Role of the Vishniac Magnetic Helicity Flux in Mean-field Galactic Dynamos

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Introduction

MAGNETIC FIELDS IN GALAXIES

- How does large scale fields originate and survive in galaxies?
  - **Dynamo processes**

- What is a dynamo?
  - Spontaneously amplifies or sustains magnetic fields by converting kinetic energy to magnetic energy.

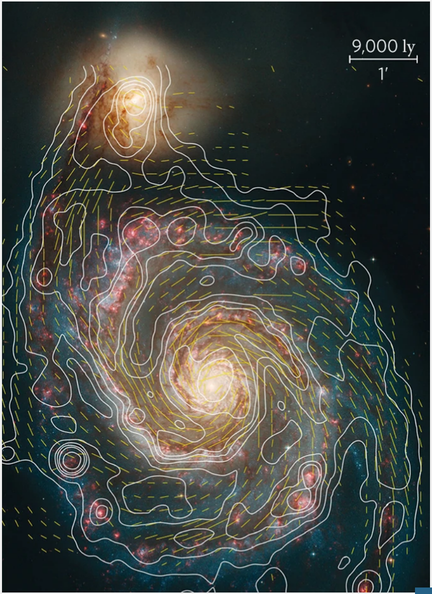


Image credits: Rainer Beck (MPIfR Bonn) and Andrew Fletcher (University of Newcastle)[4]

HOW TO STUDY THESE PROCESSES?

1. **MHD simulations** are an effective strategy for studying dynamo processes.

2. Their **limited dynamical range** hampers capturing the full complexity of dynamo processes.

3. To address this, a **combination of analytical methods and numerical simulations** is used to model dynamos more effectively.

MEAN FIELD DYNAMO THEORY

Magnetic Field Evolution and Mean-Field Theory

- **Induction Equation:**

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

- **Reynolds Averaging:**

$$\mathbf{U} = \overline{\mathbf{U}} + \mathbf{u}, \quad \mathbf{B} = \overline{\mathbf{B}} + \mathbf{b}$$

- **The mean-field induction equation:**

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times \left(\overline{\mathbf{U}} \times \overline{\mathbf{B}} + \mathcal{E} - \eta \nabla \times \overline{\mathbf{B}}\right)$$

- **Mean Electromotive Force (EMF):**

$$\mathcal{E} = \overline{\mathbf{u} \times \mathbf{b}}$$

$$\mathcal{E} = \alpha \overline{\mathbf{B}} - \eta \nabla \times \overline{\mathbf{B}}$$

The α effect

- Fluid motions **stretch and twist** the toroidal field, leading to the amplification of the poloidal field.
- Kinetic helicity, $\overline{\mathbf{u} \cdot (\nabla \times \mathbf{u})} \neq 0$
- Magnetic helicity: $\overline{\mathbf{b} \cdot (\nabla \times \mathbf{b})}$

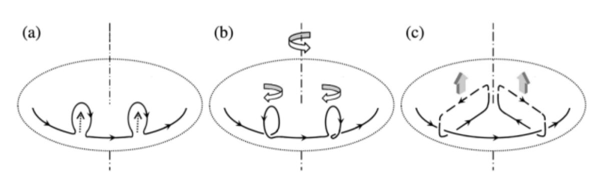


Image Source: P464 class notes.

Ω Effect

- Differential rotation stretches the poloidal field, amplifying the toroidal field.
- Shear causes the field strength to increase linearly over time.

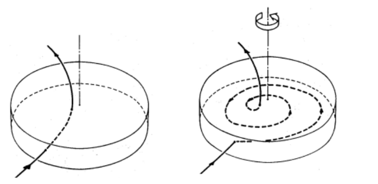


Image Sources: P464 class notes.

Combination of Effects

- The Ω-effect: poloidal fields into toroidal.
- The α-effect regenerates poloidal fields from toroidal fields.
- A positive feedback loop, the $\alpha\Omega$ dynamo.

