Studying Magnetoresistance and Hall Effect of Bismuth

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In this experiment, we investigate the Magnetoresistance and Hall effect of a sample of Bismuth. We investigate the variation of magnetoresistance with the external magnetic field and find a non-linear relation between the two. The fractional change in the magnetoresistance was found to increase exponentially with the log of the magnetic field. Furthermore, we also find the Hall coefficient of the sample at room temperature along with the carrier concentration and mobility. We then compare these results for the ones obtained for semiconductors. These results have important implications for the understanding and characterization of such conductors and their electronic properties.

I. OBJECTIVE

1. To determine magnetoresistance of Bi and its variation with the magnetic field.
2. To determine the Hall coefficient along with the carrier concentration and mobility of Bi.

II. THEORY

A. Introduction

Hall effect occurs when a current-carrying conductor is placed in a perpendicular magnetic field, causing charge carriers to experience the Lorentz force. The force deflects the carriers to one side of the conductor, creating a transverse voltage, across the conductor, called the Hall voltage. The polarity of the voltage indicates the type of charge carriers (positive for holes and negative for electrons). Magnetoresistance arises because the drift velocities of charge carriers are not uniform. In the presence of an external magnetic field, the Hall voltage balances the Lorentz force for carriers with the mean velocity. Slower carriers are overcompensated, while faster ones are under compensated, leading to trajectories deviating from the applied field. This reduces the mean free path, hence increasing resistivity.

B. Theory

Consider the Drude model of electrical conductivity. The average momentum of particles in presence of a magnetic field is,

$$\frac{d\boldsymbol{p}}{dt} = -e\left(\boldsymbol{E} + \frac{\boldsymbol{p}}{m} \times \boldsymbol{B}\right) - \frac{\boldsymbol{p}}{\tau} \tag{1}$$

where $\boldsymbol{p}$ is the average momentum of a particle with mass m and mean time before consecutive collisions τ. Here let's $\mu = \mu_0(1+\chi_m) \approx \mu_0$ for Bi (since $\chi_m \sim 10^{-4}$). Consider Fig. 1 where $\boldsymbol{E}$ is applied from the x-direction

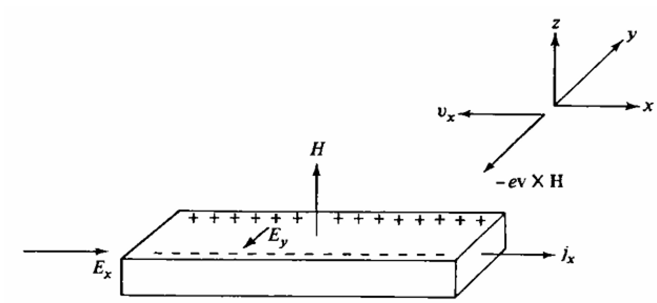

FIG. 1: Schematic diagram showing the direction of the fields and current

and $\boldsymbol{H}$ is along z-axis. Since $\frac{dp}{dt} = 0$ at steady state, we can write from Eq. ,

$$0 = -eE_x - \omega_c p_y - \frac{p_x}{\tau} \tag{2}$$

$$0 = -eE_y + \omega_c p_x - \frac{p_y}{\tau} \tag{3}$$

$$\text{where } \omega_c = \frac{\mu_0 e H}{m}$$

Now we know $\boldsymbol{J} = \sigma \boldsymbol{E}$, where σ is the conductivity from Drude model $= ne^2\tau/m$ and $\boldsymbol{J}$ is the current density. Hence we can write the above equations as,

$$\sigma E_x = \omega_c J_y \tau + J_x \tag{4}$$

$$\sigma E_y = -\omega_c J_x \tau + J_y \tag{5}$$

When Lorentz force is balanced by the electrostatic force, $J_y = 0$. Hence Eq. 5 becomes,

$$E_y = R_H B J_y, \text{ where } R_H = -\frac{1}{ne} \tag{6}$$

Here, R_H is called the Hall coefficient. Additionally, for a given magnetic field and input current, V_H is inversely proportional to the carrier density n, which is linked to

