

21. Planetary Magnetic Fields

Why are Planetary Magnetic Fields Interesting?

There are four reasons:

- (1) When a planet has a large field, this requires a dynamic process (called a *dynamo*) deep within the planet. Thus the observed field provides us with insight into the state of matter and the dynamics deep within a planet... there is no other way to do this.
- (2) When a planet has a paleofield (i.e., remanent magnetism of near surface rocks), the pattern of this magnetization may tell us about many things: The past behavior of the field (e.g. this is how we know about geomagnetic reversals), the mobility of the lithosphere (plate tectonics was deduced primarily from paleomagnetism) and perhaps something about volcanic history (since paleomagnetism in igneous rocks may well dominate).
- (3) When a body has an induced field, caused by the time variation of an external field, the magnitude and phase of this induction tells us about the conductivity structure within the body.
- (4) Field generation is a non-linear chaotic process whose dynamics are of interest in their own right (as a fundamental and very difficult problem in complex systems.)

What Fields are Observed?

In the special case of Jupiter, which is a synchrotron source of radio waves, remote detection is possible. Other giant planets (e.g. Saturn) are decametric sources though we did not use this to characterize Saturn's field before spacecraft encounter. In most cases, we learn about magnetic fields by the direct in situ detection of the field (the magnetosphere) in a flyby or orbiter spacecraft. This is usually done with a magnetometer, which is an instrument that detects change in flux through a coil. Spacecraft often have booms on which a magnetometer is placed (to keep it well away from spacecraft electrical currents). In the case of MGS, magnetometers were placed on the solar panels, either side of the main body of the spacecraft. Electron reflectometers can also provide information on field strength and field gradients by measuring the trajectories of electrons. It is not usually as useful to place magnetic field observatories on the ground; orbital data are preferred - provided you can measure (or get below) the effects of an

ionosphere. (Even on Earth, orbital data now play a crucial role; e.g. MAGSAT).

Here are the observations, with likely interpretations (explained more fully below).

<u>Planet or Satellite</u>	<u>Observed Surface Field</u> (in Gauss, approx.)	<u>Comments and</u> <u>Interpretation</u>
Mercury	0.002	Dynamo (confirmed by MESSENGER)
Venus	Unmeasurably small	No dynamo. High surface temp \Rightarrow no remanence.
Earth	0.5	Dynamo needed
Moon	Patchy; no global field.	Ancient dynamo (precessional?)
Mars	Patchy but locally strong; no global field.	Ancient dynamo, Remanent mag. Lineations.
Jupiter	4.2	Dynamo (extends to near surface)
Io	$\sim 0.01?$	Complex (deeply imbedded in Jovian field.)
Europa	~ 0.01	Induction response \Rightarrow Ocean
Ganymede	0.02	Dynamo likely + Induction also (from an ocean)
Callisto	0.005	Induction response \Rightarrow Ocean

Saturn	0.2	Dynamo
Titan	<0.001	Cassini shows no evidence of dynamo or induction
Uranus	0.2	Dynamo
Neptune	0.2	Dynamo

What is the Geometry of Large Fields?

For Mercury, Earth, Jupiter and Saturn (and probably Ganymede), the field is predominantly dipolar but with detected higher harmonics (except for Ganymede). The tilt of the dipole relative to the rotation axis is of order 10 degrees (Jupiter and Earth) and zero (Saturn). Data for Mercury are consistent with a large offset of the dipole along the rotation axis and a relatively small tilt. (An offset of this kind is also expressible as higher harmonics). Mercury's dynamo is unusually small among dynamos when taking into account its large core.

For Uranus and Neptune, the field is about equally dipole and quadrupole and the tilt of the dipole is 40-60 degrees. In the general context of astrophysics and planetary science this is unusual (and not yet understood), but we'll return to this later.

Where do Magnetic Fields Come from?

Magnetic fields, unlike electric fields, do not come from monopoles, i.e. there is no evidence for free magnetic poles (analogous to free electric charges) in the universe. Instead, the fields come from the movement of charges, or from the fundamental magnetic moments (i.e. dipoles) of elementary particles (which can often be thought of quasi-classically as rotating or orbiting electric charges, but which are sometimes unavoidably quantum mechanical effects). In everyday experience, substantial fields arise either from permanent magnets where the magnetism arises at the microscopic level and is a thermodynamic property of the material, or through macroscopic currents (e.g., Helmholtz coil). Permanent magnetism is a satisfactory explanation for modest amounts of observed magnetism

(e.g. Moon, Mars, maybe Mercury), but it requires low temperature (outer regions only of a planet) and it requires an adequate abundance of the minerals that exhibit permanent magnetization (e.g. magnetite, metallic iron). On Earth, permanent magnetization (in the crust) accounts for typically 1 part in ten thousand or 1 part in one thousand of the observed field. *Localized* fields of up to 1 Gauss or more are possible from permanently magnetized materials; this happens very rarely on Earth but may be common in the Southern hemisphere of Mars. It is likely (but we don't know for sure) that *global* fields of around 0.0001G or 0.001G are about the most one can expect from permanently magnetized shells. (The simplest magnetized shell configurations produce *zero* external field. This sometimes referred to as Runcorn's theorem.).