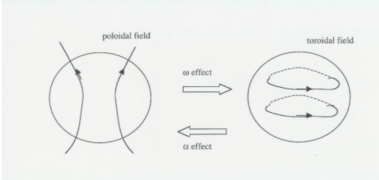


Image Sources: Caltech GE131 Notes.

Magnetic α-Effect and Dynamo saturation

$$\alpha = \alpha_k + \alpha_m; \quad \alpha_k = -\frac{\tau}{3}\overline{\mathbf{u}\cdot(\nabla\times\mathbf{u})}, \quad \alpha_m = \frac{\tau}{12\pi\rho}\overline{\mathbf{b}\cdot(\nabla\times\mathbf{b})}$$

- Early exponential growth of magnetic fields.
- Magnetic back-reaction suppresses α_k
- α_m saturates field growth.

Evolution of α_m:

$$\frac{\partial\alpha_m}{\partial t} = -\frac{2\eta}{l^2 B_{\mathrm{eq}}^2}\boldsymbol{\mathcal{E}}\cdot\overline{\mathbf{B}} - \nabla\cdot\mathcal{F}$$

Here B_{eq} is the equipartition field strength, $B_{\mathrm{eq}} = \sqrt{4\pi\rho}u$.
Fluxes contributing to $\mathcal{F}$:

$$\mathcal{F} = \mathcal{F}^A + \mathcal{F}^D + \boxed{\mathcal{F}^{NV}}$$

The Vishniac Flux

- Derived by Gopalakrishnan & Subramanian (2023):

$$\mathcal{F}^{NV} = (\nabla \times \overline{\mathbf{U}}) \left[C_1 \frac{\tau^2}{8\pi\rho} (\langle \mathbf{b}^2 \rangle)^2 + C_2 \tau^2 \langle \mathbf{u}^2 \rangle \langle \mathbf{b}^2 \rangle + C_4 \lambda^2 \langle \mathbf{b}^2 \rangle \right]$$
$$+ \tau^2 (C_3 - C_2)(\overline{\mathbf{U}} \times \langle \mathbf{u}^2 \rangle \nabla \langle \mathbf{b}^2 \rangle)$$

- Define:

$$\overline{\mathbf{U}} = r\Omega\hat{\phi} + U_z\hat{\mathbf{z}} \quad \lambda = \mathbf{u}\tau \quad \xi \equiv \frac{\mathbf{b}^2}{B_{\mathrm{eq}}^2} \quad q = -\frac{d\ln\Omega}{d\ln r}.$$

- Simplified expression: $\mathcal{F}^{NV} = 16\pi f \Omega \rho \eta^2$

- $f = \frac{\xi}{4} \left[\frac{1103}{300} \left(1 - \frac{q}{2}\right) - \frac{7\xi}{5} \left(1 - \frac{q}{2}\right) \right]$

- $\xi_{\mathrm{crit}} = 2.63$, $q_{\mathrm{crit}} = 2$

THE MODEL

Axisymmetry

The system does not depend on the azimuthal angle ϕ, thus neglecting ϕ-derivatives.

The slab approximation

Reducing the problem to 1D along the vertical direction z, allowing for local dynamo solutions.

Equations Solved

1. **Radial field evolution**

$$\frac{\partial \overline{B}_r}{\partial t} = -\frac{\partial}{\partial z}\left(\alpha \overline{B}_\phi\right) + \eta \frac{\partial^2 \overline{B}_r}{\partial z^2} - \frac{\partial}{\partial z}(\overline{U}_z \overline{B}_r)$$

2. **Azimuthal field evolution**

$$\frac{\partial \overline{B}_\phi}{\partial t} = -\Omega \overline{B}_r + \eta \frac{\partial^2 \overline{B}_\phi}{\partial z^2} - \frac{\partial}{\partial z}(\overline{U}_z \overline{B}_\phi)$$

3. **Magnetic helicity evolution**

$$\frac{\partial \alpha_m}{\partial t} = \frac{-2\eta}{l^2 B_{\mathrm{eq}}^2}\left[\alpha\left(\overline{B}_r^2 + \overline{B}_\phi^2\right) - \eta\left(\frac{\partial \overline{B}_r}{\partial z}\overline{B}_\phi - \frac{\partial \overline{B}_\phi}{\partial z}\overline{B}_r\right) + \nabla \cdot (16\pi f \Omega \rho \eta^2)\right]$$

$$+\kappa \frac{\partial^2 \alpha_m}{\partial z^2} - \frac{\partial}{\partial z}(\overline{U}_z \alpha_m)$$

- **Spatial derivatives: 6th order finite differencing**
  - Higher-order schemes showed only marginal gains.
  - Optimal choice with respect to run time and memory usage.

