We consider the ranked-NNI graph (the distance on ranked topologies inherited from $\tau$-space).

**Lemma 1.** *Let $T$ be a ranked tree on $n > 2$ leaves. Then*

$$n - 1 \leq \deg(T) \leq 2(n-2).$$

*Proof.* Obvious. $\square$

**Lemma 2.** *(1)* $\frac{\deg(T)}{deg(R)} > \frac{1}{2}$.

*(2)* $deg(T) - deg(R) > n - 3$.

*Proof.* Follows from previous Lemma. $\square$

**Lemma 3.** $d_\tau(T, R) = 1 \Rightarrow |N_1(T) \cap N_1(R)| \in \{0, 1\}$.

*Proof.* Obvious. $\square$

**Lemma 4.** *Assume uniform random walk and $d_\tau(T, R) = 1$. Then*

$$\kappa(T, R) \leq \frac{1}{2(n-2)}.$$

*Proof.* Follows from Lemma 5.2 in [**?**]. $\square$

What does this Lemma say about full and topological $\tau$-spaces? In full $\tau$-space, $\kappa(T, R) \leq 0$ [**?**] for some random walk. For which one?

**Question 1.** *Assume some (e.g. uniform, lazy, ...) random walk. What are the $m$ and $M$ such that*

$$m(\deg x, \deg y, \ldots) \leq \kappa(x, y) \leq M(\deg x, \deg y, \ldots)?$$