Experiments with SEELab Part II: Determination of the Band Gap of Germanium and Silicon using the Temperature Variation of Reverse Current

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(Dated: April 1, 2025)

In this experiment, we use a simple and cheap experimental set-up to determine the band gap of germanium using the SeeLab kit and a diode. We observe the temperature dependence of the reverse current through diodes at different temperatures, and use the diode equation to estimate the band-gap. Our results agreed with the literature value to a certain extent. We also discuss various shortcomings of this particular method in the estimation of band-gap.

I. THEORY

The band gap of a material is the minimum difference in energy between the highest energy state of the valence band and the lowest energy state of the conduction band. Determination of the band gap of a semiconductor is one of the important and basic thing in solid state physics as it provides critical information for understanding the band structure of material.

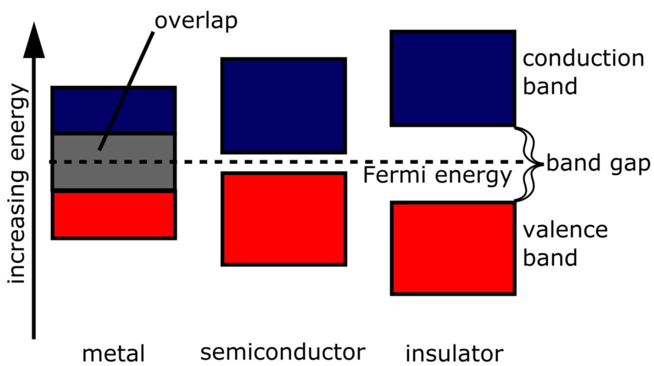

FIG. 1: Illustration depicting the conduction and valence bands along with the corresponding bandgaps for different types of materials

There are several well accepted ways to determine the bandgap of semiconductors including the four-probe method, UV-spectroscopy, by the temperature variation of resistivity or forward voltage (keeping the forward current constant) etc. In this experiment, we have presented a way to determine the band-gap of semiconductors by the studying the temperature variation of reverse current in the diode, following up on Biswas (2022).

A. Mathematical Formulation

When a pn junction diode is operated under reverse bias conditions, a very small amount of current flows through the diode, called the *reverse current.* The magnitude of this current is almost negligible ($\sim \mu$A) and is generated due to the thermally generated minority charge carriers on each side of the junction.

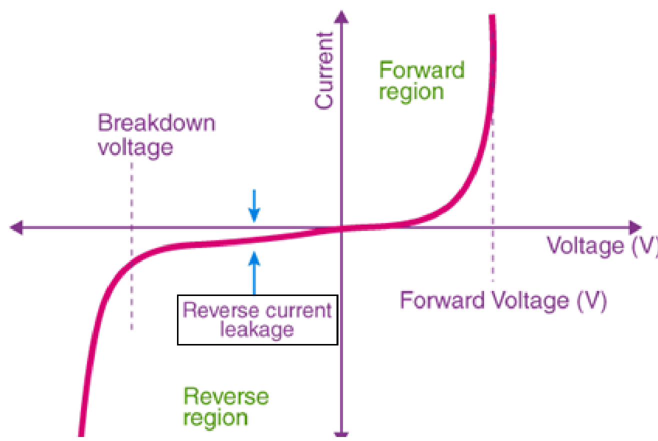

FIG. 2: Typical I-V characterstic of a pn junction. The reverse current is highlighted and is quite negligible under normal conditions.

For a fixed reverse bias potential, the number of minority charge carriers generated thermally now purely depends on the temperature of the system.

Mathematically, we can write the reverse current I_r as,

$$I_r = A \exp\left(-\frac{E_g}{\eta k_B T}\right) \tag{1}$$

where A is a constant, E_g is the band-gap of the material used, k_B is the Boltzmann constant, T is the temperature and η is the diode quality factor. Typically $\eta = 1$ for Ge and for Si, $\eta = 2$.

