Data Structures and Algorithms

Semester 2 year 2 (2021)
Tutorial 10
Dynamic Programming

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1) What is meant by optimal substructure in Dynamic Programming?

2) Given a chain $(A_1, A_2, ..., A_{n-1}, A_n)$ of n matrices, where for i = 1,2, …, n matrix A_i has dimension $p_{i-1} \times p_i$. Assume that $m[i,j]$ is the minimum number of scalar multiplications needed to compute the matrix $A_{i...j} = A_i \times A_{i+1} \times ..... \times A_{j-1} \times A_j$ and it is defined below

$$m[i,j] = \begin{cases} 0 & \text{if } i = j \\ \min\limits_{i \le k < j} \{m[i,k] + m[k+1,j]\} + p_{i-1} p_k p_j & \text{otherwise} \end{cases}$$

Consider the following set of metrics A_1, A_2, A_3 and A_4 with their dimensions of 2×5, 5×3, 3×10 and 10×4 respectively.

   a) Draw the *m* and *s* table to find the optimal parenthesizing of the matrices for the above sequence of matrices using the Dynamic Programming algorithm.
   b) Hence find the optimal parenthesizing and optimal number of scalar multiplications of the above matrices