Monash University
FIT5097 Business Intelligence Modelling
2nd Semester 2020
Major Assignment Solutions

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Question 1 – Linear Programming and variants

1a)
ANSWER:
Spreadsheet: See 1b

Variables :
X_1 = Amount of Product 1 used
X_2 = Amount of Product 2 used
X_3 = Amount of Product 3 used
X_4 = Amount of Product 4 used
X_5 = Amount of Product 5 used

Objective Function:
Here, we need to Maximise our Profit based on the quantity of products used
$MAX(510X_1 + 300X_2 + 510X_3 + 270X_4 + 810X_5)$

Constraints:
Non Negativity Constraints:
$X_1, X_2, X_3, X_4, X_5 >= 0$

Constraints as per Resource Availability:
$2X_1 + 10X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$
$6X_1 + 3X_2 + 6X_3 + 3X_4 + 10X_5 = 3030$
$2X_1 + 3X_2 + 10X_3 + 6X_4 + 2X_5 = 5217$
$2X_1 + 10 X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$
$2X_1 + 10 X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$
$2X_1 + 10 X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$

1b)
ANSWER:
Excel Spreadsheet Model for this problem:
Spreadsheet : See 1b
Snapshot :

	Product 1	Product 2	Product 3	Product 4	Product 5	Total Profit
Profit of the product	$510.00	$300.00	$510.00	$270.00	$810.00	$0.00
Units to mix	0	0	0	0	0	

= SUMPRODUCT(C7:G7,C8:G8)

= SUMPRODUCT(C14:G14,$C$8:$G$8)

	Amount of Resource in the products						
Resources	Product 1	Product 2	Product 3	Product 4	Product 5	Amount of resource	Maximum Resource Available
Resource 1	2	10	2	3	6	0	2487
Resource 2	6	3	6	3	10	0	3030
Resource 3	2	3	10	6	2	0	5217
Resource 4	7	6	5	4	3	0	4000
Resource 5	5	6	3	10	2	0	4999
Resource 6	10	3	5	3	4	0	2769

1c)
ANSWER:
Sensitivity Report generated from the solver as follows:

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [Aniruddha-30945305-2ndSem2020FIT5097.xlsx]1b
Report Created: 8/10/2020 1:23:03 PM

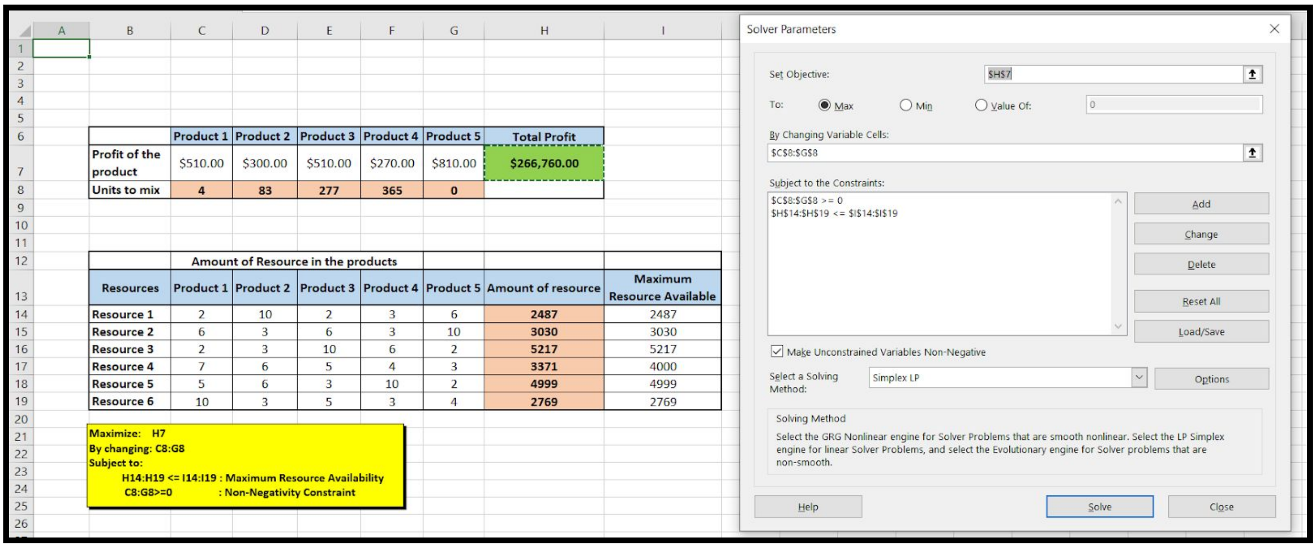
Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$8	Units to mix Product 1	4	0	510	1.333333333	27.69230769
$D$8	Units to mix Product 2	83	0	300	22.5	38.14285714
$E$8	Units to mix Product 3	277	0	510	8.780487805	1.487603306
$F$8	Units to mix Product 4	365	0	270	48.39622642	5
$G$8	Units to mix Product 5	0	-48.33922261	810	48.33922261	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$H$14	Resource 1 Amount of resource	2487	4.717314488	2487	105.3023256	3.39647E-12
$H$15	Resource 2 Amount of resource	3030	82.84452297	3030	2.98944E-13	20.00589102
$H$16	Resource 3 Amount of resource	5217	0.159010601	5217	33.54074074	9.09495E-13
$H$17	Resource 4 Amount of resource	3371	0	4000	1E+30	629
$H$18	Resource 5 Amount of resource	4999	0.636042403	4999	3.23213E-12	174.1538462
$H$19	Resource 6 Amount of resource	2769	0	2769	1E+30	5.31074E-13

