Published as a separate and in *The Journal of Psychology,* 1951, **31**, 201-231.

THE EFFECT OF DIFFERENTIAL REINFORCEMENT ON THE DISCRIMINATION OF VISUAL NUMBER*[1]

Department of Psychology, Mount Holyoke College

A. L. MINTURN AND T. W. REESE

A. DISCUSSION OF THE PROBLEM

This experiment is concerned with the effect of differential reinforcement on the perception of visual numerousness. Let us tentatively adopt the following definition of numerousness: Numerousness is "that property of a group of objects which we can discriminate, without counting, under instruction to judge how many objects the group contains."[2]

Suppose, for instance, that a person sees a fleet of ships. It is possible for him to give a report of the number of ships even if he is unable to count them. This report may be in error; that is, it may differ from the number of ships arrived at by another set of operations, namely counting. But the point is that an observer *can,* under appropriate instructions, give a report of the number of ships. Error is another problem.

The present experiment was designed to discover the way in which differential reinforcement affects this kind of report. Briefly, this is what we did. We presented fields of dots to a group of *S*'s. The number of dots in these fields ranged from 1 to 210. The exposure-time was so short that counting was impossible. The *S's* were asked to report how many dots there were. In the second half of the experiment, they were told the actual number of dots immediately after they wrote down each report.

Our use of the term *differential reinforcement* may be clarified by a brief discussion of verbal operants. A report of number is a verbal operant of the class "tacts". Tacts, as defined by Skinner, have two main characteristics: (*a*) they are closely correlated with a discriminable stimulus; (*b*) their reinforcements are generalized (8). We assume that telling the *S* the

*Received in the Editorial Office on December 22, 1950, and published immediately at Provincetown, Massachusetts. Copyright by The Journal Press.

[1]This experiment has been published under Contract *N onr*-131(01) between the Psychophysical Research Unit and Special Devices Center, Office of Naval Research. The Psychophysical Research Unit formerly operated as sub-contractor under Contract *N5 ori*-166, Task Order No. 1 between Special Devices Center, Office of Naval Research and Systems Research, The Johns Hopkins University. This experiment was done under that sub-contract. This is report No. 131(01) MHC 8.

[2]This definition was taken from Kaufman, Lord, Reese, and Volkmann (2), p. 498. It is an amplification of the earlier definition offered by Stevens (9).

correct number has two effects: it reinforces reports that are substantially correct and punishes the others.

If this is true, two things should happen. First, the number of substantially correct responses should increase. Secondly, the number of other responses should decrease. Why do we say "substantially correct"? Let us take an example. Suppose we present a field of 190 dots to an *S*. He may be so far off as to call this "550 dots." Now suppose we tell him it actually was 190 dots. He probably won't say "190" the next time he is shown 190 dots, and maybe he never will. But it is equally likely that he will never again say "550." In short, there will be a reduction in the size of the error, even though the correct response does not occur. The *S* may, on repeated exposures of 190 dots, say "200," "195," "200," "200," etc. In other words, the number of more nearly correct responses has increased. They are not the correct responses, but they are not in error by as much as the original ones. This is why we use the term "substantially correct." If only because of limited differential sensitivity, no *S* could be conditioned to give a uniformly correct response to any considerable number of stimulus-dots.

What did we expect to happen as a result of differential reinforcement? What we expect depends upon our view of the discrimination of numerousness. There is evidence that there are two mechanisms for the discrimination of visual numerousness. One mechanism operates for numbers of objects up to and including 6, and the other for numbers above 6. Kaufman *et al.* (2) have used the word "subitizing" for the mechanism which operates for numbers of objects up to 6, and they have reserved the word "estimating" for the discrimination of numbers above 6. In general, subitizing is quicker, more accurate, less variable, and more confident than estimating. The chief evidence for the existence of two mechanisms depends upon the discontinuity of the first derivative of the function relating report-time to the number of dots presented. The reader is referred to Taves (10), Kaufman *et al.*, and Jensen, Reese, and Reese (1) for a complete discussion of this evidence.

If there are two mechanisms, then differential reinforcement may affect them differently. In fact, we would expect a difference. Subitizing is much more accurate and much less variable than estimating. It follows that differential reinforcement will probably have less effect on subitizing than it will on estimating. But there is another question. Does differential reinforcement largely eliminate the differences between the two processes so that estimating comes to look like subitizing? Finally, are the effects of differential reinforcement maintained over an extended period of time?

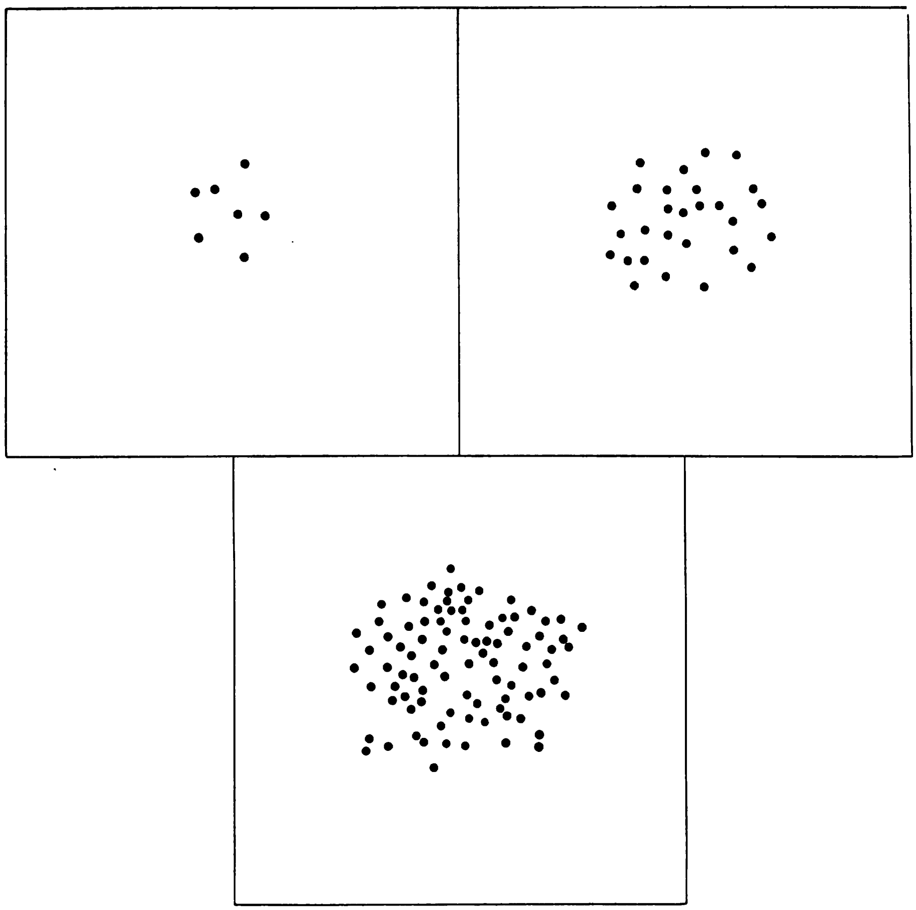

FIGURE 1
REPRESENTATIVE STIMULUS-FIELDS OF 7, 28, AND 89 DOTS

B. Experimental Conditions

1. *Stimulus-material*

The stimulus-material was the same as that used by Kaufman *et al.* (2, p. 503f). Their description follows:

> The stimulus-objects were groups of dots projected on a large screen. The film was prepared in the following way. We drew small, solid-black circles on white paper. Regular or meaningful patterns were avoided. We tried to keep the apparent density of the dots about the same from one drawing to the next, but to vary the density somewhat in the different

parts of any one field. Figure 3[3] shows dot-fields of 7, 28, and 89 dots.

Two series of stimulus-fields were prepared. In each there were 35 fields. The number of dots in the first 15 fields progressed by ones from 1-15. The remaining fields contained the following numbers of dots: 17, 19, 22, 25, 28, 32, 37, 42, 49, 57, 66, 77, 89, 103, 118, 134, 152, 170, 191, and 210. From 1 to 15, the series was as dense as it could be, since a fractional number of dots cannot be presented. Above 15 dots, the increment was less than 1 *jnd*. (To achieve this, we plotted Taves' averaged measurements of the *jnd* for numerousness against the standard number of dots, and drew a smooth curve. Another curve was drawn parallel to the first curve, but below it, and the increments were read from this second curve.) These drawings were photographed in random order on 35-mm. negative strip film and then turned upside down and photographed again. This was done to decrease the possibility that an *S* would recognize a field as one that she had previously seen. Since the dots were photographed on negative film, they appeared white when projected.

2. *Experimental Conditions*

The experiment was conducted in an auditorium. The *S*'s sat in rows of chairs facing a large, white screen 32 feet away. The screen is 40 feet long and 15 feet high. A center space 8 feet 7 inches wide and 7.5 feet high is marked off on the lower half of the screen by thin dark lines. The stimulus-dots appeared in the approximate center of this space. As projected on the screen, each dot was 1¼ inches in diameter. Its brightness was 0.108 apparent foot-candles as measured with a Macbeth illuminometer. Two 15-watt lamps, carefully shaded and located in the corners of the auditorium behind the *S*, dimly illuminated the background. The level of this illumination at the screen was less than 0.02 foot-candles. The background was illuminated for two reasons: (*a*) to show the *S* where the dots were to be projected, and (*b*) to reduce afterimages.

A photograph of the apparatus can be found in Kaufman *et al.* (2, p. 505). The essential components were a still-film projector and a timing device that included a rotating disc with an open sector. The purpose of this apparatus was to provide a ready-signal and expose the dot-pattern for 1/5-second. The various parts worked in this way: the *E* pressed a telegraph key to start the timing device. This device, driven by a synchronous motor through a gear train, automatically rang a gong as a ready-signal, and about 1½-seconds later exposed the dot-pattern for 1/5-second.

[3]This figure appears as Figure 1 in the present article.

3. *Instructions*

The experiment was divided into three parts. The instructions for the *first* part stressed the following points:

1. The *S*'s were told that the stimuli would be presented for 1/5-second between the dark guide lines. They were warned to look up after the warning gong so as to be ready for the stimulus.
2. They were told to write down how many dots there were.
3. They were told to make their reports as quickly as possible. (This point was strongly emphasized.)

In the instructions for the *second* part, the following changes were made:

1. The *S*'s were informed that after they had written down each report, they would be told the correct number of dots in the field just presented.
2. The *S*'s were warned not to change the report they had just made when they heard the correct stimulus-number, but to make subsequent reports in the light of that information.

