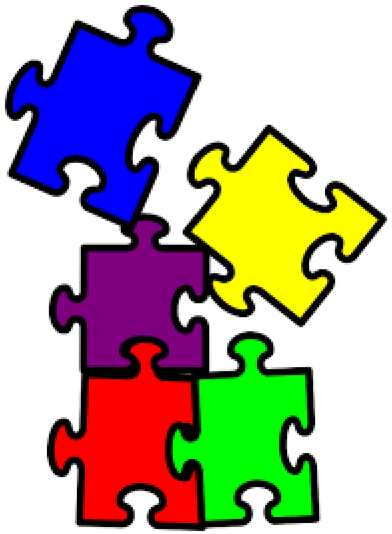
Internal factors

External factors

  - Measured using complexity – ***Computational Efficiency***
    - **Space** – memory needed to identify the solution
      - ***S(n)*** – quantity of memory used by the best algorithm A which solves a decision problem f with input data of size n
    - **Time** – time needed to identify the solution
      - ***T(n)*** – running time (number of steps) used by the best algorithm A for a decision problem f with input data of size n

Problem solving by search

- Solving problems by search can mean:
  - Build the solution step by step
  - Identify the potential optimal solution



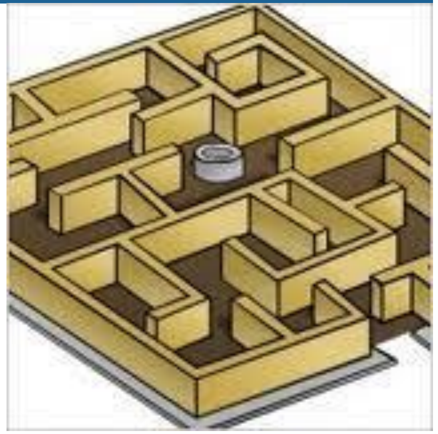
Problem solving by search

- Solving problems by search using standard methods

  - Exact methods
    - **Generate and test**
      - **Backtracking**
    - **Divide and conquer**
    - Dynamic programming

  - Heuristic methods
    - Greedy method

Generate and test



- Basic idea
  - Generate a possible solution and verify if it’s correct
  - Trial and error
  - Exhaustive search
- Mechanism
  - Generate: determine all possible solutions
  - Test: search solutions that are correct (satisfy some conditions)
- When to use it?
  - Problems that can have multiple solutions
  - Problems with restrictions (solutions need to satisfy some conditions)

Generate and test

- Algorithm

```python
#D = D(D1) = D(D1(D2))...
def generate_test(D):
    while (True):
        sol = generate_solution()
        if (test(sol) == True):
            return sol
```

1. Generate a possible solution
2. Test is solution is correct
3. Quit if a solution is found, return to step 1 otherwise

➢ This is not backtracking

Generate and test

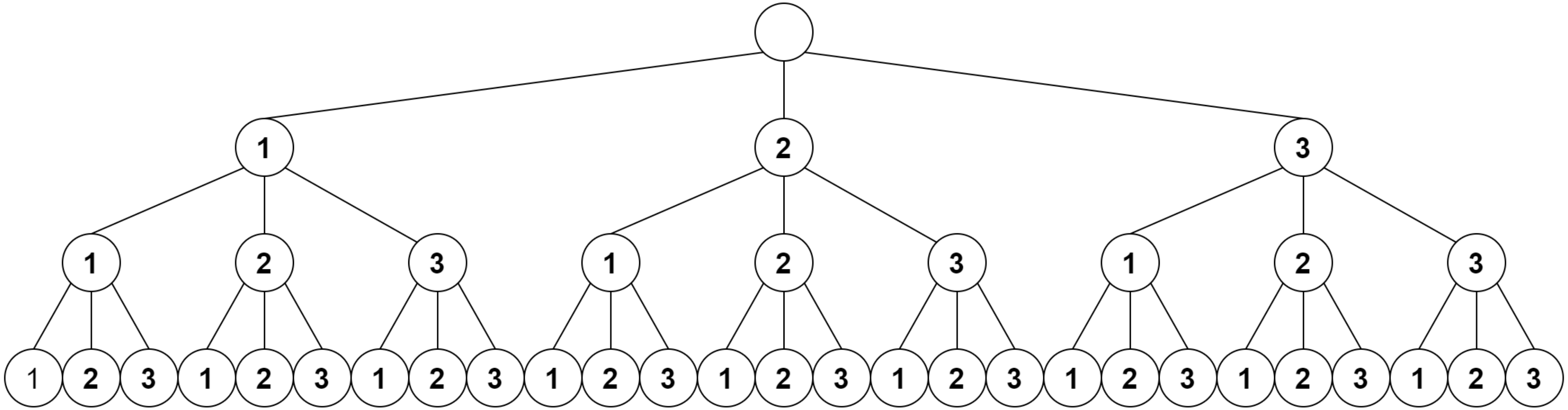
- Example: generate permutations with n=3 elements

```
def permut3():
    for i in range(1,4):
        for j in range(1,4):
            for k in range(1,4):
                # generate
                possibleSolution = [i,j,k]
                #test
                if i!=j and j!=k and k!=i:
                    yield possibleSolution

def callPermut3():
    for p in permut3():
        print(p)

callPermut3()
```

```
[1, 2, 3]
[1, 3, 2]
[2, 1, 3]
[2, 3, 1]
[3, 1, 2]
[3, 2, 1]
```