Now, taking log on both sides of the above equation,

$$\ln(I_r) = \ln(A) - \frac{E_g}{\eta k_B T} \tag{2}$$

It is to be noted that E_g itself is a function of temperature (Varshni, 1967),

$$E_g = E_0 - \frac{\alpha T^2}{\beta + T} \tag{3}$$

i.e., E_g decreases with increase in temperature where E_0 denotes the band gap at absolute zero temperature and α and β are two constants. The major reason for this is the shift in the relative position of the conduction and valence bands due to a temperature-dependant electron lattice interaction. Experimentally we have determined, $\alpha \sim 10^{-4}$ and $\beta \sim 10^3$. Due to this, fractional term is quite small compared to E_0 for the temperature range used in this experiment, we can assume $E_g \approx E_0$.

Thus, Eq. 2 simplifies to

$$\ln(I_r) \approx \ln(A) - \frac{E_0}{\eta k_B T} \tag{4}$$

Now, if we measure the value of I_r at different temperatures, we can find determine E_0 from the slope of the straight-line fit of a $\ln(I_r)$ vs $\ln(T)$ plot. That is, Eq. 4 is of the form,

$$y = mx + c \tag{5}$$

where $y = \ln(I_r)$, $x = 1/T$ and

$$m = -\frac{E_0}{\eta k_B} \tag{6}$$

$$c = \ln(A) \tag{7}$$

II. EXPERIMENTAL SETUP

A. Apparatus

1. ExpEYES-17 kit + Computer
2. Germanium Diode (1N60)
3. Resistors (one 1 kΩ and two 100 Ω)
4. Soldering iron
5. Chromel/Alumel Thermocouple for temperature measurement
6. Wooden Board to fix the setup

B. Circuit Design

The circuit diagram of the setup is shown in Fig. 3. R_s is the shunt resistance connected across R_1 to measure the voltage across the diode. If V_1 is the measured voltage across R_1 by terminal `A3`, then the reverse current through the diode is given by,

$$\begin{aligned} I_r R_1 &= \frac{V_1 R_g}{R_g + R_{\text{diode}}} \\ \implies I_r &= \frac{V_1}{G R_1} \\ \text{where, } G &= 1 + \frac{R_{\text{diode}}}{R_g} \end{aligned} \tag{8}$$

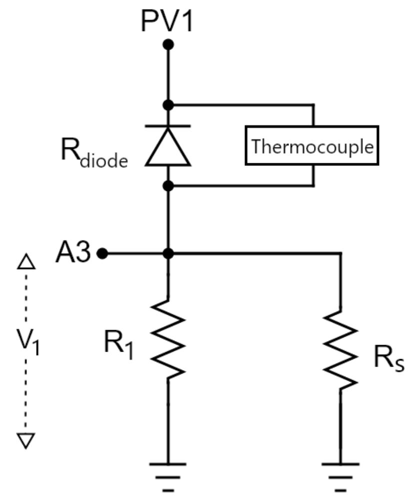

FIG. 3: Circuit diagram with connections to terminals of the SeeLab module

C. Procedure

The experimental setup and circuit diagrams are shown in Figs. 3 and 4. The procedure of the experiment is detailed below.

1. Begin by fixing the soldering iron on a wooden board and attach the diode near it so that it is not in direct contact.
2. Attach the thermocouple as close as possible to the diode to measure accurate temperature readings.
3. Connect the diode to the SeeLab breadboard setup and make connections according to the given circuit diagram. Connect the board to the computer and provide around 4V reverse bias voltage to `PV1` using following Python code:

```python
import eyes17.eyes
board = eyes17.eyes.open()
board.set_pv1(4)
```

4. Now read the voltage values at `A3` using

```python
v1 = board.get_voltage('A3')
```

5. Also note the corresponding temprature reading on the thermo-couple.
6. Calculate I_r from V_1 using Eq. 8.
7. Plot $\ln(I_r)$ vs. $\ln(T)$ and estimate E_0.

III. OBSERVATION AND CALCULATIONS

For this particular setup, we have measured $R_1 = 990$ Ω and $R_g = 195.9\,\Omega$. The diode resistance under reverse bias was measured to be $R_{\text{diode}} \approx 63$ MΩ.

