Study of Zeeman Effect
and the Determination of Bohr's Magneton

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In this experiment, we use Cadmium lamp subjected to various magnetic field intensities to study normal and anomalous Zeeman effect, with the help of a Fabry-Perot etalon. We then study the various components of Zeeman lines and their respective polarizations by varying the direction of the applied magnetic field. We also try to measure the value of Bohr's magneton from the results.

I. OBJECTIVE

1. Quantitatively study transverse normal Zeeman Effect by observing the splitting of the rings due to magnetic field resolved by Fabry-Perot etalon using a CMOS camera and evaluate the value of Bohr's magneton (μB). Observe the polarization of the rings using a polarizer
2. Observe the left circular and right circular polarized lines in anomalous normal Zeeman effect by using quarter wave plate and polarizer.
3. Observe the transverse anomalous Zeeman effect and polarization of the rings using a polarizer
4. Observe the longitudinal anomalous Zeeman effect and left circular and right circular polarized lines in anomalous normal Zeeman effect by using quarter wave plate and polarizer.

II. THEORY

Zeeman effect is the effect of splitting of a spectral line into several components in the presence of a static magnetic field. A shift in the energy level of one or both of the states involved in the transition results in a shift in the frequency/wavelength of the spectral line, which can be observed. Since the distance between the Zeeman sub-levels is a function of magnetic field strength, this effect is often used to measure magnetic field strength, for example that of stars like the sun or plasma.

Normal Zeeman Effect

The Zeeman effect that occurs for spectral lines resulting from a transition between singlet states is traditionally called the normal effect. For such states, the spin is zero and the total angular momentum J is equal to the orbital angular momentum L. When placed in an external magnetic field, the energy of the atom changes because of the energy of its magnetic moment in the field.

Mathematically, we can analyse this by considering a magnetic dipole. An external magnetic field will exert a torque on a magnetic dipole and the magnetic potential energy which results in

$$U(\theta) = -\mu \cdot B \tag{1}$$

where μ is the dipole moment. The magnetic dipole moment associated with the orbital angular momentum is given by,

$$\mu_{\text{orbital}} = \frac{e}{2m_e}L \tag{2}$$

where L is the orbital angular momentum. By the quantization of angular momentum in the solution of the Schrodinger's equation, we know that the z-component of angular momentum is given by $L_z = m_l\hbar$, where m_l is the magnetic quantum number. Then for a magnetic field in the z-direction,

$$U = \frac{e}{2m_e}L_zB = m_l\frac{e\hbar}{2m_e}B \tag{3}$$

Hence, this gives equally spaced energy levels displaced from the zero field level by

$$\Delta E = m_l\frac{e\hbar}{2m_e}B = m_l\mu_B B \tag{4}$$

$$\text{where } \mu_B = \frac{e\hbar}{2m_e} = 9.27\times10^{-24}\ J/T$$

This displacement of the energy levels gives the uniformly spaced multiplet splitting of the spectral lines. Since there are $2l+1$ values of m_l, each energy level splits into $2l+1$ levels. The selection rule $\Delta m_l = \pm1$ restricts the number of possible transition energies: $E_0+e\hbar B/2m_e$, E_0, and $E_0 + e\hbar B/2m_e$, corresponding to the transitions with $\Delta m_l = +1,\ 0,\ -1$.

In this experiment, we observe the splitting of the Cd-spectral line at 643.8 nm into three lines, the so-called *Lorentz triplets*. Cadmium has the electron structure, [Kr] $4d^{10}5s^2$, i.e. the outer shell taking part in optical transitions is composed of the two $5s^2$ electrons representing a completed electron shell. The transition seen here is $3^1d_2 \rightarrow 2^1p_1$.