On Earth, we have a much stronger argument for something else other than permanent magnetization: The field is dynamic (time varying on all time scales from years to billions of years.)

By Faraday's law, a planetary body can also have an "internal" field that is induced by a time-variable external field. These eddy currents and associated fields can be identified by their distinctive time variability and phase and amplitude. On Earth, these are called magnetotelluric currents and fields and they are very small compared to the main field

Once induced effects are removed, one is still often left with a large field that can only be due to macroscopic electrical currents deep within the planet. But unlike permanent magnetism (which never goes away provided the material stays cold), these currents dissipate energy and must thus be replenished. This process of regeneration is called a dynamo.

The Induction (Dynamo) Equation

We turn now to the dynamics of the magnetic field. For this purpose, we are *not* interested in things that move at the speed of light (conventional electromagnetic waves) but pre-Maxwell electricity and magnetism. We will write these equations in S.I. (MKS) units for a change. (The benefit is units of amps and volts for current and voltage, which you recognize from everyday experience. However, non-SI units are still often convenient. Gauss is a non-SI unit, of course.)

First, we have Ampere's Law:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} \quad (21.1)$$

where \mathbf{B} is the magnetic field strength, \mathbf{j} is the current density and μ_0 is essentially the permeability of free space ($4\pi \times 10^{-7}$; relative permeability is negligible at high temperature) . In these units, the field is measured in Tesla and $1\text{T} = 10^4$ Gauss.

We also have Faraday's law of induction:

$$\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t \quad (21.2)$$

and Ohm's law (allowing for the induced electric field due to flow):

$$\mathbf{j} = \sigma (\mathbf{E} + \mathbf{u} \times \mathbf{B}) \quad (21.3)$$

Finally we must allow for the fact that there are no magnetic monopoles:

$$\nabla \cdot \mathbf{B} = 0 \quad (21.4)$$

If we take the curl of Ampere's law and Ohm's law and then use Faraday's law to eliminate \mathbf{E} , we end up with the *dynamo equation* (sometimes just called the induction equation) :

$$\partial \mathbf{B} / \partial t = \lambda \nabla^2 \mathbf{B} + \nabla \times (\mathbf{u} \times \mathbf{B}) \quad (21.5)$$

where $\lambda \equiv 1/\mu_0 \sigma$ is known as the *magnetic diffusivity* (because it has the physical effect of allowing field to diffuse and because it has the right dimensions). This derivation assumes that λ is constant. If it varies spatially, then the first term on the RHS is $-\nabla \times (\lambda \nabla \times \mathbf{B})$.

What does the Dynamo Equation Mean?

Suppose we have some field $\mathbf{B} \sim \mathbf{B}_0 \exp[\mathbf{i} \mathbf{k} \cdot \mathbf{r} + \sigma t]$. Now because $\mathbf{k} \cdot \mathbf{B}_0 = 0$ it follows that $\mathbf{k} \times \mathbf{B}_0$ is non-zero (provided \mathbf{k} and \mathbf{B} are non-zero). Since the current $\sim \nabla \times \mathbf{B} \sim \mathbf{k} \times \mathbf{B}_0$, this field must have an associated current. Then in the absence of fluid motion (i.e. no emf by induction), we have $\sigma = -\lambda k^2$, which means that the field must decay. The physical origin is obvious....

associated with the current, there is dissipation into heat because of the finite electrical resistance (proportional to λ). This must be at the expense of the energy in the magnetic field. This is just free induction decay, like in an electrical circuit where you have a coil and resistance but no battery.

The *free decay time* of the field is $\tau \sim \sigma^{-1}$ and if we think of the currents associated with the field being confined to a sphere or box of size L , then $k \sim \pi/L$. Thus,

$$\tau \sim \frac{L^2}{\pi^2 \lambda} \sim (3000 \text{ yr}) \left(\frac{L}{10^3 \text{ km}} \right)^2 \left(\frac{1 \text{ m}^2 / \text{s}}{\lambda} \right) \quad (21.6)$$

and this is less than the age of the solar system for any planetary body and plausible values of magnetic diffusivity. For Earth's core, this timescale is ten thousand years or so.

