Determination of Coefficient of Thermal Expansion of Metals by Fizeau's Method

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(Dated: November 13, 2023)

In our everyday life, all of us have come across the expansion of metals upon heating - a quite familiar example would be the expansion of mercury in a thermometer on rising temperature. In this report, we will be studying the linear thermal expansion of brass and will determine its coefficient of thermal expansion. We observe that for small changes in temperature, the change in length of a metal is proportional to its original length and the change in temperature - the consant of proportionaity involved here is called the coefficient of thermal expansion(α). We will be using Fizeau's method - an optical method - to find the change in length of the metal bar and using that, we determine α.

I. OBJECTIVE

To determine the coefficient of thermal expansion (α) for a brass rod using Fizeau's interferometer.

II. THEORY

Consider a metal bar of length L and $\alpha(T)$ be its coefficient of thermal expansion. Then the rate of change of its length with respect to temperature is,

$$\frac{dL}{dT} = \alpha(T)L \tag{1}$$

For small changes in temperature, we can see that $\alpha(T)$ remains almost a constant and we can make an approximation

$$\frac{\Delta L}{\Delta T} \approx \alpha L \tag{2}$$

From the above expression, we can deduce the working formula

$$\alpha = \frac{1}{L}\left(\frac{\Delta L}{\Delta T}\right) \tag{3}$$

In order to determine α, we need to have changes in length of rod for different temperature changes. But this length changes are of the order of 10^{-3}, which is not easily measurable. This problem can be tackled by employing **Fizeau's method**.

In this method, we have two glass plates fixed together at one end and placed one (long plate) above the other (short plate). the non-fixed side of the long plate is allowed to rest on an insulator, which rests on the expanding rod. So the expansion of the rod will cause the upper glass plate to rise, creating a wedge shaped air film between the glass plates. The air film is then illuminated by a monochromatic light source of wavelength λ at normal incidence. Light rays reflected from the upper and lower glass plates will have a path difference (as shown in Fig. 1) and they interfere to produce a fringe pattern consisting of hyperbolic lines.The instrument that we would use to carry out the above experiment is Fizeau's interferometer — which consists of a monochromatic source, a glass that reflects the light rays towards the inclined glass plates, a holder for the metal bar and a travelling microscope to see the fringe pattern and measure fringe width.

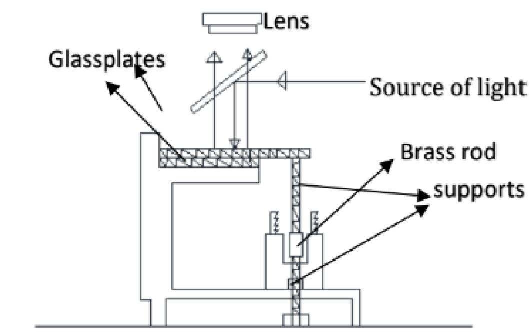

FIG. 1: Experimental Setup

Let t be the thickness of the air film between the plates at some point. The optical path difference between the direct and reflected rays at that point would be,

$$\Delta = 2t + \frac{\lambda}{2} \tag{4}$$

For minima, we have

$$\begin{aligned} \Delta &= (2n+1)\frac{\lambda}{2}, \;\; n = 0, 1, 2, ... \\ \implies 2t + \frac{\lambda}{2} &= (2n+1)\frac{\lambda}{2} \\ \text{or, } t &= \frac{n\lambda}{2} \end{aligned} \tag{5}$$

Now, consider two consecutive dark fringes - let t_1 and t_2 be the thickness of the air film at the points where dark fringes are formed. This is shown in Fig (2).

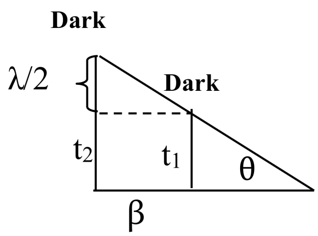

FIG. 2: Figure showing the points where two consecutive dark fringes are formed.

$$t_1 = m\frac{\lambda}{2}$$
$$t_2 = (m+1)\frac{\lambda}{2}$$
$$\implies t_2 - t_1 = \frac{\lambda}{2} \tag{6}$$

From the triangle shown in Fig. (2),

$$\tan\theta = \frac{t_2 - t_1}{\beta} = \frac{\lambda}{2\beta} \tag{7}$$

where, β is the fringe width, i.e, the distance between two consecutive minima or maxima.

