

# Sustaining glacial-fed irrigation networks with artificial ice reservoirs

---

Suryanarayanan Balasubramanian

*July 4, 2022*  
Version: My First Draft





UNIVERSITÉ DE FRIBOURG  
UNIVERSITÄT FREIBURG

Department of Geosciences  
Cryosphere Research Group

Artificial Ice Reservoirs

# Sustaining glacial-fed irrigation networks with artificial ice reservoirs

Suryanarayanan Balasubramanian

*1. Reviewer*      Martin Hoelzle

Department of Geosciences  
University Of Fribourg

*2. Reviewer*      John Doe

Department of Clean Thesis Style  
Clean Thesis Style University

*Supervisors*      Martin Hoelzle and John Smith

July 4, 2022

**Suryanarayanan Balasubramanian**

*Sustaining glacial-fed irrigation networks with artificial ice reservoirs*

Artificial Ice Reservoirs, July 4, 2022

Reviewers: Martin Hoelzle and John Doe

Supervisors: Martin Hoelzle and John Smith

**University Of Fribourg**

*Cryosphere Research Group*

Cryosphere Research Group

Department of Geosciences

Av. de l'Europe 20

CH-1700 and Fribourg

# Abstract

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

## Abstract (different language)

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.



# Acknowledgement

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

This is the second paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.



# Contents

<b>1</b>	<b>Ice Reservoirs</b>	<b>1</b>
1.1	Introduction . . . . .	1
1.2	Nomenclature and Classification . . . . .	3
1.3	Objectives . . . . .	4
1.4	Structure . . . . .	4
<b>2</b>	<b>Religion of ice reservoirs</b>	<b>5</b>
2.1	An old history . . . . .	5
2.2	The marriages of glaciers . . . . .	5
2.3	From folklore to science . . . . .	7
<b>3</b>	<b>Science of ice reservoirs</b>	<b>9</b>
3.1	Study sites and data . . . . .	9
3.1.1	Study sites . . . . .	9
3.1.2	Meteorological data . . . . .	10
3.1.3	Fountain observations . . . . .	11
3.1.4	Drone flights . . . . .	11
3.2	AIR Model . . . . .	12
3.2.1	Surface area calculation . . . . .	12
3.2.2	Energy balance calculation . . . . .	14
3.2.3	Mass balance calculation . . . . .	21
3.3	Model extensions . . . . .	22
3.3.1	Discharge rate scheduler . . . . .	22
3.3.2	Location classifier . . . . .	23
3.4	Suggestions . . . . .	25
3.4.1	Model limitations . . . . .	25
3.4.2	Spray radius . . . . .	25
3.4.3	Further data acquisition . . . . .	25
3.4.4	Improvement of model algorithm . . . . .	26
<b>4</b>	<b>Habitat of ice reservoirs</b>	<b>27</b>
4.1	Requirements for AIR construction . . . . .	27

4.1.1	Water supply . . . . .	27
4.1.2	Weather conditions . . . . .	27
4.1.3	Topography . . . . .	28
4.1.4	Checklist to identify AIR construction regions . . . . .	28
4.1.5	Metrics to rank sites within the same region . . . . .	29
<b>5</b>	<b>Technology of ice reservoirs</b>	<b>31</b>
5.1	Ice terraces . . . . .	31
5.1.1	Construction strategy . . . . .	31
5.1.2	Application . . . . .	32
5.1.3	Drawbacks . . . . .	33
5.2	Ice stupas . . . . .	33
5.2.1	Construction strategy . . . . .	34
5.2.2	Application . . . . .	35
5.2.3	Drawbacks . . . . .	35
5.3	Improvement of construction strategies through weather-sensitive automation systems . . . . .	35
5.3.1	Perpetual ice reservoirs . . . . .	35
<b>6</b>	<b>Heritage of ice reservoirs</b>	<b>37</b>
6.1	Summary . . . . .	37
6.2	Conclusions . . . . .	38
6.3	Discussion . . . . .	39
6.4	Recommendations . . . . .	39
6.5	Suggestions for future research . . . . .	39
6.6	Final thoughts . . . . .	39
<b>Bibliography</b>		<b>41</b>
<b>A Example Appendix</b>		<b>51</b>
A.1	Appendix Section 1 . . . . .	51
A.2	Appendix Section 2 . . . . .	52
<b>Declaration</b>		<b>55</b>

# Ice Reservoirs

## 1.1 Introduction

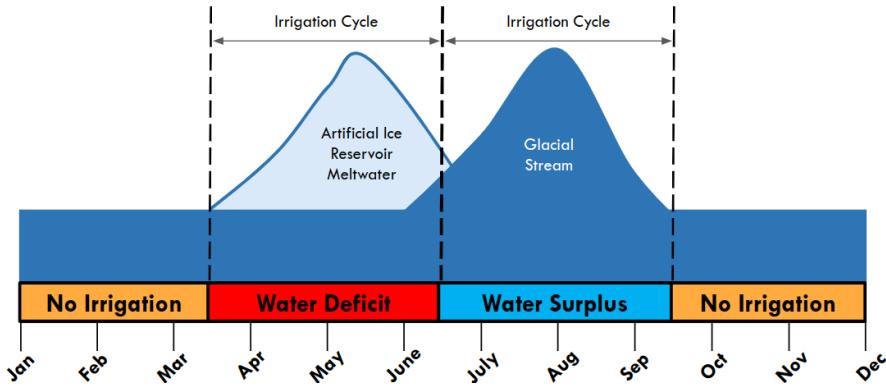
Cryosphere-fed irrigation networks in arid mountain regions are completely dependent on the timely availability of meltwater from glaciers, snow and permafrost [Imm+20; Far+15; Tve07]. With the accelerated decline of glaciers due to climate change [NB16], these regions are experiencing water scarcity, particularly during spring [NT15; MK15]. This seasonal water scarcity warrants supplementary irrigation in order to sustain agricultural output and utilize the complete growing season [NB16; Vin09b].

For example, in Ladakh, a cold arid desert in northern India, there is a typical shortage of water at the onset of the agricultural season for about 2 months until a sufficient and reliable supply of meltwater from glaciers becomes available (see Fig. 1.1) [NT15; NB16; Vin09b]. This has reduced their annual crop cycles from two to one [Nüs+19].

To cope with this recurrent water scarcity, villagers have developed two types of artificial ice reservoirs (AIRs): ice stupas and ice terraces. Both the ice reservoirs capture water in the autumn and winter, allowing it to freeze, and hold it until spring, when it melts and flows down to the fields [IPC19; Vin09a; CAS17; Nüs+19]. In this way, they retain a previously unused portion of the annual flow and facilitate its use to supplement the decreased flow in the next spring (see Fig. 1.1). This study focuses on one form of AIRs that are locally called "ice stupas".

Over the past decade, several ice stupas have been built to supplement irrigation water supply of mountain villages in India [Wan20; Pal22; Agg+21], Kyrgyzstan [BBC20] and Chile [Reu21]. Despite this widespread adoption, only a few publications examine the role of AIRs in the water resource management of these regions. None of these publications study AIRs outside Ladakh. Moreover, the published quantifications of the water storage capacity of AIRs just in Ladakh also vary widely between these studies [NT15; Bag98].

Quantifying the water storage capacity of AIRs is not straightforward since the formative processes of AIRs are complex. These processes are controlled by local

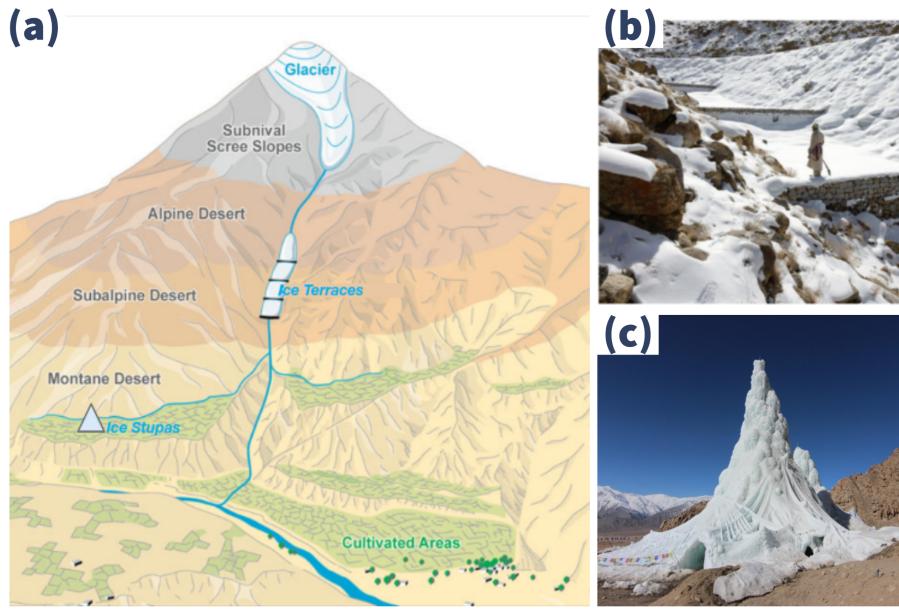


**Fig. 1.1.:** Seasonal variation in the availability of irrigation water. The graph highlights the crucial role of AIRs in bridging the critical gap in water availability. Adapted from: [NB16]

topography, meteorology and construction strategies used. However, methodologies used to quantify meteorological influences on glaciers may also be applicable on these ice reservoirs since their surface processes are similar. But due to their limited size, and comparatively more variable surface area, this assessment requires a modelling approach which is optimally constrained with comprehensive data from in-situ field measurements.

A spirit of improvisation guides the construction strategies of AIRs. This has resulted in ice reservoirs exhibiting significant volume variations despite experiencing similar meteorological conditions. For example, ice terraces have attained volumes upto 30 times larger than ice stupas built in Ladakh, India [Nüs+19]. However, the processes driving these differences can only be resolved if the complete methodology of the construction strategy used is available.

This thesis fulfills both of these requirements as it provides a new set of AIR-specific volume and area measurements from drone flights along with meteorological data during the construction period. All these datasets were produced through construction strategies using fountain systems that are quantified via in-situ observations of the fountain characteristics and discharge rate measurements. In a first step, this thesis attempts to formulate a one-dimensional AIR model in order to calibrate and validate it with the AIR datasets. In a second step, it attempts to use this model as a tool to propose a construction strategy that makes AIRs efficient and effortless to build. While this thesis reviews published AIR research and presents a comprehensive quantitative study of their water storage potential, we acknowledge that substantial additional knowledge is held by farming communities building these structures since mid-1800s.



**Fig. 1.2.:** (a) Schematic overview of the position of artificial ice reservoirs. These constructions are located at altitudes between the glaciers and the irrigation networks in the cultivated areas. (b) Ice terraces at 3900 m, located above the village of Nang, Ladakh. The cascade is composed of a series of loose masonry walls ranging in height from 2 to 3 m, which help freeze water for storage. (c) Ice stupas at 3600 m, located above the village of Phyng, Ladakh. They are made using fountain systems. Adapted from: [NB16]

## 1.2 Nomenclature and Classification

In spite of the popularity of the term "artificial glacier", we deliberately use the term "ice reservoir" since it conveys character and function of these structures more accurately [Nüs+19]. Man-made ice structures typically have a lifetime in the order of months and a size million times smaller than typical glaciers. Therefore, any comparison between these ice structures can be misleading. Since glaciers are considered as natural ice reservoirs, we use the terminology artificial ice reservoirs (AIRs) to distinguish man-made ice structures described in this thesis.