The instructions for the *third* part were the same as for the *first*—but the *third* part was held eight months after the first and second parts.

4. *Subjects*

Ten female college students were used for this experiment. None of them had served in an experiment of this kind before. The *S*'s served for six sessions of one to 1¼-hours. Each *S* made 42 judgments for each stimulus-number—20 during the control series, 16 during the first experimental series, and 6 during the second experimental series eight months later. There were, therefore, 200 reports of each of the 35 stimulus-values during the control series, or a total of 7,000 reports. Similarly, there were 160 reports for each stimulus-value during the first experimental session, a total of 5,600 reports. Only nine of the original 10 *S*'s could be brought back eight months later so that there were 54 reports for each stimulus-number during the second experimental series, or a total of 1,890 reports. The total number of reports for the whole experiment was 14,490.

5. *Procedure*

The *E* sat beside the projector and ran the film. She started the apparatus by pressing a telegraph-key. The ready-signal sounded, and about 1½-seconds later the dots flashed on the screen. The *E* then turned the film to the next frame, and the procedure was repeated.

During the second part of the experiment, a second E sat at the side of the room. After the S's looked up from writing each report, she told them the correct number of dots. For audibility and emphasis, she spoke over a public address system.

C. Results

1. *Percentage Error*

The first statistic we shall discuss is percentage error.[4] Percentage error was calculated in the following way. The median of all the reports for all the S's was found for each presented number. This was done separately

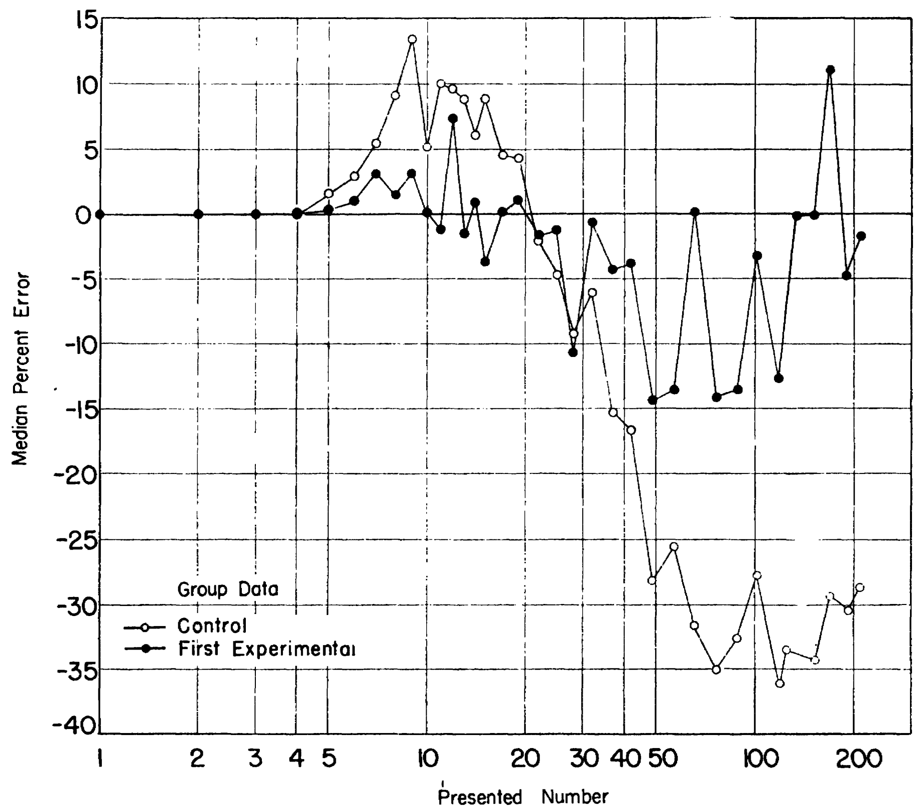

FIGURE 2
MEDIAN PERCENTAGE ERROR AS A FUNCTION OF NUMBER OF DOTS PRESENTED: GROUP RESULTS; CONTROL AND FIRST EXPERIMENTAL SESSIONS PLOTTED SEPARATELY; COORDINATES SEMI-LOGARITHMIC

[4]For the tables for this article order Document 3040 from American Documentation Institute, 1719 N Street, N. W., Washington 6, D. C., remitting $1.00 for microfilm or $1.20 for photo copies readable without optical aid.

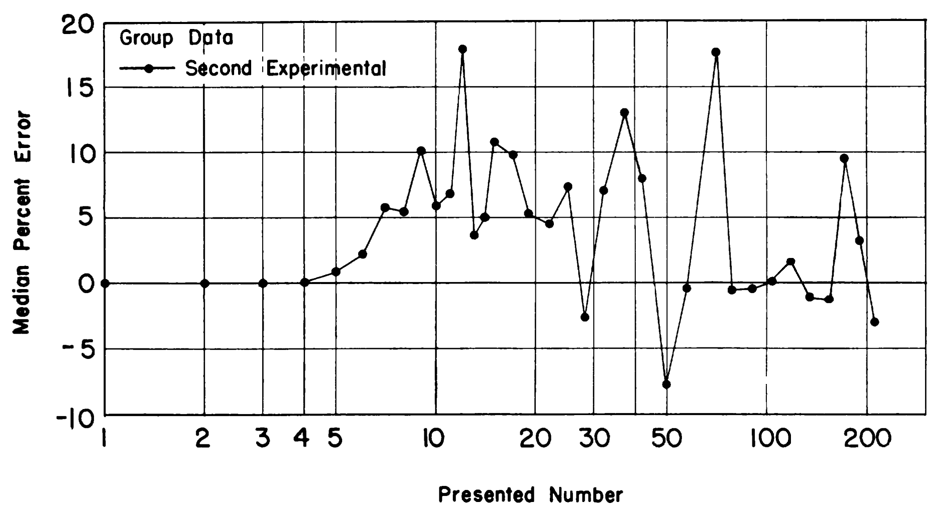

FIGURE 3
MEDIAN PERCENTAGE ERROR AS A FUNCTION OF NUMBER OF DOTS PRESENTED: GROUP RESULTS; SECOND EXPERIMENTAL SESSION (AFTER EIGHT MONTHS); COORDINATES SEMI-LOGARITHMIC

for the control and the two experimental series. Then each presented number was subtracted from the median report and the remainder divided by the presented number. This gave the percentage by which the number of dots reported differed from the number of dots presented. If the *S*'s reported more dots than were presented, the percentage error was positive; if they reported fewer dots than were presented, the percentage error was negative.

These percentage errors were then plotted as a function of presented number (Figures 2 and 3). The open circles in Figure 2 show the *control series,* and the closed circles show the *first experimental series.* Figure 3 shows the percentage errors for the *second experimental series.* First, let us look at the control series (Figure 2). The *S*'s made no, or minimal, errors up to and including 6 dots. From there up to 20 dots the errors are positive, then the curve passes through zero and the errors become negative. These negative errors increase up to about 60 or 70 dots and then begin to level off. This curve is very similar to those found by Kaufman *et al.,* which allows us to believe that our *S*'s were similar to theirs in the way they reported number. It also adds one more piece of evidence in support of a general shape for such curves.

Immediately after differential reinforcement the curve has the same general *shape* (Figure 2 closed circles).[5] The chief difference is that it lies closer to the line of zero error throughout its entire length.

Eight months after differential reinforcement (Figure 3) there is still a considerable effect. The percentage error curve up to about 15 dots is very much like it was before differential reinforcement, but the positive percentage errors continue up to 42 dots. After 42 dots the percentage errors are, in general, even smaller than they were immediately after differential reinforcement.

2. *Individual Median Reported Number*

Each *S*'s reports were divided up in the following way. The 20 reports for each presented number made during the control series were divided into four sets of five—the first five reports made on a particular number constituted the first set, the next five the next set, and so on. Similarly the 16 reports on each presented number made during the experimental series were divided into three sets—the first six reports made on a particular number constituted the first set, the next six the second set, and the last four the third set. The second experimental session consisted of one set of six reports.[6] Medians were found for each *S*, for each set.

Figure 4 shows the medians of the last set of five reports for each *S* in the *control series.* There were 10 *S*'s; thus there are 10 medians plotted for each presented number, although many of these coincide or are so close that it is impossible to plot them as distinguishably separate points. Inspection of the figure shows that the *S*'s median reports are accurate and coincide perfectly up to and including 4 dots. The *S*'s medians differ *slightly* for presented numbers of 5 and 6. But at 7 dots there is an abrupt increase in this variability between the medians of the individual *S*'s. From this point on they vary more and more, forming a funnel-like pattern, up to 210 dots. This pattern is not "solid" for its whole length. At the stimulus-number 66 the medians split. Two trends become visible: one toward increasing overestimation and the other toward increasing underestimation. The density of points in the center of the funnel is correspondingly decreased. We shall return to this point.

[5]Strictly speaking we should perhaps say "during differential reinforcement" because Figure 2 describes all the data obtained during the first experimental series; Figure 3 also represents all the data obtained during the second experimental series.

[6]The control series took two days, with 10 reports for each number made on each day. The grouping of data for the control series was decided on because, after each number had been presented five times, the *E* either called a rest period or ended the experimental session. The first experimental series took three days, with six reports on each of the first two days and four on the last. The grouping for this series was made accordingly. The second experimental series took one day.

