DATA STRUCTURES AND ALGORITHMS

LECTURE 5

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2017 - 2018

In Lecture 4...

- Linked Lists
  - Singly Linked Lists
  - Doubly Linked Lists

Today

- Sorted Lists
- Circular Lists
- Linked Lists on Arrays

Algorithmic problems using Linked Lists

- Find the n^{th} node from the end of a SLL.

Algorithmic problems using Linked Lists

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- Simple approach: go through all elements to count the length of the list. When we know the length, we know at which position the n^{th} node from the end is. Start again from the beginning and go to the computed position.
- Can we do it in one single pass over the list?

Algorithmic problems using Linked Lists

- Find the n^{th} node from the end of a SLL.
- Simple approach: go through all elements to count the length of the list. When we know the length, we know at which position the n^{th} node from the end is. Start again from the beginning and go to the computed position.
- Can we do it in one single pass over the list?
- We need to use two auxiliary variables, two nodes, both set to the first node of the list. At the beginning of the algorithm we will go forward $n-1$ times with one of the nodes. Once the first node is at the n^{th} position, we move with both nodes in parallel. When the first node gets to the end of the list, the second one is at the n^{th} element from the end of the list.

N-th node from the end of the list

function findNthFromEnd (sll, n) **is:**
//pre: sll is a SLL, n is an integer number
//post: the n-th node from the end of the list or NIL
    oneNode ← sll.head
    secondNode ← sll.head
    position ← 1
    while position < n **and** oneNode ≠ NIL **execute**
        oneNode ← [oneNode].next
        position ← position + 1
    end-while
    if oneNode = NIL **then**
        findNthFromEnd ← NIL
    else
    //continued on the next slide...

N-th node from the end of the list

```
    while [oneNode].next ≠ NIL execute
      oneNode ← [oneNode].next
      secondNode ← [secondNode].next
    end-while
    findNthFromEnd ← secondNode
  end-if
end-function
```

N-th node from the end of the list

```
    while [oneNode].next ≠ NIL execute
        oneNode ← [oneNode].next
        secondNode ← [secondNode].next
    end-while
    findNthFromEnd ← secondNode
  end-if
end-function
```

- Is the second approach more efficient than the first one?

Think about it

- Given the first node of a SLL, determine whether the list ends with a node that has NIL as *next* or whether it ends with a cycle (the *last* node contains the address of a previous node as *next*).
- If the list from the previous problems contains a cycle, find the length of the cycle.
- Find if a SLL has an even or an odd number of elements, without counting the number of nodes in any way.
- Reverse a SLL non-recursively in linear time using $\Theta(1)$ extra storage.

Sorted Lists

- A *sorted list* (or ordered list) is a list in which the elements from the nodes are in a specific order, given by a *relation*.
- This *relation* can be $<$, $\leq$, $>$ or $\geq$, but we can also work with an abstract relation.
- Using an abstract relation will give us more flexibility: we can easily change the relation (without changing the code written for the sorted list) and we can have, in the same application, lists with elements ordered by different relations.

The relation

- You can imagine the *relation* as a function with two parameters (two *TComp* elems):

$$relation(c_1, c_2) = \begin{cases} -1, & c_1 \text{ should be in front of } c_2 \\ 0, & c_1 \text{ and } c_2 \text{ are equal for this relation} \\ 1, & c_2 \text{ should be in from of } c_1 \end{cases}$$

Sorted List - representation

- When we have a sorted list (or any sorted structure or container) we will keep the relation used for ordering the elements as part of the structure. We will have a field that represents this relation.

- In the following we will talk about a *sorted singly linked list* (representation and code for a *sorted doubly linked list* is really similar).

Sorted List - representation

- We need two structures: *Node - SSLLNode* and *Sorted Singly Linked List - SSLL*

SSLLNode:
info: TComp
next: ↑ SSLLNode

SSLL:
head: ↑ SSLLNode
rel: ↑ Relation

SSLL - Initialization

- The relation is passed as a parameter to the *init* function, the function which initializes a new SSLL.
- In this way, we can create multiple SSLLs with different relations.

subalgorithm init (ssll, rel) **is:**
//pre: rel is a relation
//post: ssll is an empty SSLL
  ssll.head $\leftarrow$ NIL
  ssll.rel $\leftarrow$ rel
end-subalgorithm

- Complexity: $\Theta(1)$

SSLL - Operations

- The main difference between the operations of a SLL and a SSLL is related to the insert operation:
  - For a SLL we can insert at the beginning, at the end, at a position, after/before a given element (so we can have multiple insert operations).
  - For a SSLL we have only one insert operation: we no longer can decide where an element should be placed, this is determined by the relation.
- We can still have multiple delete operations.
- We can have search and get element from position operations as well.

