Studying the Hall Effect of Semiconductors

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In this experiment, we investigate the Hall effect of semiconductors and calculate the value of the Hall coefficient for both n- and p-type Germanium and n-type Silicon samples at room temperature. In the second part of the experiment, we examine the dependence of the Hall coefficient on temperature for p-type germanium, and observe an inversion in the Hall coefficient at high temperatures. Furthermore, we find that the Hall coefficient exhibits a nonlinear decrease with increasing temperature. These results have important implications for the understanding and characterization of semiconductors and their electronic properties. Hence the Hall effect provides insights into properties like carrier type, mobility, and concentration, which conductivity measurements alone usually cannot reveal.

I. OBJECTIVE

1. To determine the Hall coefficient of certain semiconductors at room temperature.
2. To study the variation of Hall coefficient with temperature.

II. THEORY

When a current-carrying semiconductor or metal is kept in a magnetic field, the charge carriers of the semiconductor experience a force in a direction perpendicular to both the magnetic field and the current. At equilibrium, a voltage appears at the semiconductor edges. Measurement of Hall effect is an important tool is determining mobilities of electrons and holes in semiconductors where simple measurements of conductivity is insufficient to distinguish the both. Let us consider two scenarios: (a) that there is only one kind of carrier present, and (b) that both types of carriers are present.

A. One type of Carrier

Consider a crystal, with contacts 1, 2 and 3 perpendicular to the magnetic field H in the z-direction. When a voltage V_x is applied between contacts 1 and 2 to induce current flow through the crystal in x-direction, a voltage will arise across contacts 3 and 4 in the y-direction (Fig. 1).

Under the Hall voltage, the magnetic force on the carriers is $\vec{F}_m = e\vec{E}_m = e(\vec{v} \times \vec{H})$ and is compensated by the force $\vec{F_h}$ due to the Hall fields $\vec{E_h}$. The electric field $\vec{E_m}$ is along the y-axis and is provided by $E_m = vH = \mu E_x H$, where the carrier mobility μ is determined by $v = \mu E_x$, and E_x is the applied electric field along the x-axis. This is because $\vec{v}$ is along the x-axis, and $\vec{H}$ is along the z-axis. $\sigma E_x = J_x$ describes the relationship between the electric field, current density, and conductivity. The Hall coefficient R_H is given by:

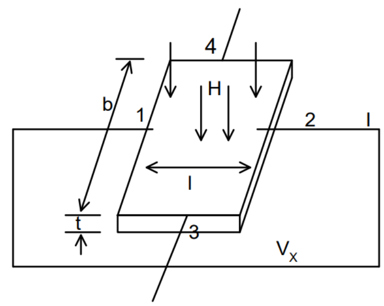

FIG. 1: Schematic arrangement for the measurement of the Hall Effect of a crystal

$$|R_H| = \frac{E_m}{J_x H} = \frac{1}{ne} = \frac{V_y t}{I_x H} \tag{1}$$

here t is the thickness of the sample. Thus, for a fixed magnetic field and input current, the Hall voltage is proportional to $1/n$.

B. Two Types of Carriers

We observe that the Hall voltage for p-type carriers (holes) has the opposite sign from that for n-type carriers (electrons) for the same electric field E_x. Consequently, the sign of the Hall coefficient R_H is also different for the two types of carriers. Both types of carriers experience a transverse motion due to the Hall field E_y, which fails to counteract the magnetic force acting on them. However, since no current flows through contacts 3, 4, and 5, the net transverse transfer of charge remains zero.

In the x-direction, we have:

$$e(v_x^+ p - v_x^- n) = J_x, \tag{2}$$

where v_x^+ has the opposite sign from v_x^-. The carrier mobility μ is always a positive number, and the total current density σ is given by:

$$e(\mu^+ p + \mu^- n) = \sigma. \tag{3}$$

The velocity of carriers in the y-direction is defined as:

$$v_y^- = \frac{F\tau}{2m^*}, \tag{4}$$

where m^* is the effective mass of carriers, and τ is the mean time between collisions. The Hall coefficient is given by:

$$R_H = \frac{E_H}{J_x H} = \frac{E_H}{\sigma E_x H} = \frac{(\mu_h^2 p - \mu_e^2 n)}{e(\mu_h p + \mu_e n)^2}, \tag{5}$$

where E_H is the Hall electric field, and H is the magnetic field strength.