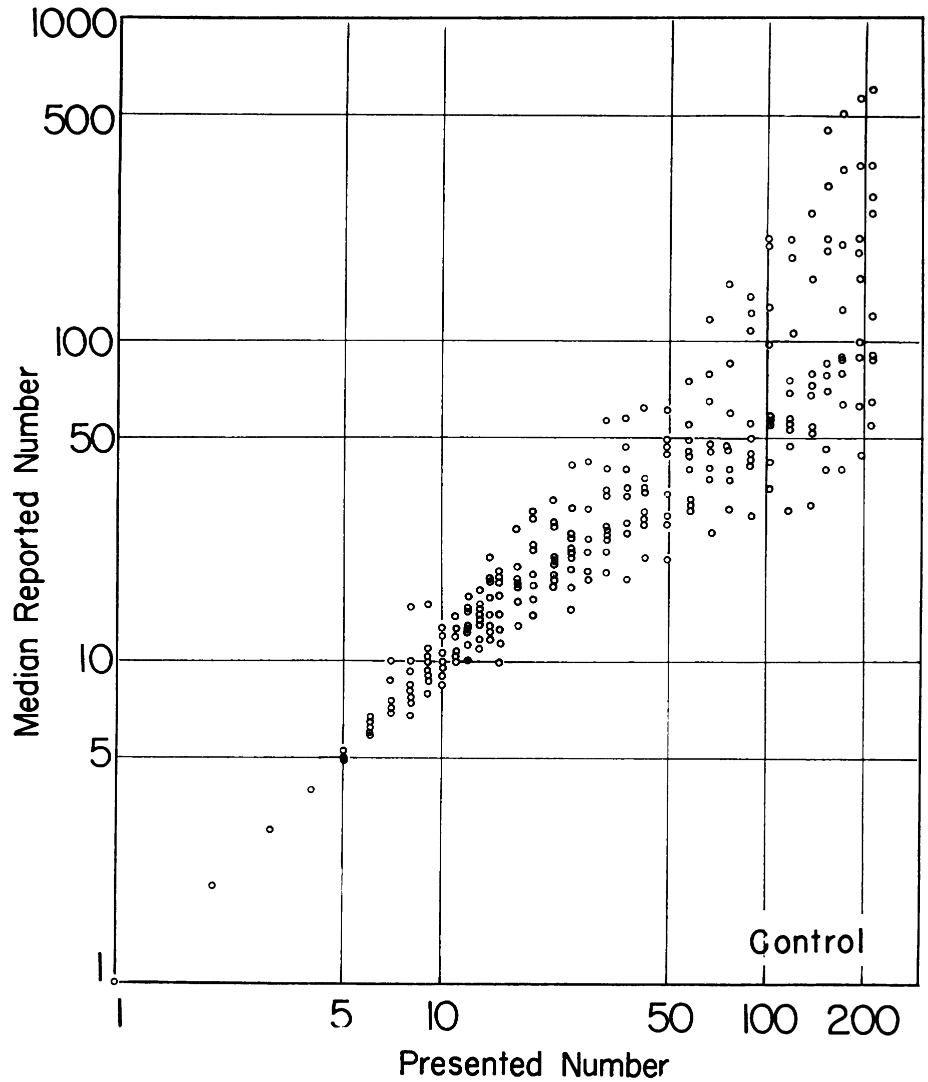

FIGURE 4

Median Reported Number as a Function of Number of Dots Presented: Medians for Each of 10 Individual Subjects; Control Session; Coordinates Logarithmic

Figure 5 shows the median of the last set of 4 reported numbers for all *S*'s in the *first experimental series*. The *S*'s median reports coincide perfectly up to and including 5 stimulus-dots. Furthermore, the median reports are accurate up to this point. They vary slightly at 6. But again there is an

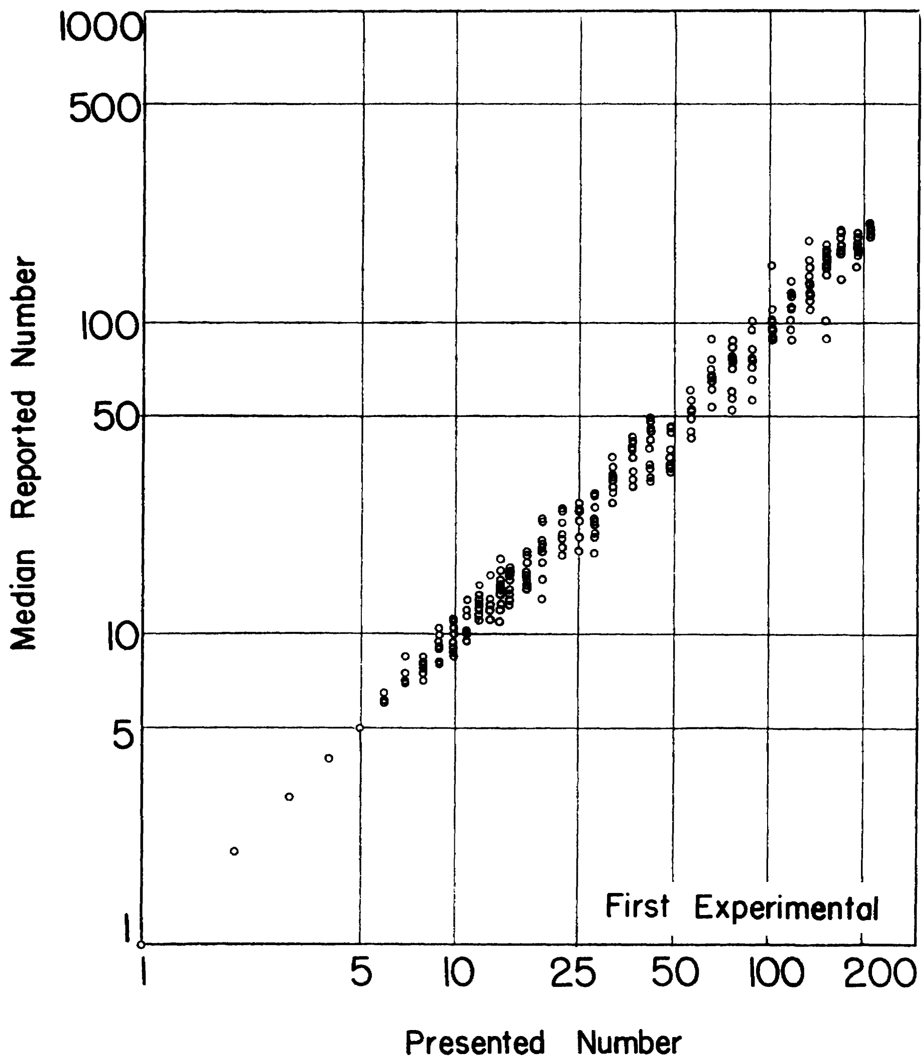

FIGURE 5
MEDIAN REPORTED NUMBER AS A FUNCTION OF NUMBER OF DOTS PRESENTED: MEDIANS FOR EACH OF 10 INDIVIDUAL SUBJECTS; FIRST EXPERIMENTAL SESSIONS; COORDINATES LOGARITHMIC

abrupt increase in this variability at 7 dots. From there on they form a narrow band tending to converge at the last two values of the stimulus-range.

Figure 6 shows the median of the six reports for each of the *S*'s for each of the presented numbers in the *second experimental series.* There is perfect

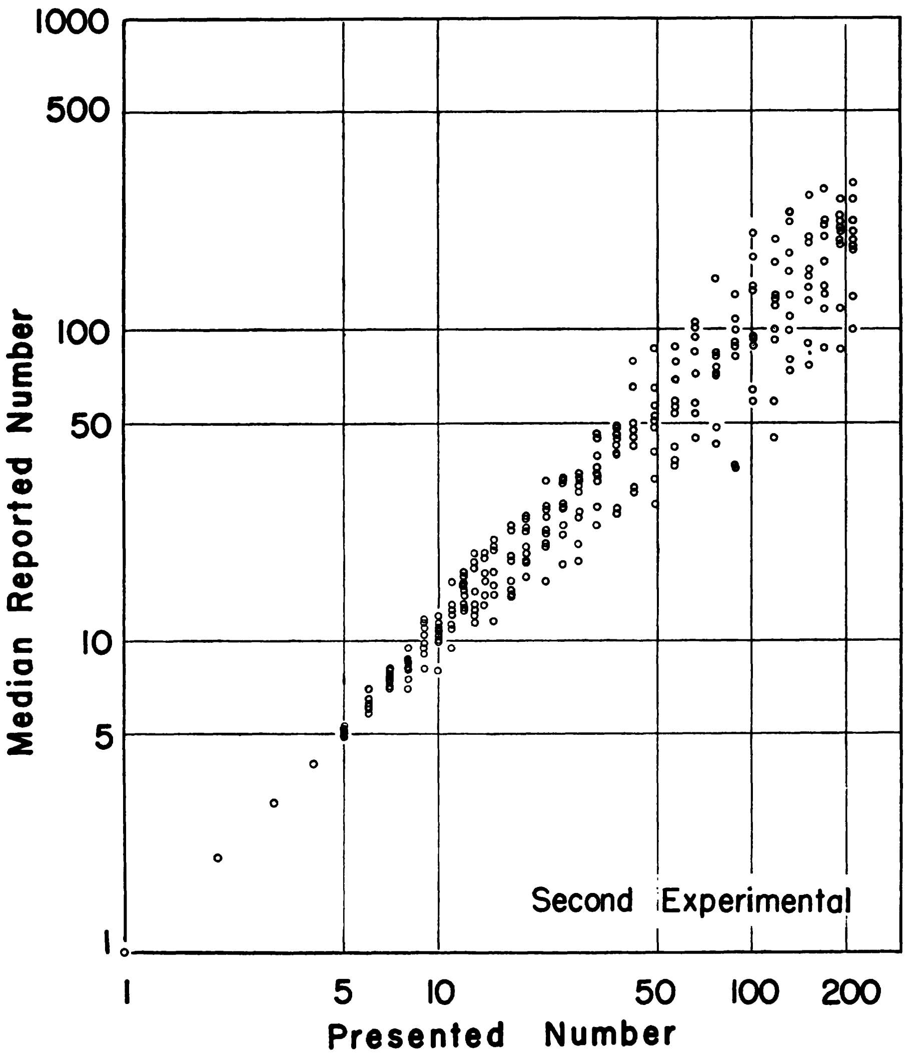

FIGURE 6

MEDIAN REPORTED NUMBER AS A FUNCTION OF NUMBER OF DOTS PRESENTED: MEDIANS FOR EACH OF NINE INDIVIDUAL SUBJECTS; SECOND EXPERIMENTAL SESSION (AFTER EIGHT MONTHS); COORDINATES LOGARITHMIC

coincidence up to and including 4 dots. There is some variability at 5 dots and this increases fairly regularly up to the end of the stimulus-range, but with some decrease at the last two values. The dispersion of points in Figure 6 is, in general, midway between the dispersion in Figures 4 and 5.

Since Figure 4 shows the last five reports in the control series and Figure 5 the last four reports in the experimental series, they contrast the maximum observed effect of unreinforced practice with the maximum observed effect of reinforced practice.

