EXPERIMENT - 4

SPLITTING OF SODIUM D-LINES USING GRATING

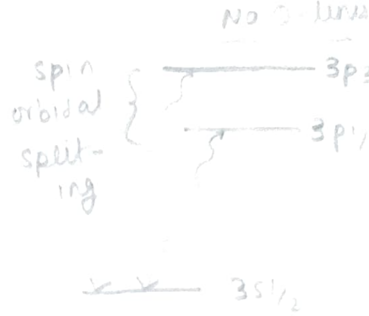
Submitted By:

Gayatri P
2211185
Int. M.Sc. (B22)

Objective :

Measurement of the wavelength separation of sodium D-lines using a diffraction grating and to calculate the angular dispersive power of the grating.

Theory



The sodium spectrum is dominated by the bright doublet - Sodium D-lines - which are at wavelengths 589.6nm. Using a diffraction grating, we can find the wavelength separation of these two.

A diffraction grating is an arrangement with a large number of parallel slits of the same width & separated by equal opaque space.

For N parallel slits, each with a width e & separated by an opaque space of width b, the diffraction pattern on the screen consists of a pattern of diffraction modulated interference fringes. The quantity (e+b) is calling grating element and N $\left(=\frac{1}{e+b}\right)$ is the number of slits per unit length. For a large no. of slits, the pattern consists of extremely sharp principle maxima with maxima of different orders having lesser intensity than the principle maxima. They are differentiated if the source of light is not monochromatic.

For a given light source with wavelength λ, the maxima are given by,

$$\boxed{(e+b)\sin\theta = m\lambda} \quad (1), \quad m = 0, \pm 1, \pm 2, \ldots$$

where m = order of maxima
& θ = angle of diffraction

By measuring the angular position of different wavelengths, one can then determine λ using

$$\lambda = \frac{(e+b)\sin\theta}{m} = \frac{\sin\theta}{Nm} \quad (2)$$

Angular dispersion power

It is defined as the rate of change of the angle of diffraction wrt the change in wavelength of the light, for a given order of maxima.

$$(e+b)\sin\theta = m\lambda$$

$$\Rightarrow (e+b)\cos\theta \, d\theta = m \, d\lambda$$

$$\Rightarrow \frac{d\theta}{d\lambda} = \frac{m}{(e+b)\cos\theta} = \frac{Nm}{\cos\theta} \quad -(3)$$

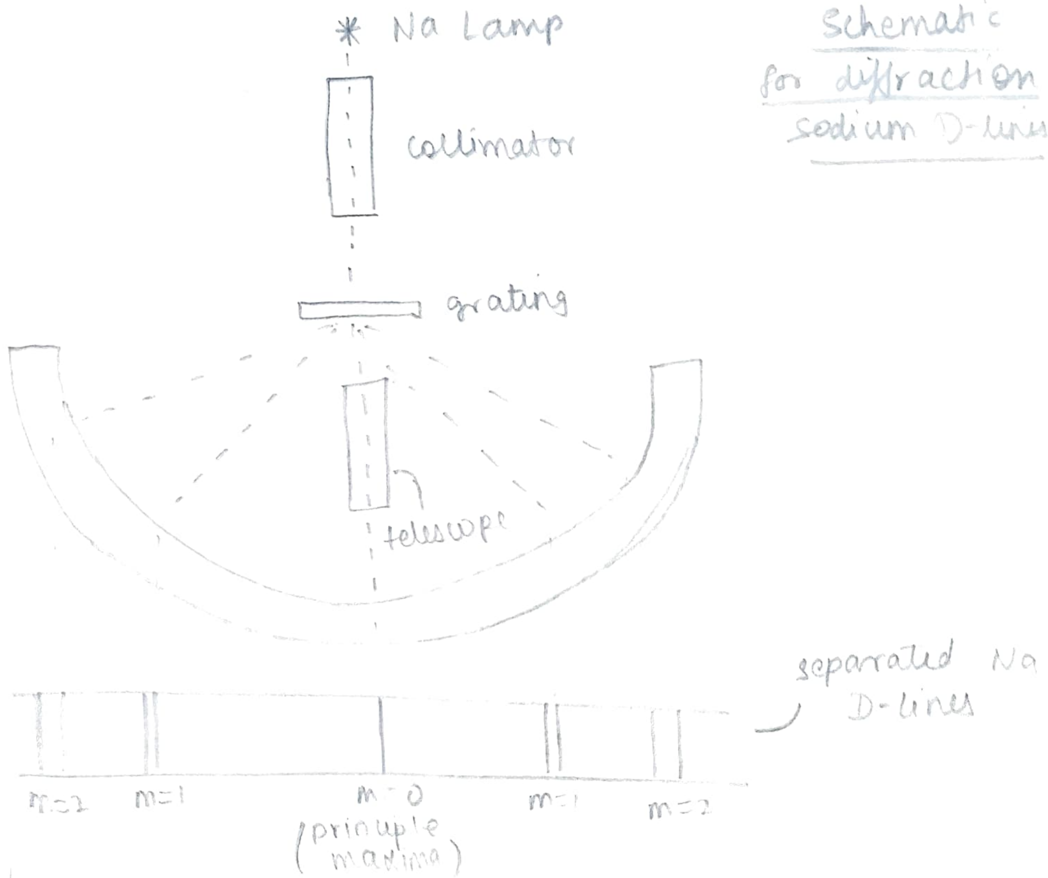
$d\theta/d\lambda$ is fixed for a given diffraction grating for different orders.

