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Disappointment and Dynamic Consistency in Choice under Uncertainty
Author(s): Graham Loomes and Robert Sugden
Source: *The Review of Economic Studies*, Vol. 53, No. 2 (Apr., 1986), pp. 271-282
Published by: Oxford University Press
Stable URL: https://www.jstor.org/stable/2297651
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Review of Economic Studies (1986) LIII, 271-282 0034-6527/86/00170271$02.00
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Disappointment and Dynamic Consistency in Choice under Uncertainty

GRAHAM LOOMES
University of York

and

ROBERT SUGDEN
University of East Anglia

The central proposition of disappointment theory is that an individual forms expectations about uncertain prospects, and that if the actual consequence turns out to be worse than (or better than) that expectation, the individual experiences a sensation of disappointment (or elation) generating a decrement (or increment) of utility which modifies the basic utility derived from the consequence. By incorporating a simple disappointment-elation function into a model of individual choice, many observed violations of conventional expected utility axioms—including violations of Savage's sure-thing principle and the "isolation effect"—can be predicted and defended as rational and dynamically consistent behaviour.

1. INTRODUCTION

A theory of rational choice under uncertainty called "regret theory" was presented simultaneously by ourselves (1982) and by David Bell (1982).[1] We showed that a number of frequently replicated violations of conventional expected utility can be predicted by a regret-rejoice model and can be defended as rational behaviour.

However, not all violations of conventional theory can be explained by regret alone. Nor does regret seem to be the only psychological experience that enters into choice behaviour. In this paper we try to show how "disappointment"—and its counterpart, "elation"—may also have an important part to play. We shall argue that although our disappointment model may explain violations which our regret model does not, and vice-versa, they have a number of features and predictions in common, and are complementary rather than competing elements in a developing alternative theory of rational choice.

2. THE MODEL

The basic propositions underlying our model are as follows. It is suggested that, when considering any uncertain prospect, an individual forms some *prior expectation* about that prospect. After the uncertainty is resolved, the individual experiences one particular consequence of the prospect: if that consequence falls short of the prior expectation, then in addition to the utility derived from the consequence itself, the individual also experiences some degree of disappointment; whereas if the consequence is better than the prior expectation, the individual also feels some measure of elation.

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This general intuition, and the terms disappointment and elation, were first suggested by Bell (1985). However, as we shall later show, our formulation of disappointment-elation, and some implications of our model, are rather different from Bell's.

To maintain compatibility with the basic framework of regret theory, we propose that the ith prospect be represented as an *action*, A_i, which is an n-tuple of state-contingent consequences. The probability that the jth state will occur is p_j where $0 < p_j \leqq 1$ and $\sum_{j=1}^n p_j = 1$. The consequence of the ith action under the jth state is denoted x_{ij}, and we assume that for any individual there exists a *basic utility*[2] function, $C(\cdot)$, unique up to an increasing linear transformation, which assigns a real-valued utility index to every conceivable consequence. For convenience, we write $C(x_{ij})$ as c_{ij}. This index is a classical cardinal measure of the ex post utility derived from any consequence in any circumstances where regret, rejoicing, disappointment or elation do not arise.

For the purposes of this paper, we shall suspend all consideration of regret and rejoicing in order to concentrate on the impact of disappointment and elation. To ease the exposition, we *initially* consider only *one-stage* gambles, although later we shall extend the analysis to include multi-stage gambles.

As a measure of the *prior expectation* of A_i, we take the value of expected basic utility, $\sum_{j=1}^n p_j c_{ij}$ which we denote $\bar{c}_i$. We represent disappointment and elation by a single differentiable real-valued function $D(\cdot)$ which assigns a decrement or increment of utility to every possible value of $c_{ij} - \bar{c}_i$, the difference between the basic utility of the consequence if the jth state occurs, and the prior expectation.

This increment or decrement of utility modifies the basic utility of any consequence in an action, so that the overall, or *modified* utility experienced under the jth state is $c_{ij} + D(c_{ij} - \bar{c}_i)$. We then denote the *expected modified utility* of the ith action as E_i where

$$E_i = \sum_{j=1}^n p_j [c_{ij} + D(c_{ij} - \bar{c}_i)]. \tag{1}$$

Our theory is that an individual tries to anticipate any disappointment or elation, and chooses so as to maximise expected modified utility. Denoting the relations of strict preference, weak preference and indifference as $\succ$, $\succcurlyeq$ and $\sim$, our theory is that for all A_i, A_k: $A_i \gtreqless A_k \Leftrightarrow E_i \gtreqqless E_k$.

Notice that since equation (1) assigns a real-valued index E_i to every action A_i, the relation $\succcurlyeq$ of weak preference must be complete, reflexive and transitive. It is clear, therefore, that this model of choice under uncertainty—unlike regret theory—cannot explain observations of non-transitive choices.