Generate and test

- Example: generate permutations with n=3 elements
- Complexity
  - Number of possible solutions: 3^3 (which is n^n)



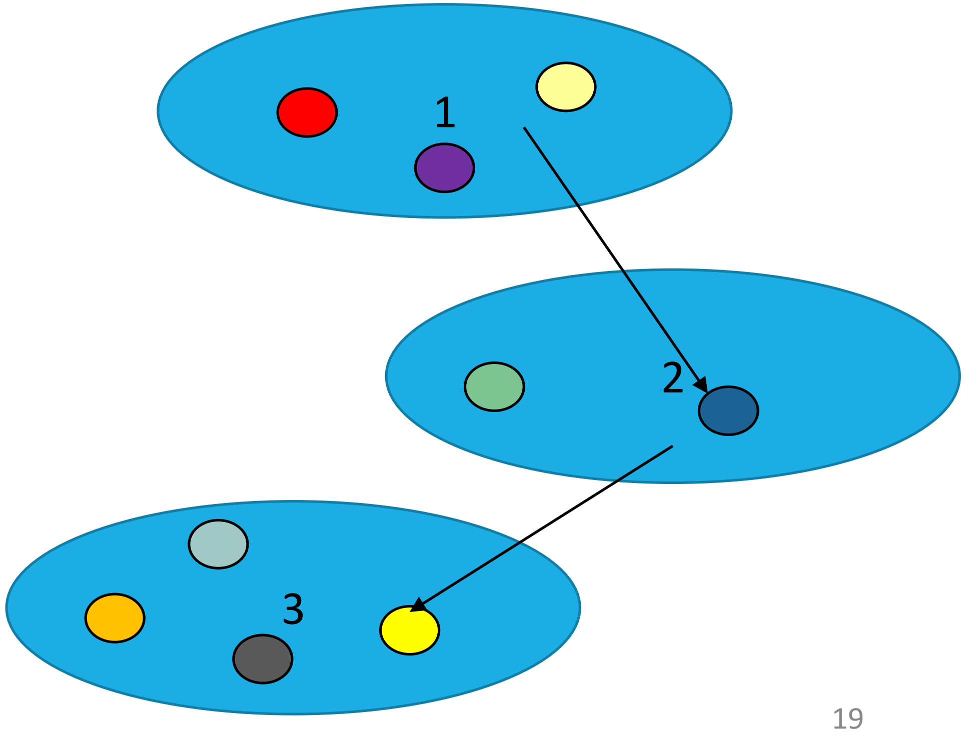
Generate and test

- Possible improvements
  - Do not explore all possible solutions
    - Example: when i= 1 there is no point to verify j=1 and k=1 because this can not lead to a possible solution
  - Build (partially) correct solutions
    - That satisfy certain conditions

```
#D = D(D1) = D(D1(D2))...
def generate_test(D):
    while (True):
        sol = generate_solution_cond()
        if (test(sol) == True):
            return sol
```

Backtracking

- Brute-force technique for finding solutions, with the main characteristic that it has the ability to undo – *backtrack* - when a potential solution is not valid
- Basic idea:
  - Try every possibility to see if it's a solution
    - unless we already know it's not valid
  - Sequence of choices
    - Once a choice is selected....another choice
    - If bad choice => backtrack
    - Until the solution is perfectly valid



Backtracking

- Search space of a solution **s** is ***S*** (definition domain)
- A solution is formed of several elements *s[0], s[1], s[2],...*
- *init:* function that generates an empty value for the definition domain of the solution
- *getNext:* function that returns the next element from the definition domain
- *isConsistent:* function that verifies if a (partial) solution is consistent
- *isSolution:* function that verifies if a (partial) solution is a final (complete) solution of the problem

