8. STATISTICAL INFERENCE [IT2110]
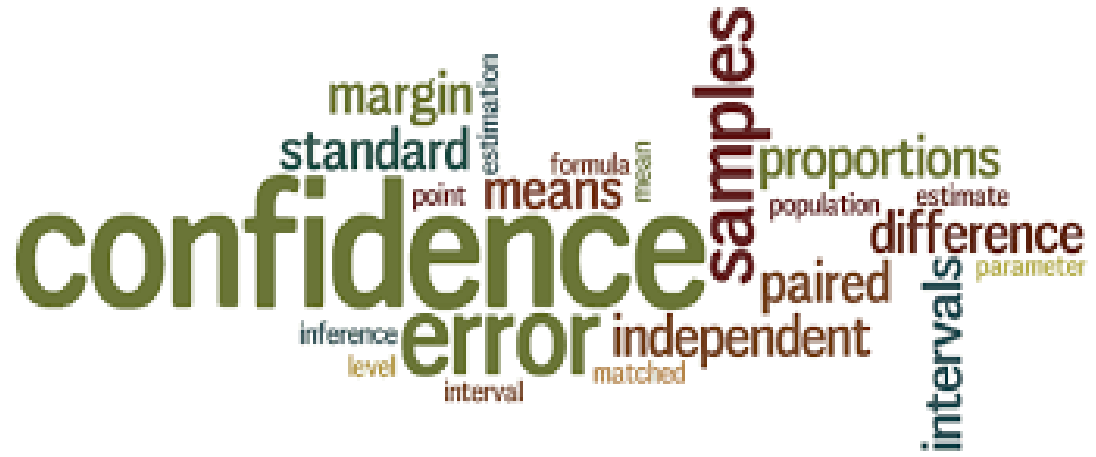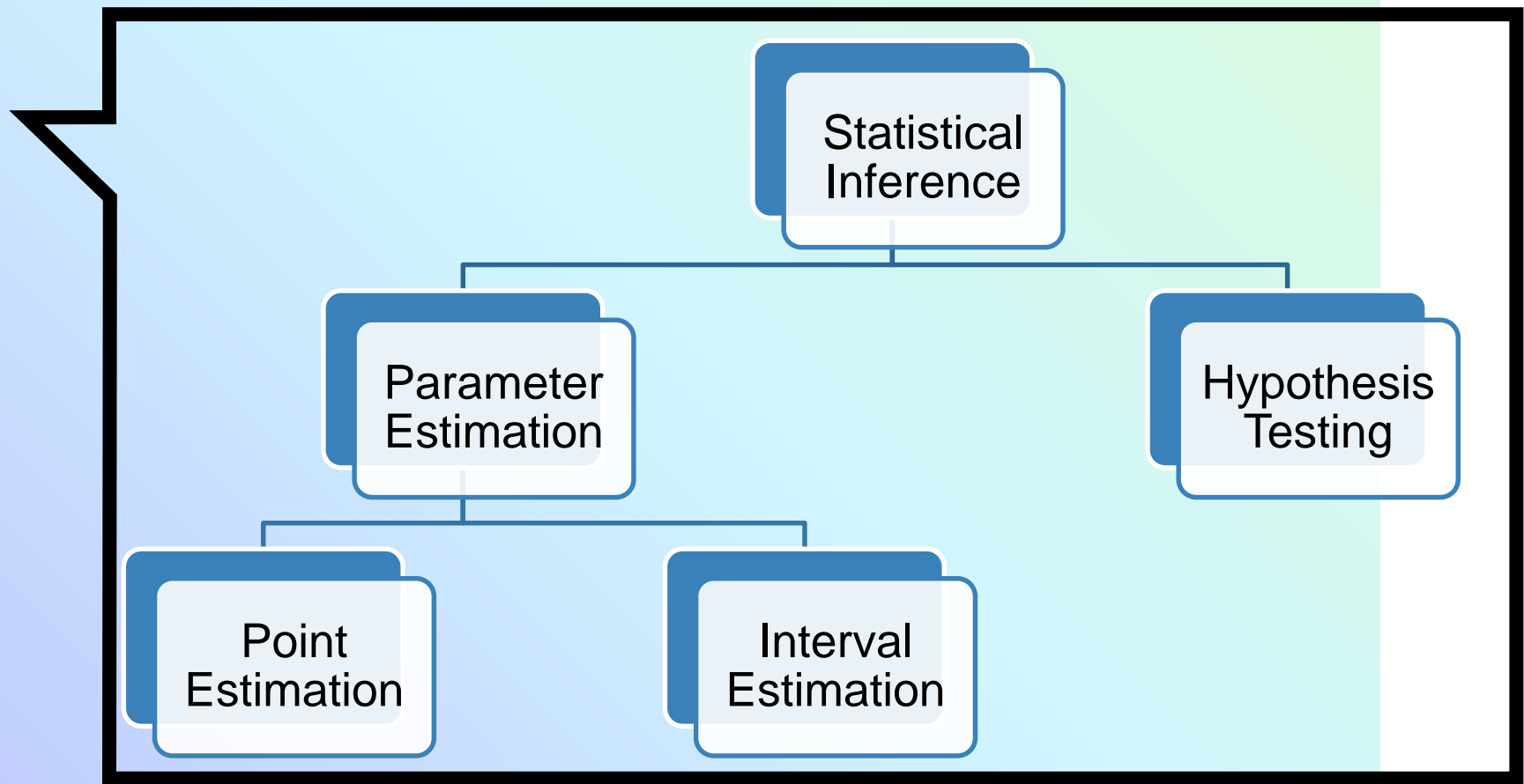
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8.2 CONFIDENCE INTERVALS