When the magnetic field is in the transverse direction, $\Delta m_l = \pm1$ transitions give σ-lines which are polarized

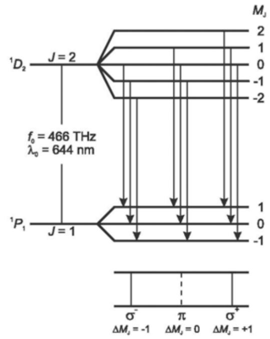
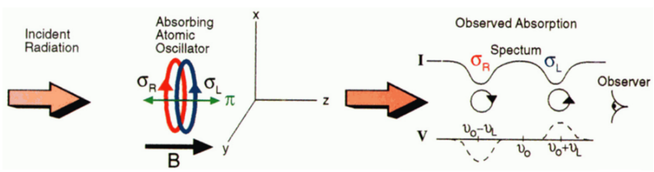
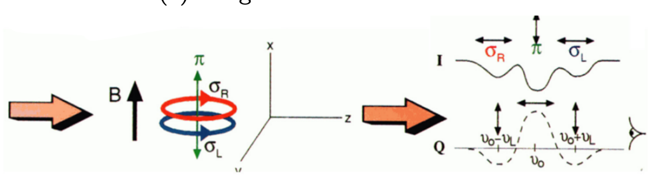


FIG. 1: Level splitting and transitions of the normal Zeeman effect in Cadmium



(a) Longitudinal Zeeman Effect



(b) Transverse Zeeman Effect

FIG. 2: Schematic representation of the polarization of the Zeeman components

vertically to the magnetic field. $\Delta m_l = 0$ transition gives a π-line in the middle, which is polarized parallel to the direction of the field. However in a longitudinal field, we see only σ lines which are right circular and left circular polarized (σ^+ and σ^-).

Anomalous Zeeman Effect

The anomalous Zeeman effect appears on transitions where the net spin of the electrons is non-zero. Here the electron spins do not cancel each other and the energy of an atomic state in a magnetic field depends on both the magnetic moments of electron orbit and electron spin. In this experiment, we use the Cd $2^3s_1 \rightarrow 2^3p_2$ transition at 508.58 nm to observe two groups of three σ lines (in vertical polarization) and one group of three π lines in horizontal polarization.

III. EXPERIMENTAL SETUP

In this experiment, we use a Fabry-Perot interferometer to study the spectral lines. This instrument uses the phe-

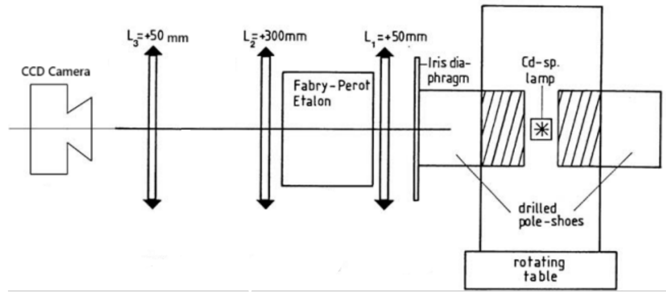


FIG. 3: Schematic representation of the the optical components of the experimental setup

nomenon of multiple beam interference that arises when light shines through a cavity bounded by two reflective parallel surfaces. Each time the light encounters one of the surfaces, a portion of it is transmitted out, and the remaining part is reflected back. The net effect is to break a single beam into multiple beams which interfere with each other. If the additional optical path length of the reflected beam (due to multiple reflections) is an integral multiple of the light's wavelength, then the reflected beams will interfere constructively. More is the number of reflection inside the cavity, sharper is the interference maximum. It makes use of multiple reflections which follow the interference condition for thin films.

The net phase change is zero for two adjacent rays, so the relation to find a maxima is:

The difference in wave numbers of one of the σ lines with respect to central π line of the same order is $\Delta k/2$. For this case,

$$\Delta E = hc\frac{\Delta k}{2} \tag{5}$$

Combining this with Eq. 4 with $\Delta m_l = 1$, we get

$$\mu_B = hc\frac{\Delta k}{2B} \tag{6}$$

Now, Δk can be calculated using,

$$\Delta k = \frac{1}{2\mu t}\left(\frac{\delta}{\Delta}\right) \tag{7}$$

The étalon consists of a quartz glass plate of $t = 3$ mm thickness coated on both sides with a partially reflecting layer (90% reflection, 10 % transmission). Refractive index of quartz at 509 nm is $\mu = 1.4519$ and at 644 nm is $\mu = 1.4560$. Here δ is mean of difference of squares of radii of different lines of same order of interference and Δ is the mean of difference of squares of radii of different order, given by

$$\delta_{n,xy} = R^2_{n,y} - R^2_{n,y} \text{ and } \Delta^x_n = R^2_{n+1,x} - R^2_{n,x} \tag{8}$$

Here, n refers to the order of the ring and x, $y \in (a, b, c)$ where a and c are outer σ lines and the middle b is π line.

