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EE M146

① 2 urns

* Pick a urn & take two balls

A	B
4 red	2 red
3 blue	4 blue
3 white	4 white

Possibility to pick A urn is $\frac{4}{10}$

Possibility to pick B urn is $\frac{6}{10}$

ⓐ when two balls are red. → $\boxed{\text{case } X}$

$$P(X) = \frac{4}{10}\cdot\frac{{}_4C_2}{{}_{10}C_2} + \frac{6}{10}\cdot\frac{{}_2C_2}{{}_{10}C_2} = \frac{4}{10}\cdot\frac{6}{45} + \frac{6}{10}\cdot\frac{1}{45} = \frac{30}{450} = \frac{1}{15}$$

ⓑ when the second ball is blue. → $\boxed{\text{case } Y}$

Pick A urn: r — b ①, b — b ②, w — b ③

pic B urn: r — b ④, b — b ⑤, w — b ⑥

$$P(Y) = ① + ② + ③ + ④ + ⑤ + ⑥$$

$$= \underbrace{\frac{4}{10}}_{\text{PickA}}\Big(\underbrace{\frac{4}{10}\cdot\frac{3}{9}}_{rb} + \underbrace{\frac{3}{10}\cdot\frac{2}{9}}_{bb} + \underbrace{\frac{3}{10}\cdot\frac{3}{9}}_{wb}\Big) + \underbrace{\frac{6}{10}}_{\text{Pick B}}\Big(\underbrace{\frac{2}{10}\cdot\frac{4}{9}}_{rb} + \underbrace{\frac{4}{10}\cdot\frac{3}{9}}_{bb} + \underbrace{\frac{4}{10}\cdot\frac{4}{9}}_{wb}\Big)$$

$$= \frac{324}{900} = \boxed{0.36}$$

ⓒ P(second is blue | first is red)

$$= \frac{P(\text{first is red, second is blue})}{P(\text{first is red})} = \frac{\frac{4}{10}\left(\frac{4}{10}\cdot\frac{3}{9}\right) + \frac{6}{10}\left(\frac{2}{10}\cdot\frac{4}{9}\right)}{\frac{4}{10}\cdot\frac{4}{10} + \frac{6}{10}\cdot\frac{3}{10}} = \boxed{\frac{8}{21}}$$

② Tossing a coin → 4 times, Z : # of head

$$P_Z(z) = \begin{cases} \frac{1}{16} & z=0 \\ \frac{4}{16} & z=1 \\ \frac{6}{16} & z=2 \\ \frac{4}{16} & z=3 \\ \frac{1}{16} & z=4 \end{cases}$$

✓ $E[Z] = 0\cdot\frac{1}{16} + 1\cdot\frac{4}{16} + 2\cdot\frac{6}{16} + 3\cdot\frac{4}{16} + 4\cdot\frac{1}{16} = \frac{4+12+12+4}{16} = \frac{32}{16} = 2$

✓ $E[Z^2] = 0^2\cdot\frac{1}{16} + 1^2\cdot\frac{4}{16} + 2^2\cdot\frac{6}{16} + 3^2\cdot\frac{4}{16} + 4^2\cdot\frac{1}{16} = \frac{4+24+36+16}{16} = \frac{80}{16} = 5$

✓ $var(Z) = E[X^2] - E[X]^2 = 5 - 2^2 = 1$

∴ a) Expected value of Z is 2

b) variance of Z is 1

③ student (study regularly) to get A : $P = \frac{85}{100}$, B : $P = \frac{10}{100}$, C(↓) : $P = \frac{5}{100}$ — $\frac{90}{100}$

student (study not regularly) to get A $P = \frac{50}{100}$, to get B $P = \frac{15}{100}$, to get C $P = \frac{35}{100}$ — $\frac{10}{100}$