- **Time stepping: Implicit RK-3**
  - Allows larger time steps while maintaining stability.
  - RK-4 vs RK-3: No notable improvement.

- **Implementing boundary conditions: Ghost zones**
  - Additional grid points beyond the physical domain.
  - Prevent numerical instabilities that might arise from directly enforcing boundary conditions.

- **Resolution: 101 physical grid points + 3×2 ghost cells**

Initial conditions

Gaussian random field with dimensionless amplitude 10^{-2} for $\overline{B}_r$ and $\overline{B}_\phi$, $\alpha_m = 0$

Boundary conditions

$$\overline{B}_r = \overline{B}_\phi = 0, \quad \frac{d^2\alpha_m}{dz^2} = 0 \quad \text{at } z = \pm h.$$

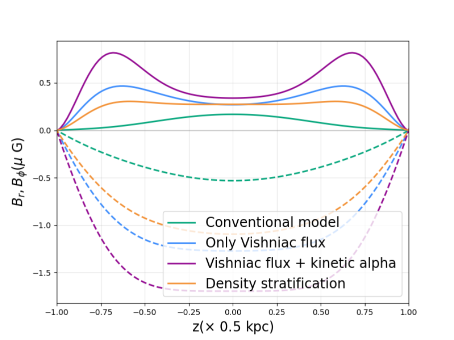
Parameter	A	B	C	D
Kinetic α effect	✓	✗	✓	✓
Advective flux	✓	✓	✓	✓
Diffusive flux	✓	✗	✓	✓
Vishniac flux	✗	✓	✓	✓
u stratification	✓	✓	✓	✓
ρ stratification	✗	✗	✗	✓

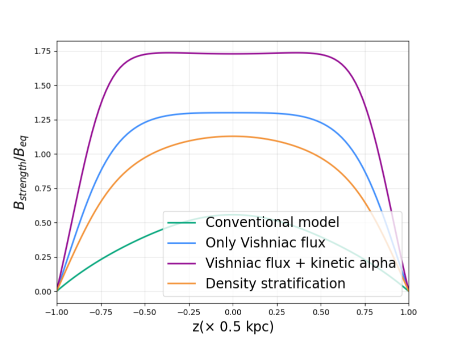
Table: Different models tested in the simulation. A is the conventional dynamo model, it is a baseline used to assess all other models which have the Vishniac flux.

A is a replication of the result from Chamandy et. al, 2014[2]

Results

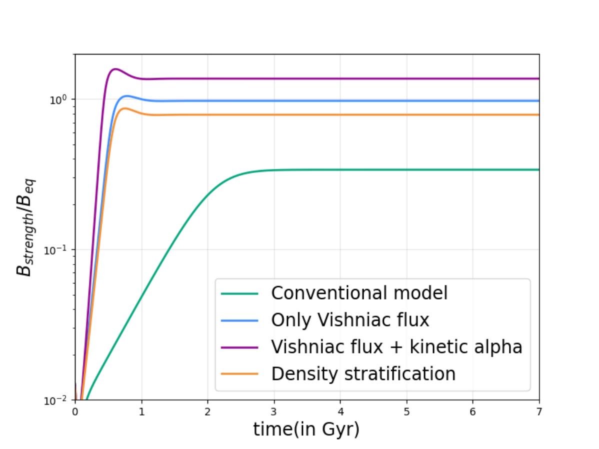
Magnetic field profiles





Plots of $\overline{B_r}$ and $\overline{B_\phi}$ against z (left) and total field strength vs z (right)

