Measurement of Elementary charge by Millikan oil drop method

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(Dated: November 2, 2024)

The experiment aimed to measure the elementary charge (e) using the Millikan oil drop method. By balancing gravitational, buoyant, and electric forces on small oil droplets suspended between charged plates, we were able to determine the charge on individual droplets. Repeated measurements of multiple droplets allowed for the identification of the smallest common charge, corresponding to the elementary charge.

I. OBJECTIVE

To demonstrate that electrical charge is quantised in discrete multiples of the electronic charge e, and to measure the value of e

II. THEORY

The measurement of the elementary charge, denoted by e, is a fundamental milestone in physics that provides deep insights into the quantized nature of electric charge. The Millikan oil drop experiment, first conducted by Robert A. Millikan in 1909, is one of the most famous experiments designed to measure this quantity with precision. Prior to Millikan's work, the existence of a smallest unit of charge was hypothesized but not conclusively proven. Millikan's experiment provided the first direct and accurate measurement of e, supporting the atomic theory of matter and the discrete nature of electric charge.

In the experiment, tiny oil droplets are introduced into a chamber where they are subjected to gravitational and electric forces. By adjusting the electric field, a droplet can be suspended in equilibrium, allowing the charge on the droplet to be determined. By observing multiple droplets, Millikan was able to show that the charges on the droplets were all integer multiples of a fundamental unit of charge, which he identified as the elementary charge. The significance of this experiment extends beyond the measurement of e; it also confirmed the quantization of charge, a key principle in modern physics. The Millikan oil drop method remains a classic demonstration of experimental ingenuity and precision in the study of fundamental physical constants.

Experimental Process

Consider a spherical oil droplet of radius r and density ρ falling under the gravitational force. This droplet in air is acted upon by a constant force and soon reaches a terminal velocity given by Stoke's law,

$$F_\nu = 6\pi\nu r v_f \tag{1}$$

where ν is the coefficient of viscosity of air and v_f is the terminal velocity during the fall. The gravitational and buoyancy forces acting on the droplet are balanced by F_ν (Fig. 1).

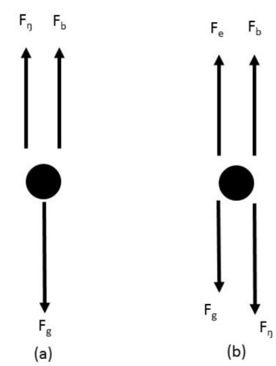

FIG. 1: Different forces acting on the oil droplet during (a) free fall and (b) rise of the droplet in the presence of an electric field.

$$\frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \rho_a g = 6\pi\nu r v_f \tag{2}$$

where ρ_a is the density of air. The falling velocity v_f is hence given by,

$$v_f = \frac{2}{9}\frac{gr^2}{\nu}(\rho - \rho_a) \tag{3}$$

If the droplet carries a charge ne and is moving upward with terminal velocity v_f under the influence of the applied electric field V/d between the parallel plate electrodes separated by the distance d and potential difference V, the force equation is

$$\frac{4}{3}\pi r^3(\rho - \rho_a)g + 6\pi\nu r v_f = \frac{Vne}{d} \tag{4}$$

Solving for ne, we subtract Eq. 2 from Eq. 4,

$$ne = \frac{6\pi\nu r d}{V}(v_f - v_r) \tag{5}$$

Dividing Eq. 5 by Eq. 3,

$$\boxed{ne = \frac{4\pi}{3}\frac{gd}{V}r^3(\rho - \rho_a)\left(1 + \frac{v_r}{v_f}\right)} \tag{6}$$

The Stoke"'s law used in obtaining Eqs. 2 to 6 assumes that the droplets are moving slowly, that there is no slipping of the medium over the surface of the droplet, that the medium is of quite large extent compared to the size of the droplet and that the inhomogeneities in the medium are of a size small compared to the size of the droplets. In the present case all the assumptions except the last one are reasonably valid. The radii of the droplets are of the order of one micron and therefore not much greater than the mean free path of the air molecules. The droplets will tend to fall more quickly in the free space between the air molecules. The expression for the falling velocity vf corrected for this effect on the basis of kinetic theory is

$$v_f = \frac{2}{9}\frac{gr^2}{\nu}(\rho - \rho_a)\left(1 + \frac{C}{Pr}\right) \tag{7}$$

where $C = 6.17 \times 10^{-8}$ m of Hg-m is a correction factor and P (in m of Hg) is the atmospheric pressure. If we write,