To make predictions about choice behaviour, we need to impose some restrictions on the shape of $D(\cdot)$. Our fundamental intuition, that disappointment is painful and elation pleasurable, requires that $D(c_{ij} - \bar{c}_i) \gtreqqless 0 \Leftrightarrow c_{ij} - \bar{c}_i \gtreqqless 0$. Notice that with $D(0) = 0$, an individual choosing under certainty behaves as though maximising basic utility.

Our intuition also suggests that the degree of disappointment (elation) is a non-decreasing function of the magnitude of the negative (positive) gap between outcome and prior expectation: that is, $D'(c_{ij} - \bar{c}_i) \geqq 0$.

Individuals for whom $D(\cdot)$ is linear will behave as though maximising expected basic utility, so that their behaviour will be consistent with conventional expected utility theory. However, individuals whose $D(\cdot)$ functions take any other form are liable to violate the conventional theory. In our model we assume a particular kind of non-linearity: that $D(c_{ij} - \bar{c}_i)$ is convex for all positive values of $c_{ij} - \bar{c}_i$ and concave for all negative values. The intuition here is that the intensity of disappointment and elation both increase at the margin. To preserve first order stochastic dominance preference[3] we also assume that $D'(c_{ij} - \bar{c}_i) < 1$ for all $c_{ij} - \bar{c}_i$.

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We shall make two more assumptions about the functions $D(\cdot)$ and $C(\cdot)$ mainly to simplify the exposition. Neither are *necessary* for our general results, but both are convenient.

First, we have no strong a priori argument for supposing that disappointment is in general a more intense sensation than elation, or vice versa, and so we provisionally adopt the neutral assumption that for all c_{ij}, $\bar{c}_i$: $D(c_{ij}-\bar{c}_i) = -D(\bar{c}_i - c_{ij})$.

Second, although it may be widely believed that, when consequences are increments or decrements of wealth, for many individuals $C(\cdot)$ is concave (i.e. diminishing marginal utility of wealth) we shall assume initially that $C(\cdot)$ is linear. The reason is simply to highlight the work being done in the model by the anticipation of disappointment and elation.

We shall now show how this small number of fairly weak assumptions can explain and/or predict a range of violations of conventional theory.

3. VIOLATIONS OF THE INDEPENDENCE AXIOM

Although some violations of the independence axiom, such as the common consequence and common ratio effects in the case of statistically independent actions, can be explained by regret theory, violations of the independence axiom in the form of Savage's (1954) "sure-thing principle" cannot (see Loomes and Sugden (1982 p. 813 and p. 819)). Nevertheless, such violations have been observed by Moskowitz (1974) and Slovic and Tversky (1974), among others.

To see how disappointment-elation may explain *this* evidence, consider the pair of actions depicted in Table I. All consequences are given in the form of basic utilities; $c>0$, $0<\lambda<1$, $0<p\leqq 1$. To analyse violations of the sure-thing principle, we shall hold all probabilities and consequences constant, *except the common consequence* c^*.

TABLE I

	States of the world and probabilities		
Action	S_1 λp	S_2 $(1-\lambda)p$	S_3 $1-p$
A_1	λc	λc	c^*
A_2	c	0	c^*

In Table I both actions have the same prior expectation, which we shall denote $\bar{c}$. This condition will later be dropped, but it is a convenient starting point. Applying equation (1) and simplifying, we get:

$$A_1 \gtreqless A_2 \Leftrightarrow D(\lambda c - \bar{c}) \gtreqless \lambda D(c-\bar{c}) + (1-\lambda)D(-\bar{c}). \tag{2}$$

Notice that c^* influences expression (2) only through $\bar{c}$. To see the significance of this, consider Figure 1, which portrays a disappointment-elation function that corresponds to our assumptions.[4]

A_1 is preferred to, indifferent to, or less preferred than A_2 according to whether the broken line P_1P_3 passes below, through, or above the point P_2. Figure 1 shows the case where $A_1 \sim A_2$.

It is clear that if all three points lie on the convex segment of the function—which is assured if $c^* \leqq -\lambda cp/(1-p)$ so that $-\bar{c} \geqq 0$—then $A_1 < A_2$; likewise if all three points lie on the concave segment—i.e. if $c^* \geqq c(1-\lambda p)/(1-p)$ so that $c-\bar{c} \leqq 0$—then $A_1 > A_2$.

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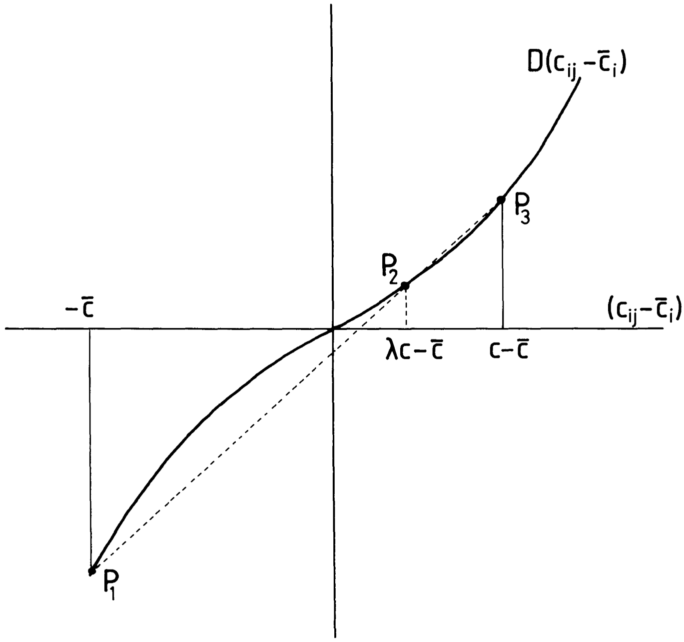

