Determination of Specific Charge of an Electron

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This experiment presents a method for estimating the specific charge of an electron, which is the ratio of its charge and its mass, using the fine beam tube method. Using the property of a Helmholtz coil to produce a uniform magnetic field, we study the path of an electron beam accelerated through a known potential under varied external magnetic fields to measure this value.

I. OBJECTIVE

To determine the specific charge of an electron (e/m_e) from the path of an electron beam and a Helmholtz coil setup.

II. THEORY

Introduction

The direct measurement of mass of the electron is difficult by experiments. It is easier to determine the specific charge of the electron e/m_e, from which the mass m can be calculated if the elementary charge e is known. This ratio was first calculated by J. J. Thompson.

A. Charged Particle in a Magnetic Field Accelerated by Potential

An electron moving at velocity $\vec{v}$ perpendicularly to a uniform magnetic field $\vec{B}$, is subject to the Lorentz force $\vec{F}$,

$$\vec{F} = e(\vec{v} \times \vec{B}) \tag{1}$$

where the force is perpendicular to the velocity and the magnetic field. Due to this, the electron will travel in a circular orbit where the axis is parallel to the direction of the magnetic field.

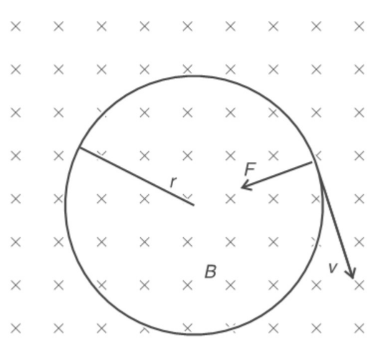

FIG. 1: Electric field lines between the capacitor plates

When the Lorentz force is balanced by the centripital force, the electron is forced into an orbit of radius r (Fig 1). Which means,

$$F = evB = \frac{m_e v^2}{r} \tag{2}$$

$$\implies \frac{e}{m_e} = \frac{v}{rB} \tag{3}$$

where m_e is the mass of an electron and hence e/m_e is the specific charge of the electron.

B. Electrons Accelerated by a Potential U

The electrons in this experiment are accelerated in a beam tube by applying a potential U. The kinetic energy gained by the electron due to U would be given by,

$$eU = \frac{1}{2} m_e v^2 \tag{4}$$

$$\implies v^2 = \frac{2eU}{m_e} \tag{5}$$

When combined with Eq. (3), Eq. (5) becomes,

$$\frac{e}{m_e} = \frac{2U}{(rB)^2} \tag{6}$$

C. Magnetic Field Generated by a Pair of Helmholtz Coils

The magnetic field generated by a pair of Helmholtz coils is twice the field generated by a single coil. If R is the radius of each coil and I is the current flowing through each of them having N turns, then the magnetic field due to both the coils at a distance $x = R/2$ is given by,

$$B = \mu_o N I \frac{R^2}{(R^2 + x^2)^{3/2}} = \frac{8\mu_o I N}{5\sqrt{5}R} = kI \tag{7}$$

where, $k = \dfrac{8\mu_o N}{5\sqrt{5}R}$

and, $\mu_o = 1.2566 \times 10^{-6} N/A^2$

From Eq. (6) and (7), the final expression for e/m_e can be given by,

$$\frac{e}{m_e} = \frac{2U}{(rkI)^2} \tag{8}$$

III. EXPERIMENTAL SETUP

Apparatus Required

1. Narrow electron beam tube of diameter 0.16m, filled with hydrogen gas at 1Pa
2. Pair of Helmholtz coils of radii 0.15m each (N = 130, current limit 2A)
3. Measuring scale for beam diameter
4. Holder for the entire assembly
5. DC power supply for Helmohltz coils (0-3A, 20V)
6. DC power supply for electron beam system (0 - 300V): Heating voltage: 6.3 V, Heating current: $\approx$ 0.7-0.8 A, Anode voltage: 150-300 V, DC Wehnelt voltage: $\pm$20 V, Plate voltage: 0-300 V DC

