# Algorithmic Congressional Redistricting

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**Introduction**

In the United States, the borders set between congressional districts have enormous influence on the outcomes of elections. The practice of intentionally altering these borders in one way or another to benefit a specific party, known as gerrymandering, has been widely regarded as damaging to Congress's ability to accurately represent its constituents, and has been considered a degradation of the democratic process as a whole. In May of 2018, Ohio voters passed the Congressional Redistricting Procedures Amendment.[1] This is a constitutional amendment that will require the state legislature to develop new congressional borders with bipartisan support. In addition, it imposes additional criteria on the redistricting process such as limits on the number of times a municipality can be split based on its population and stipulations that the new districts be designed in a way that maximizes compactness. For my honors thesis project, I will: perform a comparative analysis of several existing redistricting tools using flexible, user-defined criteria; develop a set of mathematical tools that can be used to generate a politically viable boundary proposal that satisfies a given configuration of prerequisites; and develop a tool that can be used to generate new datasets of arbitrary size for benchmarking and

[1] Wendel, Emily E. "LSC Analysis of Sub. S.J.R. 5." Ohio Legislative Service Commission, 16 Jan. 2018.

evaluation of redistricting methods. Using these toolkits, I will evaluate redistricting proposals for the state of Ohio and develop a proposal to present to the Ohio General Assembly.

The process of congressional redistricting fits perfectly into the structure of a mathematical optimization problem. A "cost function" $J(S)$ can take as input a border proposal $S$ and compute a value based on the extent to which the proposal fails to satisfy any number of weighted criteria. An algorithm then generates a proposal $S$ in a way that decreases $J(S)$ to some optimal value. Many potential solutions have already been proposed, and the proposed toolkit will compare existing algorithms to a greedy beam-search algorithm that I will develop as a part of this project.

In addition, I will develop a stochastic process for generating artificial regional configurations that could be used to construct map databases of arbitrary size. This will serve as a useful tool both for enhanced comparison of existing algorithms, and as a training dataset for potential future algorithms that will require a wider variety of regions than is currently available. For the specific purposes of state redistricting, no such tool yet exists.

The proposed toolkit will include a cost function that will have several variable weight values, essentially "sliders" that can be increased or decreased to prioritize one configuration over another (if, for example, we wanted to give special priority to producing compact districts or wanted to maximize the number of competitive districts). In this way, the toolkit will serve as a flexible tool that can be used to produce a variety of outcomes, instead of a single absolute solution. This differentiates it from several previous attempts to provide one-size-fits-all algorithmic solutions to congressional redistricting. At the same time, however, it is important that the model be (relatively) simple and easy to interpret. It is vital to be able to justify and understand why it makes certain decisions, because accountability and transparency are paramount in an environment that is as vulnerable to subversion as the redistricting process has been in the past. There are several things the model must not do (that can be incorporated into the cost function): it must not favor one party over another; it must not infringe on federal laws that protect the voting rights of politically cohesive racial minorities[2]; it must stay in keeping with the conditions imposed by the Congressional Redistricting Procedures Amendment; and it must not in any way violate the statutes of the state and federal constitution.

**How Does Gerrymandering Work?**

Gerrymandering can be used in a couple of different ways to accomplish partisan goals. By redrawing the districts in a specific way, state legislatures can give massive advantages to one chosen party. The main tactics used to do this are “cracking” and “packing.” “Cracking” involves