A spirit of improvisation guides the construction strategy of AIRs making it difficult to classify them. However, it has been found that construction strategies using fountain systems form AIRs which tend towards a conical shape and those without form flat sheets of ice. Therefore, this thesis classifies all the AIRs produced based on whether or not they use fountain systems. AIRs using fountain systems are called "ice stupas" and those without are called "ice terraces" since this terminology denotes the resulting shape of the respective AIRs appropriately.

## 1.3 Objectives

The main objective of this thesis is to quantify the water storage potential of AIRs based on the construction site and fountain chosen.

An integrated study approach including field measurements and modelling is applied to answer the following research questions:

1. What is the influence of construction location and fountain characteristics on AIR volume evolution?
2. How can ice stupa fountain systems be engineered to reduce water loss and maintenance of AIRs?

An energy and mass balance model for artificial ice reservoirs was set up to answer the first research question (paper I and II). Since in-situ measurements were required to run this model, a measurement campaign was executed in Switzerland and India during the past 4 winters. These datasets provided the necessary input, calibration and validation data to model the evolution of AIRs and study their sensitivity to meteorological conditions and fountain characteristics (paper II).

Two weather-sensitive construction strategies were developed to answer the second research question. These construction strategies employed fountains whose discharge rate was regulated by automation system using the AIR model developed before. Their advantages over traditional construction strategy are quantified in paper III.

## 1.4 Structure

Chapter 1 introduces the motivation of this work and provides a summary of the state of knowledge about AIRs prior to this thesis. Chapter 2 describes the origins of this technology as a religious practice. Chapter 3 gives an overview about the study sites and introduces the different field techniques applied. The influence of the construction location through its meteorological and topographical conditions are presented in Chapter 4. The engineering design of AIR technologies are showcased in Chapter 5 along with suggestions for their improvement. Chapter 6 concludes the thesis with a synthesis and future outlook. Papers I, II and III are included in the Appendix.

# Religion of ice reservoirs

For centuries, in the Himalayan mountain ranges, local cultures have believed that glaciers are alive. And what's more, that certain glaciers can have different genders including male and female. These people 'breed' new glaciers by grafting together—or marrying—fragments of ice from male and female glaciers, then covering them with charcoal, wheat husks, cloths, or willow branches so they can reproduce in privacy. These glacierets transform into fully active glaciers that grow each year with additional snowfall. Those then serve as lasting reserves of water that farmers can use to irrigate their crops. Over the years, these practices have inspired other cultures, where people are creating their own artificial ice reservoirs (AIRs) and applying them to solve serious modern challenges around water supplies.

## 2.1 An old history

According to legend, when the people of Baltistan learnt of the Mongol army advancing towards them from the north in the early 13th century, they came up with an ingenious way to stop them. As the inhabited valleys were only accessible through narrow passes, they decided to block the entry way by building a glacier. This successfully prevented the Mongol invasion and, crucially, it also solved the locals' other big problem: water scarcity.

## 2.2 The marriages of glaciers

The people of Gilgit Baltistan believe that glaciers are living entities. That's why a combination of female and male ice was absolutely necessary. The male glacier – called 'po gang' locally – gives off little water and moves slowly, while a 'female glacier' – or 'mo gang' – is a growing glacier that gives off a lot of water.

The glaciers that people help to grow are the fruit of the sacred union between a mother glacier and a father glacier. They get married and have offspring. The selection of an appropriate site for this marriage is of utmost importance, and

a suitable spot must fulfil a list of conditions. It should be located at an altitude of at least 4000 or 5000 metres above sea level; it should be on a gentle slope, where it should have minimal exposure to sunlight, thus a north-facing mountain side is preferable. For most of the expert glacier grafters, the presence of permafrost or ice on the site is another key requirement.

Once a suitable spot is selected, the expedition can be planned. The bride and the groom – the female and the male glacier, preferably from different villages – are chosen and the marriage can be planned. The glacier grafting usually takes place in November, when the local temperatures oscillate around zero. A 12-man party carries the pieces of female ice in woven baskets, another 12 men carry the male ice, the water drawn from the Indus river is carried traditionally in 12 gourd bottles, but sometimes clay pots or goatskins are also required, as well as charcoal and wheat husks or sawdust, which act as insulators for the ice. The last ingredient is salt, which, according to some glacier grafters, helps protect the new glacier from impurities. The bride and groom party walk from different sites and meet in a certain spot to climb together to the glacier growing site, but no greetings are exchanged, as the people involved in the ceremony must remain silent until the ice is deposited in its new home. They walk continuously without having a break, but if the distance is too much and rest is required, they do not put their loads on the ground, instead hanging the baskets on trees, or on walking sticks if nothing else is available. Each man has to carry around 15 to 25 kilograms of ice, walking in cold air, silently up the mountains, for a day or more. Once they reach the glacier growing site, they deposit their valuable loads. The ice lumps and water bottles are placed in between the boulders, or in a small cave, or sometimes in a specially dug pit, and covered with layers of salt, charcoal and sawdust. The silence is broken as religious leaders recite verses of the Quran and say prayers for the success of the glacier marriage and for protection from the djinns. Once the male and female glaciers are placed in their new home and covered, a man from the party of glacier grafters stands up and offers his life for the success of the process. His symbolic sacrifice is matched by the actual sacrifice of a goat – its meat is distributed to a charity, because prayers are more likely to be answered if accompanied by an act of charity. They will not visit the place for at least three years, so as not to disturb the glacier. It is said that a person who disturbs the glacier before its maturation will die. The celebrations continue in the village with traditional songs and prayers, alongside festive food and the joy of the accomplished mission.

## 2.3 From folklore to science

Myths, legends and superstitions are ways of knowing. But they need to be translated to the language of science. However, when it comes to past projects, there is only anecdotal evidence available.

According to Ingvar Tveiten, a researcher from Norway, the account of the glacier development process presented by a glacier grafter from Balghar bears a strong resemblance to the definition of the formation of rock glaciers. According to a description by a Balghar local: “First the ice slips down into the rocks where it grows roots. Then it starts to break the rocks bringing them up. Then the glacier comes forward. This has happened where they did the glacier growing.” Tveiten, who conducted field research in Baltistan, concludes that “glacier growing is typically performed [...] in a terrain that is conducive to the accumulation of snow by avalanching and snow slips. The presence of permafrost at these locations is likely to contribute to ice accumulating [...] Thus, glacier growing is conducted at locations which are already very prone to ice accumulation, and may explain why glacier growing is perceived to work.” Here it is, traditional knowledge translated into the language of science.

Even the choice of the glacier grafting site suggests that the technique was developed as a result of the local people’s deep understanding of local environmental processes. The view of glaciers as animate implies that humans can influence on the lives of glaciers, just as glaciers can influence on the lives of people.



# Science of ice reservoirs

This chapter provides the methodology used to estimate the ice volume evolution and water-use efficiency of AIRs. The equations governing the mass and energy balance of vertical AIRs (or Icestupas) is explained along with the associated datasets required for forcing, calibration and validation of this AIR model.

In a first step, the influence of the chosen location and fountain used was quantified by feeding weather and fountain data to an energy balance model and validating the ice volume estimates produced with volume observations from drone flights. In a second step, the model was extended to serve as a tool for recommending discharge rates and identifying favourable construction locations worldwide.

## 3.1 Study sites and data

### 3.1.1 Study sites

We chose two villages in the Swiss Alps and the Indian Himalayas called Guttannen and Gangles to collect the required datasets described above. The study period starts when the fountain was first switched on and ends when the respective AIR either melted or broke into several ice blocks. These two dates are denoted as start and expiry dates henceforth. Each AIR dataset was abbreviated based on the construction strategy used, prefix of the country code and the suffix of the year of its expiry date. The construction strategies are distinguished based on whether they used weather-sensitive approaches to regulate water supply. Those that did were codenamed automated construction strategy whereas the rest were codenamed traditional construction strategy. In total, five AIRs were studied in these two locations across three winters (see Table 3.2). Only one was built in Gangles. The rest were built in Guttannen. All except one construction campaign used traditional construction strategies. Therefore, traditional AIRs are referred to without explicitly specifying their construction strategy henceforth.

The Guttannen site ( $46.66^{\circ}\text{N}$ ,  $8.29^{\circ}\text{E}$ ) is situated in the Berne region, Switzerland and has an altitude of  $1047\text{ m a.s.l}$ . In the winter (Oct-Apr), mean daily minimum

**Tab. 3.1.:** Summary of the weather observations for AIRs built during the repective study period. The weather measurements are shown using their mean ( $\mu$ ) and standard deviation ( $\sigma$ ) during the study period as  $\mu \pm \sigma$ .

Name	Symbol	IN21	CH21	Units
Air temperature	$T_a$	$0 \pm 7$	$2 \pm 6$	°C
Relative humidity	$RH$	$35 \pm 20$	$79 \pm 18$	%
Wind speed	$v_a$	$3 \pm 1$	$2 \pm 2$	m/s
Direct Shortwave	$SW_{direct}$	$246 \pm 333$	$80 \pm 156$	$W m^{-2}$
Diffuse Shortwave	$SW_{diffuse}$	$0 \pm 0$	$58 \pm 87$	$W m^{-2}$
Hourly Precipitation	$ppt$	$0 \pm 0$	$139 \pm 457$	mm
Pressure	$p_a$	$623 \pm 3$	$794 \pm 9$	hPa

and maximum air temperatures vary between -13 and 15 °C. Clear skies are rare, averaging around 7 days during winter. Daily winter precipitation can sometimes be as high as 100 mm. These values are based on 30 years of hourly historical weather data measurements [Met21]. Several AIRs were constructed by the Guttannen Bewegt Association, the University of Fribourg and the Lucerne University of Applied Sciences and Arts during the winters of 2020-22.

The Gangles site (34.22 °N, 77.61 °E) is located around 20 km north of Leh city in the Ladakh region, lying at 4025 m a.s.l.. The mean annual temperature is 5.6 °C, and the thermal range is characterized by high seasonal variation. During January, the coldest month, the mean temperature drops to -7.2 °C. During August, the warmest month, the mean temperature rises to 17.5 °C [NSD12]. Because of the rain shadow effect of the Himalayan Range, the mean annual precipitation in Leh totals less than 100 mm, and there is high interannual variability. Whereas the average summer rainfall between July and September reaches 37.5 mm, the average winter precipitation between January and March amounts to 27.3 mm and falls almost entirely as snow. AIRs were constructed here as part of the Ice Stupa Competition by the Himalayan Institute of Alternatives, Ladakh (HIAL).

### 3.1.2 Meteorological data

Air temperature, relative humidity, wind speed, pressure, longwave and global shortwave radiation are required to calculate the surface energy balance of an AIR. The resulting dataset highlights the difference in meteorological influences driving ice volume evolution in the two study sites (see Table 3.1).

**Tab. 3.2.:** List of all the studied AIRs. The study period starts when the fountain was first switched on (denoted as Start Date) and ends when the respective AIR either melted or broke into several ice blocks (denoted as Expiry Date).

