(8) Assignment - (1) Gayathri

a) Test the consistency and solve.

i) $2x-3y+7z=5$, $3x+y-3z=13$, $2x+19y-47z=32$

$$\begin{bmatrix} 2 & -3 & 9 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix} \qquad AX \Rightarrow B$$

C is combination of matrixes A & B $\qquad C=[A:B]$

$$\Rightarrow \begin{bmatrix} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & -3 & 7 & 5 \\ 0 & 11 & -27 & 11 \\ 2 & 19 & -47 & 32 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & -3 & 7 & 5 \\ 0 & 11 & -27 & 11 \\ 0 & 22 & -54 & 27 \end{bmatrix}$$

$R_2 \to 2R_2 - 3R_1 \qquad R_3 \to R_3 - R_1 \qquad R_3 \to R_3 - 2R_2$

$$\Downarrow$$

$$A \Rightarrow \begin{bmatrix} 2 & -3 & 7 \\ 0 & 11 & -27 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 2 & -3 & 7 & 5 \\ 0 & 11 & -27 & 11 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

Rank(A) = 2 $\qquad$ Rank(C) = 3

Rank(A) $\neq$ Rank(C)

$\therefore$ The given system is consistent and there exist no solution.

b) $2x - y + 3z = 8$

$-x + 2y + z = 4$

$3x + y - 4z = 0$

$$\begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix} \qquad AX = B$$

(8)

Gayathri

8. $[6,0,3,1,4,2]$ $[0,1,2,7,0,5]$ $[12,3,0,-19,8,-11]$

Let c_1, c_2, c_3 be scalars.

Consider,

$$c_1v_1 + c_2v_2 + c_3v_3 \Rightarrow 0$$

$$6c_1 + 0c_2 + 12c_3 \Rightarrow 0$$
$$0c_1 + c_2 + 3c_3 \Rightarrow 0$$
$$3c_1 + 2c_2 + 0c_3 \Rightarrow 0$$
$$c_1 + 7c_2 - 19c_3 \Rightarrow 0$$
$$4c_1 + 0c_2 + 8c_3 \Rightarrow 0$$
$$2c_1 + 5c_2 - 11c_3 \Rightarrow 0$$

$$\begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 3 & 2 & 0 \\ 1 & 7 & -19 \\ 4 & 0 & 8 \\ 2 & 5 & -11 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to 2R_3 - R_1$

$$\begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 0 & 4 & -12 \\ 1 & 7 & -19 \\ 4 & 0 & 8 \\ 2 & 5 & -11 \end{bmatrix}$$

$R_4 \to 6R_4 - R_1$

$$\Rightarrow \begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 0 & 4 & -12 \\ 0 & 42 & -102 \\ 4 & 0 & 8 \\ 2 & 5 & -11 \end{bmatrix}$$

$R_5 \to 6R_5 - 4R_1$

$$\Downarrow$$

$$\begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 0 & 4 & -12 \\ 0 & 42 & -102 \\ 0 & 0 & 0 \\ 2 & 5 & -11 \end{bmatrix}$$

$R_5 \leftrightarrow R_6$

$$\begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 0 & 4 & -12 \\ 0 & 42 & -102 \\ 2 & 5 & -11 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 0 & 4 & -12 \\ 0 & 42 & -102 \\ 0 & 15 & -45 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_5 \to 3R_5 - R_1$

$R_3 \to R_3 + 4R_2$

$$\Downarrow$$

$$\begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 0 & 0 & 0 \\ 0 & 42 & -102 \\ 0 & 15 & -45 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_5 \leftrightarrow R_3$

$$\Leftarrow \begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 0 & 15 & -45 \\ 0 & 42 & -102 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 + 15R_1$

$$\Downarrow$$

$$\begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 0 & 0 & 0 \\ 0 & 42 & -102 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 0 & 42 & -102 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_3 \leftrightarrow R_4$

$R_3 \to R_3 + 42R_2$

$$\Downarrow$$

$$\begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 0 & 0 & 24 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore c_1, c_2, c_3$ are zeroes

So, the given vectors are linearly independent.

