Study of Compton Scattering

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(Dated: March 16, 2025)

Compton scattering describes the inelastic interaction of high-energy photons with electrons, leading to a wavelength shift dependent on the scattering angle. In this experiment we try to study the energy calibration of scintillator and its calibration factor using compton scattering. We also study the wavelength shift using both copper and aluminum as scatter bodies. In both situations, we found that the relative energy and intensity of scattered photons dropped with increasing scattering angle.

I. THEORY

Compoton scattering is a physical event that validated the dual nature of light, which had been previously established in photoelectric effect.This phenomenon is essential in medical imaging, astrophysics, and material science, aiding in X-ray diagnostics, cosmic radiation studies, and non-destructive testing. The phenomenon involves the scattering of a photon from a metal object. The photon collides with an electron within the target, losing part of its energy due to the collision principle of conservation of energy and momentum (Fig. 1). Similarly, the photon changes its propagation path, resulting in varying intensities of detection at different angles, but the electron is expelled from its starting state and, following energy absorption, excites to a higher energy level.

A. Wave Particle Duality

Light acts as both a wave and a particle, according to the wave-particle duality of light, and the Compton effect offered experimental proof that photons, which are light particles, have both wave-like and particle-like qualities. Using the conservation of momentum and energy in particle-photon collisions, we find that the wavelength of the scattered ray is greater than the wavelength of the incident ray, and that the observed shift in the wavelength of scattered photons is directly related to the energy and momentum exchanged between the photon and the electron.

Compton scattering is an example of inelastic collision and can be expressed in following way:

$$\gamma + e \rightarrow \gamma^{'} + e^{'}$$

where γ and $\gamma^{'}$ are the incident and scattered photons respectively, and e and $e^{'}$ are the initial and final electrons respectively.

The derivation of Compton scattering is based on two principles conservation of energy and conservation of momentum to the interaction between the photon and the electron. Let's assume that the photon has an initial energy of E and is incident on an electron at rest. After the interaction, the scattered photon has an energy of $E^{'}$ and moves in anew direction, while the electron recoils with a final momentum $p^{'}$. Using conservation of energy, we can write:

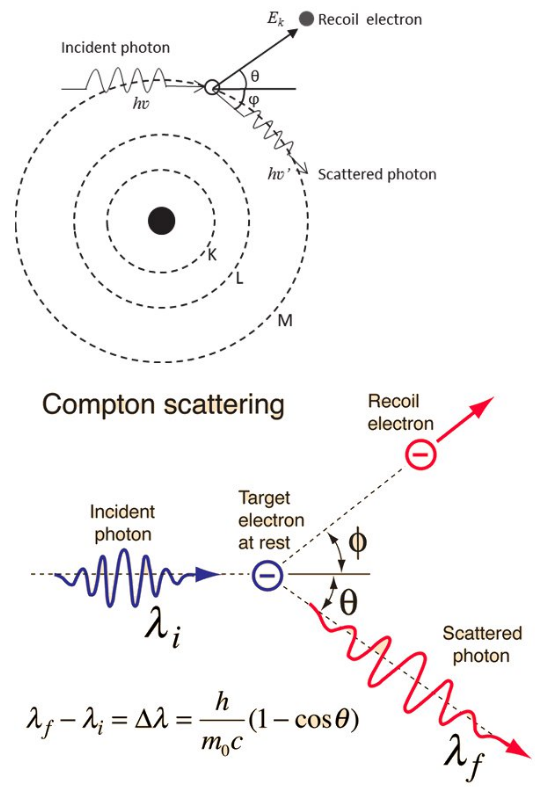


FIG. 1: An illustration depicting Compton scattering

$$E + mc^2 = E^{'} \sqrt{m^2c^4 + p^{'2}c^2} \tag{1}$$

where m is the mass of the electron, and c is the speed of light. Using conservation of momentum, we can write:

$$p_\gamma = p_\gamma^{'} + p_e^{'} \tag{2}$$

where p_γ and $p_\gamma^{'}$ are the momenta of the incident and scattered photons, respectively, and $p_e^{'}$ is the momentum of the recoiling electron. The shift of the wavelength ($\Delta\lambda$) increased with scattering angle according to the Compton formula as more scattering angle implies greater loss in energy:

