Study of Magnetic Resonance with a Compass

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This experiment derived the resonance condition for a magnetic compass in an oscillating magnetic field. By systematically adjusting the distance between a magnet and the compass, we observed resonance. To provide a qualitative understanding, we also illustrated the relationship between resonance frequency and magnetic field. This experiment showcases a classical demonstration of magnetic resonance.

I. OBJECTIVE

Understanding the concept of magnetic resonance through a simple table-top experiment.

II. THEORY

Magnetic resonance is a quantum phenomenon that describes the resonant interaction between spins and electromagnetic fields. It has significant applications in fields such as medicine, science, and technology. Magnetic resonance imaging (MRI) is an indispensable non-invasive technique used in medical diagnosis and research. Nuclear magnetic resonance (NMR) spectroscopy is widely employed in physics, chemistry, biology, and materials science for examining the physical and chemical properties of substances, as well as for identifying their distinct characteristics.

When a magnetic needle with a magnetic moment $\vec{m}$ is placed in a magnetic field composed of a strong permanent magnet $\vec{B}_{PM}$ and the Earth's magnetic field $\vec{B}_E$, the torque acting on the needle is given by,

$$\vec{T} = \vec{m} \times (\vec{B}_{PM} + \vec{B}_E) \tag{1}$$

However, since the magnetic field of the permanent magnet is much stronger than that of the Earth's magnetic field, $\vec{B}_E$ can be neglected. Therefore, the torque simplifies to

$$\vec{T} = \vec{m} \times \vec{B}_{PM} \tag{2}$$

Now, consider an alternating magnetic field $\vec{B}_{\text{drive}}$ produced by a coil with alternating current. This drive field will cause the magnetic needle to oscillate in the horizontal plane (the compass plane). Since the permanent magnet is aligned along the x-axis, the sum of the torques due to the permanent magnet and the coil's field is equal to the time-derivative of the angular momentum of the needle,

$$\vec{m} \times (\vec{B}_{PM} + \vec{B}_{\text{drive}}) = J\frac{d\theta}{dt}\hat{z} \tag{3}$$

where J is the moment of inertia of the needle, and θ is the deflection angle. For small angles, we can approximate $\sin\theta \approx \theta$ and $\sin(\pi/2 - \theta) \approx 1$. Substituting these into the z-component of the torque equation,

$$mB_{PM}\sin\theta + mB_{\text{drive}}\sin\left(\frac{\pi}{2} - \theta\right) = J\frac{d\theta}{dt} \tag{4}$$

$$\implies mB_{PM}\theta + mB_{\text{drive}} = J\frac{d^2\theta}{dt^2} \tag{5}$$

This is a differential equation for a driven harmonic oscillator. The equation can be written as

$$\frac{d^2\theta}{dt^2} + \omega_0^2\theta = \omega^2 \tag{6}$$

where,

$$\omega_0 = \sqrt{\frac{mB_{PM}}{J}} \text{ and } \omega = \sqrt{\frac{mB_{\text{drive}}}{J}} \tag{7}$$

This equation describes a harmonic oscillator wit a natural frequency ω_0. When the driving frequency ω matches the natural frequency ω_0, resonance occurs, leading to a significant increase in the oscillation amplitude.

In the experiment, the frequency of the coil's alternating current is kept constant. By adjusting the distance d between the permanent magnet and the compass, the magnetic field $\vec{B}_{PM}$ experienced by the compass changes. The magnetic field of the permanent magnet at a distance d is given by the dipole approximation

$$B_{PM} = \frac{\mu_0}{4\pi}\frac{2m_{PM}}{d^3} \tag{8}$$

where m_{PM} is the magnetic moment of the permanent magnet. The resonance frequency is then

$$f_{\text{res}} = \frac{1}{2\pi}\sqrt{\frac{\mu_0}{4\pi}\frac{m_{PM}}{d^3}\frac{m}{J}} \tag{9}$$

This formula shows how the resonance frequency depends on the distance between the permanent magnet and the compass as well as on the magnetic moment of the permanent magnet. The final working formula is given by

$$f_{\text{res}} = \frac{1}{2\pi}\sqrt{B_{PM}\frac{m}{J}} \tag{10}$$

III. EXPERIMENTAL SETUP

Apparatus

1. Neodymium magnet
2. Function Generator
3. Circular Coiled wire
4. Platform with distance label
5. Compass

When we place a compass on a table, the needle naturally aligns with the Earth's magnetic field. If we then introduce a much stronger permanent magnet near the compass, the needle will realign itself towards the magnet. A similar effect occurs if we position the compass near an electromagnetic coil carrying a constant current.