Apparatus Required

1. Spectrometer
2. Sodium Lamp
3. Prism
4. Diffraction Grating

Experimental Setup

Schematic for diffraction of sodium D-lines



Observations

No. of lines on grating = 600 lines/mm

Table 1 : Calculation of angle of dispersion of different orders for Na D-lines

Line # & order	Left side						Right side						2θ (Right vernier) (°)	$2\theta'$ (Left vernier) (°)	$\bar{\theta} = \frac{\theta'+\theta}{2}$ (°)	$\lambda = \frac{\sin\theta}{Nm}$ (nm)
	Right vernier			Left vernier			Right vernier			Left vernier						
	MSR	VSR	Total (°)	MSR	VSR	Total (°)	MSR	VSR	Total (°)	MSR	VSR	Total (°)				
m=2 D_2	141° 50'	54	141.983	322° 20'	59	322.497	232° 40'	45	232.792	53° 10'	14	54.539	90.808	92.042	45.713	596.538
m=2 D_1	141° 0'	23	141.064	322° 20'	39	322.442	232° 40'	24	232.733	53° 0'	49	53.136	91.669	90.694	45.591	595.302
m=1 D_2	166° 20'	36	166.433	346° 40'	55	346.819	208° 10'	7	208.186	28° 30'	24	28.567	41.753	41.747	20.875	593.884
m=1 D_1	166° 20'	53	166.481	346° 50'	22	346.894	208° 10'	1	208.169	28° 30'	14	28.539	41.689	41.644	20.833	592.751

29/10/24

Least count of the spectroscope = 10''

Calculations & Error Analysis

① For sodium line D2 (m = 2)

$\theta_2^{(2)} = 45.713^\circ$

$N = \frac{1}{600}\ \text{mm} = \frac{10^{-5}}{600}\ \text{m}$

from eqⁿ ②, $\lambda_2^{(2)} = \frac{\sin(45.713)}{2 \times 600} \times 10^{-5}$

$\Rightarrow \lambda_2^{(2)} = 596.538\ \text{nm}$

Error in measurement of $\lambda_2^{(2)}$:

using $\delta\lambda = \sqrt{\left(\frac{\partial \lambda}{\partial \theta}\delta\theta\right)^2}$

$= \frac{\cos\theta}{Nm}\delta\theta$

where $\delta\theta$ = least count of the instrument

$= 10'' = \frac{10}{3600} = \left(\frac{1}{360}\right)^\circ$

$$\Rightarrow \boxed{\delta\lambda = \frac{\cos\theta}{Nm}\left(\frac{\pi}{180}\right)\left(\frac{1}{360}\right)} \quad — ④$$

where the $\pi/180$ factor is due to the conversion to radians

using eqⁿ ④ with $\theta_2^{(2)}$,

$\delta\lambda_2^{(2)} = 0.028\ \text{nm}$

② For sodium line D_1 (m = 2)

$\theta_1^{(2)} = 45.591^\circ$

from eqⁿ ②, $\lambda_1^{(2)} = 595.302\ \text{nm}$

using eqⁿ ④, $\delta\lambda_1^{(2)} = 0.028\ \text{nm}$

③ For sodium line D_2 (m = 1)

$\theta_2^{(1)} = 20.875^\circ$

from eqⁿ ②, $\lambda_2^{(1)} = 593.884$

from eqⁿ ④, $\delta\lambda_2^{(1)} = 0.075$

(4) For sodium line D_1 $(m=1)$

$\theta_1^{(1)} = 20.833°$

From eq^n (2), $\lambda_1^{(1)} = \cancel{593}\cdot 592.751$ nm

From eq^n (4), $\delta\lambda_2^{(2)} = 0.076$ nm

Difference in wavelength b/w D_1 & D_2

$$\lambda_2 = \frac{\lambda_2^{(1)} + \lambda_2^{(2)}}{2} = 595.211 \text{ nm}$$

$$\lambda_1 = \frac{\lambda_1^{(1)} + \lambda_1^{(2)}}{2} = 594.027 \text{ nm}$$

$$\Rightarrow \Delta\lambda = \lambda_2 - \lambda_1 = 1.184 \text{ nm}$$

Error in $\Delta\lambda$,

$$\delta(\lambda_2) = \frac{1}{2}\sqrt{(\delta\lambda_2^{(2)})^2 + (\delta\lambda_2^{(1)})^2} = 0.040 \text{ nm}$$