1d)
ANSWER:
Spreadsheet: See 1d
Snapshot:



	Product 1	Product 2	Product 3	Product 4	Product 5	Total Profit
Profit of the product	$510.00	$300.00	$510.00	$270.00	$810.00	$266,760.00
Units to mix	4	83	277	365	0	

Resources	Product 1	Product 2	Product 3	Product 4	Product 5	Amount of resource	Maximum Resource Available
	Amount of Resource in the products						
Resource 1	2	10	2	3	6	2487	2487
Resource 2	6	3	6	3	10	3030	3030
Resource 3	2	3	10	6	2	5217	5217
Resource 4	7	6	5	4	3	3371	4000
Resource 5	5	6	3	10	2	4999	4999
Resource 6	10	3	5	3	4	2769	2769

Maximize: H7
By changing: C8:G8
Subject to:
H14:H19 <= I14:I19 : Maximum Resource Availability
C8:G8>=0 : Non-Negativity Constraint

From our Spreadsheet, after using the Solver and from the sensitivity report generated, we can clearly see that the
Optimal Production Plan:
X_1 = Amount of Product 1 used = 4
X_2 = Amount of Product 2 used = 83
X_3 = Amount of Product 3 used = 277
X_4 = Amount of Product 4 used = 365
X_5 = Amount of Product 5 used = 0

Working:
Total Profit
= (Product 1 Amount)*(Product 1 Profit) +
(Product 2 Amount)*(Product 2 Profit) +
(Product 3 Amount)*(Product 3 Profit) +
(Product 4 Amount)*(Product 4 Profit) +
(Product 5 Amount)*(Product 5 Profit) +
= (4)*(510) + (83)*(300) + (277)*(510) + (365)*(270) + (0)*(810)
= $ 266,760

Associated Total Profit = $266,760

1e)
ANSWER:
Spreadsheet: See 1e
Snapshot:

Microsoft Excel 16.0 Answer Report
Worksheet: [Aniruddha-30945305-2ndSem2020FIT5097.xlsx]1b
Report Created: 8/10/2020 4:29:05 PM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 0.094 Seconds.
Iterations: 6 Subproblems: 0
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$H$7	Profit of the product Total Profi	$266,760.00	$266,760.00

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$C$8	Units to mix Product 1	4	4	Contin
$D$8	Units to mix Product 2	83	83	Contin
$E$8	Units to mix Product 3	277	277	Contin
$F$8	Units to mix Product 4	365	365	Contin
$G$8	Units to mix Product 5	0	0	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$H$14	Resource 1 Amount of resource	2487	$H$14<=$I$14	Binding	0
$H$15	Resource 2 Amount of resource	3030	$H$15<=$I$15	Binding	0
$H$16	Resource 3 Amount of resource	5217	$H$16<=$I$16	Binding	0
$H$17	Resource 4 Amount of resource	3371	$H$17<=$I$17	Not Bindin	629
$H$18	Resource 5 Amount of resource	4999	$H$18<=$I$18	Binding	0
$H$19	Resource 6 Amount of resource	2769	$H$19<=$I$19	Binding	0
$C$8	Units to mix Product 1	4	$C$8>=0	Not Bindin	4
$D$8	Units to mix Product 2	83	$D$8>=0	Not Bindin	83
$E$8	Units to mix Product 3	277	$E$8>=0	Not Bindin	277
$F$8	Units to mix Product 4	365	$F$8>=0	Not Bindin	365
$G$8	Units to mix Product 5	0	$G$8>=0	Binding	0

In binding constraints, all resources are used up.

Here, we can also generate an answer report from the solver, in order to have a look at the binding constraints.