Here are some estimates of the magnetic diffusivity relevant to planets:

<u>Material</u>	<u>Relevant planet</u>	<u>Magnetic diffusivity</u> (m^2 / sec)
Liquid iron	All terrestrial planet cores; putative cores of satellites	1 or 2
Salty water(low P)	Icy Satellites	$\sim 10^4$ to 10^6
Conducting hydrogen at ~ 1.5 Megabar	Jupiter at $0.9 \times$ radius Saturn at $0.7 \times$ radius	~ 30 -50 (but increasing as you go up)
Monatomic metallic hydrogen	Jupiter at $0.7 \times$ radius? Saturn at $0.4 \times$ radius?	$\sim 1?$
Water, ammonia and methane at 0.1 Megabar and 1000K or more	Uranus and Neptune at $0.7 \times$ radius?	~ 200

Notice that these diffusivities are higher than any of the other diffusivities previously discussed for the core (viscous diffusion, thermal diffusion,

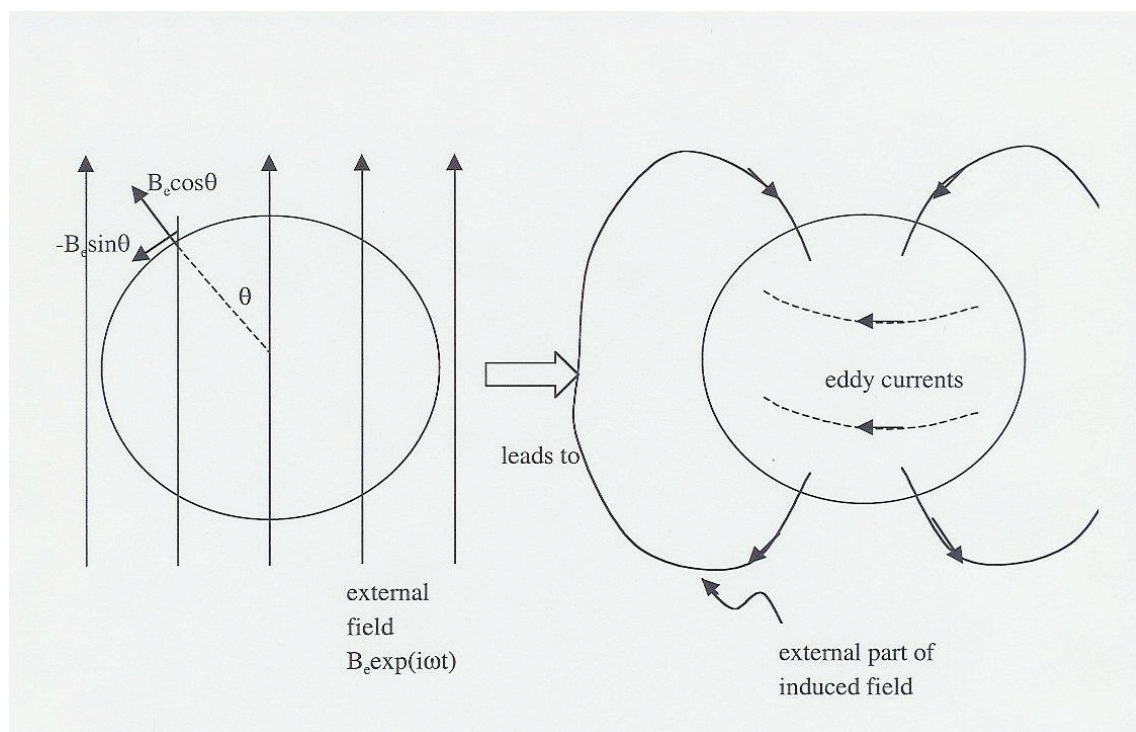
solute diffusion.... even if these are eddy diffusivities.) So fields can diffuse much more easily than heat for example. Very high diffusion occurs in regions of negligible conductivity (e.g. Earth's mantle).

Free decay times are generally geologically short; this means that if a planet has a large field now then it must have a means of generating the field now.