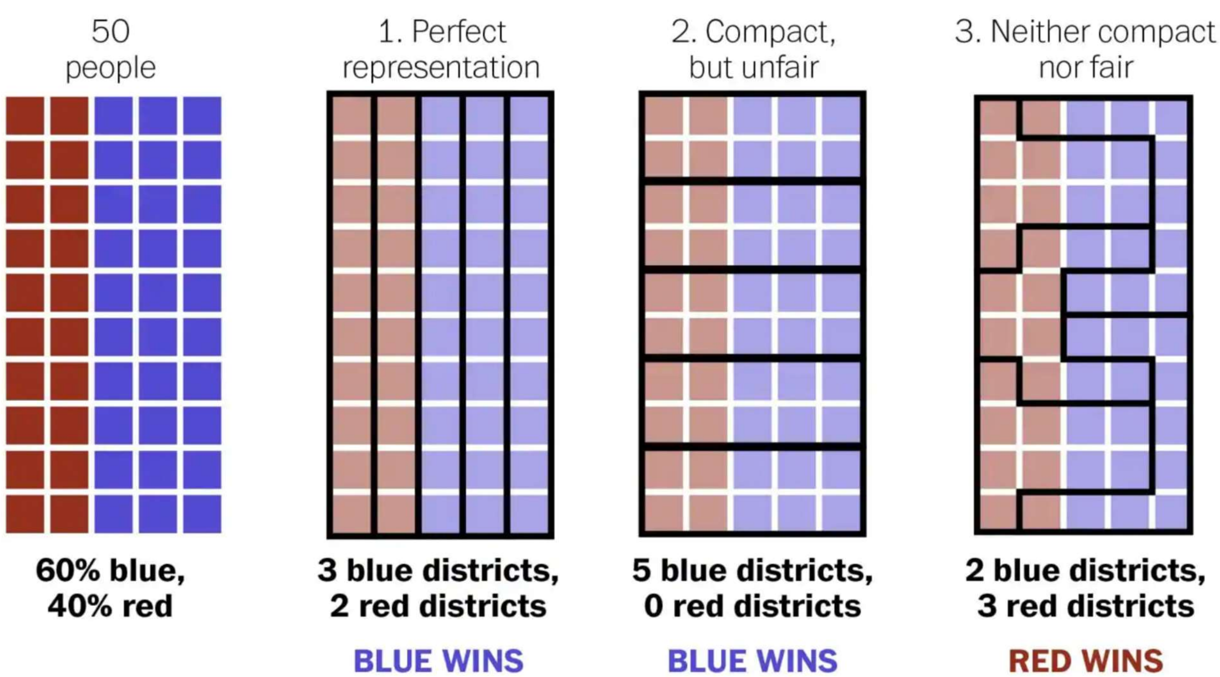


Figure 1: Four different possible outcomes, depending on the rearrangement of boundaries.[3]

[2] Carlson, David. “Voting Rights Act.” LII / Legal Information Institute, Cornell Law School, 17 June 2015.
[3] Ingraham, Christopher. “This Is the Best Explanation of Gerrymandering You Will Ever See.” The Washington Post, WP Company, 1 Mar. 2015.

dispersing members of a specific party between districts to dilute their overall impact. “Packing” is when a district is filled overwhelmingly with one group of voters, meaning that neighboring districts favor the opposing party disproportionately.

In Figure 1, item 2, dubbed “compact, but unfair,” red voters are “cracked” into five different districts. Since they are a minority in all districts, they receive no seats even though they make up 40% of the population. In the next arrangement, “neither compact nor fair,” blue voters are “packed” into two districts which results in decreased representation in the other three. This results in red voters taking home 60% of the congressional votes, even though they make up less than half the population.

Other forms of gerrymandering include “hijacking,” in which districts are rearranged to contain the houses of two incumbents from the same party, and

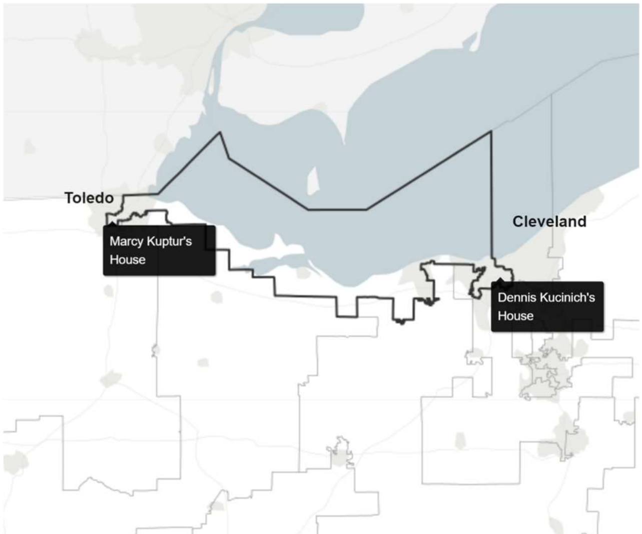


Figure 2: An example of "hijacking" in Ohio. Representative Dennis Kucinich's house in Cleveland was brought into the same district as Toledo representative Marcy Kuptur, forcing them to run against each other. Republican legislators responsible for the changes have defended the adjustments, stating that they “don’t break any laws” [4]