Since the mobilities μ_h and μ_e are functions of temperature, the Hall coefficient is also a function of temperature and may become zero and even change sign. In general, $\mu_e > \mu_h$, so Hall coefficient inversion can only occur if $p > n$. Therefore, Hall coefficient inversion is characteristic of only p-type semiconductors. At the point of zero Hall coefficient, it is possible to determine the ratio of mobilities.

The carrier concentration n can be calculated from the hall coefficient using,

$$R_H = \frac{1}{ne} \tag{6}$$

Mobility μ is a measure of how quickly charge carriers respond to an electric field and is calculated using,

$$\mu = \frac{\sigma}{ne} \tag{7}$$

III. EXPERIMENTAL SETUP

Fig. 2 shows the experimental setup for observing Hall effect. A detailed list of all the apparatus required is listed below.

Apparatus

1. Hall probe (Ge: p & n types, Si: n-type)
2. Oven
3. Temperature sensor
4. Hall Effect Set-up (DHE-22)
5. Electromagnet (EMU-50V)
6. Constant Current Power Supply (DPS-50)
7. Digital Gaussmeter (DGM-102)

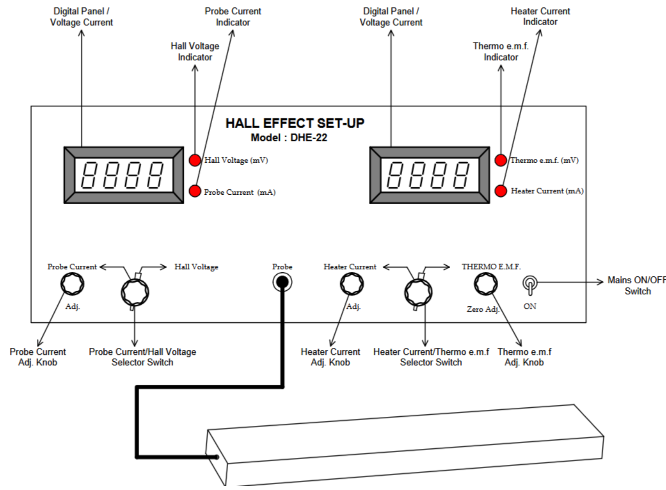

FIG. 2: Panel Diagram of the Hall effect set up

IV. OBSERVATION AND CALCULATIONS

A. Calibration of Magnetic Field with Coil Current

By placing the probe inside the electromagnet of 500 turns, we obtain a linear relationship between the current supplied and the magnetic field $\vec{H}$ between the electromagnets (Table I, Fig. 3).

I (A)	B (Gauss)	I (A)	B (Gauss)
0.00	0	1.50	1820
0.01	10	1.61	1940
0.10	120	1.70	2060
0.20	230	1.80	2190
0.31	350	1.90	2310
0.40	460	2.00	2440
0.50	570	2.10	2560
0.60	680	2.20	2690
0.71	810	2.30	2810
0.79	910	2.40	2920
0.90	1040	2.51	3060
0.99	1150	2.61	3200
1.09	1280	2.70	3300
1.19	1420	2.80	3420
1.30	1560	2.91	3560
1.41	1690	3.00	3660

