

# Emission spectra of Hydrogen (Balmer series) and determination of Rydberg's constant

Gayatri P

3rd year, Integrated M.Sc. Physics

Roll No.: 2211185

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An excited hydrogen electron emits energy when it returns to lower energy states. This energy is released in the form of light waves. From the wavelength of the light emitted, we can calculate the energy difference between its energy levels.

## I. OBJECTIVE

1. To measure the wavelengths of visible spectral lines in Balmer series of atomic hydrogen
2. To determine the value of Rydberg's constant

## II. THEORY

Hydrogen is the simplest atom. It has only one electron which revolves around a single proton (the natural abundance of deuterium and tritium is very low). When this electron gets some energy, it gets excited to higher energy states. Now, when this electron returns to lower energy states, it emits the energy back in the form of electromagnetic waves. The energy carried by this wave is equal to the energy difference between the two states. Different energies correspond to different wavelengths of electromagnetic waves. It has been experimentally observed that when the electron jumps from a higher energy state to the second energy state of the Hydrogen atom, the light emitted falls in the visible range of the human eye. These lights were first seen by Swiss mathematician *Johann Jakob Balmer*. That is why they are called Balmer lines of Hydrogen.

To find the wavelengths of spectral lines, one can start with equating the centripetal force and the electrostatic force for an electron,

$$F_c = \frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} = F_E \quad (1)$$

Hence, the total energy (K.E. + P.E.) of the electron would be,

$$E = \frac{1}{2}mv^2 - \frac{1}{4\pi\epsilon_0} \frac{e^2}{r} \quad (2)$$

$$= \frac{-1}{8\pi\epsilon_0} \frac{e^2}{r} \quad (3)$$

Now, according to Bohr's model of the Hydrogen atom, angular momenta of the electrons are integral multiples of  $h/2\pi$

$$mvr = \frac{nh}{2\pi} \implies r = \frac{\epsilon_0 n^2 h^2}{\pi m e^2} \quad (4)$$

$$\implies E_n = \frac{-me^4}{8\epsilon_0^2 h^2 n^2} \quad (5)$$

Hence, the wavelengths of spectral lines are given by

$$\frac{1}{\lambda} = R_y \left[ \frac{1}{n^2} - \frac{1}{m^2} \right] \quad (6)$$

Here,  $\lambda$  = wavelength released,  $R_y$  = Rydberg's constant,  $n = 2$  and  $m = 3, 4, 5, \dots$

Where,

$$R_y = \frac{e^4 m_e}{8\epsilon_0^2 h^3 c} = 1.097 \times 10^7 \text{ m}^{-1}$$

where  $e$  is the charge of 1 electron,  $m_e$  is mass of one electron,  $\epsilon_0$  is permittivity of air =  $8.85 \times 10^{-12}$ ,  $h$  is Planck's constant and  $c$  is velocity of light in vacuum.

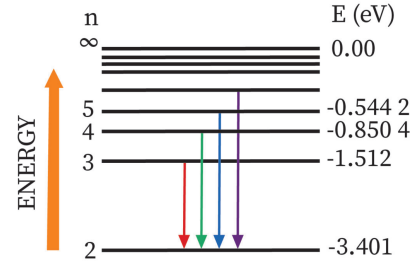


FIG. 1: Balmer series of Hydrogen

## III. EXPERIMENTAL SETUP

In this experiment, Hydrogen or mercury spectral tubes are used as the source of light. The spectral tube is fixed between two high voltage electrodes. A grating and a spectrometer will be used to analyse different spectra.

The spectral beam is a collection of beams of different wavelengths. To study the spectrum, we must split the beam into components. We do that with the help of a diffraction grating. The principle is that if a monochromatic light of wavelength  $\lambda$  falls normally on an amplitude diffraction grating with a periodicity of lines given

by  $g$  ( $=\frac{1}{N}$ , where  $N$  is the number of grating lines per unit length), the intensity peaks due to principal maxima occur under the condition:

$$g \sin \theta = p\lambda \quad (7)$$

Here  $\theta$  is the diffraction angle and  $p$  is the order of diffraction.