From the answer report generated named as :"1e Answer Report", we can see that :
The binding constraints are:
Amounts of the following resources used

- Resource 1
- Resource 2
- Resource 3
- Resource 5
- Resource 6

1f)
ANSWER:

Spreadsheet: See 1c Sensitivity Report and 1f
Snapshot:

Constraints

Cell	Name	Final Value	Shadow Price
$H$14	Resource 1 Amount of resource	2487	4.717314488
$H$15	Resource 2 Amount of resource	3030	82.84452297
$H$16	Resource 3 Amount of resource	5217	0.159010601
$H$17	Resource 4 Amount of resource	3371	0
$H$18	Resource 5 Amount of resource	4999	0.636042403
$H$19	Resource 6 Amount of resource	2769	0

In order to solve this problem, we use the concept of **"Shadow Prices"**
The shadow price gives us the amount by which our objective function changes its value for an unit increase in the RHS value of our constraint. It is the amount of money that we would be willing to pay in order to acquire additional units of that resource.

Here, the shadow prices of the resources are as follows:
(Rounding off the prices to three decimal places)

Resource	Shadow Price	Additional Units
R1	4.717	-10
R2	82.844	+1
R3	0.159	-5
R4	0	+10
R5	0.636	+100
R6	0	-3

Thus,
10 units decrease in R1 --- decreases the Objective Function by 47.17
1 units increase in R2 --- increases the Objective Function by 82.844
5 units decrease in R3 --- decreases the Objective Function by 0.795
10 units increase in R4 --- HAS NO EFFECT on the Objective Function
100 units increase in R5 --- increases the Objective Function by 63.6
-3 units decrease in R6 --- HAS NO EFFECT on the Objective Function
Snapshot : 1f

	Product 1	Product 2	Product 3	Product 4	Product 5	Total Profit
Profit of the product	$510.00	$300.00	$510.00	$270.00	$810.00	$266,759.03
Units to mix	3.88172	83.37634	280.5269	358.1398	0	

Maximize: H7
By changing: C8:G8
Subject to:
H14:H19 <= I14:I19 : Maximum Resource Availability
C8:G8>=0 : Non-Negativity Constraint

	Amount of Resource in the products						
Resources	Product 1	Product 2	Product 3	Product 4	Product 5	Amount of resource	Maximum Resource Available
Resource 1	2	10	2	3	6	2477	2477
Resource 2	6	3	6	3	10	3031	3031
Resource 3	2	3	10	6	2	5212	5212
Resource 4	7	6	5	4	3	3362.623656	4010
Resource 5	5	6	3	10	2	4942.645161	5099
Resource 6	10	3	5	3	4	2766	2766

Objective Function:
Here, we need to Maximise our Profit based on the quantity of products used
$MAX(510X_1 + 300X_2 + 510X_3 + 270X_4 + 810X_5)$

Constraints:
Non Negativity Constraints:
$X_1, X_2, X_3, X_4, X_5 >= 0$

Constraints as per Resource Availability:
$2X_1 + 10X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$
$6X_1 + 3X_2 + 6X_3 + 3X_4 + 10X_5 = 3030$
$2X_1 + 3X_2 + 10X_3 + 6X_4 + 2X_5 = 5217$
$2X_1 + 10 X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$
$2X_1 + 10 X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$
$2X_1 + 10 X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$

Now, we cannot really make out if this has any increase or decrease on the objective function. Hence, we directly add and subtract the required number of resources from the overall total resources available column and solve the equation by applying the same set of constraints.

Thus, the maximum number of resources now available are as follows:
Resource 1 : 2477
Resource 2 : 3031
Resource 3 : 5212
Resource 4 : 4010
Resource 5 : 5099
Resource 6 : 2766

Now, from the solver,
Optimal Production Plan:
X_1 = Amount of Product 1 used = 3.882
X_2 = Amount of Product 2 used = 83.376
X_3 = Amount of Product 3 used = 280.527
X_4 = Amount of Product 4 used = 358.14
X_5 = Amount of Product 5 used = 0

Working:
Total Profit
= (Product 1 Amount)*(Product 1 Profit) +
(Product 2 Amount)*(Product 2 Profit) +
(Product 3 Amount)*(Product 3 Profit) +
(Product 4 Amount)*(Product 4 Profit) +
(Product 5 Amount)*(Product 5 Profit) +
= ()3.882*(510) + (83.376)*(300) + (280.527)*(510) + (358.14)*(270) + (0)*(810)
= $ 266, 759.03

Associated Total Profit = $266, 759.03
Thus, the company would lose $1,. Thus, the company should not accept the offer.