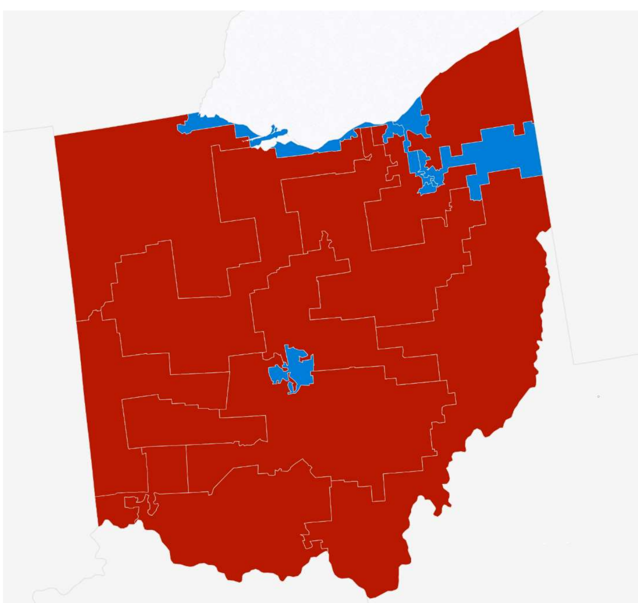

Figure 3: Current Ohio congressional district boundaries – red are districts with Republican incumbents, blue with Democrats.[5]

"kidnapping," in which a different district's borders grow to swallow the house of an incumbent, leaving their constituents behind.[4] These small-scale methods target individual candidates rather than overall trends but have the same effect of weakening one party's ability to effectively represent its constituents.

Even still, these are not the only methods available. "Sweetheart gerrymandering," for instance, does not favor one party over the other but instead packs districts with landslide majorities in order to help incumbents and minimize the number of competitive races. "Prison gerrymandering" also can occur in states with large populations of prisoners. Although they are counted as residents for census purposes, they do not have the right to vote; this means that the remaining members of that district have a disproportionately large amount of influence in the election of their representative, and given a sufficiently large prison population, this necessarily violates the principle of "one person, one vote" that is essential to proportional congressional representation.[6]

This should demonstrate *why* gerrymandering is a problem; fundamentally, it erodes the democratic process. Any concerted attempt at gerrymandering is designed directly to ensure that citizens are not represented fairly. This can only result in a worse political environment; some are not represented, some have a disproportionate impact, some candidates are not as strongly beholden to their constituents, and some candidates can no longer represent their home districts. These circumstances can arise as a result of chance, too, but gerrymandering constitutes a direct attempt to bring about these circumstances and undermine the structure of our nation.

[4] Pierce, Olga, et al. "Redistricting, a Devil's Dictionary." *ProPublica*, 2011.
[5] "Ohio Election Results 2016: House Live Map by District, Real-Time Voting Updates." POLITICO, POLITICO, 2016.
[6] "Prison-Based Gerrymandering - New York Times Editorial." Prison Gerrymandering Project, 2006.

**Prior Algorithmic Approaches**

Several algorithms have been proposed that would resolve (or rather, replace) the gerrymandering problem if implemented, almost all of which take the form of an optimization problem. One of the earliest and simplest redistricting algorithms, known as the splitline algorithm and proposed by the Center for Range Voting,[7] only ever generates a single output. Its procedure is as follows:

1. Start with the boundary of the state.
2. Split the region into two equally-populated halves along the shortest possible boundary line. If there is a tie for shortest line, use the line closest to the North-South axis. If there is still a tie, choose the line whose center is furthest west.
3. Repeat step 2 on the newly-formed "hemi-states." Repeat this until you have broken the state into the appropriate number of districts.

The algorithm has additional stipulations for odd numbers of districts, etc., but its fundamental appeal is that it is by far the simplest approach to congressional redistricting. It does not guarantee any specific quality of outcome, or even a fair outcome, but it is so simple that it leaves no doubt that it has not been influenced by external forces. Drawbacks of this algorithm make themselves apparent quickly, however: inherent in the algorithm is a complete disregard for community boundaries, racial demographics, communities of shared interest, political affiliation, and political competitiveness. This lack of consideration can be perceived as an advantage (if it doesn't take these into consideration, it can't use them to one party's advantage), but ultimately results in a generally unusable outcome that is thus far untenable on the political stage.

[7] "Splitline Districtings of All 50 States + DC + PR." RangeVoting.org - Gerrymandering and a Cure - Shortest Splitline Algorithm.

Many variants of the splitline algorithm exist, including Brian Langstraat's "Horizontal-Vertical Only" variant in which lines must be parallel to latitude and longitudinal axes, and Noam Yorav-Raphael's "Min-Variance Splitline" version which minimizes the variance of the populations inside of regions instead of analyzing the length of the boundaries.[8] Both have many of the same advantages and disadvantages: they use a simple algorithm that is inherently incorruptible but is nowhere near detailed enough to take into account the individual needs of a community.