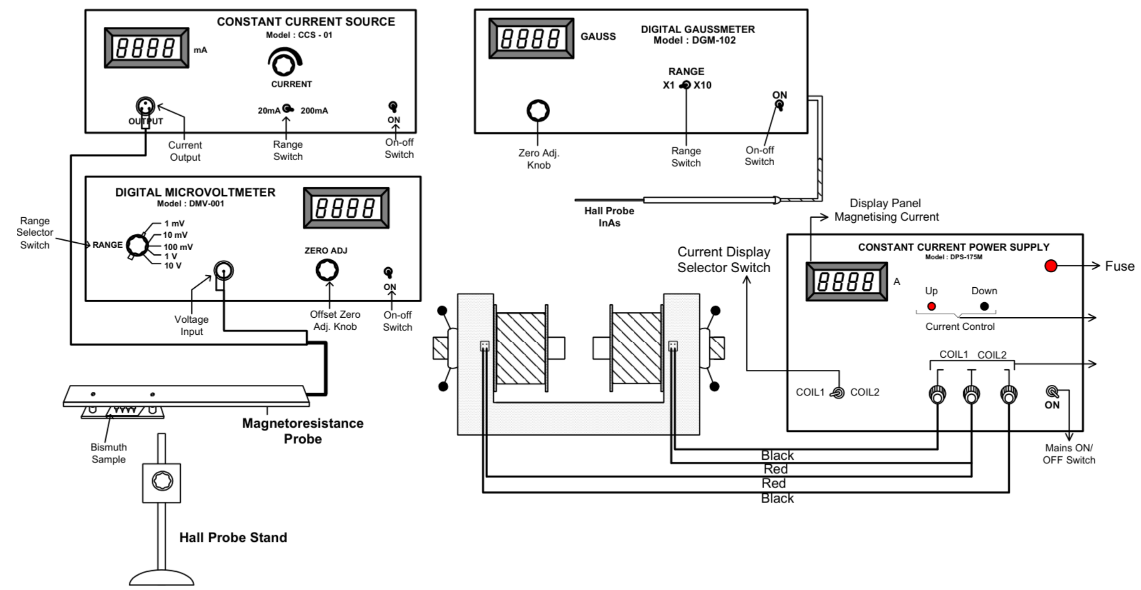

FIG. 2: Panel Diagram of the experimental setup

the material's resistivity. The conductivity σ of a material is given by $\sigma = ne\mu$, where μ is the carrier mobility. We can rewrite the above equation to use experimentally as,

$$R_H = \frac{tV_H}{IB} \tag{7}$$

where V_H is the Hall voltage, t is the thickness of the sample, I is the current and H is the external magnetic field applied. Similarly from Eq. 4, we can get the magnetoresistance (R_m) as the ratio of the voltage V and current I,

$$R_m = \frac{V}{I} \tag{8}$$

Since Hall voltage is generally not a constant, but a function of the magnetic field, so will be the Hall coefficient.

Ordinary magnetoresistance can be classified into three distinct cases. In metals with closed Fermi surfaces, electrons are constrained to their orbits in k-space, and the magnetic field increases the cyclotron frequency of the electron in its closed orbit. For metals with equal numbers of electrons and holes, the magnetoresistance increases with H up to the highest fields measured and is independent of crystallographic orientation, as seen in materials like bismuth. Metals containing Fermi surfaces with open orbits in certain crystallographic directions exhibit large magnetoresistance for fields applied in those directions, whereas resistance saturates in other directions where the orbits are closed.

Experimentally, R provides a means to measure mobility of the charge carrier in a material. By the combination of Hall coefficient and resistivity measurements, information about carrier density, mobility, and impurity concentrations can be obtained. However, $R = 1/nq$ derived from Hall effect measurements may not always match directly measured values due to the energy-dependent distribution of carriers, as those with higher velocities experience greater deviations in a magnetic field.

III. EXPERIMENTAL SETUP

Fig. 2 shows the experimental setup for observing Hall effect. A detailed list of all the apparatus required is listed below.

Experimental procedure is briefed below in the next section.