Seven points seem clear from an inspection of these figures:

1. The individual medians for both the first experimental series and the control series are accurate for the low end of the stimulus-range—up to and including 4 dots for the control series and 5 dots for the experimental series.
2. The individual medians differ slightly at 5 and 6 dots for the control series and 6 dots for the experimental series. *This variability increases abruptly at 7 dots in both series.*
3. The pattern of the remaining points is not the same for the two series. The control series shows a steady increase in the variability between the medians up to the top of the stimulus-range. However, the experimental series shows an increase in variability followed by a decrease at the very top of the stimulus-range.
4. The variability between the medians above 7 dots is, in almost all cases, greater in the control series.
5. The medians for the control series fall into two trends which diverge upward and downward respectively, from the line of accuracy.
6. The second experimental series has the same general shape as the control series up to the last two presented numbers. There are two differences, however. First there is less variability after 6 dots in the second experimental series. Secondly, there is no evidence for the funnel-like shape at the higher stimulus-values.
7. Finally, the second experimental series shows, though to a lesser degree, the reduction in variability at the two highest stimulus-values.

There is another way of presenting these facts. We can consider the 10 individual medians[7] for each presented number as a separate frequency distribution, and we can calculate the standard deviation of this distribution for each presented number in each part of the experiment. Figures 7 and 8 show these standard deviations, plotted as a function of presented number. In Figure 7 the open circles represent the control series, and the filled circles represent the first experimental series. Straight lines have been drawn by eye to include most, but not all, of the points. They show that from 7 to 170 dots, the points in each series form a straight band on logarithmic coordinates. Figure 7 leads us to these findings:

[7]Nine in the second experimental series.

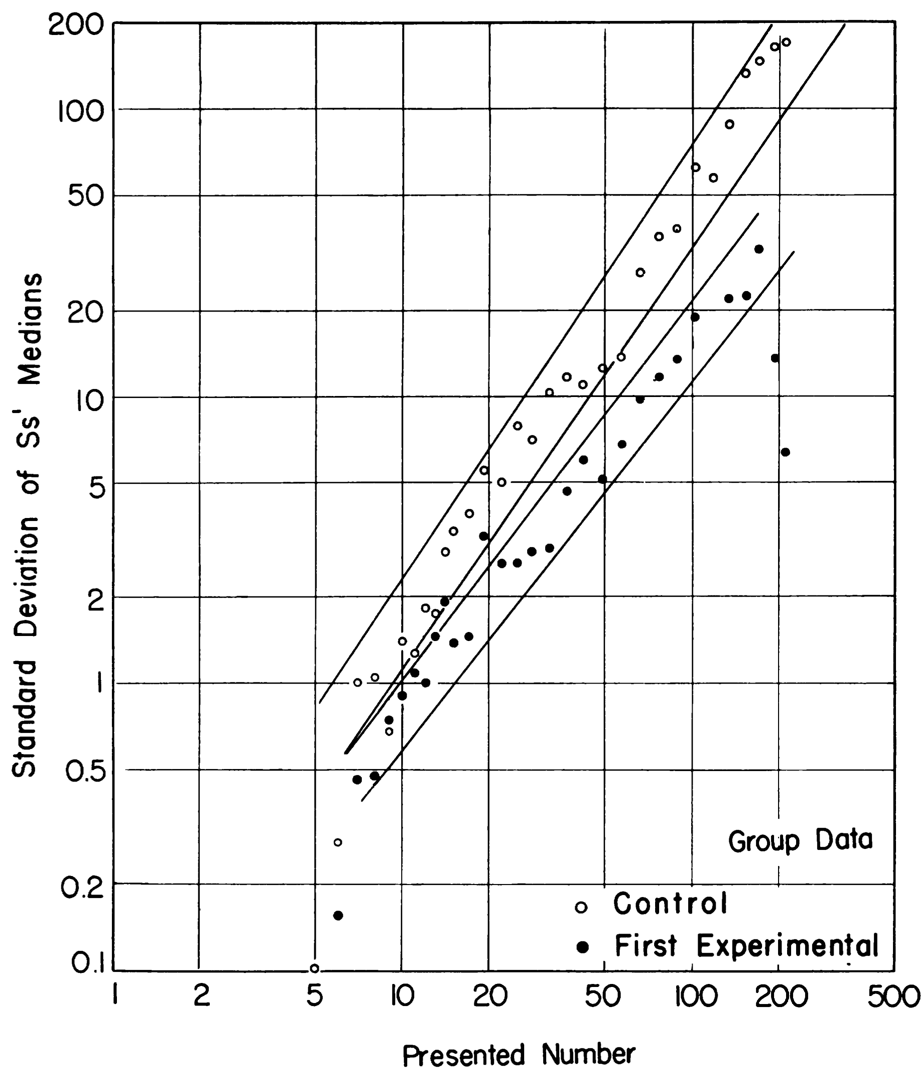

FIGURE 7
STANDARD DEVIATION OF THE SUBJECTS' MEDIANS AS A FUNCTION OF NUMBER OF DOTS PRESENTED: A MEASURE OF VARIABILITY BETWEEN THE SUBJECTS; CONTROL AND FIRST EXPERIMENTAL SESSIONS PLOTTED SEPARATELY; COORDINATES LOGARITHMIC

1. The variability between *S*'s is plainly less in the first experimental series than in the control series. This effect of differential reinforcement is clear.
2. The variability between *S*'s increases abruptly at 7 dots in the first

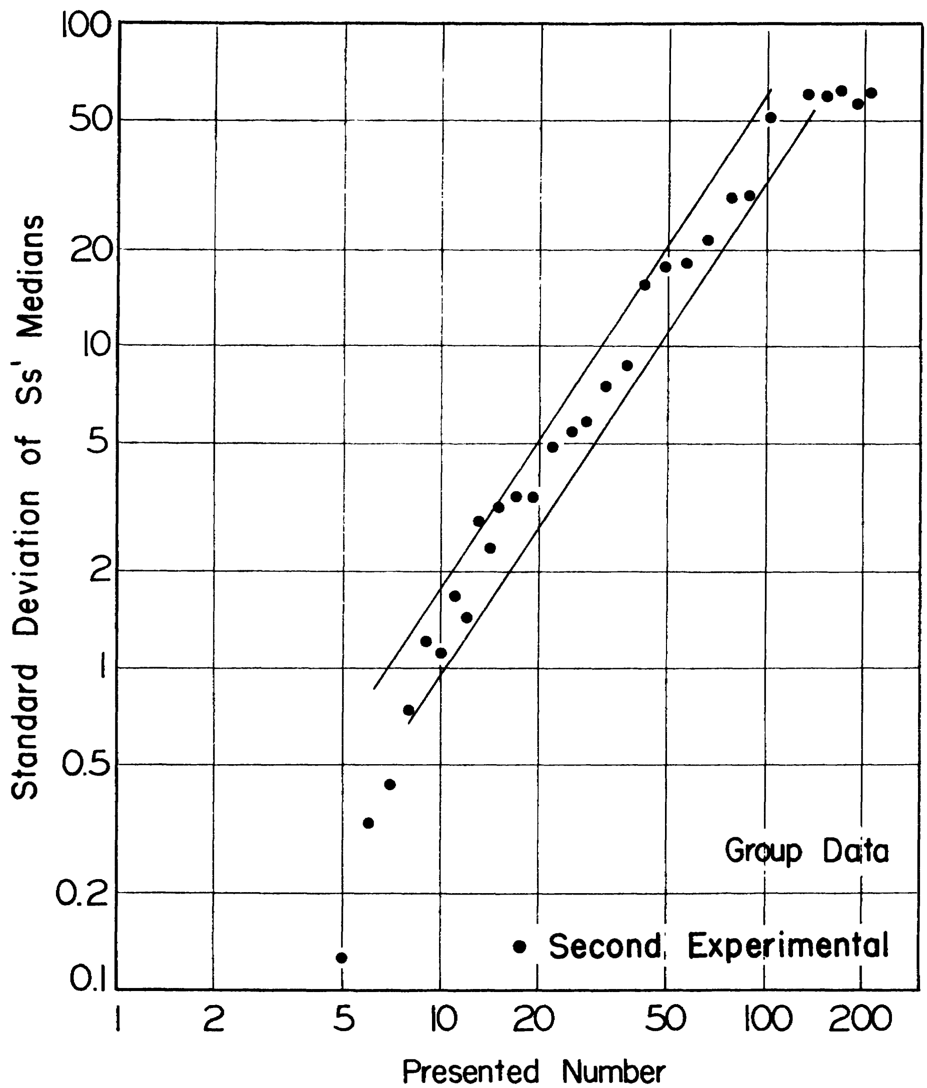

FIGURE 8

STANDARD DEVIATION OF THE SUBJECTS' MEDIANS AS A FUNCTION OF NUMBER OF DOTS PRESENTED: A MEASURE OF VARIABILITY BETWEEN THE SUBJECTS; SECOND EXPERIMENTAL SESSION (AFTER EIGHT MONTHS); COORDINATES LOGARITHMIC

experimental series as well as in the control series. (The standard deviations for 1 to 4 dots in the control series, and 1 to 5 dots in the first experimental series are zero, and therefore cannot be plotted on logarithmic coördinates.)

Differential reinforcement has not obscured or greatly altered the difference between subitizing and estimating. From the evidence of this figure it would seem that no more dots are regularly subitized after differential reinforcement than before it.

3. In the first experimental series variability between *S*'s drops at 191 dots, and the last two plotted points lie well below the band of the other data. This does not happen in the control series. It will be discussed later under the heading: *The Effectiveness of Anchoring Stimuli.*

Figure 8 shows the variability between the *S*'s after eight months, in the second experimental series. A detailed comparison of this band of data with the two bands in Figure 7 will show a small but definite after-effect of differential reinforcement. The variability is slightly lower over-all than it was before differential reinforcement, and definitely higher than it was at the end of differential reinforcement. The points at the upper end of the band no longer drop abruptly, nor do they reasonably fall within the band. They fall between what would be predicted by the control session and what would be predicted by the experimental session.

The variability of presented number 7 does not lie within the band as it might be expected to. Note, however, that it is no lower than it was during differential reinforcement (Figure 7).

3. *Overestimation and Underestimation*

Questions arise concerning the funnel-shaped distribution of individual median reports in the control series (Figure 4). Is this distribution evidence for bi-modality? Are there, roughly speaking, overestimators and underestimators? And if there are, does differential reinforcement affect them differently?

The number of *S*'s in this experiment is too small to enable us to demonstrate the existence of bi-modality. However, quite apart from the possibility of bi-modality, we can discover whether or not there are *S*'s who consistently overestimate or underestimate. If there are, we can see if differential reinforcement affects them differently.

An inspection of the data for all 10 *S*'s showed that four of them consistently overestimated the higher numbers, and six of them consistently underestimated the higher numbers. Therefore, we divided the *S*'s into two groups—overestimators and underestimators. The medians for these two groups were calculated separately and plotted as a function of presented number (Figure 9).