1g)
ANSWER:
Spreadsheet: See 1g, 1g_Sensitivity Report and 1g part ii

Snapshot:
1g :

	Product 1	Product 2	Product 3	Product 4	Product 5	Product 6	Total Profit
Profit of the product	$510.00	$300.00	$510.00	$270.00	$810.00	$155.00	$266,760.00
Units to mix	4	83	277	365	0	0	

	Amount of Resource in the products							
Resources	Product 1	Product 2	Product 3	Product 4	Product 5	Product 6	Amount of resource	Maximum Resource Available
Resource 1	2	10	2	3	6	0	2487	2487
Resource 2	6	3	6	3	10	2	3030	3030
Resource 3	2	3	10	6	2	0	5217	5217
Resource 4	7	6	5	4	3	4	3371	4000
Resource 5	5	6	3	10	2	5	4999	4999
Resource 6	10	3	5	3	4	0	2769	2769

Maximize: I7
By changing: C8:H8
Subject to:
I14:I19 <= J14:J19 : Resource Availability
C8:H8 >= 0 : Non - Negativity Constraints

1g : Sensitivity Report

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [Aniruddha-30945305-2ndSem2020FIT5097.xlsx]1h
Report Created: 8/10/2020 9:31:15 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$8	Units to mix Product 1	4	0	510	1.333333333	26.94508009
$D$8	Units to mix Product 2	83	0	300	22.5	38.14285714
$E$8	Units to mix Product 3	277	0	510	8.780487805	1.487603306
$F$8	Units to mix Product 4	365	0	270	48.39622642	5
$G$8	Units to mix Product 5	0	-48.33922261	810	48.33922261	1E+30
$H$8	Units to mix Product 6	0	-13.86925795	155	13.86925795	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$I$14	Resource 1 Amount of resource	2487	4.717314488	2487	105.3023256	3.39647E-12
$I$15	Resource 2 Amount of resource	3030	82.84452297	3030	2.98944E-13	20.00589102
$I$16	Resource 3 Amount of resource	5217	0.159010601	5217	33.54074074	9.09495E-13
$I$17	Resource 4 Amount of resource	3371	0	4000	1E+30	629
$I$18	Resource 5 Amount of resource	4999	0.636042403	4999	3.23213E-12	174.1538462
$I$19	Resource 6 Amount of resource	2769	0	2769	1E+30	5.31074E-13

1g part ii

	A	B	C	D	E	F	G	H	I	J
3										
4										
5										
6			Product 1	Product 2	Product 3	Product 4	Product 5	Product 6	Total Profit	
7		Profit of the product	$510.00	$300.00	$510.00	$270.00	$810.00	$155.00	$266,746.13	
8		Units to mix	3.485277	83.17197	277.3157	364.5595	0	1		
9										
10										
11										
12			Amount of Resource in the products							
13		Resources	Product 1	Product 2	Product 3	Product 4	Product 5	Product 6	Amount of resource	Maximum Resource Available
14		Resource 1	2	10	2	3	6	0	2487	2487
15		Resource 2	6	3	6	3	10	2	3030	3030
16		Resource 3	2	3	10	6	2	0	5217	5217
17		Resource 4	7	6	5	4	3	4	3372.244994	4000
18		Resource 5	5	6	3	10	2	5	4999	4999
19		Resource 6	10	3	5	3	4	0	2764.625442	2769

Maximize: I7
By changing: C8:H8
Subject to:
I14:I19 <= J14:J19 : Resource Availability Constraint
C8:H8 >=0 : Non - Negativity Constraint
I7 = 1 (Producing Product 6)

Manual Calculations:
Now, a new product Product 6 is considered.
It generates a profit of $155.
Resources Required:

- 2 of Resource 2 (Shadow Price = 82.844)
- 4 of Resource 4 (Shadow Price = 0)
- 5 of Resource 5 (Shadow Price = 0.636)

Let's see if it would be profitable to produce Product 6:
We will be using the concept of Reduced Cost,
It is the per-unit profit minus the per unit value of the resources it consumes.
Reduced Cost

= Profit - $\sum$(shadow price)*(units of resources required)

= 155 - [(2*82.844) + (4*0) + (5*0.636)]
= -13.868 < $0
Thus, we have a negative reduced cost.
Hence, This is not profitable!

Observations from the Solver:
Now, when we introduce a unit of Product 6, we see that the overall profit decreases to $ 266,746.13. Thus, the profit decreases by an amount of -13.87!
The same value can be observed from that of the Solver calculations!

Hence, we do not expect Product 6 to be produced.

Product 6 has to be around $ 13.868 more profitable for it to be produced.

1h)
ANSWER:
Spreadsheet: See 1h
Snapshot:

	Product 1	Product 2	Product 3	Product 4	Product 5	Total Profit
Profit of the product	$512.00	$301.00	$511.00	$269.00	$811.00	$266,763.00
Units to mix	4	83	277	365	0	

	Amount of Resource in the products						
Resources	Product 1	Product 2	Product 3	Product 4	Product 5	Amount of resource	Maximum Resource Available
Resource 1	2	10	2	3	6	2487	2487
Resource 2	6	3	6	3	10	3030	3030
Resource 3	2	3	10	6	2	5217	5217
Resource 4	7	6	5	4	3	3371	4000
Resource 5	5	6	3	10	2	4999	4999
Resource 6	10	3	5	3	4	2769	2769

