Gawon Kim

CS161 HW7

Question 1

$$Pr(\alpha_1 \cdots, \alpha_n \mid \beta) = Pr(\alpha_1 \mid \alpha_2 \cdots \alpha_n, \beta)\, Pr(\alpha_2 \mid \alpha_3 \cdots \alpha_n, \beta) \cdots Pr(\alpha_n \mid \beta)$$

when $n=2$, $Pr(\alpha_1, \alpha_2 \mid \beta) = Pr(\alpha_1 \mid \alpha_2, \beta)\, Pr(\alpha_2 \mid \beta)$ product rule.

when $n=3$ $Pr(\alpha_1, \alpha_2, \alpha_3 \mid \beta) = Pr(\alpha_1 \mid \alpha_2, \alpha_3, \beta)\, Pr(\alpha_2, \alpha_3 \mid \beta)$

$$= Pr(\alpha_1 \mid \alpha_2, \alpha_3, \beta)\, Pr(\alpha_2 \mid \alpha_3, \beta)\, Pr(\alpha_3 \mid \beta)$$

when $n=n-1$.

$$Pr(\alpha_1, \underbrace{\alpha_2, \alpha_3 \cdots \alpha_n}_{\gamma} \mid \beta) = Pr(\alpha_1, \gamma \mid \beta)$$

$$= Pr(\alpha_1 \mid \gamma, \beta)\, Pr(\gamma \mid \beta)$$

$$= Pr(\alpha_1 \mid \alpha_2, \alpha_3, \cdots \alpha_n, \beta)\, Pr(\alpha_2, \alpha_3 \cdots \alpha_n \mid \beta)$$

Keep repeating it

$$= Pr(\alpha_1 \mid \alpha_2, \alpha_3, \cdots \alpha_n, \beta) \cdot Pr(\alpha_2 \mid \alpha_3 \cdots \alpha_n, \beta) \cdots \cdots Pr(\alpha_n \mid \beta)$$

$\therefore$ QED.

Question 2

$P(\text{Oil}) = 0.5$

$P(\text{Natural Gas}) = 0.2$

$P(\text{Neither}) = 0.3$

$P(\text{Positive} \mid \text{Oil}) = 0.9$

$P(\text{Positive} \mid \text{Natural Gas}) = 0.3$

$P(\text{Positive} \mid \text{Neither}) = 0.1$

(Oil) & (Natural Gas) cannot be existed at the same time.

Q: $P(\text{Oil} \mid \text{Positive})$??

$$= \frac{P(\text{Oil}) \cdot P(\text{Positive} \mid \text{Oil})}{P(\text{Positive})}$$

$$P(\text{Positive}) = P(\text{Positive} \mid \text{Oil})P(\text{Oil}) + P(\text{Positive} \mid \text{Natural Gas})P(\text{Natural Gas}) + P(\text{Positive} \mid \text{Neither})P(\text{Neither})$$

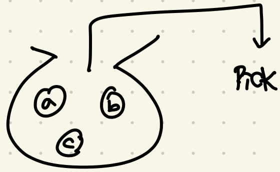
$$= (0.9)(0.5) + (0.3)(0.2) + (0.1)(0.3) = 0.54$$

$$\frac{P(\text{Oil}) \cdot P(\text{Positive} \mid \text{Oil})}{P(\text{Positive})} = \frac{(0.5)(0.9)}{0.54} = 0.83333$$

$$\boxed{P(\text{Oil} \mid \text{Positive}) \approx 0.83}$$

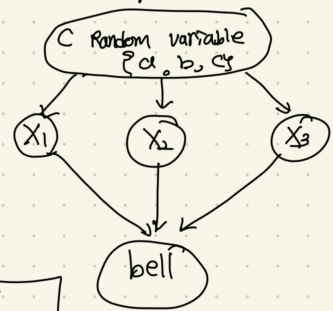
Question 3

$P(H\_a) = 0.2$
$P(H\_b) = 0.4$
$P(H\_c) = 0.8$



Pick one $\longrightarrow$ flip 3 times

Bayesian Network $\longrightarrow$



$C = \{a, b, c\}$

	C
P(a)	1/3
P(b)	1/3
P(c)	1/3

$1 \leq i \leq 3$

	X_i	P
a	Head	0.2
b	Head	0.4
c	Head	0.8

	X_1	X_2	X_3	$P(bell \mid X_1, X_2, X_3)$
a	T	T	T	1
	T	T	H	0
	T	H	T	0
	H	T	T	0
	T	H	H	0
	H	T	H	0
	H	H	T	0
	H	H	H	1
b	T	T	T	1
	T	T	H	0
	T	H	T	0
	H	T	T	0
	T	H	H	0
	H	T	H	0
	H	H	T	0
	H	H	H	1
c	T	T	T	1
	T	T	H	0
	T	H	T	0
	H	T	T	0
	T	H	H	0
	H	T	H	0
	H	H	T	0
	H	H	H	1