Backtracking: Iterative version

Generate permutations with n elements

```
def init():
    return 0

def getNext(sol, pos):
    return sol[pos] + 1

def isConsistent(sol):
    isCons = True
    i = 0
    while (i<len(sol)-1) and (isCons==True):
        if (sol[i] == sol[len(sol) - 1]):
            isCons = False
        else:
            i = i + 1
    return isCons

def isSolution(solution, n):
    return len(solution) == n
```

```
def permut_back(n):
    k = 0; solution = []
    initValue = init()
    solution.append(initValue)
    while (k >= 0):
        isSelected = False
        while (isSelected==False) and (solution[k]<n):
            solution[k] = getNext(solution, k)
            isSelected = isConsistent(solution)
        if (isSelected == True):
            if (isSolution(solution,n) == True):
                yield solution
            else:
                k = k + 1
                solution.append(init())
        else:
            del(solution[k])
            k = k - 1

def callPermut():
    for p in permut_back(3):
        print(p)

callPermut()
```

Backtracking: Recursive version

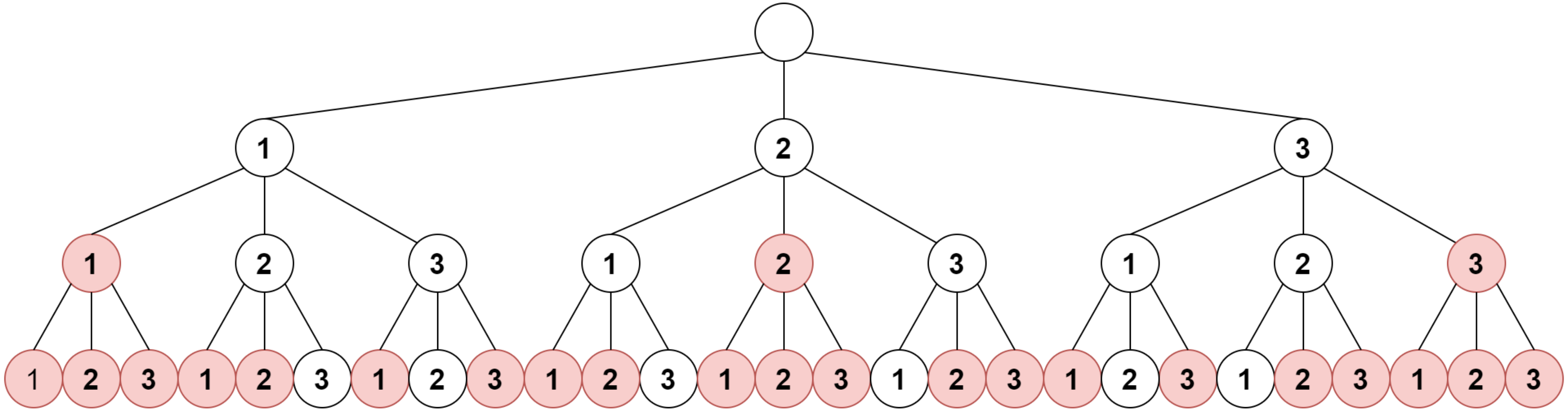
Generate permutations with n elements

```python
def init():
    return 0

def getNext(sol, pos):
    return sol[pos] + 1

def isConsistent(sol):
    isCons = True
    i = 0
    while (i<len(sol)-1) and (isCons==True):
        if (sol[i] == sol[len(sol) - 1]):
            isCons = False
        else:
            i = i + 1
    return isCons

def isSolution(solution, n):
    return len(solution) == n
```

```python
def permut_back_rec(n, solution):
    initValue = init()
    solution.append(initValue)
    elem = getNext(solution, len(solution) - 1)
    while (elem <= n):
        solution[len(solution) - 1] = elem
        if (isConsistent(solution) == True):
            if (isSolution(solution, n) == True):
                yield solution
            else:
                yield from permut_back_rec(n, solution[:])
        elem = getNext(solution, len(solution) - 1)

def callPermutRec():
    for p in permut_back_rec(3, []):
        print(p)

callPermutRec()
```

Backtracking

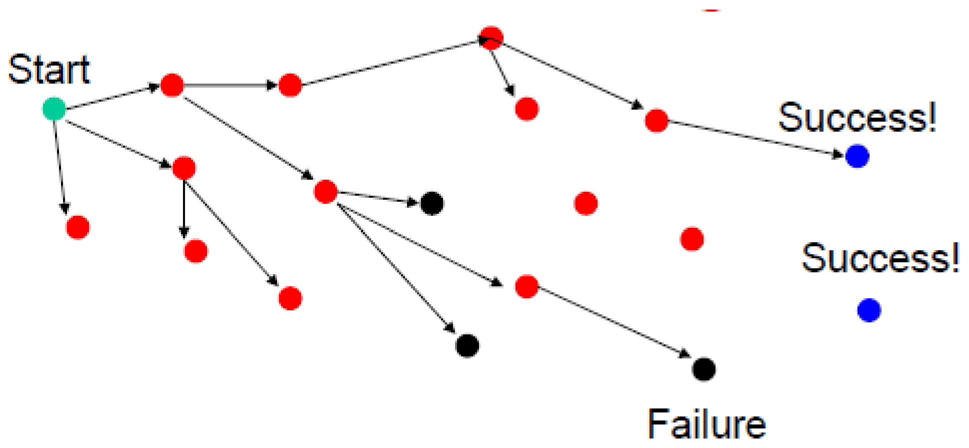
- Nodes explored for generating permutations with n=3 elements