Both groups overestimate from 5 to 19 stimulus-dots. (Oddly enough, the

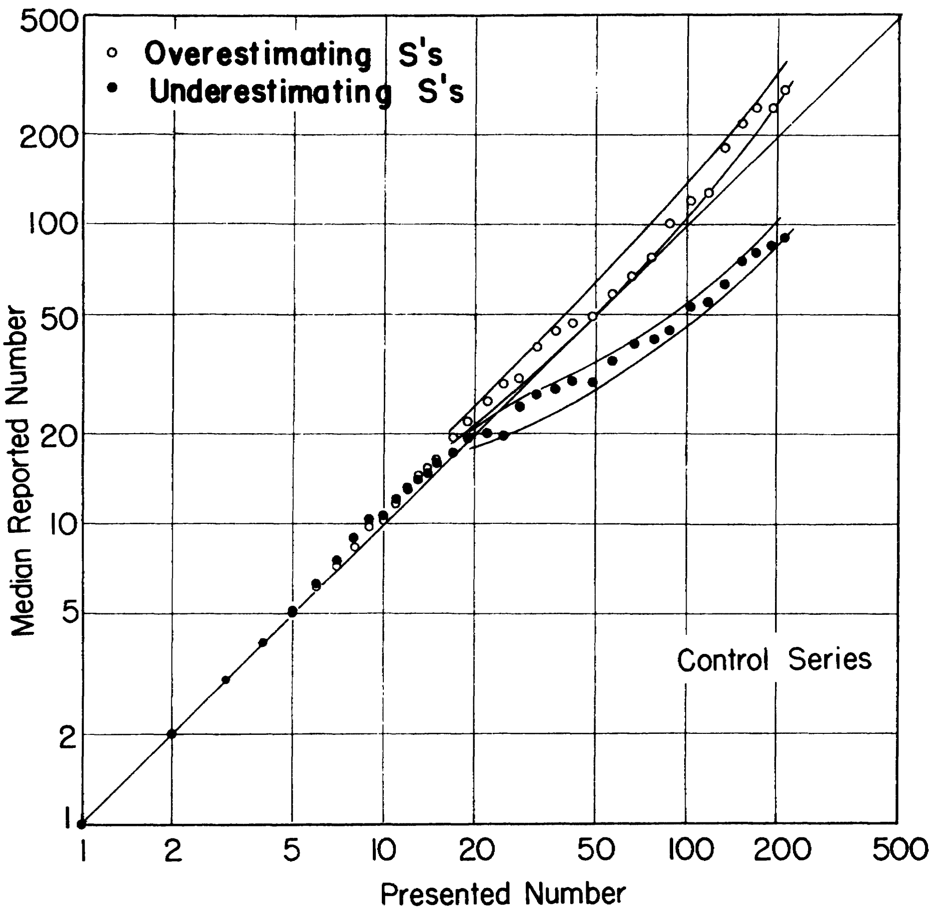

FIGURE 9

MEDIAN REPORTED NUMBER AS A FUNCTION OF NUMBER OF DOTS PRESENTED: MEDIANS FOR OVERESTIMATING SUBJECTS AND UNDERESTIMATING SUBJECTS PLOTTED SEPARATELY; CONTROL SESSION; COORDINATES LOGARITHMIC

underestimators gave higher reports than the overestimators in the limited range from 5 to 12 dots.) The two curves have approximately the same shape up to 15 dots. At this point the curve for the underestimators turns downward and actually crosses the line of accuracy between 20 and 22 dots. The curve continues to diverge downward from the line of accuracy as the presented number increases. On the other hand, the overestimators continue to overestimate by roughly the same proportion (15 per cent) up to 42 dots. Then there is a decrease in the amount of overestimation around 50 dots—

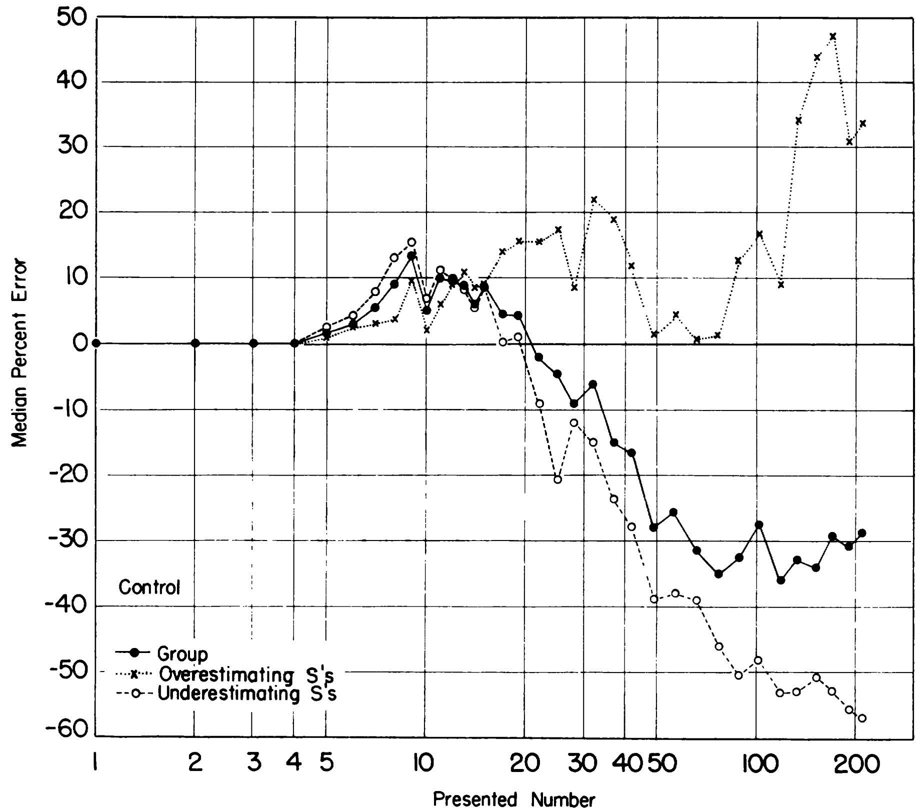

FIGURE 10

MEDIAN PERCENTAGE ERROR AS A FUNCTION OF NUMBER OF DOTS PRESENTED: MEDIAN PERCENTAGE ERROR FOR THE GROUP AND FOR OVERESTIMATING SUBJECTS AND FOR UNDERESTIMATING SUBJECTS PLOTTED SEPARATELY; CONTROL SERIES; COORDINATES SEMI-LOGARITHMIC

followed by an increase at 134.[8] These two curves are suggestive. When replotted as percentage error curves, they may be used to analyze the group percentage error curve of Figure 2.

Figure 10 shows the percentage error curves (from the control series) for the overestimators and underestimators plotted separately. Also plotted in this figure is the corresponding group curve taken from Figure 2, which plainly reflects throughout its length the combined trends of the other two curves. After 19 dots the percentage errors of the underestimators become negative and rapidly decrease in size. At the same time the percentage errors

[8]The decrease at 50 dots may be seen more clearly in Figure 10.

of the overestimating group remain approximately constant. Therefore the group percentage error curve is pulled down by the underestimators. This downward trend of the group curve continues up to 89 dots. At this point the curve for the overestimators suddenly rises, while the curve for the underestimators levels off slightly, beginning at 89 dots. Consequently the group curve levels off and even starts a slight upward trend.

Figure 11 shows the percentage error curves for the overestimators and

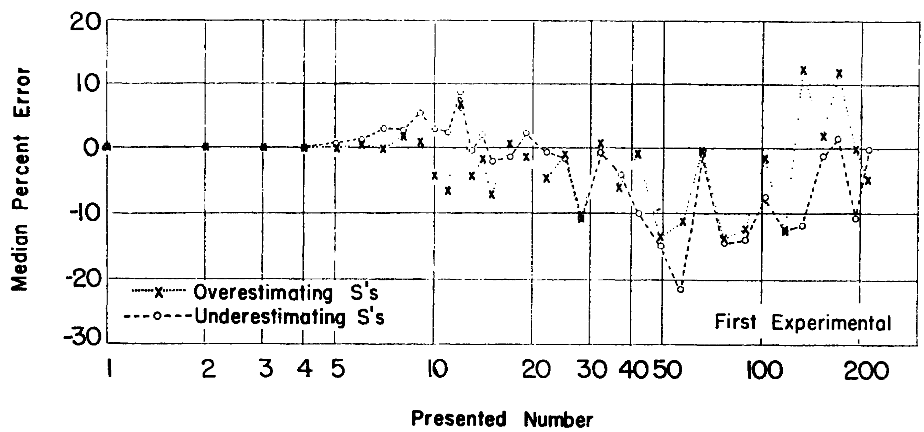

FIGURE 11

MEDIAN PERCENTAGE ERROR AS A FUNCTION OF NUMBER OF DOTS PRESENTED: MEDIAN PERCENTAGE ERROR FOR THE OVERESTIMATING SUBJECTS AND FOR THE UNDERESTIMATING SUBJECTS; FIRST EXPERIMENTAL SERIES; COORDINATES SEMI-LOGARITHMIC

underestimators *after differential reinforcement.* The effects of differential reinforcement can be most clearly seen if we compare Figure 10 with Figure 11. We have already discussed these effects with respect to the group curve. Let us now discuss them with respect to the overestimators and underestimators. What changes have occurred with differential reinforcement?

The most obvious change is the marked reduction in percentage error, seen in both curves. Both curves have, as it were, been compressed and swung toward the line of zero error. This reduction in percentage error is greatest for the highest stimulus-values. It can be seen further that differential reinforcement has had the most profound effect upon the behavior of the overestimators, since their curve now lies in the region of negative error throughout much of the stimulus-range. Also, the curves for the overestimators and the underestimators more closely resemble each other than they did before differential reinforcement. In fact the trend of the points is almost identical.

Apart from these striking changes, however, a careful comparison of Figures 10 and 11 will reveal some definite ways in which the curves have not changed. Differential reinforcement has not altered the position of the two curves, relative to each other. The curve for the underestimators lies above that of the overestimators from 5-15 stimulus-dots. From this point on, with but few exceptions, the curve of the overestimators still lies above that of the underestimators even though the overestimators' errors are now negative. The curve for the overestimators still shows a sharp rise near the top of the stimulus-range.

The two curves after differential reinforcement are like the two before differential reinforcement. It is as if the two curves before differential reinforcement had been flattened with no change in the general shape or the relation between the curves for the overestimators and underestimators.