Apparatus

1. Cd-spectral lamp on rotating table, with drilled pole pieces
2. A CCD camera
3. A Fabry-Perot etalon
4. Lenses (+50 mm and +300 mm)
5. Polarizing filters
6. Quarter-wave plate
7. Software to analyse CCD output
8. Optical Bench

IV. PROCEDURE

A. Normal Zeeman Effect w/ Transverse Magnetic Field

1. Switch on the Cd lamp and wait for it to warm up.
2. Align the lamp, first 50 mm lens and Fabry-perot tube. Use a red filter so that we only see the splitting of the 643.8 nm line. Observe the ring pattern coming out from etalon with your naked eye.
3. Now place the 300 mm lens and 50 mm lens along with camera and align the entire setup. Adjust the camera settings to make sure the rings of first three orders in normal Zeeman effect are visible on the screen.
4. Adjust the CMOS camera settings to get the ring pattern at the center of the screen.
5. One can observe that the splitting is function of magnetic field by varying the distance between pole pieces.
6. After confirming the ring pattern of Normal Zeeman effect with the ring pattern up to 3 orders, take pictures using the software for different values of magnetic field.
7. Now, place the polarizer and notice that middle line of each order is horizontally polarized and outer rings are vertically polarized. Identify the σ and π lines.

B. Normal Zeeman Effect w/ Longitudinal Magnetic Field

1. Now, rotate the magnetic field such that it is in the longitudinal direction and realign the optical components.
2. Place the quarter wave place and align it. This can convert linearly polarized light to circularly polarized light and vice-versa.
3. By rotating the polarizer, one can identify the left and right circularly polarized light.

C. Anomalous Zeeman Effect w/ Transverse Magnetic Field

1. Now, rotate the magnetic field back to the transverse direction. Replace the red filter with a green filter such that we are observing the 508.58 nm line.
2. Align the optics and observe 8 rings for each order (due to the insufficient resolution of Fabry perot etalon, instead of 9 rings, we see only 8 or sometimes 7 rings.)
3. Identify the σ and π lines and record pictures.

D. Anomalous Zeeman Effect w/ Longitudinal Magnetic Field

1. Now, rotate the magnetic field along the longitudinal direction and realign the optical components.
2. Place the quarter wave place and align it as well.
3. By rotating the polarizer, identify the left and right circularly polarized σ lines.

V. OBSERVATION AND CALCULATIONS

For normal Zeeman effect with transverse magnetic field, for different values of pole separation (i.e. different values of magnetic field), we have measured the radii of the spectral rings upto 3rd order.

	Radii (μm)		
	R_1 (1st order)	R_2 (2nd order)	R_3 (3rd order)
Pole separation = 40 mm			
a	30.54	139.47	170.47
b	72.16	140.53	180.27
c	98.66	159.69	188.19
Pole separation = 41 mm			
a	38.30	128.51	168.97
b	71.16	139.47	180.44
c	98.66	150.08	183.59
Pole separation = 42 mm			
a	28.37	126.90	168.48
b	63.49	135.73	175.78
c	85.02	145.07	182.98
Pole separation = 44 mm			
a	38.98	128.66	168.82
b	63.85	136.56	173.26
c	82.02	143.55	178.39
Pole separation = 45 mm			
a	47.91	132.32	173.78
b	67.57	139.61	177.49
c	82.58	144.96	182.39

TABLE I: Radii measured for each order for different values of pole separation. Here, Components of each order rings are designated as a, b and c, where the outer a and c are σ lines and the middle b is π line