$$\Delta\lambda = \lambda_\theta - \lambda_0 = \frac{h}{m_e c}(1-\cos\theta) \tag{3}$$

where λ_θ and λ_0 are wavelengths of scattered and initial photons respectively, h is Planck's constant, m_e is the rest mass of the electron, c is the velocity of light, θ and ϕ are angles of scattered photon and recoil electron respectively. The value of ($h/m_e c = 0.02426$ Å) is known as the Compton wavelength of the electron.

In terms of energy, Eq. 3 can be rewritten as:

$$E_\theta = \frac{E_0}{(1+\gamma)(1-\cos\theta)} \tag{4}$$

As a result, Compton scattering is significantly energy-dependent, and the relevant energy scale is determined by the ratio of input photon energy to electron rest energy. The fractional shift in energy is considerable if this ratio is big, and negligible if this ratio is small. Compton scattering is important only when the incoming photon energy is a considerable proportion of the electron's rest energy. The equation also demonstrates that the change in wave length of the scattered photon is proportional to the scattering angle and the photon's original energy. For high-energy photons ($\lambda << 0.02$Å or $E >> 511$ keV), the wavelength of the scattered photon is comparable to the Compton wavelength, while for low-energy photons ($E << 511$ keV), the Compton shift is negligible. In the non-relativistic regime, Compton scattering closely aligns with the predictions of classical Thomson scattering.

It also illustrates that the scattered photon loses energy, and this lost energy is passed to the recoiling electron.

Using quantum mechanical calculations, Klein-Nishina correctly formulated the differential Compton scattering cross-section formula. This equation is written as follows:

$$\begin{aligned} \frac{d\sigma}{d\Omega} =& r_o^2\left[\frac{1+\cos^2\theta}{2(1+\gamma(1-\cos\theta)^2)}\right]\times \\ &\left[1+\frac{\gamma^2(1-\cos\theta)^2}{(1+\cos^2\theta)(1+\gamma(1-\cos\theta))}\right] \end{aligned} \tag{5}$$

where, $r_o = \frac{e}{4\pi\epsilon_o m_e c^2} = 2.8179\times10^{-15}$ m is the classical electron radius.

For dispersed photons, gamma rays from a Cs^{137} source are used in this experiment. A calibrated scintillation detector set at varying scattering angles determines differences in incoming and scattered energy and wavelength of photons. By computing the calibration factor C using the method below, the relative intensities I_θ of the scattered radiation peaks may be compared with the predictions of the Klein-Nishina formula for the differential effective cross-section $\frac{d\sigma}{d\Omega}$. Thus:

$$C = \frac{1}{n}\sum_{\theta=0}^{n}\frac{I_\theta}{\left(\frac{d\sigma}{d\Omega}\right)} \tag{6}$$

II. EXPERIMENTAL SETUP

Apparatus

1. Cs-137 radioactive gamma source
2. Mixed radioactive source for calibration (Am-241 and Cs-137)
3. Lead block source holder with a 12 mm diameter hole at the center to accommodate radioactive sources, and an additional blind hole for inserting a steel pin as an angular direction indicator
4. NaI scintillation detector with a holder and lead shielding to define the direction of incoming gamma radiation
5. High voltage power supply (1.5 kV)
6. Cylindrical pure aluminum or copper rod to be used as a scattering body
7. Movable lead shielding to reduce the intensity of unscattered gamma radiation
8. Multichannel analyzer (256 channels) with associated software (CASSY Lab 2) installed on a desktop PC.
9. Experimental panel with a graduated angular scale

A radioactive Cs-137 source emits 662 keV gamma rays, which escape through a small opening in the shielded cavity (Fig. 2). The collimated beam is directed onto an aluminum rod, serving as the target or scatterer. Electrons in the target scatter a portion of the gamma rays, which are then detected and counted by the scintillation detector.