Apparatus

1. Magnetoresistance and Hall probes
2. Samples of Bismuth ($t = 0.5$ mm)
3. Hall Effect Set-up (DHE-22)
4. Electromagnet (EMU-75V)
5. Constant Current Source (CCS-01)
6. Constant Current Power Supply (DPS-175)
7. Digital Gaussmeter (DGM-102)
8. Digital Microvoltmeter (DMV-001)

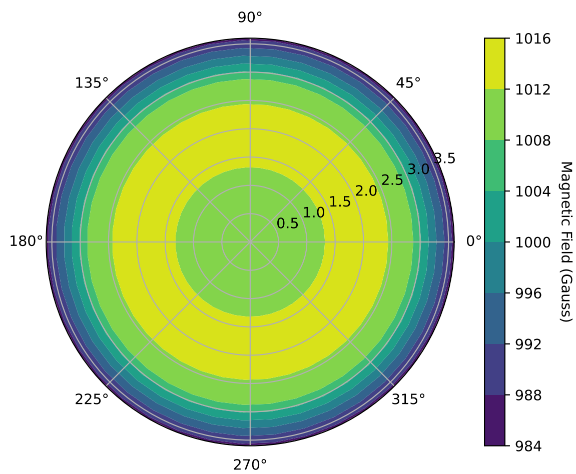

FIG. 3: Contour Plot of the Magnetic Field Distribution across the circular surface at Coil current = 0.5 A

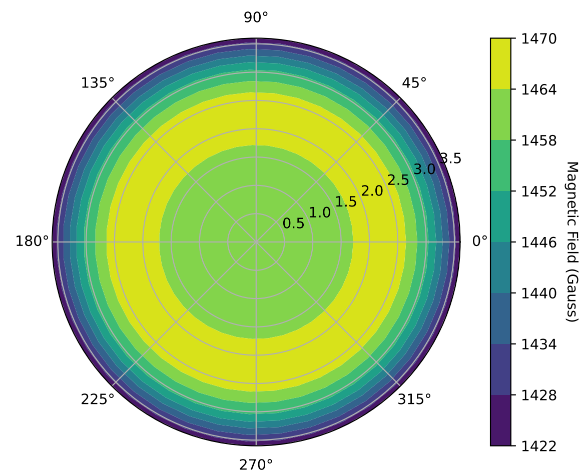

FIG. 4: Contour Plot of the Magnetic Field Distribution across the circular surface at Coil current = 1 A

IV. OBSERVATION AND CALCULATIONS

A. Magnetoresistance

Spatial Distribution of the Magnetic Field

In the first part of the experiment, we set the pole piece distance of the electromagnet to approximately 19 mm, ensuring sufficient space for the magnetoresistance probe. We verified that the electromagnet produced a magnetic field by supplying current and measuring it using a Hall sensor. Due to the geometry of the setup, the magnetic field might not be constant across all points between the coils, and the measurement between the Hall probe and the actual magnetic field experienced by the Bi sample might vary. So by taking two sets of magnetic field data by varying the position of the Hall probe we obtained a contour map with a radial distribution of the magnetic field across the geometry of diameter 7 cm (Fig. 3, 4).

We can see that the variation in the inner region is within 10 Gauss, which is the least count of the Gaussmeter. So any variation would be undetectable with our current measuring setups. Hence we have neglected this deviation in the following sections.

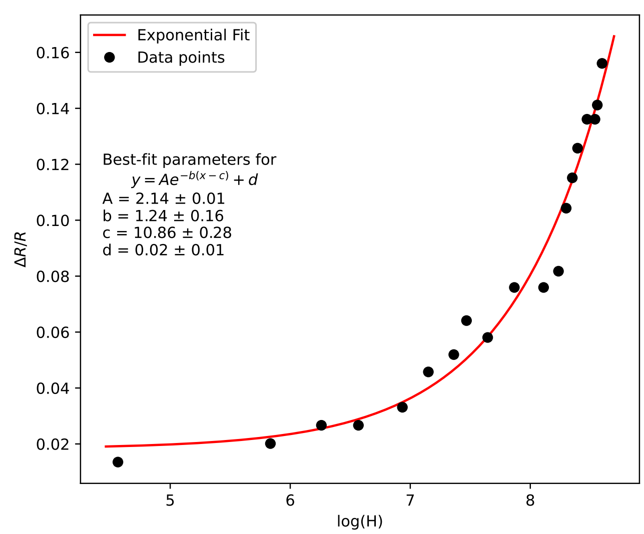

FIG. 5: Plot depicting change of resistance w.r.t magnetic field (in log scale) for CCS of 198.0 mA