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C is matrix combination of A & B matrix

$C = [A : B]$

$$\Rightarrow \begin{bmatrix} 2 & -1 & 3 & 8 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & -4 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & -1 & 3 & 8 \\ 0 & 5 & -1 & 0 \\ 3 & 1 & -4 & 0 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to 2R_3 - 3R_1$

$$\Downarrow$$

$$\begin{bmatrix} 2 & -1 & 3 & 8 \\ 0 & 5 & -1 & 0 \\ 0 & 0 & -16 & -24 \end{bmatrix} \Leftarrow \begin{bmatrix} 2 & -1 & 3 & 8 \\ 0 & 5 & -1 & 0 \\ 0 & 5 & -17 & -24 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

Rank

$$\Rightarrow \begin{bmatrix} 2 & -1 & 3 & 8 \\ 0 & 5 & -1 & 0 \\ 0 & 0 & -16 & -24 \end{bmatrix} \Rightarrow (3)$$

$\text{Rank}[A : B] = 3$

; Rank $\Rightarrow \begin{bmatrix} 2 & -1 & 3 \\ 0 & 5 & -1 \\ 0 & 0 & -16 \end{bmatrix}$ Rank$(A) = 3$

Rank $[A : B]$ = Rank $[B]$ and it is equal to no. of unknown then we get unique solution.

(iii)

$$4x - y = 12$$
$$-x + 5y - 2z = 0$$
$$-2x + 4z = -8$$

$$\Rightarrow \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -2 & 0 & 4 \end{bmatrix} \begin{bmatrix} 12 \\ 0 \\ 8 \end{bmatrix}$$

$AX \Rightarrow B$

$$\Downarrow$$

$$\begin{bmatrix} 4 & -1 & 0 & 12 \\ 0 & 21 & -8 & -12 \\ 0 & 1 & 8 & 4 \end{bmatrix} \Leftarrow \begin{bmatrix} 4 & -1 & 0 & 12 \\ 0 & 21 & -8 & -12 \\ -2 & 0 & 4 & 8 \end{bmatrix} \Leftarrow \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -2 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 12 \\ 0 \\ 8 \end{bmatrix}$$

$R_3 \to 21R_3 - R_2$ $\qquad$ $R_3 \to 2R_3 - R_1$ $\qquad$ $R_2 \to 4R_2 - R_1$

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$$\begin{bmatrix} 4 & -1 & 0 & 12 \\ 0 & 21 & -8 & -12 \\ 0 & 0 & 172 & 96 \end{bmatrix} \quad \text{Rank}[A:B] = 3 \qquad \Rightarrow \begin{bmatrix} 4 & -1 & 0 \\ 0 & 21 & -8 \\ 0 & 0 & 172 \end{bmatrix} \quad \text{Rank}[A] = 3$$

Rank [A : B] = Rank [A] and it is equal to number of unknown then we get unique solution.

2)

$$\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 10 \\ x + 2y + \lambda z &= \mu \end{aligned} \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$$

$C = [A : B]$ $\qquad R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 3-3 & 8-10 \end{bmatrix} \Leftarrow \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{bmatrix} \Leftarrow \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$$

other than 10 $\qquad R_3 \to R_3 - R_2$

Here

$$A \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & \lambda-3 \end{bmatrix}$$

If $\lambda = 3$ then $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ Rank = 2

Case I :- If $\lambda = 3$, $\mu \neq 10$ then $R(A) = 2$

$R(C) = 3$ $\quad R(A) \neq R(C)$ $\quad \therefore$ The system is inconsistente & there exists no solution.

Case II :- If $\lambda \neq 3$, $\mu \neq 10$

then $R(A) = 3$; $R(C) = 3$ $\qquad R(A) = R(C)$; n is also 3.

The system is consistent & there exists unique solution.

Case III :- If $\lambda = 3$; $\mu = 10$

then $R(A) = 2$ & $R(C) = 2$ $\to$ so many solutions.

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c) Find the what values of λ the given solutions $x+y+z=1$ $x+2y+4z=\lambda$; $x+4y+10 \Rightarrow \lambda^2$ have a solution and solve them completely in each case.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}$$

$A : B \Rightarrow C$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 3 & 9 & \lambda^2-1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-1-3(\lambda-1) \end{bmatrix}$$

$R_2 \to R_2 - R_1$ $\quad R_3 \to R_3 - R_1$ $\quad R_3 \to R_3 - 3R_1$

$$\Downarrow$$

$$\text{Rank}(A) \neq \text{Rank}(B) \neq n \Leftarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-3\lambda+2 \end{bmatrix} \Leftarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-1-3\lambda+3 \end{bmatrix}$$

$\lambda^2 - 3\lambda + 2 = 0$

$\lambda^2 - \lambda - 2\lambda + 2 \Rightarrow 0$

$\lambda(\lambda-1) - 2(\lambda-1) \Rightarrow 0$

$(\lambda-2)(\lambda-1) \Rightarrow 0$

$\lambda = 1$; $\lambda = 2$

Case 1 :- $\lambda \Rightarrow 1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x+y+z \Rightarrow 1$; $x+y+k=1$

$y+3z \Rightarrow 1$ $\quad x+y=-k$

Let $z \Rightarrow k$ $\quad y=-3k$

$\boxed{x \Rightarrow 2k+1}$

Case - 2 :- $\boxed{\lambda=2}$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x+y+z=1$

