EXPERIMENT - 05

STUDY OF MICHELSON'S INTERFEROMETER AND DETERMINATION OF WAVELENGTH OF DIFFERENT LIGHT SOURCES.

Submitted By:

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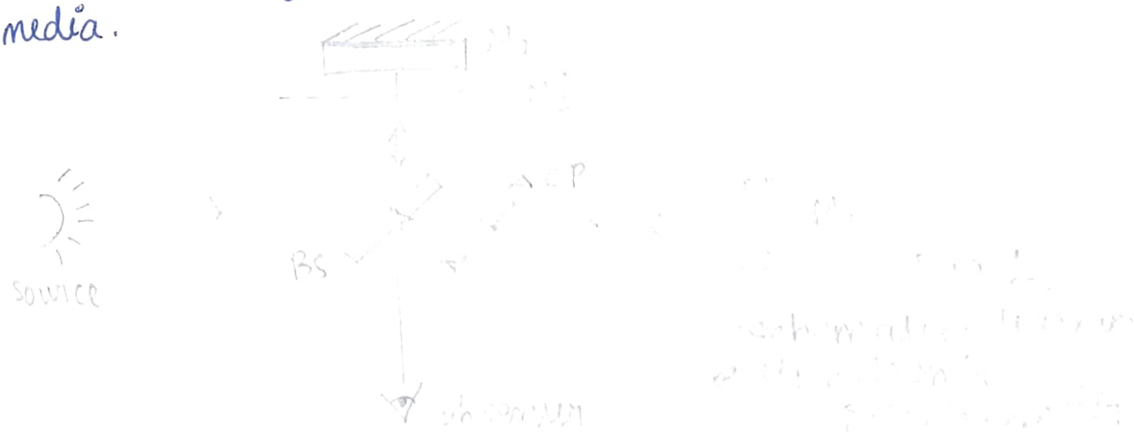
Objectives:

1. Alignment of Michelson's interferometer using He-Ne laser to observe concentric circular fringes.
2. Measurement of wavelength of He-Ne laser and Na lamp using circular fringes.

Theory

Michelson's interferometer is an important optical instrument used for measuring wavelengths of unknown light sources, to measure extremely small distances and to investigate optical media.



This instrument is based on the concept of interference of light. In the optical setup, two glass plates are kept parallel to each other -

- the beam splitter, where light from the source is equally transmitted and reflected onto mirrors M_1 & M_2
- the compensatory glass plate, which is used to compensate for the extra $2(\mu-1)t$ [μ = refractive index, t = thickness of plate] path difference of the transmitted light hitting M_2.

After reflection from the M_1 & M_2 respectively, the reflected and transmitted wave will interfere constructively or destructively at the detector based on the following condition,

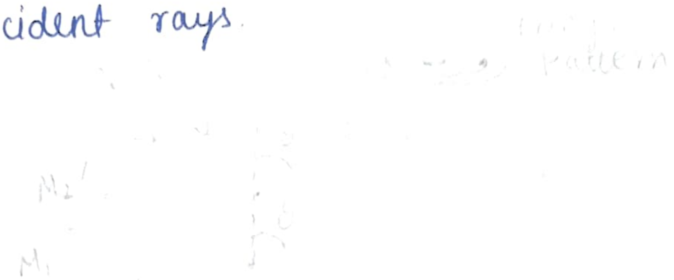
path difference $\Delta = n\lambda \Rightarrow$ constructive interference

$\Delta = (n+\frac{1}{2})\lambda \Rightarrow$ destructive interference

Different kinds of fringes can be observed using this instrument:

→ <u>concentric circular fringes</u> → when the image of M_2 as seen through the beam splitter is parallel to M_1 (i.e. $M_1 \perp' M_2$) since width of the air film b/w M_1 & the image are constant, fringes are formed depending on the inclination of reflected & incident rays.

Fig 2
Fringes of equal inclination



If the inclination angle is θ, the ray reflecting from the mirror further away will have to travel an extra distance of $2d\cos\theta$. Bands will be formed for same inclination of light source corresponding to circular fringes.