In summary, we can draw the following conclusions from the comparison of Figures 10 and 11:

1. Differential reinforcement has reduced the size of the percentage errors for both overestimators and underestimators, particularly at the higher stimulus-values.
2. Differential reinforcement has increased the resemblance between the two curves.
3. Although both curves have been greatly flattened, differential reinforcement has not altered the general trend of either curve.

4. *Trend of Individual Reports*

There are three further questions concerning the effect of differential reinforcement upon error. First, is its effect sudden or gradual? Secondly, is its effect specific or does it spread to reports on other stimulus-values? Thirdly, is there a limit to the reduction in error? To answer these questions we present data from two *S*'s: one overestimator, *Lu,* and one underestimator, *Be.* The data for the other *S*'s are similar. We plotted, for these two *S*'s, the last four reports before differential reinforcement, the first four reports during differential reinforcement, and the first four reports made eight months later. (That is, we plotted the last four reports of the control series, the first four of the first experimental series, and the first four of the second experimental series.)

Figures 12, 13, 14, 15, and 16 show these individual reports plotted in succession for the stimulus-values 7, 25, 66, 152, and 210, respectively. The two dark, vertical lines mark off the three different groups of four reports—control, first experimental, and second experimental.

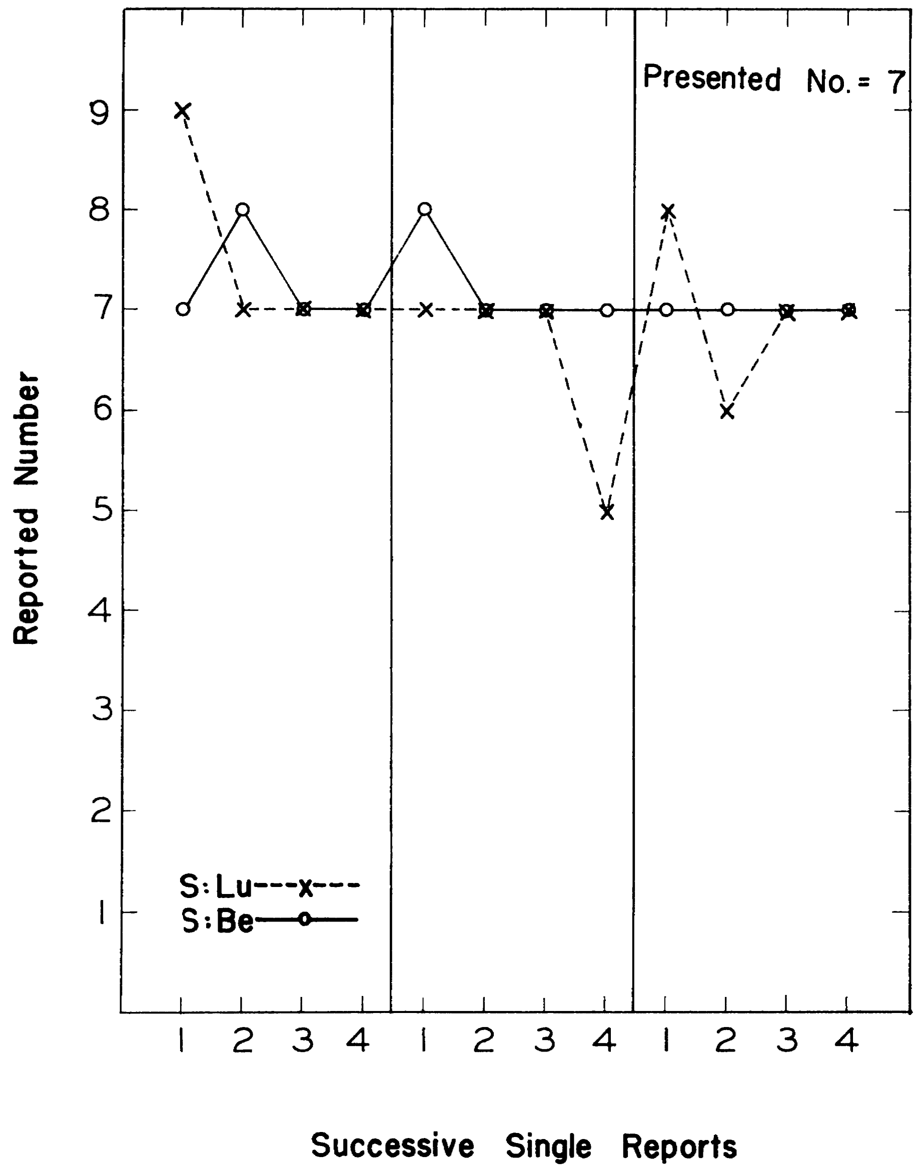

FIGURE 12

Single Reports of Number on 12 Different Presentations of Dot-Fields Containing Seven Dots; Two Subjects, *LU* and *BE*

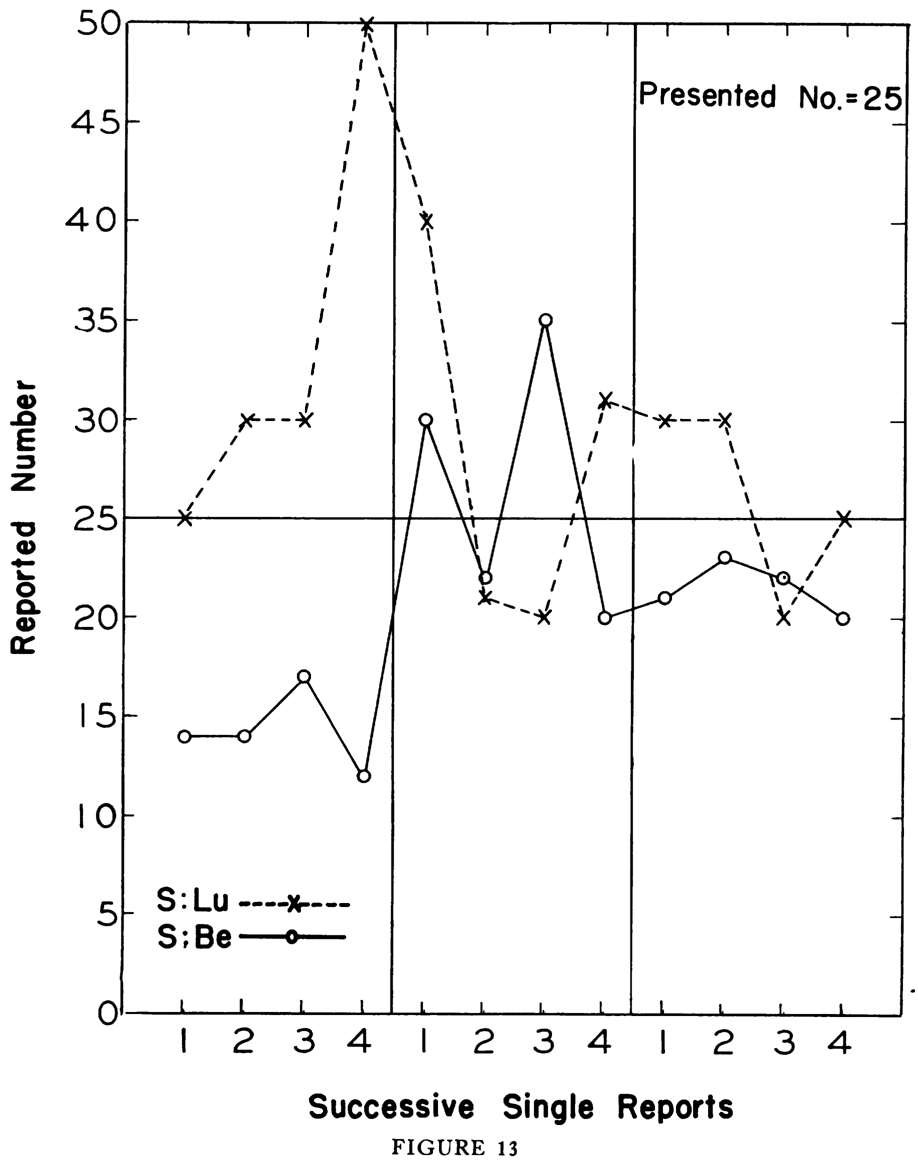

FIGURE 13

SINGLE REPORTS OF NUMBER ON 12 DIFFERENT PRESENTATIONS OF DOT-FIELDS CONTAINING 25 DOTS: TWO SUBJECTS, *LU* AND *BE*

For the presented number 7 (Figure 12), differential reinforcement has little effect. As we have said, the reports for the low stimuli are accurate, or so nearly accurate that there is little room for improvement.

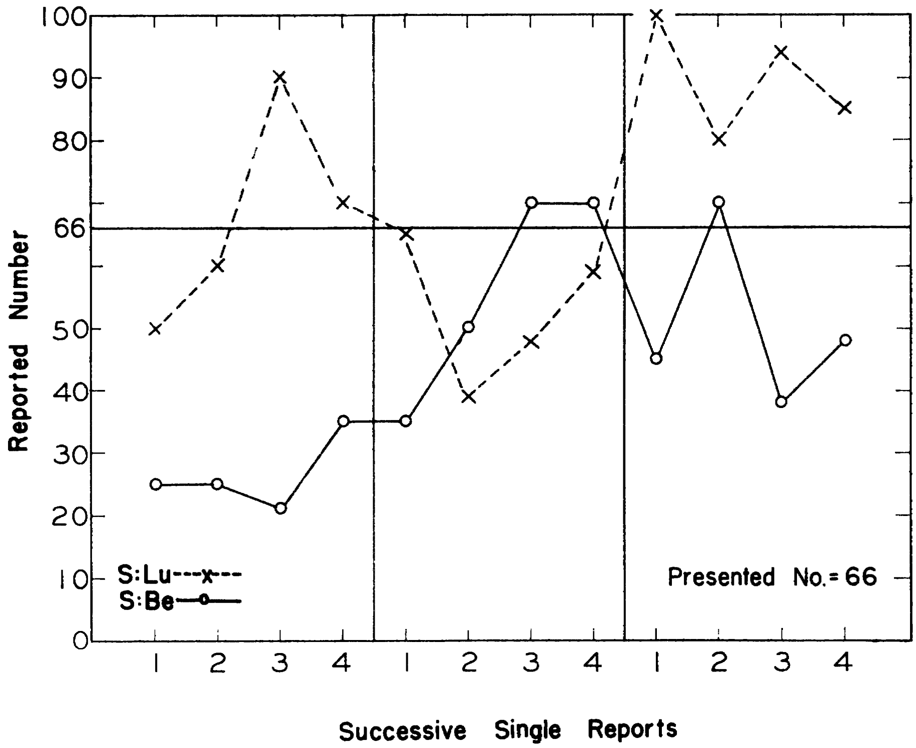

FIGURE 14
SINGLE REPORTS OF NUMBER ON 12 DIFFERENT PRESENTATIONS OF DOT-FIELDS CONTAINING 66 DOTS: TWO SUBJECTS, *LU* AND *BE*

For the remaining figures (13 through 16) the reduction in error after differential reinforcement can be clearly seen. The effect is, in nearly all cases, sudden. In these figures, it is seen in the first or second report of the first experimental series. The effect of differential reinforcement upon the *first* report in the experimental series deserves some comment. It will be remembered that the *S* was not told the correct stimulus-number until after she had made her report. Obviously there can be no effect at all until at least one report has been differentially reinforced. Very frequently, however, the shift in reported number comes before differential reinforcement for that *particular* report. This shows that the effect of these reinforcements is not specific for each stimulus-value but is spread to other stimulus-values.

