Tute 05

01) X – time that elapses between the start and the the time the officer signs.

$$f(x) = \begin{cases} kx^2 & 0 < x < 10 \\ 0 & \end{cases}$$

a) $\int_{-\infty}^{\infty} f(x).dx = 1$

$\int_{-\infty}^{0} 0.dx + \int_{0}^{10} kx^2.dx + \int_{10}^{\infty} 0.dx = 1$

$k\left[\frac{x^3}{3}\right]_0^{10} = 1$

$\frac{k}{3}\left[10^3 - 0^3\right] = 1$

$K \times \frac{1000}{3} = 1$

$K = \frac{3}{1000}$

b) $C.D.F = \int_{-\infty}^{x} f(t).dt = \int_{0}^{x} k\cancel{t}^2.dt$

$= k\left[\frac{\cancel{t}^3}{3}\right]_0^x$

$= \frac{\cancel{3}}{1000} \times \frac{x^3}{\cancel{3}} = \frac{x^3}{1000}$.

c) $P(X<3) = \int_{0}^{3} kx^2.dx =$

$= \frac{3}{1000}\int_{0}^{3} x^2.dx = \frac{3}{1000}\left[\frac{x^3}{3}\right]_0^3 = \frac{3}{1000} \times \left[\frac{27}{3} - \frac{0}{3}\right]$

$= \frac{27}{1000} = 0.027$

d) $V(x) = E(X^2) - [E(x)]^2$

$$E(x) = \int_{-\infty}^{+\infty} x \cdot f(x) \cdot dx = \int_0^{10} x \cdot k \cdot x^2 \cdot dx$$

$$= \frac{3}{1000} \int_0^{10} x^3 \cdot dx$$

$$= \frac{3}{1000} \left[\frac{x^4}{4} \right]_0^{10} = \frac{3}{\cancel{1000}} \times \frac{10\cancel{000}}{4}$$

$$= \frac{30}{4} = \underline{\underline{7.5}}$$

$$E(x^2) = \int_{-\infty}^{+\infty} x^2 \cdot f(x) \cdot dx = \int_0^{10} x^2 k x^2 \cdot dx$$

$$= k \int_0^{10} x^4 \cdot dx$$

$$= k \left[\frac{x^5}{5} \right]_0^{10}$$

$$= \frac{3}{\cancel{1000}} \times \frac{100\cancel{000}}{5}$$

$$= \frac{300}{5} = \underline{\underline{60}}$$

$$V(x) = E(x^2) - [E(x)]^2$$

$$= 60 - (7.5)^2 = \underline{\underline{3.75}}$$

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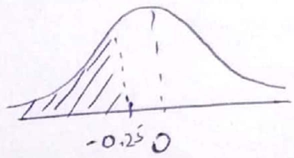
(02) X - time taken to assemble a car

$X \sim N(20, 2^2)$

a) $P(X < 19.5) = P\left(\frac{X-\mu}{\sigma} < \frac{19.5-\mu}{\sigma}\right)$

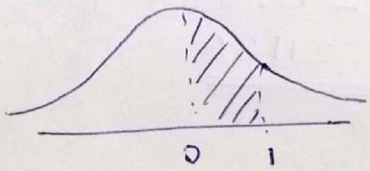
$= P\left(Z < \frac{19.5-20}{2}\right)$



$= P(Z < -0.25)$

$= \underline{\underline{0.40129}}$

b) $P(20 < X < 22) = P\left(\frac{20-20}{2} < Z < \frac{22-20}{2}\right)$



$= P(0 < Z < 1)$

$= P(Z > 0) - P(Z > 1)$

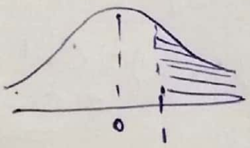
$= 0.5 - 0.15866$

$= \underline{\underline{0.34134}}$

(03) X - Marks for physics

$X \sim N(70, 10^2)$

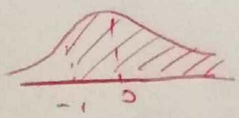
a) $P(X > 80) = P\left(Z > \frac{80-70}{10}\right) = P\left(Z > \frac{10}{10}\right)$



$= P(Z > 1)$

$= \underline{\underline{0.15866}}$

b) $P(X \geq 60) = P\left(Z \geq \frac{60-70}{10}\right) = P(Z \geq -1)$



$= \underline{\underline{0.84134}}$

c) $P(X < 60) = 1 - P(X \geq 60)$

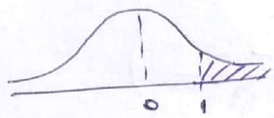
$= \underline{\underline{0.15866}}$

(04) X - speed of a car

$X \sim N(90, 10^2)$

$P(X > 100) = P\left(Z > \frac{100-90}{10}\right) = P(Z > 1)$

$= \underline{0.15866}$.



(05) X - birth weight of a baby.

$X \sim N(3500, 500^2)$

$P(X < 3100) = P\left(Z < \frac{3100-3500}{500}\right)$

$= P(Z < -0.8) = 1 - P(Z \geq -0.8)$

$= 1 - 0.78814$

$= \underline{0.21186}$.

