EXPERIMENT – 6

STUDY OF FABRY- PEROT INTERFEROMETER AND DETERMINATION OF WAVELENGTH OF LASER DIODE & DIFFERENCE IN WAVELENGTH OF SODIUM D-LINES

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Objectives:

1. Alignment of Fabry-Perot Interferometer to observe concentric circular fringes.
2. Measurement of the wavelength of a diode laser.
3. Determination of difference in wavelengths of sodium doublet.

Theory

(A) Working principle of a Fabry-Perot interferometer

A Fabry-Perot interferometer utilizes the phenomenon of multiple beam interference. The setup essentially consists of two optically flat partially reflecting surfaces held parallely. The light coming from the source is broken down into multiple beams as it continuously reflects between the highly reflecting plates, while transmitting a portion after every reflection. The fringes are formed due to interference between these reflected beams.

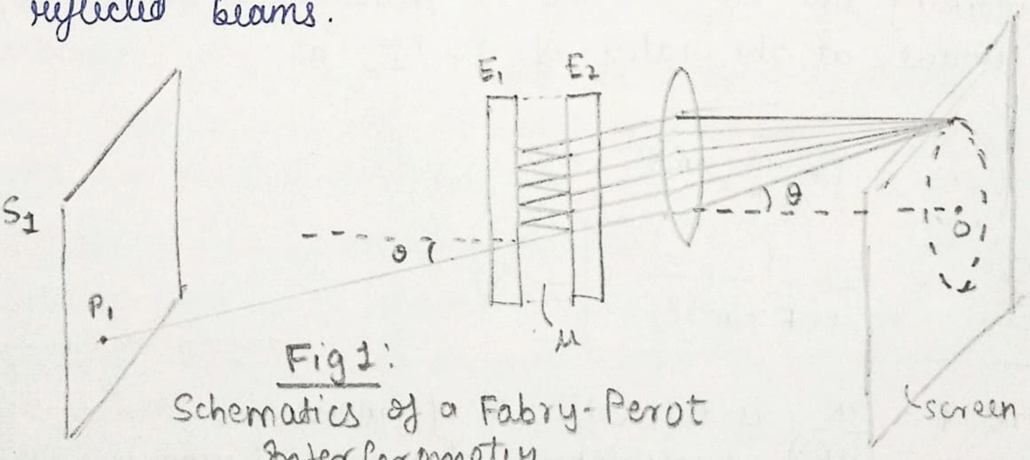


Fig 1:
Schematics of a Fabry-Perot Interferometer

In the above setup, E_1 & E_2 are two partial mirrors aligned parallel to each other. Let a ray from point P on the source be incident at an angle θ, producing multiple reflections in the cavity. The transmitted rays are parallel and focused using a convex lens at point P' on the screen, at the same angle θ. Let n be the refractive index of the medium inside the cavity. From fig. 2, the path difference between the two beams are

$$\Delta = n(AB + BC) - AD$$

we know, $AB = \frac{d}{\cos\theta'} = BC$

and $AD = AC \sin\theta$

using $AC = 2(d\tan\theta')$ & $\sin\theta = n\sin\theta'$ (Snell's law).

$\Rightarrow AD = 2d\tan\theta' . n\sin\theta'$

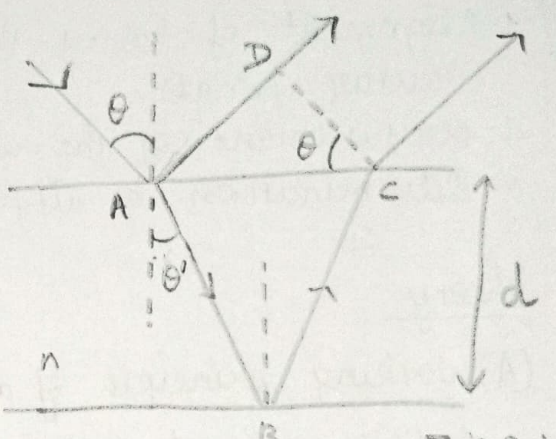


Fig 2: path difference

$$\Rightarrow \Delta = n\left(\frac{2d}{\cos\theta'}\right) - 2nd\sin\theta'\tan\theta'$$

