Differentiation, Integration and Active Filtering using an Op-Amp

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In this experiment we try to construct differentiator and integrator circuits using op-amps and study their properties. We also discuss their faults and ways to rectify them. Furthermore, we also build and study active high-pass and low-pass filters using op-amps and their frequency response curves.

I. OBJECTIVE

1. To study Op-amp as a Differentiator
2. To study Op-amp as an Integrator
3. To construct active low and high pass filters using Op-amp

II. THEORY

By introducing an RC network across the Operational Amplifier, we can perform mathematical functions that include integration and differentiation. Because of the RC component, these circuits can also behave as filters and have a frequency response curve.

In particular, op-amp filters are *active filters*, meaning that due to the applied power source, they can apply additional gain to the signal. This is different from passive filters built using just resistors, capacitors and inductors, which attenuate the entire signal and hence the maximum gain one can achieve is 1.

A. The Integrator Amplifier

If the feedback resistor in an inverting amplifier is replaced by a capacitor as shown in Fig. 10, the new op-amp circuit is known as an integrator.

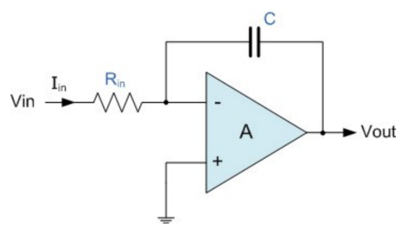

FIG. 1: Integrator Circuit Diagram

When a voltage, V_{in} is firstly applied to the input, the uncharged capacitor C has very little resistance and acts as a short circuit (voltage follower circuit) giving an overall gain of less than 1, thus resulting in zero output. As the feedback capacitor begins to charge up, the ratio of Z_f/R_{in} increases producing an output voltage that continues to increase until the capacitor is fully charged. At this point the ratio Z_f/R_{in} is infinite resulting in infinite gain and the output of the amplifier goes into saturation (Fig.11).

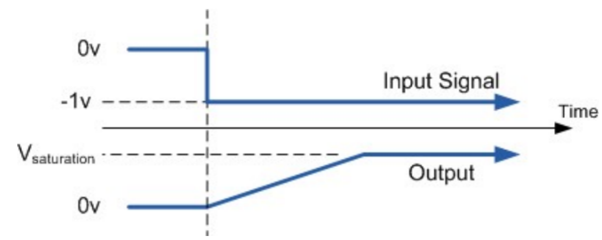

FIG. 2: Integrator circuit sample input and output curves

The rate at which the output voltage increases is determined by the RC time constant. By changing this value, the time in which it takes V_{out} to reach saturation can be changed.

Mathematically, we can express current flowing through the feedback capacitor as,

$$I_{\text{in}} = \frac{V_{\text{in}}}{R} = -C\frac{V_{\text{out}}}{dt} \quad (\because Q_{\text{in}} = CV_{\text{out}})$$
$$\implies V_{\text{out}}(t) = -\frac{1}{RC}\int V_{\text{in}}(t)\,dt \tag{1}$$

Here, the gain can be expressed as,

$$A_V = -\frac{X_C}{R} \tag{2}$$

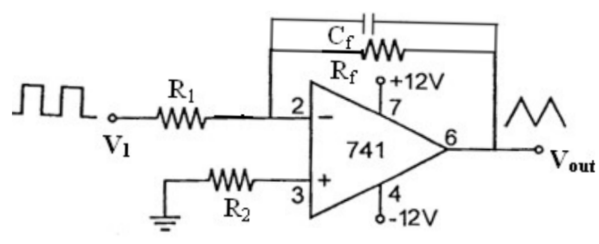

FIG. 3: **Practical Integrator Circuit** with additional components – (i) R_f to produce stable biasing at DC and limits the gain of the circuit and (ii) R_2, which is the offset minimizing resistor which reduces output offset voltage due to input bias current

B. Active Low Pass Filter

From Eq. 2, one can infer that as $f \to \infty$, $X_C \to 0$ and hence the gain $A_V \to 0$. Thus the circuit can attenuate high-frequency signals while letting low-frequency signal pass. However, the circuit behaves like a high-gain inverting amplifier at DC, when $f = 0$. Hence it is viable to introduce a feedback resistance in parallel to the capacitor which produces stable biasing by restoring feedback at DC, so that the output voltage does not saturate.