Recap: How to use backtracking

- Represent the solution as a vector: *s[0], s[1], s[2],...*
- Define what a valid solution candidate is

(filter out candidates that will not lead to a solution)



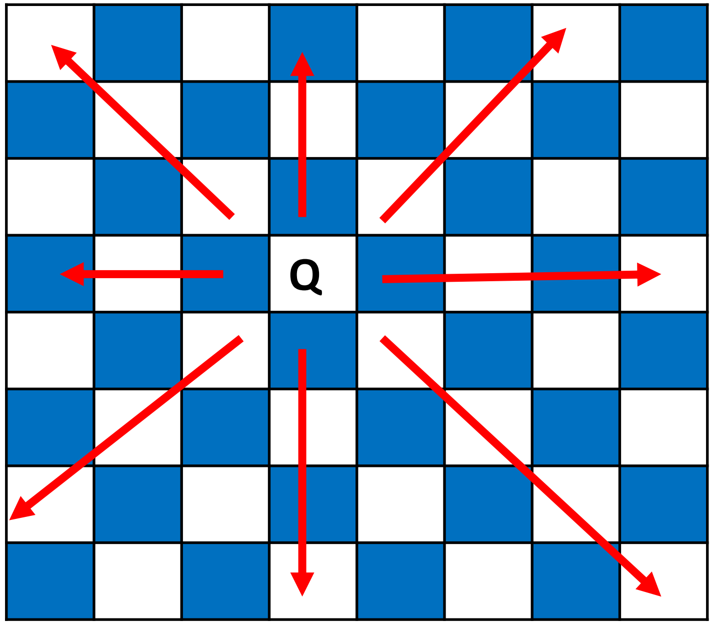
Remember:

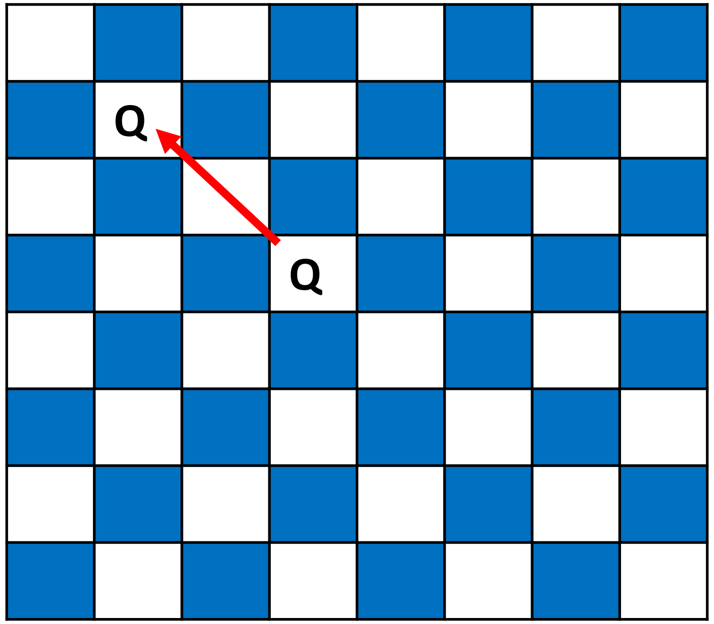
- Problem space: states (nodes) and actions (paths that lead to new states)
- If a node leads to failure go back and try other alternatives

Backtracking: Example

8 queens

- 8 queens
  - Classic backtracking problem
  - Place 8 queens on an 8x8 chessboard so that no queen can attack another





Backtracking: Example

8 queens

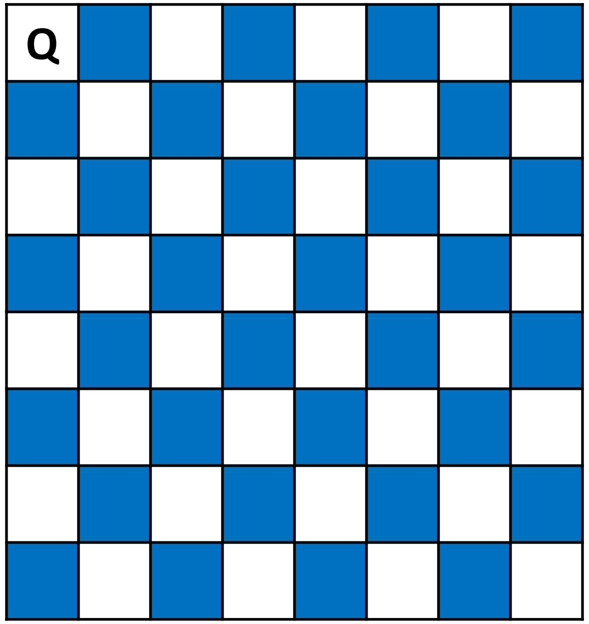
- 8x8 chessboard => 64 locations
  - After placing one queen => 63 locations to choose from
  - ....
  - 64*63*62*61*60*59*58*57 = 178,462,987,637,760 possibilities