Maximize: H7
By changing: C8:G8
Subject to:
H14:H19 <= I14:I19 : Resource Availability
C8:G8>=0 : Non - Negativity Constraint

Here, we use the 100% Rule to determine whether the optimal solution changes or not as we have a linear programming problem.
The reduced cost of four variables is 0.
The reduced cost of one variable X5 = Units required for Product 5 is non-zero.
Hence, we calculate the r_j value:

rj = $\frac{\Delta cj}{Ij}$, $\Delta cj >= 0$

rj = $\frac{-\Delta cj}{Dj}$, $\Delta cj < 0$

I_j = Allowable Increase for coefficient j
D_j = Allowable Decrease for coefficient j

The current situation remains optimal when the sum of r_j <= 1
If the sum of r_j values >1, the solution may still be optimal but it is not guaranteed.

Here, The sum of r_j values = 1.879 >1

Now, based on these new values of coefficients,
The Optimal Function is
MAX($512X_1 + 301X_2 + 511X_3 + 269X_4 + 811X_5$)

Working:
Total Profit
= (Product 1 Amount)*(Product 1 Profit) +
(Product 2 Amount)*(Product 2 Profit) +
(Product 3 Amount)*(Product 3 Profit) +
(Product 4 Amount)*(Product 4 Profit) +
(Product 5 Amount)*(Product 5 Profit) +
= (4)*(512) + (83)*(301) + (277)*(511) + (365)*(269) + (0)*(811)
= $ 266,763

From the Solver, we can see that

- The optimal amount to be produced for each of the products does not change.
- This is because we are evenly increasing and decreasing the profitability of the various products.
- The New Maximum Profit = $ 266,763
- This optimal function value **SLIGHTLY INCREASES BY 3 $**

1i)
ANSWER:
Spreadsheet: See 1i
Snapshot:

	Product 1	Product 2	Product 3	Product 4	Product 5	Total Profit
Profit of the product	$1,020.00	$600.00	$1,020.00	$540.00	$1,620.00	$533,520.00
Units to mix	4	83	277	365	0	

	Amount of Resource in the products						
Resources	Product 1	Product 2	Product 3	Product 4	Product 5	Amount of resource	Maximum Resource Available
Resource 1	2	10	2	3	6	2487	2487
Resource 2	6	3	6	3	10	3030	3030
Resource 3	2	3	10	6	2	5217	5217
Resource 4	7	6	5	4	3	3371	4000
Resource 5	5	6	3	10	2	4999	4999
Resource 6	10	3	5	3	4	2769	2769

Maximize : H7
By changing: C8:G8
Subject to:
H14:H19 <= I14:I19 : Resource Availability
C8:G8>=0 : Non - Negativity Constraint

Now, based on these new values of coefficients,
The Optimal Function is
MAX($1020X_1 + 600X_2 + 1020X_3 + 540X_4 + 1620X_5$)

Calculations:
Now, again from the 100% rule we can see that the sum of r_j values > 1
Here, it is :
$\frac{510}{1.333} + \frac{300}{22.5} + \frac{510}{8.781} + \frac{270}{48.396} + \frac{810}{48.339}$ = 476.344 >>>1

Working:
Total Profit
= (Product 1 Amount)*(Product 1 Profit) +
(Product 2 Amount)*(Product 2 Profit) +
(Product 3 Amount)*(Product 3 Profit) +
(Product 4 Amount)*(Product 4 Profit) +
(Product 5 Amount)*(Product 5 Profit) +
= (4)*(1020) + (83)*(600) + (277)*(1020) + (365)*(540) + (0)*(1620)
= $ 533, 520

From the Solver, we can see that
- The optimal amount to be produced for each of the products does not change.
- Again, this is because we have uniformly increased the different profitability values of all the products used.
- The New Maximum Profit = $ 533,520
- This optimal function value **HAS DOUBLED!**

1j)
ANSWER:
Spreadsheet: See 1j
Snapshot:

	Product 1	Product 2	Product 3	Product 4	Product 5	Total Profit
Profit of the product	$255.00	$150.00	$255.00	$135.00	$405.00	$133,380.00
Units to mix	4	83	277	365	0	

	Amount of Resource in the products						
Resources	Product 1	Product 2	Product 3	Product 4	Product 5	Amount of resource	Maximum Resource Available
Resource 1	2	10	2	3	6	2487	2487
Resource 2	6	3	6	3	10	3030	3030
Resource 3	2	3	10	6	2	5217	5217
Resource 4	7	6	5	4	3	3371	4000
Resource 5	5	6	3	10	2	4999	4999
Resource 6	10	3	5	3	4	2769	2769

Minimize: H7
By changing: C8:G8
Subject to:
H14:H19 <= I14:I19 : Resource Availability
C8:G8>=0 : Non - Negativity Constraints