$$\xi = \frac{9\nu}{2g}\frac{v_f}{\rho - \rho_a} \tag{8}$$

$$\zeta = \frac{C}{2P} \tag{9}$$

$$\text{Eq. 7 becomes, } r^2 + 2\zeta r - \xi = 0 \tag{10}$$

The radius of the droplet is given by the positive root of this equation

$$r = -\zeta + \sqrt{\zeta^2 + \xi} \tag{11}$$

The charge ne may be obtained by first calculating ξ and ζ from Eqs. 8 and 9, then calculating the radius r from Eq. 11 and finally ne from Eq. 6. Equation 5 above is for the Dynamic method. In the Balancing method, the droplet is kept stationary by adjusting the potential. The upward velocity is thus equal to zero. The final equation for ne in the Balancing method is therefore

$$\boxed{ne = \frac{4\pi}{3}\frac{gd}{V_b}(\rho - \rho_a)r^3} \tag{12}$$

where, V_b is the balancing potential Time for free-fall of the droplets under gravity between the preset points is measured to obtain the velocity for free- fall. For observing the effect of the electric field on the charge carried by the droplets, there are two alternatives. In the Dynamic method, the velocity for the vertical upward motion is measured for a fixed potential difference.

III. EXPERIMENTAL SETUP

Apparatus

1. A oil drop chamber mounted on top of the panel. It has
   - A pair of horizontal parallel plate electrodes separated by about 5 mm thick ebonite ring with a hole for viewing the oil droplets.
   - The upper plate has a small hole in its centre for the admission of the droplets which are produces by spraying oil with an atomizer.
   - A device to illuminate the space between the plate electrodes.
2. Three levelling screws at the base of the panel to make the parallel plate electrodes perfectly horizontal (perpendicular to the gravitational field) and a water-level placed on top of the panel to check it.
3. A microscope with CCD camera head to view and transmit image of oil droplets between the plate electrodes to the monitor.
4. A power pack to supply continuously variable voltage in the range 0 - 800 V to the upper plate electrode when the electric field is to be created between the plates. The lower plate is permanentl grounded.
5. A digital voltmeter to measure the potential applied to the upper plate.
6. A 'Time Meter' or a stopwatch to display the time for which the oil droplet is allowed to move. A monitor with graduated screen. The horizontal lines on the monitor screen help in setting the distance through which the droplets move.
7. An atomizer to spray droplets.

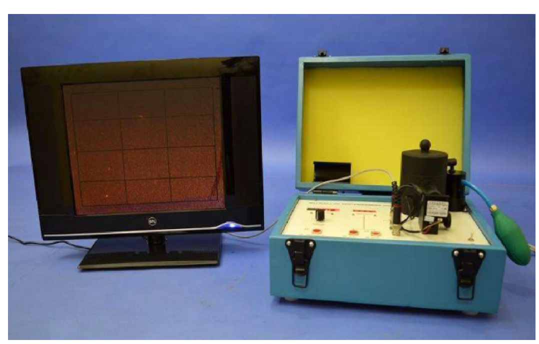

FIG. 2: The experiment set-up

IV. PROCEDURE

A. Dynamic Method

1. Set the timer and voltage to zero.
2. Spray oil droplets from the atomizer. The drifting down of oil droplets can be viewed from the monitor.
3. Note down the time the oil droplet takes to reach the second line from the bottom line starting from the second line from the top (the preset lines).
4. Switch on the voltage to make the oil droplet rise through air (if it is positively charged) and measure the time it takes to rise between the preset lines at a constant voltage. Note down the voltage and rise time.
5. Repeat the above two steps for 6-9 times. The data obtained in this manner is then used to calculate the value of e by the procedure described in the previous section.

B. Balancing Method

1. Spray oil droplets from the atomizer.
2. By varying the voltage, try to balance any of the oil droplets. Note down the balancing voltage.
3. By repeating step (3) of the dynamic method, fall time of the oil droplets can be found out, which will be used to calculate the radius of the oil droplet.

V. OBSERVATION AND CALCULATIONS

- Distance between parallel plates, d $= 5 \times 10^{-3}$ m
- Distance where the velocity is measured, L $= 1 \times 10^{-3}$ m
- Density of oil, $\rho = 929$ kg m^{-3}
- Density of air, $\rho_a = 1$ kg m^{-3}
- Room temperature, $T = 25°$ C
- Atmospheric Pressure, $P = 1$ atm
- Coefficient of viscosity of air, $\eta = 1.8480 \times 10^{-5}$ kg/m.sec
- For calculation purposes,

$$C = \frac{4}{3}\pi dg\left(\rho - \rho_a\right) = 190.13 \tag{13}$$

$$D = \frac{9\eta}{2g}\pi dg\left(\rho - \rho_a\right) = 9.04 \times 10^{-9} \tag{14}$$

$$\zeta = c/2P = 4.06 \times 10^{-8} \tag{15}$$

where $c = 6.17 \times 10^{-8}$ m of Hg m

Tables I and II show the observational data and calculation of ne for dynamic and balancing methods respectively using Eqns. 6 and 12. For simplification, we used

$$\eta = Dv_f$$

$$ne = \frac{CTr^3}{V} \text{ or } ne = \frac{Cr^3}{V_b}$$

For tables III and IV, ne divided was calculated by dividing the value of the charge ne on all the droplets by minimum value, which was then rounded off to the nearest integer as n_{eff}. This is the number of electrons present on a particular droplet. Now, by dividing ne by n_{eff} for each droplet, we can measure the value of e.