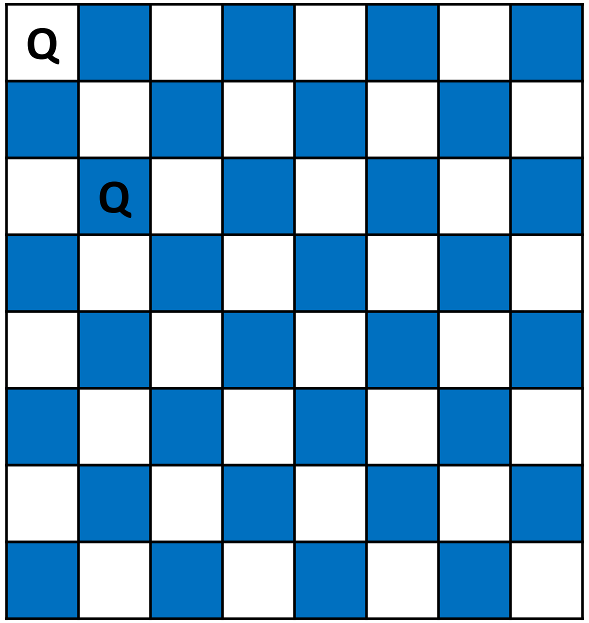
- However:
  - A valid solution has:
    - exactly 1 queen in each row and exactly 1 queen in each column
  - Explore 1 queen per column (not per cell)
  - Possibilities reduced to 8^8 = 16,777,216

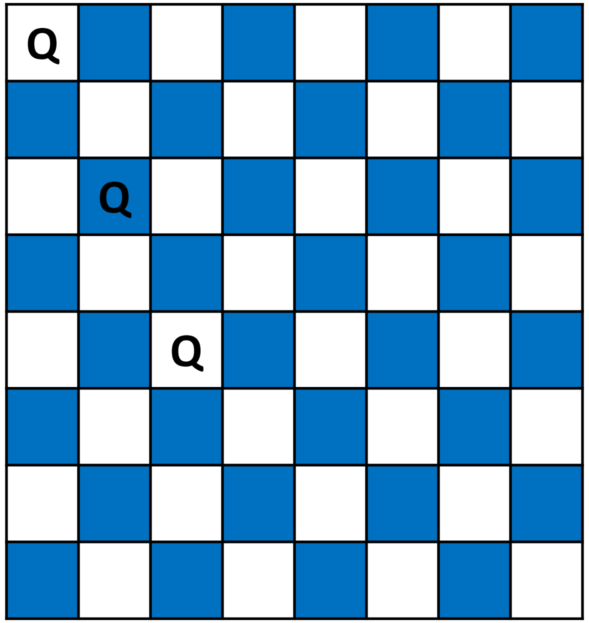
Backtracking: Example

8 queens

- Make a choice for first column
- The second choice is affected by the first choice, etc







Backtracking: Example

N queens

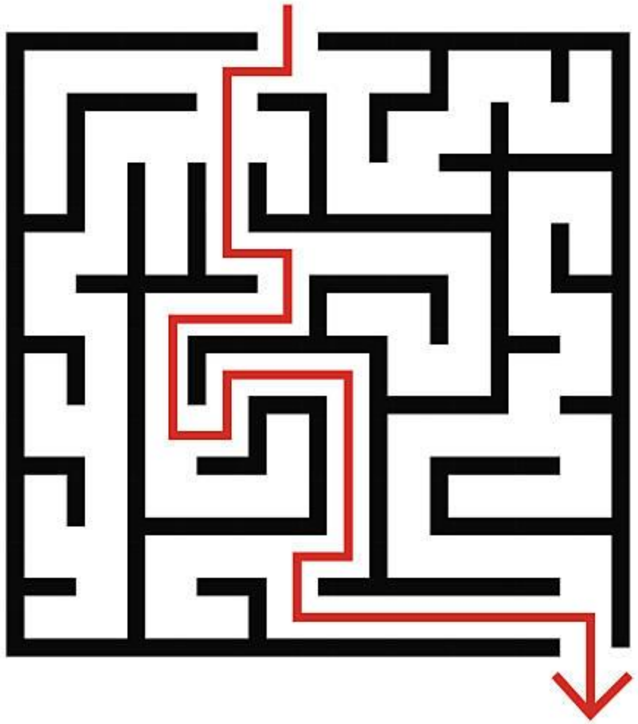
```
def initSolution():
    solution = [[0 for i in range(n)] for j in range(n)]
    return solution

def printSolution(solution):
    for row in solution:
        print(row)
```