Now, based on these new values of coefficients,
The Optimal Function is
MAX($255X_1 + 150X_2 + 255X_3 + 135X_4 + 405X_5$)

Calculations:
Now, again from the 100% rule we can see that the sum of r_j values > 1
Here, it is :
$\frac{255}{1.333} + \frac{150}{22.5} + \frac{255}{8.781} + \frac{135}{48.396} + \frac{405}{48.339}$ = >>1

Working:
Total Profit
= (Product 1 Amount)*(Product 1 Profit) +
(Product 2 Amount)*(Product 2 Profit) +
(Product 3 Amount)*(Product 3 Profit) +
(Product 4 Amount)*(Product 4 Profit) +
(Product 5 Amount)*(Product 5 Profit) +
= (4)*(255) + (83)*(150) + (277)*(255) + (365)*(135) + (0)*(405)
= $ 133,380

From the Solver, we can see that
- The optimal amount to be produced for each of the products does not change.
- This is because we have uniformly decreased the profitability of the various products used.
- The New Maximum Profit = $ 133,380

- This optimal function value **HAS DECREASED BY HALF!**

1k)
ANSWER:
Spreadsheet: 1(k and l) Sensitivity Report
Snapshot:

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [Gayu_Copy.xlsx]1(k and l)
Report Created: 16/10/2020 5:06:53 AM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$8	Units to mix Product 1	9.684910086	0	510	1.176470588	375
$D$8	Units to mix Product 2	9.684910086	0	300	1.176470588	375
$E$8	Units to mix Product 3	271.3838937	0	510	24.19354839	0.397350993
$F$8	Units to mix Product 4	405.8944488	0	270	986.146789	2.857142857
$G$8	Units to mix Product 5	9.684910086	0	810	1.176470588	375

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$C$8	Units to mix Product 1	9.684910086	-3.143080532	0	6.515151515	14.42025611
$D$8	Units to mix Product 2	9.684910086	34.03440188	0	6.323529412	16.96849315
$H$14	Resource 1 Amount of resource	1934.779515	0	2487	1E+30	552.2204848
$H$15	Resource 2 Amount of resource	3030	84.04222048	3030	34.37669377	151.0609756
$H$16	Resource 3 Amount of resource	5217	0.04691165	5217	242.8823529	776.6326531
$H$17	Resource 4 Amount of resource	3135.455825	0	4000	1E+30	864.5441751
$H$18	Resource 5 Amount of resource	4999	1.759186865	4999	469.8148148	2064.5
$H$19	Resource 6 Amount of resource	2739.246286	0	2769	1E+30	29.75371384

From the sensitivity report, we can see that after adding the new constraints, the values of "Allowable Increase" have decreased for most of the products. This clearly shows that the application of the new constraints decreases the feasible region.

Solver Analysis:
Here, we have added one more constraint in the Solver.
We have made sure that the amount of Product 1, Product 2 and Product 5 must be produced in equal amounts.

The value of the Optimal Function has slightly decreased.
Hence, **ADDING** this requirement, makes the **FEASIBLE REGION SMALLER!**

1l part 1)
ANSWER:
Spreadsheet: See 1(k and l)
Snapshot :

We now add one more constraint in the Solver in order to ensure that equal amounts of Product 1, Product 2 and Product 5 are produced.
From the Solver, we can clearly see that, the optimal amount to be produced for each of the products are as follows:

Optimal Amount produced for Product 1 = 9.685
Optimal Amount produced for Product 2 = 9.685
Optimal Amount produced for Product 3 = 271.384
Optimal Amount produced for Product 4 = 405.894
Optimal Amount produced for Product 5 = 9.685

11 part 2)
ANSWER:
Spreadsheet: See 1(k and l)

	Product 1	Product 2	Product 3	Product 4	Product 5	Total Profit
Profit of the product	$510.00	$300.00	$510.00	$270.00	$810.00	$263,686.84
Units to mix	9.68491	9.68491	271.3839	405.8944	9.68491	

Maximize: H7
By changing: C8:G8
Subject to:
H14:H19 <= I14:I19 : Resource availability
C8:G8>=0 : Non Negativity Constraint
C8 = D8 : Product 1 Amount = Product 2 Amount
D8 = G8 : Product 2 Amount = Product 5 Amount

	Amount of Resource in the products						
Resources	Product 1	Product 2	Product 3	Product 4	Product 5	Amount of resource	Maximum Resource Available
Resource 1	2	10	2	3	6	1934.779515	2487
Resource 2	6	3	6	3	10	3030	3030
Resource 3	2	3	10	6	2	5217	5217
Resource 4	7	6	5	4	3	3135.455825	4000
Resource 5	5	6	3	10	2	4999	4999
Resource 6	10	3	5	3	4	2739.246286	2769

Calculations:
Total Profit
= (Product 1 Amount)*(Product 1 Profit) +
(Product 2 Amount)*(Product 2 Profit) +
(Product 3 Amount)*(Product 3 Profit) +
(Product 4 Amount)*(Product 4 Profit) +
(Product 5 Amount)*(Product 5 Profit) +
= (9.685)*(510) + (9.865)*(300) + (271.384)*(510) + (405.894)*(270) + (9.685)*(810)
= $ 263, 686. 84

Solver:
From the Solver, we can see that the **RESULTANT PROFIT** is $263,658.84
Clearly, we can see that after adding constraints the profit decreases.