(06) X - height of adult male.

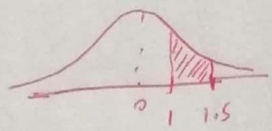
$X \sim N(70, 2^2)$

a) $P(X > 73) = P\left(Z > \frac{73-70}{2}\right) = P(Z > 1.5)$

$= \underline{0.06681}$.

b) $P(72 < X < 73) = P\left(\frac{72-70}{2} < Z < \frac{73-70}{2}\right)$

$= P(1 < Z < 1.5)$

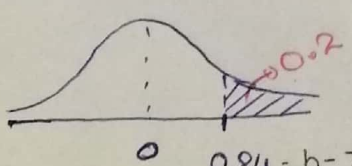


$= P(Z > 1) - P(Z > 1.5) = 0.15866 - 0.06681$

$= \underline{0.09185}$.

c) $P(X > h) = 20\%$.

$P\left(Z > \frac{h-70}{2}\right) = 0.2$



$0.84 = \frac{h-70}{2}$

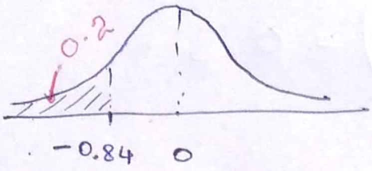
$\frac{h-70}{2} = 0.84$

$h = 70 + 1.68$

$= \underline{71.68}$

d) $P(X<h) = 0.2$

$P\left(Z < \frac{h-70}{2}\right) = 0.2 \Rightarrow P\left(Z > \frac{h-70}{2}\right) = 0.8$



$-0.84 = \frac{h-70}{2}$

$-1.68 = h-70$

$70-1.68 = h$

$h = 68.32$

(07)

X - Speed of a vehicle

$X \sim N(71, 8^2)$

a) $P(X \leq 65) = P\left(Z \leq \frac{65-71}{8}\right) = P(Z \leq -0.75)$

$= 1 - P(Z > -0.75)$

$= 1 - 0.77337$

$= 0.22663$

b) $P(X < 50) = P\left(Z < \frac{50-71}{8}\right)$

$= P(Z < -2.625)$

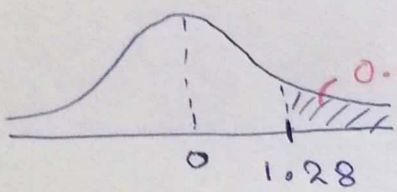
$= 1 - P(Z \geq -2.625)$

$= 1 - 0.99573$

$= 4.27 \times 10^{-3}$

c) $P(X > S) = 0.1$

$P\left(Z > \frac{S-71}{8}\right) = 0.1$



$\frac{S-71}{8} = 1.28$

$S-71 = 10.24$

$S = 81.24$

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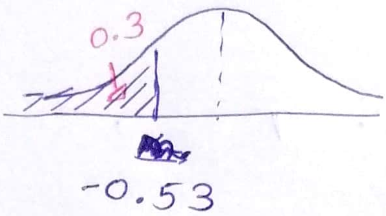
(Q8)

X - mark of a student



$$X \sim N(120, 17^2)$$

$$P(X<m) = 0.3$$

$$P\left(Z < \frac{m-120}{17}\right) = 0.3$$

$$P\left(Z > \frac{m-120}{17}\right) = 0.7$$

$$\frac{m-120}{17} = -0.53$$

$$m - 120 = -9.01$$

$$m = \underline{\underline{110.99}}$$

Passing mark $= \underline{\underline{111}}$

Question 09

Let X – Waiting time until the next customer arrives in minutes. (X≥0)

Given that λ = 20 per hour. Then, λ = 20/60 per minute = 1/3 per minute

Thus, X ~ Exp(λ=1/3)

Then, P(Next customer arrives within 6 minutes) = P(X≤6) = $\int_0^6 f_X(x)\,dx$

= $\int_0^6 \lambda e^{-\lambda x}\,dx$ = $\lambda \int_0^6 e^{-\lambda x}\,dx$

= $\lambda * [\frac{e^{-\lambda x}}{-\lambda}]_0^6$

= $[-e^{-\lambda x}]_0^6$

= $\left(-e^{-\left(\frac{1}{3}\right)*6}\right) - (-e^0)$

= $(-e^{-2}) + 1$

= $\underline{\underline{0.8647}}$

Question 10

Let X – Time required to repair a machine in hours. (X≥0)

Given that λ = 0.5 downs per hour and X ~ Exp(λ=0.5)

a) Then, P(Repair time exceeds two hours) = P(X>2) = $\int_2^\infty f_X(x)\,dx$

= 1 – P(X≤2) = 1 - $\int_0^2 \lambda e^{-\lambda x}\,dx$

= $1 - \lambda \int_0^2 e^{-\lambda x}\,dx$

= $1 - \lambda * [\frac{e^{-\lambda x}}{-\lambda}]_0^2$

= $1 - [-e^{-\lambda x}]_0^2$

= $1 - ((-e^{-0.5*2}) - (-e^0))$

= $1 + (e^{-1}) - 1$

= (e^{-1})

= $\underline{\underline{0.3679}}$

b) This problem is out of syllabus.