The Physics of Large Scale Magnetic Fields in Astrophysical Objects

With some comments about S. Chandrasekhar's role in scientific publishing.

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The Astrophysical Journal 1952-1971

Subrahmanyan Chandrasekhar served as the sole editor during this period and oversaw the establishment of the Letters (1967). This period covers the growth of the ApJ from the in-house journal of Yerkes Observatory to a leading international publication.

The incident of J.H. Waddell III, covered in the biography written by Professor Wali, illustrates both his high standards and his compassion. (ApJ 152, 577, 1968)

- Establishment of high scientific standards for The Astrophysical Journal
- American Astronomical Society acquisition of the ApJ (nonprofit, noncommercial) in 1971.
- Legal separation of the journal from the University of Chicago

Dynamos

Consider the limit of small resistivity, and ignore plasma effects (a dense medium, or scales large enough that these effects are ignorable).

$$\partial_t \vec{B} = \nabla \times \left(\vec{V} \times \vec{B} - \eta \vec{J} \right)$$

$$D_t \bar{B} = \nabla \times \overline{\left(u \times b \right)} + \left(\bar{B} \bullet \nabla \right) \bar{V}$$

α ---- Ω Dynamo

$$\overline{\langle u \times b \rangle} = \alpha_{ij} \bar{B}_j + \beta_{ijk} \partial_j \bar{B}_k + \ldots$$

This raises the question – how do we determine these matrices in a turbulent medium?

Take the time derivative, use the induction and force equations and multiply by the correlation time τ .

$$\alpha_{ij} \approx \varepsilon_{ikl}\left(\overline{u_k \partial_j u_l} - \overline{b_k \partial_j b_l}\right)\tau$$

Or

(current helicity-kinetic helicity) x correlation time

$$h_k \equiv \overline{u \bullet \nabla \times u}$$

$$h_j \equiv \overline{b \bullet j}$$

Magnetic Helicity Conservation

$$\partial_t H_B = -\nabla \bullet \left[H_B \vec{V} + \vec{B}(\Phi - \vec{A} \bullet \vec{V} \right]$$

$$H_B \equiv \vec{A} \bullet \vec{B}$$

H_B depends on our choice of gauge. Does this conservation law have any physical significance?

If we take $\vec{A} = \int \frac{\vec{j}\left(\vec{r}\,' + \vec{r}\right)}{4\pi r'} d^3 r'$ then the current helicity and the magnetic helicity are closely related.

$$\boxed{\overline{j \bullet b} \approx k^2 \overline{a \bullet b}}$$

"α" suppression

If we only consider h, the eddy scale contribution to the magnetic helicity, the conservation equation becomes:

$$\partial_t h + 2\bar{B} \bullet \left(\overline{u \times b}\right) = -\nabla \bullet \vec{j}_h$$

If the RHS is zero, then the electromotive force drives an accumulation of h, which turns off the electromotive force. **This is not really α suppression**. This is suppression of the electromotive force, regardless of its origin. (Gruzinov and Diamond)

To drive a dynamo beyond this point we need $\vec{j}_h \neq 0$

Vishniac, Cho 2001

This suggests a fairly drastic reordering of causality for dynamos, at least for fields strong enough that the magnetic helicity on small scales becomes saturated.

The turbulence drives a magnetic helicity flux, which then determines the parallel component of the electromotive force. On dimensional grounds

$$j_h \sim D_{Turb} B^2 (\Omega\tau)$$

aligned with the spin of the system. It can be estimated more precisely following the same procedure we used to get the electromotive force.

In the nonlinear limit

$$\left(\overline{u \times b}\right)_{\parallel} = \frac{-1}{2B} \nabla \bullet \vec{j}_h$$

This gives an electromotive force proportional to the derivative of the magnetic field, i.e. a "β - Ω" dynamo.