ne ($\times 10^{-19}$ C)	ne divided	n_{eff}	ne/n_{eff} ($\times 10^{-19}$ C)
51.51	23.46	23	2.24
60.47	27.54	28	2.16
8.33	3.79	4	2.08
8.12	3.70	4	2.03
9.94	4.53	5	1.99
9.90	4.51	5	1.98
9.33	4.25	4	2.33
2.20	1.00	1	2.20
Average ne/n_{eff}			2.13

TABLE III: Calculation of e for from dynamic method data

ne ($\times 10^{-19}$ C)	ne divided	n_{eff}	ne/n_{eff} ($\times 10^{-19}$ C)
5.40	2.38	2	2.70
9.21	4.07	4	2.30
5.61	2.48	2	2.80
5.75	2.54	3	1.92
49.43	21.82	22	2.25
5.58	2.46	2	2.79
2.27	1.00	1	2.27
9.38	4.14	4	2.35
Average ne/n_{eff}			2.42

TABLE IV: Calculation of e for from balancing method data

VI. ERROR ANALYSIS & SOURCES OF ERROR

We can calculate the error in our measurement of e using the standard deviation method as follows,

$$\sigma_y = \sqrt{\frac{\sum(y_i - y_{avg})^2}{N-2}} \tag{16}$$

Using this for tables III and IV, the uncertainities in e are,

- For dynamic method, $\sigma_e = 0.13 \times 10^{-19}$ C
- For balancing method, $\sigma_e = 0.31 \times 10^{-19}$ C

Drop no.	Sl no.	Free Fall time (s)	Rise time (s)	Mean Free fall time t_f (s)	Mean Rise time time t_r (s)	Mean free fall velocity (v_f) (in mm/s)	Voltage (V)	η ($\times 10^{-12}$)	r ($\times 10^{-7}$) (in m)	T ($1 + t_f/t_r$)	ne ($\times 10^{-19}$ C)
1	1	10.20	2.40	9.36	2.54	0.11	145	0.97	9.43	4.69	51.51
	2	9.10	2.70								
	3	9.10	2.70								
	4	9.20	2.30								
	5	9.20	2.60								
2	1	3.30	3.90	2.92	3.82	0.34	282	3.10	17.19	1.76	60.47
	2	2.80	3.80								
	3	2.90	3.60								
	4	2.60	3.80								
	5	3.00	4.00								
3	1	14.10	6.30	14.36	6.32	0.07	320	0.63	7.54	3.27	8.33
	2	15.30	6.30								
	3	14.20	6.10								
	4	14.40	6.50								
	5	13.80	6.40								
4	1	16.30	3.91	16.998	3.816	0.06	419	0.53	6.90	5.45	8.12
	2	15.79	3.98								
	3	17.83	3.66								
	4	17.47	3.60								
	5	17.60	3.93								
5	1	17.57	2.77	17.16	2.144	0.06	557	0.53	6.86	9.00	9.94
	2	16.87	2.09								
	3	17.08	2.08								
	4	16.33	2.08								
	5	17.95	1.70								
6	1	13.59	2.19	13.71	2.142	0.07	655	0.66	7.72	7.40	9.90
	2	13.89	2.04								
	3	14.20	2.15								
	4	13.28	2.20								
	5	13.59	2.13								
7	1	8.31	6.43	8.09	6.298	0.12	490	1.12	10.17	2.28	9.33
	2	8.15	6.18								
	3	7.97	6.21								
	4	7.86	6.42								
	5	8.16	6.25								
8	1	15.85	13.84	15.554	13.98	0.06	691	0.58	7.23	2.11	2.20
	2	15.60	14.83								
	3	15.37	13.59								
	4	15.40	13.68								
	5	15.55	13.96								

TABLE I: Observational data and calculation of ne for the dynamic method

As we can see from our error analysis that our experiment was precise but not accurate. Measurements were consistent but systematically deviated from the true value of the elementary charge (e). This suggests the presence of systematic errors rather more than random errors. Possible sources of such errors include:

- The experiment relies on an accurate value of the viscosity of air to calculate the charge on the droplets. If the air temperature or humidity was not controlled or the viscosity value was incorrect, this could lead to systematic errors.
- Errors in determining the radius of the droplets, perhaps due to optical issues or incorrect assumptions about the droplet's shape, could result in incorrect calculations of the charge.
- Changes in atmospheric pressure could affect the buoyant force on the droplets, leading to errors in the calculation of the charge.
- If the voltage supply or the distance between the plates is not accurately calibrated, the calculated charge values will be systematically off, leading to consistent but inaccurate results.
- If the oil droplets were contaminated with dust or other particles, this could alter their mass and the calculated charge, leading to consistently skewed results.
- An inaccurate voltmeter or fluctuating voltage supply could lead to an incorrect calculation of the electric field, affecting the determination of the charge on the droplets.