FIGURE 1

Thus we see that there exists some value θ in the range $-\lambda cp/(1-p)<\theta<c(1-\lambda p)/(1-p)$ such that $A_1 \gtrless A_2 \Leftrightarrow c^* \gtreqless \theta$: in other words, as c^* falls through this range, there will at some point be a switch from $A_1 > A_2$ to $A_1 < A_2$, which is the violation of Savage's sure-thing principle that has been so frequently observed.

Note that this result holds even if $D(\xi)$ is not equal to $-D(-\xi)$ for all ξ, so long as the other assumptions hold. However, if we do assume that $D(\xi)=-D(-\xi)$ for all ξ, we can be more precise about θ and its relationship with λ and p. If $c^*=\lambda c$, $\lambda>0{\cdot}5$ is sufficient to ensure $A_1 > A_2$; and if $c^*=0$, $p<0{\cdot}5$ is sufficient to ensure $A_1 < A_2$. Thus if $0<p<0{\cdot}5<\lambda<1$, θ will be in the range $0<\theta<\lambda c$, so that reducing c^* from λc to 0 will generate a reversal from $A_1 > A_2$ to $A_1 < A_2$.

This result is based on the case where the expected basic utility is the same for both actions and where our assumptions about $D(\cdot)$ all hold. However if we concentrate on cases where $0<p<0{\cdot}5<\lambda<1$, we may relax some of the other conditions without undermining our explanation of the empirical evidence.

In particular, we can consider cases where $\bar{c}_1 \neq \bar{c}_2$. Given $p<0{\cdot}5$ we can construct an action A_3 differing from A_2 only by offering a less valuable consequence under S_1 with a basic utility of $c-d$. Now $\bar{c}_3<\bar{c}_1$, but for sufficiently small d it will still be the case that $A_1 < A_3$ when $c^*=0$; but since A_3 is dominated by A_2, it must still be the case that $A_1 > A_3$ when $c^*=\lambda c$. Hence the reversal will still occur as c^* is reduced from λc to 0.

By a similar argument, given $\lambda>0{\cdot}5$ we can construct an action A_4 offering a consequence under S_1 with a basic utility $c+e$. For sufficiently small e it will still be the case that $A_1 > A_4$ when $c^*=\lambda c$; but since A_4 dominates A_2 it must also be the case that $A_1 < A_4$ when $c^*=0$, again entailing a reversal as c^* is reduced from λc to 0.

So we can envisage a range of cases where we observe reversals even though the expected basic utilities, and hence the expected money values, of the actions may differ.

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If we also relax the assumption that $C(\cdot)$ is linear, and instead take that function to be concave, the disparities between expected money values may be even wider. And indeed it appears that the pattern of violation is observable across quite wide disparities in expected value: Kahneman and Tversky (1979, Problems 1 and 2) observed substantial reversals in a case where the expected value of the gamble was very close to the value of the certain prospect—2409 Israeli pounds compared with 2400; but similarly substantial numbers of reversals have been observed with the Allais (1953) problem where the expected value of the gamble—Fr. 1·39 million—is considerably greater than the value of the certainty—Fr. 1 million. Perhaps more significant is that in both cases λ and p lie within the ranges discussed above: in the Kahneman and Tversky experiment, $\lambda = 0{\cdot}97$ and $p = 0{\cdot}34$, while in the Allais problem $\lambda = 0{\cdot}91$ and $p = 0{\cdot}11$.

We do not claim that reversals will *not* occur under other conditions, such as $\lambda < 0{\cdot}5$ and $p > 0{\cdot}5$.[5] The purpose of this section has been simply to illustrate how the disappointment-elation model works, and how, under conditions where $\lambda > 0{\cdot}5$ and $p < 0{\cdot}5$, it generates fairly strong predictions about violations of the sure-thing principle which appear to be consistent with a substantial body of observed behaviour.

Essentially the same argument can be used to show that the common ratio effect is consistent with our theory. As before, A_1 and A_2 are the actions described in Table I, but we set $c^* = 0$ and let p become the variable. The empirical evidence (see, for example, Kahneman and Tversky 1979, 1981) shows a systematic tendency for individuals to switch from $A_1 > A_2$ to $A_1 < A_2$ as p decreases. Bearing in mind that much of the empirical evidence involves choices where the expected values of the alternatives are the same or similar, that it is often the case in these experiments that $\lambda > 0{\cdot}5$, and that changes in p, like the changes in c^* in the common consequence effect, influence expression (2) through their impact on $\bar{c}$, it is not difficult to see that the common ratio effect is predicted by disappointment theory in essentially the same way as the violation of the sure-thing principle in the form of the common consequence effect.