Name	Start Date	Expiry Date	No. of flights	Spray radius
Traditional CH20			2	
Traditional CH21	Nov 22 2020	May 10 2021	8	6.9 m
Traditional IN21	Jan 18 2021	June 20 2021	6	10.2 m
Traditional CH22	Dec 8 2021	April 12 2022	8	
Automated CH22	Dec 8 2021	April 12 2022	6	

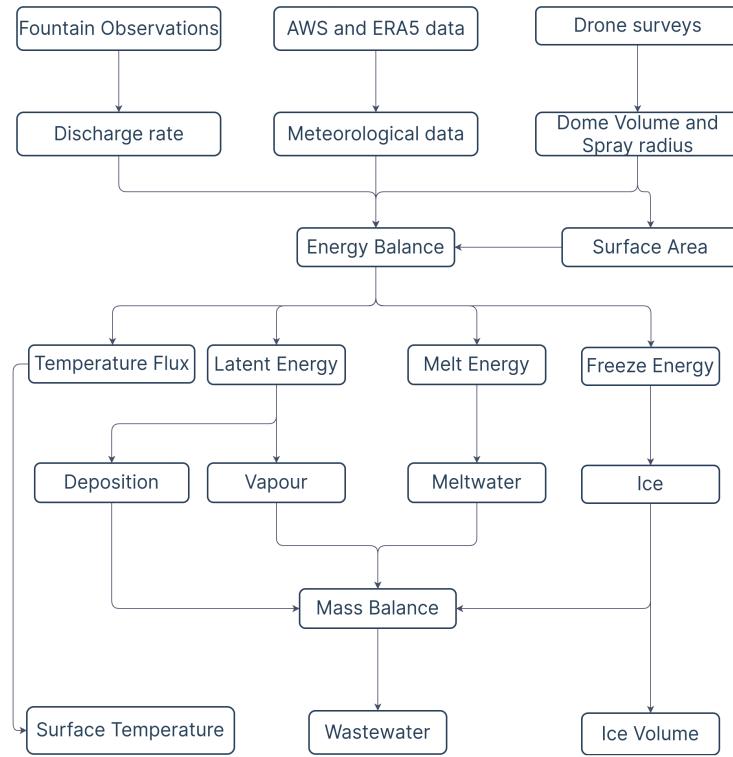
### 3.1.3 Fountain observations

The fountain consists of a pipeline and a nozzle. The pipeline has three attributes, namely : discharge rate ( $Q$ ), height ( $h$ ) and water temperature ( $T_F$ ). Discharge rate represents the discharge rate of the water in the fountain pipeline. Height denotes the height of the fountain pipeline installed. Fountain water temperature is the temperature of water droplets produced by the fountain.

The fountain nozzle has three characteristics, namely : the aperture diameter ( $dia$ ), the spray radius ( $r$ ) and pressure loss ( $P$ ) . Spray radius denotes the observed ice radius formed from the fountain water droplets. Pressure loss denotes the loss of water head caused due to the fountain nozzle.

### 3.1.4 Drone flights

Several photogrammetric surveys were conducted for each of the AIRs. The details of these surveys and the methodology used to produce the corresponding outputs are explained in paper I. The digital elevation models (DEMs) generated from the obtained imagery were analysed to document the ice radius, the surface area and the volume of the ice structures. Ice radius measurements of drone flights which observed either an increase in AIR circumference or volume were averaged to determine the fountain's spray radius. The number of drone surveys conducted for each of the AIRs and the corresponding spray radius observed is shown in Table 3.2.



**Fig. 3.1.:** Model schematic showing the workflow used in the model at every time step.

## 3.2 AIR Model

A bulk energy and mass balance model is used to calculate the amounts of ice, meltwater, water vapour and wastewater of the AIR. In each hourly time step, the model uses the AIR surface area, energy balance and mass balance calculations to estimate its ice volume, surface temperature and wastewater as shown in Fig. 3.1

.

### 3.2.1 Surface area calculation

The model assumes the AIR shape to be a cone and assigns the following shape attributes:

$$A_{cone}^i = \pi \cdot r_{cone}^i \cdot \sqrt{(r_{cone}^i)^2 + (h_{cone}^i)^2} \quad (3.1a)$$

$$V_{cone}^i = \pi/3 \cdot (r_{cone}^i)^2 \cdot h_{cone}^i \quad (3.1b)$$

$$j_{cone}^i = \frac{\Delta M_{ice}^i}{\rho_{water} * A_{cone}^i} \quad (3.1c)$$

where  $i$  denotes the model time step,  $r_{cone}^i$  is the radius;  $h_{cone}^i$  is the height;  $A_{cone}^i$  is the surface area;  $V_{cone}^i$  is the volume and  $j_{cone}^i$  is the AIR surface normal thickness change as shown in Fig. 3.2.  $M_{ice}^i$  is the mass of the AIR and  $\Delta M_{ice}^i = M_{ice}^{i-1} - M_{ice}^{i-2}$ . Henceforth, the equations used display the model time step superscript  $i$  only if it is different from the current time step.

AIR density can be defined as:

$$\rho_{cone} = \frac{M_F + M_{dep} + M_{ppt}}{(M_F + M_{dep})/\rho_{ice} + M_{ppt}/\rho_{snow}} \quad (3.2)$$

where  $M_F$  is the cumulative mass of the fountain discharge;  $M_{ppt}$  is the cumulative precipitation;  $M_{dep}$  is the cumulative accumulation through water vapour deposition;  $\rho_{ice}$  is the ice density ( $917 \text{ kg m}^{-3}$ ) and  $\rho_{snow}$  is the density of wet snow ( $300 \text{ kg m}^{-3}$ ) taken from [CP10].

AIR volume can also be expressed as:

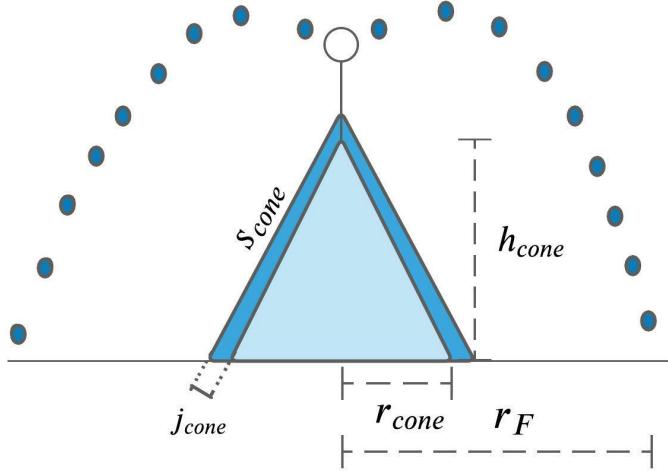
$$V_{cone} = \frac{M_{ice}}{\rho_{cone}} \quad (3.3)$$

The initial radius of the AIR is assumed to be  $r_F$ . The initial height  $h_0$  depends on the dome volume  $V_{dome}$  used to construct the AIR as follows:

$$h_0 = \Delta x + \frac{3 \cdot V_{dome}}{\pi \cdot (r_F)^2} \quad (3.4)$$

where  $\Delta x$  is the surface layer thickness (defined in Section 3.2.2)

During the subsequent time steps, the dimensions of the AIR evolve assuming a uniform thickness change ( $j_{cone}$ ) across its surface area with an invariant slope  $s_{cone} = \frac{h_{cone}}{r_{cone}}$ . During these time steps, the volume is parameterised using Eqn. 3.1b as:



**Fig. 3.2.:** Shape variables of the AIR.  $r_{cone}$  is the radius,  $h_{cone}$  is the height,  $j_{cone}$  is the thickness change and  $s_{cone}$  is the slope of the ice cone.  $r_F$  is the spray radius of the fountain.

$$V_{cone} = \frac{\pi \cdot (r_{cone})^3 \cdot s_{cone}}{3} \quad (3.5)$$

We define the Icestupa boundary through its spray radius, i.e. we assume ice formation is negligible when  $r_{cone} > r_F$ . Combining Eqns. 3.1b, 3.3, 3.4 and 3.5, the geometric evolution of the Icestupa at each time step  $i$  can be determined by considering the following rules:

$$(r_{cone}, h_{cone}) = \begin{cases} (r_F, h_0) & \text{if } i = 0 \\ (r_{cone}^{i-1}, \frac{3 \cdot M_{ice}}{\pi \cdot \rho_{ice} \cdot (r_{cone}^{i-1})^2}) & \text{if } r_{cone}^{i-1} \geq r_F \text{ and } \Delta M_{ice} > 0 \\ (\frac{3 \cdot M_{ice}}{\pi \cdot \rho_{ice} \cdot s_{cone}})^{1/3} \cdot (1, s_{cone}) & \text{otherwise} \end{cases} \quad (3.6)$$

### 3.2.2 Energy balance calculation

We approximate the energy balance at the surface of an AIR by a one-dimensional description of energy fluxes into and out of a (thin) layer with thickness  $\Delta x$ :

$$\rho_{cone} \cdot c_{ice} \cdot \frac{\Delta T}{\Delta t} \cdot \Delta x = q_{SW} + q_{LW} + q_L + q_S + q_F + q_R + q_G \quad (3.7)$$

Upward and downward fluxes relative to the ice surface are positive and negative, respectively. The first term is the energy change of the surface layer, which can be translated into a phase change energy should phase changes occur.  $q_{SW}$  is the net shortwave radiation;  $q_{LW}$  is the net longwave radiation;  $q_L$  and  $q_S$  are the turbulent latent and sensible heat fluxes.  $q_F$  and  $q_R$  represent the heat exchange of the fountain water droplets and rain droplets with the AIR ice surface respectively.  $q_G$  represents ground heat flux between the AIR surface and its interior.

The density of the AIR  $\rho_{cone}$  was parameterised as follows:

$$\rho_{cone} = \frac{M_F + M_{dep} + M_{ppt}}{(M_F + M_{dep})/\rho_{ice} + M_{ppt}/\rho_{snow}} \quad (3.8)$$

where  $M_F$  is the cumulative mass of the fountain discharge;  $M_{ppt}$  is the cumulative precipitation;  $M_{dep}$  is the cumulative accumulation through water vapour deposition;  $\rho_{ice}$  is the ice density ( $917 \text{ kg m}^{-3}$ ) and  $\rho_{snow}$  is the density of wet snow ( $300 \text{ kg m}^{-3}$ ) taken from [CP10] .

The energy flux acts upon the AIR surface layer, which has an upper and lower boundary defined by the atmosphere and the ice body of the AIR, respectively. Here, we define the surface temperature  $T_{ice}$  to be the modelled average temperature of the icestupa surface layer.

### Net Shortwave Radiation $q_{SW}$

The net shortwave radiation  $q_{SW}$  is computed as follows:

$$q_{SW} = (1 - \alpha) \cdot (SW_{direct} \cdot f_{cone} + SW_{diffuse}) \quad (3.9)$$

where  $SW_{direct}$  and  $SW_{diffuse}$  are the direct and diffuse shortwave radiation,  $\alpha$  is the modelled albedo and  $f_{cone}$  is the area fraction of the ice structure exposed to the direct shortwave radiation.

The albedo varies depending on the water source that formed the current AIR surface layer. During the fountain runtime, the albedo assumes a constant value corresponding to ice albedo. However, after the fountain is switched off, the albedo can reset to snow albedo during snowfall events and then decay back to ice albedo. We use the scheme described in [OK98] to model this process. The scheme records the decay of albedo with time after fresh snow is deposited on the surface.  $\delta t$  records

the number of time steps after the last snowfall event. After snowfall, albedo changes over a time step,  $\delta t$ , as

$$\alpha = \alpha_{ice} + (\alpha_{snow} - \alpha_{ice}) \cdot e^{(-\delta t)/\tau} \quad (3.10)$$

where  $\alpha_{ice}$  is the bare ice albedo value (0.25),  $\alpha_{snow}$  is the fresh snow albedo value (0.85) and  $\tau$  is a decay rate (16 days), which determines how fast the albedo of the ageing snow recedes back to ice albedo. Discharge events decrease the decay rate by a factor of  $\alpha_{ice}/\alpha_{snow}$ .