$y+3z=1$

$y+3k=1$

$y \Rightarrow 1-3k$

$x+1-3k+k=1$

$x-2k=0$

$\boxed{x=2k}$

$\boxed{y \Rightarrow 1-3k}$

$\boxed{z=k}$

d) Find the solution of the system of equations $x+3y-2z=0$, $2x-y+4z=0$; $x-11y+14z=0$

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$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$C = A : B$

$$\begin{bmatrix} 1 & 3 & -2 & 0 \\ 2 & -1 & 4 & 0 \\ 1 & -11 & 14 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 1 & -11 & 14 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & -14 & 16 & 0 \end{bmatrix}$$

$\Rightarrow$ $R_2 \to R_2 - 2R_1$ $\quad R_3 \to R_3 - R_1$ $\quad R_3 \to R_3 + 2R_2$

$$\Downarrow$$

$$\begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank of $[A : B] \Rightarrow 2$

Rank of $[A] \Rightarrow 2$

And not equal to number of unknown. So, the system of equation is consistent, it will have infinite solutions.

2nd assignment

Find the eigen values and eigen vectors of following matrices.

Gayathri ①

1. $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

The characteristic equation is

$|A - \lambda I| = 0$

$\Rightarrow \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow 0 \Rightarrow \begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix} = 0$

$\Rightarrow (-2-\lambda)\Big((1-\lambda)(-\lambda) - 12\Big) - 2\Big(2(-\lambda) - (6)\Big) - 3\Big(-4 + (1)(1-\lambda)\Big) = 0$

$\Rightarrow (-2-\lambda)\left(-\lambda + \lambda^2 - 12\right) - 2(-2\lambda - 6) - 3(-4 + 1 - \lambda) = 0$

$\Rightarrow -\lambda(-2-\lambda) + \lambda^2(-2-\lambda) - 12(-2-\lambda) + 4\lambda + 12 + 9 + 3\lambda = 0$

$\Rightarrow 2\lambda + \lambda^2 - 2\lambda^2 - \lambda^3 + 24 + 12\lambda + 4\lambda + 12 + 9 + 3\lambda = 0$

$\Rightarrow -\lambda^3 + \lambda^2 - 21\lambda - 45 = 0$

This is characteristic polynomial. Now, we have to find roots of this polynomial.

The equation is $-\lambda^3 + \lambda^2 - 21\lambda - 45 = 0$

Hit & Trial Method

Substitute "-1" in place of λ

$\Rightarrow (-1)^3 + (-1)^2 - 21(-1) - 45 = 0$

$\Rightarrow -1 + 1 + 21 - 45 = 0$

$\Rightarrow 21 - 45$

$\Rightarrow -24 \neq 0$

Substitute "-2" in place of λ

$\Rightarrow (-2)^3 + (-2)^2 - 21(-2) - 45 = 0$

$\Rightarrow 8 + 4 + 42 - 45 = 0$

$\Rightarrow 57 + 42 = 0$

$\Rightarrow 99 \neq 0$

Gayathri ①

Substitute "-3" in λ

$\Rightarrow (-3)^3 + (-3)^2 - 21(-3) - 45 = 0$

$\Rightarrow -27 + 9 + 63 - 45 = 0$

$\Rightarrow -72 + 72 = 0$

$0 = 0$

So, -3 is one root for of equation

Now, by doing long division, we finding remaining two roots.

$$\begin{array}{c|cccc} -3 & 1 & 1 & -21 & -45 \\ & 1 & -3 & +6 & +45 \\ \hline & 1 & -2 & -15 & 0 \end{array} \Rightarrow x^2 - 2x - 15 = 0$$

For roots, $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{2 \pm \sqrt{4 - 4(1)(-15)}}{2(1)}$

$\Rightarrow \frac{2 \pm \sqrt{64}}{2} \Rightarrow \frac{2 \pm 8}{2} \Rightarrow \frac{2+8}{2}$ & $\frac{2-8}{2}$

$\Rightarrow \frac{10}{2}$ & $\frac{-6}{2}$

5 & -3

Therefore, roots are -3, -3, 5

So, they are eigen values of given matrix.

Let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be the eigen vectors corresponding to eigen value λ.