A similar approach by Brian Olson sought instead to maximize the compactness of each congressional district.[9] By minimizing the average distance to the geographic center of each district, the method produces much more attractively-shaped districts. It uses census blocks, the smallest areas of land recognized by the US Census, as guidelines to ensure that each district would have more realistic boundaries and fit into a practical context. This is an improvement; the splitline method or its descendants could potentially have resulted in a house being split down the middle due to its location. Still, this algorithm incorporates very few strictly political considerations. This does present one of the most feasible semi-objective methods that could be used to redraw congressional districts from a purely mathematical basis. Once again, by ignoring political demographics it makes itself immune to partiality, but the political leanings of each district would then be left to chance and could potentially result in an extremely unfair result (See Figure 1: the "compact but unfair" model is a legitimate possibility with devastating consequences for the minority party). In response to a hypothetical question about whether he had analyzed the political impact of his proposed districts, Olson wrote "No. In part, I don't want

[8] Smith, Warren D. "The Shortest Splitline Algorithm – and Variants." RangeVoting.org - Gerrymandering and a Cure - Shortest Splitline Algorithm, 2018.
[9] Olson, Brian. "BDistricting - About." BDistricting,

to. I don't want this to be a discussion about winners and losers, but about good government and what our democracy is for and how it should be structured." In the end, however, the discussion about "winners and losers" was covered by FiveThirtyEight anyways – the results found that Olson's districts were slightly less Republican and slightly more competitive, on average.[10]

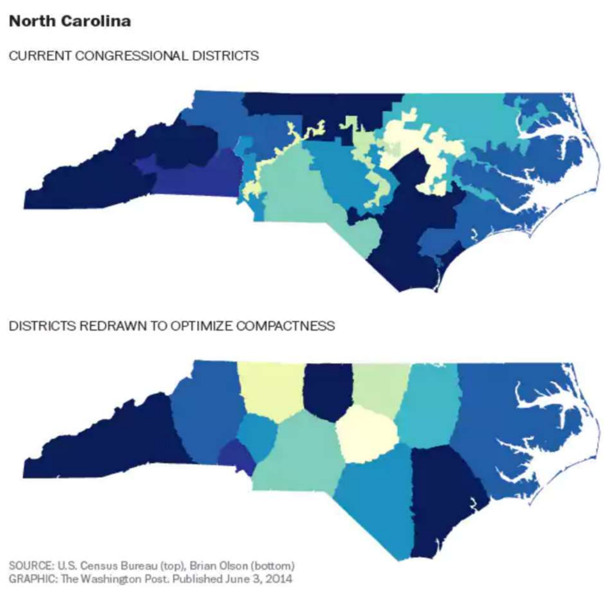


Figure 4: Above, North Carolina's current district borders. They are some of the least compact districts in the country. Below, Olson's revision: the new borders look much nicer, and minimize geographic distance to the center of their district.[11]

Another important consideration that poses an obstacle to these more simplistic redistricting processes is the Voting Rights Act. By ignoring demographic data about the citizens of each state, these algorithms expose themselves to violations of Section 2 of the Voting Rights Act, which forbids redistricting systems that would "crack" minority populations, also known as vote dilution. In the 1986 case *Thornburg v. Gingles*, the Supreme Court ruled that in order for plaintiffs to successfully sue on the basis of vote dilution, three conditions were necessary:

1. It must be possible to draw a "geographically-compact" district where members of the minority group would constitute a majority.
2. The minority group must be "politically cohesive" – generally, they must share common political interests, and support similar candidates.

[10] Bycoffe, Aaron et al. "The Atlas of Redistricting." FiveThirtyEight, 25 Jan. 2018.
[11] Ingraham, Christopher. "This Computer Programmer Solved Gerrymandering in His Spare Time." The Washington Post, WP Company, 3 June 2014.

3. A white majority must also vote "as a bloc" to defeat minority candidates.[12]

These conditions make no mention of the intentions of the redistricting authority. Since it is conceivable that these three conditions could arise were these objective, mathematical systems to be imposed on a national (or even statewide) level, redistricting authorities using these algorithms could put themselves at risk of civil rights litigation. This means that most of the proposed systems would require either a rewriting of the Voting Rights Act (a controversial proposition to say the least), or they will need to be replaced with more socially conscious alternatives. For my project, I have determined that lobbying for a rewriting of the Voting Rights Act would be most likely out of scope, so I will as a result need to take minority populations (specifically, "politically cohesive" communities) into account as a part of the implemented redistricting algorithm. To this end, I will ensure that each political party receives something that approximates proportional representation. Obviously, this doesn't guarantee a perfect approach to minority representation, but the inclusion of racial demographic data into the algorithm will necessarily increase its complexity and generate a new result that still will not guarantee total protection of political "communities of interest." By looking specifically at party membership, we can ensure at the very least that members of both parties are receiving a fair deal independent of race, and that a sufficiently large politically cohesive minority community of interest will not be cracked and packed unfairly.