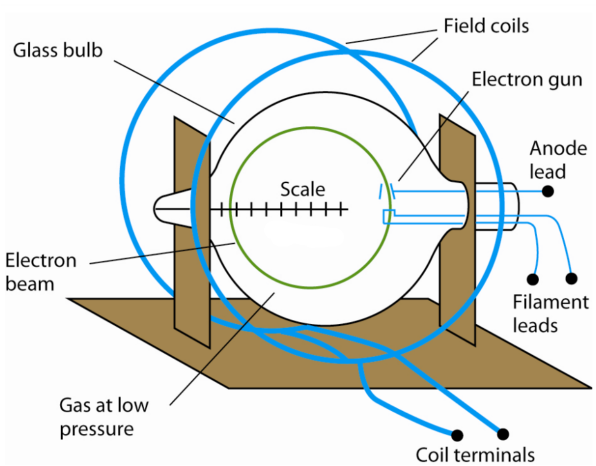

FIG. 2: Schematic of the apparatus

This experiment uses the fine beam tune method. A heater heats a cathode, which emits electrons which are accelerated through a known potential. The collision between the electrons and the Hydrogen gas inside the beam tube produces a visible trail. The Helmholtz coil produces a uniform magnetic field perpendicular to the electron beam, which causes the beam to move in a circular path.

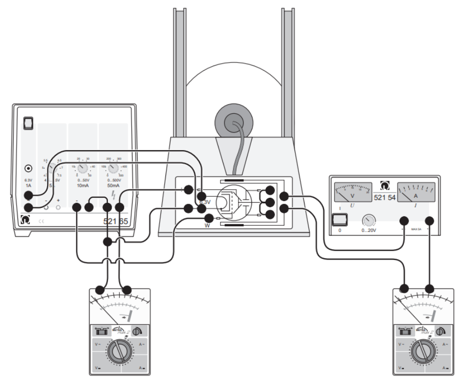

FIG. 3: Complete schematics of the set up with electrical connections to DC power supplies and multimeters.

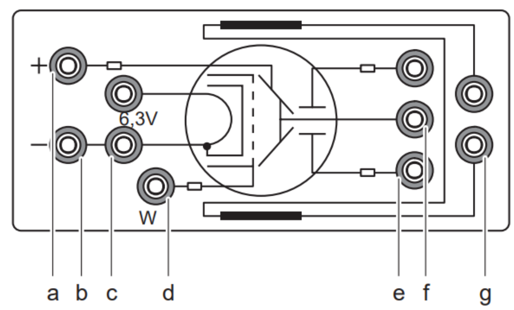

FIG. 4: Details of the connection sockets on the setup: (a)Anode (b) Cathode (c) Heating filament (d) Wehnelt cylinder (e) Deflection plates (f) Anode, for symmetrical adjustment of the deflection voltage (g) Helmholtz coils

IV. OBSERVATIONS

I(A)	$2r$ (cm)	r (cm)
1.333	12.2	6.10
1.436	10.9	5.45
1.542	10.1	5.05
1.634	9.7	4.85
1.736	9.0	4.50
1.840	8.5	4.25
1.943	8.1	4.05
2.041	7.7	3.85
2.142	7.2	3.60
2.239	6.9	3.45
2.320	6.7	3.35
2.449	6.3	3.15
2.613	6.0	3.00
2.720	5.7	2.85
2.817	5.6	2.80

TABLE I: I vs r data for a fixed $U = 300$ V

I(A)	$2r$ (cm)	r (cm)
1.286	11.2	5.60
1.385	10.1	5.05
1.497	9.3	4.65
1.594	8.7	4.35
1.699	8.1	4.05
1.802	7.7	3.85
1.892	7.4	3.70
2.023	6.8	3.40
2.107	6.5	3.25
2.205	6.2	3.10
2.291	5.9	2.95