<u>Determination of wavelength</u>

As we change the distance b/w the mirrors (d), say N fringes appear or disappear near the center (where $\theta = 0$).

$$\Rightarrow 2(d + \Delta d) = (n + N)\lambda$$

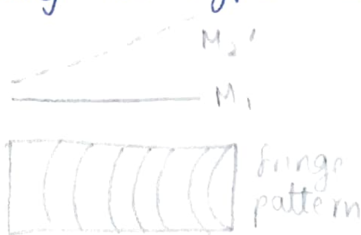
since $2d = n\lambda$ is the initial path difference

$$\Rightarrow 2\Delta d = N\lambda \Rightarrow \boxed{\lambda = \frac{2\Delta d}{N}} \quad \text{— (1)}$$

→ <u>curved fringes</u> (fringes of equal thickness)

occurs when M_1 is inclined at an angle w.r.t. M_2 forming a wedge shaped air column. Fringes will appear for path difference leading to hyperbolic fringe pattern

Fig 3
curved fringes



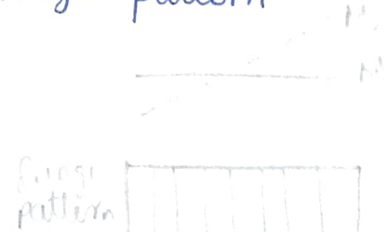


Fig 4
Straight Line fringes

→ <u>straight line fringes</u>

when M_1 & M_2' intersect straight line fringes are formed around the point of intersection where path difference is zero.

Instruments Required

1. Michelson interferometer setup
2. A screen
3. A He-Ne laser
4. A Na lamp
5. A grounded glass plate

Observations & Calculations

(A) He-Ne Laser

TABLE 1: Observation table for Δd vs N plot for HeNe laser

N	p (×0.01mm)	q (×0.0001 mm)	d_i (mm) p+q	Δd (mm)	λ (nm)
0	10	82	0.1082		
5	10	65	0.1065	0.0017	~~680~~ 700
0	97	95	0.9795		
5	97	78	0.9778	0.0017	~~680~~ 700
0	99	23	0.9923		
20	98	70	0.9870	0.0053	~~530~~ 500
0	98	70	0.9870		
4	98	56	0.9856	0.0014	700
0	98	56	0.9856		
5	98	39	0.9839	0.0017	~~680~~ 700
0	80	20	0.8020		
15	79	75	0.7975	0.0045	600
0	79	75	0.7975		
8	79	52	0.7952	0.0023	~~575~~ 600
0	50	15	0.5015		
25	49	25	0.4925	0.0090	~~720~~ 700
0	49	25	0.4925		
2	49	17	0.4917	0.0008	800
0	40	30	0.4030		
12	39	95	0.3995	0.0035	~~580~~ 600
0	30	20	0.3020		
18	29	58	0.2958	0.0062	~~690~~ 700

Here the least count of main scale = 0.01mm

" " of circular scale = 0.0001 mm

Least square fitting for ~~a~~ ~~N vs~~ Δd vs N plot

From eq^n 1, we can see that $\Delta d = \frac{\lambda N}{2}$ is a eq^n of the form $y = mx + b$, where $m = \frac{\lambda}{2}$, is the slope & b = y-intercept.

For the least square fitting, we know that the measurement of y_i (Δd_i) is governed by a normal distribution around the true value, given by $\frac{1}{\sigma_y^N} e^{-\chi^2/2}$.

Here, the exponent $\chi^2 = \sum_{i=1}^{N} \frac{(y_i - m x_i - b)^2}{\sigma_y^2}$ has to be minimised wrt. m & b.

$\therefore$ putting $\frac{\partial \chi^2}{\partial m} = \frac{-2}{\sigma_y^2} \sum_{i=1}^{N} (y_i - m x_i - b) = 0$ — 2

and $\frac{\partial \chi^2}{\partial b} = \frac{-2}{\sigma_y^2} \sum_{i=1}^{N} x_i (y_i - m x_i - b) = 0$ — 3

Solving for m & b, we ~~get~~ can write two eq^{n}s from 2 & 3,

$$\sum y_i = m \sum x_i + bN \quad — 4$$

$$\& \quad \sum x_i y_i = m \sum x_i^2 + b \sum x_i \quad — 5$$

Solving eq^{n}s 4 & 5 for m & b, we get

$$m = \frac{N \sum x_i y_i - \sum x_i \sum y_i}{\Delta} \quad — 6$$

$$b = \frac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i}{\Delta} \quad — 7, \text{ where } \Delta = N \sum x_i^2 - (\sum x_i)^2$$