$[A - \lambda I] X = 0$

$$\begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & 6 \\ -1 & -2 & 0-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Gaydhri (2)

2. $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

$A \Rightarrow \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

The characteristic equation is $|A - \lambda I| = 0$

$$\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow 0 \Rightarrow \begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix} \Rightarrow 0$$

$$\Rightarrow (4-\lambda)\left((1-\lambda)(1-\lambda) - 0\right) - 0(-2(1-\lambda) + 0) + 1(0 + 2(1-\lambda))$$

$$\Rightarrow (4-\lambda)(1-\lambda-\lambda+\lambda^2) + 2 - 2\lambda = 0$$

$$\Rightarrow (4-\lambda)(1-2\lambda+\lambda^2) + 2 - 2\lambda = 0$$

$$\Rightarrow (4-\lambda) - 2\lambda(4-\lambda) + \lambda^2(4-\lambda) + 2 - 2\lambda = 0$$

$$\Rightarrow 4 - \lambda - 8\lambda + 2\lambda^2 + 4\lambda^2 - \lambda^3 + 2 - 2\lambda = 0$$

$$\Rightarrow \boxed{-\lambda^3 + 6\lambda^2 - 11\lambda + 6 = 0}$$

This is characteristic polynomial.

Now, we have to find roots of this polynomial.

The equation,

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

Hit Ef trail method

Substitute "-1" in place of "λ".

$(-1)^3 - 6(-1)^2 + 11(-1) - 6 = 0$

$-1 - 6 - 11 - 6 \neq 0$ ⊗

Not a root.

Substitute "1" in place of "λ".

$(1)^3 - 6(1)^2 + 11(1) - 6 = 0$

$1 - 6 + 11 - 6 = 0$

$12 - 12 = 0$

$0 = 0$

∴ Therefore, one of root is 1.

Gayathri ②

Now, consider $\lambda = 5$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-7x_1 + 2x_2 - 3x_3 = 0$ — ①

$2x_1 - 4x_2 - 6x_3 = 0$ — ②

$-x_1 - 2x_2 - 5x_3 = 0$ — ③

Considering ① & ②

$$\frac{x_1}{\begin{vmatrix} 2 & -3 \\ -4 & -6 \end{vmatrix}} \Rightarrow \frac{-x_2}{\begin{vmatrix} -7 & -3 \\ 2 & -6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -7 & 2 \\ 2 & -4 \end{vmatrix}}$$

$$\frac{x_1}{-12-12} = \frac{-x_2}{42+6} = \frac{x_3}{28-4} \Rightarrow \frac{x_1}{-24} = \frac{-x_2}{48} = \frac{x_3}{24}$$

$$\frac{x_1}{-1} = \frac{-x_2}{2} = \frac{x_3}{1}$$

$[-1, -2, 1] \rightarrow$ These are eigen vectors for $\lambda = 5$.

Now, $\lambda = -3$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & 6 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow$$

$x_1 + 2x_2 - 3x_3 = 0$ — ④

$2x_1 + 4x_2 + 6x_3 = 0$ — ⑤

$-1x_1 - 2x_2 + 3x_3 = 0$ — ⑥

Considering ④ & ⑤

$$\frac{x_1}{\begin{vmatrix} 2 & -3 \\ 4 & 6 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & -3 \\ 2 & 6 \end{vmatrix}} \Rightarrow \frac{x_3}{\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix}}$$

$$\frac{x_1}{24} = \frac{-x_2}{12} = \frac{x_3}{0}$$

$$\frac{x_1}{2} = \frac{-x_2}{1} = \frac{x_3}{0}$$

$[2, -1, 0] \rightarrow$ eigen vectors for $\lambda = -3$.

Gayathri (3)

Nao, $\lambda = 2$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2x_1 + 0x_2 + x_3 = 0$$
$$-2x_1 - x_2 + x_3 = 0$$
$$-2x_1 + 0x_2 - x_3 = 0$$

$$\frac{x_1}{\begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 2 & 1 \\ -2 & 0 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 2 & 0 \\ -2 & -1 \end{vmatrix}}$$

$x_1 = 1$, $x_2 = -2$, $x_3 = -2$

$[1, -2, -2] \longrightarrow$ These are eigen vectors for $\lambda = 2$.

Now, $\lambda = 3$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 0 & 1 \\ -2 & 0 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & 1 \\ -2 & 0 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 0 \\ -2 & -2 \end{vmatrix}}$$

$$x_1 + 0x_2 + x_3 = 0$$
$$-2x_1 - 2x_2 + 0x_3 = 0$$
$$-2x_1 + 0x_2 - 2x_3 = 0$$

$$\frac{x_1}{2} = \frac{-x_2}{2} = \frac{x_3}{-2}$$

$x_1 = 2$; $x_2 = -2$; $x_3 = -2$.

$[2, -2, -2] \longrightarrow$ These are eigen vectors for $\lambda = 3$.

3. $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$ $\quad A \Rightarrow \begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$

The characteristic equation is $|A - \lambda I| = 0$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix}$$

Gayathri ③ 7

$$\begin{array}{c|cccc} 1 & 1 & -6 & 11 & -6 \\ & & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

$x^2 - 5x + 6 = 0$

$x^2 - 2x - 3x + 6 = 0$

$x(x-2) - 3(x-2) = 0$

$(x-2)(x-3) = 0$

$\Rightarrow x = 2$ & $x = 3$

$\therefore$ The roots of characteristic polynomial are 1, 2, 3.