TABLE I: Calibration of magnetic field with current

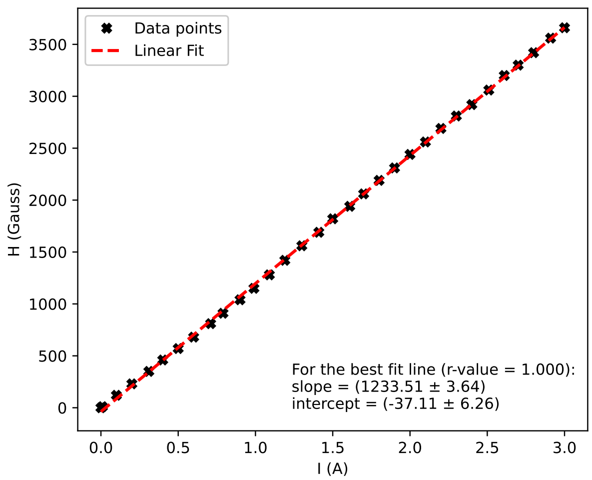

FIG. 3: Calibration plot for Magnetic field vs. Coil Current

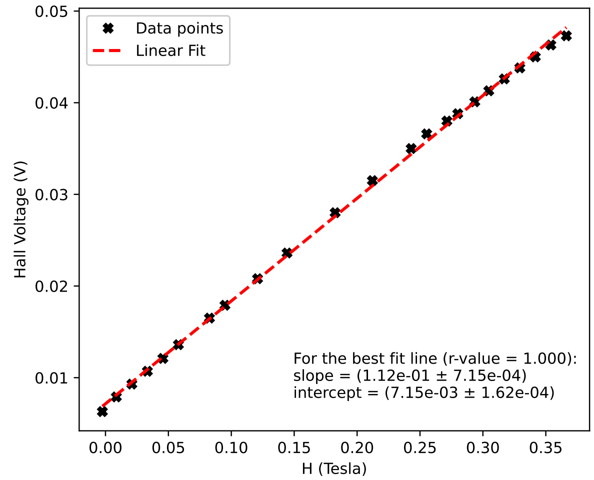

FIG. 4: Hall voltage vs. Magnetic Field for Ge p-type semiconductor

B. Estimation of Hall Coefficient

At constant room temperature, we measured the Hall voltage at different values of coil currents for three different semiconductors. H was calculated for each I from using $H = (1233.51 \cdot I - 37.11)$ Gauss (Fig. 3).

1. *Ge p-type*

The Hall voltage against different values of coil currents is shown in Table II. The Hall voltage as a function of the magnetic field obtained through the calibration curve plot is shown in Fig. 4.

- Probe current = 4.31 mA
- Resistivity $\sigma = (7 \pm 1)\,\Omega$ cm
- Sample thickness $t = 0.5$ mm

Current (A)	Hall Voltage (mV)	Current (A)	Hall Voltage (mV)
0.01	6.3	2.00	35.0
0.10	7.9	2.10	36.6
0.20	9.3	2.23	38.0
0.30	10.7	2.30	38.8
0.40	12.1	2.41	40.1
0.50	13.6	2.50	41.3
0.70	16.5	2.60	42.6
0.80	17.9	2.70	43.8
1.01	20.8	2.80	45.0
1.51	28.0	2.90	46.3
1.20	23.6	3.00	47.3
1.75	31.5		

TABLE II: Current supplied to the electromagnet and corresponding hall voltage produced for Ge p-type semiconductor

We can rewrite Eq. 1 as,

$$R_H = \left(\frac{V}{H}\right)\frac{t}{I} \tag{8}$$

where the V/H is the slope of the above plot, I refers to the probe current and t is the thickness of the sample. Plugging in the values, the Hall coefficient comes out to be 1.300×10^4 cm^3/C.

Using Eq. 6, the charge carrier density comes out to be 4.81×10^{14} cm^{-3}. Similarly using Eq. 7, the carrier mobility (μ_e) comes out to be, $\mu_e = R_H\sigma = R_H/7 = 1.86^3$ cm^2/Vs

2. *Ge n-type*

The Hall voltage against different values of coil currents is shown in Table III. The Hall voltage as a function of the magnetic field obtained through the calibration curve plot is shown in Fig. 5. For this setup,

- Probe current = 3.88 mA
- Resistivity $\sigma = (7 \pm 1)\,\Omega$ cm
- Sample thickness $t = 0.5$ mm

Note that since this is an n-type semiconductor, the majority charge carriers are electrons. Hence, the Hall voltage increases in the negative direction as opposed to the previous case. Using the same methods used for Ge p-type semiconductor, the Hall coefficient of Ge n-type comes out to be -2.154×10^4 cm^3/C.

Using Eq. 6, the charge carrier density comes out to be 2.90×10^{14} cm^{-3}. Similarly using Eq. 7, the carrier mobility (μ_e) comes out to be, $\mu_e = R_H/7 = 3.08 \times 10^3$ cm^2/Vs

Current (A)	Hall Voltage (mV)	Current (A)	Hall Voltage (mV)
0.00	1.2	1.70	36.4
0.10	3.2	1.91	40.8
0.30	7.2	2.01	43.1
0.50	11.3	2.16	46.0
0.70	15.3	2.30	48.9
0.90	19.7	2.50	52.8
1.01	21.7	2.72	57.1
1.10	24.0	2.89	60.7
1.30	28.0	3.01	63.0
1.50	32.1	3.12	64.9

