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Assignment 5
CS161

Question 1

$P \to Q$, $\neg Q \to \neg P$

P	Q	$P \to Q$	$\neg Q$	$\neg P$	$\neg Q \to \neg P$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

$\therefore$ $P \to Q$ & $\neg Q \to \neg P$ are equivalent.

$P \leftrightarrow \neg Q$, $((P \land \neg Q) \lor (\neg P \land Q))$

P	$\neg P$	Q	$\neg Q$	$P \leftrightarrow \neg Q$	$P \land \neg Q$	$\neg P \land Q$	$(P \land \neg Q) \lor (\neg P \land Q)$
T	F	T	F	F	F	F	F
T	F	F	T	T	T	F	T
F	T	T	F	T	F	T	T
F	T	F	T	F	F	F	F

$\therefore$ $P \leftrightarrow \neg Q$ & $((P \land \neg Q) \lor (\neg P \land Q))$ are equivalent.

Question 2

S: smoke F: Fire.

• $(S \to F) \Rightarrow (\neg S \to \neg F)$

S	¬S	F	¬F	$S \to F$	$\neg S \to \neg F$	$(S \to F) \to (\neg S \to \neg F)$
T	F	T	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	T	F	T	T	T	T

∴ Satisfiable

S: smoke F: Fire. H: Heat

• $(S \to F) \to ((S \vee H) \to F)$

S	F	H	$S \to F$	$S \vee H$	$(S \vee H) \to F$	$(S \to F) \to ((S \vee H) \to F)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	T	T	F	F
F	F	F	T	F	T	T

∴ Satisfiable

• $[(S \wedge H) \to F] \leftrightarrow [(S \to F) \vee (H \to F)]$

S	F	H	$S \wedge H$	① $(S \wedge H) \to F$	$S \to F$	$H \to F$	② $(S \to F) \vee (H \to F)$	① ↔ ②
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	T	F	F	F	F	T
T	F	F	F	T	F	T	T	T
F	T	T	F	T	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	F	T	T	F	T	T
F	F	F	F	T	T	T	T	T

Valid. ← all satisfied.

Question 3

Myth : A
immortal : B
Mammal : C
Horned : D
magical : E

a)

myth → immortal.
¬ myth → mortal mammal
immortal ∨ mammal → horned
horned → magic al

$A \to B$
$\neg A \to (\neg B \land C)$
$(B \lor C) \to D$
$D \to E$

b)

$A \to B$
$\neg A \to (\neg B \land C)$
$(B \lor C) \to D$
$D \to E$

CNF →

$\neg A \lor B$
$A \lor (\neg B \land C)$
$\neg(B \lor C) \lor D$
$\neg D \lor E$

simplify.

$(\neg A \lor B)$
$(A \lor \neg B) \land (A \lor C)$
$(\neg B \lor D) \land (\neg C \lor D)$
$(\neg D \lor E)$

c)

1) $\neg A \lor B$ given
2) $(A \lor \neg B) \land (A \lor C)$ given
3) $(\neg B \lor D) \land (\neg C \lor D)$ given
4) $\neg D \lor E$ given.
5) $A \lor \neg B$ 2
6) $A \lor C$ 2
7) $\neg B \lor D$ 3
8) $\neg C \lor D$ 3

9) $\neg A \lor B$, $\neg B \lor D$ 1 7
$\neg A \lor D$
10) $\neg A \lor D$, $A \lor C$ 9 6
$D \lor C$
11) $D \lor C$, $\neg C \lor D$ 10 8
D
12) D, $\neg D \lor E$ 11 4
E

Unicorn is horned and magical. , but there is not enough information to be mythical

O Horn
O Magical
X Mythical

Question 4

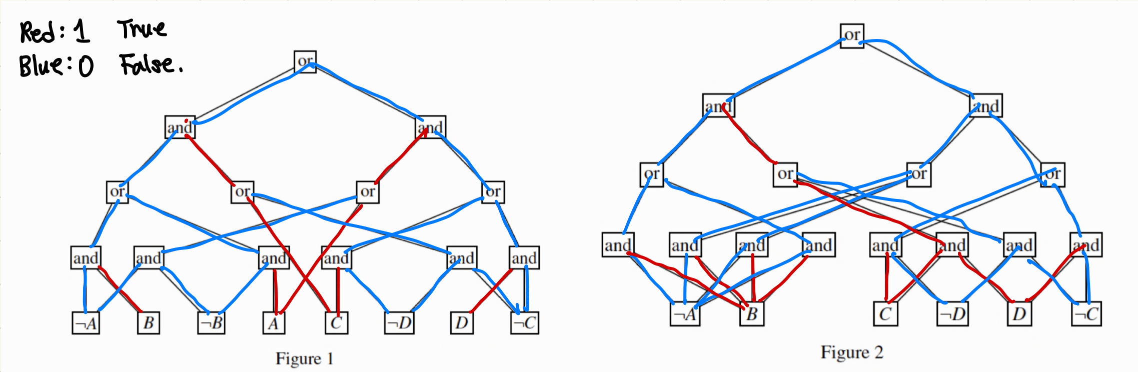

Figure 1 1) deterministic $\boxed{OR}$ is not allowed indeterministic
and the fig1 does not have like this.
1 1

2) decomposable. there is no $\boxed{\text{and}}$ to share variable in children.

3) smooth each or's children has same kind of literals
ABCD

Figure 2: decomposable O – It shares no variables; satisfy α and B, $var(\alpha) \cap var(B) = \emptyset$

deterministic X – When inserting input $(\neg A, B, C, D)$

$(\neg(\neg A) \cap (B)) \cup (\neg(\neg A) \cap (B))$

$(A \cap B) \cup (A \cap B)$ → which is two high inputs.

So from the or gate, It is not deterministic.

smooth O – $\alpha \cup B$; $var(\alpha) = var(B)$ it is satisfied
so it is smooth.

Question 5

Suppose we have the following literal weights: ω(A)=0.2, ω(¬A)=0.8, ω(B)=0.4, ω(¬B)=0.6, ω(C)=0.6, ω(¬C)=0.4 ω(D)=0.8, ω(¬D)=0.2.

a) $(\neg A \land B) \lor (\neg B \land A) = W(\neg A)W(B) + W(\neg B)W(A)$

$= 0.8 \cdot 0.4 + 0.6 \cdot 0.2$

$= 0.32 + 0.12 = \boxed{0.44}$

b) the W of true assignment ← product of literal weight

$W(A, B, C) = W(A)W(B)W(C)$

which is $W(A \land B \land C)$

They are same.

c) $[W(\neg A)W(B) + W(A)W(\neg B)][W(C)W(D) + W(\neg C)W(\neg D)]$

$+$

$[W(A)W(B) + W(\neg A)W(\neg B)][W(C)W(\neg D) + W(\neg C)W(D)]$

$= [(0.8)(0.4) + (0.2)(0.6)] \cdot [(0.6) \cdot (0.8) + (0.4)(0.2)]$

$+$

$[(0.2)(0.4) + (0.8)(0.6)] \cdot [(0.6)(0.2) + (0.4)(0.8)] = 0.4928$