Notes: Preliminary results with the Vishniac flux

November 8, 2011

These notes contain some first results using the new Vishniac flux.

Basic Equations

The helicity evolution equation :

$$\frac{\partial \chi}{\partial t} = -2\,\overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\boldsymbol{B}} - 2\overline{\boldsymbol{j}\cdot\boldsymbol{b}} - \nabla\cdot\boldsymbol{F} \tag{1}$$

Using the following relations, already defined in SSS07, $\alpha_{\rm m} = \tau\,\chi/3\,l_0^2\rho$, $\overline{\boldsymbol{j}\cdot\boldsymbol{b}} \approx l_0^{-2}\,\overline{\boldsymbol{a}\cdot\boldsymbol{b}}$, $\eta_{\rm t} = \tau\,u^2/3$ and $B_{\rm eq}^2 = \rho\,u^2$ and rearranging equation (1) we have,

$$\frac{\partial \alpha_{\rm m}}{\partial t} = -\frac{2\,\eta_{\rm t}}{l_0^2}\left[\frac{\overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\boldsymbol{B}}}{B_{\rm eq}^2} + \frac{\alpha_{\rm m}}{R_{\rm m}}\right] - \frac{\eta_{\rm t}}{B_{\rm eq}^2\,l_0^2}\,\nabla\cdot\boldsymbol{F} \tag{2}$$

Using $\overline{\boldsymbol{\mathcal{E}}} = \alpha\overline{\boldsymbol{B}} - \eta_{\rm t}\overline{\boldsymbol{J}}$, we get

$$\frac{\partial \alpha_{\rm m}}{\partial t} = -\frac{2\,\eta_{\rm t}}{l_0^2}\left[\frac{\alpha\overline{B}^2}{B_{\rm eq}^2} - \eta_{\rm t}\frac{\overline{\boldsymbol{J}}\cdot\overline{\boldsymbol{B}}}{B_{\rm eq}^2} + \frac{\alpha_{\rm m}}{R_{\rm m}}\right] - \frac{\eta_{\rm t}}{B_{\rm eq}^2\,l_0^2}\,\nabla\cdot\boldsymbol{F} \tag{3}$$

In dimensionless form using $t = \tilde{t}\,h^2/\eta_{\rm t}$, $z = \tilde{z}\,h$, $\alpha = \alpha_0\tilde{\alpha}$ and $\overline{B} = B_{\rm eq}\tilde{\overline{B}}$ equation (3) can be expressed as,

$$\frac{\partial \alpha_{\rm m}}{\partial t} = -2\left(\frac{h}{l_0}\right)^2\left[\alpha\overline{B}^2 - \frac{\eta_{\rm t}}{\alpha_0\,h}\left(\overline{B}_\phi\frac{\partial\overline{B}_r}{\partial z} - \overline{B}_r\frac{\partial\overline{B}_\phi}{\partial z}\right) + \frac{\alpha_{\rm m}}{R_{\rm m}}\right] - \frac{h^2}{\alpha_0\,l_0^2\,B_{\rm eq}^2}\frac{\partial F_{\rm z}}{\partial z} \tag{4}$$

Vishniac motivates a contribution to the flux of the form

$$j_h \approx D_T\,\overline{b^2}\,\Omega\,\tau \tag{5}$$

arising from the fact that in an inhomogeneous environment, the properties of turbulence will vary with location. (To show analytically the exact form of any such contribution)

We solve the z-dependent galactic dynamo equations with the Vishniac flux (z-component) of the form (see Vishniac's Chandra conference talk)

$$F_z = \frac{1}{2}\,u^2\,\tau^2\,G\,\left\langle b^2\right\rangle \tag{6}$$

Here u is the turbulent velocity, b, the small-scale magnetic field, τ is the relaxation time of the turbulence and G is the shear. Therefore,

$$\frac{\partial F_z}{\partial z} = \frac{1}{2}\,\tau^2\, G\,\left\langle b^2\right\rangle\,\frac{\partial u^2}{\partial z} \tag{7}$$