Now, the average values of Δ and δ calculated from

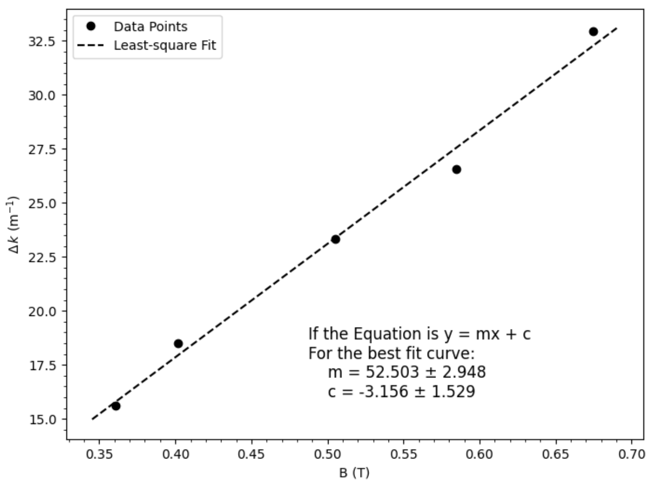


FIG. 4: Least-square fitting for the Δk vs. B plot

Table I using Eq. 8 are outlined in Table II. These values are then used to measure the difference in wave number, Δk, using Eq. 7 (Table III).

Pole Separation (mm)	B (mT)	Δk (m^{-1})	μ_B ($\times 10^{-24}$ J/T)
40	675	32.93	4.85
41	585	26.56	4.51
42	505	23.35	4.59
44	402	18.46	4.56
45	361	15.62	4.30

TABLE III: Values of pole separation and the corresponding values for B, Δk and μ_B

Rearranging Eq. 6, we get

$$\Delta k = \left(\frac{2\mu_B}{hc}\right) B \tag{9}$$

which is of the form $y = mx + b$. By performing a least square fit on the values of Table III shown in Fig. 4, we get the slope, $m = 52.503\ \mathrm{m}^{-1}\ \mathrm{T}^{-1}$ which translates to $\mu_B = 5.215 \times 10^{-24}$ J/T.

The rest of the observations for normal and anomalous Zeeman effect are provided at the end (Figs. 6, 7)

Error Analysis

Using Eq. 9, we can write the uncertainity in μ_B as,

$$\frac{\Delta \mu_B}{\mu_B} = \frac{\Delta m}{m} \tag{10}$$

where m is the slope. Hence, we get

$$\Delta \mu_B = \frac{5.215 \times 10^{-24} \times 2.948}{52.503} = 0.293 \times 10^{-24}\ \mathrm{J/T}$$

VI. RESULT AND DISCUSSION

Using a a Cd lamp source and a Fabry Perot etalon, we have observed Normal and Anomalous Zeeman effect in action. By observing upto 3 orders of rings for normal Zeeman effect with magnetic field in transverse direction (Fig. 5), we have calculated the value of Bohr's Magneton μ_B as,

$$\mu_B = (5.215 \pm 0.293) \times 10^{-24}\ \mathrm{J/T}$$

The value of μ_B obtained however is quite away from the actual value. The primary reason could be beacuse these magnets are not properly calibrated with the data graph provided to us. Maybe these magnets are old and have degraded over time.

Furthermore on observing the rings through a polarizing filter, we can observe that the middle line of each order is horizontally polarized and outer rings are vertically polarized. Identify the σ and π lines. Hence, one can identify the outer ones as σ lines and the middle one as a π line.

In the case of normal longitudinal Zeeman Effect, we first convert the circularly polarized light into linearly polarization using a quarter-wave plate. We observe that only the σ lines are now visible. Now, by using the polarizing filter, one can identify the left and right circularly polarized light as σ^+ and σ^- lines (Fig. 6). The reason we don't see the π line is beacause in this case the light propagates along B, and the incident electromagnetic wave has no oscillatory component along the linear oscillator parallel to B (the π component), so absorption of that component cannot occur (Fig. 2).

In the case of anomalous transverse Zeeman effect, we were able to identify 7 to 8 lines per order (Fig. 7). After using the polarization filter, we could distinguish between the 5 σ lines and 3 π lines by rotating the polarizer. For the anomalous transverse Zeeman effect, we could only observe the 5 σ lines (Fig. 7d). The reason for it is the same as that for normal Zeeman effect.