Statistical Inference
- Parameter Estimation
  - Point Estimation
  - Interval Estimation
- Hypothesis Testing

Introduction

- Estimates will differ from the true parameter values by varying amounts depending on the samples obtained.

- Point estimates do not convey any measure of reliability.

Interval Estimation

- Interval estimation states that a **population parameter** is **within two values** (an interval) with a **certain probability (Confidence Level)**.

- Interval Estimation is also known as **Confidence Interval**.

- For a good interval estimate,
  - *The probability that the parameter is within the interval should be high.*
  - *The length of the interval should be small.*

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- A confidence level for the interval should be defined first.

$$Confidence\ Level = 1 - \alpha$$

  where **α** is the significance level discussed in hypothesis testing.

- Let L and U be the lower and upper confidence limits for a parameter θ based on a random sample $X_1,\ldots,X_n$. Both L and U are functions of the sample. We can write the interval estimate of θ as,
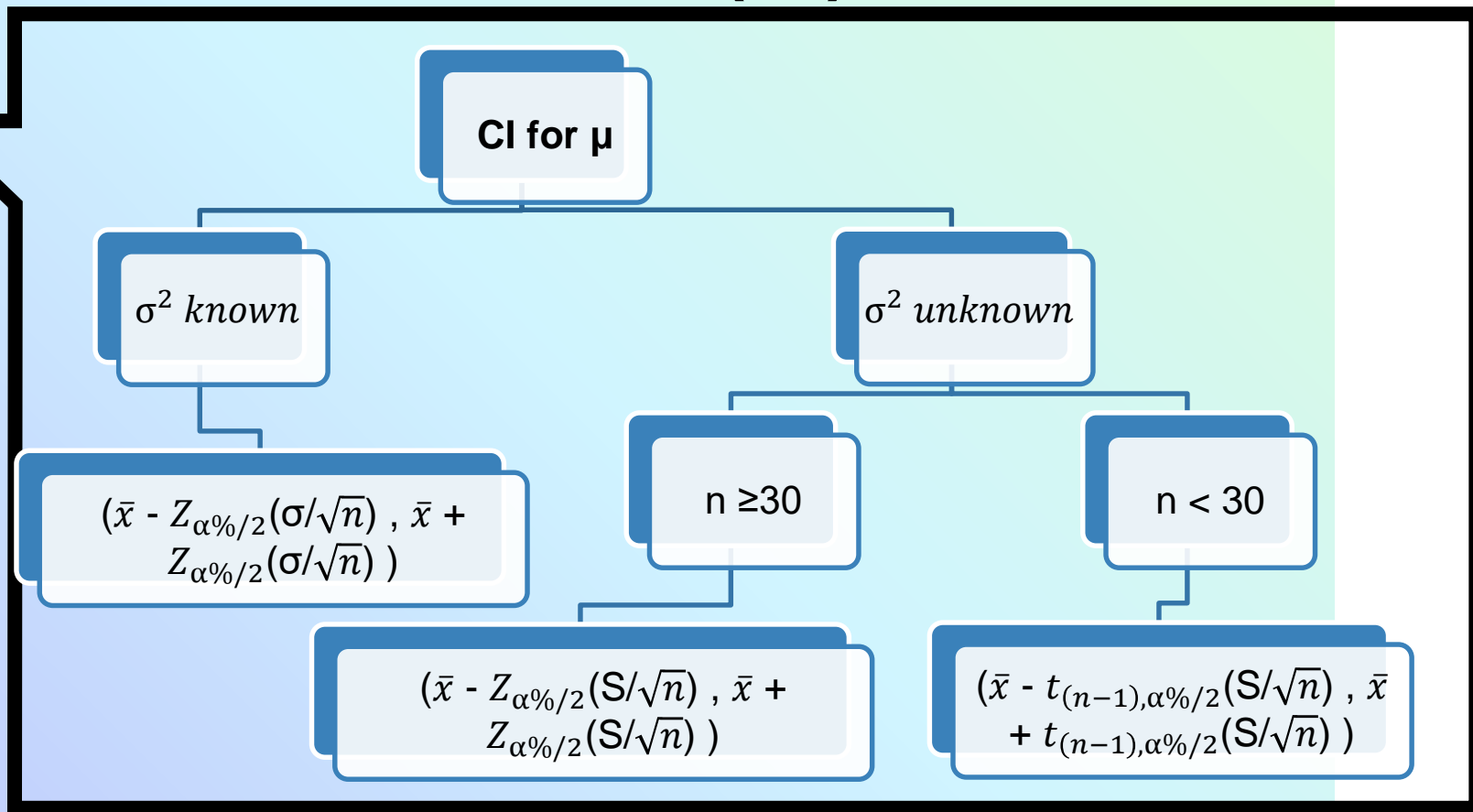
$$Pr\ (L \leq \theta \leq U) = 1\text{-}\alpha$$

- **Interpretation:**

  We are $(1 - \alpha)$ % confident that the true parameter θ is located in the interval (L, U).

- In this session, we will discuss confidence intervals for population mean (μ) only.

Confidence Intervals (CI)

CI for μ

σ² known

σ² unknown

$(\bar{x} - Z_{\alpha\%/2}(\sigma/\sqrt{n})\ ,\ \bar{x} + Z_{\alpha\%/2}(\sigma/\sqrt{n})\ )$

n ≥30

n < 30

$(\bar{x} - Z_{\alpha\%/2}(S/\sqrt{n})\ ,\ \bar{x} + Z_{\alpha\%/2}(S/\sqrt{n})\ )$

$(\bar{x} - t_{(n-1),\alpha\%/2}(S/\sqrt{n})\ ,\ \bar{x} + t_{(n-1),\alpha\%/2}(S/\sqrt{n})\ )$

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Example:

A company that manufactures cars claims that the gas mileage for its new line of hybrid cars, on the average, is 60 miles per gallon (mpg) with a standard deviation of 4 mpg. It was also found out that the mpg was normally distributed. A random sample of 16 cars yielded a mean of 57 miles per gallon. What is the interval estimation for the population mean at a 95% confidence level?

Thanks!

Any questions?