Hence G comes out to be 3.216×10^4. The observed values of V_1 along with the corresponding temperature is

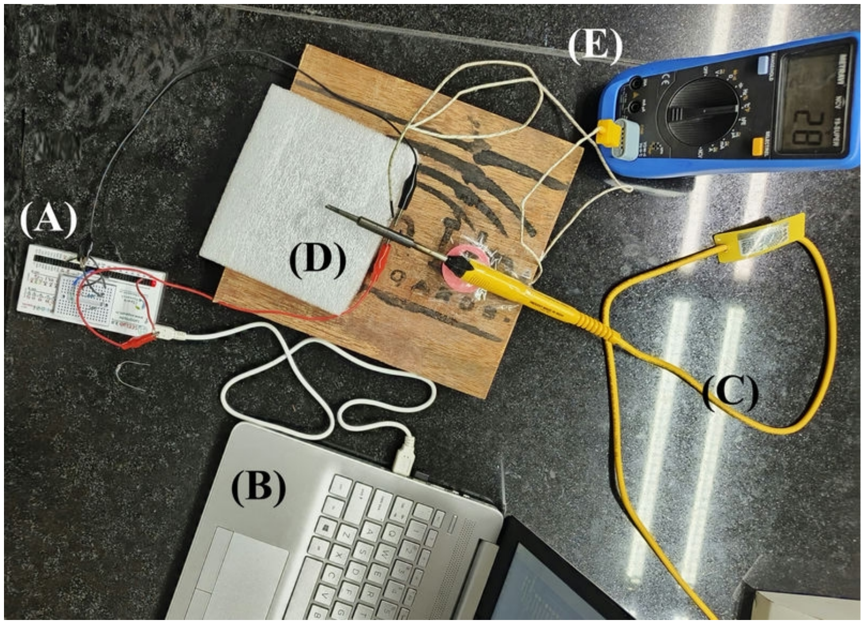

FIG. 4: Experimental setup where (A) is the SeeLab module connected as per the circuit diagram to the (B) computer. (C) is the soldering iron attached to a wooden board (D) on which the diode is also attached. (E) is the multimeter connected to the thermocouple which reads the temperature of the diode.

Temperature (°C)	V_1 (V)	I_r (10^{-10} A)
35	0.02189120	6.875
38	0.02812214	8.832
42	0.03284560	10.316
45	0.03966955	12.459
46	0.05946561	18.677
48	0.06635798	20.841
50	0.08912472	27.992
54	0.08676519	27.251
57	0.09966206	31.302
61	0.15966206	50.147

TABLE I: Observed temperature and V_1 along with the corresponding I_r

given in Table I.

Fig. 5 shows the plot between $\log(I_r)$ and T^{-1}. Using linear regression, we have obtained a best-fit straight line with slope and intercept as shown in the figure.

We can now use Eq. 6 to equate the slope and intercept to the corresponding physical parameters.

$$\begin{aligned} m &= -7.763 \times 10^3 = -\frac{E_0}{\eta k_B} \\ \implies E_0 &= -7.763 \times 10^3 \times \eta k_B \\ &= 0.669 \text{ eV} \end{aligned}$$

using $k_B = 8.6173 \times 10^{-5}$ eV K^{-1}. Similarly, we can also obtain

$$\begin{aligned} c &= 4.082 = \ln(A) \\ \implies A &= 59.27 \text{ Amp} \end{aligned}$$

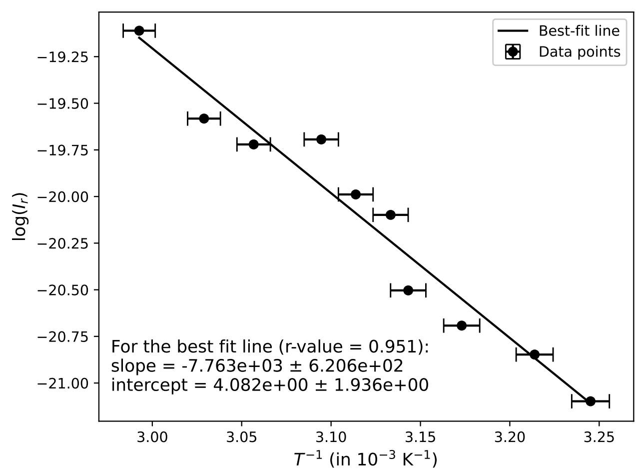

FIG. 5: $\log(I_r)$ vs. T^{-1} plot

For the purposes of this experiment, we do not discuss the value of A, which represents the *reverse saturation current*.