The solar area fraction  $f_{cone}$  of the ice structure exposed to the direct shortwave radiation depends on the shape considered. Using the solar elevation angle  $\theta_{sun}$ , the solar beam can be considered to have a vertical component, impinging on the horizontal surface (semicircular base of the AIR), and a horizontal component impinging on the vertical cross section (a triangle). The solar elevation angle  $\theta_{sun}$  used is modelled using the parametrisation proposed by [Woo68]. Accordingly,  $f_{cone}$  is determined as follows:

$$f_{cone} = \frac{(0.5 \cdot r_{cone} \cdot h_{cone}) \cdot \cos\theta_{sun} + (\pi \cdot (r_{cone})^2/2) \cdot \sin\theta_{sun}}{\pi \cdot r_{cone} \cdot ((r_{cone})^2 + (h_{cone})^2)^{1/2}} \quad (3.11)$$

The diffuse shortwave radiation is assumed to impact the conical AIR surface uniformly.

### Net Longwave Radiation $q_{LW}$

The net longwave radiation  $q_{LW}$  is determined as follows:

$$q_{LW} = LW_{in} - \sigma \cdot \epsilon_{ice} \cdot (T_{ice} + 273.15)^4 \quad (3.12)$$

where  $T_{ice}$  is the modelled surface temperature given in  $^{\circ}\text{C}$ ,  $\sigma = 5.67 \cdot 10^{-8} \text{ J m}^{-2} \text{s}^{-1} \text{K}^{-4}$  is the Stefan-Boltzmann constant,  $LW_{in}$  denotes the incoming longwave radiation and  $\epsilon_{ice}$  is the corresponding emissivity value for the Icestupa surface (0.97).

The incoming longwave radiation  $LW_{in}$  for the Indian site, where no direct measurements were available, is determined as follows:

$$LW_{in} = \sigma \cdot \epsilon_a \cdot (T_a + 273.15)^4 \quad (3.13)$$

here  $T_a$  represents the measured air temperature and  $\epsilon_a$  denotes the atmospheric emissivity. We approximate the atmospheric emissivity  $\epsilon_a$  using the equation suggested by [Bru82], considering air temperature and vapor pressure (Eqn. 3.14). The vapor pressure of air over water and ice was obtained using Eqn. 3.17. The expression defined in [Bru75] for clear skies (first term in equation 3.14) is extended with the correction for cloudy skies after [Bru82] as follows:

$$\epsilon_a = 1.24 \cdot \left( \frac{p_{v,w}}{(T_a + 273.15)} \right)^{1/7} \cdot (1 + 0.22 \cdot cld^2) \quad (3.14)$$

with a cloudiness index  $cld$ , ranging from 0 for clear skies to 1 for complete overcast skies. For the Indian site, we assume cloudiness to be negligible.

### Turbulent fluxes

The turbulent sensible  $q_S$  and latent heat  $q_L$  fluxes are computed with the following expressions proposed by [Gar92]:

$$q_S = \mu_{cone} \cdot c_a \cdot \rho_a \cdot p_a / p_{0,a} \cdot \frac{\kappa^2 \cdot v_a \cdot (T_a - T_{ice})}{\left( \ln \frac{h_{AWS}}{z_0} \right)^2} \quad (3.15)$$

$$q_L = \mu_{cone} \cdot 0.623 \cdot L_s \cdot \rho_a / p_{0,a} \cdot \frac{\kappa^2 \cdot v_a (p_{v,w} - p_{v,ice})}{\left( \ln \frac{h_{AWS}}{z_0} \right)^2} \quad (3.16)$$

where  $h_{AWS}$  is the measurement height above the ground surface of the AWS (around 2 m for all sites),  $v_a$  is the wind speed in [ $m s^{-1}$ ],  $c_a$  is the specific heat of air at constant pressure ( $1010 J kg^{-1} K^{-1}$ ),  $\rho_a$  is the air density at standard sea level ( $1.29 kg m^{-3}$ ),  $p_{0,a}$  is the air pressure at standard sea level ( $1013 hPa$ ),  $p_a$  is the measured air pressure,  $\kappa$  is the von Karman constant (0.4),  $z_0$  is the surface roughness (3 mm) and  $L_s$  is the heat of sublimation ( $2848 kJ kg^{-1}$ ). The vapor pressure of air with respect to water ( $p_{v,w}$ ) and with respect to ice ( $p_{v,ice}$ ) was obtained using the formulation given in [Hua18] :

$$p_{v,w} = e^{\frac{(34.494 - \frac{4924.99}{T_a + 237.1})}{(T_a + 105)^{1.57 \cdot 100}}} \cdot \frac{RH}{100}$$

$$p_{v,ice} = e^{\frac{(43.494 - \frac{6545.89}{T_{ice} + 278})}{(T_{ice} + 868)^2 \cdot 100}}$$
(3.17)

The dimensionless parameter  $\mu_{cone}$  is an exposure parameter that deals with the fact that AIR has a rough appearance and forms an obstacle to the wind regime. This factor accounts for the larger turbulent fluxes due to the roughness of the surface [Oer+21], and is a function of the AIR slope as follows:

$$\mu_{cone} = 1 + \frac{s_{cone}}{2}$$
(3.18)

A possible source of error is the fact that wind measurements from the horizontal plane at the AWS are used, which might be different from those on a slope. However, without detailed datasets from the AIR surface, we retain this assumption.

### Fountain discharge heat flux $q_F$

The fountain water temperature  $T_F$  is assumed to cool to 0 °C after contact with the ice surface.  $T_F$  is equal to the measured source water temperature. But during time periods when the ambient temperature is subzero,  $T_F$  is assumed to be 0 °C. Thus, the heat flux caused by this process is:

$$q_F = \begin{cases} \frac{\Delta M_F \cdot c_{water} \cdot T_F}{\Delta t \cdot A_{cone}} & \text{if } T_{temp} > 0 \\ 0 & \text{otherwise} \end{cases}$$
(3.19)

with  $c_{water}$  as the specific heat of water (4186 J kg<sup>-1</sup>K<sup>-1</sup>).

### Rain heat flux $q_R$

The influence of rain events on the albedo and the energy balance was assumed to be similar to that of discharge events. However, the water temperature of a rain event was assumed to equal to the air temeperature. Accordingly, the heat flux generated due to a rain event was equal to:

$$q_R = \frac{\Delta M_{ppt} \cdot c_{water} \cdot T_a}{\Delta t \cdot A_{cone}} \quad (3.20)$$

### Bulk Icestupa heat flux $q_G$

The bulk Icestupa heat flux  $q_G$  corresponds to the ground heat flux in normal soils and is caused by the temperature gradient between the surface layer ( $T_{ice}$ ) and the ice body ( $T_{bulk}$ ). It is expressed by using the heat conduction equation as follows:

$$q_G = k_{ice} \cdot (T_{bulk} - T_{ice})/l_{cone} \quad (3.21)$$

where  $k_{ice}$  is the thermal conductivity of ice ( $2.123 \text{ W m}^{-1} \text{ K}^{-1}$ ),  $T_{bulk}$  is the mean temperature of the ice body within the icestupa and  $l_{cone}$  is the average distance of any point in the surface to any other point in the ice body.  $T_{bulk}$  is initialised as  $0^\circ\text{C}$  and later determined from Eqn. 3.21 as follows:

$$T_{bulk}^{i+1} = T_{bulk}^i - (q_G \cdot A \cdot \Delta t) / (M_{ice} \cdot c_{ice}) \quad (3.22)$$

Since AIRs typically have conical shapes with  $r_{cone} > h_{cone}$ , we assume that the center of mass of the cone body is near the base of the fountain. Thus, the distance of every point in the AIR surface layer from the cone body's center of mass is between  $h_{cone}$  and  $r_{cone}$ . Therefore, we calculate  $q_G$  assuming  $l_{cone} = (r_{cone} + h_{cone})/2$ .

### Phase changes

In this section, the numerical procedures to model phase changes at the surface layer are explained. Let  $T_{temp}$  be the calculated surface temperature. Therefore, Eqn. 3.7 can be rewritten as:

$$q_{total} = \rho_{ice} \cdot c_{ice} \cdot \frac{(T_{temp} - T_{ice})}{\Delta t} \cdot \Delta x$$

where  $q_{total}$  represents the total energy available to be redistributed. Even if the numerical heat transfer solution produces temperatures which are  $T_{temp} > 0^\circ\text{C}$ , say from intense shortwave radiation, the ice temperature must remain at  $T_{temp} = 0^\circ\text{C}$ . The “excess” energy is used to drive the melting process. Moreover, the energy input is used to melt the surface ice layer, and not to raise the surface temperature

to some unphysical value. Similarly, for freezing to occur, three conditions are required. Firstly, fountain water is present ( $\Delta M_F > 0$ ) and secondly the calculated temperature of the ice,  $T_{temp}$ , is below  $0^\circ C$ . However, these two conditions are not sufficient as the latent heat turbulent fluxes can only contribute to temperature fluctuations. Therefore, an additional condition, namely,  $(q_{total} - q_L) < 0$ , is required. Depending on the above conditions, the total energy  $q_{total}$  can be redistributed for the melting ( $q_{melt}$ ), freezing ( $q_{freeze}$ ) and surface temperature change ( $q_T$ ) processes as follows:

$$q_{total} = \begin{cases} q_{freeze} + q_T & \text{if } \Delta M_F > 0 \text{ and } T_{temp} < 0 \text{ and } (q_{total} - q_L) < 0 \\ q_{melt} + q_T & \text{otherwise} \end{cases} \quad (3.23)$$

Henceforth, time steps when the total energy is redistributed to the freezing energy are called freezing events and the rest of the time steps are called melting events.