4. OTHER MODELS

If regret and rejoicing are set aside, our theory makes the expected modified utility of any single-stage action depend only on the nature of the consequences of that action and their respective probabilities. In this respect, our theory is one of a class of theories in which individuals have preference orderings over probability distributions of consequences but in which these orderings do not necessarily satisfy the independence axiom. Models with such properties were suggested by Samuelson (1950, p. 170) and Allais (1953), and later variants were proposed by (among others) Hagen (1979), Chew and MacCrimmon (1979), Fishburn (1983), Machina (1982) and Bell (1985).

We shall not discuss all those other theories here, but concentrate on two: Bell's (1985) model of disappointment and elation; and Machina's (1982) Generalized Expected Utility analysis.

Bell presents a theory of preference orderings over two-outcome lotteries, which offer the consequences x, y with probabilities p, $1-p$; consequences are increments or decrements of wealth such that $x > y$. Such a lottery is written (x, p, y). Bell assumes that the marginal (basic) utility of wealth is constant, which allows us to measure utility on the same scale as wealth.

Bell defines two functions $c_1(\cdot, \cdot, \cdot)$ and $c_2(\cdot, \cdot, \cdot)$ so that $c_1(x, p, y)$ represents the certainty-equivalent value of the experience of winning x in the lottery (x, p, y) and $c_2(x, p, y)$ represents the certainty-equivalent value of the experience of winning y in the

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same lottery. These values may, of course, also be interpreted as utility indices. Bell then assumes that the certainty-equivalent value of the lottery itself is equal to $pc_1(x, p, y) + (1-p)c_2(x, p, y)$.

Applying our own theory to the same two-outcome lottery, and making the same assumption of linearity in the basic utility function, the certainty-equivalent value of the lottery is $p[x + D(x-\bar{c})] + (1-p)[y + D(y-\bar{c})]$ where $\bar{c} = px + (1-p)y$. The two theories would be equivalent, as far as this case is concerned, if $c_1(x, p, y) = x + D(x-\bar{c})$ and $c_2(x, p, y) = y + D(y-\bar{c})$.

Bell then imposes a restriction that he calls "linear expectations". This says that for all x, p and y, and for any constants a and b (where $b > 0$), $c_1(a+bx, p, a+by) = a + bc_1(x, p, y)$ and $c_2(a+bx, p, a+by) = a + bc_2(x, p, y)$. Given that $C(\cdot)$ is linear, it is easy to check that our model would satisfy this condition only if $D(\xi)$ was linear for all $\xi > 0$ *and* linear for all $\xi < 0$.

Retaining *our* assumption that $D(\cdot)$ is differentiable, Bell's restriction would require $D(\xi)$ to be linear for all ξ so that the theory would yield the same predictions as expected utility theory. However, if it is assumed only that $D(\cdot)$ is continuous, Bell's restriction would allow the function to be kinked at the origin.

Bell considers this case explicitly, assuming (to use our terminology) that $D'(\xi)$ is greater for $\xi < 0$ than for $\xi > 0$. This formulation is consistent with *some* observations of the common consequence and common ratio effects, but it cannot explain these effects when individuals are choosing between actions with the same expected money value; whereas our non-linear $D(\cdot)$ function is consistent with such observations, as we showed in the previous section.

Bell extends his model beyond the simple kinked-linear formulation by making disappointment and elation depend on more than the difference between the individual's prior expectation and the consequence subsequently experienced.

In effect, Bell assumes that for given values of x and $\bar{c}$, $c_1(x, p, y)$ is also a function of p, decreasing as p increases; likewise, for given values of y and $\bar{c}$, $c_2(x, p, y)$ decreases as p increases. Thus in Bell's extended model, unlike ours, there is no unique index of elation or disappointment associated with particular values of $(x-\bar{c})$ or $(y-\bar{c})$. In this respect, our formulation is both simpler than Bell's and more easily able to encompass lotteries with more than two outcomes.

So it is clear that although we share Bell's basic intuition about disappointment and elation, we model them rather differently. To bring out the differences between our theory and a number of other models in the same class, we now turn to the work of Machina.

Machina (1983) has shown that the models of Chew and MacCrimmon (1979) and Fishburn (1983) both imply a "fanning out" of indifference curves over probability distributions (for an earlier example of this functional form, see Samuelson (1950)). Consider the case where there are three consequences with basic utilities $c_3 > c_2 > c_1$ occurring with probabilities p_3, $1-p_1-p_3$, and p_1 respectively. For any set $\{c_3, c_2, c_1\}$ a triangle diagram such as in Figure 2 can be constructed. Conventional expected utility axioms require that an individual's preferences correspond to a set of indifference curves all of which have the same slope, so that the individual's preference between any two prospects f_1 and f_2 depends on whether the slope of the line connecting them is less han, equal to, or greater than the slope of the indifference curves. Hence conventional expected utility theory entails that if we construct any other pairs of prospects, such as f_3 and f_4, or f_5 and f_6, which lie on lines with the same slope as the one connecting f_1 and f_2, then the preferences between those pairs will be the same as the preference between f_1 and f_2.