For example, suppose that the first stimulus-value to be shown in the experimental series was a field of 66 dots. The *E* would show this field, allow the *S's* to make their reports, and then tell them that the field actually

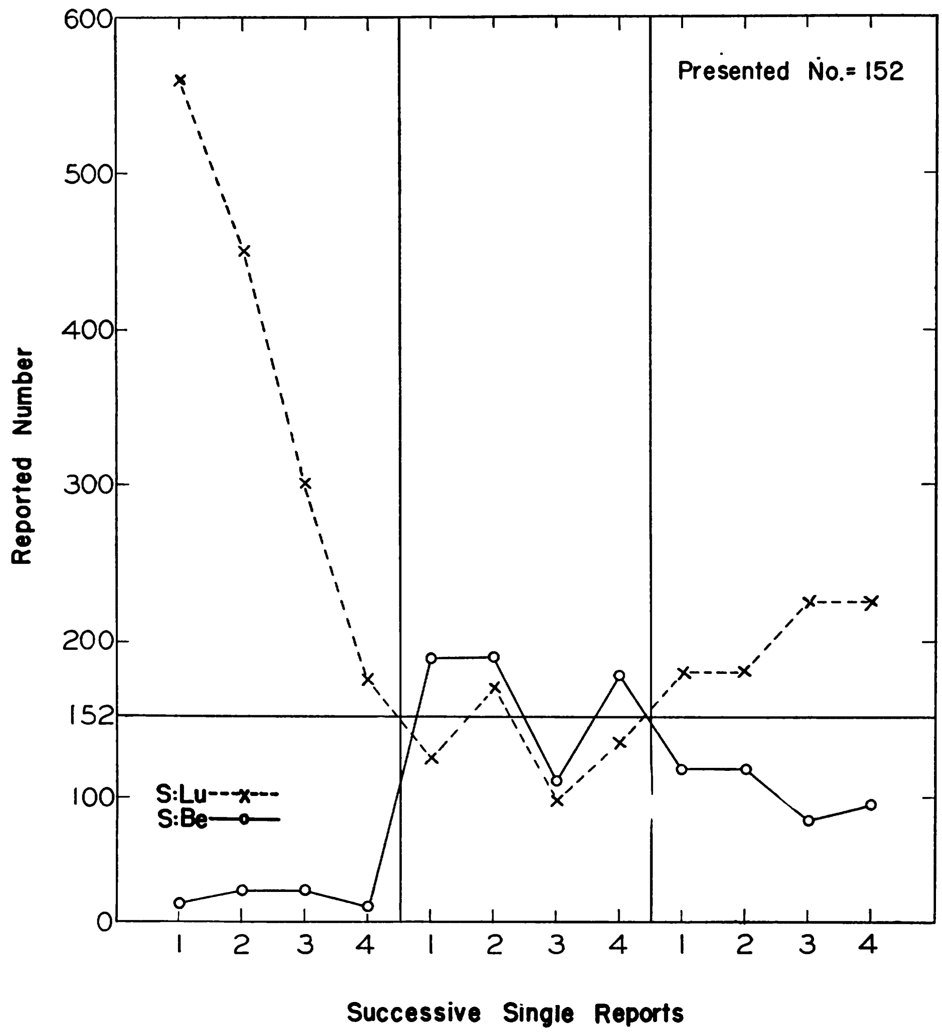

FIGURE 15
SINGLE REPORTS OF NUMBER ON 12 DIFFERENT PRESENTATIONS OF DOT-FIELDS CONTAINING 152 DOTS: TWO SUBJECTS, *LU* AND *BE*

contained 66 dots. Then the *E* would present a field of 118 dots. The knowledge that the former field contained 66 dots immediately affected the way that the *S*'s reported this field of 118 dots. The reports were very much more accurate; the overestimators lowered theirs, the underestimators raised theirs.

In general, the errors of the last few trials during differential reinforcement

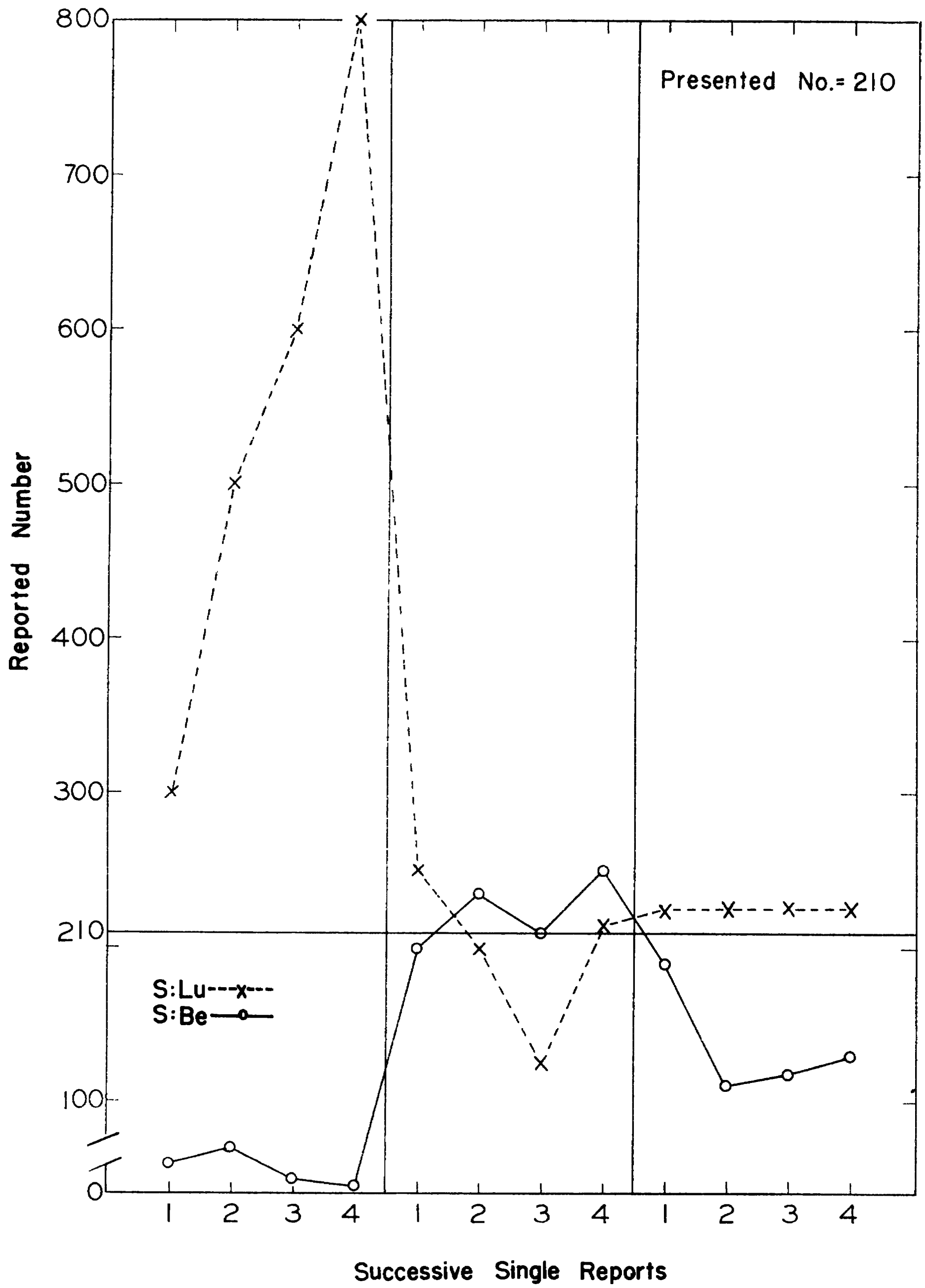

FIGURE 16
SINGLE REPORTS OF NUMBER ON 12 DIFFERENT PRESENTATIONS OF DOT-FIELDS CONTAINING 210 DOTS: TWO SUBJECTS, *LU* AND *BE*

are no longer decreasing. That is to say, after the first effect of differential reinforcement, the *S*'s show little further improvement. The presented number 210 is an exception. It will be discussed later as a special case of anchoring.

After eight months—the last four points in these curves—the size of the errors of the individual reports is greater than at the end of differential reinforcement, but in general not so large as it was in the control series. One of the few exceptions to this last statement can be seen in *Lu*'s curve in Figure 14.

There are three other aspects of these curves which are worth mentioning. In complete accordance with the expectations of reinforcement-theory, practice without differential reinforcement does not necessarily reduce error. For instance, the curve of *Lu* in Figures 13 and 16 shows a large increase in error during the last four trials before differential reinforcement.

While differential reinforcement in general reduces the errors, the *S*'s occasionally make errors at the end of differential reinforcement which are as great as or greater than those made before. Figures 12, 13, and 14 show instances of this kind.

Finally, there is a marked tendency for the *S*'s to "overcompensate" during differential reinforcement. By this we mean only that the overestimators tend to underestimate, and the underestimators tend to overestimate. This tendency is reversed again after eight months. The overestimators are overestimating again, and the underestimators are underestimating. This can be clearly seen in Figures 13-16.

D. Discussion

1. *Differential Reinforcement and Anchoring*

As we have just seen, the effects of differential reinforcement are abrupt rather than gradual and soon reach their limit. They also spread immediately to reports on stimuli that are remote from the stimulus just presented. These facts suggest strongly the phenomenon of *anchoring,* often encountered in the psychology of judgment. What, then, is the relation between differential reinforcement and anchoring?