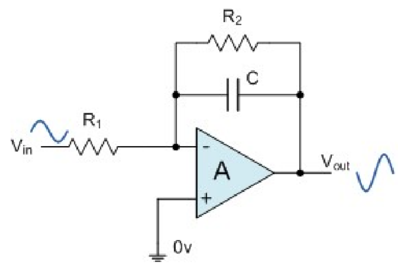

FIG. 4: Circuit diagram for an Active Low Pass Filter

Maximum gain is achieved when $f = 0$, i.e. $A_V = R_2/R_1$. Hence, the frequency response would be something like Fig. 5.

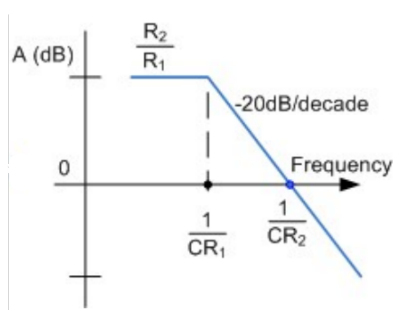

FIG. 5: Typical Frequency Response curve for an active low pass filer

C. The Differentiator Amplifier

Here, the capacitor is connected to the input terminal of the inverting amplifier while the resistor, R_f forms the negative feedback element across the operational amplifier. So the circuit produces an output voltage which is proportional to the rate-of-change of the input voltage and the current flowing through the capacitor.

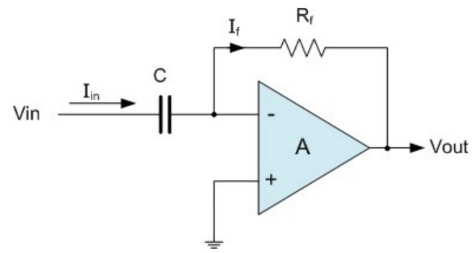

FIG. 6: Differentiator Circuit Diagram

From the circuit (Fig. 6), we can express,

$$I_f = -\frac{V_{\text{out}}}{R_f} \text{ and } I_{\text{in}} = C\frac{dV_{\text{in}}}{dt} \tag{3}$$

Since we assume that no current passes through the op-amp,

$$\begin{aligned} I_f &= I_{\text{in}} \\ \implies V_{\text{out}} &= -R_f C\ \frac{dV_{\text{in}}}{dt} \end{aligned} \tag{4}$$

Hence, the output depends on the rate of change of the input signal. The gain of the amplifier is thus,

$$A_V = -\frac{R_f}{X_C} \tag{5}$$

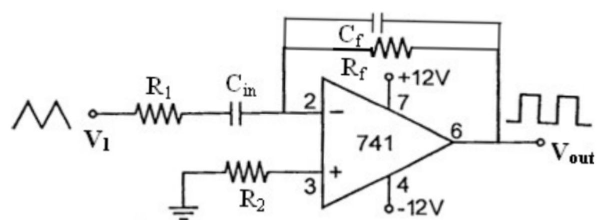

FIG. 7: **Practical Differentiator Circuit** with additional component – (i) C_f in parallel with R_f to control gain at high frequencies, (ii) R_1 at the input in series with C_{in} to drop the noise at the input and (iii) R_2, the offset minimizing resistor which reduces output offset voltage due to input bias current

D. Active High Pass Filter

Similar to the active low pass filter one can infer from Eq. 5 that the gain is directly proportional to the frequency of the input signal. Thus the circuit can attenuate low-frequency signals while letting high-frequency signals pass. Thus, the capacitor blocks any DC content only allowing AC-type signals whose frequency is dependent on the rate of change of the input signal.