Effect of Time-Varying External Fields

This is a special case in which induction from outside a conductor induces eddy currents. Here is a simplified analysis of this situation for the limiting case of a “good” conductor (with the definition of “good” defined after we solve the problem):

Consider a spherical conducting body of radius R placed at the origin, in the presence of an external field $B_0 \exp(i\omega t)$ which is directed along the axis of coordinates (we are always at liberty to do this). So at a particular colatitude θ , the local radial field is $B_0 \cos\theta \exp(i\omega t)$ and the local θ -component is $-B_0 \sin\theta \exp(i\omega t)$. The field induced in the sphere creates an external field that decays in accordance with Laplace's equation and with the right angular dependence; clearly this is a dipole in which the radial component is $2B_e(R/r)^3 \cos\theta \exp(i\omega t)$ and the θ -component is $-B_e(R/r)^3 \sin\theta \exp(i\omega t)$.



Assuming that we can adopt a local Cartesian coordinate system, with z vertically upwards and x in the θ -direction, each component of the internal field must satisfy

$$\lambda(d^2B/dz^2 + d^2B/dx^2) = i\omega B \quad (21.7)$$

because it must share the same time-variation to match the external field. Assuming the z -variation is more rapid (i.e., ignoring x -derivatives) the two components of the internal field are accordingly

$$\begin{aligned} B_{\text{int},x} &= B_i \sin\theta \cdot \exp\left[\frac{(1+i)z}{\delta}\right] e^{i\omega t} \\ B_{\text{int},z} &= -\frac{\delta}{R(1+i)} B_i \cos\theta \cdot \exp\left[\frac{(1+i)z}{\delta}\right] e^{i\omega t}; \quad \delta \equiv \sqrt{2\lambda/\omega} \end{aligned} \quad (21.8)$$

Notice that this satisfies the differential equation and has zero divergence (as it must). It also decays as one goes to large negative z . The lengthscale δ is called the electromagnetic skin depth and our assumptions of local Cartesian coordinates and neglect of the x -component of the Laplace operator will be correct provided this skin depth is small compared to planetary radius. Requiring continuity of field components at the planet surface:

$$\begin{aligned} B_i &= -B_e - B_o \\ -\frac{\delta}{R(1+i)} B_i &= 2B_e + B_o \\ \Rightarrow B_{\text{radial}}(r=R) &\equiv 2B_e + B_o \approx B_o \cdot \frac{\delta}{R(1+i)} \end{aligned} \quad (21.9)$$

In other words, the total radial field at the surface is small if $\delta \ll R$. This means that the external field has induced an internal field whose effect is to almost cancel the radial component of the external field. (The latitudinal component is not cancelled).

For a good metal ($\lambda \sim 1 \text{ m}^2/\text{sec}$) and $\omega \sim 10^{-4} \text{ sec}^{-1}$ (corresponding to Jupiter's spin) $\delta \sim 100$ meters. For an electrolyte such as salty water, one gets ~ 10 km. So a "good" conductor for this induction effect is defined not just by conductivity but by the thickness of the conducting layer. Indeed, a quite

poor conductor (salty water) is plenty good enough even for a layer thickness much smaller than the size of the body.

This means that the induction effect should not be thought of as a detector of metals but a detector of quite low but nonetheless interesting conductivity. Solid silicates and water ice are still insulators in this context because they have electromagnetic skin depths comparable to or exceeding a planetary radius.

The “cancellation” of the vertical field is observed for Europa and Callisto and appears to have also been detected for Ganymede. On this basis, a salty water ocean is inferred for these bodies. *This is the strongest argument for an ocean in Europa.*

The following figure (from Khurana, K. K., Kivelson, M. G., Stevenson, D. J. , Schubert, G., Russell, C. T., Walker, R. J., and Polanskey, C. Induced magnetic fields as evidence for subsurface oceans in Europa and Callisto. *Nature* **395**, 777-780, 1998) shows a comparison of the predicted induction response with the observed field, during an encounter with Europa. Obviously some encounters are better than others for testing the model: This is not surprising, since the induction field varies with the phase (system III longitude) of Jupiter’s field during the encounter.

The data are consistent with a “perfect” induction response (meaning the response for a conductor good enough so that the induced field almost perfectly cancels the background field). Thus, no phase shift (“delay”) is observed in the response.

These data do not tell us the thickness of the ice overlying the ocean. They also do not provide more than a lower bound for the thickness of the ocean (even if one assumed a particular saltiness).