In dimensionless units where $z = \tilde{z}\, h$ and dropping the $\tilde{}$ notation,

$$\frac{\partial F_z}{\partial z} = \frac{1}{2}\,\tau^2\, G\,\left\langle b^2\right\rangle\,\frac{u_0^2}{h}\,\frac{\partial u^2}{\partial z} \tag{8}$$

Now for a choice of u of the form $u = \exp(-z^2/2)$,

$$\frac{\partial F_z}{\partial z} = -\frac{G\,\tau^2\, u_0^2}{h}\,\left\langle b^2\right\rangle\, z\exp(-z^2/2) \tag{9}$$

Substituting equation (9) in the equation (4) and rearranging, we get,

$$\frac{\partial \alpha_{\rm m}}{\partial t} = -2\left(\frac{h}{l_0}\right)^2\left[\alpha\overline{B}^2 - \frac{1}{R_\alpha}\left(\overline{B}_\phi\frac{\partial \overline{B}_r}{\partial z} - \overline{B}_r\frac{\partial \overline{B}_\phi}{\partial z}\right) + \frac{\alpha_{\rm m}}{R_{\rm m}}\right] + \left(\frac{R_\omega}{R_\alpha}\right)\, f\, z\exp(-z^2) \tag{10}$$

where we assume that the small-scale energy is a certain fraction of the equipartition value, i.e., $\langle \boldsymbol{b}^2\rangle = f\, B_{\rm eq}^2$. In the runs, we vary the parameter f.

Explicit evaluation of the coefficient of the last term of equation (4) is as follows :

$$C = \frac{h^2}{\alpha_0\, l_0^2\, B_{\rm eq}^2}\,\frac{G\,\tau^2\, u_0^2}{h}\,\left\langle b^2\right\rangle = \frac{9\,h\,G\,\eta_{\rm t}^2}{\alpha_0 l_0^2\, u_0^2}\, f = 9\,\left(\frac{R_\omega}{R_\alpha}\right)\, f\,\left(\frac{\eta_{\rm t}^2}{l_0^2\, u_0^2}\right) = \left(\frac{R_\omega}{R_\alpha}\right)\, f \tag{11}$$

using $\eta_{\rm t}^2 = \tau^2\, u_0^4/9$.

Equation (10) solved along with the following :

$$\frac{\partial \overline{B}_r}{\partial t} = -R_\alpha\frac{\partial}{\partial z}(\alpha\,\overline{B}_\phi) + \frac{\partial^2 \overline{B}_r}{\partial z^2} \tag{12}$$

$$\frac{\partial \overline{B}_\phi}{\partial t} = R_\omega\,\overline{B}_r + \frac{\partial^2 \overline{B}_\phi}{\partial z^2} \tag{13}$$

Numerical experiment

- One dimensional z-dependent $\alpha\omega$-mean-field equations.
- No kinetic helicity (except in one case) and no advective flux.
- Solar neighborhood values used, Ra = 1.0 in all runs and R_ω is varied.
- Initial random seed fields of $B_0 = 10^{-5}$ used throughout.
- Because of 'sign' uncertainty, we check both positive and negative signs of the flux.
- Resolutions varied from 64 to 128 grid points.