So, they are eigen values of given matrix.

Let $X \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be eigen vectors corresponding to eigen value λ.

$[A - \lambda I] X = 0$

$$\begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix} - \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now, consider 1,

$$\begin{bmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{array}{l} 3x_1 + 0x_2 + x_3 = 0 \\ -2x_1 + 0x_2 + 0x_3 = 0 \\ -2x_1 + 0x_2 + 0x_3 = 0 \end{array}$$

$$\frac{x_1}{\begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix}} \Rightarrow \frac{-x_2}{\begin{vmatrix} 3 & 1 \\ -2 & 0 \end{vmatrix}} \Rightarrow \frac{x_3}{\begin{vmatrix} 3 & 0 \\ -2 & 0 \end{vmatrix}}$$

$$\Rightarrow \frac{x_1}{0} \Rightarrow \frac{-x_2}{0+2} \Rightarrow \frac{x_3}{0}$$

$x_1 = 0$; $x_2 = -2$; $x_3 = 0$

$[0, -2, 0] \rightarrow$ These are eigen vectors for $x = 1$.

Gayathri (9)

$5x_1 \Rightarrow 0$
$-x_1 + x_3 \Rightarrow 0$

By above equations,
$x_1 \Rightarrow 0;\ x_2 \Rightarrow 0;\ x_3 \Rightarrow 0$

Eigen vectors for $\lambda \Rightarrow 0 \rightarrow [0, 0, 0]$

Now, $\lambda \Rightarrow 3$

$$\begin{bmatrix} 5-3 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 3+3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow 2x_1 \Rightarrow 0$
$-3x_2 \Rightarrow 0$
$-x_1 + 6x_3 \Rightarrow 0$

By the above equations,
Eigen vectors for $\lambda \Rightarrow 3$,
$\Rightarrow [0, 0, 0]$

4) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix}$ $A \Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix}$ The characteristic equation is $|A - \lambda I| = 0$

$$\Rightarrow \begin{bmatrix} -\lambda & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 0 & -2-\lambda \end{bmatrix} \Rightarrow 0 \quad \Rightarrow -\lambda\big((3-\lambda)(-2-\lambda)\big) \Rightarrow 0$$

$\Rightarrow \lambda \Rightarrow 0, 3, -2$

So, the eigen values of given matrix is $0, 3, -2$.

Let $X \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be the eigen vectors corresponding to eigen value λ.

$[A - \lambda I]X \Rightarrow 0$

$$\begin{bmatrix} -\lambda & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 0 & -2-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow (5-\lambda)\left(-\lambda(3-\lambda)\right)$

$\Rightarrow (5-\lambda)(-3\lambda + \lambda^2) \Rightarrow -15\lambda + 5\lambda^2 + 3\lambda^2 - \lambda^3$

$\Rightarrow -\lambda^3 + 8\lambda^2 - 15\lambda = \lambda^3 - 8\lambda^2 + 15\lambda.$

This is characteristic polynomial.

Now, we have to find roots of this polynomial.

The equation,

$\lambda^3 - 8\lambda^2 + 15\lambda = 0 \rightarrow$ we can write this in form of

~~Hit~~ ~~Ef~~ ~~Trial~~ Method

$(5-\lambda)(-\lambda)(3-\lambda) = 0$

Therefore, the roots are $\lambda = 5, 0, 3$

Now,

$x_k \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be eigen vectors corresponding to eigen value λ.

$[A - \lambda I]\, X = 0$

$$\Rightarrow \begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow 0$$

Now, $\lambda \Rightarrow 5$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ -1 & 0 & 3-5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow 0$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ -1 & 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

By the above equations,

$\Rightarrow -5x_2 \Rightarrow 0$

$-x_1 - 2x_3 \Rightarrow 0$

$x_1 = 0;\ x_2 = 0;\ x_3 = 0.$

Eigen vectors for $\lambda \Rightarrow 5 \Rightarrow [0, 0, 0]$

Now, $\lambda \Rightarrow 0$

$$\begin{bmatrix} 5-0 & 0 & 0 \\ 0 & -0 & 0 \\ -1 & 0 & 3-0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow 0 \Rightarrow \begin{bmatrix} 5 & 0 & 0 \\ 0 & -0 & 0 \\ -1 & 0 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow 0.$$

Assignment (3)

Gayathri (5)

Are the followings sets of vectors linearly independent (or) dependent?

1) [1 0 0], [1 1 0], [1 1 1]

Let $\lambda_1, \lambda_2, \lambda_3$ are scalars.