As the temperature of the metal bar increases, it expands and pushes up the top glass plate, increasing the wedge angle. Let θ_1, θ_2 and β_1, β_2 be the respective wedge angles and fringe widths at temperatures T_1 and T_2. Then,

$$\tan\theta_1 = \frac{\lambda}{2\beta_1} \implies \theta_1 = \tan^{-1}\left(\frac{\lambda}{2\beta_1}\right)$$
$$\tan\theta_2 = \frac{\lambda}{2\beta_2} \implies \theta_2 = \tan^{-1}\left(\frac{\lambda}{2\beta_2}\right)$$

Therefore, by measuring fringe widths at different temperatures we can find out the corresponding wedge angles. Let $\Delta\theta$ be the change in the wedge angle caused by a ΔT change in temperature.

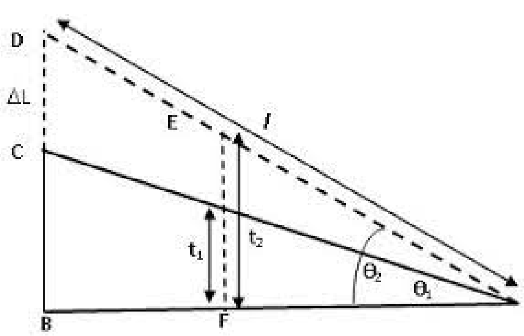

FIG. 3: Change in wedge angle with temperature increase

Let l be the length of the top glass plate. Since $\Delta\theta$ is small, we can see from ΔACB in Fig (3) that,

$$\Delta L = l\Delta\theta \tag{8}$$

Once we have found out ΔL, we can find α using,

$$\alpha = \frac{l}{L_{RT}}\left(\frac{\Delta L}{\Delta T}\right) \tag{9}$$

Using Eq. (8) we have,

$$\alpha = \frac{l}{L_{RT}}\left(\frac{\Delta\theta}{\Delta T}\right) \tag{10}$$

where, L_{RT} is the length of the metal plate at room temperature. Also, $\Delta\theta/\Delta T$ is given by the slope of θ vs T plot. Hence, we can find out α.

Typical values of α for aluminium, copper and brass at room temperature are 23.1×10^{-6} K^{-1}, 16.6×10^{-6} K^{-1} and 20.3×10^{-6} K^{-1} respectively.

III. EXPERIMENTAL SETUP

Apparatus

1. Two Glass plates
2. Fizeau's interferometer
3. Thermocouple and temperature indicator
4. Travelling Microscope
5. Variable transformer
6. Sodium vapour lamp
7. Brass rod
8. Heater

The schematic design of the experimental set up is shown in Fig. (1). It consists of an interferometer assembly along with a travelling microscope attached. The sample is usually a small metal rod — brass in this case — which is sorrounded by a heater, which is a small cement tube over which a heating element is wound. A thermocouple is inserted such that it is close to the sample. As the metal expands, the support rod on top of the metal will move up, essentially creating a wedge between the two glass plates.

IV. OBSERVATIONS

- Room temperature = 24.4 °C = 297.55 K
- Length of glass plate (from the point of suspension) = 5.5 cm
- Wavelength of light used (λ)= 589.3×10^{-9} m

Measurement of length of rod at room temperature

Least count of vernier calipers = 0.01 cm

MSR (cm)	VSR (cm)	Total (cm)	L_{RT} (cm)
2	0.06	2.06	
2	0.07	2.07	2.063
2	0.06	2.06	

TABLE I: Measurement of the length of rod using a vernier caliper

Measurement of fringe width (β)

Least count of the travelling microscope = 0.01 mm Data given in Table (II).

V. DATA ANALYSIS AND CALCULATIONS

Using the data shown in Table (II), we make the following θ vs T plot using least-square fitting.

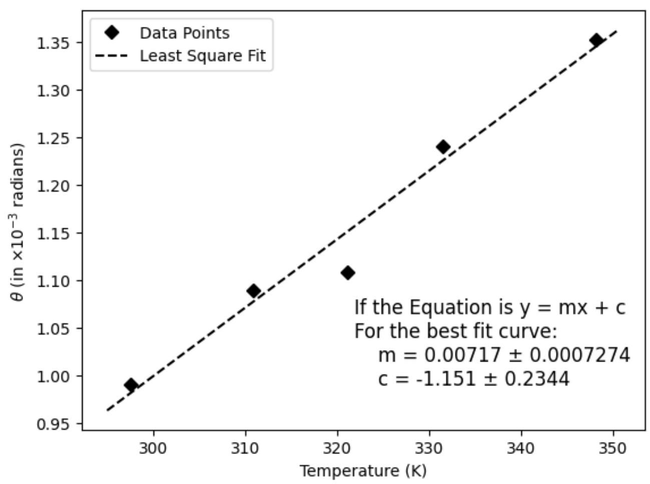

FIG. 4: θ vs T

Here slope of the θ vs T plot comes out to be $m = 0.007145 \times 10^{-3}$ K^{-1}. Using Eq. (8), we get,

