Study of Rutherford Scattering

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In this experiment, we investigate Rutherford's α-particle scattering experiment using a gold foil. The primary objective is to validate the dependence of the scattering cross-section on $\sin^{-4}(\theta/2)$, using an Am-241 α decay source in our experimental setup. Additionally, we also estimate the nuclear charge of Aluminium by performing the scattering experiment with similar parameters for both gold and aluminium and then comparing the two.

I. THEORY

A. Introduction

When α particles strike a thin gold foil, some of the particles experience Coulombic repulsion from the Au nuclei and get scattered from the original direction depending on their impact parameter (the perpendicular distance from the axis of the nucleus) (Fig. 1). However, the number of particles which experience significantly large scattering angles (θ) or even get scattered back ($\theta = 180°$) are quite small $\sim 1/2000$. This was explained by Rutherford as the entire positive charge of the atom being concentrated in a very tiny region at the center, which led to the discovery of the nucleus.

B. Scattering Rate and Cross Section

The angular distribution of scattering cross section is given by $\frac{d\sigma}{d\Omega}$, which is the amount of scattering per soild angle. Let us denote $N(\theta)$ as the number of particles scattered per unit time per unit solid angle.

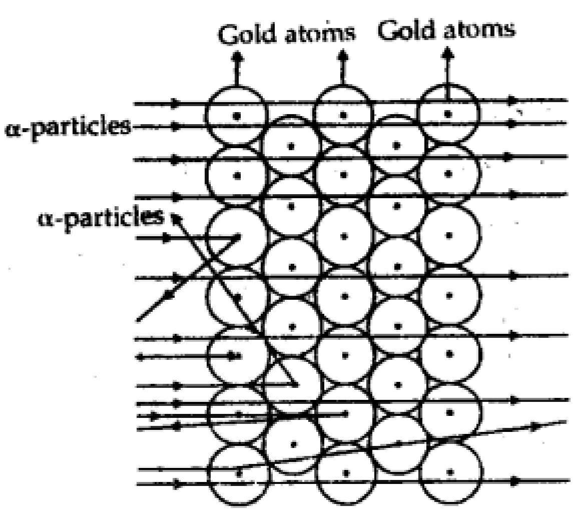

FIG. 1: Scattering of α-particles on a block of Gold atoms visualised

Rutherford's scattering formula (in a non-relativistic scenario) can be derived as,

$$\frac{d\sigma}{d\Omega} = \left(\frac{Z_1 Z_2 e^2}{4\pi\epsilon_0}\frac{1}{4E}\right)^2 \frac{1}{\sin^4(\frac{\theta}{2})} \tag{1}$$

where Z_1 and Z_2 are the atomic numbers of the two interacting particles, E is the kinetic energy of the incoming particle and θ is the scattering angle. If α particles ($Z = 2$) are emitted at a rate N_0 with energy E on a material with atomic number Z, the scattering rate is given by

$$N(\theta) = \frac{N_0 Z e^4}{(8\pi\epsilon_0)^2 E^2}\frac{c_F d_F}{\sin^4(\frac{\theta}{2})} \tag{2}$$

where c_F is the atomic concentration in the foil and d_F the thickness of the foil.

Due to the dependence of the scattering rate on the inverse 4th power of $\sin(\theta/2)$, the value of $N(\theta)$ rapidly declines after the singularity at $\theta = 0$. Higher scattering angles result in very tiny counting rates, hence in order to achieve an acceptable level of precision, the gate times $t(\theta)$ for calculating the counting rate $N(\theta)$ must be raised with rising angle θ. As a result, in the initial portion of the experiment, we begin at 5° and gradually increase the gate duration as we climb higher until 30°.

Space correction for Scattering Rate

The scattering rates $N_d(\theta)$ are determined by recording the pulse counts $n(\theta)$ for a given angle θ over a gate time t.

$$N_d(\theta) = \frac{n(\theta)}{t} \tag{3}$$

However, because of the design of the chamber used in this experiment, this $N_d(\theta)$ is for a flat scattering geometry. However, Rutherford's formula indicates that the theoretical function is connected to a three-dimensional geometry. The relationship between them is illustrated in the Fig. 2.