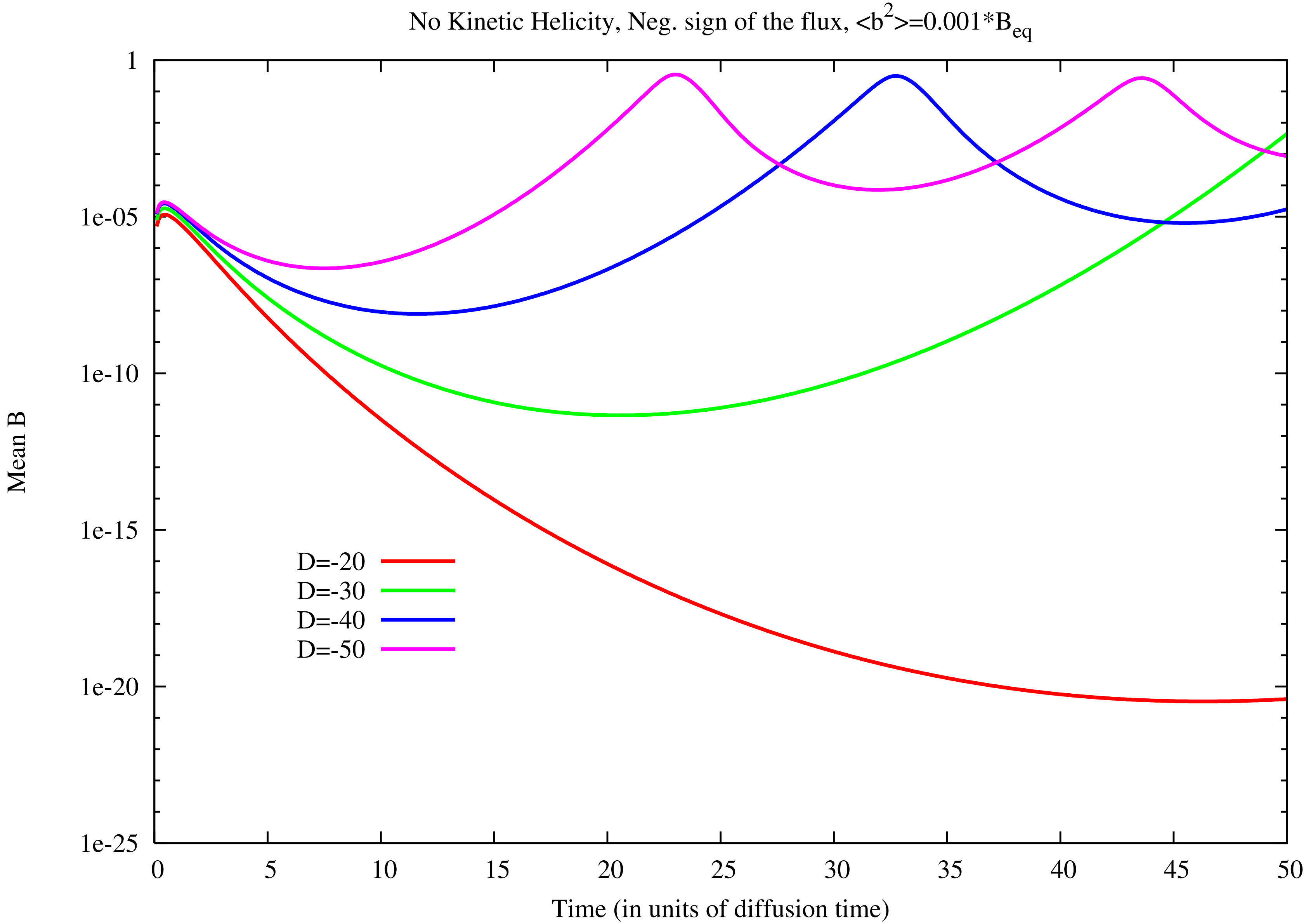

Figure 1: Time evolution for different dynamo numbers for runs with zero α_k, negative sign of the flux and $< b^2 >= 10^{-3} B_{eq}^2$