A scintillation detector is a device used to detect and measure ionizing radiation. It operates by utilizing the excitation effect of incoming radiation on a scintillating material, which produces light pulses that are subsequently detected by photodetectors such as a photomultiplier tube (PMT), a charge-coupled device (CCD) camera.

The resulting signal is processed by a multichannel analyzer (MCA), and the corresponding spectrum is displayed on the computer. By adjusting the source to various angles on the angular scale of the experimental panel, the scattered radiation is measured to analyze the angular dependence of Compton scattering.

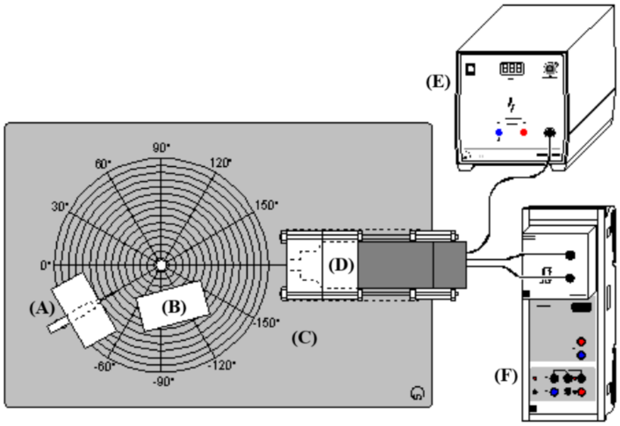


FIG. 2: The experimental setup, with components (A) the Lead block source holder (B) Additional Lead block for shielding (C) graduated angular scale (D) the scintillation detector (E) High-voltage power supply and (F) MCA connected to a computer

III. OBSERVATION AND CALCULATIONS

A. Energy calibration of scintillation detector

At first, the scintillation detector is calibrated using a mixed source of Americum and Cesium. The resulting spectra is shown in Fig. 3. The two peaks are calibrated to 59.54 keV (due to Am-241) and 661.66 keV (due to Cs-137).

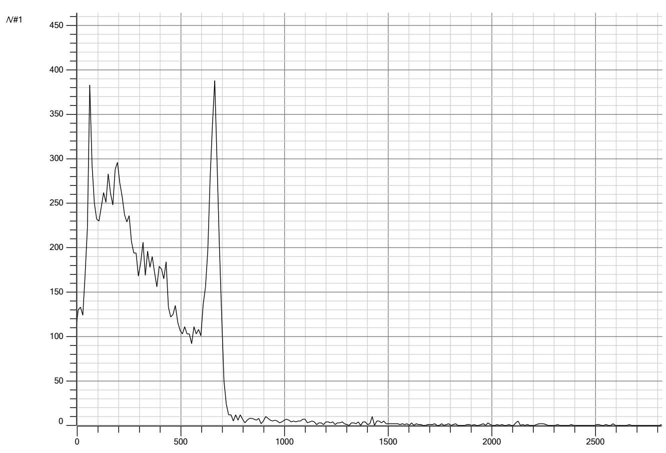


FIG. 3: Energy Callibration Curve

B. Change in wavelength of the scattered γ radiation as a function the θ

The scattering spectra observed for different scattering angles are shown in Fig. 4. A gaussian is fitted for each peak and the best-fit μ value corresponds to E_θ. Tables I and II summarise the corresponding ΔE and $\Delta\lambda$ values for both the scattering bodies.

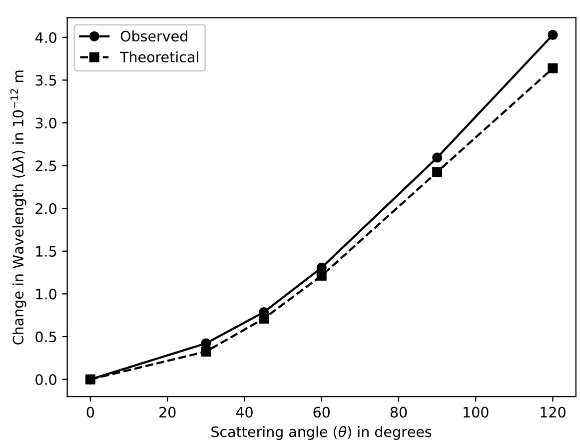
θ (°)	E (keV)	$\Delta E_{\rm obs}$ (keV)	$\Delta E_{\rm th}$ (keV)	λ (pm)	$\Delta\lambda_{\rm obs}$ (pm)	$\Delta\lambda_{\rm th}$ (pm)
0	665.0	0.0	0.0	1.86	0	0.000
30	542.4	122.6	97.9	2.29	0.421	0.325
45	467.9	197.1	182.1	2.65	0.785	0.711
60	391.0	274.0	260.2	3.17	1.307	1.213
90	278.1	386.9	373.6	4.46	2.594	2.426
120	210.4	454.6	437.1	5.89	4.029	3.639