$$= 2nd\left[\frac{1-\sin^2\theta'}{\cos\theta'}\right]$$

$$\Rightarrow \boxed{\Delta = 2nd\cos\theta'} \text{ — (1)}$$

For our case, the medium inside the cavity is air, hence $n=1$

$\Rightarrow \sin\theta = \sin\theta' \Rightarrow \theta = \theta'$

$$\therefore \boxed{\Delta = 2d\cos\theta} \text{ — (2)}$$

By working out the incident & transmitted intensities, one can arrive at the ratio of I_T / I_0 as,

$$\frac{I_T}{I_0} = \frac{1}{1+\frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2}}$$

or
$$\boxed{\frac{I_T}{I_0} = \frac{1}{1+F\sin^2\delta/2}} \text{ — (3)}$$

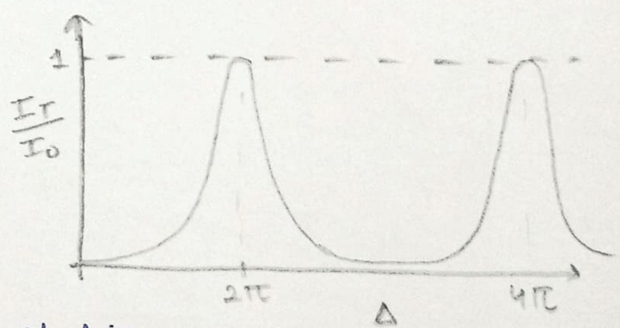


Fig. 3 Plot of Airy f^n for I_T/I_0

where $F = \frac{4R}{(1-R^2)}$ is the coefficient of finesse describes the sharpness of rings.

& R = Reflectance of the plates.

& $\delta = \frac{2\pi}{\lambda}\Delta$

from eq^n (3), we can see that I_T/I_0 is maximum when

$\delta = 2m\pi \Rightarrow \boxed{\Delta = m\lambda}$ (bright fringe)

correspondingly, dark fringes occur when

$\delta = (2m+1)\pi \Rightarrow \boxed{\Delta = (2m+1)\lambda/2}$

Rays with same inclination on the plate will form a circular fringe pattern (fringes of equal inclination).

(B) Determination of Wavelength

For light rays with the same inclination (θ), the path difference changes as the distance between the plates change. For increasing d, a maxima will appear while on decreasing d, it will vanish. For small values of θ [near the center of the fringe pattern),

$$\Delta m = \frac{2\Delta d}{\lambda} \Rightarrow \boxed{\lambda = \frac{2\Delta d}{\Delta m}} \quad \text{—(4)}$$

where Δd = change in distance b/w the plates

& Δm = no. of fringes appearing/disappearing at the center.

(C) Determination of difference in wavelength of the Sodium Doublet

The sodium doublet consists of two wavelengths very close to each other, 589.0 & 589.6 nm. This arises due to the spin-orbit splitting of 3p level into states w/ angular momentum $j = 3/2$ & $j = 1/2$ by the magnetic energy of electron spin & in the presence of internal magnetic field caused by orbital motion.

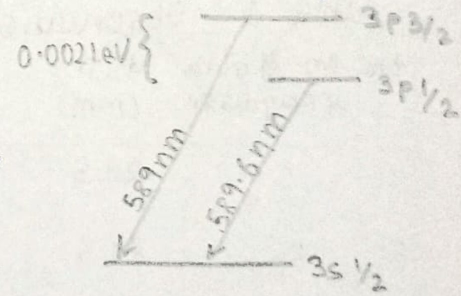


Fig. 4
sodium D-lines

During the process of moving the plates, two yellow lines will appear periodically clear and blurry (due to splitting).