TABLE III: Current supplied to the electromagnet and corresponding hall voltage produced for Ge n-type semiconductor

Current (A)	Hall Voltage (mV)	Current (A)	Hall Voltage (mV)
0	-2.9	1.7	24.4
0.1	-1.6	1.9	27.7
0.31	1	2.02	29.9
0.5	3.5	2.12	31.8
0.71	6.3	2.3	35.9
0.9	11.1	2.54	39.4
1	12.9	2.71	42.7
1.11	14.6	2.9	45.9
1.31	17.9	3.01	47.3
1.5	21.1		

TABLE IV: Current supplied to the electromagnet vs. hall voltage produced for Si n-type semiconductor

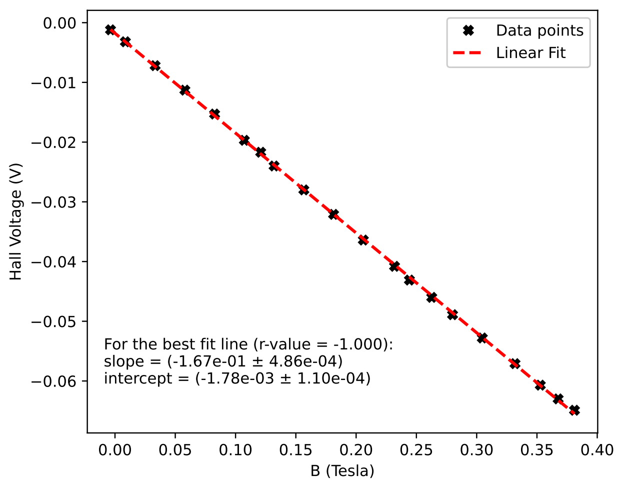

FIG. 5: Hall voltage vs. Magnetic Field for Ge n-type semiconductor

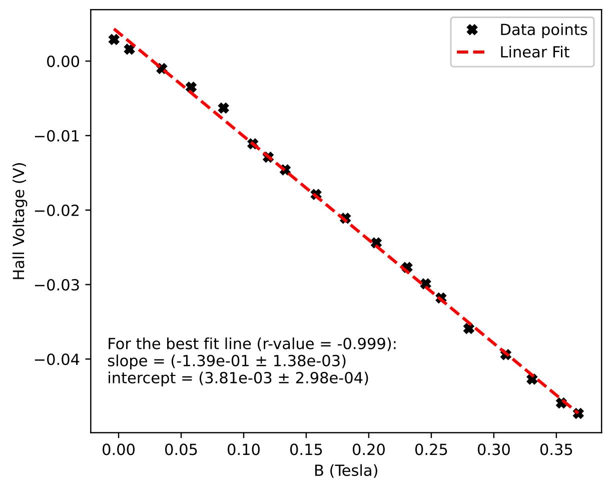

FIG. 6: Hall voltage vs. Magnetic Field for Si n-type semiconductor

3. *Si n-type*

The Hall voltage against different values of coil currents is shown in Table IV. The Hall voltage as a function of the magnetic field obtained through the calibration curve plot is shown in Fig. 6. For this setup,

- Probe current = 3.92 mA
- Resistivity $\sigma = (24 \pm 1)\,\Omega$ cm
- Sample thickness $t = 0.5$ mm

Using the same methods used for Ge n-type semiconductor, the Hall coefficient of Ge n-type comes out to be -1.791×10^3 cm^3/C.

Using Eq. 6, the charge carrier density comes out to be 3.49×10^{14} cm^{-3}. Similarly using Eq. 7, the carrier mobility (μ_e) comes out to be, $\mu_e = R_H/24 = 0.75$ cm^2/Vs

C. Temperature variation of Hall Coefficient

Using a Ge p-type semiconductor fixed with an oven, we were able to vary the temperature on the semiconductor by varying the EMF supplied to the oven. Temperature is measured with Cromel-Alumel thermocouple with its junction at a distance of 1 mm from the crystal. After waiting for about 10 minutes after each change in thermal EMF, we measured the Hall voltage and corresponding Hall coefficient (Table V) to obtain a relationship as shown in Fig. 7.

As we can see, the Hall coefficient is observed to decrease with increasing temperature, and a significant drop is seen at higher temperatures roughly after 50°C to 70°C. After that, it more or less stabilises.