1m part 1)
ANSWER:

Spreadsheet: See 1m

	Product 1	Product 2	Product 3	Product 4	Product 5	Total Profit
Profit of the product	$510.00	$300.00	$510.00	$270.00	$810.00	$266,760.00
Units to mix	4	83	277	365	0	

Maximize: H7
By changing: C8:G8
Subject to:
H14:H19 <= I14:I19 : Resource Availability
C8:G8>=0 : Non - Negativity Constraint
C8:G8 = integer : Integer Constraint

	Amount of Resource in the products						
Resources	Product 1	Product 2	Product 3	Product 4	Product 5	Amount of resource	Maximum Resource Available
Resource 1	2	10	2	3	6	2487	2487
Resource 2	6	3	6	3	10	3030	3030
Resource 3	2	3	10	6	2	5217	5217
Resource 4	7	6	5	4	3	3371	4000
Resource 5	5	6	3	10	2	4999	4999
Resource 6	10	3	5	3	4	2769	2769

Now, we add an additional constraint, that all the amounts of the various products have to be integer values.
From the solver, we can see that the :
Optimal Amount produced for Product 1 = 4
Optimal Amount produced for Product 2 = 83
Optimal Amount produced for Product 3 = 277
Optimal Amount produced for Product 4 = 365
Optimal Amount produced for Product 5 = 0

We can see that the optimal amounts produced for the products have not changed.
They are the same as the original amounts as the original LP also gave integer values.

1m part 2)
ANSWER:
Spreadsheet: See 1m
Snapshot:

	Product 1	Product 2	Product 3	Product 4	Product 5	Total Profit
Profit of the product	$510.00	$300.00	$510.00	$270.00	$810.00	$266,760.00
Units to mix	4	83	277	365	0	

Maximize: H7
By changing: C8:G8
Subject to:
H14:H19 <= I14:I19 : Resource Availability
C8:G8>=0 : Non - Negativity Constraint
C8:G8 = integer : Integer Constraint

	Amount of Resource in the products						
Resources	Product 1	Product 2	Product 3	Product 4	Product 5	Amount of resource	Maximum Resource Available
Resource 1	2	10	2	3	6	2487	2487
Resource 2	6	3	6	3	10	3030	3030
Resource 3	2	3	10	6	2	5217	5217
Resource 4	7	6	5	4	3	3371	4000
Resource 5	5	6	3	10	2	4999	4999
Resource 6	10	3	5	3	4	2769	2769

Working:
Total Profit
= (Product 1 Amount)*(Product 1 Profit) +
(Product 2 Amount)*(Product 2 Profit) +
(Product 3 Amount)*(Product 3 Profit) +
(Product 4 Amount)*(Product 4 Profit) +
(Product 5 Amount)*(Product 5 Profit) +
= (4)*(510) + (83)*(300) + (277)*(510) + (365)*(270) + (0)*(810)
= $ 266,760

From the Solver, we can see that the value of the objective function now is : $266,760
We can see that the value of the objective function has not changed as the original optimal amounts of the products were also integer values.

1n)

Product	Product 1	Product 2	Product 3	Product 4	Product 5
Unit Profit	$510	$300	$510	$270	$810
Fixed-cost (Start-up cost)	2000	4000	8000	16000	1000

1n part 1)
ANSWER:
Spreadsheet: See 1n

Snapshot :

	Product 1	Product 2	Product 3	Product 4	Product 5	
Units to mix (X)	0	164	423	0	0	
Binary Variables (Y)	0	1	1	0	0	
Linking Constraints	0	-84.7	-82	0	0	
Set Up Cost	2000	4000	8000	16000	1000	Total Profit
Profit of the product	$510.00	$300.00	$510.00	$270.00	$810.00	$252,930.00

Maximize: H13
By changing: C8:G9
Subject to:
H18:H23 <= I18:I23 : Resource Availability Constraint
C8:G9 >= 0 : Non-Negativity Constraint
C8:G8 = integer : Integer Constraint
C11:G11 <= 0 : Linking Constraint
C9:G9 binary : Binary Constraint

	Amount of Resource in the products						
Resources	Product 1	Product 2	Product 3	Product 4	Product 5	Amount of resource	Maximum Resource Available
Resource 1	2	10	2	3	6	2486	2487
Resource 2	6	3	6	3	10	3030	3030
Resource 3	2	3	10	6	2	4722	5217
Resource 4	7	6	5	4	3	3099	4000
Resource 5	5	6	3	10	2	2253	4999
Resource 6	10	3	5	3	4	2607	2769