For a given separation (2d), the maxima of the two wavelengths coincide as,

$$2d = m_1\lambda_1 = m_2\lambda_2 \quad \text{—(5)}$$

Due to difference in λ, on changing d the corresponding fringes will not move equally and the pattern will be blurry. When $(m_1+m)^{th}$ order of λ_1 coincides with $(m_1+m+1)^{th}$ order of λ_2, ($\lambda_1 > \lambda_2$) after changind 2d by $2\Delta d$, eq^n (5) becomes,

$$2(d+\Delta d) = (m_1+m)\lambda_1 = (m_2+m+1)\lambda_2$$

approximating $\lambda_1\lambda_2 \approx \lambda^2$, $\Delta\lambda$ becomes,

$$\boxed{\Delta\lambda = \frac{\lambda^2}{2\Delta d}} \quad \text{—(6)}$$

Apparatus Required

1. Optical Breadboard
2. Laser Diode
3. Optical post & holder
4. Beam expander
6. Fabry-Perot interferometer (w/ kinematic adjustment)
7. Linear translation stage
8. Diffusing screen
9. Sodium Lamp.

Observations & Calculations

(A) Determination of wavelength of ~~Dio~~ Laser Diode

Least count of the coarse micrometer = 0.5 mm

" " " fine micrometer $= \frac{\text{pitch}}{\text{divs.}} = \frac{0.5}{50} = 0.01\,mm$

Lever ratio of the micrometer = 0.03:1 (LR = 0.03)

Table 1: Observation table for Δd as a fn of Δm

S.no.	No. of fringes appear/disapp.	MSD (mm)	CSD (mm)	d_i (mm)	Δd_i (mm)	$\Delta d_i \times$ L.R. (mm)	λ (nm)
1.	0	20.5	31	20.81			
2.	20	20.5	12	20.62	0.19	0.0057	570
3.	30	20.5	4	20.54	0.27	0.0081	540
4.	40	20.0	43	20.43	0.38	0.0114	570
5.	50	20.0	34	20.34	0.47	0.0141	564
6.	60	20.0	25	20.25	0.56	0.0168	560
7.	70	20.0	16	20.16	0.65	0.0195	557
8.	80	20.0	7	20.07	0.74	0.0222	555
9.	90	~~20.0~~ 19.5	47	19.97	0.84	0.0252	560
10.	100	~~20.1~~ 19.5	39	19.89	0.92	0.0276	552
11.	110	19.5	30	19.80	1.01	0.0303	551

Least square fitting for Δd vs. Δm plot

We can rearrange eqn (4) to form, $\Delta d = \left(\frac{\lambda}{2}\right)\Delta m$
which is of the form $y = mx + b$, where slope, $m = \frac{\lambda}{2}$
& b = y-intercept.

By defining $\chi^2 = \sum_i \left(\frac{y_i - mx_i - b}{\sigma_i^2}\right)^2$ and minimising χ^2 wrt m & b,
we get the eqns

$$m\sum_i \frac{x_i}{\sigma_i^2} + b\sum_i \frac{1}{\sigma_i^2} = \sum_i \frac{y_i}{\sigma_i^2} \quad \text{— (7)}$$

$$\& \quad m\sum_i \frac{x_i^2}{\sigma_i} + b\sum_i \frac{x_i}{\sigma_i^2} = \sum_i \frac{x_i y_i}{\sigma_i^2} \quad \text{— (8)}$$

or, $bS + mS_x = S_y$
$bS_x + mS_{xx} = S_{xy}$, where $S = \sum_i \frac{1}{\sigma_i^2}$, $S_x = \sum_i \frac{x_i}{\sigma_i}$, $S_y = \sum_i \frac{y_i}{\sigma_i}$

$S_{xx} = \sum_i \frac{x_i^2}{\sigma_i^2}$, $S_{xy} = \sum_i \frac{x_i y_i}{\sigma_i^2}$

