8. STATISTICAL INFERENCE [IT2110]

By SLIIT Mathematics Unit

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- In most researches, we collect data through a sample survey over a census survey.
- Statistical inference is used when sample survey is conducted over a census survey.
- ***Inference:*** A conclusion reached on the basis of evidence and reasoning.

  - Oxford University Press -
- ***Statistical Inference:*** Drawing conclusions about population parameters by using sample statistics.

- Statistical Inference
  - Parameter Estimation
    - Point Estimation
    - Interval Estimation
  - Hypothesis Testing

PARAMETER ESTIMATION

Parameter Estimation

- In distribution theory we assumed that distribution parameters are known.
- But practically they should be found or estimated.
- If estimated parameters are wrong, all calculated probabilities will be inaccurate.
- Estimation can be done in two methods.
    - *Point estimation*
    - *Interval estimation*

Parameter Estimation

- Point estimation gives a single estimated value for the parameter.
- Interval estimation gives a range of values (interval) as the estimate.
- There are many point and interval estimation methods with their own criteria for use.
- Some interval estimates will be discussed later in this chapter.

HYPOTHESIS TESTING

Hypothesis Testing

- ***Hypothesis:*** A supposition or proposed explanation made on the basis of limited evidence as a starting point for further investigation.

  -Oxford University Press-

- Hypothesis testing is all about checking whether assumptions (research hypothesis) are correct.

- These assumption should be regarding population parameters.

Major Steps under Hypothesis Testing

1. Define the hypothesis (H_0 & H_1)
2. Test statistic and its distribution
3. Define the significance level (α)
4. Define the rejection region.
5. Conduct the test (Calculate test statistic value)
6. Conclusion

- There are various cases under hypothesis testing. The test statistic that you should use depends on the case.
- In this session, we will discuss the hypothesis testing for one population mean.

- Hypothesis Testing
  - One Population
    - Population Variance
    - Population Mean
      - Population variance known
      - Population variance unknown
        - Large Sample (n≥30)
        - Small Sample (n<30)
    - Population Proportion
  - Two Populations
    - Two dependent
    - Two independent

To get more details go to the following link:
http://goo.gl/4we99Q

Defining Hypothesis

- The assumption should be clearly stated in order to test.
- Two statements, null hypothesis (H_0) and an alternative hypothesis (H_1or H_a) are used for that.
- $H_0$and H_1 can be considered as opposites of each other.
- The statement with the equal (=) should always come to H_0. Usually if a claim is made, it is selected for H_1.

Defining Hypothesis

Three possible Hypothesis

$H_0: \mu = a$

$H_1: \mu \neq a$

Two-tailed Hypothesis

$H_0: \mu \geq a$

$H_1: \mu < a$

$H_0: \mu \leq a$

$H_1: \mu > a$

One-tailed Hypothesis

Examples

1) In a coin tossing experiment, it should be found whether
    a) it's fair coin or not.
    b) it's biased in favor of heads.
    c) it's biased in favor of tails.

2) A company that manufactures cars claims that the gas mileage for its new line of hybrid cars, on the average, is 60 miles per gallon (mpg) with a standard deviation of 4 mpg. It was also found out that the mpg was normally distributed. A random sample of 16 cars yielded a mean of 57 miles per gallon. Is the company's claim about the mean gas mileage per gallon of its cars, correct?

Test Statistic

- ***Recap:*** A function of observable r.v.s that does not depend on any unknown parameters is called a statistic.
- A test statistic is a quantity associated with the sample.
- The test statistic will depend on the ***parameter of interest*** as well as the ***characteristics of the population***.
- We assume that the assumption (H_0) is correct and find a sampling distribution for the test statistic.