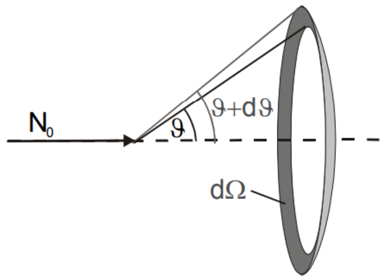

FIG. 2: Angular deflection of α-particles, scattered into an angular region $\theta + d\theta$ and ϕ from 0 to 2π

The plane angular differential $d\theta$ corresponds in three dimentions to a $d\Omega = 2\pi \sin\theta d\theta$. Hence the space correction to $N_d(\theta)$ and the spatial scattering rate $N(\theta)$ is

$$N(\theta) = \frac{N_d(\theta)}{2\pi \sin\theta} \tag{4}$$

C. Determination of Unknown Nuclear Charge

By keeping all quantities unchanged and by swapping the goild foil with a foil of an unknown material, we can find the atomic number of this unknown material. In this experiment for example, we have used Aluminium as the unknown element. From Eq. 2, we can see that

$$N(\theta) \propto dZ^2$$

where d is the thickness of the foil and Z is its atomic number. Hence for two materials of different thickness and atomic number (say Au and Al),

$$\frac{N_{\mathrm{Al}}}{N_{\mathrm{Au}}} = \frac{d_{\mathrm{Al}} Z_{\mathrm{Al}}^2}{d_{\mathrm{Au}} Z_{\mathrm{Au}}^2}$$
$$\implies N_{\mathrm{Al}} = Z_{\mathrm{Au}} \sqrt{\frac{d_{\mathrm{Au}} N_{\mathrm{Al}}}{d_{\mathrm{Al}} N_{\mathrm{Au}}}} \tag{5}$$

II. EXPERIMENTAL SETUP

Apparatus

1. Rutherford Scattering chamber
2. Au and Al foil with frames
3. Vacuum pump
4. Centering ring
5. Discriminator preamplifier
6. Counter
7. Plug-in power supply unit 12 V
8. Measuring cable BNC

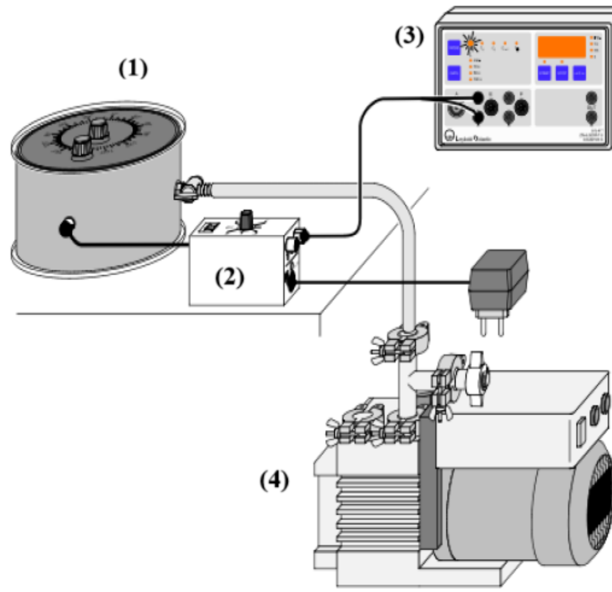

FIG. 3: The experimental setup with the (1) scattering chamber (2) discriminator preamplifier (3) counter and (4) vacuum pump

The experimental set-up consists of a scattering chamber which holds the gold leaf and the detector, a discriminator preamplifier, and a counter (Fig. 3). There is a vacuum pump attached to the scattering chamber as there might be a small number of alpha-particles in the air. The gold foil receives the alpha particles that are released from the Am-241 preparation through a slit aperture, which exit the gold foil at varied scattering angles. A semiconductor detector is used to determine which alpha particles were dispersed. With the arrangement we're using, the gold foil, slit, and preparation – which are all mounted to a standard swivel arm – are what are swung, not the detector. The chamber's side wall is securely fastened to the detector. The discriminator level should be set halfway between the spots where the noise is masking out and where the alpha count rate starts to decline.