TABLE I: E and the corresponding ΔE and $\Delta\lambda$ values as function of θ for the Aluminium scattering body based on Fig. 4.

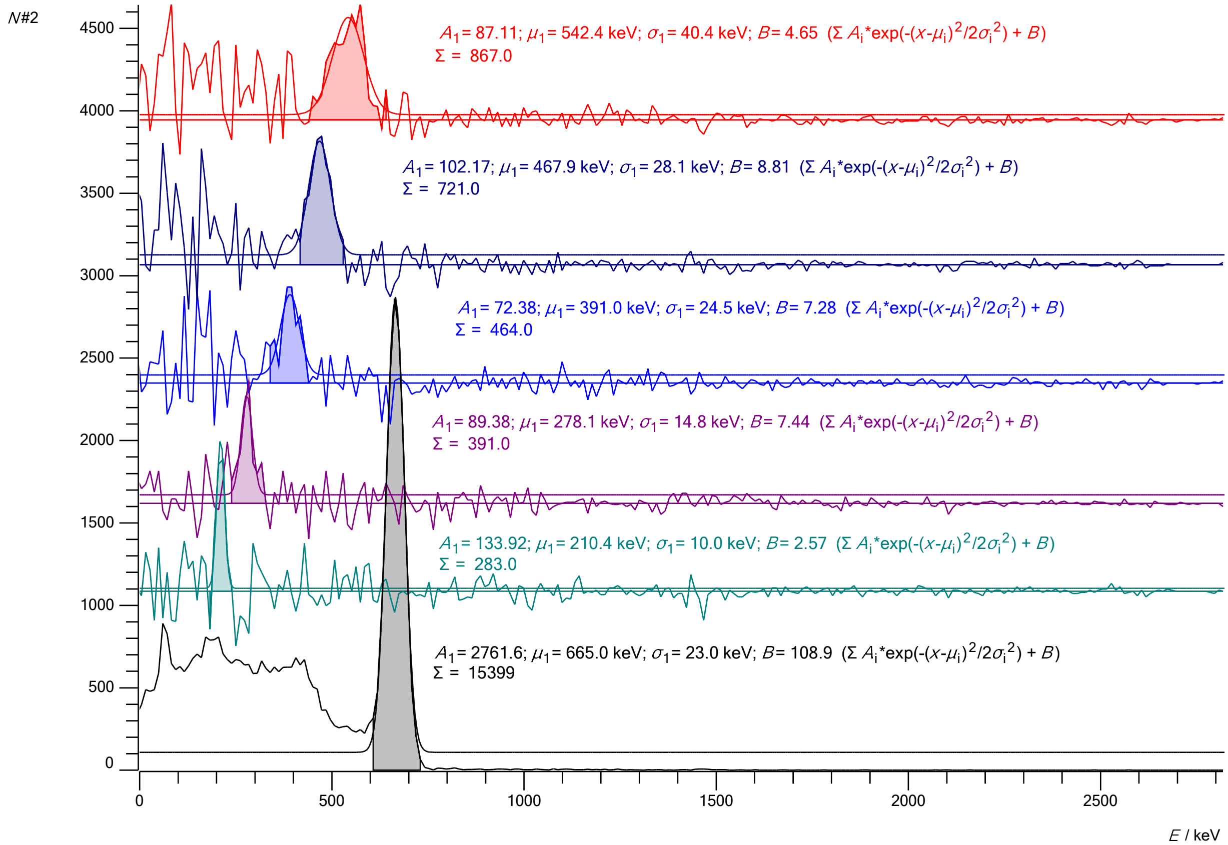
θ (°)	E (keV)	$\Delta E_{\rm obs}$ (keV)	$\Delta E_{\rm th}$ (keV)	λ (pm)	$\Delta\lambda_{\rm obs}$ (pm)	$\Delta\lambda_{\rm th}$ (pm)
0	666.6	0.0	0.0	1.86	0	0.000
30	551.4	115.2	97.9	2.25	0.389	0.325
45	469.8	196.8	182.1	2.64	0.779	0.711
60	399.1	267.5	260.2	3.11	1.247	1.213
90	280.0	386.6	373.6	4.43	2.568	2.426
120	215.4	451.2	437.1	5.76	3.897	3.639