Solving for m & b, we get

$$m = \frac{SS_{xy} - S_x S_y}{\Delta} \quad (9), \qquad b = \frac{S_{xx}S_y - S_x S_{xy}}{\Delta} \quad (10)$$

Table 2: Least square fitting parameters for Δd vs. Δm plot

S.no.	x_i (Δm_i)	y_i (Δd_i) (mm)	x_i^2	$x_i y_i$ (mm)	$[y_i - (mx_i + b)]^2$ ($\times 10^{-10}$)
1.	20	0.0057	400	0.1140	1.19
2.	30	0.0081	900	0.2430	1309.12
3.	40	0.0114	1600	0.4560	350.71
4.	50	0.0141	2500	0.7050	185.95
5.	60	0.0168	3600	1.0080	73.02
6	70	0.0195	4900	1.3650	11.93
7.	80	0.0222	6400	1.7760	2.68
8.	90	0.0252	8100	2.2680	541.62
9.	100	0.0276	10000	2.7600	139.67
10.	110	0.0303	12100	3.3330	2085.92
Sums	650	0.1809	50500	14.028	2.90×10^{-7}

eq^n ⑦ & ⑧ become

$$650m + 10b = 0.1809 \quad \& \quad 50500m + 650b = 14.028$$

Here, $\Delta = S_{xx}S - S_x^2$

From eq^n ⑨ & ⑩, putting in the value of Δ,

$$m = \frac{N\sum_i x_i y_i - \sum_i x_i \sum_i y_i}{N\sum_i x_i^2 - (\sum_i x_i)^2} = 2.7509 \times 10^{-4}$$

$$\& \; b = \frac{\sum_i x_i^2 \sum_i y_i - \sum_i x_i \sum_i x_i y_i}{N\sum_i x_i^2 - (\sum_i x_i)^2} = 2.0909 \times 10^{-4}$$

using $m = \frac{\lambda}{2}$, we get $\lambda = 5.502 \times 10^{-4}$ mm

≈ 0.0006 mm $= 600$ nm

(upto correct S.Gs)

(B) Difference in wavelength of Sodium D-lines

Least count of the micrometer screw = 0.01mm

Using eq^n ⑥, $\lambda^2 = (589nm)(589.6nm) = 3.47 \times 10^{-7}\ mm^2$

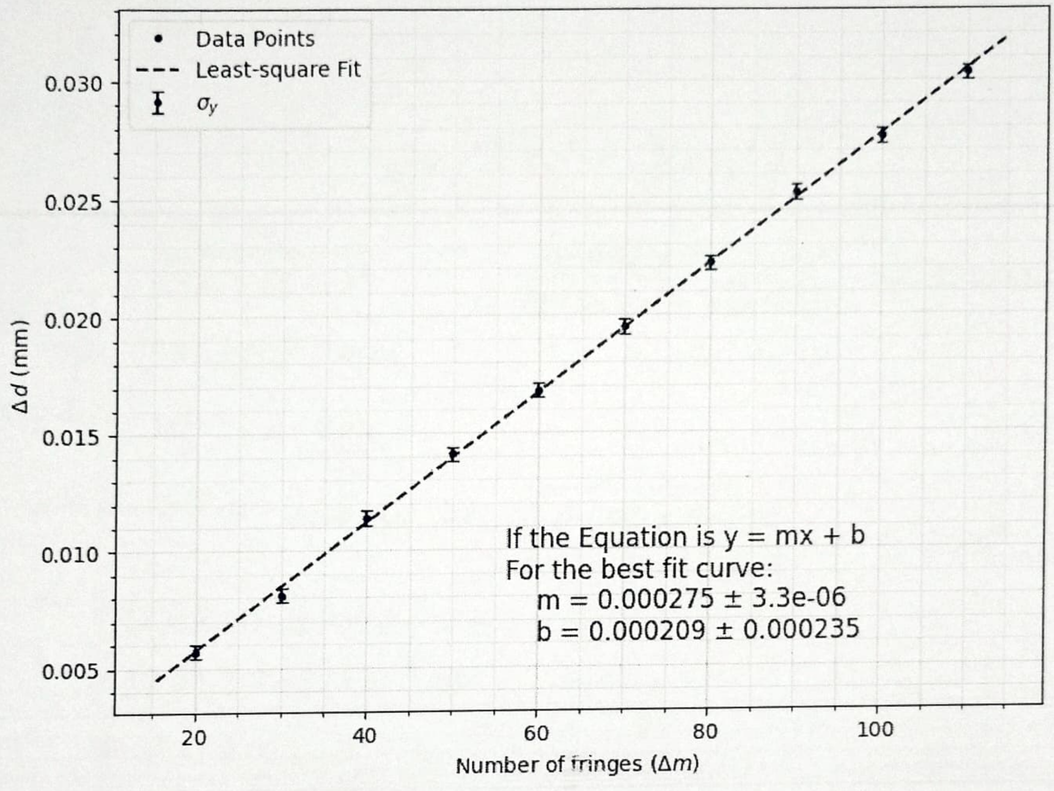
Table 3 : $2\Delta d$ values measured for sodium D-lines