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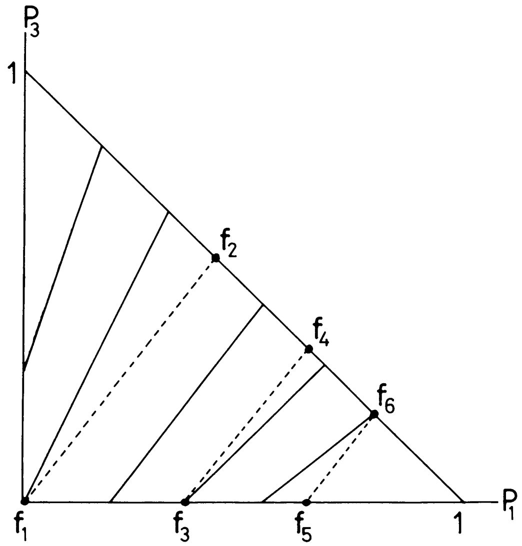

FIGURE 2

However if the indifference curves fan out, as in Figure 2, an individual may both prefer f_1 to f_2 and prefer f_4 to f_3, which is consistent with the common ratio effect. Other violations, such as the common consequence effect, can likewise be shown to be consistent with indifference maps of this kind.

Machina (1982) has offered a general non-parametric characterisation of the "fanning out" of the indifference curves, in the form of his Hypothesis II. He defines a "local utility function" $U(x; F)$ where x is any consequence and F is any probability distribution of consequences. When ranking differential shifts from F, the individual behaves as if his local utility function was a von Neumann–Morgenstern function. Hypothesis II is that for any x, and for any F, F^* where F^* stochastically dominates F, the local utility function is at least as concave at F^* as at F.

Since our theory predicts the common consequence and common ratio effects, it is natural to ask whether it is simply a special case of Machina's general characterisation. Clearly, our model fits into Machina's framework to the extent that it proposes a preference functional over (single-stage) probability distributions. The corresponding local utility function is:

$$U(x_{ij}; F) = c_{ij}[1 - \textstyle\sum_{j=1}^{n} p_j D'(c_{ij} - \bar{c}_i)] + D(c_{ij} - \bar{c}_i). \tag{3}$$

(To derive this function, notice that $U(x; F)$ measures the rate at which utility changes as the probability associated with x changes. Thus $U(x_{ij}, F) = \partial E_i / \partial p_{j\cdot}$)

Equation (3) shows that our assumptions that $C(\cdot)$ is an increasing function and that $0 \leqq D'(c_{ij} - \bar{c}_i) < 1$ for all $c_{ij} - \bar{c}_i$ are sufficient for $U'(x_{ij}; F) > 0$ and hence for first-order stochastic dominance preference. Our assumptions about $D(\cdot)$ suggest that the local utility function may be concave over low values of x_{ij} and convex over high values. (It will certainly have these properties if $C(\cdot)$ is linear.) This would be consistent with the phenomenon of simultaneous insurance and gambling.

However, our formulation does not in general satisfy Hypothesis II. Consider again Figure 2. The implication of "fanning out" is that if a reversal of preference occurs,

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there will be no "re-reversal": that is to say, the reduction in the slope of the indifference curves which tends to occur as we move rightwards from $\{f_1, f_2\}$ to $\{f_3, f_4\}$, continues as we move further to the right so that if $f_1 > f_2$ and $f_3 < f_4$ it cannot be the case that $f_5 > f_6$.

However, this is not a necessary implication of our disappointment model. It *does* hold if f_1 and f_2 have the same expected basic utilities; but if we relax that assumption, it is possible to construct examples where $f_1 > f_2$, $f_3 < f_4$ *and* $f_5 > f_6$.[6]

We should stress that our theory does not predict that such re-reversals *must* occur; they are no more than *possibilities.* We know of no evidence of systematic re-reversals. (Most experimenters investigating the common consequence and common ratio effects have confronted their subjects with a *couple* of pairwise choice problems; but to observe re-reversals, it would be necessary to give subjects at least a *triple.*) So we do not claim that in this respect our theory is *superior* to the class of theories encompassed by Hypothesis II—merely that it is *different.*

A second respect in which our model differs from Machina's concerns the analysis of multi-stage gambles. Machina's approach is based on general propositions about *all* probability distributions over final consequences, and therefore invokes the compound probability axiom—the principle of reducing complex prospects to simple ones by applying the calculus of probabilities. Until now, we have framed our theory in terms of simple single-stage gambles. However, as we show in the next section, when we extend our theory to include multi-stage gambles, we draw a distinction between single- and multi-stage gambles and predict the violation of the compound probability axiom known as the "isolation effect" which lies outside the scope of Machina's model.