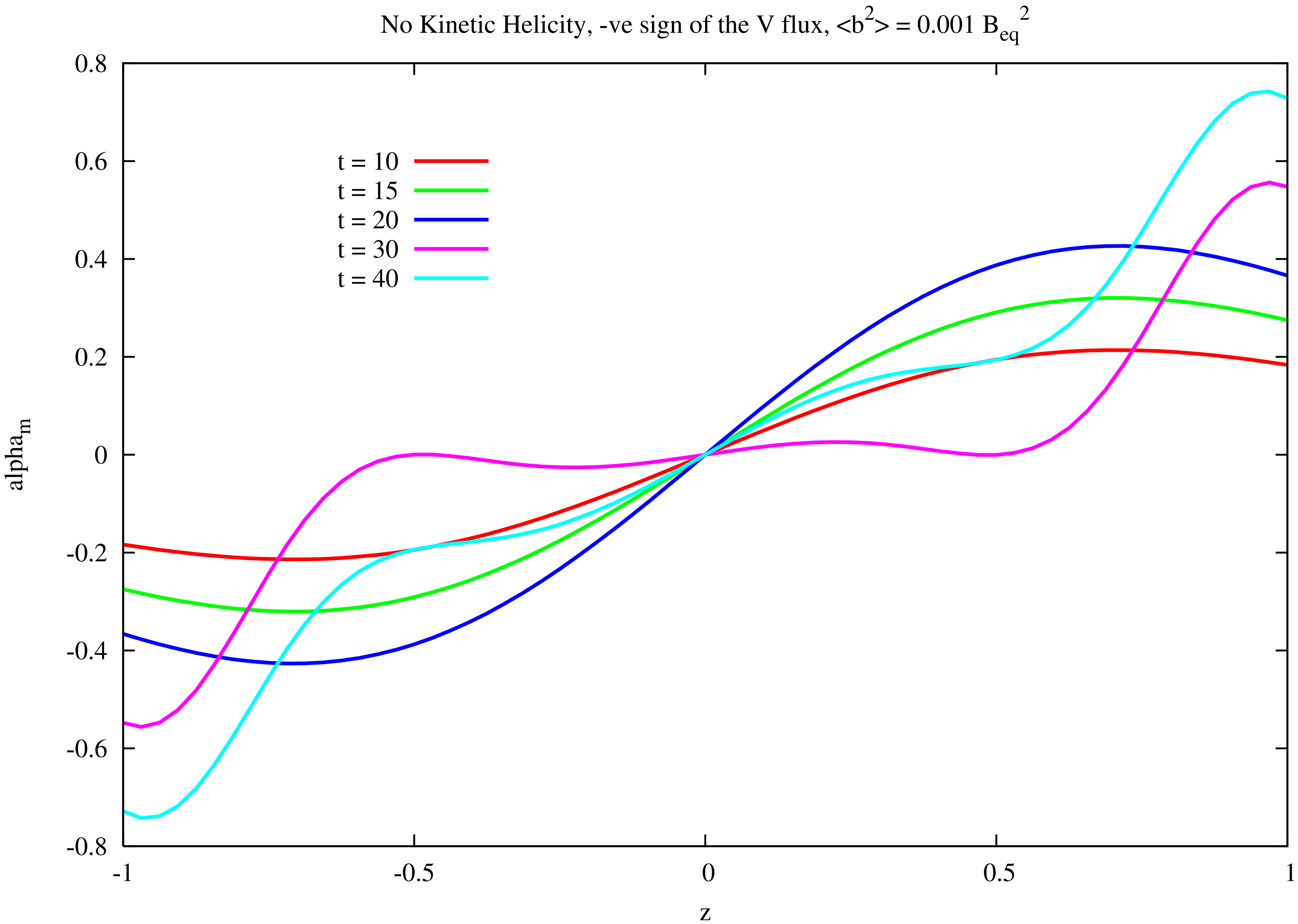

Figure 2: Profile of $\alpha_{\rm m}$ at different times for $D = -50$ for a run with zero $\alpha_{\rm k}$ and negative sign of the flux.