An ideal high pass filter will have a frequency response curve that looks like Fig. 8.

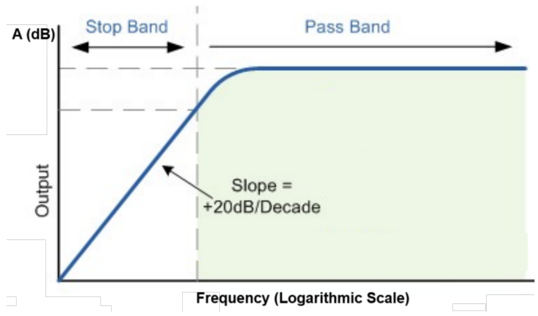

FIG. 8: Typical frequency response curve of an active high pass filter

Practically, the basic single resistor and single capacitor differentiator circuits are not widely used because of the two inherent faults – Instability and Noise.

At high frequencies, a differentiator circuit becomes unstable and will start to oscillate. To avoid this, the high-frequency gain of the circuit needs to be reduced by adding an additional small value capacitor, C_f, across the feedback resistor R_f. Also, the capacitive input makes it very susceptible to random noise signals and any noise or harmonics present in the circuit will be amplified more than the input signal itself. This is because the output is proportional to the slope of the input voltage. So some means of limiting the bandwidth to achieve closed-loop stability is required. In order to reduce the overall closed-loop gain of the circuit at high frequencies, an extra Resistor, R_{in} is added to the input. Thus, the new circuit acts like a Differentiator amplifier at low frequencies and an amplifier with resistive feedback at high frequencies giving much better noise rejection (Fig. 9).

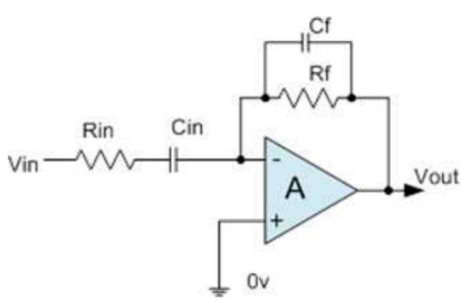

FIG. 9: Circuit diagram for an Active High Pass Filter

III. EXPERIMENTAL SETUP

Apparatus

1. OPAMP 741 Chip
2. Resistors
3. DC Power Supply
4. Function Generator
5. Breadboard
6. Oscilloscope
7. Connecting Wires
8. Multimeters

IV. OBSERVATION AND CALCULATIONS

A. Op-amp as an integrator

Refer Fig. 3.

- $R_1 = 9.85$ kΩ, $R_2 = 9.90$ kΩ, $R_f = 99.10$ kΩ
- $C_f = 102.2$ nF

The following waveforms were obtained on the oscilloscope.

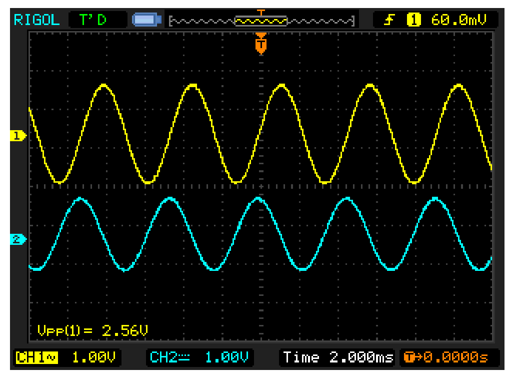

FIG. 10: Sinusoidal input waveform (above) and its corresponding phase shifted output waveform (below) by an Integrator Circuit

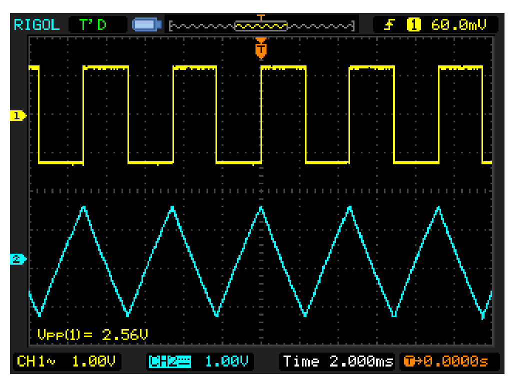

FIG. 11: Square wave input signal (above) and its corresponding output waveform (below) by an Integrator Circuit.