VII. RESULTS & DISCUSSION

In the dynamic method, the minimum value of ne measured came out to be 2.20×10^{-19} C. Using this the mean value of e was measured to be,

$$e = (2.13 \pm 0.13) \times 10^{-19}\ C$$

In the balancing method, the minimum value of ne measured came out to be 2.27×10^{-19} C. Using this the mean value of e was measured to be,

$$e = (2.42 \pm 0.31) \times 10^{-19}\ C$$

Drop no.	Sl no.	Free Fall time (s)	Mean Free fall time t_f (s) (s)	Mean free fall velocity (v_f) (in mm/s)	Balancing Voltage (V)	η ($\times 10^{-13}$)	r ($\times 10^{-7}$) (in m)	ne ($\times 10^{-19}$ C)
1	1	6.72	6.82	0.15	484	13.26	11.12	5.40
	2	6.97						
	3	6.69						
	4	6.81						
	5	6.90						
2	1	8.78	8.32	0.12	208	10.87	10.03	9.21
	2	8.47						
	3	8.03						
	4	8.22						
	5	8.09						
3	1	8.09	8.05	0.12	360	11.24	10.20	5.61
	2	8.03						
	3	8.09						
	4	7.97						
	5	8.05						
4	1	6.34	6.22	0.16	523	14.53	11.65	5.75
	2	6.22						
	3	6.22						
	4	6.12						
	5	6.21						
5	1	1.45	1.46	0.68	562	61.75	24.45	49.43
	2	1.48						
	3	1.42						
	4	1.50						
	5	1.47						
6	1	15.35	14.37	0.07	146	6.29	7.54	5.58
	2	14.22						
	3	13.73						
	4	14.46						
	5	14.07						
8	1	15.85	15.55	0.06	317	5.81	7.23	2.27
	2	15.60						
	3	15.37						
	4	15.40						
	5	15.55						
7	1	8.31	8.07	0.12	214	11.20	10.18	9.38
	2	8.15						
	3	7.86						
	4	7.89						
	5	8.16						

TABLE II: Observational data and calculation of ne for the balancing method

The standard value of e is 1.6×10^{-19} C. Our measured values are 33% and 51% higher for each method respectively. As discussed above, since the values are systematically deviated, there could be systematic errors present in the experiment. Despite that, our values are precise based on the error bars we obtained, especially in the case of dynamic method.

VIII. PRECAUTIONS

1. Maintain a constant temperature in the experiment setup to prevent changes in air viscosity, which can alter the droplet's behavior.
2. Use a stable and precise voltage source for the electric field. Fluctuations in the voltage can lead to inaccurate measurements of the droplet's charge.
3. Ensure the oil droplets are small enough to be influenced by the electric field but large enough to be visible under the microscope. Accurate focusing is crucial for precise measurements.
4. Use an accurate stopwatch or timing device to measure the time intervals as the droplet moves, which is critical for calculating the charge.
5. Ensure that the chamber, plates, and all equipment are free of dust and contaminants to avoid interference with the motion of the oil droplets.

IX. CONCLUSION

The Millikan oil drop experiment successfully measured the elementary charge e, and provided robust evidence for the quantization of electric charge. By balancing the gravitational, buoyant, and electric forces acting on tiny oil droplets, we were able to determine of the charge on individual droplets, and hence the charge of the electron.

Historically, the Millikan oil drop experiment significantly contributed to the development of quantum theory by providing direct evidence for the quantization of electric charge. Before this experiment, the idea that certain physical properties, like charge, existed in discrete units was still a hypothesis. Millikan's work demonstrated that electric charge is not continuous but rather comes in indivisible packets, specifically multiples of the elementary charge (e). This finding was crucial in validating the concept of quantization, which is a cornerstone of quantum mechanics.

The precise determination of the elementary charge is crucial for calculations in electromagnetism, such as those involving Coulomb's law, and for determining other fundamental constants, such as the fine-structure constant. Understanding the quantization of charge also has been essential in developing technologies such as semiconductors, transistors, and various electronic components, which rely on the controlled movement of charge.

[1] SPS, *Measurement of Elementary charge by Millikan oil drop method*, NISER (2023).

[2] R. A. Millikan, On the elementary electrical charge and the avogadro constant, Rev. 32 349 (1911).