Consider $\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 \Rightarrow 0 \Rightarrow \lambda_1(1,0,0) + \lambda_2(1,1,0) + \lambda_3(1,1,1)$

$\lambda_1 + \lambda_2 + \lambda_3 \Rightarrow 0$

$\lambda_2 + \lambda_3 \Rightarrow 0$ → in matrix form $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$\lambda_3 \Rightarrow 0$

By the equations,

$\lambda_1 \Rightarrow 0$; $\lambda_2 \Rightarrow 0$; $\lambda_3 \Rightarrow 0$

The three scalars are zero, so the given vectors are linearly independent there exists no solution.

2. [7, -3, 11, -6] [-56 24 -88 48]

$X_1 \Rightarrow [7, -3, 11, -6]$ $X_2 \Rightarrow [-56, 24, -88, 48]$

Let λ_1, λ_2 be scalars. Consider $\lambda_1 x_1 + \lambda_2 x_2 \Rightarrow 0$

$7\lambda_1 - 56\lambda_2 \Rightarrow 0$

$-3\lambda_1 + 24\lambda_2 \Rightarrow 0$

$11\lambda_1 - 88\lambda_2 \Rightarrow 0$

$-6\lambda_1 + 48\lambda_2 \Rightarrow 0$

$$\Rightarrow \begin{bmatrix} 7 & -56 & 0 \\ -3 & 24 & 0 \\ 11 & -88 & 0 \\ -6 & +48 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 7 & -56 & 0 \\ 0 & 0 & 0 \\ 11 & -88 & 0 \\ -6 & 48 & 0 \end{bmatrix}$$

$R_2 \to 7R_2 + 3R_1$ $R_2 \leftrightarrow R_4$

$$\Downarrow$$

$$\begin{bmatrix} 7 & -56 & 0 \\ 11 & -88 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 7 & -56 & 0 \\ 0 & 0 & 0 \\ 11 & -88 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Leftarrow \begin{bmatrix} 7 & -56 & 0 \\ -6 & 48 & 0 \\ 11 & -88 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to 7R_2 + 6R_1$

$R_2 \to 3R_2 - 11R_1$

Gayathri (6)

Gayathri (5)

Now, $\lambda \Rightarrow 0$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Longrightarrow \begin{array}{l} x_1 \Rightarrow 0 \\ 3x_2 + 4x_3 \Rightarrow 0 \\ -2x_3 \Rightarrow 0 \end{array}$$

By solving above three equations, $x_1 \Rightarrow 0;\ x_2 \Rightarrow 0;\ x_3 \Rightarrow 0$

Eigen vectors for $\lambda \Rightarrow 0$ is $[0,0,0]$.

Now, $\lambda \Rightarrow 3$

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{array}{l} -3x_1 \Rightarrow 0 \\ 4x_3 \Rightarrow 0 \\ -5x_3 \Rightarrow 0 \end{array}$$

By solving these equations $x_1 \Rightarrow 0;\ x_2 \Rightarrow 0\ \ x_3 \Rightarrow 0$

Eigen vectors for $\lambda \Rightarrow 3$ is $[0,0,0]$

Now, $\lambda \Rightarrow -2$

$$\begin{bmatrix} -2 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{array}{l} -2x_1 \Rightarrow 0 \\ 5x_2 + 4x_3 \Rightarrow 0 \\ -7x_3 \Rightarrow 0 \end{array}$$

By solving these equations

$x_1 \Rightarrow 0;\ x_2 \Rightarrow 0;\ x_3 = 0$

Eigen vectors for $\lambda \Rightarrow -2$ is $[0,0,0]$

5) For following matrix find one eigen value without calculation and justify your answer.

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

Ans:- Calculating det $\Rightarrow 1(6-6) - 2(3-3) + 3(2-2) \Rightarrow 0+0+0 \Rightarrow 0$

As the determinant of matrix is zero, product of all the eigen values is determinant of matrix.

Atleast one eigen value must be zero.

$R_2 \to 7R_2 - 11R_1$

Gayathri (6)

$$\begin{bmatrix} 7 & -56 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 7 & -56 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow \begin{aligned} 7\lambda_1 - 56\lambda_2 &\Rightarrow 0 \\ 0\lambda_1 + 0\lambda_2 &\Rightarrow 0 \\ 0\lambda_1 + 0\lambda_2 &\Rightarrow 0 \\ 0\lambda_1 + \lambda_4 &\Rightarrow 0 \end{aligned}$$

$\boxed{\lambda_1 \Rightarrow 0}$ $\boxed{\lambda_2 \Rightarrow 0}$

Two scalars are zero, the given two vectors are linearly independent.

3) $[-1\ 5\ 0]$ $[16\ 8\ -3]$ $[-64\ 56\ 9]$

c_1, c_2, c_3 are scalars.