SSLL - insert

- Since we have a singly-linked list we need to find the node *after* which we insert the new element (otherwise we cannot set the links correctly).
- The node we want to insert after is the first node whose successor is *greater than* the element we want to insert (where *greater than* is represented by a value 1 returned by the relation).
- We have two special cases:
  - an empty SSLL list
  - when we insert before the first node

SSLL - insert

subalgorithm insert (ssll, elem) **is:**
//pre: ssll is a SSLL; elem is a TComp
//post: the element elem was inserted into ssll to where it belongs
    newNode ← allocate()
    [newNode].info ← elem
    [newNode].next ← NIL
    if ssll.head = NIL **then**
    //the list is empty
        ssll.head ← newNode
    else if ssll.rel(elem, [ssll.head].info) = -1 **then**
    //elem is "less than" the info from the head
        [newNode].next ← ssll.head
        ssll.head ← newNode
    else
//continued on the next slide...

SSLL - insert

```
        cn ← ssll.head //cn - current node
        while [cn].next ≠ NIL and ssll.rel(elem, [[cn].next].info) > 0 execute
            cn ← [cn].next
        end-while
        //now insert after cn
        [newNode].next ← [cn].next
        [cn].next ← newNode
    end-if
end-subalgorithm
```

- Complexity: $O(n)$

SSLL - Other operations

- The search operation is identical to the search operation for a SLL (except that we can stop looking for the element when we get to the first element that is "greater than" the one we are looking for).
- The delete operations are identical to the same operations for a SLL.
- The return an element from a position operation is identical to the same operation for a SLL.
- The iterator for a SSLL is identical to the iterator to a SLL (discussed in Lecture 3).

SSLL in action

- We define a function that receives as parameter two integer numbers and compares them:

```
function compareGreater(e1, e2) is:
//pre: e1, e2 integer numbers
//post: compareGreater returns -1 if e1 < e2; 0 if they are equal;
//and 1 if e1 > e2
    if e1 < e2 then
        compareGreater ← -1
    else if e1 = e2 then
        compareGreater ← 0
    else
        compareGreater ← 1
    end-if
end-function
```

SSLL in action

- We define another function that compares two integer numbers based on the sum of their digits

```
function compareGreaterSum(e1, e2) is:
//pre: e1, e2 integer numbers
//post: compareGreaterSum returns -1 if the sum of digits of e1 is less than
//that of e2; 0 if the sums are equal; 1 if sum for e1 is greater
    sumE1 ← sumOfDigits(e1)
    sumE2 ← sumOfDigits(e2)
    if sumE1 < sumE2 then
        compareGreaterSum ← -1
    else if sumE1 = sumE2 then
        compareGreaterSum ← 0
    else
        compareGreaterSum ← 1
    end-if
end-function
```

SSLL in action

- Suppose that the *sumOfDigits* function - used on the previous slide - is already implemented
- We define a subalgorithm that prints the elements of a SSLL using an iterator:

```
subalgorithm printWithIterator(ssll) is:
//pre: ssll is a SSLL; post: the content of ssll was printed
    iterator(ssll, it) //create an iterator for ssll
    while valid(it) execute
        getCurrent(it, elem)
        write elem
        next(it)
    end-while
end-subalgorithm
```

SSLL in action

- Now that we have defined everything we need, let's write a short main program, where we create a new SSLL and insert some elements into it and print its content.