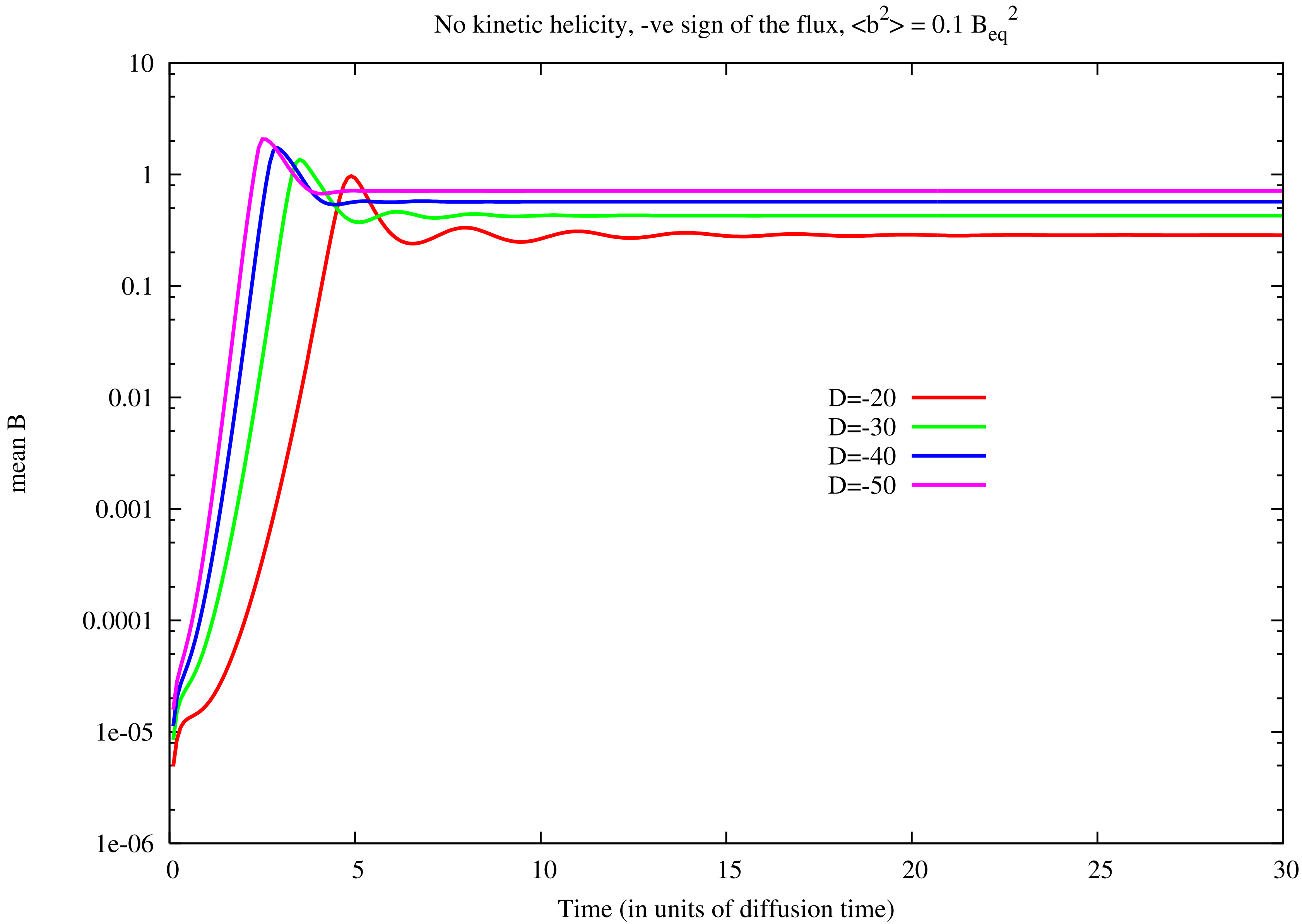

Figure 3: Time evolution for different dynamo numbers for runs with zero α_k, negative sign of the flux and $< b^2 >= 0.1B_{eq}^2$.

