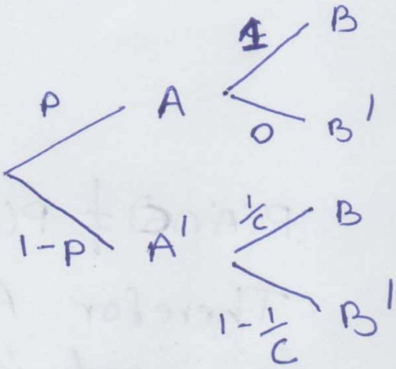
Tute-03

(01) A – Event of knowing the answer

B – Answer is correct

$P(A|B) = ?$ $\qquad P(A) = P$

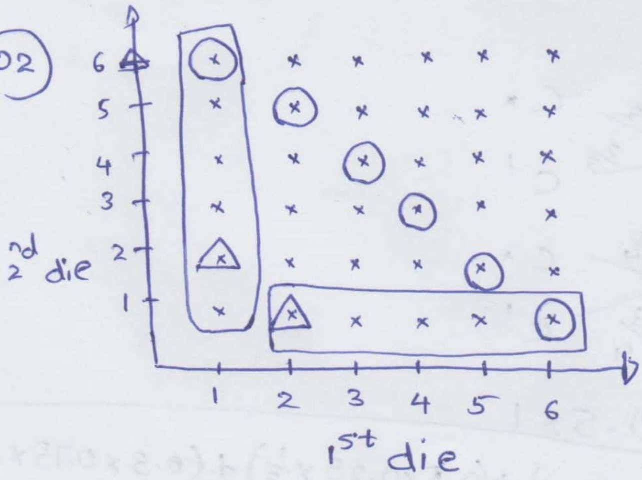


$$P(A\cap B) = P(A)\cdot P(B|A)$$
$$= P \times 1 = \underline{\underline{P}}$$

$$P(B) = P\times 1 + (1-P)\times \frac{1}{c}$$
$$= P + \frac{1}{c} - \frac{P}{c}$$
$$= \frac{PC+1-P}{C}$$

$$= P(A|B) = \frac{P(A\cap B)}{P(B)}$$
$$= \frac{PC}{(PC+1-P)}$$

(02)



A – Sum of 2 dice equal 3

B – " " " " " 7

C – at least one of the dice shows 1

△ – A

○ – B

□ – C

a) $P(A|C) = \frac{P(A\cap C)}{P(C)}$

$$= \frac{2/36}{11/36} = \underline{\underline{\frac{2}{11}}}$$

b) $P(B|C) = \dfrac{P(B \cap C)}{P(C)}$

$= \dfrac{2/36}{11/36} = \underline{\underline{\dfrac{2}{11}}}$.

c) A & C

$P(A \cap C) = \dfrac{2}{36}$

$P(A) \times P(C) = \dfrac{2}{36} \times \dfrac{11}{36}$

$P(A \cap C) \neq P(A) \times P(C)$

Therefor A & C are not independant

04 $P(A) = 0.4 \quad P(B) = 0.3 \quad P((A\cup B)^c) = 0.42$

$$P(A\cup B) = 1 - P((A\cup B)^c) = 1 - 0.42$$
$$= 0.58$$

$$P(A\cap B) = P(A) + P(B) - P(A\cup B)$$
$$= 0.4 + 0.3 - 0.58$$
$$= \underline{\underline{0.12}}$$

$$A(A) \times P(B) = 0.4 \times 0.3 = \underline{\underline{0.12}}$$

$$P(A\cap B) = P(A) \times P(B)$$

$\therefore$ A & B are independant.

05 A - First Check point is busy
B - 2^{nd} checkpoint is busy.

$$P(A\cup B)^c = 1 - P(A\cup B)$$
$$= 1 - [P(A) + P(B) - P(A\cap B)]$$
$$= 1 - [0.4 + 0.2 - 0.08]$$
$$= \underline{\underline{0.48}}$$

06 A - Event that rating is very good

B - Event that rating is good

$P(A) = 0.22$ $\quad P(B) = 0.35$

a) $P(A^c) = 1 - P(A)$

$= 1 - 0.22 = \underline{\underline{0.78}}$

b) $P(A \cup B) = P(A) + P(B)$ (Since two events are mutually exclusive)

$= 0.22 + 0.35$

$= \underline{\underline{0.57}}$

c) $P(A \cap B) = 0$

07 A - Event that a student fails in maths.

B - " " " " " " English.