TABLE II: I vs r data for a fixed $U = 249.7$ V

U (V)	$2r$ (cm)	r (cm)
299.9	7.8	3.90
290.4	7.7	3.85
279.1	7.6	3.80
270.2	7.4	3.70
260.3	7.3	3.65
250.7	7.0	3.50
240.5	6.8	3.40
230.6	6.7	3.35
219.9	6.5	3.25
210.8	6.4	3.20
200.2	6.3	3.15

TABLE III: U vs r data for a fixed $I = 2$ A

U (V)	$2r$ (cm)	r (cm)
300.6	10.4	5.20
289.2	10.2	5.10
279.3	9.8	4.90
269.6	9.4	4.70
259.2	9.3	4.65
249.0	9.1	4.55
238.9	9.0	4.50
229.1	8.8	4.40
219.5	8.7	4.35
209.5	8.3	4.15
199.9	8.1	4.05

TABLE IV: U vs r data for a fixed $I = 1.5$ A

V. DATA ANALYSIS AND CALCULATIONS

A. For a Fixed Acceleration Potential

We can rearrange Eq. (8), to show,

$$\frac{I^2}{1/r^2} = \frac{2U}{(e/m_e)k^2}$$
$$\text{or, } \frac{e}{m_e} = \frac{2U}{k^2 \cdot m_1} \tag{9}$$

where m_1 is the slope of the I^2 vs $1/r^2$ plot.

From table I,

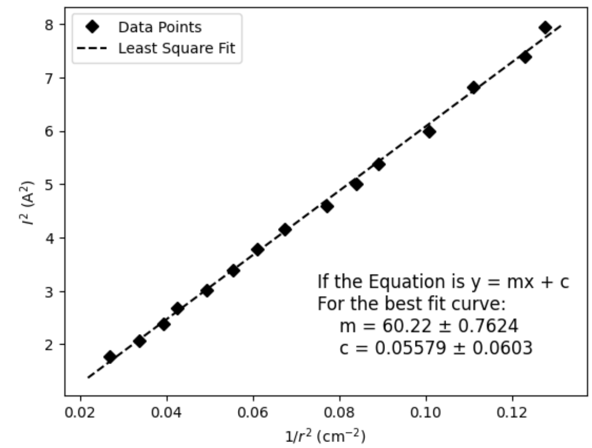

FIG. 5: I^2 vs $1/r^2$ plot for $U = 300$ V

Here, $m_1 = 60.22$ A^{2}cm^2 and $U = 300.0$ V. Hence using Eq. (9), specific charge of an electron, $e/m_e = 1.641 \times 10^{11}$ C kg^{-1}.

Similarly, from table II,

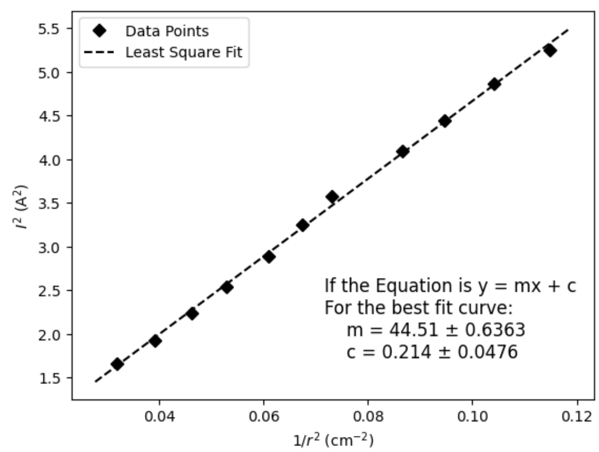

FIG. 6: I^2 vs $1/r^2$ plot for $U = 249.7$ V

Here, $m_1 = 44.51$ A^{2}cm^2 and $U = 249.7$ V. Hence using Eq. (9), specific charge of an electron, $e/m_e = 1.847 \times 10^{11}$ C kg^{-1}.

