EXPERIMENT- 2

DIFFRACTION OF LIGHT DUE TO ULTRASONIC WAVE PROPAGATION IN LIQUIDS

Submitted By:

Gayatri P
2211185
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Objectives :

1. To study the diffraction of light due to propagation of ultrasonic waves in a liquid
2. To determine the speed of sound in turpentine oil at room temperature
3. To determine the compressibility of the given fluid (turpentine oil)

Theory

Acoustic waves in closed liquids can cause periodic high and low density regions in them. The spacing between high & low-density regions ~~in~~ of ultrasonic waves in the MHz range is similar to the spacing in diffraction gratings. Since these density changes in liquids will cause changes in index of refraction of the liquid, it can be shown that parallel light passed through the liquid will be diffracted as much as if it had passed through a grating and can be used as an indirect method to measure the velocity of sounds in various liquids. This phenomenon of interaction between light & sound waves in liquid is called the Debye-Sears effect.

(A) Velocity of sound in fluid

Due to reflections at the sides of the container with the liquid, ~~the~~ a stationary wave pattern is formed with nodes and anti-nodes at regular intervals inside the liquid, as periodic high and low density regions.

If λ_u = wavelength of sound in liquid, λ = wavelength of incident light in air & θ_n = angle of diffraction of n^{th} order, we have,

$$d \sin\theta_n = n\lambda \quad \text{--- (1)}$$

where $d = \frac{1}{N} = \lambda_u$, since the liquid acts as the diffraction grating.

$$\Rightarrow \lambda_u \sin\theta_n = n\lambda \quad \text{--- (2)}$$

Now, if ν is the frequency of ~~att ultrasonic wave~~ crystal, ~~the~~ its velocity in the liquid can be calculated by,

$$\boxed{v_u = \nu\lambda_u} \quad \text{--- (3)}$$

Thus, the velocity of sound in the liquid can be calculated from eqⁿ (3) as,

$$\boxed{v_u = \frac{n\lambda v}{\sin\theta_n}} \quad — (4)$$

(B) Compressibility of the liquid

The speed of sound ~~both~~ depends on both the inertial property of the medium (to store kinetic energy) an elastic property (potential energy storage), given by the relⁿ.

$$v_u \propto \sqrt{\frac{\text{elastic property}}{\text{inertial property}}} \quad — (5)$$

For a liquid, its bulk modulus (E) quantifies the extent to which an element changes in volume when pressure is applied,

$$E = -\frac{\Delta P}{\Delta V/V} \quad — (6)$$

where -ve sign accounts for the inverse relⁿ b/w P & V.

∴ The speed of sound in a liquid can be written as,

$$v_u = \frac{n\lambda v}{\sin\theta_n} = \sqrt{\frac{E}{\rho}} \quad — (7)$$

where ρ = density of the liquid.

$$\Rightarrow \frac{E}{\rho} = v_u^2 \Rightarrow v_u^2\rho = E = \frac{1}{K} \quad — (8)$$

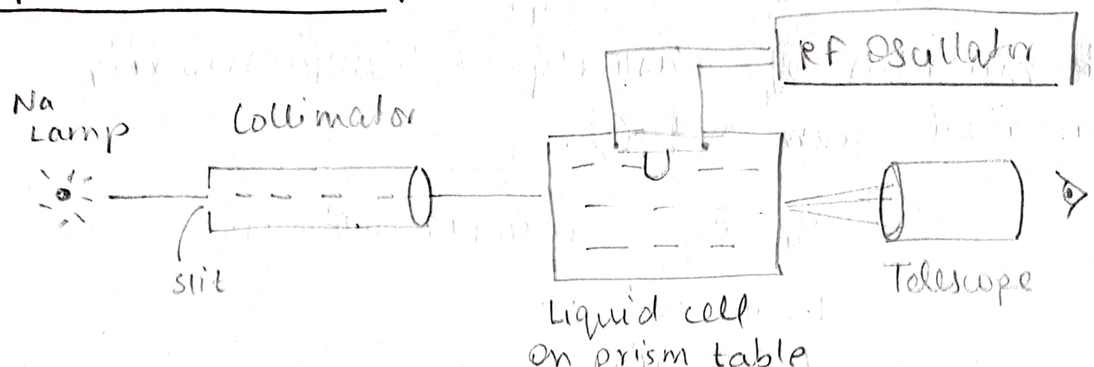
where K = compressibility of the liquid.

eqⁿ (7) is also called the Newton-Laplace eqⁿ.

Apparatus Required

1. Radio Frequency oscillator w/ frequency meter
2. Quartz crystal slab w/ two leads
3. Spectrometer
4. Glass cell w/ turpentine oil
5. Sodium Lamp
6. Spirit Level.

Experimental setup



Schematic Diagram of the Experimental Setup

Observations & Calculations

Least count of the spectrometer $= \dfrac{\text{min. division on MS}}{\text{no. of divisions on VC}}$

$= \dfrac{30'}{30} = 1' = \dfrac{1}{60}^\circ$

Frequency of the vibrating crystal $(f) = 3.9691$ MHz

Density of turpentine oil $(\rho) = 865\ kg/m^3$

Order (n)	Left of central line (degree)		Right of central line (degree)		2θ [a-b] (degree)	2θ' (a'-b') (degree)	Mean θ	d_u (mm)	v_u (m/s)
	a	a'	b	b'					
~~2~~	~~92.733~~	~~272.583~~	~~92.983~~	~~272.767~~	~~0.250~~	~~0.483~~			
2	92.733	272.583	93.033	272.933	0.300	0.350	0.163	0.415	1648.569
1	92.867	272.700	93.150	272.900	0.283	0.200	0.121	0.279	1108.520
2	92.717	272.650	93.133	273.067	0.417	0.417	0.208	0.324	1285.885
1	92.900	272.733	93.050	272.883	0.250	0.250	0.100	0.337	1339.461