VII. PRECAUTIONS AND SOURCES OF ERROR

1. Handle the filters carefully.
2. While setting the optical path don't look to the Cd-lamp in naked eye for long.
3. Make sure the optical components are properly aligned and the pattern in is the center before taking measurements.

VIII. CONCLUSION

By subjecting a Cadmium lamp to various magnetic field intensities we have studied normal and anomalous

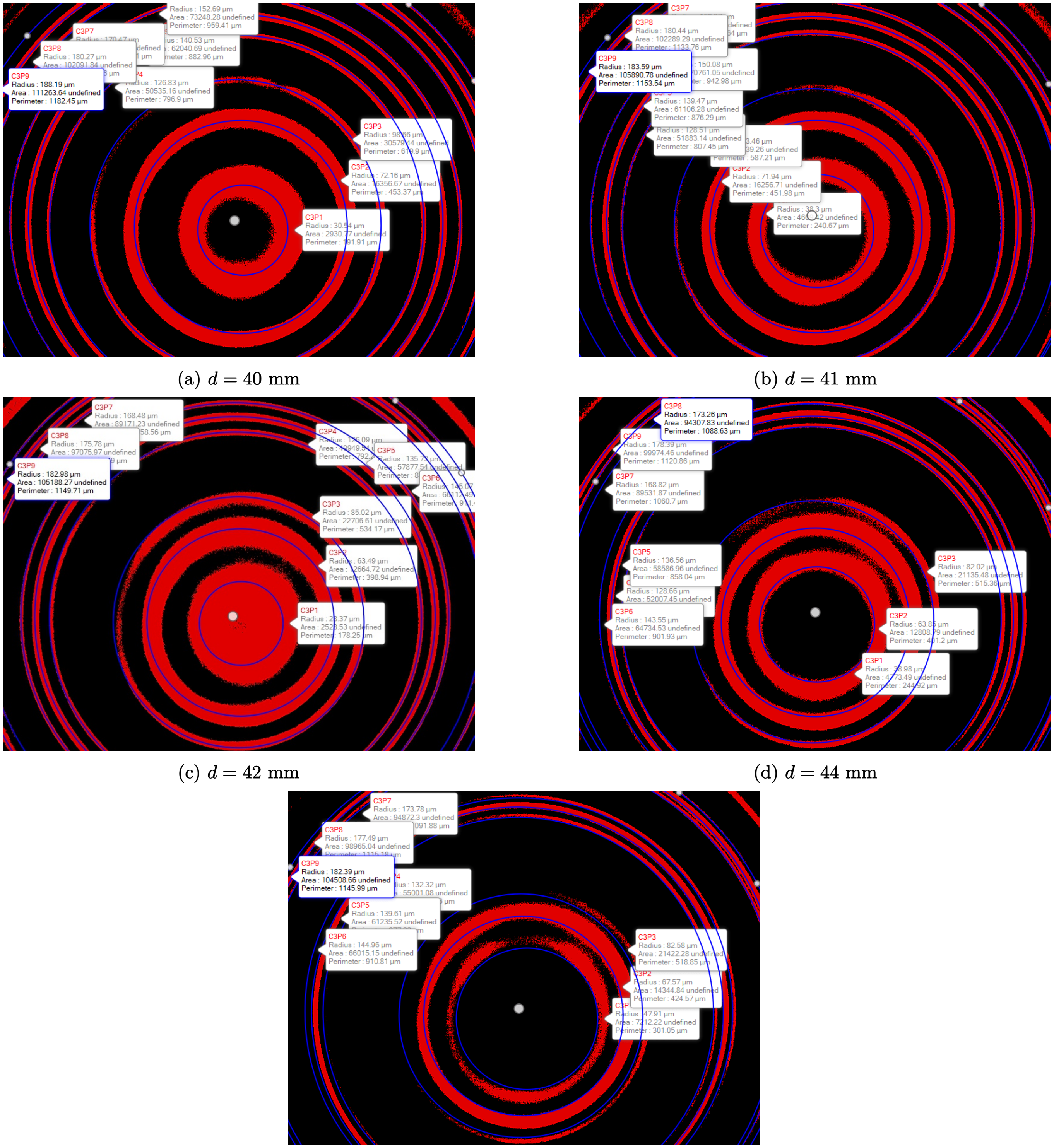
	$\delta_{n,xy} = R^2_{n,y} - R^2_{n,x}$ (mm^2)					$\Delta^x_n = R^2_{n+1,x} - R^2_{n,x}$			
	$n = 1$	$n = 2$	$n = 3$	Avg. $\delta_{n,xy}$		$x = a$	$x = b$	$x = c$	Avg. Δ^x_n
Pole separation = 40 mm									
$x = a$, $y = b$	4274.37	3662.83	3437.25	4095.27	$n = 1$	15153.16	14541.62	15767.10	14236.93
$x = b$, $y = a$	4526.73	5752.22	2918.20		$n = 2$	12974.17	12478.59	9914.58	
Pole separation = 41 mm									
$x = a$, $y = b$	3708.47	2937.06	4007.73	3071.92	$n = 1$	15047.93	14276.52	13789.24	13239.62
$x = b$, $y = a$	3559.41	3072.13	1146.69		$n = 2$	12036.04	13106.71	11181.28	
Pole separation = 42 mm									
$x = a$, $y = b$	3226.12	2319.02	2513.10	2743.57	$n = 1$	15298.75	14391.65	13816.91	13450.26
$x = b$, $y = a$	3197.42	2622.67	2583.07		$n = 2$	12281.90	12475.98	12436.38	
Pole separation = 44 mm									
$x = a$, $y = b$	2557.38	2095.24	1518.84	2097.31	$n = 1$	15033.96	14571.81	13879.32	13003.11
$x = b$, $y = a$	2650.46	1957.97	1803.96		$n = 2$	11946.80	12011.75	9914.58	
Pole separation = 45 mm									
$x = a$, $y = b$	2270.34	1982.37	1303.21	1849.25	$n = 1$	15213.21	14925.25	14193.95	13547.96
$x = b$, $y = a$	2253.75	1522.45	1763.41		$n = 2$	12690.91	12011.75	12252.71	