Thus, the mean value of tthe specific charge of an electron, calculated using this method is $e/m_e = 1.744 \times 10^{11}$ C kg^{-1}.

B. For a Fixed Current in the Helmholtz Coil

We can rearrange Eq. (8), to show,

$$\frac{r^2}{U} = \frac{2}{(e/m_e)k^2I^2}$$
$$\text{or, } \frac{e}{m_e} = \frac{2m_2}{k^2I^2} \tag{10}$$

where m_2 is the slope of the U vs r^2 plot.

From table III,

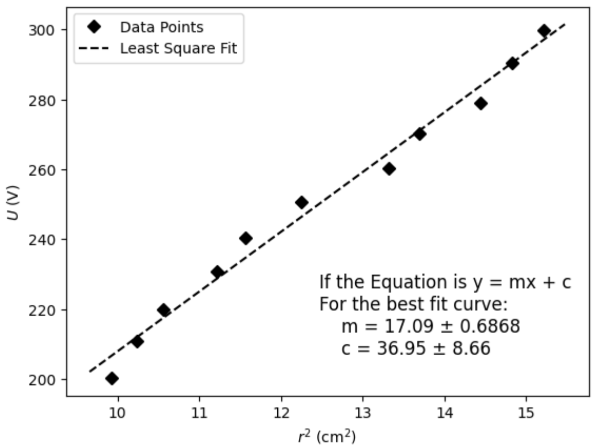

FIG. 7: U vs r^2 plot for $I = 2.0$ A

Here, $m_2 = 17.09$ V cm^{-2} and $I = 2.0$ A. Hence using Eq. (10), specific charge of an electron, $e/m_e = 1.408 \times 10^{11}$ C kg^{-1}.

Similarly, from table IV,

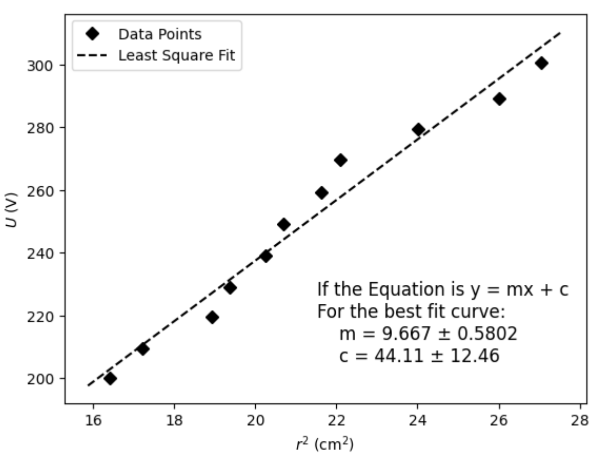

FIG. 8: U vs r^2 plot for $I = 1.5$ A

Here, $m_2 = 9.667$ V cm^{-2} and $I = 1.5$ A. Hence using Eq. (10), specific charge of an electron, $e/m_e = 1.411 \times 10^{11}$ C kg^{-1}.

Thus, the mean value of the specific charge of an electron, calculated using this method is $e/m_e = 1.410 \times 10^{11}$ C kg^{-1}.

VI. ERROR ANALYSIS

Let δ be the uncertainity in the measurement of specific charge of an electron.

From Eq. (9), we can show that

$$\frac{\delta}{e/m_e} = \sqrt{\left(\frac{\Delta U}{U}\right)^2 + \left(\frac{\Delta m_1}{m_1}\right)^2}$$
$$\text{or, } \delta = \frac{e}{m_e}\sqrt{\left(\frac{\Delta U}{U}\right)^2 + \left(\frac{\Delta m_1}{m_1}\right)^2} \tag{11}$$

From Fig. 5,

$$\delta_1' = 1.641 \times 10^{11}\sqrt{\left(\frac{0.1}{300.0}\right)^2 + \left(\frac{0.7624}{60.22}\right)^2}$$
$$= 0.016 \times 10^{11} \text{ C kg}^{-1}$$