Predicted features of this dynamo:

1. The magnetic field has to exceed the rms turbulent velocity by about $(\Omega\ \tau)^{-1}$
2. The growth rate will be roughly $\Gamma \sim \frac{u_{rms}}{L}\Omega\tau$
3. Saturation due to the stiffness of the field lines once the field energy exceeds the turbulent energy density. Suggests limit around

$$B_{sat} \sim L\Omega$$

4. Works fine in a periodic box. No ejection of h necessary.
5. Electromotive force will depend on $h_j B$

How (what) did we do? (Shapovalov & Vishniac, submitted)

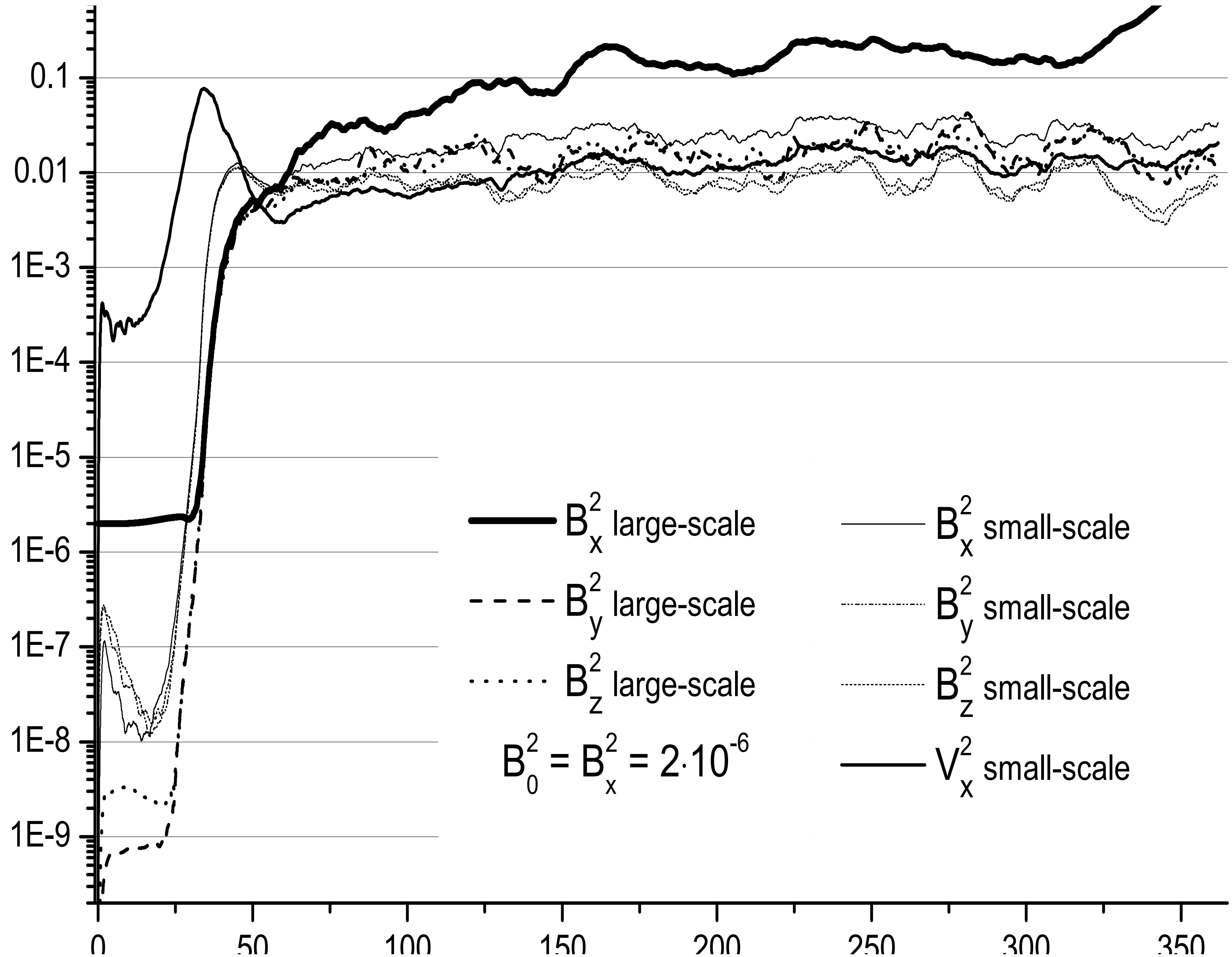
- Periodic box simulations (256 and 512 cubed)
- Explicit viscosity and resistivity, tuned to be just above the level of the grid effects
- Large scale sinusoidal forcing to create periodic shear at $k_x=1$
- Small scale forcing (varied, but typically k~25) with a typical eddy turn over rate close to the large scale shear. (Non-helical forcing)
- Small scale turbulence was anisotropic.