TABLE II: E and the corresponding ΔE and $\Delta\lambda$ values as function of θ for the Copper scattering body based on Fig. 4.

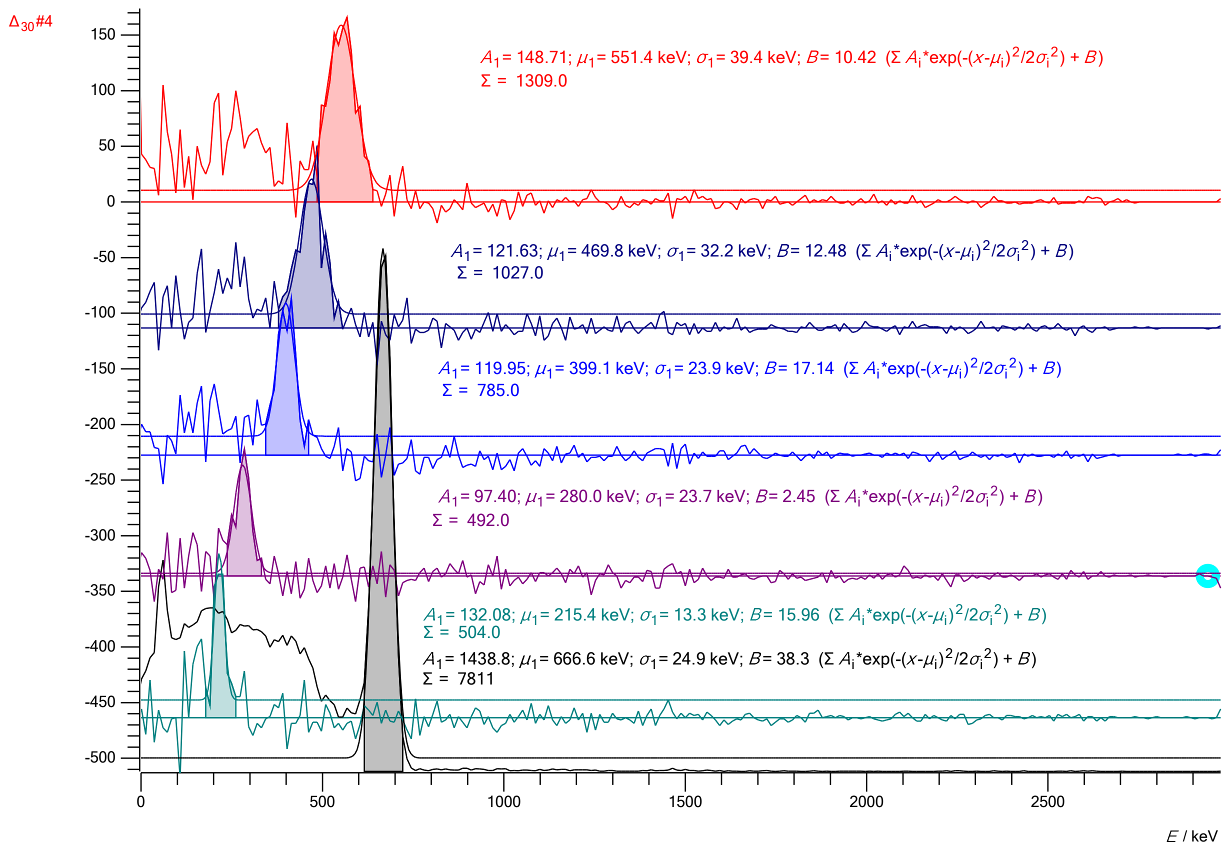
Fig. 5 shows the variation in experimental and theoretical $\Delta\lambda$ values as a function of θ.



