Turbulence and the Generation of Large Scale Magnetic Fields

Or what, if anything, would convince me that we need primordial magnetic fields to explain observations?

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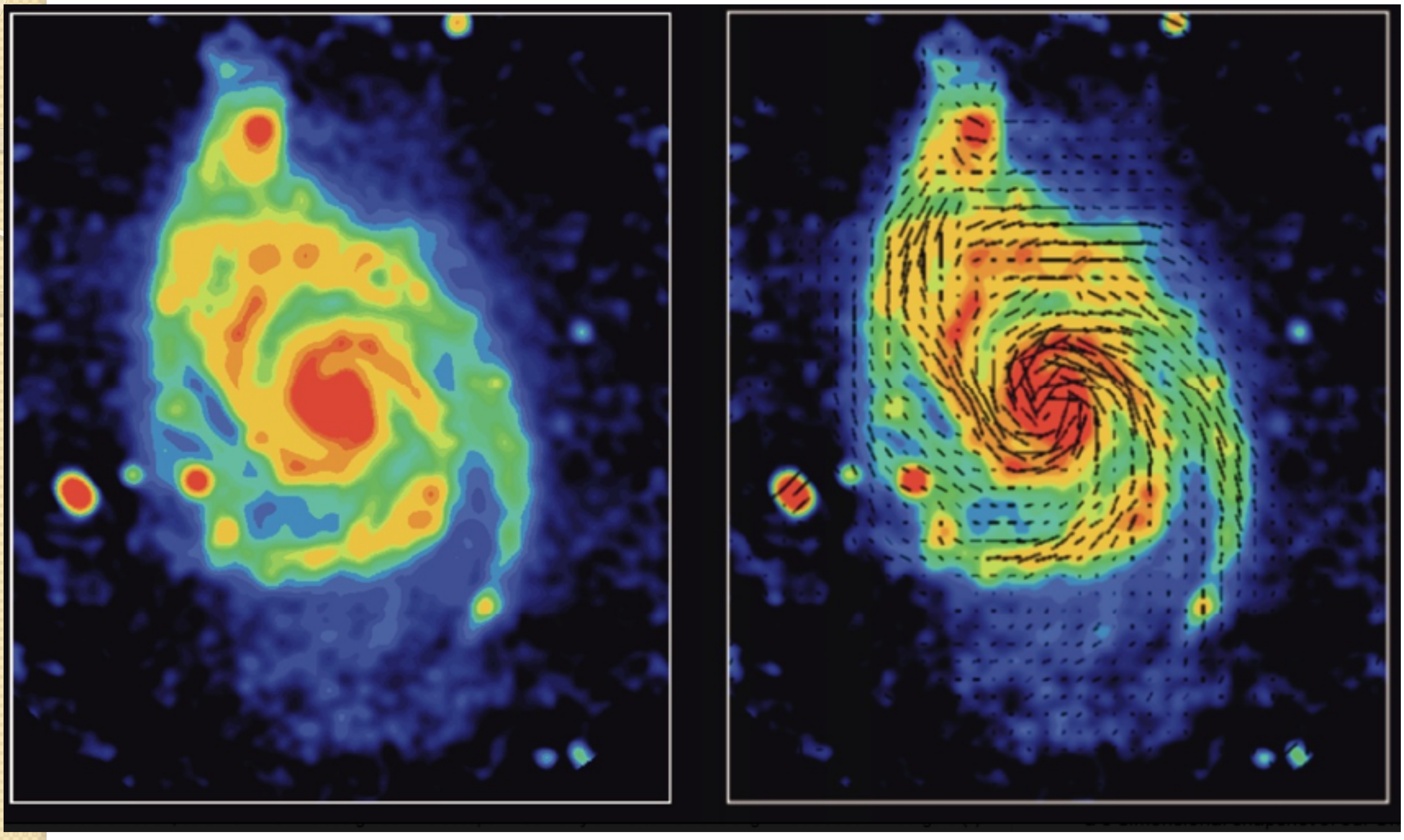

Diffuse radio emission with polarization vector - Beck and Horellou

Disk galaxies typically show large scale (somewhat) organized magnetic fields ~ several microgauss strength.

Such fields are present from early times at roughly comparable strength.

Cosmological seed fields are pathetically weak, and basically irrelevant (unless we invoke new physics)

Large Scale Dynamos

Consider the limit of small resistivity, and ignore plasma effects (a dense medium, or scales large enough that these effects are ignorable).

$$D_t \bar{B} = \nabla \times \overline{(u \times b)} + (\bar{B} \bullet \nabla) \bar{V}$$

α ---- Ω Dynamo

$$\overline{\langle u \times b \rangle} = \alpha_{ij} \bar{B}_j + \beta_{ijk} \partial_j \bar{B}_k + \ldots$$

This raises the question – how do we determine these matrices in a turbulent medium?

Take the time derivative, use the induction and force equations and multiply by the correlation time τ.

$$\alpha_{ij} \approx \varepsilon_{ikl}\left(\overline{u_k \partial_j u_l} - \overline{b_k \partial_j b_l}\right)\tau$$

Or

(current helicity-kinetic helicity) x correlation time

$$h_k \equiv \overline{u \bullet \nabla \times u}$$ ← Not conserved (imposed by environment)

$$h_j \equiv \overline{b \bullet j}$$ ← Related to the magnetic helicity $\vec{A} \bullet \vec{B}$

Magnetic Helicity Conservation

$$\partial_t H_B = -\nabla \bullet \left[H_B \vec{V} + \vec{B}(\Phi - \vec{A} \bullet \vec{V} \right]$$

$$H_B \equiv \vec{A} \bullet \vec{B}$$

H_B depends on our choice of gauge. (Current helicity does not.) Does this conservation law have any physical significance?