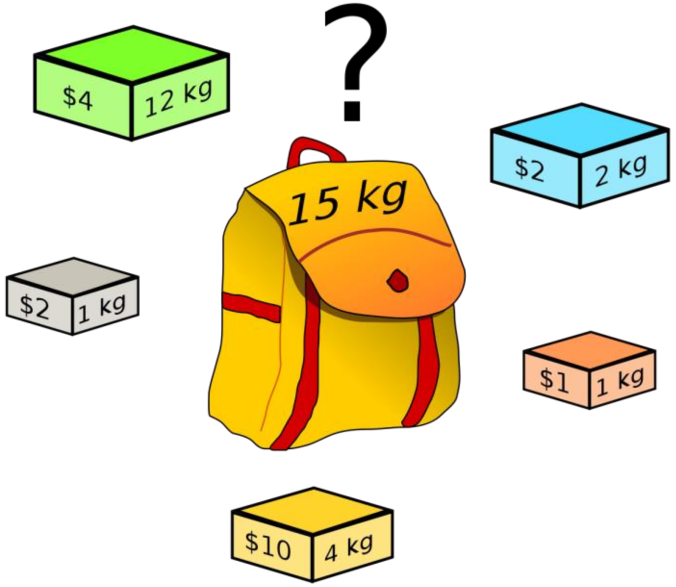
```
def isConsistent(solution, row, column):
    # check the row
    for j in range(column):
        if solution[row][j] == 1:
            return False

    # check the first diagonal to left (up)
    for i,j in zip(range(row,-1,-1), range(column,-1,-1)):
        if solution[i][j] == 1:
            return False

    # check the second diagonal to left (down)
    i = row + 1
    j = column - 1
    while (i < len(solution)) and (j >= 0):
        if solution[i][j] == 1:
            return False
        i = i + 1
        j = j - 1

    return True
```

```
def solveProblem(solution, column):
    if column >= n:
        print("COMPLETE solution:")
        printSolution(solution)
        return True

    for i in range(n):
        if isConsistent(solution, i, column):
            solution[i][column] = 1
            print("Partial correct solution:")
            printSolution(solution)
            if solveProblem(solution, column + 1) == True:
                return True
            else:
                solution[i][column] = 0
    return False

n=8
solveProblem(sol, 0)
```

Backtracking: other examples

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9





Divide et impera – Divide and conquer

- Basic idea
  - Divide the problem in several independent sub-problems similar to the initial problem but smaller in size and determine the final solution by combining sub-solutions
- Mechanism
  - **Divide**: breaking the problem in sub-problems
  - **Conquer**: solve the sub-problems
  - **Combine**: combine sub-solutions to obtain final solution
- When it can be used
  - A problem ***P*** with the input data ***D*** can be solved by solving the same problem ***P*** but with input data ***d***, where ***d* < *D***

Divide et impera – Divide and conquer

- Algorithm

```
#D = d1 U d2 U d3...U dn
def div_imp(D):
    if (size(D) < lim):
        return rez
    rez1 = div_imp(d1)
    rez2 = div_imp(d2)
    ...
    rezn = div_imp(dn)
    return combine(rez1, rez2, ..., rezn)
```

Divide et impera – Divide and conquer

- Example: find the maximum of a list
  - Size of problem = n
  - First version
    - Size of sub-problem 1 = n-1
    - Size of sub-problem 2 = n-2
    - ...
    - *meaning:*
      - D = l = [l1,l2,..,ln]
      - d1=[l2,..,ln]
      - d2=[l3,..,ln]
      - ...
  - *O(n)*

```python
def findMax(l):
    '''
    Descr: finds the maximum elem of a list
    Input: a list
    Output: the maximum elem of list
    '''
    if (len(l) == 1):
        return l[0]
    max = findMax(l[1:])
    if (max > l[0]):
        return max
    else:
        return l[0]

def test_findMax():
    assert findMax([2,5,3,6,1]) == 6
    assert findMax([12,5,3,2,1]) == 12
    assert findMax([2,5,3,6,11]) == 11

test_findMax()
```

Divide et impera – Divide and conquer

- Example: find the maximum of a list
  - Size of problem = n
  - Second version
    - Size of sub-problem 1 = n/2
    - Size of sub-problem 2 = n/2