Stephanopoulos and McGee published a paper in 2014[13] which proposes a new metric: the efficiency gap. By looking at the number of "wasted votes," votes which were cast for a losing candidate, or votes which were unnecessarily cast for a winning candidate, they posit this

[12] Carlson, Ibid.

[13] Nicholas O. Stephanopoulos, Eric M. McGhee, "Partisan gerrymandering and the efficiency gap," University of Chicago Law Review, 2014.

as an acceptable measurement of the extent to which a district has been gerrymandered. In a political environment where it is assumed that only two major parties exist (a safe bet, considering that the last third-party congressional candidate elected in Ohio was in 1854[14]) and that congressional districts are of equivalent populations (which is constitutionally required), this can be calculated as:

$$E_p = S_p - (2 * V_p)$$

where $S_p$ is the "seat margin" – the share of all seats held by party *p*, minus 50%, and $V_p$ is the "vote margin" – the share of all votes held by party *p*, minus 50%. A positive efficiency gap indicates a bias in favor of party $p$. So, for example, if party A wins 55% of the vote and 85% of the seats, its efficiency gap is (85%-50%)-2*(55%-50%) = 5%, indicating that the current congressional framework is slightly in favor of party A. The formula is based on the implicit assumption that, ideally, a party should have a seat margin equivalent to twice its voting margin. This is a product of the "winner's bonus" that is a naturally acknowledged result of the single-member district voting system.

Many other proposed algorithms exist, but one of the most sophisticated algorithms is PEAR, the Parallel Evolutionary Algorithm for Redistricting.[15] Published in *Swarm and Evolutionary Computation*, it expounds a heavily optimization-based method that can meet several criteria simultaneously using an evolutionary model. They introduce a number of metrics that can be used to evaluate a proposed set of boundaries, including the compactness metric, defined as

[14] "List of Third Party Performances in United States Elections." Wikipedia, Wikimedia Foundation, 28 Aug. 2018.
[15] Liu, Yan Y., Wendy K. Tam Cho, and Shaowen Wang. 2016. ''PEAR: A Massively Parallel Evolutionary Computational Approach for Political Redistricting Optimization and Analysis.'' *Swarm and Evolutionary Computation* 30.

$$C = \frac{4\pi a}{p^2}$$

with $a$ the area of the district, and $p$ the perimeter of the district. The $4\pi$ coefficient normalizes this value between 0 (a line with no area) and 1 (a perfect circle); a proposal with a high average district compactness is generally preferable, but the extent to which this is prioritized over other metrics is to be accounted for in the final formulation of the project. They also stipulate that all districts must have populations that are as close as possible, and that no district may be entirely contained within another district. They also include a competitiveness metric (that serves as a simpler alternative to the efficiency gap) that measures the competitiveness of each district's congressional election based on Democratic and Republican voter registration.

**Project Structure**

The first part of this project will be the construction of a comparative toolkit that takes any boundary proposal $S$ with $N$ districts, each with perimeter $p$ and area $a$, and evaluates it with the parameterized cost function $J_\theta(S)$.

We define the set of all possible boundary proposals as $\boldsymbol{S}$. For any proposal $S \in \boldsymbol{S}$, we quantify $S$ as:

$$S = \{\{a_1, \dots, a_N\}, \{p_1, \dots, p_N\}, \{d_1, \dots, d_N\}, \{r_1, \dots r_N\}\}$$

with

- $a_n > 0$, the area of district $n$
- $p_n > 0$, the perimeter of district $n$
- $d_n \geq 0$, the number of votes cast for Democrat candidates in region $n$
- $r_n \geq 0$, the number of votes cast for Republican candidates in region $n$

With $\theta = \{\theta_1, \theta_2, \dots, \theta_K\}$, $\theta_k \in \boldsymbol{R}$ chosen by the user, we define $J_\theta : \boldsymbol{S} \to \boldsymbol{R}$ as

$$J_\theta(S) = \theta_1 C + \theta_2 E + ...$$

using (initially) a weighted sum of the compactness metric C:

$$C = \frac{1}{N}\sum_{n=1}^{N}\frac{4\pi a_n}{p_n}$$

and efficiency gap metric E:

$$E = |E_r| = |S_r - 2V_r|$$

(We use the Republican Party to calculate the efficiency gap, but since $|E_d| = |E_r|$, we don't lose information). We define the Republican seat margin $S_r$ as:

$$S_r = \left(\frac{1}{N}\sum_{n=1}^{N} S_{r|n}\right) - 0.5$$

where $S_{r|n} = \{1 \; if \; r_n > d_n, else \; 0\}$. The vote margin $V_r$ is defined as:

$$V_r = \left(\frac{1}{N}\sum_{n=1}^{N}\frac{r_n}{r_n + d_n}\right) - 0.5$$

Additional metrics, ($K$ in total) including the mean-median metric[16] (a relatively simple statistical test that checks for skewed distributions with regards to party representation), convex hull metric[17] (a test that compares the perimeter of the smallest convex polygon that surrounds the district to the district's area – this would require a reformulation of our current representation of a district), and others, could be implemented as well with their own corresponding weight values.