The discriminator preamplifier is set to a voltage such that the count rate is zero in around 100s for an angle $\sim$ 30, to eliminate any electrical noise.

III. OBSERVATION AND CALCULATIONS

A. Recording Scattering Rate as a function of θ

The counts observed for different values of scattering angles are given in Table I. Using this, we have plotted θ vs. $N(\theta)$ and have fitted a best-fit plot of the form,

$$f(\theta) = \frac{A}{\sin^4\left(\frac{\theta - B}{2}\right)} \tag{6}$$

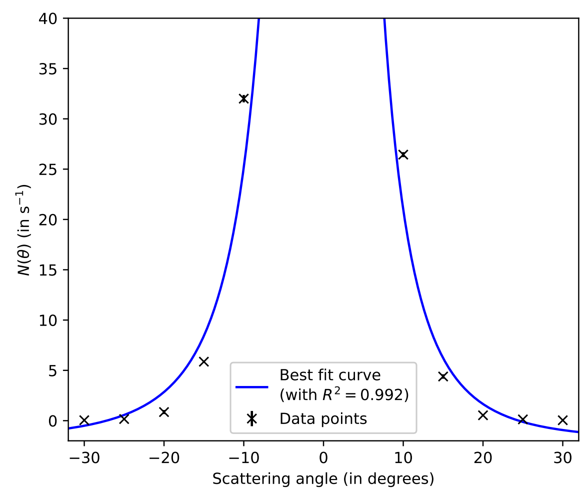

FIG. 4: $N(\theta)$ vs. scattering angle plot. The standard deviation in $N(\theta)$ is too small to be noticed.

Since the function blows up at $\theta = 0$ angle, our analysis focuses on measurements taken in the range $|\theta| > 5°$. The data is plotted along with the best-fit curve according to $f(\theta)$ in Fig. 4. The best-fit values of A and B obtained are:

$$A = (0.014 \pm 0.006)\ \mathrm{s}^{-1}$$
$$B = (-0.026 \pm 0.014)°$$

Hence, we can say that our experiment results validate Rutherford's scattering formula as they roughly obey a $\sin^{-4}(\theta/2)$ relation.

B. Determination of the atomic number of Aluminium

The count rates for Au and Al foils of slit width 1mm are given in Table II. The following parameters are known to us:

- Thickness of the gold foil, $d_{\mathrm{Au}} = 2\mu$m
- Thickness of the aluminium foil, $d_{\mathrm{Al}} = 8\mu$m
- Atomic concentration approximation, $c_{\mathrm{Al}} = c_{\mathrm{Au}}$
- Nuclear charge of gold, $Z_{\mathrm{Au}} = 79$
- Slit width (for both Au and Al) = 1 mm.

Using Eq. 5, we can plug in the above values to get,

$$\implies Z_{\mathrm{Al}} = 79\sqrt{\frac{2 \cdot 0.320}{8 \cdot 0.0325}} = 12.6$$

IV. ERROR ANALYSIS

From Eq. 5, we can get the uncertainity in N_{Al} as,

θ (°)	$t(\theta)$ (s)	$n(\theta)$	$n_{\mathrm{avg}}(\theta)$	$N_d(\theta)$ (in s^{-1})	$N(\theta)$ (in s^{-1})
5	100	5082	5070.3	50.70	92.59
		5024			
		5105			
10	100	2880	2885.0	28.85	26.44
		2912			
		2863			
15	100	712	714.0	7.14	4.39
		693			
		737			
20	200	220	230.7	1.15	0.54
		225			
		247			
25	600	205	205.0	0.34	0.13
		205			
30	900	127	124.0	0.14	0.04
		121			
-5	100	5295	5278.3	52.78	96.39
		5281			
		5259			
-10	100	3466	3492.0	34.92	32.01
		3518			
-15	100	947	953.5	9.54	5.86
		960			
-20	200	340	368.5	1.84	0.86
		397			
-25	600	252	261.0	0.44	0.16
		270			
-30	900	133	131.0	0.15	0.05
		129			

TABLE I: Measured counts n for different scattering angles for Au foil of width 5mm. N_d refers to counts per second. N refers to the space corrected count rate as per Eq. 4.