$P(A) = 0.2$

$P(B) = 0.15$

$P(A \cap B) = 0.03$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.20 + 0.15 - 0.03$

$= \underline{\underline{0.32}}$

(08) A - Drawing a king
B - " " Diamond.

No. These two events are not mutually exclusive. These two events have an intersection.

ex: Diamond king.

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$
$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \underline{\underline{\frac{16}{52}}}$$

(09) ~~A~~ R - Event of getting a red ball
~~B~~ Y - " " " " yellow "
G - " " " " green "

$P(R) = \frac{4}{18}$ $P(Y) = \frac{8}{18}$ $P(G) = \frac{6}{18}$

These three events are mutually exclusive.

$$\therefore P(R\cup G) = P(R) + P(G) = \frac{4}{18} + \frac{6}{18} = \frac{\cancel{10}^{5}}{\cancel{18}_{9}} = \underline{\underline{\frac{5}{9}}}$$

(11) R - 2, b - 4, W - 5

R - Getting a Red ball.
B - " " blue ball
W - " " White "

a) $P(R) = \frac{2}{11}$

b) $P(R') = \frac{9}{11}$

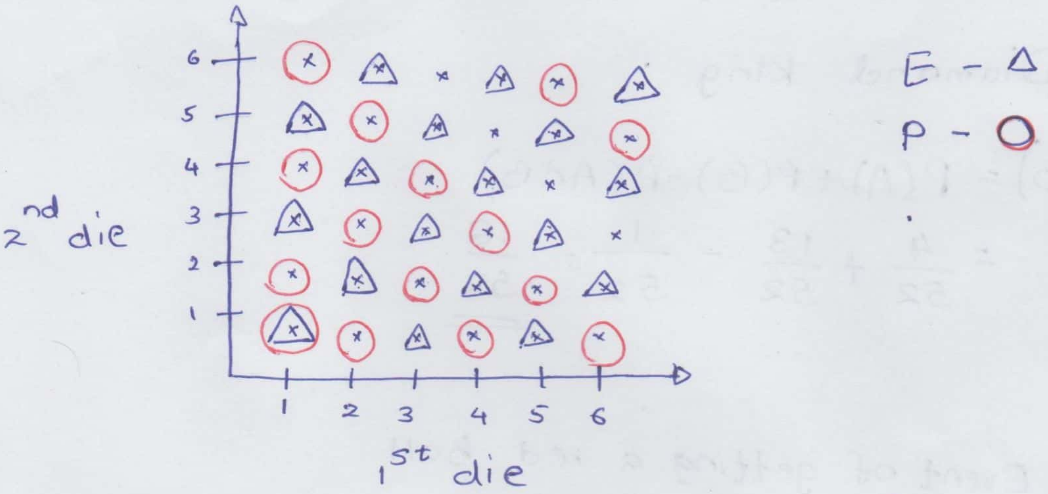
c) these three events are mutually exclusive.

$$P(R\cup B) = P(R) + P(B)$$
$$= \frac{2}{11} + \frac{4}{11} = \underline{\underline{\frac{6}{11}}}$$

(12)

E - Event of rolling a sum that is an even number.

P - Event of rolling a sum that is a prime number.



$$P(P \cup E) = P(P) + P(E) - P(P \cap E)$$

$$= \frac{15}{36} + \frac{18}{36} - \frac{1}{36}$$

$$= \frac{32}{36}$$

(14) G - ~~Wa~~ Event of watching gymnastic

B - " " " base ball

S - " " " soccer.

$P(G) = 0.28$, $P(B) = 0.29$, $P(S) = 0.19$,

$P(G \cap B) = 0.14$, $P(B \cap S) = 0.12$, $P(G \cap S) = 0.1$

$P(G \cap B \cap S) = 0.08$

~~P(A~~ $P(G \cup B \cup S)' = 1 - P(G \cup B \cup S)$

We know, $P(G \cup B \cup S) = P(G) + P(B) + P(S) - P(G \cap B) - P(G \cap S) - P(B \cap S) + P(B \cap G \cap S)$

$P(G \cup B \cup S) = 0.28 + 0.29 + 0.19 - 0.14 - 0.1 - 0.12 + 0.08$

$= \underline{\underline{0.48}}$.

$P(G \cup B \cup S)' = 1 - 0.48$

$= \underline{\underline{0.52}}$.