$P(bell)$
$= P(bell \mid a)P(a)$
$+ P(bell \mid b)P(b)$
$+ P(bell \mid C)P(C)$

$= \frac{1}{3}(0.2)^3 \cdot \times 2$ ← pick A

$+ \frac{1}{3}(0.4)^3 \times 2$ ← pick B

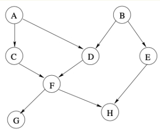
$+ \frac{1}{3}(0.8)^3 \times 2$ ← pick C

$= \boxed{0.389\overline{3}}$

Question 4

ⓐ Markovian assumption asserted by the DAG.

$I(V, \text{Parent}(V), \text{non-descendants}(V))$

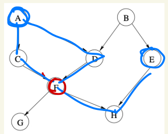


$I(A, \varnothing, BE)$
$I(B, \varnothing, AC)$
$I(C, A, BDE)$
$I(D, AB, CE)$
$I(E, B, ACDFG)$
$I(F, CD, ABE)$
$I(G, F, ABCDEH)$
$I(H, FE, ABCDG)$

ⓑ T or F

- d_separated (A, F, E)

$Z = \{F\}$



$C \rightarrow F \rightarrow H$

variable $W \in Z$ so that close

∴ d_separation & True

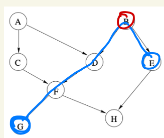
There are three possible paths

① A C F H E → Close ($C \rightarrow F \rightarrow H$) is serial and $W (F) \in Z (F)$
② A D F H E → Close ($D \rightarrow F \rightarrow H$) is serial and $W (F) \in Z (F)$
③ A C F D B E → Open ($C \rightarrow F \leftarrow D$) is convergent and it is not satisfied ($W \notin Z$) so open

False ← in the path, it is not entirely separated. so not d_separated.

- d_separated (G, B, E)

True



G F D B E

$$\begin{tikzcd} & B \arrow[dl] \arrow[dr] & \\ D & & E \end{tikzcd}$$

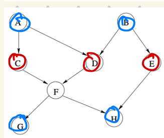
$W (B) \in Z$

so that close & there is no other path.

∴ d_separated

- d_separated (AB, CDE, GH)

True



C, D, E are all connected with serial. with all path from (A, B) to (GH) (so all close).

ⓒ $P(a, b, c, d, e, f) = P(a)$
$* P(b)$
$* P(C \mid a)$
$* P(d \mid a, b)$
$* P(e, b)$
$* P(f \mid cd)$
$* P(G \mid f)$
$* P(H \mid E, f)$

ⓓ $P(A=1, B=1) = P(A=1) \cdot P(B=1) = 0.2 \cdot 0.7 = \boxed{0.14}$

$P(E=0 \mid A=0)$ since A and E are independent.

$\Rightarrow P(E=0) = P(E=0 \mid B=0) P(B=0) + P(E=0 \mid B=1) P(B=1)$
$= (0.1)(0.3) + (0.9)(0.7)$
$= \boxed{0.66}$

Question 5)

$\alpha: A \Rightarrow B \;=\; \neg A \vee B$

ⓐ

	A	B	α	P(A,B)
w_0	T	T	T	0.3
w_1	T	F	F	0.2
w_2	F	T	T	0.1
w_3	F	F	T	0.4

$M(\alpha) = \{w_0, w_2, w_3\}$

ⓑ $P(\alpha) = 0.3 + 0.1 + 0.4$
$= 0.8$

ⓒ

$$\frac{P(w_0 \wedge \alpha)}{P(\alpha)} = \frac{0.3}{0.8}$$

3 worlds

$$\frac{P(w_2 \wedge \alpha)}{P(\alpha)} = \frac{0.1}{0.8}$$

$$\frac{P(w_3 \wedge \alpha)}{P(\alpha)} = \frac{0.4}{0.8}$$

A	B	α	$P(A,B\mid\alpha)$
T	T	T	0.375
T	F	T	0
F	T	T	0.125
F	F	T	0.5
T	T	F	0
T	F	F	1
F	T	F	0
F	F	F	0

ⓓ $A \Rightarrow \neg B \;\rightarrow\; \neg A \vee \neg B$

	A	B	$\neg A$	$\neg B$	$\neg A \vee \neg B$	
w_0	T	T	F	F	F	
w_1	T	F	F	T	T	✓
w_2	F	T	T	F	T	✓
w_3	F	F	T	T	T	✓

$P(w_1 \mid \alpha) + P(w_2 \mid \alpha) + P(w_3 \mid \alpha)$
$= 0 + 0.125 + 0.5 \quad = 0.625$