Estimation of R_m

Now, after checking that the magnetoresistance setup, preloaded with bismuth, responded to the supplied current, we connect the outer probe pair connected to the CCS-01 terminals and the inner pair to the DMV-001 terminals. We then positioned the magnetoresistance probe and Hall sensor vertically aligned. Using the constant current source, we recorded the magnetic field versus voltage, and determined the resistance (R_m) in the presence of the magnetic field for two different sets of probe current. We measured the voltage at zero magnetic field to calculate the resistance (R) of the sample. The data is shown in Tables I I.

First the probe resistance is calculated for a finite probe current and voltage in presence of zero magnetic field. Then this resistance is kept as the reference for calculating the fractional change in resistance due in applied magnetic field. Calculations for magnetoresistance is performed using Eq. 8. Log scale is used in both Fig. 5 and Fig. 6 to highlight the change in magnetoresistance across large variation of magnetic field.

Probe current = 99.5 mA						Probe current = 198.0 mA					
Magnetic Field H (G)	Voltage V_m (mV)	R_m ($\times 10^{-4}\,\Omega$)	ΔR ($\times 10^{-4}\,\Omega$)	$\Delta R/R$	$\log(H)$	Magnetic Field H (G)	Voltage V_m (mV)	R_m ($\times 10^{-4}\,\Omega$)	ΔR ($\times 10^{-4}\,\Omega$)	$\Delta R/R$	$\log(H)$
0	0.063	6.332				0	0.146	7.374			
86	0.065	6.533	0.201	0.031	4.454	96	0.148	7.475	0.101	0.014	4.564
250	0.064	6.432	0.101	0.016	5.521	342	0.149	7.525	0.152	0.020	5.835
431	0.065	6.533	0.201	0.031	6.066	523	0.150	7.576	0.202	0.027	6.260
602	0.065	6.533	0.201	0.031	6.400	712	0.150	7.576	0.202	0.027	6.568
1088	0.066	6.633	0.302	0.045	6.992	1027	0.151	7.626	0.253	0.033	6.934
1408	0.068	6.834	0.503	0.074	7.250	1275	0.153	7.727	0.354	0.046	7.151
1797	0.068	6.834	0.503	0.074	7.494	1576	0.154	7.778	0.404	0.052	7.363
2580	0.069	6.935	0.603	0.087	7.856	1752	0.156	7.879	0.505	0.064	7.469
2800	0.070	7.035	0.704	0.100	7.937	2090	0.155	7.828	0.455	0.058	7.645
3060	0.071	7.136	0.804	0.113	8.026	2610	0.158	7.980	0.606	0.076	7.867
3210	0.073	7.337	1.005	0.137	8.074	3330	0.158	7.980	0.606	0.076	8.111
3460	0.074	7.437	1.106	0.149	8.149	3770	0.159	8.030	0.657	0.082	8.235
3820	0.074	7.437	1.106	0.149	8.248	4020	0.163	8.232	0.859	0.104	8.299
3990	0.075	7.538	1.206	0.160	8.292	4230	0.165	8.333	0.960	0.115	8.350
4440	0.077	7.739	1.407	0.182	8.398	4420	0.167	8.434	1.061	0.126	8.394
4960	0.078	7.839	1.508	0.192	8.509	4780	0.169	8.535	1.162	0.136	8.472
						5110	0.169	8.535	1.162	0.136	8.539
						5210	0.170	8.586	1.212	0.141	8.558
						5420	0.173	8.737	1.364	0.156	8.598