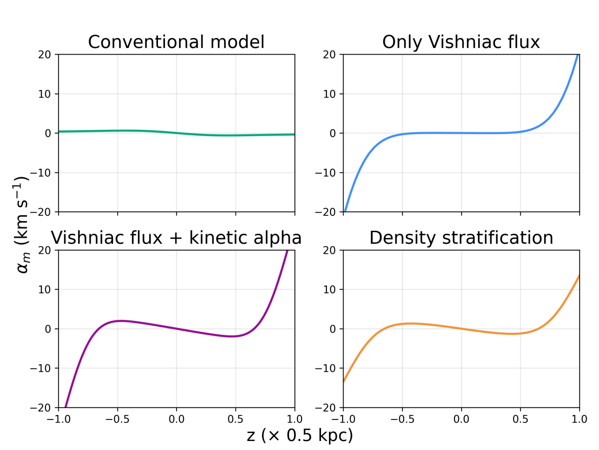
- Models with $\mathcal{F}^{NV}$ have an almost flat profile.
- $\mathcal{F}^{NV}$ makes, saturation strength is higher.



Plots of total field strength vs time

- Saturation strength high with $\mathcal{F}^{NV}$.
- Higher growth rate with $\mathcal{F}^{NV}$.
- Highest at when $\alpha_k + \mathcal{F}^{NV}$.

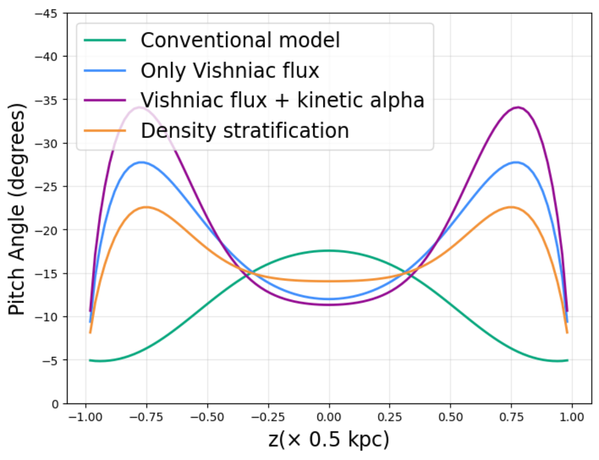
The α_m profiles



Plots of α_m vs z

- High values of α_m at $\pm z$ with $\mathcal{F}^{NV}$.

Pitch angle profile



Plots of pitch angles vs z

- $p = \tan^{-1}\left(\frac{\overline{B}_r}{\overline{B}_\phi}\right)$
- 0°: purely azimuthal.
- $\pm 90^\circ$: purely radial.

Summarizing the Results

$\mathcal{F}^{NV}$ flux is unstable with negative sign, which arises for critical values of $\xi > \xi_{\text{crit}} = 2.62$ and $q > q_{\text{crit}} = 2$.

This flux alone can sustain a dynamo, with significantly higher growth rates than conventional models.

Addition of this flux results in a more negative pitch angle as distance from the midplane increases.

Vishniac flux results in unphysical growth of α_m at disk boundaries.

Conclusion

Preliminary inferences about the Vishniac flux

- $\mathcal{F}^{NV}$ flux results in stable solutions for a typical disc galaxy.
- $\mathcal{F}^{NV}$ strengthens the dynamo.
- This flux allows large-scale magnetic fields to arise very rapidly and already be present near equipartition level in young galaxies at high cosmological redshift.
- Pitch angle profiles suggest tightly wound fields at the midplane, becoming less tight at boundaries.
- Vishniac flux affects α_m significantly near disk boundaries.

Limitations and Future Plans

Current Limitations

- **Boundary Effects**: The absence of advection causes α_m to take unphysical values at the boundaries, limiting accurate modeling at the edges.
- **Simplified Model**: The current model relies on approximations, such as the slab assumption, which restricts the generalization of the results.

Future Directions

- **Incorporating a Gaseous Halo**: The next step is to expand the simulation by adding a gaseous halo.
- **2D Model**: A 2D simulation is necessary to study the global solutions of dynamo.

