Sri Lanka Institute of Information Technology

Year 02 – Semester II – 2020

Probability and Statistics – IT2110

Tutorial 04 - Answers

1) Since, $\sum_{i=1}^{4} P(X = x_i)$ = 1

c + 4c + 9c + 16c = 1

30c = 1

C = 1/30

E(X) = $\sum_{all\ x} x * P(X = x)$

E(X) = (1*c) + (2*4c) + (3*9c) + (4*16c)

E(X) = 100c

E(X) = 100*(1/30)

E(X) = 3.33

5) Let X – No of passengers in a car (X = 0,1,2,3,4)

a) P(X≥2) = P(X=2) + P(X=3) + P(X=4)

P(X≥2) = 0.1 + 0.05 + 0.05

P(X≥2) = 0.2

b)

X	0	1	2	3	4
P(X=x)	0.7	0.1	0.1	0.05	0.05
$F_X(x)$ / P(X≤x) (c.d.f)	0.7	0.8 (0.7+0.1)	0.9(0.7+0.1+0.1)	0.95	1

No need to sketch the c.d.f. (cumulative distribution function)

c)

i. E(X) = $\sum_{all\ x} x * P(X = x)$

E(X) = (0*0.7) + (1*0.1) + (2*0.1) + (3*0.05) + (4*0.05)

E(X) = 0.65

ii. E(X^2) = $\sum_{all\ x} x^2 * P(X = x)$

E(X^2) = (0*0.7) + (1*0.1) + (4*0.1) + (9*0.05) + (16*0.05)

E(X^2) = 1.75

iii. V(X) = E(X^2) – $(E(X))^2$

$V(X) = 1.75 - 0.65^2$

$\underline{\underline{V(X) = 1.3275}}$

iv. $E(3X-2) = E(3X) – E(2)$ *(Apply the properties of the expected value)*

$E(3X-2) = 3*E(X) – 2$

$E(3X-2) = 3*0.65 – 2$

$\underline{\underline{E(3X-2) = -0.05}}$

v. $V(2X+6) = V(2x) + V(6)$ *(Apply the properties of the variance. Note that covariance between a variable and a constant is zero)*

$V(2X+6) = 2^2 * V(x) + 0$

$V(2X+6) = 4 * 1.3275$

$\underline{\underline{V(2X+6) = 5.31}}$

2) Given that X ~ Bin(100, 0.08).
   Mean (E(X)) = np = 100*0.08 = $\underline{\underline{8}}$
   Variance (V(X)) = np(1-p) = 100*0.08*(1-0.08) = $\underline{\underline{7.36}}$

3) Let X – No of faulty components out of 50
   When p 0.95,
   X ~ Bin(n = 50, p’ = 0.05) [where p’ (Probability of being faulty) = 1 – p = 1 – 0.95 = 0.05]

   Probability that there are fewer than 4 faulty components [P(X<4)] = P(X≤3)

   $P(X<4) = 1 – P(X\geq4)$

   $P(X<4) = 1 – 0.23959$ *(Get the value from binomial table)*

   $\underline{\underline{P(X<4) = 0.76041}}$

   When p 0.75,
   X ~ Bin(n = 50, p’ = 0.25) [where p’ (Probability of being faulty) = 1 – p = 1 – 0.75 = 0.25]

   Probability that there are fewer than 10 faulty components [P(X<10)] = P(X≤9)

   $P(X<10) = 1 – P(X\geq10)$

   $P(X<10) = 1 – 0.83632$ *(Get the value from binomial table)*

   $\underline{\underline{P(X<10) = 0.16368}}$

4) Given that X ~ Bin(400, 0.05)
   Since n = 400 > 50 and p = 0.05 < 0.1, we can approximate X into a Poisson distribution.
   Then, X ~ Poisson (λ = 20) *[λ = np = 400*0.05 = 20]*
   So, $\underline{\underline{P(X\geq5) = 0.99998}}$ *(Get the value from Poisson table)*

6) Let X – No of surf rescues per day
   Given that X ~ Poisson (λ = 2) [Because as an average two surf rescues per day]

   a) P (X>2) = P(X≥3)
      $\underline{\underline{P(X>2) = 0.32332}}$ *(Get the value from Poisson table)*

b) For a 3 day period, $\lambda = 2*3 = 6$

Let Y - No of surf rescues for three days and Y ~ Poisson ($\lambda = 6$)

Then, $P(Y=5) = P(Y\geq5) - P(Y\geq6)$

$P(Y=5) = 0.71494 - 0.55432$ *(Get the values from Poisson table)*

$\underline{\underline{P(Y=5) = 0.16062}}$

7) Let X – The demand for a particular item per day

Given that X ~ Poisson ($\lambda = 5$) [Because as an average demand for the item is 5 per day]

a) $P(X>5) = P(X\geq6) = \underline{\underline{0.38404}}$ *(Get the value from Poisson table)*

b) $P(X=0) = P(X\geq0) - P(X\geq1) = 1 - 0.99326 = \underline{\underline{0.00674}}$

8) Let X – No of traffic accidents per month at a certain intersection

Given that X ~ Poisson ($\lambda = 3$) [Because as an average 3 traffic accidents occur per month]

a) $P(X=5) = P(X\geq5) - P(X\geq6) = 0.18474 - 0.08392 = \underline{\underline{0.10082}}$

b) $P(X<3) = P(X\leq2) = 1 - P(X\geq3) = 1 - 0.57681 = \underline{\underline{0.42319}}$

c) $P(X\geq2) = \underline{\underline{0.80085}}$

9) Let X – No of defective gear boxes out of 140 cars

Then X ~ Bin(n = 140, p = 0.02)

Since n = 140 > 50 and p = 0.02 < 0.1, we can approximate X into a Poisson distribution [Or since np = 140*0.02 = 2.8 < 5, we can approximate X into a Poisson distribution].

Then, Y ~ Poisson ($\lambda = np = 2.8$) *[$\lambda = np = 140*0.02 = 2.8$]*

a) $P(X=2) = P(X\geq2) - P(X\geq3) = 0.76892 - 0.53055 = \underline{\underline{0.23837}}$ *(Get the values from Poisson table)*

b) $P(X>5) = P(X\geq6) = \underline{\underline{0.06511}}$ *(Get the value from Poisson table)*

c) $P(X<4) = P(X\leq3) = 1 - P(X\geq4) = 1 - 0.30806 = \underline{\underline{0.69194}}$