What did we see?

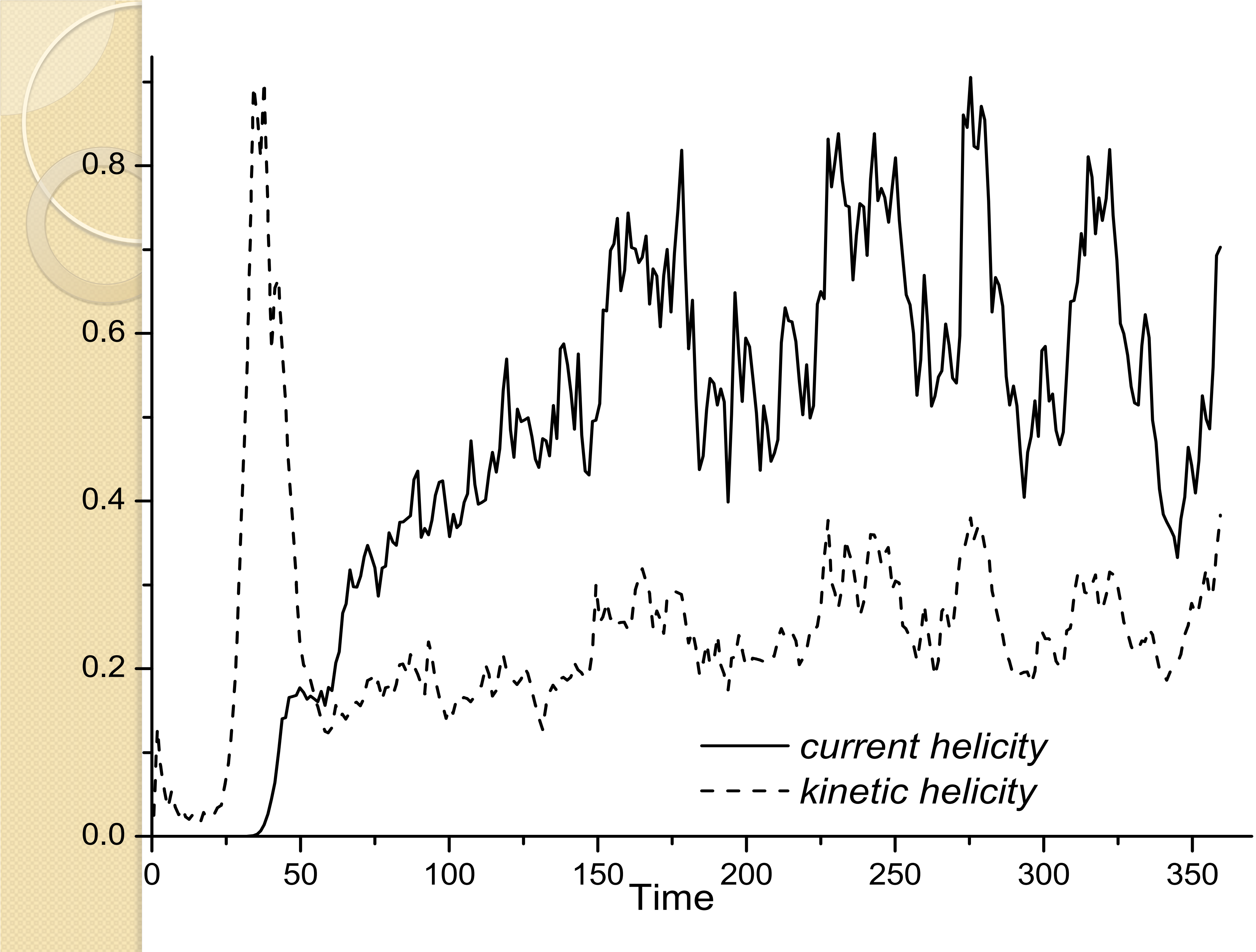
- Good (enough) magnetic helicity conservation. Dissipative losses were a bit more than an order of magnitude lower than all the other terms in the conservation equation including the div(j) term.
- Sufficiently anisotropic small scale turbulence would produce a strong dynamo.
- This was not a particularly good MHD turbulence simulation. Not enough dynamic range on small scales, but the flow was chaotic. (Doubling the resolution produced similar results, i.e. different, but not systematically different.)