The project's second contribution will be a greedy beam-search algorithm for district redrawing that will (roughly) conform to the following outline:

[16] Whitaker, Rob. "Princeton Gerrymandering Project." Princeton University, The Trustees of Princeton University.
[17] Barnes, Richard, and Justin Solomon. "Gerrymandering and Compactness: Implementation Flexibility and Abuse." Computer Science and Artificial Intelligence Laboratory, MIT, 2018.

Greedy Beam Search Algorithm

Given a set of weights $\theta$, initialization algorithm $a$, optional early stopping criterion $c$, and number of epochs $K$:

1. Let $k = 0$. Using $a$, generate $P_k = \{S_0\}$. (at first, only a single border proposal)
2. Repeat (until k = K or $c$ is satisfied):
    a. $P_{k+1} = P_k$ (the set of new proposals)
    b. For each border proposal $S$ in $P_k$:
        i. Construct $P_m(S)$, the top $m$ (in terms of $J_\theta$) ***local alterations*** to $S$.
        // A *local alteration* would include, for example, a transfer of a census block on the border between region $a_i$ and region $a_j$; if $b \in a_i$, transfer $b$ to $a_j$ and if $b \in a_j$, transfer $b$ to $a_i$.
        ii. $P_{k+1} = P_{k+1} \cup P_m(S)$ // add $P_m(S)$ to $P_{k+1}$
    c. Remove all but the top $m$ (in terms of $J_\theta$) proposals in $P_{k+1}$.
    d. $k = k + 1$.
3. Return $S_{FINAL} = max(P_K)$, the highest-rated border proposal produced by the algorithm.

One thing that must be considered while applying this algorithm would be the constitutional requirement of equivalent populations per district; one way to ensure this would be to apply an additional highly-weighted metric to the cost function $J_\theta$ that penalizes large differences in district populations.

This algorithm will be compared against a variety of industry standards which could include, but would not be limited to the algorithms listed above. Of those options, it is likely that a politically viable proposal could be developed and presented to the General Assembly.

Finally, the project's third goal will be to develop a method of procedural map generation that can be used to augment training datasets. One conceivable method could involve the application of Voronoi diagrams to generate semi-natural district outlines. Lloyd's algorithm can be applied to the diagrams to modulate the uniformity of the borders, and Perlin noise can be used on the individual borders for a fuzzier, more realistic appearance. Political leanings of a given district could be sampled from random distributions. The algorithm could also generate population distributions by selecting a randomly sampled number of urban centers in the area and distributing population of regions according to their distance from these centers. These artificial maps would be simple but vital in the construction of arbitrarily large datasets that could then be used to train more sophisticated models in the future.

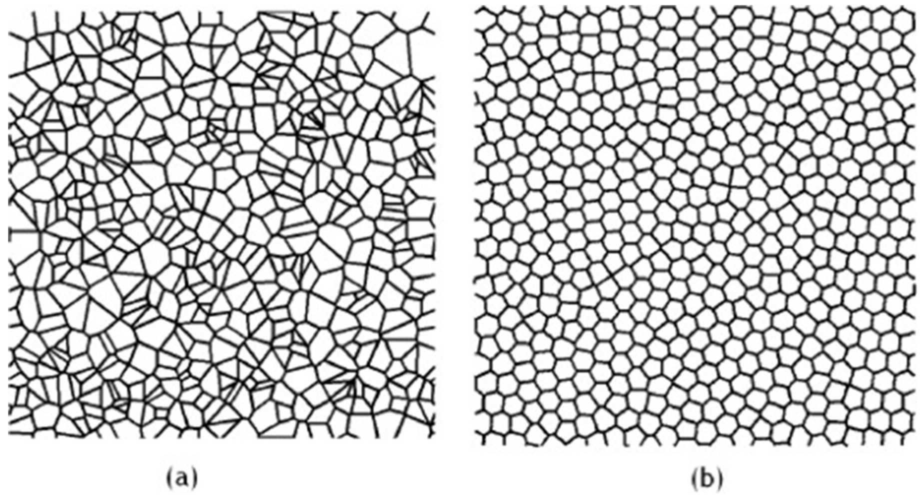


Figure 5a: Randomly generated Voronoi diagram with no application of Lloyd's algorithm. Figure 5b: The same diagram after 50 iterations of Lloyd's algorithm. Fewer iterations can be used to generate more irregular-looking maps.[18]