ⓐ P(studied regularly | got A)

$$= \frac{P(\text{students who studied regularly got an A})}{P(\text{got an A})} = \frac{\frac{90}{100}\cdot\frac{85}{100}}{\frac{90}{100}\cdot\frac{85}{100} + \frac{10}{100}\cdot\frac{50}{100}} = \frac{153}{163} = 0.93865$$

ⓑ P(Not studied regularly | got B or lower)

$$= \frac{P(\text{student who did not study regularly got a B or lower})}{P(\text{got a B or lower})} = \frac{\frac{10}{100}\left(\frac{15+35}{100}\right)}{\frac{90}{100}\left(\frac{10}{100} + \frac{5}{100}\right) + \frac{10}{100}\left(\frac{15}{100} + \frac{35}{100}\right)}$$

$$= 0.270270270$$

④ X, Y discrete random variable

ⓐ Show $E[X+Y] = E[X] + E[Y]$

$$* \; E[X+Y] = \sum_i\sum_j (x_i + y_j) f_{XY}(x_i, y_j) = \sum_i\sum_j x_i f_{XY}(x_i, y_j) + \sum_i\sum_j y_j f_{XY}(x_i, y_j)$$

$$= E[X] + E[Y]$$

ⓑ If X & Y indep. → show $var(X+Y) = var(X) + var(Y)$

$$var(X+Y) = E[X+Y - E[X+Y]]^2 = E[(X+Y)^2 - 2(X+Y)E[X+Y] + E[X+Y]^2]$$

$$= E[(X+Y)^2] - E[X+Y]^2$$

$$= \cancel{E[X^2]} + 2E[XY] + \cancel{E[Y^2]} - (E[X+Y])^2$$

$$- (\cancel{E[X]^2} + \cancel{E[Y]^2} + 2E[X]E[Y])$$

$$= (E[X^2] - E[X]^2) + (E[Y^2] - E[Y]^2) + 2E[XY] + (-2E[X]E[Y])$$

$$= var(X) + var(Y) + \cancel{\underbrace{2E[XY]}_{2E[X]E[Y]} - 2E[X]E[Y]} = var(X) + var(Y)$$

since indepentent

⑤ The waiting time (bus arrival) is T ← exponentially distributed rv (with λ)

$$P(T \le t) = 1 - e^{-\lambda t} \quad \forall t \ge 0$$

ⓐ have waited r, will not arrive within d more seconds.

case ① $T < r+d$

case ② $T < r$

$$P(①|②) = \frac{P(T < r+d)}{P(T < r)} = \frac{[1-(1-e^{-\lambda(r+d)})]}{1-(1-e^{-\lambda r})} = \frac{e^{-\lambda(r+d)}}{e^{-\lambda r}} = e^{-\lambda d}$$

ⓑ PDF of an exponential distributed r.v

It is CDF so that it can be written as $F(t) = 1 - e^{-\lambda t}$

To get PDF : $\frac{d}{dt}F(t) = \frac{d}{dt}(1 - e^{-\lambda t}) = \lambda e^{-\lambda t}$ which is $f(t)$

$f(t) = \lambda e^{-\lambda t}$

To get average waiting time : Expected value of T : $\int_0^\infty x f(x)\,dx = \int_0^\infty x\lambda e^{-\lambda x}\,dx$

$$= [-xe^{-\lambda x}]_0^\infty - \int_0^\infty -e^{-\lambda x}\,dx$$

$$= [-xe^{-\lambda x}]_0^\infty - \left[\frac{1}{\lambda}e^{-\lambda x}\right]_0^\infty$$

$$= \frac{1}{\lambda}$$

CDF : $F(x) = 1 - e^{-\lambda x}$

$f(x) = -(-\lambda e^{-\lambda x}) = \lambda e^{-\lambda x}$

⑥ ⓐ

ⓑ * Initialize Hyperplane parameter : 0

Max iteration : 1000

In each data set, provide Hyperplane parameters. (by w and d)

total # of updates performed (u)

plot data & decision boundary. (defined by $\boxed{w^T x + b = 0}$

Based on total # → comment on the convergence of perceptron for each data set