d_1 msd (mm)	csd	tot. (mm)	d_2 msd (mm)	csd	tot. (mm)	$2\Delta d_i$ (mm)	$\Delta\lambda_i = \lambda^2/2\Delta d_i$ (mm)	$\sigma_\lambda = (\overline{\Delta\lambda} - \Delta\lambda_i)^2$ (mm^2)
5.5	7	5.57	6.0	33	6.33	0.76	4.57×10^{-7}	8.01×10^{-14}
6.0	15	6.15	6.5	8	6.58	0.43	8.08×10^{-7}	4.59×10^{-15}
6.0	2	6.02	6.0	46	6.46	0.44	7.89×10^{-7}	2.44×10^{-15}
8.0	0	8.00	8.0	40	8.40	0.40	8.68×10^{-7}	1.65×10^{-14}
9.0	14	9.14	9.5	10	9.60	0.46	7.55×10^{-7}	2.27×10^{-16}
4.5	44	4.94	5.0	47	5.47	0.53	6.55×10^{-7}	7.17×10^{-15}
4.0	30	4.30	4.5	21	4.71	0.41	8.47×10^{-7}	1.15×10^{-14}

Here mean $\Delta\lambda$, $\overline{\Delta\lambda} = 7.40 \times 10^{-7}$ mm

$\therefore$ ~~Std. dev.~~ $\cancel{= \sqrt{\frac{1}{N-1}\sum_i (\overline{\Delta\lambda} - \Delta\lambda_i)^2} = \sqrt{\frac{1.22 \times 10^{-13}}{6}} = 1.42 \times 10^{-7}\ mm}$

$\therefore$ the difference b/w sodium D-lines wavelengths are ≈ 0.74 nm


Data Points
Least-square Fit
σy
0.030
0.025
0.020
0.015
0.010
0.005
Δd (mm)
20
40
60
80
100
Number of fringes (Δm)
If the Equation is y = mx + b
For the best fit curve:
m = 0.000275 ± 3.3e-06
b = 0.000209 ± 0.000235

Error Analysis

(A) Determination of Wavelength

By minimising m & b wrt. y_i, we can find the uncertainities in m & b as σ_m & σ_b respectively.