In Fig. 11, the input may be treated as $\pm V$ for short periods of time. With time, the area swept out from underneath the input signal increases, and then decreases due to the polarity changes. It follows that the output will grow (or shrink) in a linear fashion, as the input is constant. In other words, straight ramps are produced, hence this type of circuit is also known as the *Ramp Generator*.

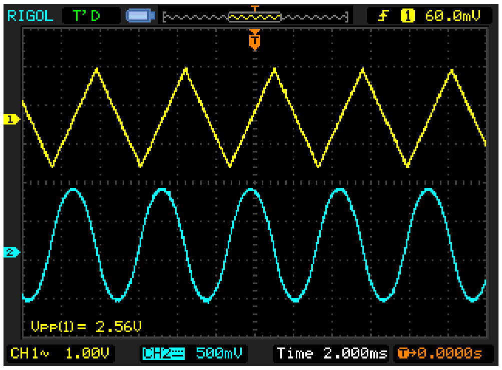

FIG. 12: Triangle waveform input (above) and its corresponding output waveform (below) by an Integrator Circuit

In Fig. 12, the triangular wave is composed of linearly raising and linearly falling parts, the integral of which will result in a quadratic term along with some constants arising due to the components of the circuit. So the output will be parabolic in nature. The output observed is a composition of +ve and -ve halves of *parabolas*, and *not a sinusoidal curve.*

B. Op-amp as a differentiator

Refer Fig. 7.

- $R_1 = 1.006$ kΩ, $R_2 = 1.002$ kΩ, $R_f = 9.90$ kΩ
- $C_f = 10.40$ nF, $C_{\text{in}} = 102.2$ nF

The following waveforms were obtained on the oscilloscope.

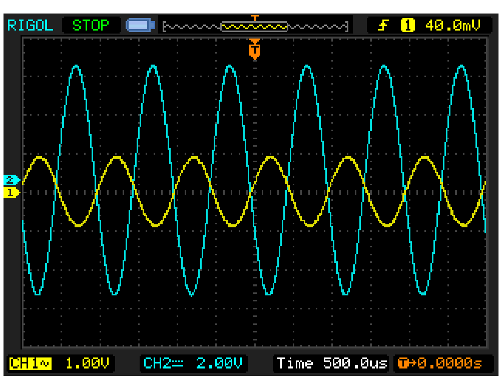

FIG. 13: Sinusoidal input waveform with its corresponding output waveform (shown here with lower amplitude), phase shifted by π, by a differentiator circuit

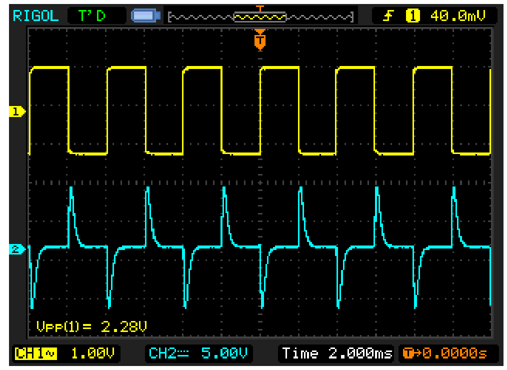

FIG. 14: Square wave input signal (above) with its corresponding output signal. The positive and negative spikes coincide with the rising/falling edge of the input square wave, which indicate increasing/decreasing slope.