5. MULTI-STAGE GAMBLES AND THE "ISOLATION EFFECT"

As part of an experiment reported by Tversky and Kahneman (1981, p. 455 and footnote 15), subjects were presented with three choice problems as follows:

Problem 1

A: The certainty of \$30.

B: 0·8 chance of \$45; 0·2 chance of 0.

Problem 2

C: A first stage, in common with D, giving a 0·75 chance of dropping out and receiving 0, and a 0·25 chance of going through to the second stage; the second stage gives the certainty of \$30.

D: A first stage, in common with C, giving a 0·75 chance of dropping out and receiving 0, and a 0·25 chance of going through to the second stage; the second stage gives a further lottery, with a 0·8 chance of \$45 and a 0·2 chance of 0.

Problem 3

E: 0·25 chance of \$30; 0·75 chance of 0.

F: 0·2 chance of \$45; 0·8 chance of 0.

Consider Problems 1 and 3. This couple of problems is one in which the common ratio effect can be observed. Tversky and Kahneman found this effect at work: 85 of their 205 subjects exhibited the conjunction of preferences $A > B$ and $E < F$.

Now consider Problems 2 and 3. C and D have, respectively, the same probability distribution of final consequences as E and F. Any theory of choice under uncertainty

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that incorporates the compound probability axiom—that complex prospects can be expressed as simple ones by applying the calculus of probabilities—entails $C \gtreqless D \Leftrightarrow E \gtreqless F$.

Many theories are based on the fundamental assumption that individuals have preference orderings over probability distributions of final consequences, and thus incorporate the compound probability axiom. Some such theories—including Chew and MacCrimmon (1979), Machina (1982) and Fishburn (1983)—are compatible with the common ratio effect. Theories of this kind would predict that the common ratio effect would be observed just as frequently between Problem 1 and Problem 2 as between Problem 1 and Problem 3.

Tversky and Kahneman, however, found that subjects tended to reverse their preferences much more frequently between Problems 1 and 3 than between Problems 1 and 2. (See also Kahneman and Tversky (1979).) They called this "the isolation effect", which they interpret as a case of the more general phenomenon of "framing effects".

Framing effects occur when a choice problem with a given logical structure elicits different responses according to the way it is presented; and there seems little doubt that such effects *do* occur.[7] However, we are reluctant to classify the isolation effect in this way since it seems to be connected with a principle of rational choice that has considerable intuitive appeal: dynamic consistency.

Consider Problems 1 and 2. In Problem 2, C and D offer the same 0·25 chance of reaching the second stage and receiving, respectively, prospects A and B as in Problem 1. Since choosing (say) C rather than D amounts to committing oneself to receiving A rather than B in the event that one is successful in the first stage, dynamic consistency requires that for any individual $A \gtreqless B \Leftrightarrow C \gtreqless D$.

Clearly, any theory that accommodates the common ratio effect must dispense either with dynamic consistency or with the compound probability axiom. If the compound probability axiom is retained, the isolation effect is left unexplained (except as a framing effect); but if dynamic consistency is retained, the isolation effect is a logical corollary of the common ratio effect.

Tversky and Kahneman's results suggest that dynamic consistency is violated much less frequently than the compound probability axiom; of the 85 subjects who exhibited the common ratio effect, 20 violated the former and 65 the latter.

To explain why people may respond sufficiently differently to one-stage and two-stage gambles that they violate the compound probability axiom, we develop another insight of Bell's (1985), namely, that the sum total of disappointment and elation may be affected by the way in which uncertainty is resolved through time.

The point may be illustrated concretely by reference to Tversky and Kahneman's prospects D and F. With F, there is a single moment when the uncertainty is resolved, whereupon an individual may expect to feel some degree of disappointment (probability 0·8) or elation (probability 0·2). However, with D the uncertainty is resolved in two stages: at the end of the first stage there may be disappointment (probability 0·75) or elation (probability 0·25)—but not necessarily the same *intensities* of disappointment and elation as with F; and for those who experience elation from winning a valuable lottery ticket in the first round, there is the possibility of subsequent disappointment or further elation depending on the outcome of the second stage. Clearly, then, although both D and F involve the same probability distribution over final consequences, the logic of our theory allows individuals to anticipate quite different patterns of disappointment and elation, and therefore provides no justification for an axiom which reduces any multi-stage gamble to an equivalent single-stage gamble simply by applying the probability calculus.

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On the other hand, the principle of dynamic consistency *does* seem compatible with our theory, and we can incorporate it quite straightforwardly into a treatment of two-stage gambles—a treatment which can be applied to any two consecutive stages, and therefore generalises quite naturally to gambles involving any number of stages.