Ideally, the final product will be able to: randomly generate an artificial map (or load an existing one); take as input preference parameters for the cost function; compare the performance of a half-dozen or so leading algorithms, in addition to the beam search algorithm outlined above; choose the best proposal for the region and display alternate options; and display the extent to which each proposal satisfies each of the criteria. This is a project which I believe both

[18] Alsayednoor, J., and P. Harrison. "Evaluating the Performance of Microstructure Generation Algorithms for 2-d Foam-like Representative Volume Elements." Mechanics of Materials, vol. 98, 2016, pp. 44–58.

is feasible in the time frame that has been set out and would prove invaluable in the redistricting process.

**Conclusion**

One difficulty in this project will be the process of representing the regions and border proposals in a more abstract manner that can be operated on easily. Currently, most census data are stored in shapefiles, so some methods to translate between image and data will need to be assembled (or adopted in some way from previous research). My experience in computational optimization, mathematics, statistics, and computer science should prove invaluable over the course of this project. I have taken tutorials on multiple methods of mathematical optimization, including calculus of variations, integer programming, and path planning, and I have taken classes in machine learning and a graduate-level class in deep learning methods which, while not directly applicable to the current project's framework, still has provided invaluable insight into the optimization process (and should prove extremely useful if I need to manually implement some of the more complex evolutionary redistricting models). I have followed through the mathematics department's standard track in calculus, differential equations, and linear algebra, and completed advanced coursework in statistical computing and stochastic processes that may be useful as well. I can see myself applying modern algebra – not group theory directly perhaps, but something analogous – to the process of abstractly representing the boundary proposals and the operations on them. Additionally, my experience in computer science using data structures, Python, and even some elements from the computational ethics class could be applied to this problem.

This is part of my reason for selecting this problem; it is an interesting mathematical problem that plays to my strengths and experience academically and connects to my future law school career, but most importantly, it also provides me with an opportunity to *make a difference* and make some small contribution to improve the democratic process as applied to the state of Ohio.

**Timeline**

By October 31: Secure regional & political datasets of the state of Ohio and process them into a computationally interpretable format. Develop the data structures that will be used to digitally represent the maps and proposals.

By November 30: Compile list of redistricting algorithms to be included in the study, including beam search. Develop the software framework required to evaluate redistricting algorithms. Speak with local officials involved in the redistricting process about the criteria involved in the development of a good district proposal. Use this to inform the construction of $J_\theta$.

By January 31: Have 4 redistricting algorithms evaluation-ready, including beam search. Implement all metrics to be included in $J_\theta$.

By March 31: Incorporate remaining redistricting algorithms. By this point, the entirety of the optimization portion should be completed. Remain in contact with local officials and get feedback on algorithmically-produced border proposals.

By April 30: Complete implementation of artificial dataset generation. Run optimizer on artificial data. Generate new datasets and publish toolkit online.

Summer 2019 (after thesis submitted): Reach out to contacts in local & state government. Work towards developing a useful proposal for the Ohio General Assembly. Eventually, develop and present a proposal to the General Assembly.

**Annotated Bibliography**

1. Wendel, Emily E. "LSC Analysis of Sub. S.J.R. 5." Ohio Legislative Service Commission, 16 Jan. 2018

This legislative analysis provides an overview of the procedures and criteria of the Congressional Redistricting Procedures Amendment. It explains the process by which the state of Ohio will be redistricted over the course of the next few years.

2. Carlson, David. "Voting Rights Act." LII / Legal Information Institute, Cornell Law School, 17 June 2015.

This webpage outlines the requirements of the Voting Rights Act, including an explanation of Section 2 which concerns minority rights in the redistricting process. In addition, it states the criteria established in *Thornburg v. Gingles* that are necessary for determining whether vote dilution has taken place.

3. Ingraham, Christopher. "This Is the Best Explanation of Gerrymandering You Will Ever See." The Washington Post, WP Company, 1 Mar. 2015.

This *Washington Post* article covers the basics of gerrymandering in an easily accessible format, and includes Figure 1, which demonstrates an example of different gerrymandering methods being employed to reach vastly different outcomes.

4. Pierce, Olga, et al. "Redistricting, a Devil's Dictionary." ProPublica, 2011.

This webpage defines several more advanced gerrymandering techniques, including "hijacking," "kidnapping," and "sweetheart gerrymandering." It is also the source of Figure 2, an illustrative example of congressional "hijacking."