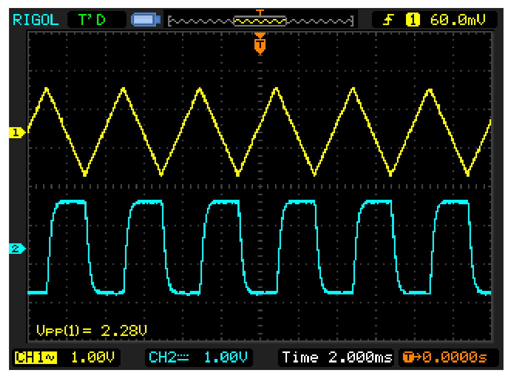

FIG. 15: Triangular wave input signal (above) with its corresponding output signal. Since the input consists of steady increase/decrease in voltage, the corresponding slopes are constant, resulting in a square wave.

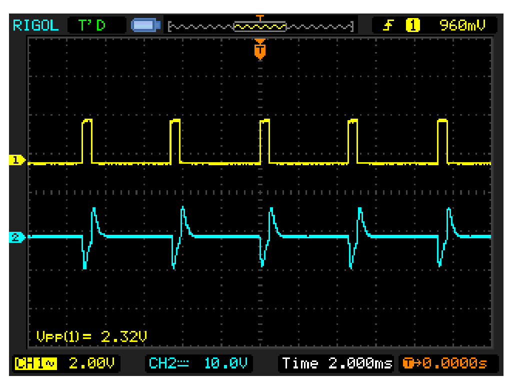

FIG. 16: Pulsating input signal (above) with the corresponding output signal by a differentiator circuit. The same explanation as Fig. 14 applies here.

C. Op-amp as an active low pass filter

Using the circuit in Fig 4, the following values voltages were obtained as a function of frequency.

- $R_1 = 9.85$ kΩ, $R_f = 99.10$ kΩ, $C = 102.2$ nF
- $f_c = 1/2\pi RC = 158$ Hz

Frequency	V_o (V)	$A_v = V_o/V_i$	Gain (in dB)
2.5 Hz	15.4	9.87	45.79
4.0 Hz	15.4	9.87	45.79
8.7 Hz	15.2	9.74	45.53
9.9 Hz	15.2	9.74	45.53
13.55 Hz	15.2	9.74	45.53
25.0 Hz	15.0	9.62	45.27
36.0 Hz	14.6	9.36	44.73
43.5 Hz	14.2	9.10	44.17
56.0 Hz	14.2	9.10	44.17
74.0 Hz	13.8	8.85	43.60
93.0 Hz	13.2	8.46	42.71
111.5 Hz	12.5	8.01	41.62
127.0 Hz	11.9	7.63	40.64
145.0 Hz	11.4	7.31	39.78
153.0 Hz	11.0	7.05	39.06
158.0 Hz	10.8	6.92	38.70
176.0 Hz	10.4	6.67	37.94
186.0 Hz	10.0	6.41	37.16
196.0 Hz	9.6	6.15	36.34
215.0 Hz	9.1	5.83	35.27
228.0 Hz	8.6	5.51	34.14
260.0 Hz	7.8	5.00	32.19
300.0 Hz	7.2	4.62	30.59
355.0 Hz	6.08	3.9	27.21
375.0 Hz	5.76	3.69	26.13
415.0 Hz	5.28	3.38	24.38
465.0 Hz	5.0	3.21	23.30
500.0 Hz	4.6	2.95	21.63
550.0 Hz	4.2	2.69	19.81
620.0 Hz	4.0	2.56	18.83
695.0 Hz	3.6	2.31	16.72
805.0 Hz	3.0	1.92	13.08
980.0 Hz	2.6	1.67	10.22
1.325 kHz	2.0	1.28	4.97
1.62 kHz	1.56	1.00	0.00
2.00 kHz	1.24	0.79	-4.59
2.55 kHz	1.00	0.64	-8.89
5.05 kHz	0.52	0.33	-21.97
8.00 kHz	0.36	0.23	-29.33
14.90 kHz	0.18	0.12	-43.19
30.75 kHz	0.108	0.07	-53.41
80.00 kHz	0.072	0.05	-61.52
307.50 kHz	0.060	0.04	-65.16
515.00 kHz	0.050	0.03	-68.81
1.00 MHz	0.050	0.03	-68.81
3.00 MHz	0.050	0.03	-68.81