(a) Aluminium scattering body

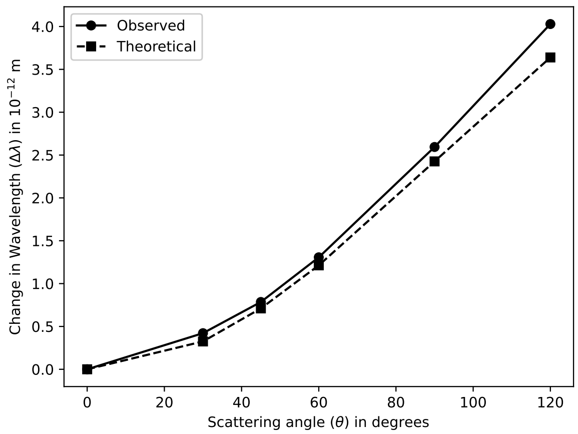


(a) Aluminium scattering body



(b) Copper scattering body

FIG. 4: Scattering Spectra due to Compton scattering for different scattering angles



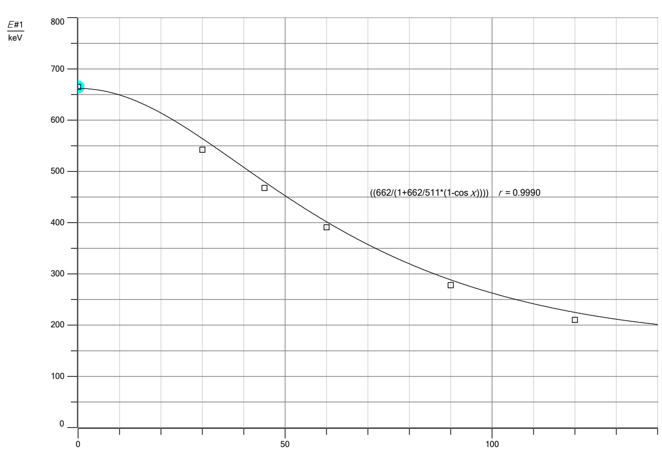
(b) Copper scattering body

FIG. 5: Comparison of Wavelength shift due to Compton scattering for copper theoretically and experimentally

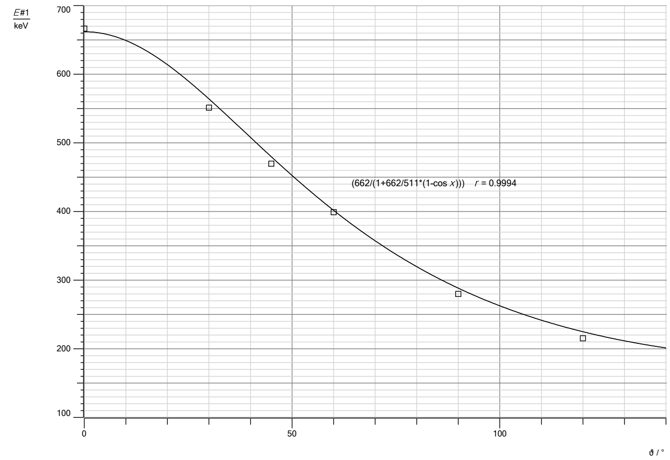
Fig. 6 shows the variation in E as a function of θ. The data points have been fitted to the function based on Eq. 4,

$$E = \frac{662}{1 + \frac{662}{511}(1 - \cos\theta)}$$

where E_0 is 662 keV and $m_e c^2$ is 511 keV. We can see that our data agrees with the theoretical predictions with a pretty good r-value.