During a freezing event, the AIR surface is assumed to warm to  $0^\circ C$ . The available energy ( $q_{total} - q_L$ ) is further increased due to this change in surface temperature represented by the energy flux:

$$q_0 = \frac{\rho_{ice} \cdot \Delta x \cdot c_{ice} \cdot T_{ice}^{i-1}}{\Delta t}$$

The available fountain discharge ( $\Delta M_F$ ) may not be sufficient to utilize all the freezing energy. At such times, the additional freezing energy further cools down the surface temperature. Accordingly, the surface energy flux distribution during a freezing event can be represented as:

$$(q_{freeze}, q_T) = \begin{cases} \left(\frac{\Delta M_F \cdot L_f}{A_{cone} \cdot \Delta t}, q_{total} + \frac{\Delta M_F \cdot L_f}{A_{cone} \cdot \Delta t}\right) & \text{if } \Delta M_F \text{ insufficient} \\ (q_{total} - q_L + q_0, q_L - q_0) & \text{otherwise} \end{cases} \quad (3.24)$$

If  $T_{temp} > 0^\circ C$ , then energy is reallocated from  $q_T$  to  $q_{melt}$  to maintain surface temperature at melting point. The total energy flux distribution during a melting event can be represented as:

$$(q_{melt}, q_T) = \begin{cases} (0, q_{total}) & \text{if } T_{temp} \leq 0 \\ \left(\frac{T_{temp} \cdot \rho_{ice} \cdot c_{ice} \cdot \Delta x}{\Delta t}, q_{total} - \frac{T_{temp} \cdot \rho_{ice} \cdot c_{ice} \cdot \Delta x}{\Delta t}\right) & \text{if } T_{temp} > 0 \end{cases} \quad (3.25)$$

### 3.2.3 Mass balance calculation

The mass balance equation for an AIR is represented as:

$$\frac{\Delta M_F + \Delta M_{ppt} + \Delta M_{dep}}{\Delta t} = \frac{\Delta M_{ice} + \Delta M_{water} + \Delta M_{sub} + \Delta M_{waste}}{\Delta t} \quad (3.26)$$

where  $M_F$  is the cumulative mass of the fountain discharge;  $M_{ppt}$  is the cumulative precipitation;  $M_{dep}$  is the cumulative accumulation through water vapour deposition;  $M_{ice}$  is the cumulative mass of ice;  $M_{water}$  is the cumulative mass of melt water;  $M_{sub}$  represents the cumulative water vapor loss by sublimation and  $M_{waste}$  represents the fountain wastewater that did not interact with the AIR. The left hand side of equation 3.26 represents the rate of mass input and the right hand side represents the rate of mass output for an AIR.

Precipitation input is calculated as shown in equation 3.27b where  $\rho_w$  is the density of water ( $1000 \text{ kg m}^{-3}$ ),  $\Delta ppt/\Delta t$  is the measured precipitation rate in [ $\text{m s}^{-1}$ ] and  $T_{ppt}$  is the temperature threshold below which precipitation falls as snow. Here, snowfall events were identified using  $T_{ppt}$  as  $1^\circ\text{C}$ . Snow mass input is calculated by assuming a uniform deposition over the entire circular footprint of the AIR.

The latent heat flux is used to estimate either the evaporation and condensation processes or sublimation and deposition processes as shown in equation 3.27c. During the time steps at which the surface temperature is below  $0^\circ\text{C}$  only sublimation and deposition can occur, but if the surface temperature reaches  $0^\circ\text{C}$ , evaporation and condensation can also occur. As the differentiation between evaporation and sublimation (and condensation and deposition) when the air temperature reaches  $0^\circ\text{C}$  is challenging, we assume that negative (positive) latent heat fluxes correspond only to sublimation (deposition), i.e. no evaporation (condensation) is calculated.

Since we have categorized every time step as a freezing or melting event, we can determine the melting/freezing rates and the corresponding meltwater/ice quantities as shown in equations 3.27e, 3.27d and 3.27f. Having calculated all other mass components, the fountain wastewater generated every time step can be calculated using Eqn. 3.26.

$$\frac{\Delta M_F}{\Delta t} = \begin{cases} \frac{60}{\rho_w \cdot \Delta t} \cdot d_F & \text{if fountain is on} \\ 0 & \text{otherwise} \end{cases} \quad (3.27a)$$

$$\frac{\Delta M_{ppt}}{\Delta t} = \begin{cases} \pi \cdot (r_{cone})^2 \cdot \rho_w \cdot \frac{\Delta ppt}{\Delta t} & \text{if } T_a < T_{ppt} \\ 0 & \text{if } T_a \geq T_{ppt} \end{cases} \quad (3.27b)$$

$$\left( \frac{\Delta M_{dep}}{\Delta t}, \frac{\Delta M_{sub}}{\Delta t} \right) = \begin{cases} \frac{q_L \cdot A_{cone}}{L_s} \cdot (1, 0) & \text{if } q_L \geq 0 \\ \frac{q_L \cdot A_{cone}}{L_s} \cdot (0, -1) & \text{if } q_L < 0 \end{cases} \quad (3.27c)$$

$$\frac{\Delta M_{water}}{\Delta t} = \frac{q_{melt} \cdot A_{cone}}{L_f} \quad (3.27d)$$

$$\frac{\Delta M_{freeze/melt}}{\Delta t} = \frac{q_{freeze/melt} \cdot A_{cone}}{L_f} \quad (3.27e)$$

$$\frac{\Delta M_{ice}}{\Delta t} = \frac{q_{freeze} \cdot A_{cone}}{L_f} + \frac{\Delta M_{ppt}}{\Delta t} + \frac{\Delta M_{dep}}{\Delta t} - \frac{\Delta M_{sub}}{\Delta t} - \frac{\Delta M_{water}}{\Delta t} \quad (3.27f)$$

Considering AIRs as water reservoirs, their net water loss can be defined as:

$$\text{Net water losses} = \frac{M_{waste} + M_{sub}}{(M_F + M_{ppt} + M_{dep})} \cdot 100 \quad (3.28)$$

### 3.3 Model extensions

The AIR model presented above was extended in order to (a) recommend fountain scheduling strategies and (b) improve its transferability to new locations.

#### 3.3.1 Discharge rate scheduler

In this section, we focus on the description of the integration of fountain scheduling processes in the model.

Two types of fountain scheduling strategies are possible depending on the constraints of water supply and favourable weather window in the respective location. Locations constrained by their available water supply require water-sensitive fountain scheduling strategies and those constrained by the duration of favourable weather windows require weather-sensitive fountain scheduling strategies. These two kinds of scheduled fountains will be referred to as water-sensitive fountain and weather-sensitive fountain henceforth.

**Tab. 3.3.:** Assumptions for the parametrisation introduced to simplify the model.

Estimation of	Symbol	IVOM	WEOM
Slope	$s_{cone}$	1	0
Albedo	$\alpha$	$\alpha_{snow}$	$\alpha_{ice}$
Cloudiness	$cld$	0	1

Water-sensitive fountains are expected to produce higher water use efficiency whereas weather-sensitive fountains are expected to produce higher ice volumes. Accordingly, we produce the discharge rate recommendations for these two kinds of fountains through two sets of model forcing assumptions for the following three model variables: (a) slope , (b) albedo and (c) cloudiness. These two kinds of models will be referred to as ice volume optimised model (IVOM) and water-use efficiency optimised model (WEOM) respectively. The slope variable increases the shortwave radiation and sensible heat impact. The albedo variable decreases the shortwave radiation impact. The cloudiness variable increases both the shortwave and the long-wave radiation impact. We associate the upper and lower bounds of these variables with the IVOM and WEOM versions depending on whether they overestimate and underestimate the freezing rate of the AIR as shown in Table 3.3.

We apply the assumptions described in Table 3.3 on the one-dimensional description of energy fluxes through Eqn. 3.7. The derivation of the individual energy and mass balance terms for the IVOM and WEOM model versions are discussed in Paper III.

### 3.3.2 Location classifier

The model, in its current form, is not expected to perform well for locations where it has not been calibrated for before. This limits its ability to identify and classify other favourable locations worldwide.

In this section, we showcase a strategy to improve the AIR model's transferability to new locations. Specifically, in paper I, we highlight the sensitivity of the model to the surface layer thickness parameter. This parameter requires prior calibration for better model performance. However, the dependence of the model on this parameter can be removed if spatial temperature fluctuations across the ice structure are resolved.

In order to remove the calibration requirements for model performance, we combine the AIR model with the COupled Snowpack and Ice surface energy and mass balance model in PYthon (COSIPY) . COSIPY is typically used for modelling distributed snow and glacier mass changes [SAS20]. However, its flexible, user-friendly and modular

framework makes it an ideal platform to implement the alternate modules required for modelling ice reservoirs. This modified COSIPY model will be referred to as COSISTUPA model henceforth.

## Model configuration

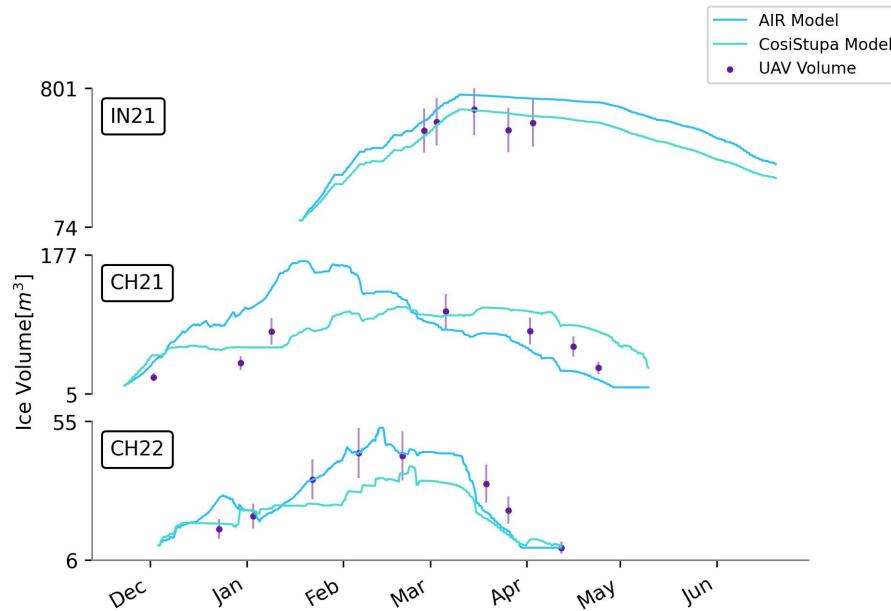
In this section, we describe all the adjustments to COSIPY modules necessary to convert it to the COSISTUPA model.

The cosy model input was extended to include discharge rate and cloudiness index measurements. Additionally, spray radius parameter was provided as input during model initialization. The model initialization of the ice dimensions were made identical to the AIR model.

Several parameterizations are available for estimating each of the surface processes in COSIPY. Most of the parameterizations used in the AIR model are among these options. But some parameterizations required minor modifications to be applicable for processes on a conical surface. Additionally, new parameterizations were required to estimate the conical shape evolution and model the freezing process due to the fountain discharge rate. To extend COSIPY into COSISTUPA, parametrizations of the following processes were modified:

- Fountain rain heat flux : The heat flux generated due to the difference in fountain water droplet temperature and surface temperature was introduced as a new energy balance component. This implementation is identical to that described in Sec. 3.2.2
- Turbulent flux scaling : The sensible and the latent heat fluxes were scaled by the  $\mu_{cone}$  factor introduced in Sec. 3.2.2
- Freezing process : Phase transition processes were introduced during time periods when the fountain discharge was active. These processes created new ice layers whenever the energy balance allowed it following the algorithm introduced in Sec. 3.2.2
- Conical shape evolution : The surface mass balance estimation was converted to the volume estimation through the methodology introduced in Sec. 3.2.1

Please note the above list of changes are not exhaustive and represent only the major modifications necessary to develop the COSISTUPA model.



**Fig. 3.3.**

### Model intercomparison

Execution time and multiprocessing workflow RMSE error User friendly and long-term maintenance

## 3.4 Suggestions

### 3.4.1 Model limitations

### 3.4.2 Spray radius

### 3.4.3 Further data acquisition

#### Meltwater quantities

The model validation set can be extended if daily AIR meltwater is measured. This dataset could serve as a superior way to validate the model compared to the drone surveys which are also used for determining the spray radius. However, the study site needs to satisfy two conditions in order to do this. First, the terrain of the

site needs to be waterproof and oriented so that most of the AIR runoff can be collected. Second, the chosen location should not have high wind speeds, otherwise a significant fraction of AIR wastewater would be dispersed in the air.