First, a mercury source is used to determine the grating element ( $g$ ). Then using this value of  $g$  and a hydrogen spectral tube the unknown lines of Balmer series of hydrogen spectra are determined.

#### Apparatus

1. Spectrometer
2. Diffraction Grating
3. Hg Spectral tube
4. Hydrogen Spectral tube
5. AC Power Supply

#### IV. PROCEDURE

##### Determination of $g$

1. Adjust the spectrometer and use a spirit level to make it parallel. Fix the grating on the prism table.
2. Bring in the Hg source close to the collimator of the and switch ON the power supply and let the Hg lamp warm up.
3. Look through the telescope to notice the first order spectral lines of Hg on both sides of the direct image of the slit at the center. Make the spectral lines vertical by turning the grating slightly in its plane.
4. Note down the positions of the cross wire for each line on one side using the two verniers on the spectrometer.
5. Repeat the above step by turning the telescope to the other side too. Determine the diffraction angle,  $\theta$ , for all the spectral lines of Hg spectrum. Using the spectral data of Hg provided, calculate  $g$ .
6. Switch off the power supply and remove the Hg spectral tube.

##### Studying hydrogen spectrum

1. Bring in the hydrogen source to the front of collimator. Then switch ON the power supply.
2. Look through the telescope to notice the three first order spectral lines of Hydrogen Balmer series (red, green and violet) on both sides of the direct image of the slit at the center.
3. Follow the step described in the above section to determine the diffraction angle for all three lines.

4. Using the value of  $g$  determined earlier, calculate the wavelength of each of the spectral lines.
5. Switch off hydrogen spectral tube.

#### V. OBSERVATION AND CALCULATIONS

$$\text{Least Count of Spectrometer} = (1 - \frac{59}{60}) \times \frac{1}{2} = 30''$$

Wavelength or Colour ( $\lambda$ nm)	Left Side ( $^\circ$ )					
	Vernier 1			Vernier 2		
	MSR	VSR	Total	MSR	VSR	Total
413	7.5	25	7.708	187.5	50	187.917
436	8.5	40	8.833	189.0	2	189.017
501	12.5	45	12.875	193.0	1	193.008
588	14.0	10	14.083	194.0	30	194.250
614	14.5	31	14.758	194.5	51	194.925
627	15.0	50	15.417	195.0	58	195.483

Wavelength or Colour ( $\lambda$ nm)	Right Side ( $^\circ$ )					
	Vernier 1			Vernier 2		
	MSR	VSR	Total	MSR	VSR	Total
413	339.5	4	339.533	159.5	31	159.758
436	338.0	45	338.375	158.5	6	158.550
501	334.5	9	334.575	154.5	38	154.817
588	333.0	54	333.450	153.5	24	153.700
614	332.5	50	332.917	153.0	10	153.083
627	332.0	39	332.325	152.5	3	152.525

$2\theta$ from V1 (deg)	$2\theta$ from V2 (deg)	Average $\theta$ (deg)	$\sin \theta$	$g$ (nm)
12.758	12.325	12.542	0.217	1901.915
12.792	12.433	12.613	0.218	1996.737
12.550	12.175	12.363	0.214	2340.071
12.467	12.050	12.258	0.212	2769.407
12.325	11.992	12.158	0.211	2915.286
12.258	11.992	12.125	0.210	2985.072

TABLE I: Determination of  $g$  for the grating

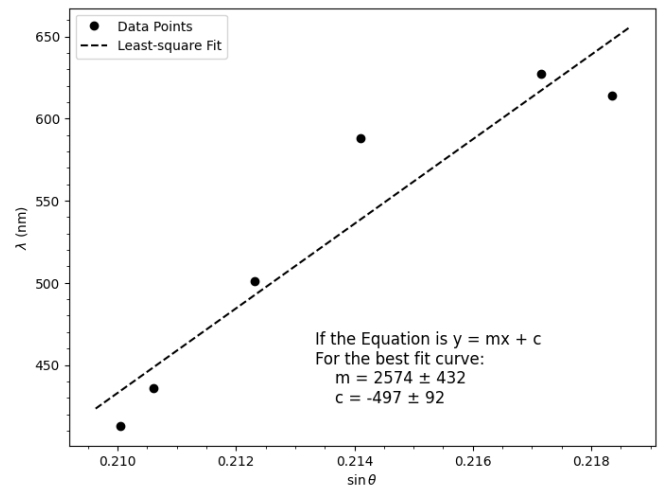


FIG. 2:  $\lambda$  vs.  $\sin \theta$  plot for Hg Spectrum

From table I, a  $\lambda$  vs.  $\sin \theta$  graph has been plotted where the known values of wavelengths are used. The datapoints have been fitted for linear regression. From this the value of  $g$  comes out to be (based on Eq. 7),  $g = (2574 \pm 432) \text{ nm}$ .