(15) S - Event that the visit is for specialist

L - " " " " " " Lab work

$P(S \cup L)' = 0.35$

$P(S) = 0.3$

$P(L) = 0.4$

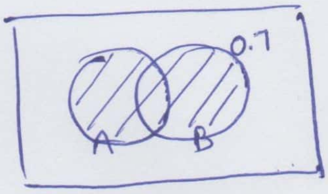
$P(S \cap L) = P(S) + P(L) - P(S \cup L)$

$= P(S) + P(L) - [1 - P(S \cup L)']$

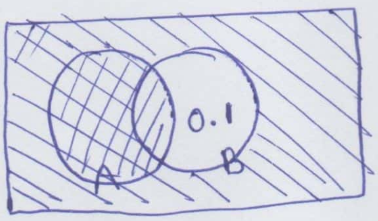
$= 0.3 + 0.4 - [1 - 0.35]$

$= \underline{\underline{0.05}}$.

(16) $P(A \cup B) = 0.7$



$P(A \cup B^c) = 0.9$



$\therefore P(A) = 0.7 - 0.1 = \underline{\underline{0.6}}$.

(10) we know $P(A \cup B) \leq 1$

$$P(A)+P(B)-P(A \cap B) \leq 1$$

$$P(A)+P(B)-1 \leq P(A \cap B).$$

$$\therefore P(A \cap B) \geq P(A)+P(B)-1$$

(13) $P(A)=0.8$, $P(B)=0.9$

if A & B are mutually exclusive,

$$P(A)+P(B) = P(A \cup B)$$

here,

$P(A)+P(B) = 0.8+0.9 = 1.7$

But $P(A \cup B) \not> 1$. Therefore, there should be an intersection.

$\therefore$ A & B are not mutually exclusive.

(17). A - Event that a patient visits chiropractor

B - " " " " physical therapist

$P(A \cap B)=0.22$, $P(A \cup B)^c = 0.12$, $P(B)=a$, $P(A)=0.14a$

$$P(A \cup B) = P(A)+P(B)-P(A \cap B)$$

$$[1-0.12] = 0.14a + a - 0.22$$

$$0.88 = 1.14a - 0.22$$

$$1.1 = 1.14a$$

$$a = 0.9649$$

$\therefore P(B) = 0.9649$

Corrected Answer for Third Question in Tutorial 03

03).

Let X - Student knows the answer

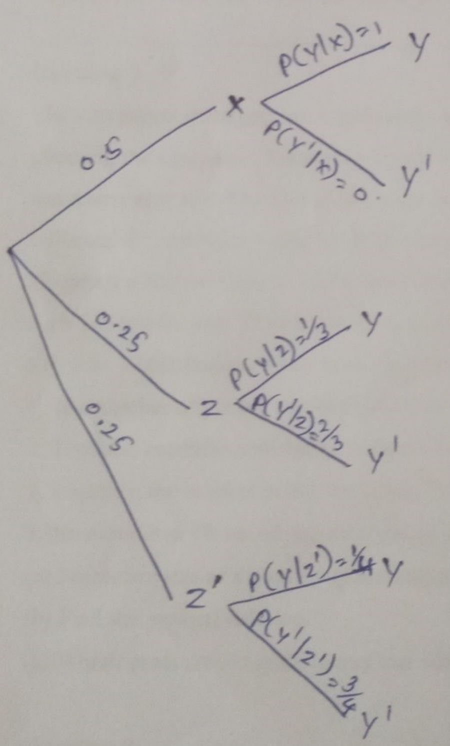
Y - The answer is correct.

Z - Student eliminate one choice.

Given that, $P(X) = 0.5$, $P(Z) = 0.25$

$P(Y|X) = 1$, $P(Y|Z) = \frac{1}{3}$, $P(Y|Z') = \frac{1}{4}$.

If we draw tree diagram,



~~$P(X|Y)$~~

$$P(X|Y) = \frac{P(X\cap Y)}{P(Y)}$$

$$= \frac{P(X\cap Y)}{P(X\cap Y) + P(Z\cap Y) + P(Z'\cap Y)}$$

$$= \frac{P(Y|X).P(X)}{P(Y|X).P(X) + P(Y|Z).P(Z) + P(Y|Z').P(Z')}$$

$$= \frac{1\times 0.5}{(1\times 0.5) + (\frac{1}{3}\times 0.25) + (\frac{1}{4}\times 0.2}$$

$$= \underline{\underline{0.7742}}$$

(Here, X, Z, and Z' are mutually exclusive & collectively exhaustive events)