TABLE I: Magnetoresistance data for both values of probe currents

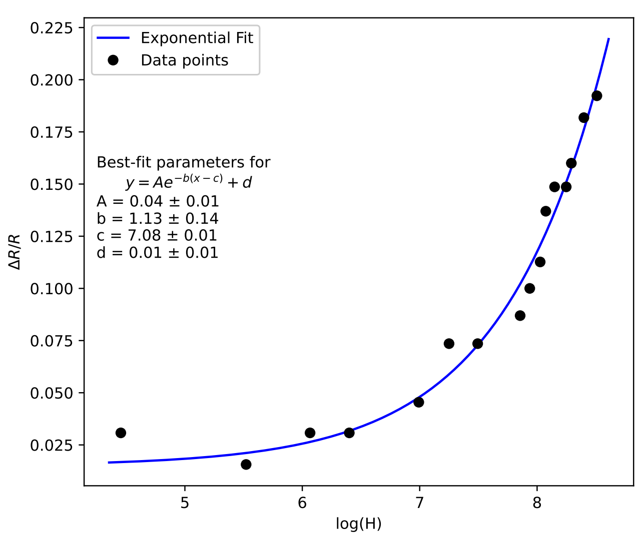

FIG. 6: Plot depicting change of resistance w.r.t magnetic field (in log scale) for CCS of 99.5 mA

B. Hall Coefficient

Calibration of Magnetic Field vs. Coil Current

First we calibrated the magnetic field values against the coil current using Table II. H was calculated for each I from using $H = (2344.45 \cdot I - 396.14)$ Gauss (Fig. 7).

I (A)	B (Gauss)	I (A)	B (Gauss)
0	0	1.76	4920
0.30	841	1.95	5410
0.55	1563	2.27	5860
0.72	2070	2.51	6220
0.99	2880	2.71	6460
1.28	3640	2.97	6740
1.54	4380		

TABLE II: Calibration of magnetic field with current

Estimation of R_H

The Hall voltage against different values of coil currents, for two different values of probe current is shown in Table III. The Hall voltage as a function of the magnetic field obtained through the calibration curve plot is shown in Figs. 8 and 9. All units have been converted to SI for simplicity.

We can rewrite Eq. 7 as,

$$R_H = \left(\frac{V}{H}\right)\frac{t}{I} \tag{9}$$

where the V/H is the slope of the above plot, I refers to the probe current and t is the thickness of the sample which is 0.5mm. Plugging in the values, the Hall coefficient comes out to be:

- For $I = 119.0$ mA, $R_H = -564.64$ cm^3/C
- For $I = 198.2$ mA, $R_H = -506.95$ cm^3/C