### **Fountain characteristics**

Fountain pressure

### **Drone flight analysis**

Identification of AIR boundary is subjective.

#### **3.4.4 Improvement of model algorithm**

##### **Initialisation of model**

##### **Better model parametrisations**

For turbulent fluxes

For other shapes/ shape evolution

Albedo variation due to fountain

# Habitat of ice reservoirs

AIRs cannot be built anywhere. They require sufficient water supply, specific topography and favourable weather conditions to amass a seasonal stock of ice. In this chapter, we attempt to quantify these requirements by examining the magnitude of these differences across the AIRs built in India and Switzerland. We also discuss strategies to judge the utility of AIRs in new locations.

## 4.1 Requirements for AIR construction

### 4.1.1 Water supply

The water source of an AIR could be either a spring or a stream. Springs are the ideal water source since they are easy to transport via pipelines to the construction site due to their relatively warm temperatures. Other water sources have higher risks of freezing events in the fountain pipeline.

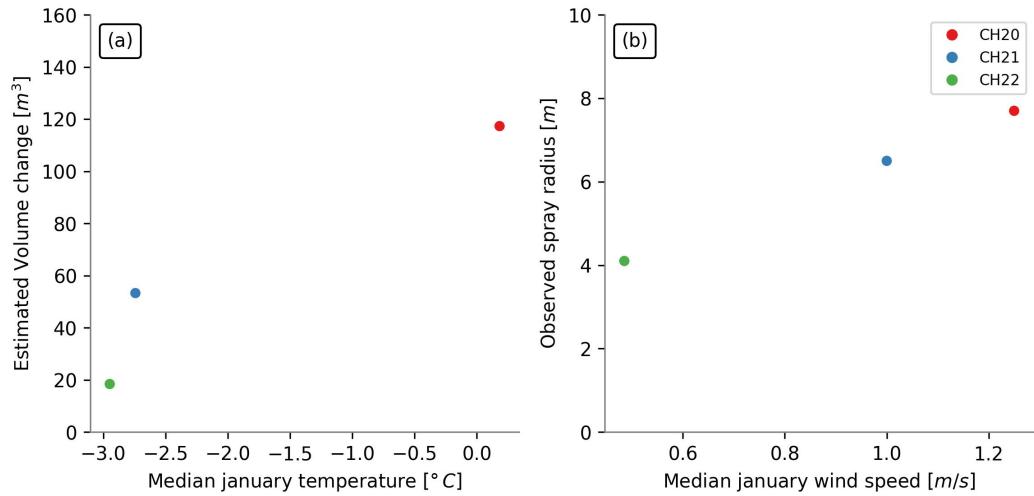
### 4.1.2 Weather conditions

AIRs prefer colder, drier and less-cloudy regions. Correspondingly, temperature, humidity and number of cloud-free days during the construction period can be used to rank different sites.

However, determining favourable construction locations is challenging due to the significant interannual, interregional and intraregional variations in weather conditions.

#### Interannual variability

Fig. 4.1 (a) shows the correlation of the ice volume changes with median temperature during January for AIRs built in the Swiss locations across three winters . Contrary to expectations, ice volume increases as median temperature increases.



**Fig. 4.1.:** (a) Estimated volume change and median temperature and (b) Observed spray radius and median wind speed during january for AIRs built across three winters.

This indicates that temperature is not the primary driver of interannual variations in the ice volume. Instead, we found that higher wind speeds drove the volume differences of these AIRs due to wind driven redistribution effects (see paper III). The influence of this process on the spray radius managed to generate AIRs six times bigger in spite of temperatures being  $3^{\circ}C$  warmer (see Fig. 4.1 (b)).

### Interregional variability

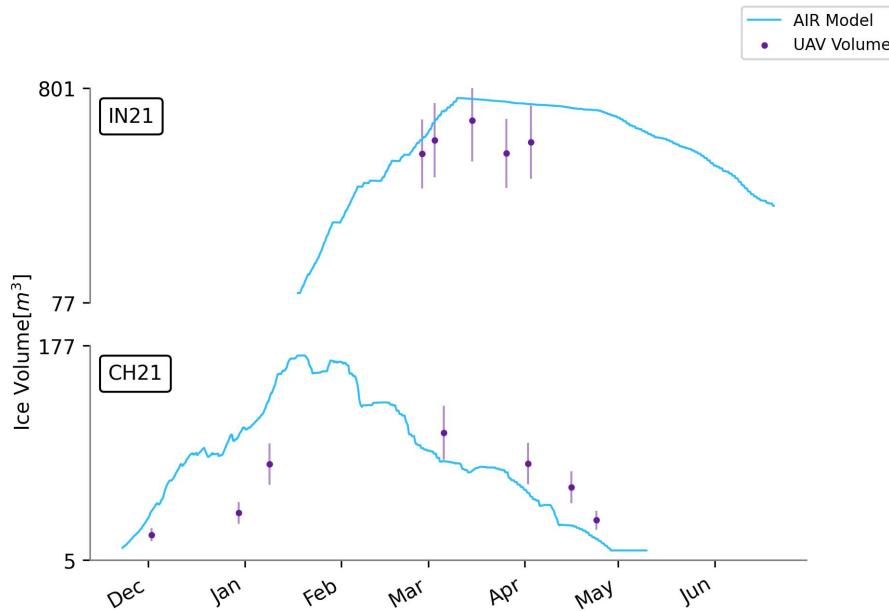
### Intraregional variability

#### 4.1.3 Topography

AIRs prefer shadowed valleys. This is because these landforms have lower sunshine hours which is the major driver of the melting rate of all the AIRs studied in this thesis.

#### 4.1.4 Checklist to identify AIR construction regions

Accordingly, we propose the following checklist to determine AIR site suitability. The values presented are determined based on past construction experiences and can be used as guidelines to avoid construction attempts in unfavourable sites.



**Fig. 4.2.:** AIRs show significant variation in volume evolution depending on the choice of construction location.

1. Minimum mean monthly temperature less than  $0^\circ C$ .
2. Water supply with mean discharge rate more than  $2l/min$ .
3. Terrain slope between water source and site greater than 10 m every km.

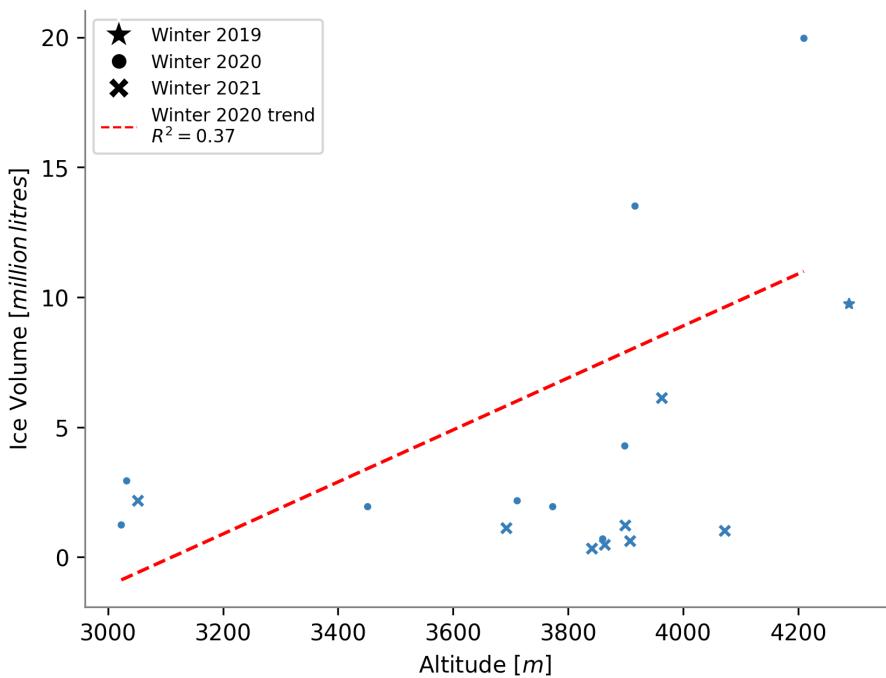
#### 4.1.5 Metrics to rank sites within the same region

Given a valley or a region satisfying the above requirements, further selection of sites around the particular water supply can be performed using the criterions below:

1. Water source temperature is higher.
2. Daylight hours are lower due to shadows.
3. Altitude is higher.

Such villages are expected to produce AIRs that supply daily meltwater in the order of thousands of litres for 2 months.

Different forms of AIRs show different sensitivities to each of these requirements. This is discussed in the next chapter.



**Fig. 4.3.:** (a) Estimated volume change and median temperature and (b) Observed spray radius and median wind speed during january for AIRs built across three winters.

# Technology of ice reservoirs

AIRs are a natural evolution of Ladakh's agricultural system. They can be related to traditional water harvesting technologies like the zing, which are small tanks where meltwater is collected through the use of an intricate network of channels. The mountain oases of the Hindu Kush and Karakoram ranges have similar irrigation networks [NB16].

## 5.1 Ice terraces

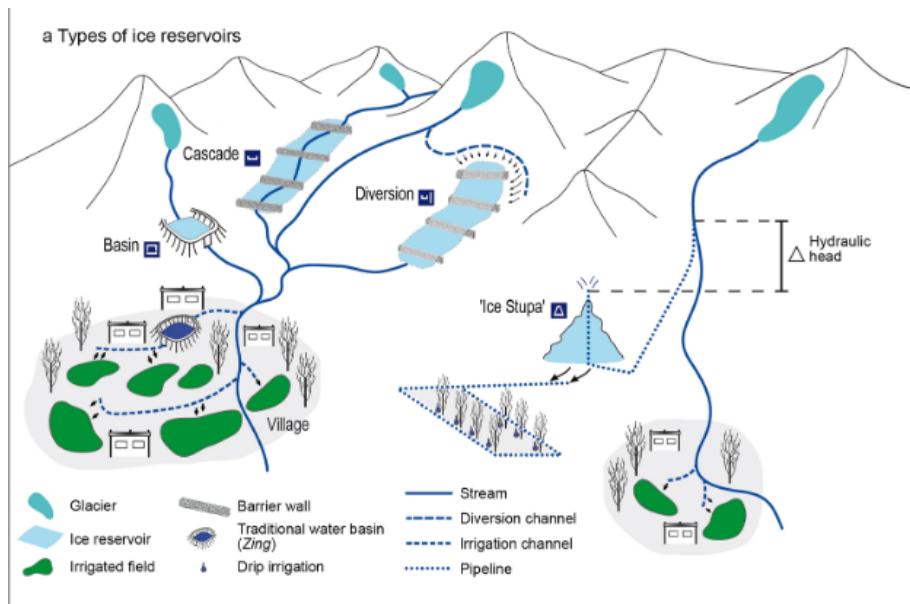
Ice terraces are the oldest form of AIRs [Nor09]. Usually situated below the glaciers at elevations where snowmelt starts end of March, these structures facilitate the freezing of stream water during winter at selected sites, usually shaded by surrounding mountains. Chewang Norphel, a well known engineer of the Leh Nutrition Project, introduced this practice to Ladakh in the 1980s and 1990s [Vin09a].