This behaviour can be explained by examining the semiconductor properties at the low temperature region. Here, the conduction is primarily due to holes (major-

Thermal EMF (mV)	Temperature (°C)	Hall Voltage (mV)	Offset Voltage (mV)	Corrected Hall Voltage (mV)	Hall Coefficient (cm^3/C)
1.00	25.00	28.5	18.9	9.6	6.24
1.25	31.25	27.4	18.1	9.3	6.04
1.71	42.75	27.3	18.6	8.7	5.65
2.30	57.50	24.3	18.0	6.3	4.09
2.82	70.50	17.9	14.8	3.1	2.01
3.51	87.75	9.6	9.4	0.2	0.13
4.30	107.50	5.0	5.5	-0.5	-0.32
4.74	118.50	4.0	4.5	-0.5	-0.32

TABLE V: The hall coefficient measured against different thermal EMF values to observe the temperature dependence of R_H. (Probe current = 3.19 mA)

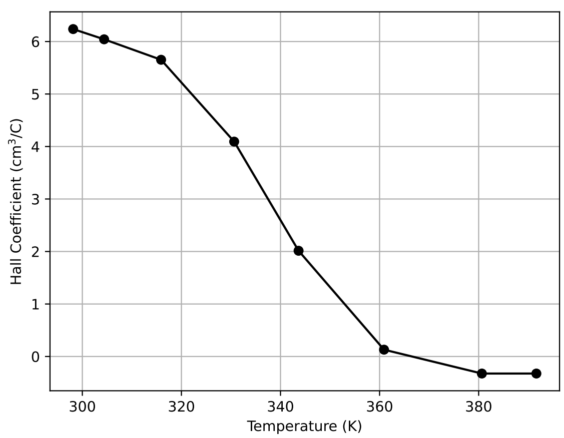

FIG. 7: Hall voltage and coefficient measured at different temperatures for Ge p-type semiconductor

ity carriers in p-type semiconductors). As the temperature rises, thermal energy of the particles increase, which causes the excitation of electrons from the valence band to the conduction band. This increases the minority carrier concentration (electrons). Since Hall coefficient is inversely proportional to the difference in carrier concentrations,

$$R_H \propto \frac{1}{q(p-n)} \tag{9}$$

where p and n are the hole and electron concentrations respectively, the overall value of R_H decreases. Hence, the semiconductor transitions from an extrinsic to intrinsic conduction regime.

At even higher temperatures around 80°C, the in trinsic carrier concentration (n_i) becomes comparable to the dopant-induced carrier concentration (p_0). When n_i dominates, conduction is influenced by both electrons (minority carriers) and holes (majority carriers). Electrons, having higher mobility than holes, significantly reduce the effective Hall coefficient. In the intrinsic conduction region, the equation for R_H is,

$$R_H = \frac{1}{q(p+n)}\left(\frac{p-n}{p+n}\right) \tag{10}$$

As $p \approx n \approx n_i$, the value of R_H sharply decreases. Moreover, after the significant drop, R_H value more or less stabilises at a temperature corresponding to approximately 110°C. This is expected as all the electrons from the valence band have been excited to the conduction band, so no further decrease of the Hall coefficient can occur.

V. ERROR ANALYSIS

The error in Hall coefficient can be dervied from Eq. 1 as,

$$\frac{\Delta R_H}{R_H} = \sqrt{\left(\frac{\Delta \text{slope}}{\text{slope}}\right)^2 + \left(\frac{\Delta I}{I}\right)^2} \tag{11}$$

The error in Charge carrier density can be derived from Eq. 6 as,

$$\frac{\Delta n}{n} = \frac{\Delta R_H}{R_H} \tag{12}$$

The error in carrier mobility can be dervied from Eq. 7 as,

$$\frac{\Delta \mu_e}{\mu_e} = \sqrt{\left(\frac{\Delta R_H}{R_H}\right)^2 + \left(\frac{\Delta \sigma}{\sigma}\right)^2} \tag{13}$$

Plugging in the corresponding values for all three semiconductors,

- Ge p-type

$$\Delta R_H = 0.008 \times 10^4 \text{ cm}^3/\text{C}$$
$$\Delta n = 0.03 \times 10^{14} \text{ cm}^{-3}$$
$$\Delta \mu_e = 0.26 \text{ cm}^2/\text{Vs}$$