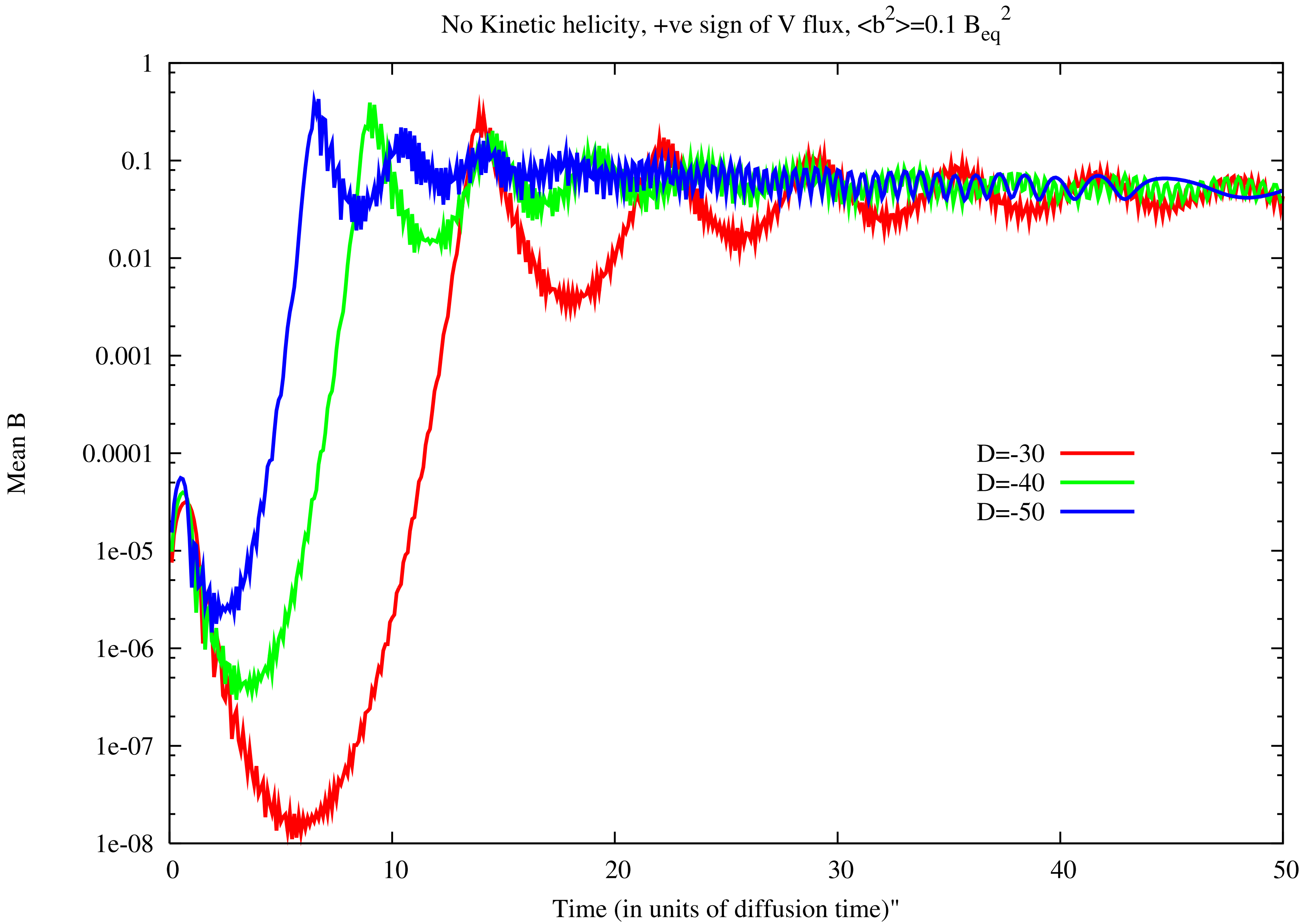

Figure 4: Time evolution for different dynamo numbers for runs with zero α_k, positive sign of the flux and $< b^2 >= 10^{-1} B_{eq}^2$