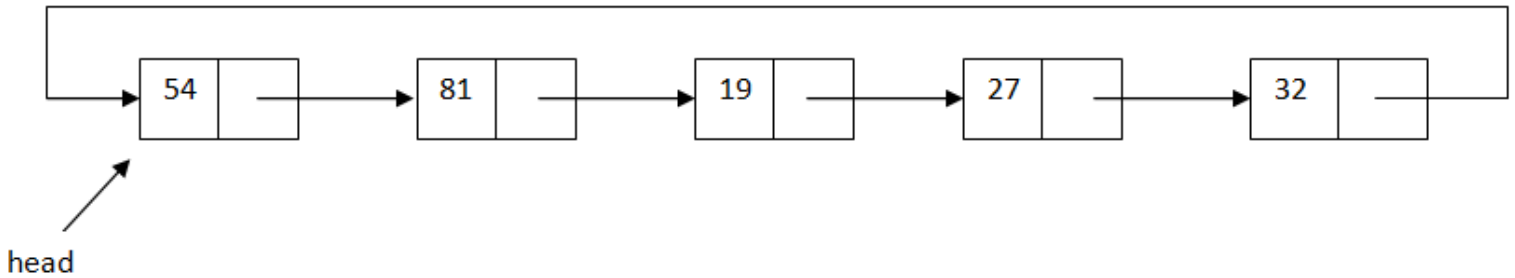
```
subalgorithm main() is:
    init(ssll, compareGreater) //use compareGreater as relation
    insert(ssll, 55)
    insert(ssll, 10)
    insert(ssll, 59)
    insert(ssll, 37)
    insert(ssll, 61)
    insert(ssll, 29)
    printWithIterator(ssll)
end-subalgorithm
```

SSLL in action

- Executing the *main* function from the previous slide, will print the following: 10, 29, 37, 55, 59, 61.
- Changing only the relation in the *main* function, passing the name of the function *compareGreaterSum*, instead of *compareGreater* as a relation, the order in which the elements are stored, and the output of the function changes to: 10, 61, 37, 55, 29, 59
- Moreover, if I need to, I can have a list with the relation *compareGreater* and another one with the relation *compareGreaterSum*. This is the flexibility that we get by using abstract relations for the implementation of a sorted list.

Circular Lists

- For a SLL or a DLL the last node has as *next* the value *NIL*. In a *circular list* no node has *NIL* as next, since the last node contains the address of the first node in its next field.



Circular Lists

- We can have singly linked and doubly linked circular lists, in the following we will discuss the singly linked version.
- In a circular list each node has a successor, and we can say that the list does not have an end.
- We have to be careful when we iterate through a circular list, because we might end up with an infinite loop (if we set as stopping criterion the case when *currentNode* or *[currentNode].next* is *NIL*.
- There are problems where using a circular list makes the solution simpler (for example: Josephus circle problem, rotation of a list)

Circular Lists

- Operations for a circular list have to consider the following two important aspects:
  - The *last* node of the list is the one whose *next* field is the *head* of the list.
  - Inserting before the head, or removing the head of the list, is no longer a simple $\Theta(1)$ complexity operation, because we have to change the *next* field of the last node as well (and for this we have to find the last node).