(a) Aluminium scattering body, r-value = 0.9990



(b) Copper scattering body, r-value = 0.9994

FIG. 6: Energy vs Scattering angle due to Compton scattering for different scattering angles

C. Calculation of Differential cross-section and the Calibration Factor

Table III shows the intensity distribution across different scattering angles measured from the area under the peaks from Fig. 4. The differential cross section has been calculated using the Klein-Nishina formula (Eq. 5), which is illustrated in Fig. 7.

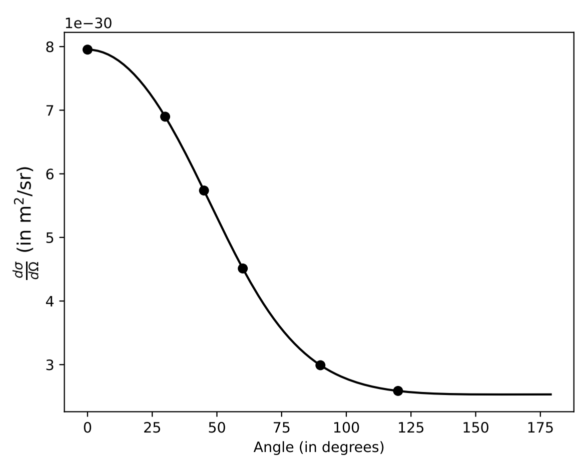


FIG. 7: $\frac{d\sigma}{d\Omega}$ calculated using the Klein-Nishina formula as a function of θ for $E_0 = 662$ keV

θ ($^\circ$)	$\frac{d\sigma}{d\Omega}$ ($\times 10^{-30}$ m^2)	I_θ for Al	I_θ for Cu
0	7.941	15399	7811
30	6.890	867	1309
45	5.733	721	1027
60	4.513	464	785
90	2.994	391	492
120	2.569	283	504

TABLE III: Differential cross section and observed intensity distribution across different scattering angles for both the scattering bodies

Now, using Eq. 6, the Calibration Factor (C) can be calculated. This comes out as,

- Al: 2.11×10^{31} m^{-2}
- Cu: 1.57×10^{31} m^{-2}

IV. ERROR ANALYSIS

The error in the calibration factor can be calculated using

$$\frac{\Delta C}{C} = \sqrt{\sum_{i=0}^{n} \frac{\Delta \frac{d\sigma}{d\Omega}}{\frac{d\sigma}{d\Omega}}} \tag{7}$$

Plugging in the values the error comes out ot be 0.017×10^{31} m^{-2}.

V. DISCUSSION & CONCLUSION

We have successfully conducted an experiment analysing Compton scattering of electrons.

We first calibrated the MCA setup using a mixed source of Americum and Caesium. Then, using different scattering bodies we observed the compton effect using a Cs-137 source.

The observed data in our experiment shows that the intensity of scattered photons decreases as the scattering angle increases. This observation agrees with the Compton formula, which describes the energy transfer from photons to electrons in the scattering process. We also found that the energy of scattered photons decreases as the scattering angle increases, which again agrees with the Compton formula. This is because the energy of scattered photons is related to the energy lost by the electrons in the scattering process.

Our experiment also revealed that the detector used in the experiment had different calibration factors for different materials. The calibration factors obtained are:

- For Aluminium: $(2.11 \pm 0.02) \times 10^{31}$ m^{-2}
- For Copper: $(1.57 \pm 0.02) \times 10^{31}$ m^{-2}

This implies that for particular cross-sections, the relative intensity for cooper is more than for aluminium.

By measuring the scattered photon energy and angle, along with the incident photon energy, it is possible to calculate the rest mass of the electron. This technique is widely used in experimental physics to measure the masses of particles.

VI. PRECAUTIONS AND SOURCES OF ERROR

1. Properly shield the setup before taking data.
2. Measure the relative intensities using the software by only taking the appropriate energy ranges.

[1] SPS, *Study of Compton Scattering*, NISER (2023).

[2] L. D. GmbH, *Quantitative observation of the Compton effect*, LD Physics leaflets (2023).