Similarly from Fig. 6,

$$\delta_1'' = 1.847 \times 10^{11}\sqrt{\left(\frac{0.1}{249.7}\right)^2 + \left(\frac{0.6363}{44.51}\right)^2}$$
$$= 0.017 \times 10^{11} \text{ C kg}^{-1}$$

Hence mean δ_1 would be,

$$\delta_1 = \frac{1}{2}\sqrt{(\delta_1')^2 + (\delta_1'')^2} = 0.012 \times 10^{11} \text{ C kg}^{-1}$$

Now, for Eq. (10), we can show that,

$$\frac{\delta}{e/m_e} = \sqrt{\left(\frac{2\Delta I}{I}\right)^2 + \left(\frac{\Delta m_2}{m_2}\right)^2}$$
$$\text{or, } \delta = \frac{e}{m_e}\sqrt{\left(\frac{2\Delta I}{I}\right)^2 + \left(\frac{\Delta m_2}{m_2}\right)^2} \tag{12}$$

From Fig. 7,

$$\delta_2' = 1.408 \times 10^{11}\sqrt{\left(\frac{2 \cdot 0.1}{2.0}\right)^2 + \left(\frac{0.6868}{17.09}\right)^2}$$
$$= 0.146 \times 10^{11} \text{ C kg}^{-1}$$

Similarly from Fig. 8,

$$\delta_2'' = 1.411 \times 10^{11}\sqrt{\left(\frac{2 \cdot 0.1}{1.5}\right)^2 + \left(\frac{0.5802}{9.667}\right)^2}$$
$$= 0.194 \times 10^{11} \text{ C kg}^{-1}$$

Hence mean δ_2 would be,

$$\delta_2 = \frac{1}{2}\sqrt{(\delta_2')^2 + (\delta_2'')^2} = 0.121 \times 10^{11} \text{ C kg}^{-1}$$

VII. RESULTS

The specific charge of an electron was estimated in this experiment using two methods.

In the first method, the acceleration potential (U) of the electron beam was fixed and the current in the Helmholtz coil (I) was varied to get electron beams of different radii. The observed value of e/m_e was found out to be,

$$e/m_e = (1.744 \pm 0.012) \times 10^{11} \text{ C kg}^{-1}$$

In the second method, the current in the Helmholtz coil was fixed and the acceleration potential of the electron beam was varied to get beams of different radii. The observed value of e/m_e was found out to be,

$$e/m_e = (1.410 \pm 0.121) \times 10^{11} \text{ C kg}^{-1}$$

The mean value of the specific charge of an electron from both the methods comes out to be,

$$e/m_e = (1.577 \pm 0.061) \times 10^{11} \text{ C kg}^{-1}$$

VIII. CONCLUSION

The literature value of e/m_e is 1.759×10^{11} C kg^{-1}. The values obtained through experiment are close to this value, within some error bars. Since mass of an electron is difficult to measure directly, once the specific charge is calculated, it is easier to find the m_e if the elementary charge e is known. This also helps us understand how electrons move under electric field especially compared to other particles. Two particles withe the same specific charge will move in the path in a vacuum, when subjected to the same electric and magnetic fields.

IX. PRECAUTIONS & SOURCES OF ERROR

1. Perform the experiment in a dark area and avoid parallax error while taking any readings.
2. Make sure the connections are proper before switching on the electricity.
3. Make sure that the electron beam is in fact perpendicular to the magnetic field.
4. Do not exceed any current or voltage values as cautioned in the apparatus.

[1] Leybold Didactic. Leybold fine beam tube instruction sheet (555 581). *Website*, 2023. `https://www.ld-didactic.de/documents/en-US/GA/GA/5/555/555571e.pdf`.

[2] SPS. Lab manual. *Website*, 2023. `https://www.niser.ac.in/sps/sites/default/files/5_Specific%20charge%20of%20electron.pdf`.