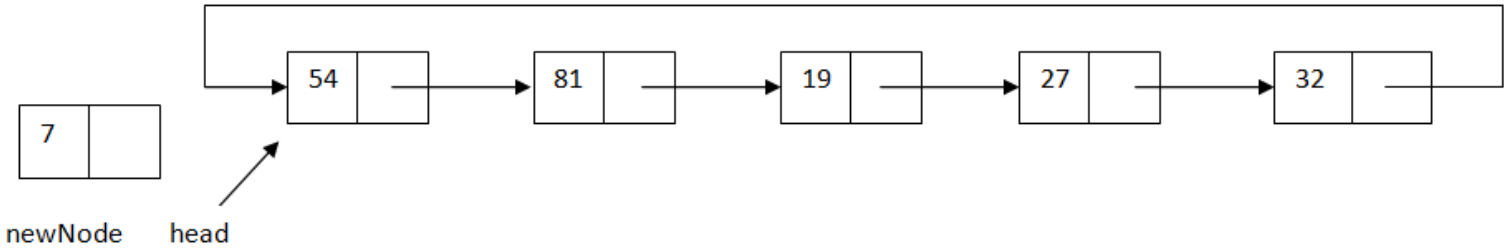
Circular Lists - Representation

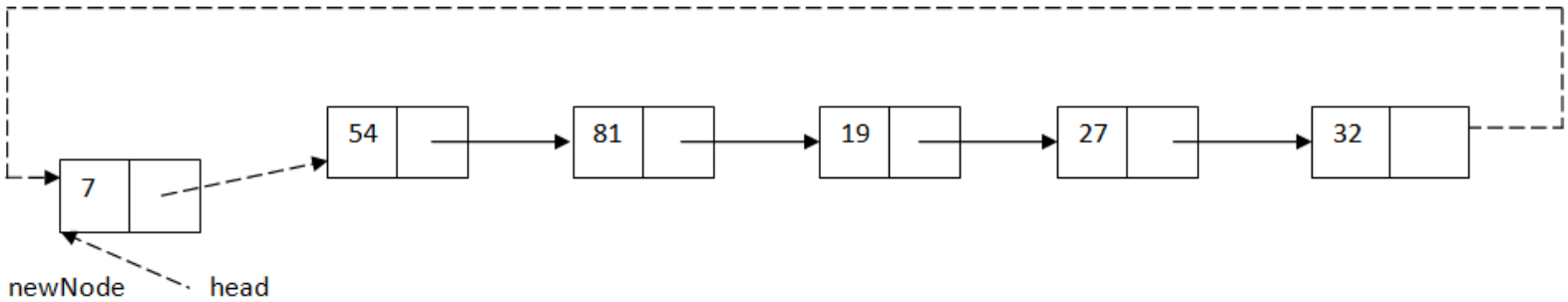
- The representation of a circular list is exactly the same as the representation of a simple SLL. We have a structure for a *Node* and a structure for the *Circular Singly Linked Lists - CSLL*.

CSLLNode:
  info: TElem
  next: ↑ CSLLNode

CSLL:
  head: ↑ CSLLNode

CSLL - InsertFirst





CSLL - InsertFirst

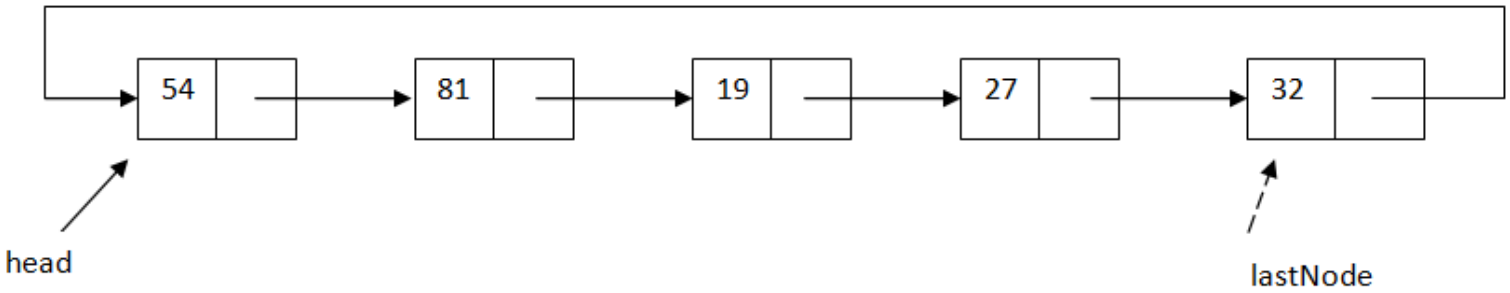
```
subalgorithm insertFirst (csll, elem) is:
//pre: csll is a CSLL, elem is a TElem
//post: the element elem is inserted at the beginning of csll
    newNode ← allocate()
    [newNode].info ← elem
    [newNode].next ← newNode
    if csll.head = NIL then
        csll.head ← newNode
    else
        lastNode ← csll.head
        while [lastNode].next ≠ csll.head execute
            lastNode ← [lastNode].next
        end-while
//continued on the next slide...
```

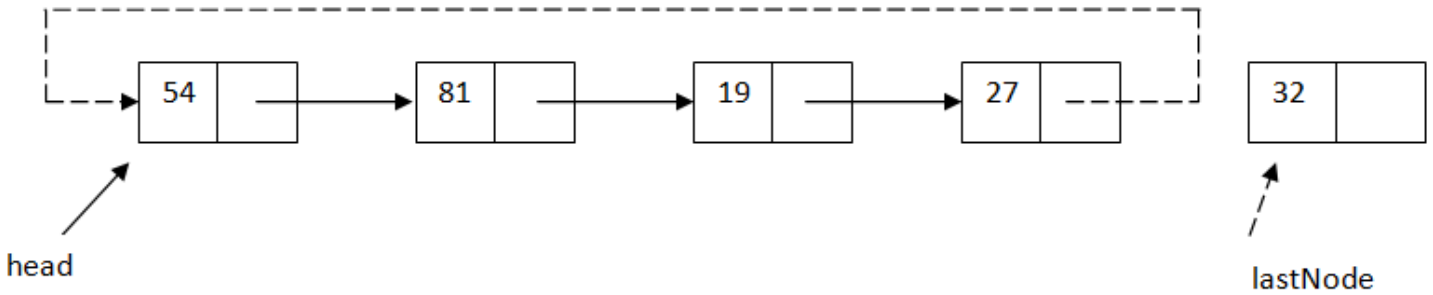
CSLL - InsertFirst

```
    [newNode].next ← csll.head
    [lastNode].next ← newNode
    csll.head ← newNode
  end-if
end-subalgorithm
```