$$\sigma_m^2 = \frac{S}{\Delta} = \frac{\sum_i \frac{1}{\sigma_i^2}}{\Delta}$$

using $\Delta = S_{xx} S - S_x^2$, $\frac{1}{\sigma_i^4}\left[N\sum_i x_i^2 - (\sum x_i)^2\right]$

$$\Rightarrow \sigma_m^2 = \frac{\sigma_i^2 N}{N\sum x_i^2 - (\sum x_i)^2} = \frac{0.003 \times 10}{10 \times 50500 - 650^2} \approx \cancel{3.3 \times 10^{-6}\ mm}$$

$$\Rightarrow \sigma_m \approx 3.3 \times 10^{-6}\ mm$$

Similarly, $\sigma_b^2 = \frac{S_{xx}}{\Delta}$

which turns out to be, $\sqrt{\frac{0.003 \times 50500}{10 \times 50500 - 650^2}} \approx 2.35 \times 10^{-4} \approx b$

$\therefore$ $m = (2.75 \pm 0.03) \times 10^{-4}$ mm

$b = (2.09 \pm 2.35) \times 10^{-4}$

since $\lambda = 2m$, $\frac{\sigma_\lambda}{\lambda} = \frac{\sigma_m}{m}$

$$\Rightarrow \sigma_\lambda = \frac{0.03}{2.75} \times 5.50 \times 10^{-4} = 0.06\ mm$$

$\Rightarrow$ $\boxed{\lambda \approx (600 \pm 6)}$ nm upto approriate sig. figures

(B) Difference in λ of sodium D-lines

Mean $\Delta\lambda$ obtained from table 2, $\overline{\Delta\lambda} = 7.40 \times 10^{-7}$ mm

std. dev in $\Delta\lambda$, $\sigma_{\Delta\lambda} = \sqrt{\frac{1}{N-1}\sum_i (\overline{\Delta\lambda} - \Delta\lambda_i)^2}$

$$= \sqrt{\frac{1.22 \times 10^{-13}}{7-1}} = 1.40 \times 10^{-7}$$

$\therefore$ $\Delta\lambda = (7.4 \pm 1.4) \times 10^{-7}$ mm

or $\boxed{\Delta\lambda = (0.74 \pm 0.14)\ nm}$

Results & Discussion

Using the Fabry-Perot interferometer we were successfully able to observe concentric circular fringes. Initially, the source was a monochromatic laser diode. After the formation of stable interference pattern on the screen, by changing the distance b/w the reflective plates, fringes either appeared (on increasing d) or disappeared (on decreasing d). By measuring the no. of fringes thus disappearing / appearing as a function of the change in distance, we were able to estimate the wavelength of the laser source. This value came out to be,

$$\lambda = (600 \pm 6) \text{ nm}.$$

The value is ~5% off from the literature value of 633 nm. However since our measurements are limited by the resolution of the instrument (~10^{-4} mm = 100 nm in our case), the wavelength calculated is far from accurate, even within the error bar. Each measurement had an error margin of ±0.0003 since we only took one reading of Δd for a particular value of Δm.

The next part of the experiment consisted of studying Sodium D-lines. We observed that by varying distance b/w the reflective plates doublet rings pattern periodically became visible and then blurry. This is because the corresponding fringes not moving equally due to the difference in wavelength.

By measuring the distance b/w two consecutive doublet pattern appearances, we were able to estimate the difference in wavelength of ~~the~~ sodium D-lines as,

$$\Delta\lambda = (0.74 \pm 0.14) \text{ nm}$$

which ~24% off from the literature value of ~0.60 nm. However, the true value is within 1 σ-deviation from the calculated value.

However, as mentioned earlier, due to the resolution of the instrument (~0.01 mm), our estimations are quite crude.
Also due to the low degree of coherence of the Sodium Lamp, the fringes could not be formed on screen, like in the case of Laser Diode. Hence this demonstrates the low coherence length of the Na Lamp as compared to the laser.

Sources of Error

1. When counting the no. of appearing disappearing one could miss / overestimate Δm,
2. The instrument's screws possessed a large amount of backlash. This can however be minimised by only turning the screw in one-direction.
3. In case of the Sodium D-lines, it was quite difficult to make out when exactly the lines became clear / blurry, hence affecting the accuracy of measurement of Δd.