### 5.1.1 Construction strategy

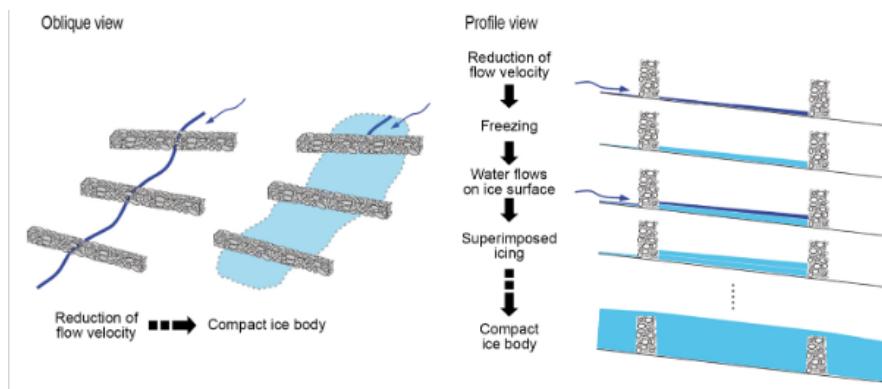
There are four distinct types of ice terraces with site-specific modifications as shown in Fig. 5.1: the first type is built as cascades on perennial streams. A series of loose rock walls in the river bed reduce reduces flow velocity, but still lets water pass through. Such cascades allow flowing water to freeze on exposed surfaces and form superimposed ice layers when temperatures drop.

The second type diverts water from streams with higher flow velocity to small side valleys, shaded by surrounding mountains. This design allows to integrate higher slope positions for additional ice formation. It consists of a series of partially cemented stone walls across the stream bed. Their dimensions are adjusted based on the valley topography. The water for the ice terrace is obtained through a long diversion channel.

The third type is a basin structure, resembling the traditional zing form of water storage, but located above the cultivated fields.



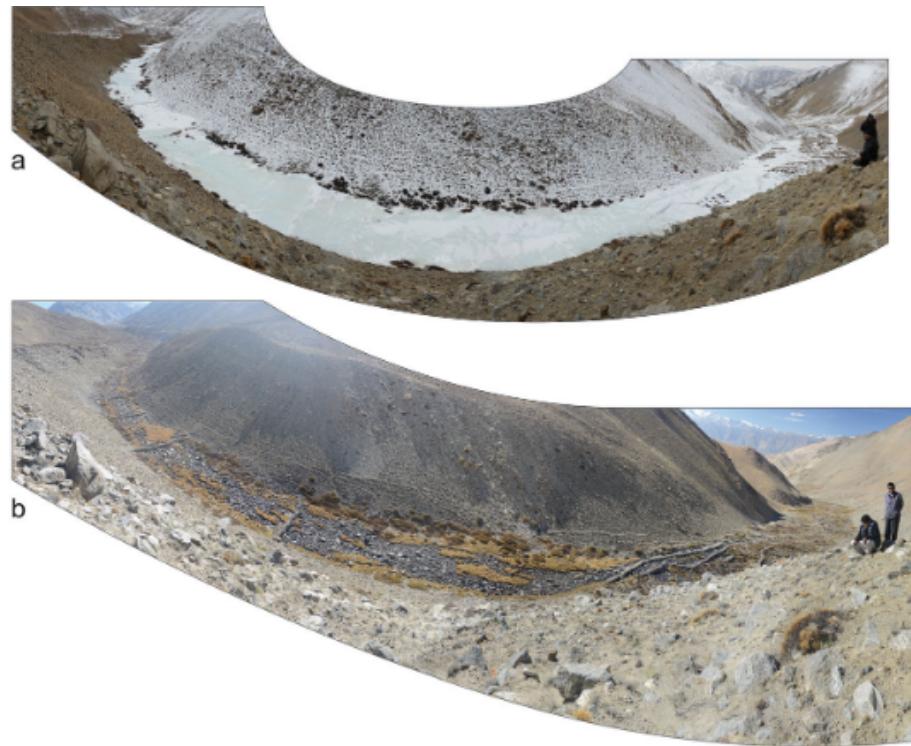
**Fig. 5.1.:** Adapted from: [Nüs+19]



**Fig. 5.2.:** The process of ice accumulation for ice terraces Adapted from: [Nüs+19]

### 5.1.2 Application

Over the past 30 years, 14 ice terraces have been constructed in central Ladakh, located in tributary valleys of the Indus. The oldest ice terrace was of the form cascades in Ladakh was built in 1987 at a favourable location between 4290 and 4640 m in Phuktse. However, according to oral history and Corona imagery from 1969, the first ice terrace of this design type are older than 50 years and can be found in Phuktse and Igoo. In February 2014, Phuktse built a successful cascade with an almost continuous stretch of ice (Fig.).



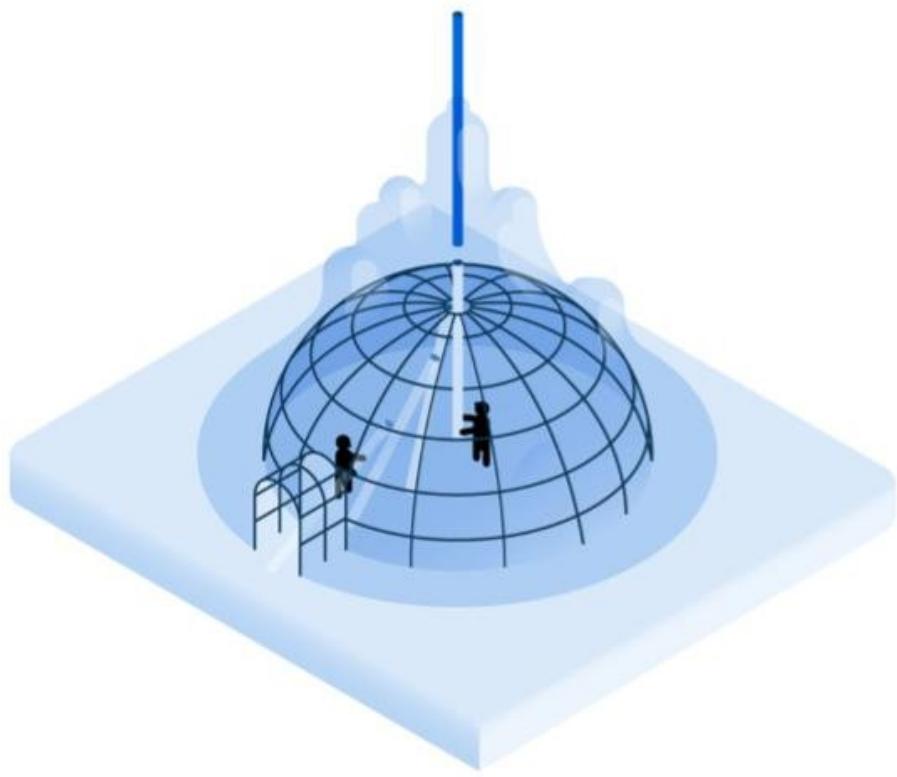
**Fig. 5.3.:** Ice terrace of Phuktse, viewpoint 4430 m. (a) February 2014 (b) October 2014  
Adapted from: [Nüs+19]

### 5.1.3 Drawbacks

However, the location requirements and the construction cost of ice terraces were prohibitive for widespread adoption.

## 5.2 Ice stupas

This prompted the invention of Ice stupas by Sonam Wangchuk in 2013 ???. Due to their shape, Ice stupas could be built adjacent to the irrigated plantations. It was also relatively cheaper. A typical Ice stupa just requires a fountain nozzle mounted on a supply pipeline. The water source is usually a spring or a glacial stream. Due to the altitude difference between the pipeline input and fountain output, water ejects from the fountain nozzle as droplets that eventually lose their energy and accumulate as ice. The fountain is manually activated during the winter nights and is raised, through addition of metal pipes, when significant ice accumulates below.



**Fig. 5.4.:** The process of ice accumulation for ice stupas. Diagram by: Francesco Muzzi

### 5.2.1 Construction strategy

A typical AIR (see Fig. ??) simply requires a fountain nozzle mounted on a supply pipeline. The water source is usually a high altitude lake or glacial stream. Due to the altitude difference between the pipeline input and fountain output, water ejects from the fountain nozzle as droplets which freeze under subzero winter conditions. The fountain is manually activated during winter nights. The fountain nozzle is raised through the addition of metal pipes when significant ice accumulates below. Typically, a dome of branches is constructed around the metal pipes so that pipe extensions can be done from within this dome. Threads, tree branches and fishing nets are used to guide and accelerate the ice formation.



**Fig. 5.5.:** Ice terrace of Phuktse, viewpoint 4430 m. (a) February 2014 (b) October 2014  
Adapted from: [Nüs+19]

### 5.2.2 Application

### 5.2.3 Drawbacks

## 5.3 Improvement of construction strategies through weather-sensitive automation systems

### 5.3.1 Perpetual ice reservoirs



# Heritage of ice reservoirs

This chapter provides conclusions based on research findings from data collected on AIRs in Switzerland and India, as well as discussion and recommendations for future research. This Chapter will review the purpose of the study, research questions, literature review, and findings of the study. It will then present conclusions, discussion of the conclusions, and recommendations for practice and for further research.

## 6.1 Summary

Cryosphere fed irrigation networks are completely dependant on the timely availability of meltwater from glaciers, snow and permafrost. With the accelerated decline of glaciers, these irrigation networks can no longer deliver adequate water to sustain agricultural output and take advantage of the complete growing season. As a consequence, some mountain villages have either been abandoned or lie on the brink of desertification [Gro15].

In the past few decades, artificial ice reservoir (AIR) technologies have provided much needed relief to these water-stressed communities. These strategies revolve around augmenting their glacial ice reservoirs with man-made ones that provide supplementary irrigation during the spring. In the context of the observed present and predicted global glacier shrinkage, the development of such water storage technologies is crucial to ensure continued sustenance of cryosphere-fed irrigation networks.

AIR observations and investigations date back to the mid-2000s [Tve07]. The vast majority have been published in the 2010s, mostly using qualitative methods. However, quantifications of their storage capacity differ widely amongst these publications. [Bag98; NT15; Nüs+19]. Because small-scale processes, complex feedbacks and non-linearities govern their evolution, modelling the volume evolution of ice stupas is only feasible if backed up with comprehensive input, calibration and validation datasets.

In response, we conducted measurement campaigns using drones, flowmeters and weather stations on almost a dozen AIRs across two locations (India and Switzerland), over four winters (2019, 2020, 2021 and 2022) and using two different construction methods (traditional and automated). Each dataset contained information on the meteorological conditions, fountain characteristics and AIR volume evolution.

With a special focus on ice stupas, a mass and energy balance model was developed and used as a tool to quantify the influence of meteorological conditions and fountain characteristics. The meteorological and fountain observations were used as model input; AIR volume observations were used to calibrate the model parameters and validate its ice volume estimations.

## 6.2 Conclusions

The main objective of this thesis was to improve our understanding about the response of AIRs to changes in their construction location and fountain characteristics. In a first experiment, the evolution of AIRs in Indian Himalayas and the Swiss Alps are compared. In a second experiment, the evolution of AIRs using different fountain scheduling strategies are compared. The model results show:

1. Volumes of ice stupas located in different regions may differ by an order of magnitude. The differences could be attributed to the accelerated sublimation process in colder and drier regions.
2. Water losses of ice stupas may be up to 80 % due to excessive use of water by traditional construction strategies. However, weather-sensitive fountain systems can produce ice stupas of similar volumes by using just one-tenth of the water supply.
3. Traditional construction strategies demand significant maintenance efforts since they are prone to freezing events in the fountain pipeline. However, automated construction strategies can prevent these events to make the construction process maintenance-free.