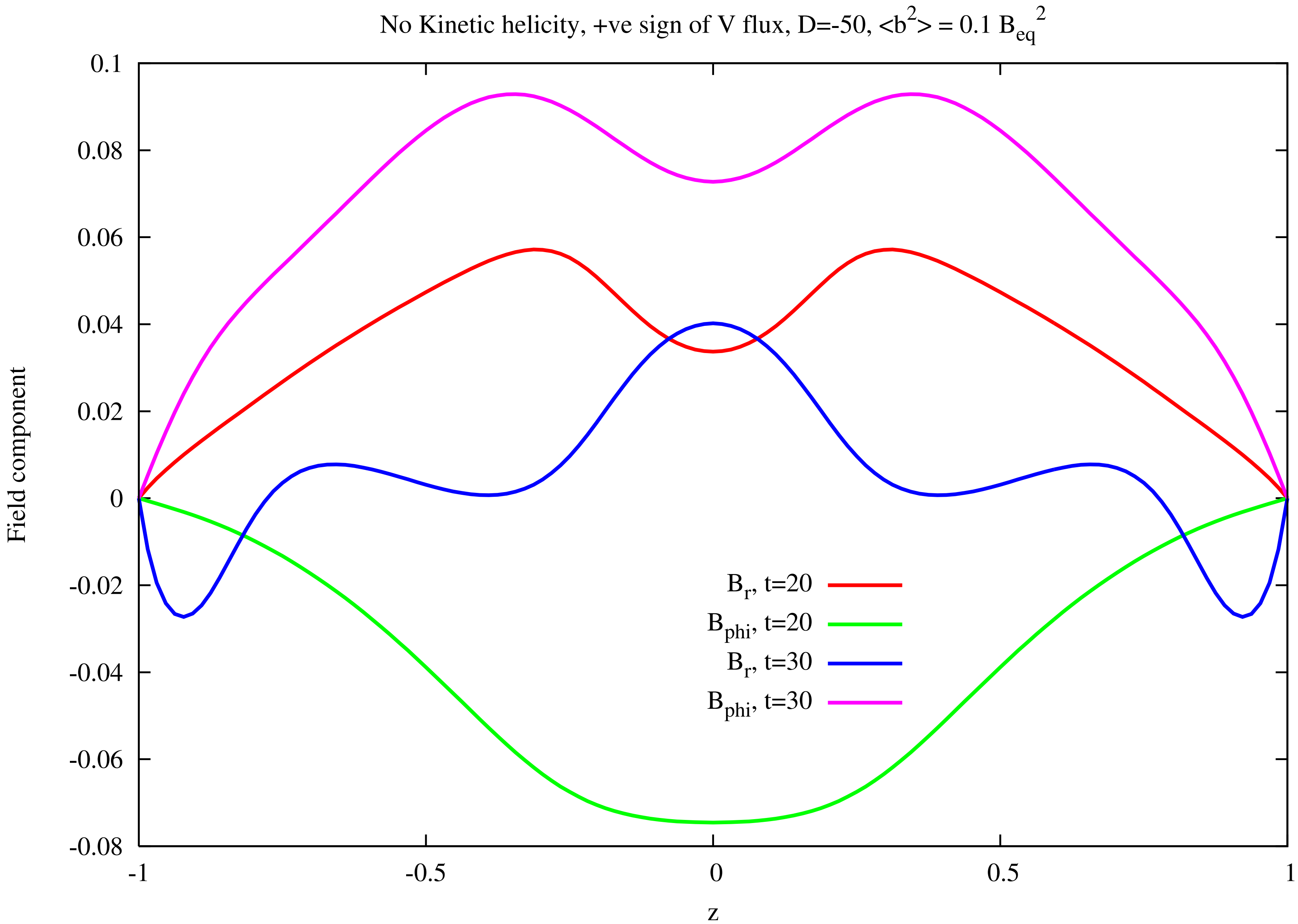

Figure 5: Profile of B_r and B_ϕ at two different times for a run with $D = -50$, zero $\alpha_{\rm k}$, positive sign of the flux and $< b^2 >= 10^{-1} B_{eq}^2$