TABLE I: Frequency response data for op-amp as an active low pass filter

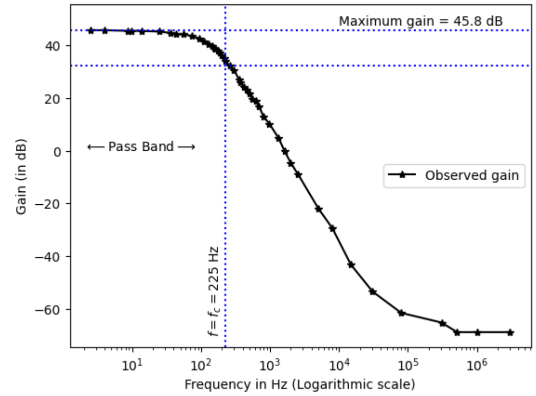

FIG. 17: Bode plot (Gain vs frequency) for Table I

From the graph, the observed value of the cutoff frequency is 225 Hz. There is some deviation in the expected value from the experimental value, which could be due to various reasons like offset voltage or error in measurement.

D. Op-amp as an active high pass filter

By constructing a high pass filter (Fig. 9), the following values voltages were obtained as a function of frequency. Note: C_f was not connected in parallel to R_f in the practical circuit used as it attenuated higher frequencies.

- $R_{\text{in}} = 1.002$ kΩ, $R_f = 9.90$ kΩ, $C_{\text{in}} = 102.2$ nF
- $f_c = 1/2\pi RC = 1554$ Hz

Frequency	V_o (V)	$A_v = V_o/V_i$	Gain (in dB)
25 Hz	0.144	0.29	-24.90
67 Hz	0.272	0.54	-12.18
90 Hz	0.336	0.67	-7.95
120 Hz	0.425	0.85	-3.25
135 Hz	0.496	0.99	-0.16
160 Hz	0.544	1.09	1.69
182 Hz	0.616	1.23	4.17
204 Hz	0.680	1.36	6.15
230 Hz	0.760	1.52	8.37
306 Hz	0.980	1.96	13.46
410 Hz	1.26	2.52	18.49
690 Hz	1.96	3.92	27.32
895 Hz	2.44	4.88	31.70
1.315 kHz	3.08	6.16	36.36
1.410 kHz	3.20	6.40	37.13
1.585 kHz	3.40	6.80	38.34
2.060 kHz	3.80	7.60	40.56
2.540 kHz	4.00	8.00	41.59
3.075 kHz	4.28	8.56	42.94
5.00 kHz	4.52	9.04	44.03
6.85 kHz	4.64	9.28	44.56
9.00 kHz	4.68	9.36	44.73
15.00 kHz	4.68	9.36	44.73

TABLE II: Frequency response data for op-amp as an active high pass filter

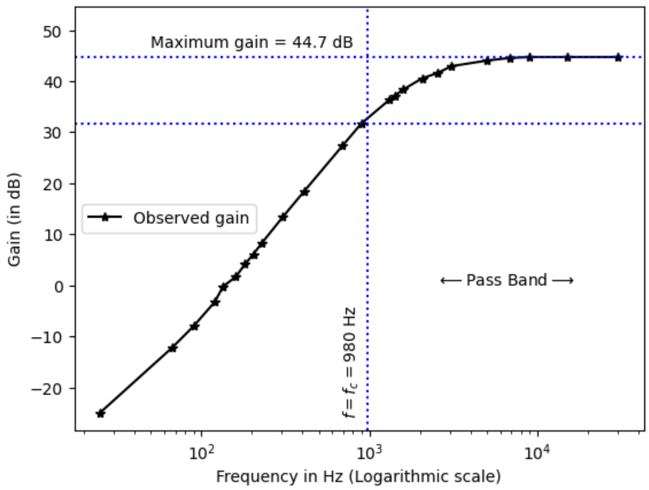

FIG. 18: Bode plot (Gain vs frequency) for Table II

From the graph, the observed value of the cutoff frequency is around 980 Hz. There is some deviation in the expected value from the experimental value, which could be due to various reasons as mentioned previously.