In our earlier formulation, there were n states of the world and all uncertainty was resolved simultaneously. Let us now interpret those states as the possible outcomes of the *first* stage of a two-stage problem, and let at least one consequence x_{ij} be able to take the form of an uncertain action defined in relation to a subsequent set of states conditional upon the occurrence of state j at the end of the first stage. Let A_i^j be that consequent action, and E_i^j its expected modified utility. We can then incorporate dynamic consistency by assuming $C(x_{ij}) = E_i^j$ which, together with the other central assumptions of our theory, entails that $\mathrm{C} \gtreqless \mathrm{D} \Leftrightarrow \mathrm{A} \gtreqless \mathrm{B}$, but entails no such implication between {C, D} and {E, F}. Thus to the extent that our model predicts the common ratio effect, it also predicts the isolation effect.

6. CONCLUSION

In this paper we have tried to combine intuitive appeal and functional simplicity to produce a model of choice under uncertainty that generates a wide range of predictions which are consistent with observed behaviour. And although there are other models which share some of the same predictions, we have shown that the implications of our theory differ in important respects from these.

However, our theory claims to do more than predict a range of violations of the sure-thing principle and the compound probability axiom: it provides an explanation for them within a framework of rational choice.

In contrast, many writers who have recognised the limitations of expected utility theory for predictive purposes have been reluctant to abandon the idea that violations of that theory are in some way irrational or normatively unacceptable (cf. Savage (1954, pp. 102–103), Morgenstern (1979, p. 180), Kahneman and Tversky (1979, p. 277), Bell (1985, p. 26–27)).

We are not, of course, the only theorists to doubt the normative appeal of the standard axioms of expected utility theory. But merely to construct a rival system of axioms, however elegant and however compatible with experimental evidence, is not to answer the question: "*Why* can it be rational to violate the sure-thing principle or the compound probability axiom?"

We believe that our analysis *does* provide an answer to that question. In our theory, people seek consistently to maximise expected satisfaction, where that expectation includes the anticipation of possible disappointment and elation. We cannot see any reason for regarding such a maximand as irrational; nor do we think that any simple experience of satisfaction, whatever its source, can be designated either rational or irrational.

This conclusion is in some degree supported by the evidence of an experiment conducted by Slovic and Tversky (1974). They found that a number of people who violated the sure-thing principle continued to wish to do so even after being exposed to the argument—due to Savage—that such behaviour is inconsistent. MacCrimmon (1968, p. 21) reported rather larger shifts towards conformity with conventional postulates during interviews with his repondents, but noted that "most of the persistent violations that did occur involved Postulate 2" (Savage's sure-thing principle). In a more recent experiment, MacCrimmon and Larsson (1979, pp. 366–369) recorded a greater preference for an

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Allais-type counter-axiom argument than for either of two Savage-type arguments, even among respondents whose observed choices were consistent with the sure-thing principle. Thus it would appear that a number of people do not subscribe to all the principles of rationality that have appealed to conventional theorists. Our theory may explain why this is so.

By basing a theory of choice on psychological assumptions rather than behavioural axioms it is sometimes possible to provide a unified explanation for patterns of behaviour that would otherwise appear to be quite unrelated. Our analysis of the isolation effect provides an example. Since this effect is a violation of the compound probability axiom rather than of the independence axiom, it appears at first sight to be unconnected with the common consequence and common ratio effects. We have been able to show that violations of the compound probability axiom and violations of the independence axiom may have a common rationale.

In a more fundamental sense, all of these violations may have the same underlying rationale as violations of the transitivity axiom. This can be seen by comparing the present theory with regret theory. Both theories assume that the individual seeks to maximise the mathematical expectation of satisfaction. In the present theory, the satisfaction derived from any consequence depends in part on a comparison between that consequence and the other consequences of the same action in different states of the world. In regret theory, the satisfaction derived from a consequence depends in part on a comparison between that consequence and the consequences of other actions in the same state of the world. These two kinds of comparison are not mutually exclusive, and at the psychological level they have much in common. Disappointment and regret are different kinds of pain that one may experience when one reflects on "what might have been". Both, we suggest, are natural human emotions, and ones that can be recognised through introspection. We can see no reason for supposing either emotion to be more natural—still less, more rational—than the other.

Because regret theory makes comparisons *across actions* but *within states of the world*, it can predict violations of the transitivity axiom but not violations of the sure-thing principle; whereas disappointment theory, which makes comparisons *across states of the world* but *within actions* can predict violations of the sure-thing principle, but not violations of transitivity. However, both theories generate many of the same predictions such as the common consequence and common ratio effects in the case of statistically independent prospects, simultaneous gambling and insurance, and the isolation effect. Given the similarity of the fundamental structure of both theories, there may be grounds for thinking that a more general theory of rational choice under uncertainty may encompass both regret and disappointment.

First version received March 1984; final version accepted June 1985 (Eds.)

An earlier version of this paper was presented at the Second International Conference on Utility and Risk Theory. We are grateful to the participants at that Conference, and in particular Mark Machina and David Bell for their comments and suggestions.