$c_1v_1 + c_2v_2 + c_3v_3 \Rightarrow [0\ 0\ 0]$

$\cancel{-c_1 + 16c_1 - 64c_1 \Rightarrow 0}$ $\Rightarrow [-1\ 5\ 0]c_1 + [16\ 8\ -3]c_2 + [-64\ 56\ 9]c_3 \Rightarrow [0\ 0\ 0]$

$\cancel{5c_2 + 8c_2 + 56c_2 \Rightarrow 0}$ $\Rightarrow$

$\cancel{0c_3 - 3c_3 + 9c_3 \Rightarrow 0}$

$$\begin{aligned} -c_1 + 16c_2 - 64c_3 &\Rightarrow 0 \\ 5c_1 + 8c_2 - 3c_3 &\Rightarrow 0 \\ 0c_1 - 3c_2 + 9c_3 &\Rightarrow 0 \end{aligned}$$

$9c_3 = 3c_2 \Rightarrow \boxed{c_2 \Rightarrow 3c_3}$

$\cancel{5c_1 + 8(3c_3) - 3c_3 \Rightarrow 0}$ $\boxed{\cancel{5c_1 \Rightarrow -21c_3}}$

$5c_1 + 24c_3 - 3c_3 \Rightarrow 0$

$-c_1 + 16c_2 - 64c_3 \Rightarrow 0$

$16c_2 \Rightarrow 64c_3 + c_1$

$16(3c_3) \Rightarrow 64c_3 + c_1$

$48c_3 \Rightarrow 64c_3 + c_1$

$c_1 \Rightarrow -16c_3$.

Vector $\Rightarrow k \begin{bmatrix} 16 \\ 3 \\ 0 \end{bmatrix}$

$\Rightarrow$

4) $[1,-1,1]$, $[1,1,-1]$, $[-1,1,1]$, $[0,1,0]$ Gayathri (6)

$\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are scalars. Consider,

$$\lambda_1 v_1 + \lambda_2 v_2 + \lambda_3 v_3 + \lambda_4 v_4 \Rightarrow 0$$

$$\lambda_1(1,-1,1) + \lambda_2(1,1,-1) + \lambda_3(-1,1,\phi) + \lambda_4(0,1,0) \Rightarrow 0$$

$$\begin{bmatrix} 1 & 1 & -1 & 0 & \phi \\ -1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & -1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2 + R_1$ $\quad$ $R_3 \to R_3 + R_1$.

By this matrix,

$\lambda_1 + 0\lambda_2 - \lambda_3 + 0\lambda_4 \Rightarrow 0$ — (1)

$0\lambda_1 + 2\lambda_2 + \lambda_3 + \phi\lambda_4 \Rightarrow 0$ — (2)

$2\lambda_1 \Rightarrow 0$ — (3)

$\lambda_1 \Rightarrow 0$ Substituting in eq.(1), we get

$2\lambda_2 \Rightarrow -\lambda_4$ $\quad$ $\lambda_3 \Rightarrow 0$

$\lambda_2 \Rightarrow -\dfrac{\lambda_4}{2}$

$\therefore$ Since there are two non-zeroes. The given vectors are linear dependent.

5) $[2\ -4]$, $[1\ 9]$, $[3\ 5]$

$2c_1 + c_2 + 3c_3 \Rightarrow 0$ — (1)

$-4c_1 + 9c_2 + 5c_3 \Rightarrow 0$ — (2)

Multiply eq.(1) with 2 & add eq.(2)

$$\begin{aligned} \Rightarrow \quad & \cancel{4c_1} + 2c_2 + 6c_3 \Rightarrow 0 \\ (+) \quad & \cancel{-4c_1} + 9c_2 + 5c_3 \Rightarrow 0 \\ \hline & 11c_2 + 11c_3 \Rightarrow 0 \end{aligned}$$

$c_2 \Rightarrow -c_3$.

Let $c_3 \Rightarrow k$

$c_2 \Rightarrow -k$

Substitute c_3 value in eq.(2)

$-4c_1 + 9(-k) + 5(k) \Rightarrow 0$

$-4c_1 + 4k \Rightarrow 0$

$4c_1 \Rightarrow +4k$

$c_1 \Rightarrow +k$

Gayathri (7)

Now,

$$c_1 \Rightarrow +c_3 \qquad c_2 \Rightarrow -c_3 \qquad c_3 \Rightarrow c_3 \quad \Rightarrow \quad k\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

→ Hence all are non-zeroes The given vectors are linear dependent.