TABLE II: δ and Δ values calculated using Eq. 8. Here, δ is difference of squares of radii of different lines of same order, calculated for both the components of σ line and take average. Δ is the difference of squares of radii of different order. Note that we have ignored the value of Δ^c_2 in the avg. since it was too deviant from the other values.

Zeeman effect, with the help of a Fabry-Perot etalon. We have also studied the various components of Zeeman lines and their respective polarizations by varying the direction of the applied magnetic field. We have estimated the value of Bohr's magneton from the results.

[1] SPS, *Zeeman Effect (Lab Manual)*, NISER (2024).

[2] Riethmuller, *Tools for Solar Observations-III Zeeman effect. Magnetic fields and polarimetry* (2013).



(a) $d = 40$ mm

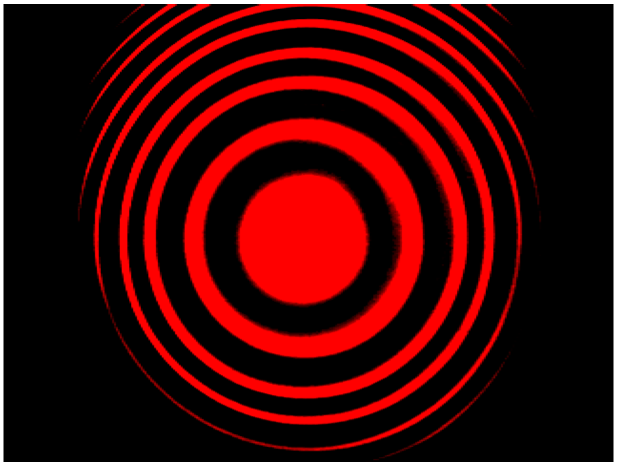
(b) $d = 41$ mm

(c) $d = 42$ mm

(d) $d = 44$ mm

(e) $d = 45$ mm

FIG. 5: Normal Zeeman effect observed for different values of magnetic field, i.e. pole separation (d)