5. "Ohio Election Results 2016: House Live Map by District, Real-Time Voting Updates." POLITICO, POLITICO, 2016.

A live map of the 2016 Ohio congressional election outcome, used in Figure 3.

**6.** “Prison-Based Gerrymandering - New York Times Editorial.” Prison Gerrymandering Project, 2006.

This editorial delves into the idea of prison gerrymandering and its political & constitutional implications. It defines prison gerrymandering and gives several examples of districts where it has affected congressional elections in the past.

**7.** Splitline Districtings of All 50 States + DC + PR.” RangeVoting.org - Gerrymandering and a Cure - Shortest Splitline Algorithm.

The Center for Range Voting’s website is a wellspring of voting reform proposals. Among these was the definition and discussion of the splitline algorithm, along with several illustrative graphics.

**8.** Smith, Warren D. “The Shortest Splitline Algorithm – and Variants.” RangeVoting.org - Gerrymandering and a Cure - Shortest Splitline Algorithm, 2018.

Another page on the Center for Range Voting’s website that explores variants on the splitline algorithm and includes debate between users on the merits and difficulties posed by each variation.

**9.** Olson, Brian. “BDistricting - About.” BDistricting.

Brian Olson’s “BDistricting” project hosts his exploration of the compactness-maximizing algorithm that he implemented to generate new congressional district proposals. It also includes his discussion of the potential implications of the project, and of possible future avenues of investigation.

**10.** Bycoffe, Aaron et al. “The Atlas of Redistricting.” FiveThirtyEight, 25 Jan. 2018.

This interactive web page compares several different redistricting algorithms and their implications for each political party. It uses a large visual graphic to display the geographic differences in the border proposals across the entirety of the United States. It also notes that many of the simpler algorithms could potentially be in violation of the Voting Rights Act.

**11.** Ingraham, Christopher. “This Computer Programmer Solved Gerrymandering in His Spare Time.” The Washington Post, WP Company, 3 June 2014.

Despite the optimistic title, this article explains many of the difficulties posed in the redistricting process and goes into further detail about the process that Olson used to develop his redistricting proposals. Figure 4 comes from this article.

13. Nicholas O. Stephanopoulos, Eric M. McGhee, “Partisan gerrymandering and the efficiency gap,” University of Chicago Law Review, 2014.

This paper compares a new metric, the efficiency gap, to previous systems of gerrymandering measurement including partisan bias and mean-median disparity. It elaborates on the intuition that underlies the metric and explains how to calculate it under a variety of conditions. In addition, the paper includes an analysis of the efficiency gaps of the current borders in the congressional election process over previous years.

14. “List of Third Party Performances in United States Elections.” Wikipedia, Wikimedia Foundation, 28 Aug. 2018.

This page simply provides a list of the positions held by third-party candidates throughout American history.

15. Liu, Yan Y., Wendy K. Tam Cho, and Shaowen Wang. 2016. ‘‘PEAR: A Massively Parallel Evolutionary Computational Approach for Political Redistricting Optimization and Analysis.’’ Swarm and Evolutionary Computation 30.

This paper serves as an example of a more complex computational method applied to the redistricting process; by using evolutionary computing and parallel processing, the authors are

able to apply a much more complex algorithm to this high-dimensional problem and achieve a satisfactory result. The paper's algorithm is stated to operate on individual census blocks, but the graphics depict operations only on entire counties for the sake of visual interpretability; the level of detail involved in the algorithm, both in terms of interpretability and time constraints, will be an important consideration in my own project as well.

16. Whitaker, Rob. "Princeton Gerrymandering Project." Princeton University, The Trustees of Princeton University.

This page was used for a brief outline of the definitions of the mean-median and partisan bias metrics. This page also features a tool that can be used to compare existing state congressional boundaries using these two metrics and the efficiency gap.

17. Barnes, Richard, and Justin Solomon. "Gerrymandering and Compactness: Implementation Flexibility and Abuse." Computer Science and Artificial Intelligence Laboratory, MIT, 2018.

zThis paper raises several important questions about different definitions of compactness and defines & compares a variety of metrics that includes the convex-hull metric and the area-perimeter ratio included above. It also discusses potential issues that can arise when these metrics are implemented; for example, states with especially rough coastlines or state borders may have very simple district borders that are by no means geographically compact.

18. Alsayednoor, J., and P. Harrison. "Evaluating the Performance of Microstructure Generation Algorithms for 2-d Foam-like Representative Volume Elements." Mechanics of Materials, vol. 98, 2016, pp. 44–58.

This article was the source of the Voronoi diagrams used in Figure 5a and 5b, to demonstrate the visual properties of Lloyd's algorithm which could be applied to procedural region drawing.