Material	θ (°)	$t(\theta)$ (s)	$n(\theta)$	$n_{\mathrm{avg}}(\theta)$	$N_d(\theta)$ (in s^{-1})
Au	-15	100	45	32.0	0.320
			35		
			26		
			27		
	15		27		
			30		
			30		
			36		
Al	-15	1000	30	32.5	0.033
			29		
	15		26		
			45		

TABLE II: Measured count rates for Au and Al foils of slit width 1mm

$$\frac{\Delta Z_{\mathrm{Al}}}{Z_{\mathrm{Al}}} = \sqrt{\left(\frac{\Delta N_{\mathrm{Au}}}{2N_{\mathrm{Au}}}\right)^2 + \left(\frac{\Delta N_{\mathrm{Al}}}{2N_{\mathrm{Al}}}\right)^2 + \left(\frac{\Delta d_{\mathrm{Au}}}{2d_{\mathrm{Au}}}\right)^2 \left(\frac{\Delta d_{\mathrm{Al}}}{2d_{\mathrm{Al}}}\right)^2} \tag{7}$$

By assuming $\Delta d = 0$ and taking the ΔN values as the standard deviation in count rates we have $\Delta N_{\mathrm{Al}} =$

$7.37/1000s = 7.37 \times 10^{-3}$ s^{-1} and $\Delta N_{\mathrm{Al}} = 6.00/100s = 6.00 \times 10^{-2}$ s^{-1},

$$\Delta Z_{\mathrm{Al}} = 1.8$$

V. DISCUSSION & CONCLUSION

We have successfully performed Rutherford's scattering experiment. In the first part, we by varying the angle between the detector and the emitter, we were able to get the count rate distribution as a function of the scattering angle. Our distribution more or less agrees with the general form of Rutherford scattering as given by Eq. 6. The fitting parameters obtained were:

The non-zero offsets B suggests inaccuracies stemming from misalignment in the collimator slit and minor horizontal angular displacement errors. From these, an angular displacement error of $(0.026 \pm 0.014)^\circ$ was identified, Since the resolution of the instrument is only 5°, this was considered almost negligible. Despite these limitations, we able to successfully confirm the $\sin^{-4}(\theta/2)$ dependence in the distribution from graphical analysis.

In the second part of the experiment, we performed Rutherford's scattering experiment at a fixed scattering angle for both gold and aluminium foils. Then, by Eq. 5, we were able to estimate the nuclear charge of Aluminium as,

$$\boxed{Z_{\mathrm{Al}} = 12.6 \pm 1.8}$$

The high amount of error the measurement is due to the extreme variation in count number for both foils. Ideally, these measurements should be taken for longer periods of time to minimise the error. Since the true value of Z_{Al} is 13, our results are within the acceptable values, with a percentage error of 3.07%.

VI. PRECAUTIONS AND SOURCES OF ERROR

1. When the vacuum pump is running, the gold leaf needs to be parallel to the nozzle. High velocity air might rupture the thin & expensive gold foil.
2. The gold foil should not be touched by the hands. The oil from the hands might be deposited on the foil, which will cause the readings to be inaccurate. It can also cause the foil to be damaged.
3. Any amount of air inside the vacuum chamber will cause the readings to be non-uniform and inaccurate.
4. The offset discriminator has to be set properly to get the correct readings.

[1] L. D. GmbH, *Rutherford scattering: measuring the scattering rate as a function of the scattering angle and the atomic number*, LD Physics leaflets (2023).