## 6.3 Discussion

## 6.4 Recommendations

## 6.5 Suggestions for future research

- Identification of favourable locations.
- Cosistupa model development.

## 6.6 Final thoughts



# Bibliography

- [Agg+21] Anubha Aggarwal, Holger Frey, Graham McDowell, et al. “Adaptation to Climate Change Induced Water Stress in Major Glaciated Mountain Regions”. In: *Climate and Development* 0.0 (Sept. 2021), pp. 1–13 (cit. on p. 1).
- [Bag98] Pallava Bagla. “Artificial Glaciers to Help Farmers”. In: *Science* 282.5389 (Oct. 1998), pp. 619–619 (cit. on pp. 1, 37).
- [BBC20] BBC News. *Bright Artificial Glacier in Naryn - BBC Kyrgyz*. <https://www.youtube.com/watch?v=Tpe> Feb. 2020 (cit. on p. 1).
- [Bru82] W. Brutsaert. *Evaporation into the Atmosphere. Theory, History and Application*. Kluwer Academic Publishers, 1982 (cit. on p. 17).
- [Bru75] W. Brutsaert. “On a Derivable Formula for Long-Wave Radiation from Clear Skies”. In: *Water Resources Research* 11.1-4 (1975), pp. 742–744 (cit. on p. 17).
- [CAS17] Carey Clouse, Naomi Anderson, and Taylor Shippling. “Ladakh’s Artificial Glaciers: Climate-Adaptive Design for Water Scarcity”. In: *Climate and Development* 9.5 (2017), pp. 428–438 (cit. on p. 1).
- [CP10] Kurt M. Cuffey and W. S. B. Paterson. *The Physics Of Glaciers*. Elsevier, 2010 (cit. on pp. 13, 15).
- [Far+15] Suhaib Bin Farhan, Yinsheng Zhang, Yingzhao Ma, Yanhong Guo, and Ning Ma. “Hydrological Regimes under the Conjunction of Westerly and Monsoon Climates: A Case Investigation in the Astore Basin, Northwestern Himalaya”. In: *Climate Dynamics* 44.11 (June 2015), pp. 3015–3032 (cit. on p. 1).
- [Gar92] J. R. Garratt. *The Atmospheric Boundary Layer*. Cambridge University Press, 1992 (cit. on p. 17).
- [Gro15] D. Grossman. *As Himalayan Glaciers Melt, Two Towns Face the Fallout*. <https://e360.yale.edu/feature/as-himalayan-glaciers-melt-two-towns-face-the-fallout> 2015 (cit. on p. 37).
- [Hua18] Jianhua Huang. “A Simple Accurate Formula for Calculating Saturation Vapor Pressure of Water and Ice”. In: *AMETSO* 57 (June 2018) (cit. on p. 17).
- [Imm+20] W. W. Immerzeel, A. F. Lutz, M. Andrade, et al. “Importance and Vulnerability of the World’s Water Towers”. In: *Nature* 577.7790 (Jan. 2020), pp. 364–369 (cit. on p. 1).
- [IPC19] IPCC. “Chapter 2: High Mountain Areas — Special Report on the Ocean and Cryosphere in a Changing Climate”. In: *IPCC Special Report on the Ocean and Cryosphere in a Changing Climate* (2019) (cit. on p. 1).

- [Met21] Meteoblue. *Climate Guttannen*. <https://www.meteoblue.com/en/weather/historyclimate/climatemodele> Feb. 2021 (cit. on p. 10).
- [MK15] Biswajit Mukhopadhyay and Asif Khan. “A Reevaluation of the Snowmelt and Glacial Melt in River Flows within Upper Indus Basin and Its Significance in a Changing Climate”. In: *Journal of Hydrology* 527 (Aug. 2015), pp. 119–132 (cit. on p. 1).
- [Nor09] Chewang Norphel. “Artificial Glacier: A High Altitude Cold Desert Water Conservation Technique”. In: Energy and Climate Change in cold regions of Asia: Proceedings of the Seminar, Ladakh, India (2009), pp. 62–64 (cit. on p. 31).
- [NT15] Chewang Norphel and Padma Tashi. “Snow Water Harvesting in the Cold Desert in Ladakh: An Introduction to Artificial Glacier”. In: *Mountain Hazards and Disaster Risk Reduction*. Ed. by Hari Krishna Nibanupudi and Rajib Shaw. Disaster Risk Reduction. Tokyo: Springer Japan, 2015, pp. 199–210 (cit. on pp. 1, 37).
- [Nüs+19] M. Nüsser, Juliane Dame, Benjamin Kraus, Ravi Baghel, and Susanne Schmidt. “Socio-Hydrology of Artificial Glaciers in Ladakh, India: Assessing Adaptive Strategies in a Changing Cryosphere”. In: *Regional Environmental Change* (2019) (cit. on pp. 1–3, 32, 33, 35, 37).
- [NB16] Marcus Nüsser and Ravi Baghel. “Local Knowledge and Global Concerns: Artificial Glaciers as a Focus of Environmental Knowledge and Development Interventions”. In: *Ethnic and Cultural Dimensions of Knowledge*. Ed. by Peter Meusburger, Tim Freytag, and Laura Suarsana. Knowledge and Space. Cham: Springer International Publishing, 2016, pp. 191–209 (cit. on pp. 1–3, 31).
- [NSD12] Marcus Nüsser, Susanne Schmidt, and Juliane Dame. “Irrigation and Development in the Upper Indus Basin: Characteristics and Recent Changes of a Socio-hydrological System in Central Ladakh, India”. In: *Mountain Research and Development* 32.1 (2012), pp. 51–61 (cit. on p. 10).
- [Oer+21] J. Oerlemans, S. Balasubramanian, C. Clavuot, and F. Keller. “Brief Communication: Growth and Decay of an Ice Stupa in Alpine Conditions – a Simple Model Driven by Energy-Flux Observations over a Glacier Surface”. In: *The Cryosphere* 15.6 (2021), pp. 3007–3012 (cit. on p. 18).
- [OK98] J. Oerlemans and W. H. Knap. “A 1 Year Record of Global Radiation and Albedo in the Ablation Zone of Morteratschgletscher, Switzerland”. In: *Journal of Glaciology* 44.147 (1998), pp. 231–238 (cit. on p. 15).
- [Pal22] Lisa Palmer. “Storing Frozen Water to Adapt to Climate Change”. In: *Nature Climate Change* 12.2 (Feb. 2022), pp. 115–117 (cit. on p. 1).
- [Reu21] Reuters. “Conservationists in Chile Aim to Freeze Water in Man-Made Glaciers”. In: *Reuters* (Oct. 2021) (cit. on p. 1).

- [SAS20] Tobias Sauter, Anselm Arndt, and Christoph Schneider. “COSIPY v1.3 – an Open-Source Coupled Snowpack and Ice Surface Energy and Mass Balance Model”. In: *Geoscientific Model Development* 13.11 (Nov. 2020), pp. 5645–5662 (cit. on p. 23).
- [Tve07] Ingvar Tveiten. “Glacier Growing - A Local Response to Water Scarcity in Baltistan and Gilgit, Pakistan”. PhD thesis. 2007 (cit. on pp. 1, 37).
- [Vin09a] Gaia Vince. “Glacier Man”. In: *Science* 326.5953 (Oct. 2009), pp. 659–661 (cit. on pp. 1, 31).
- [Vin09b] Stauffer Vincent. *Energy and Climate Change in Cold Regions of Asia*. Aug. 2009 (cit. on p. 1).
- [Wan20] Sonam Wangchuk. *Ice Stupa Competition*. <https://tribal.nic.in/IceStupa.aspx>. Dec. 2020 (cit. on p. 1).
- [Woo68] Harold M. Woolf. *On the Computation of Solar Elevation Angles and the Determination of Sunrise and Sunset Times*. National Aeronautics and Space Administration, 1968 (cit. on p. 16).



# List of Figures

1.1	Seasonal variation in the availability of irrigation water. The graph highlights the crucial role of AIRs in bridging the critical gap in water availability. Adapted from: [NB16] . . . . .	2
1.2	(a) Schematic overview of the position of artificial ice reservoirs. These constructions are located at altitudes between the glaciers and the irrigation networks in the cultivated areas. (b) Ice terraces at 3900 m, located above the village of Nang, Ladakh. The cascade is composed of a series of loose masonry walls ranging in height from 2 to 3 m, which help freeze water for storage. (c) Ice stupas at 3600 m, located above the village of Phyang, Ladakh. They are made using fountain systems. Adapted from: [NB16] . . . . .	3
3.1	Model schematic showing the workflow used in the model at every time step. . . . .	12
3.2	Shape variables of the AIR. $r_{cone}$ is the radius, $h_{cone}$ is the height, $j_{cone}$ is the thickness change and $s_{cone}$ is the slope of the ice cone. $r_F$ is the spray radius of the fountain. . . . .	14
3.3	. . . . .	25
4.1	(a) Estimated volume change and median temperature and (b) Observed spray radius and median wind speed during january for AIRs built across three winters. . . . .	28
4.2	AIRs show significant variation in volume evolution depending on the choice of construction location. . . . .	29
4.3	(a) Estimated volume change and median temperature and (b) Observed spray radius and median wind speed during january for AIRs built across three winters. . . . .	30
5.1	Adapted from: [Nüs+19] . . . . .	32
5.2	The process of ice accumulation for ice terraces Adapted from: [Nüs+19]	32
5.3	Ice terrace of Phuktse, viewpoint 4430 m. (a) February 2014 (b) October 2014 Adapted from: [Nüs+19] . . . . .	33

5.4	The process of ice accumulation for ice stupas. Diagram by: Francesco Muzzi	34
5.5	Ice terrace of Phuktse, viewpoint 4430 m. (a) February 2014 (b) October 2014 Adapted from: [Nüs+19]	35

# List of Tables

3.1	Summary of the weather observations for AIRs built during the repective study period. The weather measurements are shown using their mean ( $\mu$ ) and standard deviation ( $\sigma$ ) during the study period as $\mu \pm \sigma$ . . . . .	10
3.2	List of all the studied AIRs. The study period starts when the fountain was first switched on (denoted as Start Date) and ends when the respective AIR either melted or broke into several ice blocks (denoted as Expiry Date). . . . .	11
3.3	Assumptions for the parametrisation introduced to simplify the model. . . . .	23
A.1	This is a caption text. . . . .	51
A.2	This is a caption text. . . . .	52



## List of Listings



# A

## Example Appendix

And after the second paragraph follows the third paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

### A.1 Appendix Section 1

After this fourth paragraph, we start a new paragraph sequence. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Alpha	Beta	Gamma
0	1	2
3	4	5

**Tab. A.1.:** This is a caption text.

## A.2 Appendix Section 2

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Alpha	Beta	Gamma
0	1	2
3	4	5

**Tab. A.2.:** This is a caption text.

This is the second paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

## Colophon

This thesis was typeset with  $\text{\LaTeX}2\varepsilon$ . It uses the *Clean Thesis* style developed by Ricardo Langner. The design of the *Clean Thesis* style is inspired by user guide documents from Apple Inc.

Download the *Clean Thesis* style at <http://cleanthesis.der-ric.de/>.



# Declaration

You can put your declaration here, to declare that you have completed your work solely and only with the help of the references you mentioned.

*Fribourg, July 4, 2022*

---

Suryanarayanan  
Balasubramanian