V. DISCUSSION

A practical integrator circuit (Fig. 3) consists of two additional components, to that of an ideal integrator circuit.

- R_f: Since the integrator circuit acts as a low pass filter, at low frequencies the gain of the circuit is very high. Hence, to avoid the op-amp going into open loop configuration at low frequencies, R_f is added in parallel to C_f. When R_f was removed, the circuit was observed to be very unstable at low frequencies.

- R_2: Real op-amps have some bias current that exhibits some voltage imbalance. While these imperfections are not large, they can cause problems with circuits like integrators, in which the effect of a small error grows with time. To fix this, the non-inverting terminal of the op-amp is connected to an offset-minimizing resistor before being grounded. Removing this resistor was observed to make the output signal less clean.

Similarly, a practical Differentiator circuit (Fig. 7) consists of 3 additional components.

- R_1: Broadly there are two reasons to use R_1 in series with C_{in}:
  - At higher frequencies, C_{in} gets shorted, which means the gain goes to infinity. It means that it can amplify high-frequency noise signals. Also, the gain of the circuit increases continuously with the increase in frequency (roll-off). Connecting R_1 will thus limit the gain of the amplifier.
  - The input impedance of the circuit will be zero at high frequencies. Due to this, the input source gets loaded and the circuit draws more current from the source. Connecting R_1 will make sure that $X_C + R_1 \neq 0$

  When R_1 was removed, the output signal was observed to be very unstable.

- C_f: It is a small valued capacitor. It provides additional attenuation at high frequencies such that the circuit won't run into oscillations. It also reduces bandwidth, so high-frequency noise gets bypassed and will not appear at output. When removed, the gain was observed to decrease after a certain frequency.

- R_2: The non-inverting terminal is connected to an offset-minimizing resistor before being grounded. This is to make sure there is no offset voltage - since the differentiator circuit is very sensitive to any change in the input voltage. Removing this resistor was observed to make the output signal less clean.

VI. PRECAUTIONS AND SOURCES OF ERROR

1. Connections should be verified before switching on the circuit
2. The resistance to be chosen should be in kΩ range
3. Outputs should be observed within a suitable frequency range, about 50Hz to 100kHz

VII. CONCLUSION

We have successfully constructed differentiator and integrator circuits using op-amps and studied their property. We also discussed their faults and ways to rectify them using additional circuit components. Furthermore, we also built and studied active high-pass and low-pass filters using op-amps and studied their frequency response curves.

VIII. APPLICATIONS

Integrator circuits are most commonly used in analog to digital converters, various signal wave-shaping circuits and in ramp generators.

Differentiator circuits are most commonly used in wave shaping circuits to detect the high-frequency components in the input signal and as a rate-of-change detector in the FM demodulators.

Early analog computers used vacuum tube op-amps to build summing amplifiers and integrators to solve linear, ordinary differential equations. Differentiators were avoided because of their inherent noise problems. One can say that a differentiator is a high-pass filter that also passes broadband noise while an integrator is a quieter, low-pass filter.

Active filters are used in communication systems for suppressing noise, in audio systems, in sensitive biomedical instruments and in various other places where we have to amplify weak signals with desired frequencies.

[1] SPS, *Operational Amplifiers (Supplementary note)*, NISER (2023).
[2] R. A. Gayakwad, *Op-Amps and Linear Integrated Circuits* (Pearson, 2015).
[3] P. Horowitz and W. Hill, *The art of electronics* (Cambridge University Press, 2015).