THE END, **THANKS!**

References I

Rainer Beck, Luke Chamandy, Ed Elson, and Eric G. Blackman.
Synthesizing observations and theory to understand galactic magnetic fields: Progress and challenges, 2019.

Luke Chamandy, Anvar Shukurov, Kandaswamy Subramanian, and Katherine Stoker.
Non-linear galactic dynamos: a toolbox.
Monthly Notices of the Royal Astronomical Society, 443(3):1867–1880, 07 2014.

Kishore Gopalakrishnan and Kandaswamy Subramanian.
Magnetic helicity fluxes from triple correlators.
The Astrophysical Journal, 943(1):66, jan 2023.

Simon J. Lilly.
Magnetic fields under the lens.
Nature Astronomy, 1(9):565–566, 2017.

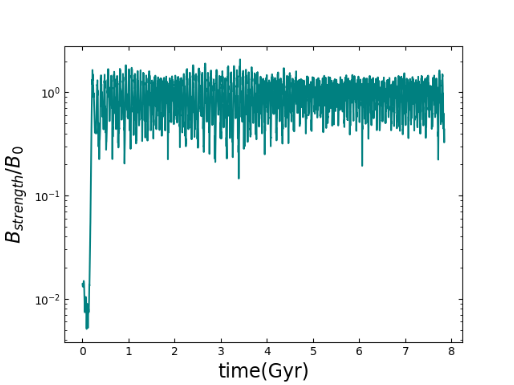
References II

Anvar Shukurov and Kandaswamy Subramanian.
Astrophysical Magnetic Fields: From Galaxies to the Early Universe.
Cambridge Astrophysics. Cambridge University Press, 2021.

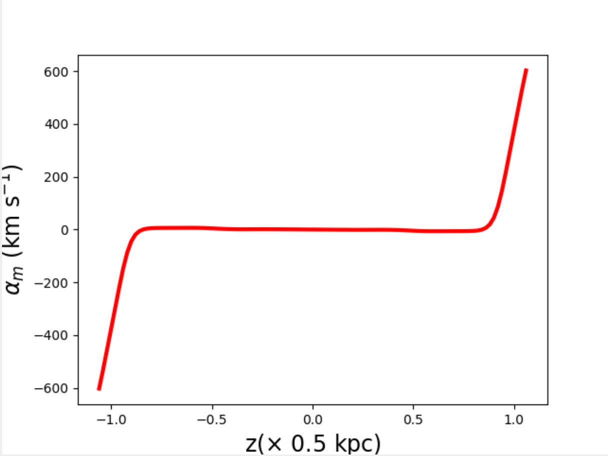
E. T. Vishniac.
A Simple Model for the Galactic Dynamo.
In *American Astronomical Society Meeting Abstracts #220*, volume 220 of *American Astronomical Society Meeting Abstracts*, page 308.05, May 2012.

WHAT HAPPENS IF THE SIGN IS NEGATIVE?

- Unstable solutions.
- Happens only when $\xi > 2.62$ or $q > 2$, less probable in disc galaxies.

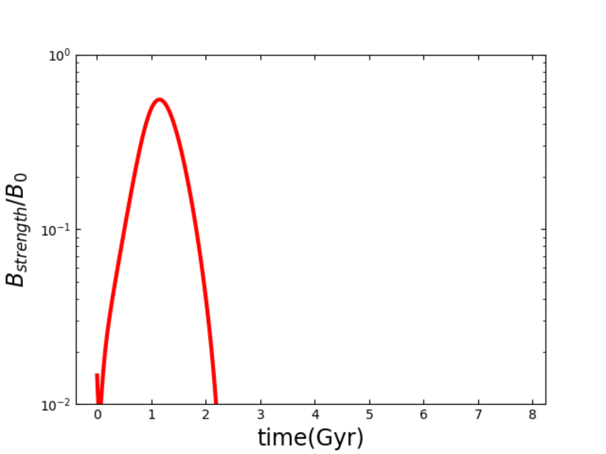


No advection



- α_m blow up at boundaries.

No α_m in the model



- Catastrophic quenching!