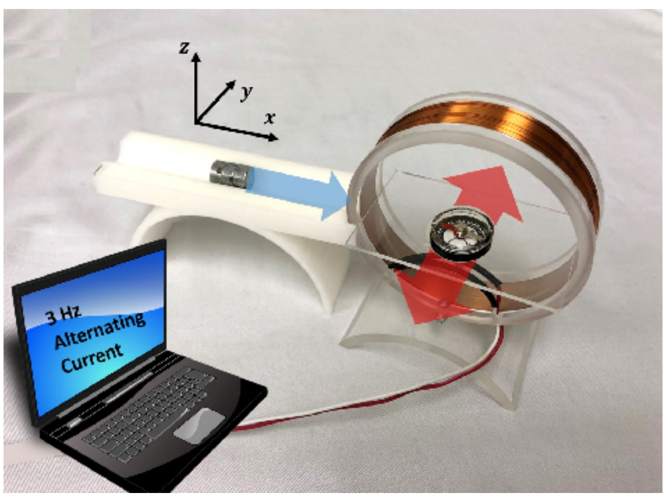

FIG. 1: Schematic Diagram of the setup

However, if we instead pass a weak alternating current through the coil, the needle won't fully follow the alternating magnetic field. Instead, it will oscillate around its initial alignment with the permanent magnet when both the permanent magnet and the electromagnetic coil are present. The oscillation of the needle remains minimal when the permanent magnet is either too close or too far from the compass. However, at an intermediate distance between the compass and the permanent magnet, the oscillations become significantly stronger, indicating resonance.

IV. PROCEDURE

1. Arrange the permanent magnet such that magnetic fields of the coil and the permanent magnet are perpendicular to each other.
2. Apply a sine wave with a frequency in the range of 1 to 3 Hz using the function generator, ensuring that the 20 dB button is pressed on the level button of the generator.
3. Slowly adjust the frequency to determine the resonance frequency for a fixed distance between the magnet and the coil.
4. Obtain the magnetic field versus distance data provided to you.

V. OBSERVATION AND CALCULATIONS

Outer radius of the coil = 8.4 cm
Least count for frequency measurement = 0.001 Hz

TABLE I: Table for Measurement of magnetic field and resonance frequency

distance (cm)	B_{PM}	f (Hz)
11.8	0.23	2.210
13.0	0.18	2.095
14.0	0.15	1.901
15.0	0.13	1.777
16.0	0.11	1.570
17.0	0.10	1.401
19.0	0.07	1.283

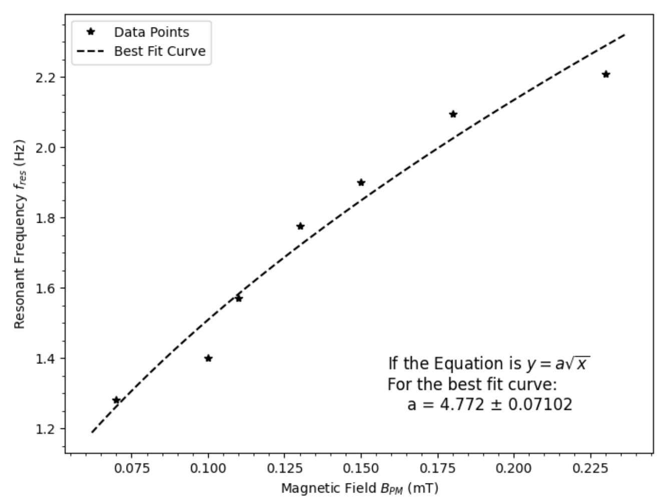

FIG. 2: Plot of f_{res} vs B_{PM}

VI. DISCUSSION AND CONCLUSION

The relation between magnetic field and the resonance frequency followed the expected trend. The fitted line is given by $f_{\text{res}} = (4.77 \pm 0.07)\sqrt{B_{PM}}$.

Additionally, knowing the fitted parameter and the moment of inertia of the compass needle will allow us to determine the magnetic moment of the compass or vice versa. This experiment qualitatively explored the relationship between resonance frequency and external magnetic field. Magnetic resonance is a phenomenon involving the resonant excitation of particle spins, which can be those of atomic nuclei or electrons. These microscopic particles possess intrinsic spin, resulting in magnetic moments. In a magnetic field, these moments experience a torque that aligns them with the field. An additional magnetic drive induces oscillations in these moments, leading to magnetic resonance.

VII. PRECAUTIONS

1. After finding the approximate distance move the magnet to and fro to get the precise location.
2. Change the position of magnet slowly in order. to get the proper resonance condition.
3. It must be ensure that magnet and coil are properly aligned with respect to each other.
4. At each position the needle must be given enough time to reach a steady state of oscillation.

[1] SPS, *Study of Magnetic Resonance with a Compass*, NISER (2023).