What did we see?

- The large scale field was not steady, but moved in space and in intensity within the box.
- The kinetic helicity showed a strong peak early on in our "typical" case, but relaxed thereafter to a fraction of the current helicity.
- The field strength saturated close to the level of the shearing velocity and far above the level of the turbulence.
- The current helicity x the large scale magnetic field was strongly correlated with the electromotive force after several eddy turn over times.



0.1
0.01
1E-3
1E-4
1E-5
1E-6
1E-7
1E-8
1E-9
0
50
100
150
200
250
300
350
B_x^2 large-scale
B_y^2 large-scale
B_z^2 large-scale
$B_0^2 = B_x^2 = 2\cdot10^{-6}$
B_x^2 small-scale
B_y^2 small-scale
B_z^2 small-scale
V_x^2 small-scale



0.0
0.2
0.4
0.6
0.8
0
50
100
150
200
250
300
350
Time
current helicity
kinetic helicity

Do we really need a large scale field?

In an inhomogeneous environment, the properties of the turbulence will vary with location. We will get a nonzero divergence even for

$$j_h \sim D_T \overline{b^2} \Omega \tau$$

Is there such a term? Go back to analytic expression for the magnetic helicity flux, assume no large scale field, except for the velocity field, and take the time derivative and multiply by the turbulent correlation time. We recover this term.

How will a realistic inhomogeneous system evolve?

- The magnetic helicity h will accumulate in separate large scale regions, growing linearly with time. (That implies a linear growth in the current helicity.)
- The large scale magnetic field will evolve via the incoherent dynamo, i.e. the random addition of eddy scale electric fields will give a root N push to B, with a sign that varies every eddy turn over time. Large scale shear will produce a net growth proportional to $t^{3/2}$. (Vishniac and Brandenberg 1997)

- The current helicity will dominate the kinetic helicity **induced by the environment** after roughly one eddy turn over time.
- In less than one diffusion time, the growth of the large scale field reaches the point where it can couple to the accumulated magnetic helicity. The latter cascades to large scales, driving a dynamo with a growth rate that increases as root(t), i.e. super-exponentially.
- This continues until the dynamo actually soaks up all the accumulating magnetic helicity, at which point the growth becomes roughly linear (the electromotive force is inversely proportional to the large scale field).

- As before the field lines start to inhibit helicity transport as their energy density passes the turbulent energy density
- Leading to a saturation much like the earlier case.
- The total time for growth to saturation varies, but is generally comparable to a few e-folding times in the exponential model of the dynamo.

Applications?

- In accretion disks with MRI driven turbulence, the large and small scale fields are comparable, and similar to the rms turbulent velocity. There is no distinction between using small and large scale fields to drive the magnetic helicity flux. We can recover a reasonable model for the dynamo if we consider the tendency of the eddies to flatten in the absence of vertical structure.
- Galaxies will grow their magnetic fields in a couple of rotations.

Conclusions:

- The dynamo process can be driven by the magnetic helicity flux, which is determined by the local properties of the turbulence and the shear/rotation of the fluid.
- In general, the growth of the large scale field is not exponential. It is much faster.
- The saturation of the field is not pegged to the local rms turbulent speed, but depends on the height and shear of the system.