IV. ERROR ANALYSIS

The error in G can be derived from the error propagation formula using

$$\Delta G = \frac{R_{\text{diode}} \Delta R_g}{R_g^2} = 0.156$$

using $\Delta R_g = 0.1\,\Omega$ and $\Delta R_{\text{diode}} \approx 0\,\Omega$. The uncertainity in I_r will be,

$$\frac{\Delta I_r}{I_r} = \sqrt{\left(\frac{\Delta V_1}{V_1}\right)^2 + \left(\frac{\Delta R_1}{R_1}\right)^2 + \left(\frac{\Delta G}{G}\right)^2} \tag{9}$$

$\Delta R_1 = 0.1\,\Omega$ and $\Delta V_1 = 10^{-8}$ A. Hence the uncertainity in $\ln(I_r)$ would be $\Delta I_r/I_r \sim 10^{-4}$ which is too small to be noticable in Fig. 5. The uncertainity in T^{-1} is $\Delta T/T^2 \sim 10^{-5}$ where $\Delta T = 1°\text{C}$.

Now, for the uncertainity in the band-gap, we get

$$\begin{aligned} \frac{\Delta E_0}{E_0} &= \sqrt{\left(\frac{\Delta\text{slope}}{\text{slope}}\right)^2} \qquad (10) \\ \implies \Delta E_0 &= 0.669 \times \frac{0.621}{7.763} \\ &= 0.053 \text{ eV} \end{aligned}$$

V. DISCUSSION & CONCLUSION

In this experiment, we fixed the reverse bias potential of a Ge diode to 4.0V and recorded reverse current I_r using SeelLab, at different temperatures ranging from 35°C to 61°C.

The reverse was found to vary slowly for the Ge diode, there was a relatively large increase for temperatures over 50°C. This could be due to the T^2 dependence of the band-gap energy.

Then, using the $\ln(I_r)$ vs. T^{-1} plot, we estimated the band-gap of Ge to be around,

$$\boxed{E_0 = (0.669 \pm 0.053) \text{ eV}}$$

While this is close to the literature value of 0.741 eV, there is still a relatively large error of around -9.7%. This could mainly be due to the improper measurement of temperature. Note that we measure the temperature by placing the thermocouple as close as possible to the diode, the actual temperature of the pn junction might be different from the temperature just outside the diode. There was also a significant amount of fluctuation present in the temperature readings. However, on calibration with a mercury thermometer, the temperature recorded by the thermocouple was found to match perfectly.

Also note that in determining the band gap, we neglected the temperature dependent term from Eq. 3. Over the temperature range of the experiment, we can estimate the theoretical change in E_g using

$$\Delta E_g = \alpha \left(\frac{T_L^2}{T_L + \beta} - \frac{T_H^2}{T_H + \beta}\right) \tag{11}$$

Using $T_L = 35°\text{C}$ and $T_H = 61°\text{C}$, along with $\alpha = 7.917 \times 10^{-4}$ eV K^{-1} and $\beta = 2220$ K [2], we get $\Delta E_g = 0.005$ eV. Thus the variation of E_g is significantly lower than the error estimated in our measurement of E_0 and hence is unlikely to be detected by the current setup. Thus, for the purposes of this experiment, we can estimate $E_g = E_0$.

This experiment was also performed using a Silicon diode to comparitively study it with the Germanium one. However, we found it difficult to measure the temperature of the diode to the closed architecture of the Si diode as compared to the Ge one. Hence, we have not included that in our analysis.

VI. PRECAUTIONS AND SOURCES OF ERROR

1. Avoid heating the diode to higher temperatures to avoid possible damage to the diode.
2. Ideally, many data points need to be collected to statistically reduce the error in the measurement of band-gap.
3. Make sure that the soldering iron does to heat too much to avoid possible damage to any nearby items. Use proper protection while using it.

[1] S. Biswas, Determination of the band gap of germanium and silicon using ExpEYES-17 kit, Physics Education **57**, 025026 (2022).
[2] Y. Varshni, Temperature dependence of the energy gap in semiconductors, Physica **34**, 149 (1967).
[3] B. P. A. Kumar and V. V. V. Satyanarayana, *The ExpEYES-17 User's Manual*, The Phoenix Project, Inter University Accelerator Center (2017).