- Complexity: $\Theta(n)$
- Note: inserting a new element at the end of a circular list looks exactly the same, but we do not modify the value of *csll.head* (so the last instruction is not needed).

CSLL - DeleteLast





CSLL - DeleteLast

```
function deleteLast(csll) is:
//pre: csll is a CSLL
//post: the last element from csll is removed and the node
//containing it is returned
    deletedNode ← NIL
    if csll.head ≠ NIL then
        if [csll.head].next = csll.head then
            deletedNode ← csll.head
            csll.head ← NIL
        else
            prevNode ← csll.head
            while [[prevNode].next].next ≠ csll.head execute
                prevNode ← [prevNode].next
            end-while
//continued on the next slide...
```

```
        deletedNode ← [prev].next
        [prev].next ← csll.head
    end-if
  end-if
  [deletedNode].next ← NIL
  deleteLast ← deletedNode
end-function
```

- Complexity: $\Theta(n)$

CSLL - Iterator

- How can we define an iterator for a CSLL?
- The main problem with the *standard* SLL iterator is its *valid* method. For a SLL *valid* returns false, when the value of the *currentElement* becomes *NIL*. But in case of a circular list, *currentElement* will never be *NIL*.
- We have finished iterating through all elements when the value of *currentElement* becomes equal to the *head* of the list.
- However, writing that the iterator is invalid when *currentElement* equals the *head*, will produce an iterator which is invalid the moment it was created.

CSLL - Iterator - Possibilities

- We can say that the iterator is invalid, when the *next* of the *currentElement* is equal to the *head* of the list.
- This will stop the iterator when it is set to the last element of the list, so if we want to print all the elements from a list, we have to call the *element* operation one more time when the iterator becomes invalid (or use a do-while loop instead of a while loop - but this causes problems when we iterate through an empty list).

CSLL - Iterator - Possibilities

- We can add a boolean flag to the iterator besides the *currentElement*, something that shows whether we are at the *head* for the first time (when the iterator was created), or whether we got back to the *head* after going through all the elements.
- For this version, standard iteration code remains the same.

CSLL - Iterator - Possibilities

- Depending on the problem we want to solve, we might need a read/write iterator: one that can be used to change the content of the CSLL.
- We can have *insertAfter* - insert a new element after the current node - and *deleteAfter* - delete the element after the current node.
- We can say that the iterator is invalid when there are no elements in the circular list (especially if we delete from it).

The Josephus circle problem

- There are n men standing a circle waiting to be executed. Starting from one person we start counting into clockwise direction and execute the m^{th} person. After the execution we restart counting with the person after the executed one and execute again the m^{th} person. The process is continued until only one person remains: this person is freed.
- Given the number of men, n, and the number m, determine which person will be freed.
- For example, if we have 5 men and $m = 3$, the 4^{th} man will be freed.

Circular Lists - Variations

- There are different possible variations for a circular list that can be useful, depending on what we use the circular list for.
  - Instead of retaining the *head* of the list, retain its *tail*. In this way, we have access both to the *head* and the *tail*, and can easily insert before the head or after the tail. Deleting the head is simple as well, but deleting the tail still needs $\Theta(n)$ time.
  - Use a *header* or *sentinel* node - a special node that is considered the *head* of the list, but which cannot be deleted or changed - it is simply a separation between the head and the tail. For this version, knowing when to stop with the iterator is easier.