NOTES

1. Peter Fishburn's SSB utility theory was also being developed at that time (see Fishburn (1982)). For further details of the relationship between regret theory and SSB utility, see Loomes and Sugden (1983) and Machina (1983).
2. The corresponding concept in regret theory was called "choiceless utility". We now believe that the more general term "basic utility" is more appropriate.
3. See Section 4 for a proof that this condition entails first-order stochastic dominance preference.

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4. For the sake of clarity, Figure 1 has been drawn with the scale of the vertical axis double the scale of the horizontal axis.

5. It is not difficult to construct numerical examples in which such reversals will occur.

6. Consider this example. As in Table I, the probabilities of states S_1, S_2 and S_3 are λp, $(1-\lambda)p$ and $1-p$ respectively, and λ is fixed at 0·8. However, c_{11}, $c_{12} \neq \lambda c_{21}$. Instead, $c_{11}=c_{12}=250$ while $c_{21}=300$; $c_{22}=c_{23}=c_{13}=0$. $D(\xi)=\xi^2/(\xi+50)$ for all $\xi \geqq 0$ and $-\xi^2/(50-\xi)$ for all $\xi<0$. Readers may check that when $p=1$, $A_1 > A_2$; when $p=0{\cdot}5$, $A_1 < A_2$: and when $p=0{\cdot}2$, $A_1 > A_2$ again.

7. See, for instance, the striking example in Tversky and Kahneman (1981) where subjects respond very differently according to whether a particular problem is presented in terms of "lives saved" or "lives lost".

REFERENCES

ALLAIS, M. (1953), "Le Comportement de l'Homme Rationnel devant le Risque; Critique des Postulats et Axiomes de l'Ecole Americaine", *Econometrica*, **21**, 503-546.

BELL, D. E. (1982), "Regret in Decision Making Under Uncertainy", *Operations Research*, **30**, 961-981.

BELL, D. E. (1985), "Disappointment in Decision Making Under Uncertainty", *Operations Research*, **33**, 1-27.

CHEW, S. and MACCRIMMON, K. (1979), "Alpha-Nu Choice Theory: A Generalization of Expected Utility Theory" (Working Paper No. 669, University of British Columbia Faculty of Commerce).

FISHBURN, P. (1982), "Nontransitive Measurable Utility", *Journal of Mathematical Psychology*, **25**, 31-67.

FISHBURN, P. (1983), "Transitive Measurable Utility", *Journal of Economic Theory*, **31**, 293-317.

HAGEN, O. (1979), "Towards a Positive Theory of Preferences under Risk", in Allais, M. and Hagen, O. (eds.) *Expected Utility Hypotheses and the Allais Paradox* (Dordrecht: Reidel) 271-302.

KAHNEMAN, D. and TVERSKY, A. (1979). "Prospect Theory: An Analysis of Decision under Risk", *Econometrica*, **47**, 263-291.

LOOMES, G. and SUGDEN, R. (1982), "Regret Theory: An Alternative Theory of Rational Choice Under Uncertainty", *Economic Journal*, **92**, 805-824.

LOOMES, G. and SUGDEN, R. (1983), "Regret Theory and Measurable Utility", *Economics Letters*, **12**, 19-21.

MACCRIMMON, K. R. (1968), "Descriptive and Normative Implications of the Decision Theory Postulates", in Borch, K. and Mossin, J. (eds.) *Risk and Uncertainty.* (I.E.A.) 3-23.

MACCRIMMON, K. R. and LARSSON, S. (1979), "Utility Theory: Axioms versus 'Paradoxes'", in Allais, M. and Hagen, O. (eds.) *Expected Utility Hypotheses and the Allais Paradox* (Dordrecht: Reidel) 333-409.

MACHINA, M. J. (1982), "'Expected Utility' Analysis Without the Independence Axiom", *Econometrica*, **50**, 277-323.

MACHINA, M. J. (1983), "The Economic Theory of Behavior Towards Risk: Theory, Evidence and New Directions" (Technical Report No. 433 of the Center for Research on Organizational Efficiency, Stanford University).

MORGENSTERN, O. (1979), "Some Reflections on Utility", in Allais M. and Hagen, O. (eds.) *Expected Utility Hypotheses and the Allais Paradox* (Dordrecht: Reidel) 175-183.

MOSKOWITZ, H. (1974), "Effects of Problem Presentation and Feedback on Rational Behavior in Allais and Morlat Type Problems", *Decision Sciences*, **5**, 225-242.

SAMUELSON, P. (1950), "Probability and the Attempts to Measure Utility", *Economic Review*, **1**, 167-173.

SAVAGE, L. J. (1954) *The Foundtions of Statistics* (New York: Wiley).

SLOVIC, P. and TVERSKY, A. (1974), "Who Accepts Savage's Axiom?", *Behavioral Science*, **19**, 368-373.

TVERSKY, A. and KAHNEMAN, D. (1981), "The Framing of Decisions and the Psychology of Choice", *Science*, **211**, 453-458.

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