- Ge n-type

$$\Delta R_H = 0.006 \times 10^4 \text{ cm}^3/\text{C}$$
$$\Delta n = 0.01 \times 10^{14} \text{ cm}^{-3}$$
$$\Delta \mu_e = 0.44 \text{ cm}^2/\text{Vs}$$

- Si n-type

$$\Delta R_H = 0.018 \times 10^4 \text{ cm}^3/\text{C}$$
$$\Delta n = 0.03 \times 10^{14} \text{ cm}^{-3}$$
$$\Delta \mu_e = 0.03 \text{ cm}^2/\text{Vs}$$

VI. RESULTS

My observing and measuring the hall effect of semiconductors, we were able to determine the Hall coefficient R_H, charge carrier density n and the carrier mobility μ_e of three different semiconductors as follows,

- Ge p-type

$$R_H = (1.300 \pm 0.008) \times 10^4 \text{ cm}^3/\text{C}$$
$$n = (4.81 \pm 0.03) \times 10^{14} \text{ cm}^3$$
$$\mu_e = (1.86 \pm 0.26) \times 10^3 \text{ cm}^2/\text{Vs}$$

- Ge n-type

$$R_H = -(2.154 \pm 0.006) \times 10^4 \text{ cm}^3/\text{C}$$
$$n = (2.90 \pm 0.01) \times 10^{14} \text{ cm}^3$$
$$\mu_e = (3.08 \pm 0.44) \times 10^3 \text{ cm}^2/\text{Vs}$$

- Si n-type

$$R_H = -(1.791 \pm 0.018) \times 10^4 \text{ cm}^3/\text{C}$$
$$n = (3.49 \pm 0.03) \times 10^{14} \text{ cm}^3$$
$$\mu_e = (0.75 \pm 0.03) \times 10^3 \text{ cm}^2/\text{Vs}$$

VII. CONCLUSION

In conclusion, our experiments for measuring the Hall coefficient have shown that the polarity of the Hall voltage is opposite for p-type and n-type samples, resulting in a change in the sign of R_H. We found that the calculated absolute value of R_H for n-type germanium is higher than that of p-type germanium, indicating a lower charge density in the former. This is because the absolute value of R_H is inversely proportional to the charge density, which is in turn proportional to the number of charge carriers. The carrier mobility of p-type semiconductor is also significantly higher than those for n-type semiconductors.

Similarly, we can also see that the carrier mobility is higher for n-type Ge than p-type Ge. This is expected behaviour as the mobility of electrons is greater than that of holes. Since conduction electrons travel in the conduction band and valence electrons (holes) travel in the valence band, the movement of valence electrons are restricted under an applied electric field. Thus, the effective mass of holes are larger and their time between scattering is lesser resulting in lesser mobility.

Our temperature-dependent analysis revealed that the value of R_H decreases with increasing temperature for the p-type sample and eventually becomes negative as stabilises, as expected due to an increase in the number of negative charge carriers with an increase in temperature. The decrease is not linear because of the significant difference between the mobilities of the two types of charge carriers at low temperatures.

VIII. PRECAUTIONS AND SOURCES OF ERROR

The fluctuations in the values obtained could be attributed to several factors like,

1. Fluctuations in the coil current during the experiment, which may result in variations in the magnetic field strength experienced by the sample. Such fluctuations could be caused by external disturbances or instabilities in the power supply.
2. Impurities in the sample, which could introduce additional sources of scattering and lead to variations in the measured value of R_H. Such impurities may be present in the bulk of the material or at the interfaces between different layers or regions of the sample.
3. Thermal effects on the sample, caused by high probe current or non-zero thermal EMF. These effects could cause changes in the temperature of the sample, leading to variations in the measured value of R_H . The magnitude of such effects may depend on the specific characteristics of the sample and the experimental setup.
4. Variations in the ambient temperature of the room, which could affect the electrical properties of the sample and lead to variations in the measured value of R_H. Such variations could be due to fluctuations in air conditioning or heating systems, or other en vironmental factors.

Note that other uncontrolled variables could also contribute to the observed variations in the measured values. More accurate methods like using the four-point probe technique or the Van der Pauw method eliminates errors from contact resistance, improving accuracy.

[1] SPS, *Hall effect of semiconductors*, NISER (2023).

[2] E. H. Putley, *Hall Effect and Related Phenomena* (1960).