Here, mean value of $v_u = \dfrac{v_{u1} + v_{u2} + v_{u3} + v_{u4}}{4}$

$= \dfrac{5382.436}{4} = 1345.609$ m/s

Similarly mean value of $\lambda_u = \frac{1.356}{4}$ mm = 0.339 mm

The Bulk modulus of elasticity & compressibility can be calculated from eqⁿ (8).

$$E = v_u^2 \rho = \cancel{100}\ (1.345.609)^2 \cdot 865$$

$$= 1.566 \times 10^9 \text{ Pa}$$

$$E = 1.566 \text{ GPa}$$

Hence compressibility, $K = \frac{1}{E} = 6.385 \times 10^{-10}$ m^2/N

Error Analysis

The error in v_u & λ_u can be calculated by taking the std. deviation of the 4 sample data points,

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^{4} (x_i - \bar{x})^2}$$

for v_u, $n=4$, $\bar{x} = 1345.609$ m/s

$$\Rightarrow \sigma_{v_u} = \sqrt{\frac{1}{3}[91784 + 56211 + 3566 + 37]}$$

$$\sigma_{v_u} = 194.680 \text{ m/s}$$

for λ_u, $\bar{x} = 0.339$ mm

$$\sigma_{\lambda_u} = \sqrt{\frac{1}{3}[0.005 + 0.003 + 0.0002 + 2.4 \times 10^{-6}]}$$

$$\sigma_{\lambda_u} = 0.057 \text{ mm}$$

Assuming the uncertainity in ρ is zero, the uncertainity in E can be calculated by,

$$\frac{\sigma_E}{E} = \sqrt{\left(\frac{2\sigma_{v_u}}{v_u}\right)^2} = \frac{2\sigma_{v_u}}{v_u}$$

$$\Rightarrow \sigma_E = 1.566 \times 2 \times \frac{194.680}{1345.609} = 0.453 \text{ GPa}$$

Similarly, the uncertainity in K can be calculated by,

$$\frac{\sigma_K}{K} = \sqrt{\left(\frac{\sigma_E}{E}\right)^2} = \frac{\sigma_E}{E}$$

$$\Rightarrow \sigma_K = K \cdot \frac{\sigma_E}{E} = \frac{6.385 \times 10^{-10} \times 0.453}{1.566} = 1.847 \times 10^{-10} \, m^2/N$$

Results & Discussion

In this experiment, we were successfully able to produce and observe diffraction patterns by passing MHz range ultrasonic waves through a liquid (turpentine oil in our case). This method uses piezoelectric effect to produce vibrations in a crystal, which in turn produce density based standing waves in the liquid which acts like a diffraction, ~~at~~ also known as the Debye-Sears method.

Using this method the measured value of the wavelength of sound in turpentile oil was,

$$\lambda_u = (0.339 \pm 0.057) \text{ mm}$$

And hence the velocity of sound in that liquid came out to be,

$$v_u = (1345.609 \pm 194.680) \text{ m/s}$$

The literature value of $v_u = 1240$ m/s is well within the error range of our measured value. The relatively large ~~%~~ error bar (~ 14%) could be attributed mainly to the fact that ~~there~~ only 4 data-points we taken. More number of measurements could've significantly decreased the std. dev. in v_u. Besides, the experiment also requires precision in taking readings, ~~a~~ especially minutes in the vernier scale, which is prone to much ~~human~~ ~~er~~ random error.

It is to be noted that velocity of sound in ~~any~~ ~~m~~ liquids is temperature dependent and the above value is measured for room temperature.

Furthermore, we were ~~ab~~ also able to calculate the bulk modulus of elasticity (E) and compressibility of turpentine oil (K) using the Newton-Laplace equation:

$$E = (1.566 \pm 0.453)\ GPa$$

$$K = (6.385 \pm 1.847) \times 10^{-10}\ m^2/N$$

The huge error bars here could again be attributed to the previously mentioned reasons.
It is to be noted that we are determining the bulk modulus of adiabatic compression because there is no energy exchange with the environment when sound waves are passed through the matter. This is different from isothermal bulk modulus.

Precautions

1. The crystal should be mounted parallel to the side walls or the standing wave will not be proper & the intensity of diffraction pattern may not be equal on both sides.
2. ~~Ex~~ Excercise caution while taking minute scale readings and prevent any backlash errors.
3. Turn the knob on the RF oscillator slowly to vary frequency.

$f = 3.9691$ MHz

Order	Left of center		Right of center		2θ	$2\theta'$
	a	a′	b	b′		
2nd order						

Observations (Data sheet)

Table: Measurement of angle of diffraction from the spectrometer. [Total = MSR + $\frac{1}{60}$ · VSR in degrees]

	Left side						Right side					
	Vernier 1			Vernier 2			Vernier 1			Vernier 2		
order	MSR	VSR	Total	MSR	VSR	Total	MSR	VSR	Tot	MSR	VSR	Tot
2	92.5	14	92.733	272.5	5	272.583	93.0	2	93.033	272.5	26	272.93
1	92.5	22	92.867	272.5	12	272.700	93.0	9	93.150	272.5	24	272.900
2	92.5	13	92.717	272.5	9	272.650	93.0	8	93.133	273.0	4	273.067
1	92.5	24	92.900	272.5	14	272.733	93.0	1	93.050	272.5	29	272.983

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