One kind of anchoring is stimulus-anchoring. The operations for producing stimulus-anchoring are almost the same as one class of operations for producing differential reinforcement. We refer to those cases in which: (*a*) The *S* is shown a set of stimuli or stimulus-objects. (These might be a set of dot-fields ranging from 1-210 dots.) (*b*) The *S* is instructed to make a differential response with respect to these stimuli. (He might be instructed to report the number of dots in each field.) (*c*) The *S* is told ahead of time what response to make to one or more of these fields. In this case we say that the *E* supplies the *S* with an anchoring stimulus. (For example, the *S* may be shown a field of 210 dots and simultaneously be told that it is a field of 210 dots.)[9]

Now let us take a look at the operations for producing differential reinforcement. They are essentially the following: (*a*) A set of discriminative stimuli (S^Ds) is provided. (This could be identical to (*a*) above—providing a set of dot-fields ranging from 1 to 210 dots.) (*b*) Some differential responses occur. (These could be identical to (*b*) above—reporting the number of dots presented.) (*c*) Reinforcement is given for one response, called the "right response." Reinforcement is not given (or punishment is given) for all the other responses, called "wrong responses." (This operation could be similar to (*c*) above in that the *S* could be presented with a field of 210 dots and could be told that it was 210 dots.)

But there is an apparent difference between these operations. In the case of differential reinforcement, the *S* is told what he should have said *after* he makes his estimate. But in the anchoring situation he is told how to respond *before* he responds. Is this difference a real difference? And are there any other operations, not immediately apparent, that could distinguish anchoring from differential reinforcement? We think we can answer these two questions, but to do so it will be necessary to take an actual example. It is a hypothetical example, and it is a simple example, but it contains all of the essential operations of differential reinforcement and of anchoring. We will use for our example an experiment in the estimation of number. The stimulus-figures consist of only two dot-fields—100 dots and 90 dots. We assume that there has been a control experiment—that the *E* knows how the *S* responds, on the average, to the two patterns before anchoring or before differential reinforcement. We also assume that the *S* is adequately instructed to report the number of dots presented, that the important independent variables are controlled, etc.

The experimental series for anchoring follows (the stimulus-field of 100 dots will be the anchoring stimulus):

1. The *E* says, "This field contains 100 dots."
2. He presents a field of 100 dots.
3. The *S* makes no overt response.

[9]It is immaterial in this analysis whether or not the response the *S* is told to make is "correct" or "true". He may be told that 210 dots is 250.

4. The *E* presents a field of 90 dots.
5. The *S* writes down his estimate.

The series could be repeated, but it is not necessary to repeat it for the purpose of our analysis. Operations (1) and (2) are sufficient to establish 100 dots as an anchoring stimulus for the following dot-field of 90 dots.

Suppose, on the other hand, that the *E* told the *S* that the first field contained 100 dots, *after* the *S* had actually made a response. Then we would have a case of differential reinforcement:

1. The *E* presents a field of 100 dots.
2. The *S* writes down his estimate.
3. The *E* says "That field contained 100 dots."
4. The *E* presents a field of 90 dots.
5. The *S* writes his estimate.

The experiment is over. It obviously produced differential reinforcement for the *S*'s estimate of 100 dots in Step 2, but it equally obviously affects his estimate of 90 dots in Step 5. To take the simplest case, suppose that the *S* has actually written down "100 dots" as an estimate in Step 2. That behavior would then have been strongly reinforced by the *E*'s statement "100 dots" in Step 3. The predicted results are at least the following: (*a*) a strengthening of the response "100 dots" to 100 stimulus-dots; (*b*) a strengthening of the response "100 dots" to neighboring stimuli, by stimulus-generalization; (*c*) a strengthening of neighboring responses (like "90 dots" or "110 dots") by response-generalization.

Indeed, we are inclined to say that in this special case of estimating 100 stimulus-dots as 100, the operations of reinforcement have also produced an anchoring effect, and have made the 100-dot-stimulus an anchoring stimulus for the following estimate (Step 5) of the 90-dot stimulus (Step 4). This is to explain anchoring in terms of reinforcement. Naturally, it is quite another matter to account in this way either for the particular effects reported in this paper or for the other complex and extensive anchoring effects reported elsewhere.[10] But modern behavior theory and the psychology of judgment must converge somewhere, and this is one of the possible routes.

What can we say about the other cases in the example of differential reinforcement, when the *S* says something other than "100 dots" to the 100-dot-stimulus? Or the example of anchoring given above, in which the *S* makes no overt response at all to the 100-dot-stimulus? One possibility is

[10]See, for example, McGarvey (4), Reed and Safford (5), Kaufman *et al.* (3), and Rogers *et al.* (6).

to assume an implicit response like "That is 100 dots" following the 100-dot-stimulus. In the anchoring example this response would be evoked jointly by the stimulus (Step 2) and by the *E*'s instruction (Step 1). In the example of differential reinforcement, it would follow the *E*'s statement "100 dots" (Step 3), if it had not occurred as a correct response already.

No operationist will be happy about assuming an implicit response, if "implicit" means "unobservable." However, one can probably find operations for making this kind of response as observable as any other piece of verbal behavior. It would be possible, for example, to apply a kind of talking-out method in the psychophysical experiment: the *S* could be instructed to keep right on talking out loud in the intervals between stimulations and judgments. Very probably one would hear such statements as "Oh, that was 100 dots," or "I must remember that that is 100 dots."

A related set of operations are those which produce introspections at the close of an experiment. These are verbal responses related to the original implicit ones that are presumed to have occurred during the experiment.[11] An introspection on an anchoring experiment would be likely to mention the occurrence of the words "That is 100 dots."

This mode of explanation reduces all of the cases mentioned above to one: at some time after the 100-dot-stimulus, and before the 90-dot-stimulus, there occurs a verbal response, "100 dots." This response may be overt or implicit. The *E*'s behavior, both before and during the experiment, evokes this response in the presence of a 100-dot-stimulus.[12] Perhaps the *E*'s behavoir even provides indirectly for the reinforcement of this response: the *S* is made to feel very confident that this particular stimulus is indeed 100 dots.

There is one other difference between the situations in which the terms *reinforcement* and *anchoring* are usually applied. One can provide reinforcement for a class of responses without instituting any further discriminations at all. Anchoring, on the other hand, requires the discrimination of some particular aspect or aspects: in our examples, numerousness.

For example, in the experiment cited above it would not be necessary to present the field of 90 dots to demonstrate that the response "100 dots" had been *reinforced* when 100 dots was presented. But it would be necessary to present some other dot-field such as 90 dots in order to demonstrate that

[11]In this case, the introspection may quite plausibly be regarded as a discrimination by an *S* of his own preceding behavior. Skinner (7) has discussed the much more complicated case in which neither the response nor the discriminative stimulus is externally observed.

[12]When the response occurs for the first time, it is not to be attributed to anchoring. It falls under the usual rubric of "suggestion," which we hope to discuss in another paper.

100 dots was acting as an *anchoring stimulus.* The process of abstracting a particular discriminable aspect plainly depends on the language-behavior of the *S,* which itself has a long history of differential reinforcement. In giving the *S* verbal instructions, the *E* relies on this long history to produce the particular discriminations that he desires.

2. *The Effectiveness of Anchoring Stimuli*

In this experiment the operation of differential reinforcement has made every stimulus potentially an anchoring stimulus. The effectiveness of the anchoring, however, is limited by the differential sensitivity of the *S*'s. For example, since the *S*'s were unable to discriminate 66 dots from 57 dots or 77 dots, they sometimes called 66 dots "57" or "77" even after differential reinforcement. If the adjacent stimuli had all been separated from one another by several *jnd*'s, we might expect that the *S*'s would eventually be as accurate for 66 dots as for 4 dots. However, with a stimulus-series as dense as the one used in this experiment, we do not believe that this would happen.

The curves of percentage error and of individual median reported number have shown that the various anchoring stimuli are not all equally effective. Percentage error was reduced most in the experimental series at the top stimulus (210 dots). Variability between *S*'s also was reduced most at the top stimulus. In short, this stimulus became the most effective anchoring stimulus.

This fact may be partially explained in the following way. Assume that in the procedure of differential reinforcement the *S*'s have learned which particular response-categories to use, to the virtual exclusion of the other categories. The highest category that they will use, then, is "210"; this is the highest number that they have heard the *E* call out. Given any discriminative capacity, the *S*'s will be more likely to call 210 dots "210" than to call 191 dots (or any other stimulus) "210." The top category "210" thus becomes anchored by the 210-dot-stimulus: a case of end-anchoring. Now 210 dots may be called "191," but it will not be called anything higher than "210"; hence the relatively low variability of estimating 210 dots after differential reinforcement. Every other dot-field could be called something either higher or lower than its true value. By similar reasoning 191 dots should also show some effect of the end-anchoring of the category "210" since it can be placed in only one higher category, "210."

E. Conclusions

Our *S*'s received brief, simultaneous, visual presentations of randomly arranged fields of dots. They were instructed to report the number of these dots. They then were differentially reinforced, i.e., they were informed of the accuracy of their reports. Eight months later they were brought back to the laboratory, shown the same dot-fields again, and asked to report the number. This time they were not informed of their accuracy. The actual number of dots presented ranged from 1-210.

Analysis of the results leads us to the following conclusions:

1. Differential reinforcement decreases the error of reporting number and decreases the variability between the *S*'s.

2. All of these functions show the same trends after differential reinforcement as before. The functions are all greatly flattened, however, so that the trends are less easily seen.

3. Differential reinforcement does not obscure the differences between subitizing and estimating. One line of evidence, the variability between *S*'s, indicates that immediately after differential reinforcement no more dots are subitized than before.

4. All of the effects of differential reinforcement are present, but in much smaller degree, after eight months.

5. The results offer some suggestive but inconclusive evidence for the existence of two populations of *S*'s: "overestimators" and "underestimators."

6. The effects of differential reinforcement are in general abrupt, and soon reach their apparent limit. The effects spread rapidly to reports on remote stimuli. These facts suggest the phenomenon of *anchoring* in the psychology of judgment.

7. One particular anchoring effect appears: the decreased error and variability of reporting the few highest stimuli. This is a case of end-anchoring.

8. The operations for producing differential reinforcement and anchoring are very similar. The process of making a stimulus into an anchoring stimulus is interpreted as a special case of reinforcement. However, one probably cannot subsume all anchoring phenomena under reinforcement theory at the present time.

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Department of Psychology
Mount Holyoke College
South Hadley, Massachusetts