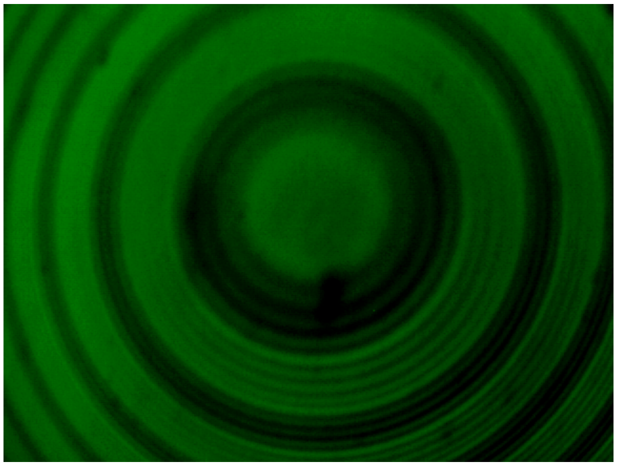


(a) Without any polarization filter

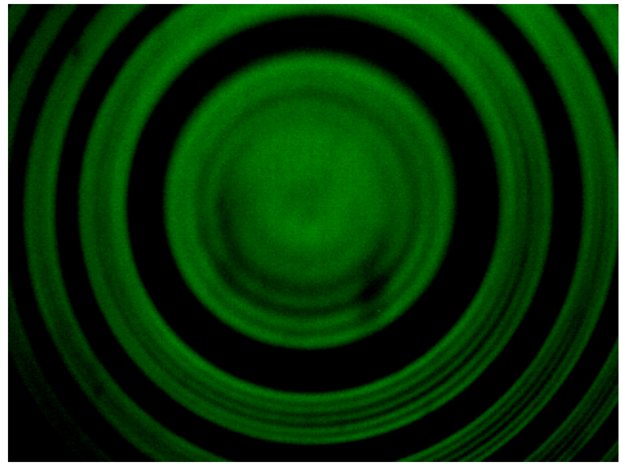
(b) Left circularly polarized light

(c) Right circularly polarized light

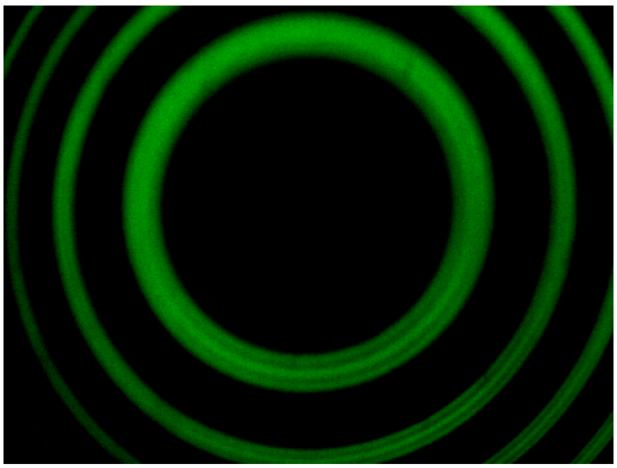
FIG. 6: Normal Zeeman Effect observed with longitudinal magnetic field



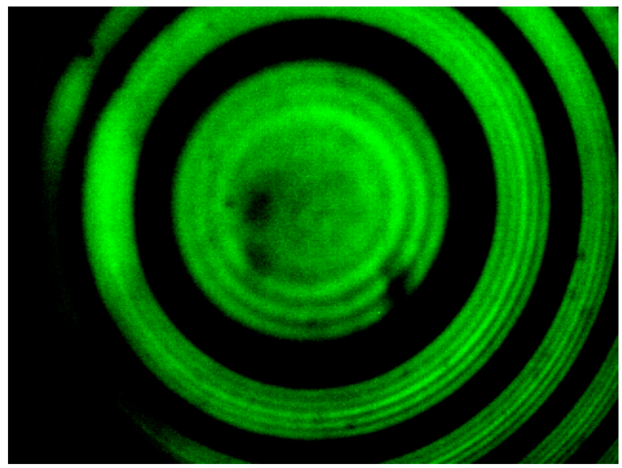
(a) Without any polarization filter



(b) σ lines observed



(c) π lines observed



(d) Only sigma lines observed even without the polarization filter in case of longitudinal magnetic field

FIG. 7: Anomalous Zeeman Effect observed in (a) to (c) transverse and (d) longitudinal magnetic fields