Table 2: Least square fit parameters for He-Ne Laser

S.no.	x_i (N_i)	y_i (Δd_i) (mm)	x_i^2	$x_i y_i$ (mm)	$[y_i - (mx_i + b)]^2$ ($\times 10^{-8}$)
1.	5	0.0017	25	0.0085	1.47
2.	20	0.0053	400	0.1060	129.57
3.	4	0.0014	16	0.0056	2.11
4.	5	0.0017	25	0.0085	1.47
5.	15	0.0045	225	0.0675	10.14
6.	8	0.0023	64	0.0184	6.28
7.	25	0.0090	625	0.2250	88.70

8	2	0.0008	4	0.0016	3.73
9	12	0.0035	144	0.0420	12.00
10	18	0.0062	324	0.1116	16.78
11	5	0.0017	25	0.0085	1.47
sum	119	0.0381	1877	0.6032	273.74

Using eqⁿs ⑥ & ⑦ w/ parameters from Table 2, we get

$m = 3.24 \times 10^{-4}$ mm $\Rightarrow$ using $m = \lambda/2$ we get

$b = -4.12 \times 10^{-5}$

$\lambda = 6.48 \times 10^{-4}$ mm

$= 648$ nm ≈ 600 nm (w/ S.G.)

Furthermore, the value $(y_i - mx_i - b)^2$ is plotted in table 2 to be used later in error analysis.

(B) Na Lamp

Table 3: Observation table for Δd vs N plot for Na lamp.

N	p (x0.01mm)	q (x0.0001mm)	d_i(mm) p+q	Δd (mm)	λ (nm)
0	75	98	0.7598		
5	75	79	0.7579	0.0019	~~760~~ 800
0	2	98	0.0298		
7	2	70	0.0270	0.0028	800
0	2	70	0.0270		
10	2	29	0.0229	0.0041	~~820~~ 800
0	2	29	0.0229		500
2	2	24	0.0224	0.0005	
0	0	74	0.0074		
12	0	42	0.0042	0.0032	~~530~~ 500
0	1	94	0.0194		
15	1	57	0.0157	0.0037	~~490~~ 500
0	1	48	0.0148		
20	0	75	0.0075	0.0073	~~730~~ 700
0	0	41	0.0041		
17	99	89	0.9989	0.0052	~~610~~ 600

Similar to part (A), we can assume ~~sd~~ vs N follows a straight line of the form $y = mx + b$.
The least square-fit parameters are as the following.

Table 4: least square fit parameters for Na Lamp

Sno.	x_i (N_i)	y_i (Δd_i) (mm)	x_i^2	$x_i y_i$ (mm)	$[y_i - (mx_i + b)]^2$ ($\times 10^{-8}$)
1.	5	0.0019	25	0.0095	3.47
2.	7	0.0028	49	0.0196	21.32
3.	10	0.0041	100	0.0410	68.03
4.	2	0.0012	4	0.0010	7.65
5.	12	0.0032	144	0.0384	48.97
6.	15	0.0037	225	0.0555	129.22
7.	20	0.0073	400	0.1460	81.30
8.	17	0.0052	289	0.0884	6.83
sum	88	0.0287	1236	0.3994	366.81

Using eq^{n}s ⑥ & ⑦ w/ parameters from table 4, we get

$\Delta = 2144$

& $m = 3.12 \times 10^{-4}$ mm $\Rightarrow$ using $m = \lambda/2$,

$b = 1.52 \times 10^{-4}$

$\lambda = 6.24 \times 10^{-4}$ mm

$= 624$ nm

≈ 600 nm (w/ s.f.)

Error Analysis

Error propagation in least-square fitting can ~~me~~ be measured by analysing χ^2,

$$\chi^2 = \sum_i \left(\frac{y_i - mx_i - b}{\sigma_i^2} \right)^2$$

where σ_i = error in measurement of y = least count of the instrument (in this case)

By minimising χ^2 wrt m & b,

$$\frac{\partial \chi^2}{\partial m} = 0 \Rightarrow \sum_i \frac{-2x_i(y_i - mx_i - b)}{\sigma_i^2} = 0 \quad — ⑧$$

and, $\frac{\partial \chi^2}{\partial b} = 0 \Rightarrow \sum_i \frac{-2(y_i - m x_i - b)}{\sigma_i^2} = 0$ — (9)

Rearranging (8) & (9), we get,

$$m \sum_i \frac{x_i}{\sigma_i^2} + b \sum_i \frac{1}{\sigma_i^2} = \sum_i \frac{y_i}{\sigma_i} \quad \text{— (10)}$$

$$\& \quad m \sum_i \frac{x_i^2}{\sigma_i^2} + b \sum_i \frac{x_i}{\sigma_i^2} = \sum_i \frac{x_i y_i}{\sigma_i^2} \quad \text{— (11)}$$