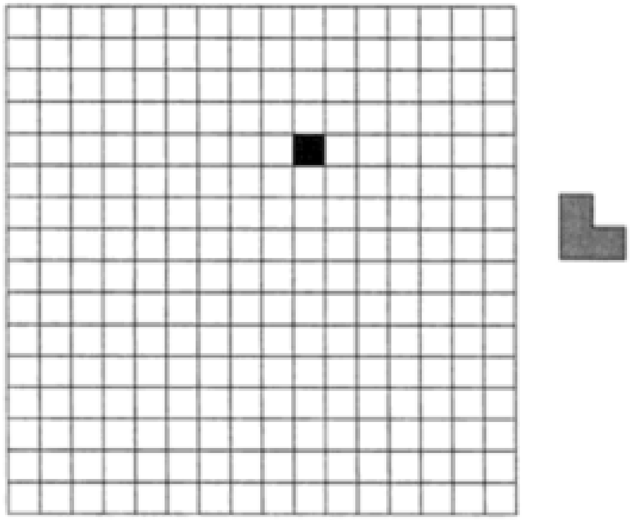
    - *meaning:*
      - D = l = [l1,l2,..,ln]
      - d1=[l2,..,ln/2]
      - d2=[ln/2+1,..,ln]

  - *O(n)*

```
def findMax_v2(l):
    '''
    Descr: finds the maximum elem of a list
    Data: a list
    Res: the maximal elem of list
    '''
    if (len(l) == 1):
        return l[0]
    middle = len(l) // 2
    max_left = findMax_v2(l[0:middle])
    max_right = findMax_v2(l[middle:len(l)])
    if (max_left < max_right):
        return max_right
    else:
        return max_left

def test_findMax_v2():
    assert findMax_v2([2,5,3,6,1]) == 6
    assert findMax_v2([12,5,3,2,1]) == 12
    assert findMax_v2([2,5,3,6,11]) == 11

test_findMax_v2()
```

Divide et impera – Example

- Consider a chessboard of size 2^m (with 2^m X 2^m cells) that contains a hole (one random cell is removed)
- We have several shapes L
- Objective: cover the chessboard with L shapes (any orientation)

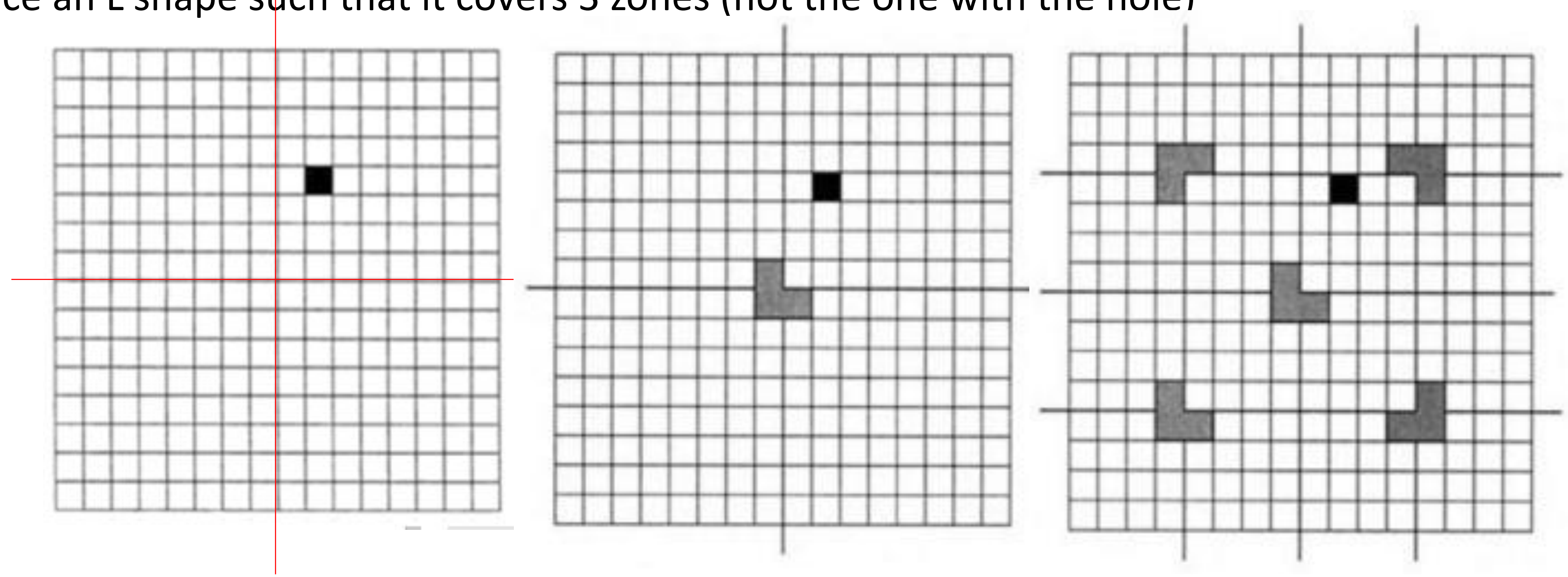


m=4 => chessboard16 X 16

- ✓ Search space: possible arrangements of L shapes on the board
- ✓ D&C is an ideal method