Test Statistic & Distribution

To Test Population Mean (μ)

- σ^2 *known*
  - $Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$
- σ^2 *unknown*
  - n < 30
    - $T = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} \sim t_{n-1}$
  - n ≥30
    - $Z = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} \sim N(0,1)$

Test Statistic [For μ - When σ^2 known]

- ***Recap:*** Let $X_1, \ldots, X_n$ be a random sample of size n from a Normal population with mean μ and variance σ^2. Then,

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

- Then,

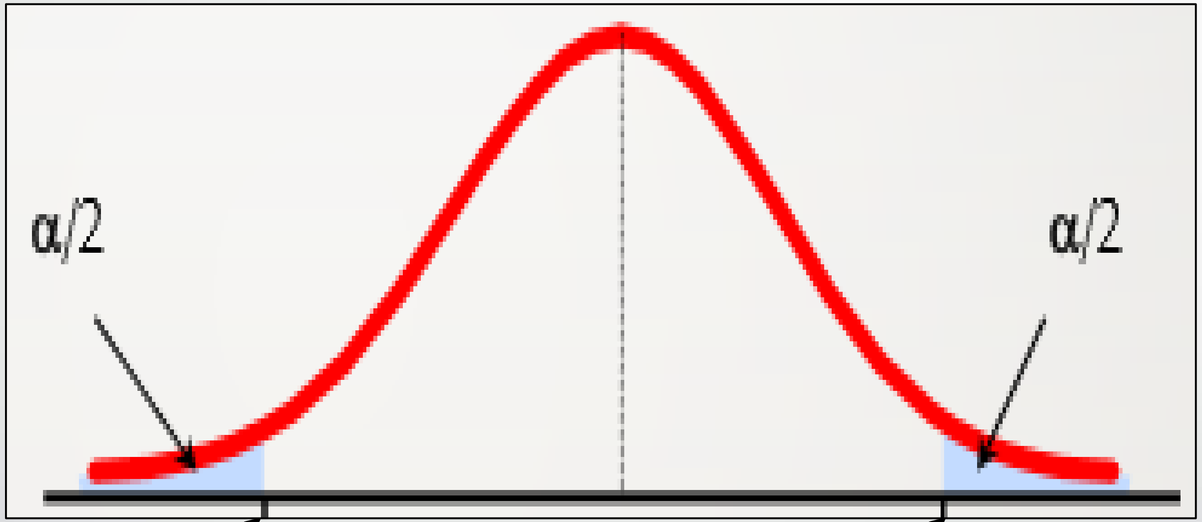
$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

- If the hypothesis is, $H_0: \mu = \mu_0$ vs. $H_1: \mu \neq \mu_0$, then under H_0,

$$\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$$

Rejection Region [For μ]

For a two-tailed hypothesis



-$Z_{\alpha/2}$ or $-t_{(n-1),\alpha/2}$ **(a1)**

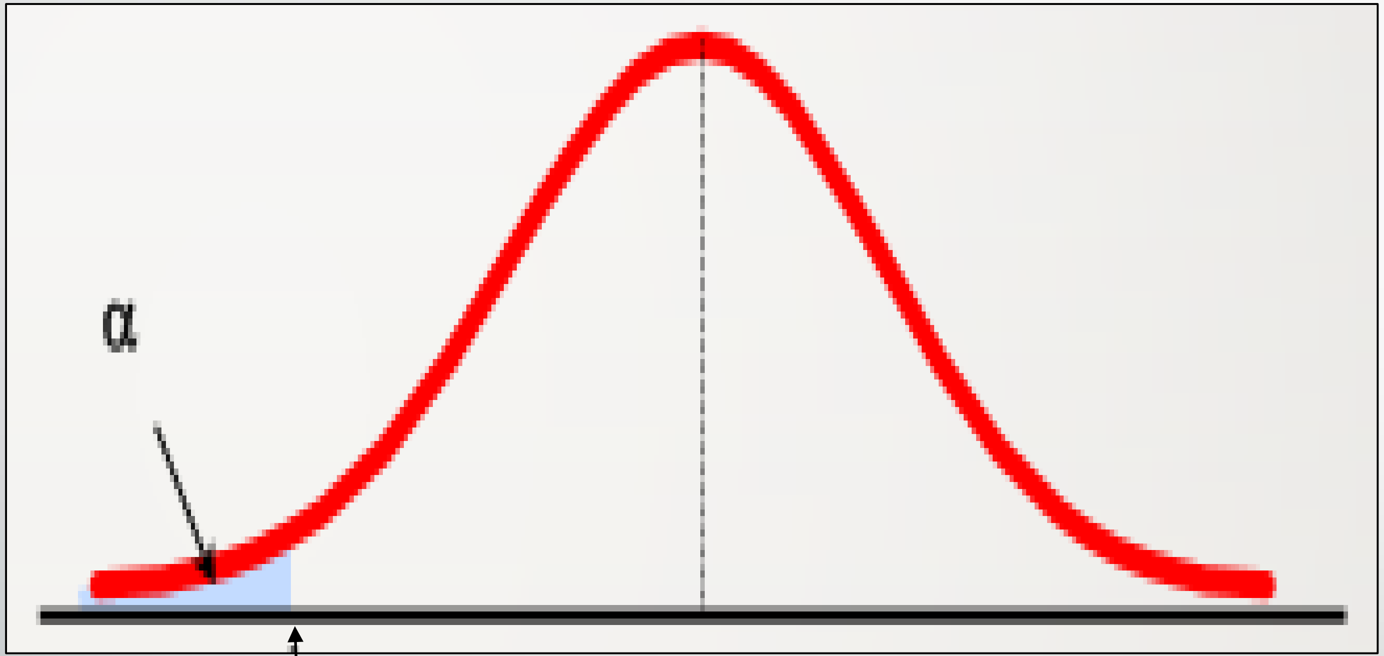
$Z_{\alpha/2}$ or $t_{(n-1),\alpha/2}$ **(a2)**