In our case, since σ_i is constant $\forall\, i$, eqⁿs (10) & (11) will simplify to form eqⁿs (4) & (5) described earlier.
By using,

$$S = \sum_i \frac{1}{\sigma_i^2}, \quad S_x = \sum_i \frac{x_i}{\sigma_i^2}, \quad S_y = \sum_i \frac{y_i}{\sigma_i^2}, \quad S_{xx} = \sum_i \frac{x_i^2}{\sigma_i^2}$$

$$\& \quad S_{xy} = \sum_i \frac{x_i y_i}{\sigma_i^2},$$

we can re-write (10) & (11) as,

$$\begin{aligned} bS + mS_x &= S_y \\ bS_x + mS_{xx} &= S_{xy} \end{aligned} \Rightarrow \begin{pmatrix} b \\ m \end{pmatrix} \begin{pmatrix} S & S_x \\ S_x & S_{xx} \end{pmatrix} = \begin{pmatrix} S_y \\ S_{xy} \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} b \\ m \end{pmatrix} = \frac{1}{\Delta} \begin{pmatrix} S_{xx} & -S_x \\ -S_x & S \end{pmatrix} \begin{pmatrix} S_y \\ S_{xy} \end{pmatrix}$$

where $\Delta = S_{xx} S - (S_x)^2$.

$$\Rightarrow \boxed{m = \frac{S S_{xy} - S_x S_y}{\Delta}} \quad \& \quad \boxed{b = \frac{S_{xx} S_y - S_x S_{xy}}{\Delta}} \quad \text{— (12) \& (13)}$$

which again reduces to eqⁿs (6) & (7).
To find the error in slope & intercept, we again minimise m & b wrt y_i.

$$\sigma_b^2 = \sum_i \left(\frac{\partial b}{\partial y_i}\right)^2 \sigma_i^2, \quad \sigma_m^2 = \sum_i \left(\frac{\partial m}{\partial y_i}\right)^2 \sigma_i^2$$

$$\Rightarrow \sigma_b^2 = \frac{S_{xx}^2 S + S_x^2 S_{xx} - 2 S_{xx} S_x^2}{\Delta^2}$$

$$= \frac{S_{xx}^2 S - S_{xx} S_x^2}{\Delta^2} = \frac{S_{xx}(S_{xx} S - S_x^2)}{\Delta^2}$$

$$\boxed{\sigma_b^2 = \frac{S_{xx}}{\Delta}} \quad \text{— (14)}$$

Similarly, $\sigma_m^2 = \dfrac{S_{xx} S^2 - S_x^2 S}{\Delta^2} = \dfrac{S(S_{xx} S - S_x^2)}{\Delta^2}$

$$\boxed{\sigma_m^2 = \frac{S}{\Delta}} \quad \text{— } (15)$$

(A) He-Ne Laser

Here $\sigma_i = 0.0001 \text{ mm} = 1 \times 10^{-4} \text{ mm}$

$$S_x = \sum_i \frac{x_i}{\sigma_i^2} = \frac{119}{10^{-8}} = 1.19 \times 10^{10}$$

$$S_{xx} = \sum_i \frac{x_i^2}{\sigma_i^2} = 1.88 \times 10^{11}$$

$$S_y = \sum_i \frac{y_i}{\sigma_i^2} = \frac{0.0381}{10^{-8}} = 3.81 \times 10^{+6}$$

$$S_{xy} = \sum_i \frac{x_i y_i}{\sigma_i^2} = 6.03 \times 10^{7}$$

$$S = \sum_i \frac{1}{\sigma_i^2} = 1.10 \times 10^{9}$$

$\Rightarrow \Delta = S_{xx} S - S_x^2 = 6.49 \times 10^{19}$

From eq^n (12), $m = 3.24 \times 10^{-4}$

From eq^n (13), $b = -4.12 \times 10^{-5}$

From eq^n (14), $\sigma_b = 5.38 \times 10^{-5}$

From eq^n (15), $\sigma_m = 4.12 \times 10^{-6}$

Since $m = \frac{\lambda}{2}$, we can get $\lambda = 2m$

$= 6.48 \times 10^{-4}$ mm

$= 6 \times 10^{-4}$ mm (upto appropriate significant digits)