$$\delta(\lambda_1) = \frac{1}{2}\sqrt{(\delta\lambda_1^{(2)})^2 + (\delta\lambda_1^{(1)})^2} = 0.040 \text{ nm}$$

$$\Rightarrow \delta(\Delta\lambda) = \sqrt{(\delta\lambda_1)^2 + (\delta\lambda_2)^2}$$

$$= \sqrt{(0.040)^2 + (0.040)^2} = 0.057 \text{ nm}$$

$$\Rightarrow \boxed{\Delta\lambda = (1.184 \pm 0.057) \text{ nm}}$$

Calculation of Angular Dispersive Power

Eq^n (3) gives us the angular dispersive power of the grating for a given order & diffraction angle,

$$D = \frac{d\theta}{d\lambda} = \frac{mN}{\cos\theta}$$

(1) For D_2 $(m=2)$

using $N = \frac{1\text{mm}}{600}$ & $\theta_2^{(2)} = 45.713$,

$$D_2^{(2)} = \frac{2 \times 1\text{mm}}{600 \times \cos(45.713)} = \cancel{1.72 \times 10^3}\ 1.718 \times 10^3 \text{ rad/mm}$$

~~② For D_1 (m = 2)~~

~~$\theta_1^{(2)} = 45.591°$~~

Error in $D_2^{(2)}$:

$$\delta D = \frac{Nm}{} \cdot \frac{-1}{\cos^2\theta} (-\sin\theta)\,\delta\theta$$

$$\delta D = \frac{Nm}{\cos\theta}\left(\frac{\sin\theta}{\cos\theta}\,\delta\theta\right) = \boxed{D \tan\theta\, \delta\theta} \quad -⑤$$

$$\Rightarrow \delta D_2^{(2)} = 1.718 \times 10^3 \times \tan(45.713)\left(\frac{1}{360}\right)\left(\frac{\pi}{180}\right)$$

$$= 0.013 \text{ rad/mm}$$

② For D_1 (m = 2)

$\theta_1^{(2)} = 45.591°$

$\Rightarrow D_1^{(2)} = 1.715 \times 10^3$ rad/mm [from eqn ③]

Using eqn ⑤, $\delta D_1^{(2)} = 0.003$ rad/mm

③ For D_2 (m = 1)

$\theta_2^{(1)} = 20.875°$

$\Rightarrow D_2^{(1)} = 6.422 \times 10^2$ rad/mm

& $\delta D_2^{(1)} = 0.015$ rad/mm

④ For D_1 (m = 1)

$\theta_1^{(1)} = 20.833°$

$\Rightarrow D_1^{(1)} = 6.419 \times 10^2$ rad/mm

& $\delta D_1^{(1)} = 0.013$ rad/mm

Results & Discussion

In this experiment, we were able to successfully measure and observe the wavelength separation of sodium D-lines using a diffraction grating as well as measure the dispersive power of the grating.

After getting familiar with the spectrometer, we first performed optical levelling of a prism and used Schuster's method to properly focus the telescope and the collimator. We then moved on the actual experiment.

In the experiment, we measured the wavelengths of sodium D-lines as follows -

$$\lambda_{D1} = (594.027 \pm 0.074)\,nm \quad \& \quad \lambda_{D2} = (595.211 \pm 0.073)\,nm$$

From which the wavelength separation was calculated as,

$$\Delta\lambda = (1.184 \pm 0.057)\,nm$$

It is to be noted that our wavelengths values calculated are close to the standard values of $\lambda_i = 589.0$ nm & 589.6 nm, but are systematically deviated by a few nms. This could primarily be due to the heavy backlash present in the spectrometer screws. The rotation axis also could have been not levelled properly resulting in the deviation.

One can also note that the wavelength values derived from order (m) = 2 fringes are more deviated from the literature values. This could be due to high diffractive power of the prism resulting in the 2nd order angle being quite large.

Regardless, the Fraunhoffer approximation for diffraction is quite accurate and the results are quite close to the literature values.

We also measured the dispersive power of the grating for different orders for both the sodium D-lines.

For $m=1$ (1^{st} order):

$$D_1 = (6.419 \times 10^2 \pm 0.013)\ \text{rad/mm}$$

$$D_2 = (6.422 \times 10^2 \pm 0.015)\ \text{rad/mm}$$

For $m=2$ (2^{nd} order):

$$D_1 = (1.715 \times 10^3 \pm 0.003)\ \text{rad/mm}$$

$$D_2 = (1.718 \times 10^3 \pm 0.013)\ \text{rad/mm}$$

As expected, the dispersive power for the second order fringes are much higher. The low error bars associated with the D values here are because the high degree of precision of the instrument, resulting in a very low least count.

Precautions & Sources of Error

1. Once adjusted for parallel rays, the focusing of the collimator & telescope should not be disturbed throughout the experiment.
2. While taking measurements, the turntable must be locked.
3. Move the fine adjustment screw only in one direction to avoid any backlash error.