$$\begin{aligned}\alpha &= \frac{l}{L_{RT}} \times m \\ &= \frac{5.5}{2.063} \times 0.00717 \times 10^{-3} \\ &= 19.114 \times 10^{-3}\,\mathrm{K}^{-1}\end{aligned}$$

VI. ERROR ANALYSIS

From Eq. (8), the uncertainity in the measurement of α is,

$$\frac{\Delta\alpha}{\alpha} = \sqrt{\left(\frac{\Delta l}{l}\right)^2 + \left(\frac{\Delta L_{RT}}{L_{RT}}\right)^2 + \left(\frac{\Delta \mathrm{slope}}{\mathrm{slope}}\right)^2} \tag{11}$$

From Fig. (4), there uncertainity in slope is 0.0007274×10^{-3} K^{-1}. Similarly, $\Delta l = 0.1$ cm and $\Delta L_{RT} = 0.01$ cm, are the least count of the respective measuring instruments. Hence, we can find $\Delta\alpha$ as,

$$\begin{aligned}\frac{\Delta\alpha}{\alpha} &= \sqrt{\left(\frac{0.1}{5.5}\right)^2 + \left(\frac{0.01}{2.063}\right)^2 + \left(\frac{0.0007274}{0.00717}\right)^2} \\ \implies \Delta\alpha &= 19.114 \times 10^{-3} \times 0.0188 \\ &= 0.360 \times 10^{-3}\,\mathrm{K}^{-1}\end{aligned}$$

VII. RESULTS AND CONCLUSION

From the experiment, the coefficient of thermal expansion of Brass was measured to be,

$$\alpha = (19.114 \pm 0.360) \times 10^{-3}\,\mathrm{K}^{-1}$$

We found that the θ vs T gives us a nearly straight-line plot, as predicted. The standard value of the coefficient of thermal expansion of Brass is,

$$\alpha_{\mathrm{std}} = 20.3 \times 10^{-3}\,\mathrm{K}^{-1}$$

Hence, there is a 5.8% deviation from the standard value. This could be due to various reasons including

(i) disturbance of the apparatus which can cause the optical paths to be disturbed hence altering the fringe widths
(ii) missing fringes during the measurement
(iii) the apparatus in which the brass rod is kept can expand on heating and hence alters the wedge angle and thereby the fringe width.

VIII. PRECAUTIONS

1. Make sure that the fringes are vertical and they remain vertical even after the plate is lifted for some wedge angle. If not, adjust the glass plates so that you obtain vertical fringes.
2. Do not disturb the apparatus while taking measurements.
3. Handle the glass plates carefully without contaminating its surface.

Temperature in °C	in K	No. of Fringes	Microscope reading (mm)	Width of 5 fringes (mm)	Avg. width of 5 fringes (mm)	Fringe width (β) (mm)	Wedge angle (θ) in 10^{-3} rad
24.3	297.45	0	8.82	-	1.488	0.298	0.990
		5	8.82	1.42			
		10	8.82	1.48			
		15	8.82	1.52			
		20	8.82	1.53			
37.7	310.85	0	8.77	-	1.353	0.271	1.089
		5	10.04	1.27			
		10	11.38	1.34			
		15	12.75	1.37			
		20	14.18	1.43			
48.0	321.15	0	8.99	-	1.330	0.266	1.108
		5	10.26	1.27			
		10	11.56	1.30			
		15	12.91	1.35			
		20	14.31	1.40			
58.4	331.55	0	9.09	-	1.188	0.238	1.241
		5	10.24	1.15			
		10	11.43	1.19			
		15	12.60	1.17			
		20	13.84	1.24			
75.0	348.15	0	8.94	-	1.090	0.218	1.352
		5	10.02	1.08			
		10	11.09	1.07			
		15	12.20	1.11			
		20	13.30	1.10			

TABLE II: Measurement of fringe width (β)

[1] SPS. *Lab Manual: Coefficient of thermal expansion of metals by Fizeau's interferometer.* NISER, 2023.