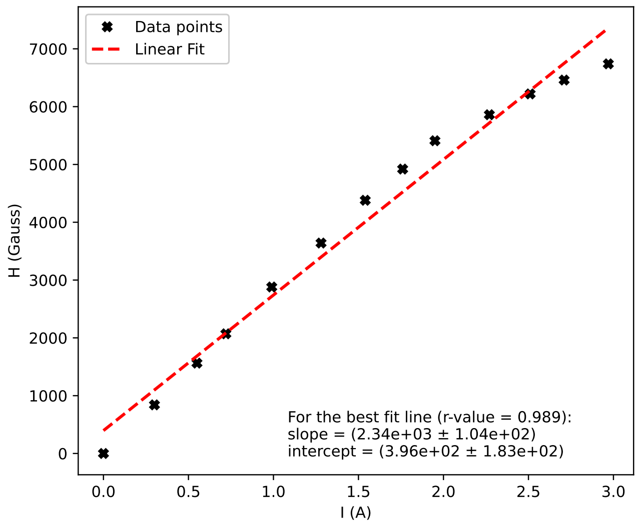

FIG. 7: Calibration plot of Magnetic field and Coil Current

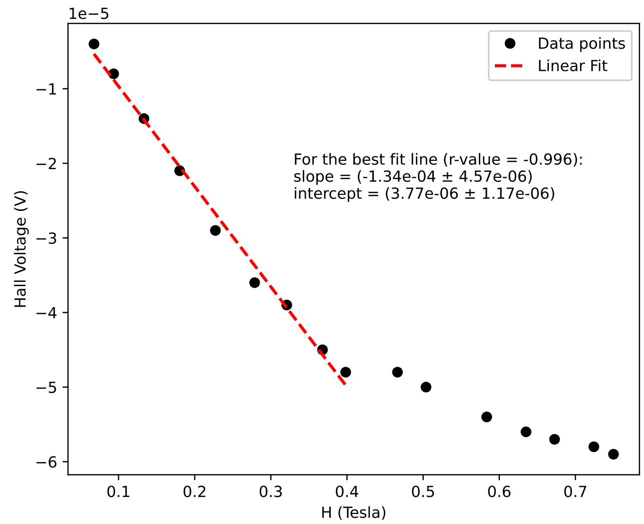

FIG. 8: Plot of Hall voltage against applied magnetic field for CCS 119.0 mA

$I = 119.0$ mA		$I = 198.2$ mA	
I (A)	Hall Voltage (mV)	I (A)	Hall Voltage (mV)
0.12	-0.004	0.00	0.000
0.23	-0.008	0.15	-0.004
0.40	-0.014	0.40	-0.018
0.60	-0.021	0.60	-0.028
0.80	-0.029	0.82	-0.042
1.02	-0.036	1.01	-0.050
1.20	-0.039	1.20	-0.055
1.40	-0.045	1.42	-0.066
1.53	-0.048	1.60	-0.073
1.82	-0.048	1.82	-0.075
1.98	-0.050		
2.32	-0.054		
2.54	-0.056		
2.70	-0.057		
2.92	-0.058		
3.03	-0.059		

TABLE III: Coil current and Hall voltage data for two different probe currents

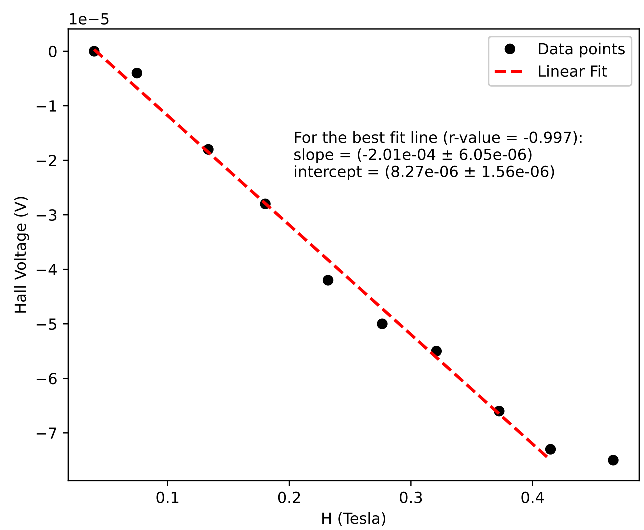

FIG. 9: Plot of Hall voltage against applied magnetic field for CCS 198.2 mA

Since $R_H = 1/ne$, we can also derive the charge carrier density to be,

- For $I = 119.0$ mA, $n = 1.11 \times 10^{16}$ cm^{-3}
- For $I = 198.2$ mA, $n = 1.23 \times 10^{16}$ cm^{-3}

Similarly using the given resistivity of the sample $1/\sigma = 1.3 \times 10^{-4}\,\Omega$ cm, we can find the mobility of charge carrier for different values of current using the formula, $\mu_e = \sigma R_H$.

- For $I = 119.0$ mA, $\mu_e = 7.34 \times 10^{-2}$ cm^2/Vs
- For $I = 198.2$ mA, $\mu_e = 6.59 \times 10^{-2}$ cm^2/Vs

V. ERROR ANALYSIS

The error in Hall coefficient can be dervied from Eq. 9 as,

$$\frac{\Delta R_H}{R_H} = \sqrt{\left(\frac{\Delta \text{slope}}{\text{slope}}\right)^2 + \left(\frac{\Delta I}{I}\right)^2} \tag{10}$$

The error in Charge carrier density can be derived from Eq. $R_H = 1/ne$ as,

$$\frac{\Delta n}{n} = \frac{\Delta R_H}{R_H} \tag{11}$$

$$\frac{\Delta\mu_e}{\mu_e} = \sqrt{\left(\frac{\Delta R_H}{R_H}\right)^2 + \left(\frac{\Delta\sigma}{\sigma}\right)^2} \tag{12}$$

Plugging in the corresponding values for both probe current values (assume $\Delta\sigma = 0$),