6) $[3\ -2\ 0\ 4]$, $[5\ 0\ 0\ 1]$ $[-6\ 1\ 0\ 1]$ $[2\ 0\ 0\ 3]$

$3c_1+5c_2-6c_3+2c_4 \Rightarrow 0$

$-2c_1 + c_3 \Rightarrow 0$

$4c_1+c_2+c_3+3c_4 \Rightarrow 0$

By row - echelon form,

$$\Rightarrow \begin{bmatrix} 3 & 5 & -6 & 2 & 0 \\ -2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 4 & 1 & 1 & 3 & 0 \end{bmatrix} \underset{R_3 \leftrightarrow R_4}{\Rightarrow} \begin{bmatrix} 3 & 5 & -6 & 2 & 0 \\ -2 & 0 & 1 & 0 & 0 \\ 4 & 1 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \underset{R_2 \to 2R_2+R_3}{\Rightarrow} \begin{bmatrix} 3 & 5 & -6 & 2 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 4 & 1 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\Downarrow$$

$$\begin{bmatrix} 3 & 5 & -6 & 2 & 0 \\ 0 & -17 & 27 & 1 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Leftarrow \begin{bmatrix} 3 & 5 & -6 & 2 & 0 \\ 4 & 1 & 1 & 3 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to 3R_2 - 4R_1$

By, the matrix,

$3c_1 + 5c_2 - 6c_3 + 2c_4 \Rightarrow 0$ — (1)

$-17c_2 + 27c_3 + c_4 \Rightarrow 0$ — (2)

$3c_3 \Rightarrow 0$ — (3) → Substituting in (2)

$c_4 \Rightarrow 17c_2$

Let $c_2 \Rightarrow k$

$c_4 \Rightarrow 17k$ Substituting in (1)

$3c_1 + 5k + 0 + 2(17k) \Rightarrow 0$

$\Rightarrow 3c_1 + 39k \Rightarrow 0$

$c_1 \Rightarrow -13k$

∴ Since there are non zeroes. The given vectors are linearly dependent.

Gayathri 7

7) $[3\ 4\ 7]\ [2\ 0\ 3]\ [8\ 2\ 3]\ [5\ 5\ 6]$

$C_1V_1 + C_2V_2 + C_3V_3 + C_4V_4 \Rightarrow 0$

$3C_1 + 2C_2 + 8C_3 + 5C_4 \Rightarrow 0$

$4C_1 + 0C_2 + 2C_3 + 5C_4 \Rightarrow 0$

$7C_1 + 3C_2 + 3C_3 + 6C_4 \Rightarrow 0$

By row-echelon form,

$$\Rightarrow \begin{bmatrix} 3 & 2 & 8 & 5 & 0 \\ 4 & 0 & 2 & 5 & 0 \\ 7 & 3 & 3 & 6 & 0 \end{bmatrix}$$

$\Rightarrow R_2 \to 3R_2 - 4R_1$

$$\begin{bmatrix} 3 & 2 & 8 & 5 & 0 \\ 0 & 0 & -26 & -5 & 0 \\ 7 & 3 & 3 & 6 & 0 \end{bmatrix}$$

$R_3 \to 3R_3 - 7R_1$

$$\Rightarrow \begin{bmatrix} 3 & 2 & 8 & 5 & 0 \\ 0 & 0 & -26 & -5 & 0 \\ 0 & -5 & -47 & -17 & 0 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\Rightarrow \begin{bmatrix} 3 & 2 & 8 & 5 & 0 \\ 0 & -5 & -47 & -17 & 0 \\ 0 & 0 & -26 & -5 & 0 \end{bmatrix}$$

$R_2 \to 2R_2 + 5R_1$

$$\Rightarrow \begin{bmatrix} 3 & 2 & 8 & 5 & 0 \\ 0 & 0 & -54 & -9 & 0 \\ 0 & 0 & -26 & -5 & 0 \end{bmatrix}$$

$R_3 \to 27R_3 - 13R_2$

$$\Rightarrow \begin{bmatrix} 3 & 2 & 8 & 5 & 0 \\ 0 & 0 & -54 & -9 & 0 \\ 0 & 0 & 0 & -18 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & 2 & 8 & 5 \\ 0 & 0 & -54 & -9 \\ 0 & 0 & 0 & -18 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow 3C_1 + 2C_2 + 8C_3 + 5C_4 \Rightarrow 0$

$-54C_3 - 9C_4 \Rightarrow 0$

$-18C_4 \Rightarrow 0$

$\boxed{C_4 \Rightarrow 0}$

$-54C_3 \Rightarrow 0$ $\boxed{C_3 \Rightarrow 0}$

$3C_1 + 2C_2 \Rightarrow 0$

$3C_1 \Rightarrow -2C_2$

$C_1 \Rightarrow \frac{-2}{3}C_2$

$\boxed{C_2 \Rightarrow k}$

$\boxed{C_1 \Rightarrow \frac{-2}{3}k}$

C_1, C_2 are non-zero values

So, the given vectors are linearly dependent.