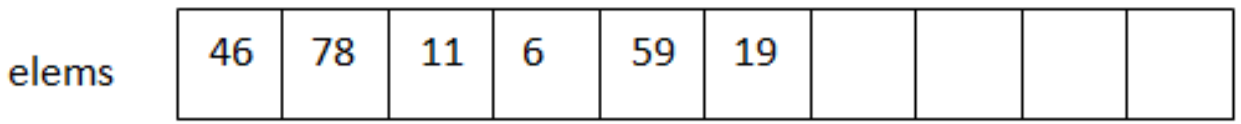
Linked Lists on Arrays

- What if we need a linked list, but we are working in a programming language that does not offer pointers (or references)?
- We can still implement linked data structures, without the explicit use of pointers or memory addresses, simulating them using arrays and array indexes.

Linked Lists on Arrays

- Usually, when we work with arrays, we store the elements in the array starting from the leftmost position and place them one after the other (no empty spaces in the middle of the list are allowed).

- The order of the elements is given by the order in which they are placed in the array.



- Order of the elements: 46, 78, 11, 6, 59, 19

Linked Lists on Arrays

- We can define a linked data structure on an array, if we consider that the order of the elements is not given by their relative positions in the array, but by an integer number associated with each element, which shows the index of the next element in the array.

elems	46	78	11	6	59	19				
next	5	6	1	-1	2	4				

head = 3

- Order of the elements: 11, 46, 59, 78, 19, 6

Linked Lists on Arrays

- Now, if we want to delete the number 46 (which is actually the second element of the list), we do not have to move every other element to the left of the array, we just need to modify the links:

elems		78	11	6	59	19				
next		6	5	-1	2	4				

head = 3

- Order of the elements: 11, 59, 78, 19, 6

Linked Lists on Arrays

- If we want to insert a new element, for example 44, at the 3^{rd} position in the list, we can put the element anywhere in the array, the important part is setting the links correctly:

elems		78	11	6	59	19		44		
next		6	5	-1	8	4		2		

head = 3

- Order of the elements: 11, 59, 44, 78, 19, 6

Linked Lists on Arrays

- When a new element needs to be inserted, it can be put to any empty position in the array. However, finding an empty position has $O(n)$ complexity, which will make the complexity of any insert operation (anywhere in the list) $O(n)$. In order to avoid this, we will keep a linked list of the empty positions as well.

elems		78	11	6	59	19		44		
next	7	6	5	-1	8	4	9	2	10	-1

head = 3

firstEmpty = 1

Linked Lists on Arrays

- In a more formal way, we can simulate a singly linked list on an array with the following:
  - an array in which we will store the elements.
  - an array in which we will store the links (indexes to the next elements).
  - the capacity of the arrays (the two arrays have the same capacity, so we need only one value).
  - an index to tell where the *head* of the list is.
  - an index to tell where the first empty position in the array is.

SLL on Array - Representation

- The representation of a singly linked list on an array is the following:

SLLA:
elems: TElem[]
next: Integer[]
cap: Integer
head: Integer
firstEmpty: Integer

SLLA - Operations

- We can implement for a SLLA any operation that we can implement for a SLL:
  - insert at the beginning, end, at a position, before/after a given value
  - delete from the beginning, end, from a position, a given element
  - search for an element
  - get an element from a position

SLLA - Init

subalgorithm init(slla) **is:**
//pre: true; post: slla is an empty SLLA
    slla.cap ← INIT_CAPACITY
    slla.elems ← @an array with slla.cap positions
    slla.next ← @an array with slla.cap positions
    slla.head ← -1
    for i ← 1, slla.cap-1 **execute**
        slla.next[i] ← i + 1
    end-for
    slla.next[slla.cap] ← -1
    slla.firstEmpty ← 1
end-subalgorithm

- Complexity: $\Theta(n)$

SLLA - Search

```
function search (slla, elem) is:
//pre: slla is a SLLA, elem is a TElem
//post: return True is elem is in slla, False otherwise
    current ← slla.head
    while current ≠ -1 and slla.elems[current] ≠ elem execute
        current ← slla.next[current]
    end-while
    if current ≠ -1 then
        search ← True
    else
        search ← False
    end-if
end-function
```

- Complexity: $O(n)$

SLLA - Search

- From the *search* function we can see how we can go through the elements of a SLLA (and how similar this traversal is to the one done for a SLL):
  - We need a *current* element used for traversal, which is initialized to the index of the *head* of the list.
  - We stop the traversal when the value of *current* becomes -1
  - We go to the next element with the instruction: *current* $\leftarrow$ *slla.next[current]*.