With that value of  $g$  we calculate Table 2:

Colour	Left Side ( $^\circ$ )					
	Vernier 1			Vernier 2		
	MSR	VSR	Total	MSR	VSR	Total
Violet	17.0	12	17.100	197.0	35	197.292
Green	10.5	21	10.675	190.5	48	190.900
Red	8.5	35	8.792	189.0	0	189.000

Colour	Right Side ( $^\circ$ )					
	Vernier 1			Vernier 2		
	MSR	VSR	Total	MSR	VSR	Total
Violet	330.5	19	330.658	150.5	41	150.842
Green	337.0	23	337.192	156.5	53	156.942
Red	338.5	12	338.600	158.5	35	158.792

From Energy State ( $m$ )	$2\theta$ from V1 (deg)	$2\theta$ from V2 (deg)	Average $\theta$ (deg)	$\lambda$ (nm)
3	12.242	11.867	12.054	537.544702
4	12.133	12.158	12.146	541.571315
5	12.608	12.208	12.408	553.094366

TABLE II: Determination of spectral lines of hydrogen

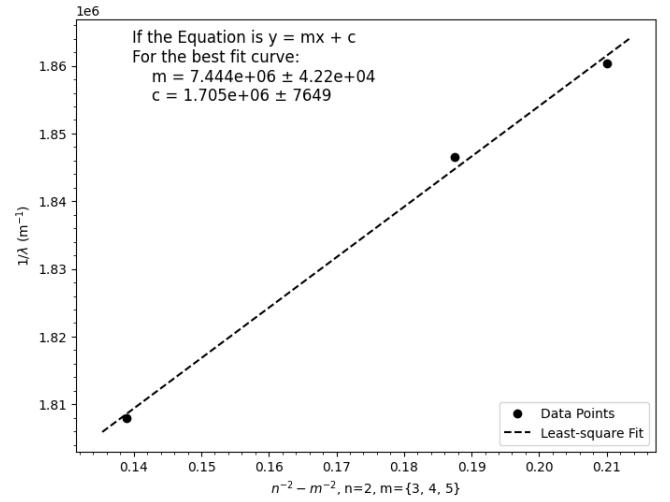


FIG. 3:  $1/\lambda$  vs.  $n^{-2} - m^{-2}$  plot for Hg Spectrum

Here, only three Balmer lines could be identified. With the wavelength values from II, we plot a  $1/\lambda$  vs.  $n^{-2} - m^{-2}$  graph and fit it for linear regression. From this and based on Eq. 6 we get the value of Rydberg's constant as:

$$R_y = (0.744 \pm 0.042) \times 10^7 \text{ m}^{-1}$$

## VI. DISCUSSION AND CONCLUSION

The wavelengths of the three Balmer lines identified in this experiment were 537.54 nm ( $H_\alpha$ ), 541.57 nm ( $H_\beta$ ) and 553.09 nm ( $H_\gamma$ ), while the literature values of these lines are 670 nm, 790 nm and 440 nm respectively. Hence our values are quite off.

The calculated value of Rydberg's constant from these Hydrogen lines is  $R_y = (0.744 \pm 0.042) \times 10^7 \text{ m}^{-1}$  while the literature value is  $R_y = 1.097 \times 10^7 \text{ m}^{-1}$ . The large error in the final values could be due to the distinct lines in the spectrum being very close to each other. This was also heavy backlash error associated with the spectrometer. This resulted in some odd values in the data.

[1] SPS, *Emission spectra of Hydrogen (Balmer series) and determination of Rydberg's constant*, NISER (2023).