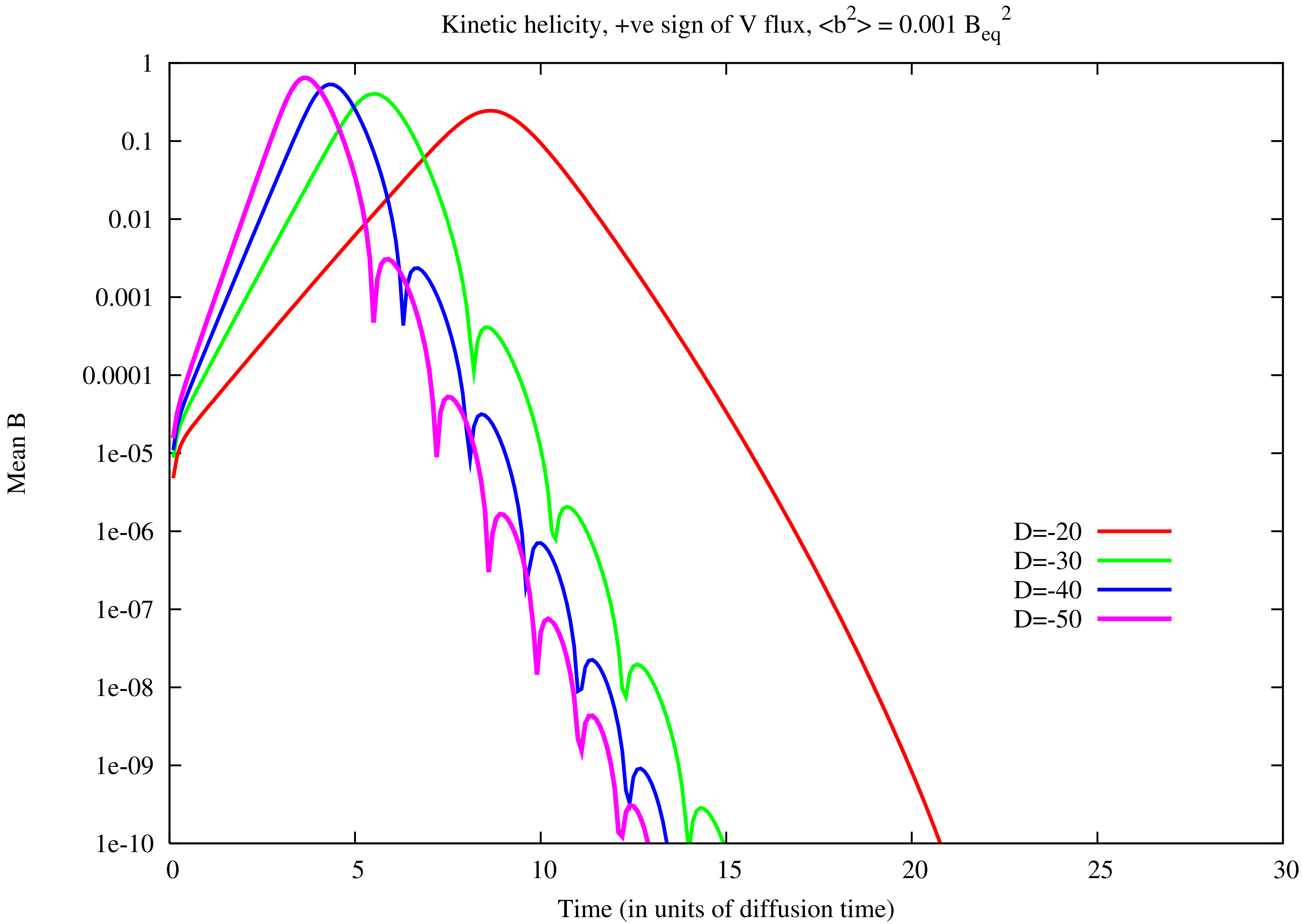

Figure 6: Time evolution for different dynamo numbers for runs with $\alpha_{\rm k}$, positive sign of the flux and $< b^2 >= 10^{-3} B_{eq}^2$