Critical Values

Reject H_0 if $Z_{cal} \geq$ $\boldsymbol{a2}$
OR if $Z_{cal} \leq$ $\boldsymbol{a1}$

For a one-tailed hypothesis

$H_0 : \mu \geq a$
$H_1 : \mu < a$



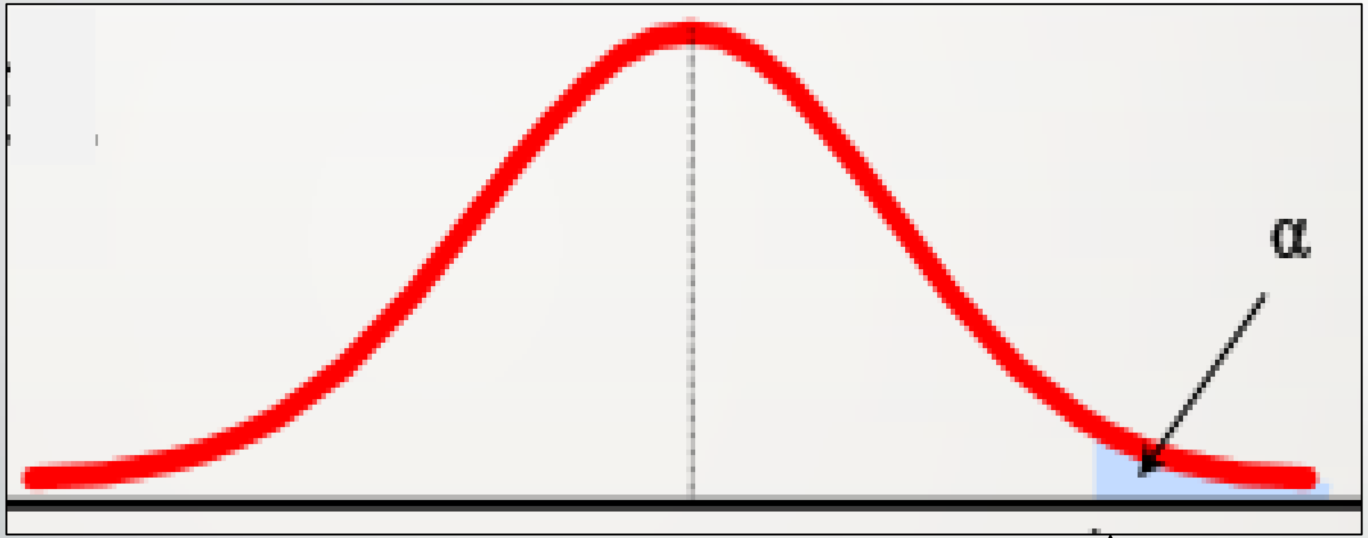
$-Z_\alpha$ or $-t_{(n-1),\alpha}$ ***(a1)***

Critical Value

Reject H_0 if $Z_{cal} < \boldsymbol{a1}$

For a one-tailed hypothesis

$H_0 : \mu \leq a$
$H_1 : \mu > a$



Critical Value → Z_{α} or $t_{(n-1),\alpha}$ **(a1)**

Reject H_0 if $Z_{cal} >$ $\boldsymbol{a1}$

Example 02:

- $\boldsymbol{H_0}: \mu = 60$
  $\boldsymbol{H_1}: \mu \neq 60$

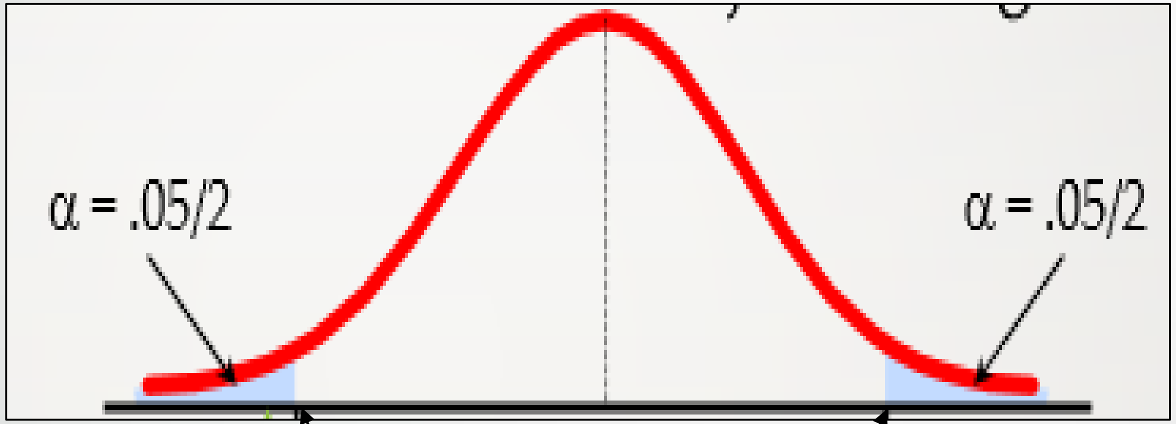
Two-tailed hypothesis

- ***Test Statistic***: Under H_0,

$$Z = \frac{\bar{x} - 60}{\frac{\sigma}{\sqrt{n}}} \sim N(0, 1)$$

- *Consider 5% level of significance.*

- ***Rejection Region:***



$-Z_{0.025}$

$Z_{0.025}$

Reject H_0 if $Z_{cal} > Z_{0.025}$ ***OR*** if $Z_{cal} < -Z_{0.025}$

$Z_{0.025} = 1.96$

- ***Test:***

$\bar{x} = 57$, σ = 4 & $n = 16$

Then, $Z_{Cal} = \frac{\bar{x}-60}{\frac{\sigma}{\sqrt{n}}}$

$$Z_{Cal} = \frac{57-60}{\frac{4}{\sqrt{16}}}$$

$$Z_{Cal} = -3$$

- ***Conclusion:***

Since Z_{Cal} = -3 < -1.96 = $Z_{0.025}$, we reject H_0 at 5% level of significance. Therefore, there is no enough evidence to suggest that company's claim about the mean gas mileage per gallon of its cars is correct.

P value Approach

- This is an alternative way of deciding the rejection criteria.
- ***P value:*** *The probability of obtaining a test statistic which is more extreme than observed test statistic value given when H_0 is true.*

P value

P value	Hypothesis
$Pr(\lvert TS\rvert > \lvert TS_{Cal}\rvert)$	*Two tailed Hypothesis*
$Pr(TS > TS_{Cal})$	$H_1: \mu > a$
$Pr(TS < TS_{Cal})$	$H_1: \mu < a$

- *For any test,*

If p value < significance level (α)	$\longrightarrow$	***Reject*** H_0
If p value ≥ significance level (α)	$\longrightarrow$	***Do not Reject*** H_0

Errors in Hypothesis Testing

Statistical Decision	*True State of the Null Hypothesis*	
	H_0 is True	**H_0 is False**
Reject H_0	Type I Error	Correct
Do not Reject H_0	Correct	Type II Error

Pr (Type I Error) = Pr (Reject $H_0 | H_0$ true) = α
Pr (Type II Error) = Pr (Do not Reject $H_0 | H_0$ false) = β

THANKS!

Any questions?