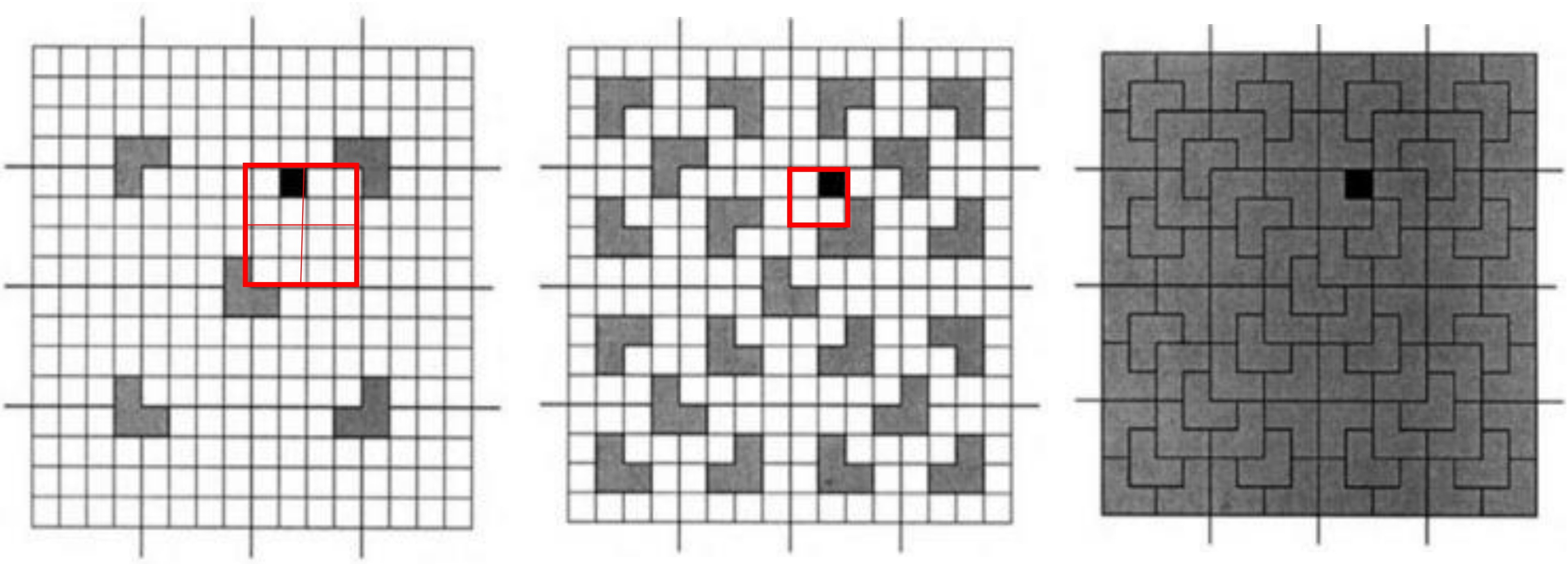
Divide et impera – Example

- Divide the chessboard in 4 equal zones
- Only one will contain the hole
- Place an L shape such that it covers 3 zones (not the one with the hole)



Divide et impera – Example

- Each square has a single black cell



Final solution

Recap today

- Problem solving methods
- Generate and test
  - Exhaustive
  - Backtracking
- Divide and conquer

Next time

- Algorithms
  - Dynamic programming
  - Greedy method

Reading materials and useful links

1. The Python Programming Language - https://www.python.org/
2. The Python Standard Library - https://docs.python.org/3/library/index.html
3. The Python Tutorial - https://docs.python.org/3/tutorial/
4. M. Frentiu, H.F. Pop, Fundamentals of Programming, Cluj University Press, 2006.
5. MIT OpenCourseWare, Introduction to Computer Science and Programming in Python, https://ocw.mit.edu, 2016.
6. K. Beck, Test Driven Development: By Example. Addison-Wesley Longman, 2002. http://en.wikipedia.org/wiki/Test-driven_development
7. M. Fowler, Refactoring. Improving the Design of Existing Code, Addison-Wesley, 1999. http://refactoring.com/catalog/index.html

Bibliography

The content of this course has been prepared using the reading materials from previous slide, different sources from the Internet as well as lectures on Fundamentals of Programming held in previous years by:

- Prof. Dr. Laura Dioşan - www.cs.ubbcluj.ro/~lauras
- Conf. Dr. Istvan Czibula - www.cs.ubbcluj.ro/~istvanc
- Lect. Dr. Andreea Vescan - www.cs.ubbcluj.ro/~avescan
- Lect. Dr. Arthur Molnar - www.cs.ubbcluj.ro/~arthur