- $I = 119.0$ mA

$$\Delta R_H = 19.19 \text{ cm}^3/\text{C}$$
$$\Delta n = 0.04 \times 10^{16} \text{ cm}^3$$
$$\Delta\mu_e = 0.25 \times 10^{-2} \text{ cm}^2/\text{Vs}$$

- $I = 198.2$ mA

$$\Delta R_H = 15.27 \text{ cm}^3/\text{C}$$
$$\Delta n = 0.03 \times 10^{16} \text{ cm}^3$$
$$\Delta\mu_e = 0.19 \times 10^{-2} \text{ cm}^2/\text{Vs}$$

VI. RESULTS & DISCUSSION

Before starting the experiment, we measured the spatial distribution of the magnetic field across the electromagnetic coils, which we use later. By obtaining a contour plot of the distribution, we observed that the change in H between the position of the Hall probe through which we measure the magnetic field, and the Bismuth sample (within 1cm of the center) is roughly of the order of a few Gauss. Since the least count of the Gaussmeter is higher than that (10 Gauss), we have neglected this effect in future calculations. Despite that, this proved a valuable exercise in understanding the non-uniform magnetic field distribution produced by electromagnetic coils.

Magnetoresistance

In the first part of the experiment, we measure the magnetoresistance of Bismuth against the applied magnetic field. Firstly, we see that the resistance of the Bismuth is very small to begin with ($\sim 10^{-4}\,\Omega$), which is not surprising given that Bismuth is a post-transition metal, which are generally very good conductors. Then for applied magnetic field, we observed the resistance increased. The relation however, between the increase in resistance with increase in applied magnetic field is not linear. By plotting $\log(H)$ vs. $\Delta R/R$, where ΔR is the change in magnetic field from zero H, we observed a roughly exponential relation between the two. This indicates high variation in R_m for larger values of magnetic field. But again the effect is very small for smaller values of H.

Hall Coefficient

In the second part of the experiment, we have measured the hall coefficient, carrier density and carrier mobility for two different values of constant probe current. The results are as follows

- $I = 119.0$ mA

$$R_H = -(564.64 \pm 19.19) \text{ cm}^3/\text{C}$$
$$n = (1.11 \pm 0.04) \times 10^{16} \text{ cm}^3$$
$$\mu_e = (7.34 \pm 0.25) \times 10^{-2} \text{ cm}^2/\text{Vs}$$

- $I = 198.2$ mA

$$R_H = -(506.95 \pm 15.27) \text{ cm}^3/\text{C}$$
$$n = (1.23 \pm 0.03) \times 10^{16} \text{ cm}^3$$
$$\mu_e = (6.59 \pm 0.19) \times 10^{-2} \text{ cm}^2/\text{Vs}$$

Negative sign in the hall coefficient suggest that the charge carries are negatively charged. The values we obtained lies within the standard value of Hall coefficient of Bismuth in literature, which is $R_H = -540$ cm^3/C [1, 2]. Thus, our measured values have a percentage error of roughly 4% and 6% respectively.

According to the Drude model the Hall coefficient should not change with the probe current. But in general, the Hall voltage is not a linear function of magnetic field applied, i.e. the Hall coefficient is not generally a constant, but a function of the applied magnetic field. The deviations in the results can also be due to the crude approximation of the linear region we took for our analysis.

The carrier densities obtained are also around 10^2 times the values we obtained for semiconductors, which is expected, as Bismuth is a good conductor and will have a large electron density. We also see that the mobility of charge carriers is higher for the lower current value which makes sense as the Hall voltage is higher for the higher current values, which slows down the carriers.

VII. PRECAUTIONS AND SOURCES OF ERROR

1. The current through the specimen should not be large enough to cause heating.
2. Make sure that the hall probe and the bismuth strip are one above the other when placed between the poles of the Helmholtz coil for accurate measurements.
3. The Hall probe must be rotated in the field until the position of maximum voltage is reached.
4. Make sure to be gentle with the Bismuth sample as it might get damaged due to its brittleness.

[1] C. Kittel, *Introduction to Solid State Physics* (Wiley, 2004).

[2] Hall coefficient of bismuth (bi): Data extracted from the landolt-börnstein book series.