If we take $\vec{A} = \int \frac{\vec{j}\left(\vec{r}\,' + \vec{r}\right)}{4\pi r'} d^3 r'$ then the current helicity and the magnetic helicity are closely related.

$$\boxed{\overline{j \bullet b} \approx k^2 \overline{a \bullet b}}$$

The Dynamo and Magnetic Helicity

If we only consider h, the eddy scale contribution to the magnetic helicity, the conservation equation becomes:

$$\partial_t h + 2\overline{B} \bullet \left(\overline{u \times b}\right) = -\nabla \bullet \vec{j}_h$$

where

$$h \equiv \left\langle \vec{a} \cdot \vec{b} \right\rangle$$

The large scale dynamo and the transfer of magnetic helicity between scales are the same process. Attempts to drive a dynamo via kinetic helicity are self-limiting. (See for example, Gruzinov and Diamond 1994.)

To drive a dynamo we need $\vec{j}_h \neq 0$

The magnetic helicity driven dynamo

This suggests a fairly drastic reordering of causality for dynamos. Turbulence drives a magnetic helicity flux, which then determines the parallel component of the electromotive force. On dimensional grounds

$$j_h \sim \pm D_T \left\langle b^2 \right\rangle \Omega \tau \hat{z}$$

It can be estimated more precisely following the same procedure we used to get the electromotive force. If we assume isotropic turbulence with weak shear and rotation in a galactic disk then since

$$j_h = \left\langle \vec{a} \times \left(\vec{v} \times \vec{b} + \nabla \phi \right) \right\rangle + \text{ Explicit shear term}$$

- The magnetic helicity flux (with no large scale magnetic field) is

$$j_h \approx -\frac{19}{30}\left\langle v^2\right\rangle_1 \left\langle b^2\right\rangle_1 \Omega\tau^2 + \frac{1}{180}\frac{\left\langle b^2\right\rangle_1^2}{4\pi\rho}\Omega\tau^2$$

- It is negative and proportional to the turbulent eddy scale squared (actually smaller in spiral arms).
- The corresponding term in the dynamo equation will dominate over the kinetic helicity term in one eddy turn over time.

There is no kinematic dynamo regime for the galactic dynamo (and probably for any realistic astrophysical dynamo).

Can we see this effect in numerical simulations?

- Simulations of stellar dynamos show that the parallel electric field is not well correlated with the kinetic helicity, but is well correlated with the current helicity.
- Simulation of turbulence in a periodic box with imposed periodic large scale shear with Shapovalov.
- Similar, but in a shearing box and modulating the strength of turbulence (Cridland)

How will the magnetic field in a galactic disk evolve?

- The magnetic helicity h will accumulate in separate large scale regions, growing linearly with time. (That implies a linear growth in the current helicity.)
- The large scale magnetic field will evolve via the incoherent dynamo, i.e. the random addition of eddy scale electric fields will give a root N push to B, with a sign that varies every eddy turn over time. Large scale shear will produce a net growth proportional to $t^{3/2}$. (Vishniac and Brandenberg 1997)

- In less than one diffusion time, the growth of the large scale field reaches the point where it can couple to the accumulated magnetic helicity. The latter cascades to large scales, driving a dynamo with a growth rate that increases as root(t), i.e. super-exponentially.
- This continues until the dynamo actually soaks up all the accumulating magnetic helicity, at which point the growth becomes roughly linear (the electromotive force is inversely proportional to the large scale field).

- Eventually the field lines start to inhibit helicity transport as their energy density passes the turbulent energy density.
- Leading to saturation when the Alfven speed is comparable to the disk thickness times the shear. (This is roughly comparable to the equipartition level in our galaxy.)
- The total time for growth to saturation is sensitive to details, but is generally comparable to a few e-folding times in the exponential model of the dynamo or several hundred million years.

What about intergalactic space?

- We know that galaxy winds spread metals into the intergalactic medium. The disk is generating a preferred magnetic helicity current which is coherent over the whole disk – even if the magnetic field is not.
- This means that the medium outside the galaxy acquires a large scale structure of magnetic helicity, which ***can*** be reflected in the existence of a large scale field.

Conclusions (for dynamo theory):

- The dynamo process is driven by the magnetic helicity flux, which is determined by the local properties of the turbulence and the shear/rotation of the fluid.
- In general, the growth of the large scale field is not exponential. It is much faster.
- The saturation of the field is not pegged to the local rms turbulent speed, but depends on the height and shear of the system, but this is the same in a galactic disk.

- The strength of this field (or more realistically, the upper limit on its strength) is

$$B_{IGM} \sim B_{disk} \left(R_{disk} / L_{IGM} \right) \left(\lambda_{turb} / L_{IGM} \right) \left(\Omega t_{wind} \right)^{1/2}$$

- For a megaparsec scale zone of influence for a single spiral galaxy, this is slightly less than 10^{-10} Gauss.

Conclusions (for cosmology):

- Galactic disks acquire their magnetic fields early, within a few rotation times. Their structure may continue to evolve.
- Disk galaxies will pollute their environment with magnetic helicity, just as they pollute it with metals.
- An upper limit on megaparsec scale fields from this process is (after a billion years) roughly 10^{-10} to 10^{-11} Gauss from a single galaxy.