Error in λ can be calculated as,

$$(\sigma_\lambda)^2 = \left(\frac{\partial \lambda}{\partial m} \sigma_m\right)^2 \Rightarrow \sigma_\lambda = 2\sigma_m$$

$$\Rightarrow \sigma_\lambda = 8.24 \times 10^{-6} \text{ mm}$$

$\therefore \lambda_{\text{He-Ne laser}} = (6 \pm 0.08) \times 10^{-4}$ mm

$= (600 \pm 8)$ nm

(B) Na Lamp

Here $\sigma_i = 0.0001\,mm = 1\times10^{-4}\,mm$

$$S_x = \sum_i \frac{x_i}{\sigma_i^2} = \frac{880}{10^{-8}} = 8.80\times10^{9}$$

$$S_{xx} = \sum_i \frac{x_i^2}{\sigma_i^2} = 1.24\times10^{11}$$

$$S_y = \sum_i \frac{y_i}{\sigma_i^2} = 2.87\times10^{6}$$

$$S = \sum_i \frac{1}{\sigma_i^2} = 8.00\times10^{8}$$

$$S_{xy} = \sum_i \frac{x_i y_i}{\sigma_i^2} = 3.99\times10^{7}$$

$$\Delta = S_{xx}S - (S_x)^2 = 2.14\times10^{19}$$

From eq^n (12), $m = 3.12\times10^{-4}$

from eq^n (13), $b = 1.52\times10^{-4}$

From eq^n (15), $\sigma_m = 6.11\times10^{-6}$

from eq^n (14), $\sigma_b = 7.59\times10^{-5}$

$\therefore$ since $\lambda = 2m$, we get $\lambda = 2\times3.12\times10^{-4}$
$= 6.24\times10^{-4}$
$\simeq 6\times10^{-4}$ (upto approoiate significant bits)

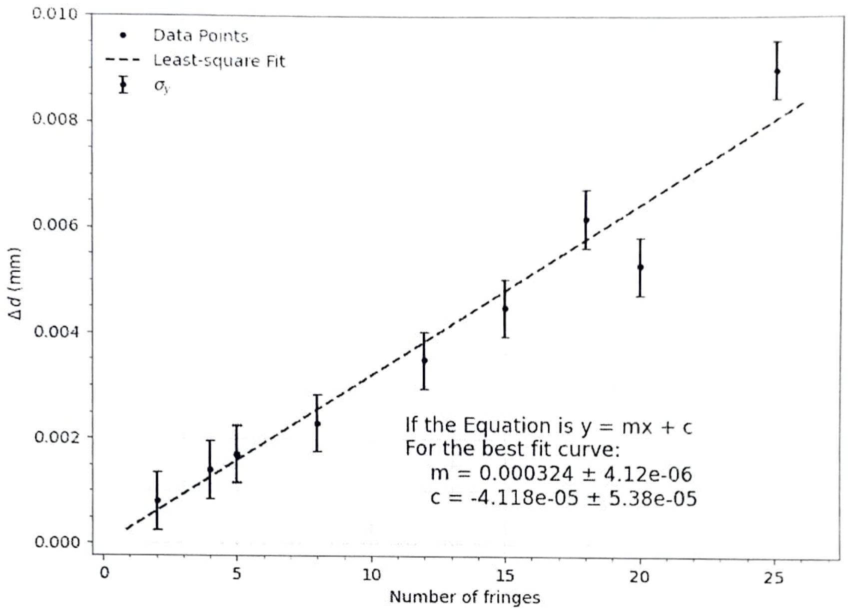
Error in λ, $\sigma_\lambda = \frac{\partial\lambda}{\partial m}\sigma_m = 2\sigma_m$

$\sigma_\lambda = 2\times6.11\times10^{-6} = 1.22\times10^{-5}\,mm$
$\approx 0.12\times10^{-4}\,mm$