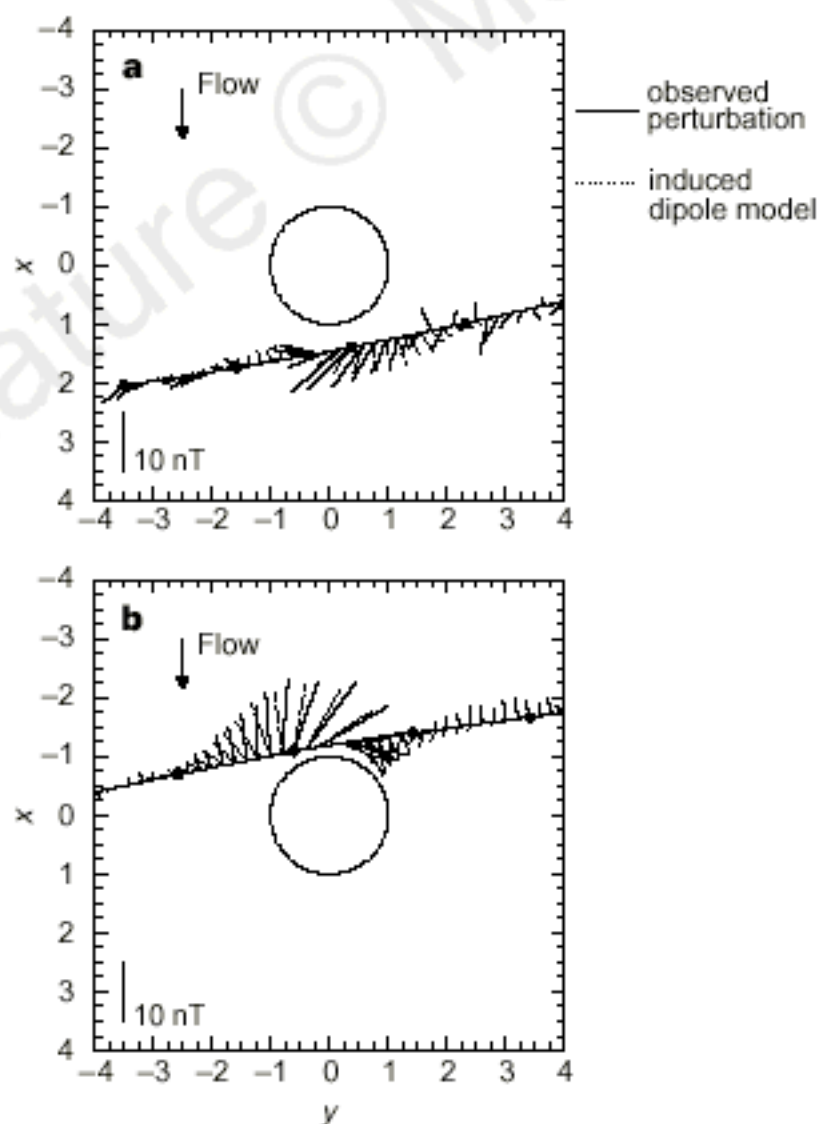


Figure 3 Magnetic field observations from the C3 and C9 passes. **a**, The magnetic field perturbations (vectors drawn with solid lines) and the modelled induction field (vectors shown dotted) along the trajectory of the C3 encounter in the x - y plane. **b**, The magnetic field perturbations and the modelled induction field for the C9 encounter. The distance scale is in units of R_C ($1R_C$ = radius of Callisto = 2,409 km).

Problem

21.1 MGS observed strong evidence for a Martian paleomagnetic field from orbital determination (a minimum of 100km above the surface). Consider a hypothetical Venus mission that attempted the same thing for that planet: Estimate by what factor the observed field would be smaller than that seen at Mars. Assume that the *only* thing determining this is the thickness of the magnetized layer, which is bounded above by the solid surface of the planet and bounded below by the depth at which the temperature reaches the blocking temperature of magnetite, assumed to be 800K. [Note: The blocking temperature is a few tens of degrees below the Curie temperature and is the temperature below which domains will preserve their magnetization for geologically interesting time scales.] You should explain in a sentence or two why the measured field is expected to be proportional to the thickness of the magnetized layer. (If you think this is obvious, you're probably not thinking about it carefully enough!) The rocks on Venus are much younger than the relevant rocks on Mars, and the geologically recent heat flow on Venus is about the same as the early heat flow on Mars, so assume a subsurface temperature gradient of 15 K/km for both bodies. Assume the mean ("zero elevation") Venus surface is at 740 K and the Martian surface is 240K. Comment on the merit of checking out Maxwell Montes on Venus, which has 10 km elevation. (The adiabatic lapse rate in the Venus atmosphere is about 7K per km elevation). *[This whole problem is computationally trivial but you benefit by thinking through the ideas and assumptions. Obviously, some of the assumptions may be completely wrong, including the most important of all - that Venus had a field at the time the current surface rocks were emplaced.]*