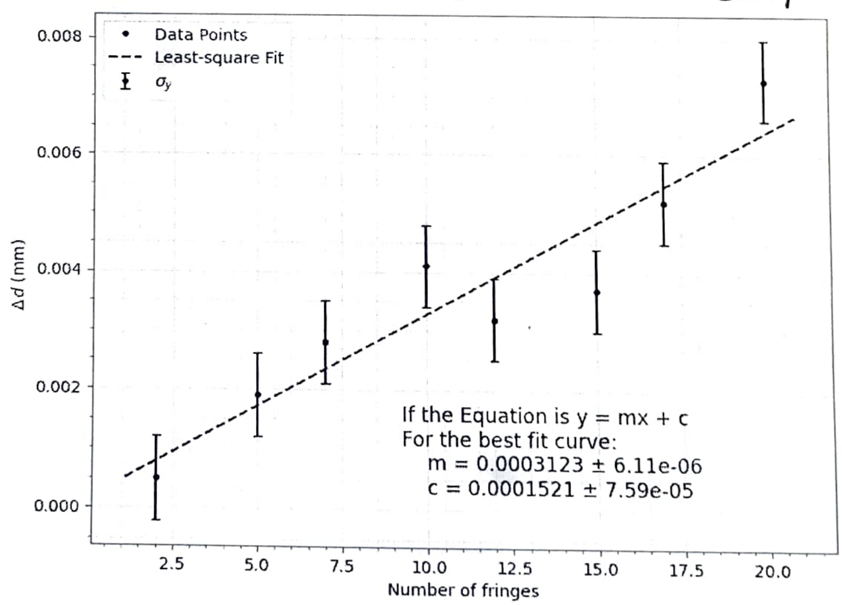
Hence, $\lambda_{\text{Na Lamp}} = (6\pm0.12)\times10^{-4}\,mm$
$= (600\pm12)\,nm$

Δd vs. N plots

GRAPH 1 : Δd vs N for He-Ne ~~Lamp~~ Laser



GRAPH 2 : Δd vs N plot for Na Lamp



In both cases, σ_y was calculated using,

$$\sigma_y = \sqrt{\frac{1}{N-2}\sum_i [y_i - mx_i - b]^2}$$

[we assume measurement of y_i is normally distributed about its true value $(mx_i + b)$ w/ σ_y std. dev.]

where $(N-2)$ represents the no. of degrees of freedom in our measurement. Using this, the values of σ_y for both datasets are :-

$$(\sigma_y)_{\text{He-Ne laser}} = 0.0005 \text{ mm}$$

$$(\sigma_y)_{\text{Na Lamp}} = 0.0008 \text{ mm}$$

Results & Discussion

Using the Michelson interferometer, by changing the distance between two mirrors & observing & appearance / disappearance of fringes, we have calculated the values of wavelengths of our two light sources as,

$$\lambda_{\text{He-Ne Laser}} = (600 \pm 8)\ \text{nm}$$

$$\lambda_{\text{Na lamp}} = (600 \pm 12)\ \text{nm}$$

On comparison with existing literature the error % (relative error) for the He-Ne Laser wavelength is 1.8% and that for the Na lamp is 5.2%.

Discussion

In this experiment, we used a He-Ne Laser and a Na Lamp to act as monochromatic light sources for a Michelson interferometer to produce a stable interference.

In the interferometer setup, since the path difference b/w the reflected and transmitted waves hitting the screen is a function of distance from the center, (Fig. 2), we can see circular dark and bright fringes which forms the locus of equal inclination (θ).

Since our measurements are limited by the resolution of the instrument (which has a least count of 10^{-4} mm = 100nm), the wavelengths calculated are far from accurate. Each reading of Δd had an error margin of ± 0.0001 mm since we only took one reading of Δd for a particular value of N.

Another possible reason for error is that we considered the value of N to be absolute while taking measurements (i.e. $\sigma_N = 0$). But ~~then~~ in reality, there could also be some uncertainity in N due to random error (or simply human error due to missing counts of fringes).

Another significant error source to mention is the fine adjustment screw which had a huge backlash. Although we took readings by rotating the knob in one-direction, the screw was prone to slipping which meant that our Δd values calculated were significantly higher than required.

When taking readings in order (eg. N = 5, 10, 15, 20 etc.), we found that this instrumental error quickly adds up and the calculated values of λ were increasing at a steady pace. (This data was later discarded).

Instead, we measured Δd by setting seemingly arbitrary reference points for different values of N, not any higher than 25 to prevent any slippage.

Additionally, after calculation of std. deviation from the least square-fits for both the light sources using $[y_i - f(x_i)]^2$, σ_y comes out to be 8×10^{-4} mm for Na Lamp & 5×10^{-4} mm for He-Ne Laser which could be because we took more readings for the latter. Also the std. deviation in y_i could not be minimised since we only took one reading each for every particular N value. To put it plainly, Here, $y_1, \dots y_N$ are not N measurements of the same quantity

Conclusion

We have performed this experiment using Michelson interferometer has gained valuable insight into it working, formation of stable fringe patterns and taking measurements to calculate the wavelengths of different sources of light.

We have also discussed various issues that rise up & possible sources of error that could tamper the readings.

Precautions

1. Always rotate the screws only in one-direction to avoid backlash error.
2. Direct eye exposure to laser should be avoided