Continuing our answer from 1b, we use two variables X_i and Y_i

Variables:
X_1 = Amount of Product 1 used
X_2 = Amount of Product 2 used
X_3 = Amount of Product 3 used
X_4 = Amount of Product 4 used

X_5 = Amount of Product 5 used

Binary Variables:
Y_i = Variables for the fixed start up cost
= 1 If $X_i > 0$ i.e when when Product X_i is chosen; where i = 1, 2, 3, 4, 5

= 0 If $X_i = 0$ i.e when product Product X_i is NOT chosen

Objective Function:
Here, we need to Maximise our Profit based on the quantity of products used
MAX : $510X_1 + 300X_2 + 510X_3 + 270X_4 + 810X_5 -$
$2000Y_1 - 4000Y_2 - 8000\ Y_3 - 16000Y_4 - 1000Y_5$
Constraints:
Non Negativity and Integer Constraints:
$X_1, X_2, X_3, X_4, X_5 >= 0$
X_1, X_2, X_3, X_4, X_5 are Integers
Binary Constraints:
All Y_i must be binary, where i = 1,2,3,4,5
Linking Constraints:
$X_1 <= M_1Y_1$ or $X_1 - M_1Y_1 = 0$
$X_2 <= M_2Y_2$ or $X_2 - M_2Y_2 = 0$
$X_3 <= M_3Y_3$ or $X_3 - M_3Y_3 = 0$
$X_4 <= M_4Y_4$ or $X_4 - M_4Y_4 = 0$
$X_5 <= M_5Y_5$ or $X_5 - M_5Y_5 = 0$
Here, M_i introduces an upper bound on X_i
Calculating M_i values:-
$M_1 = \mathrm{MIN}(\frac{2487}{2}, \frac{3030}{6}, \frac{5217}{2}, \frac{4000}{7}, \frac{4999}{5}, \frac{2769}{10}) = 279.6$
$M_2 = \mathrm{MIN}(\frac{2487}{10}, \frac{3030}{3}, \frac{5217}{3}, \frac{4000}{6}, \frac{4999}{6}, \frac{2769}{3}) = 248.7$
$M_3 = \mathrm{MIN}(\frac{2487}{2}, \frac{3030}{6}, \frac{5217}{10}, \frac{4000}{5}, \frac{4999}{3}, \frac{2769}{5}) = 505$
$M_4 = \mathrm{MIN}(\frac{2487}{3}, \frac{3030}{3}, \frac{5217}{6}, \frac{4000}{4}, \frac{4999}{10}, \frac{2769}{3}) = 499.9$
$M_5 = \mathrm{MIN}(\frac{2487}{6}, \frac{3030}{10}, \frac{5217}{2}, \frac{4000}{3}, \frac{4999}{2}, \frac{2769}{4}) = 303$
Constraints as per Resource Availability:
$2X_1 + 10X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$
$6X_1 + 3X_2 + 6X_3 + 3X_4 + 10X_5 = 3030$
$2X_1 + 3X_2 + 10X_3 + 6X_4 + 2X_5 = 5217$
$2X_1 + 10\ X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$
$2X_1 + 10\ X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$
$2X_1 + 10\ X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$

From the solver, we can see that
From the solver, we can see that the :
Optimal Amount produced for Product 1 = 0
Optimal Amount produced for Product 2 = 164
Optimal Amount produced for Product 3 = 423
Optimal Amount produced for Product 4 = 0
Optimal Amount produced for Product 5 = 0

1n part 2)
ANSWER:
Spreadsheet: See 1n
Working:
Total Profit
= (Product 1 Amount)*(Product 1 Profit) +
(Product 2 Amount)*(Product 2 Profit) +
(Product 3 Amount)*(Product 3 Profit) +
(Product 4 Amount)*(Product 4 Profit) +
(Product 5 Amount)*(Product 5 Profit) +
= (0)*(510) + (164)*(300) + (423)*(510) + (0)*(270) + (0)*(810)
= $ 252,930

Thus, we can see that adding more constraints has decreased the value of the profits produced.

From the Solver, we can see that the value of the objective function now is : $252, 930

1o) Part i
ANSWER:
Spreadsheet : See 1o
Snapshot :

	Product 1	Product 2	Product 3	Product 4	Product 5	
Units to mix (X)	**177**	**116**	**0**	**0**	**162**	
Binary Variables (Y)	1	1	0	0	1	
Linking Constraints	-99.9	-132.7	0	0	-141	
Set Up Cost	2000	4000	8000	16000	1000	**Total Profit**
Profit of the product	$510.00	$300.00	$510.00	$270.00	$810.00	**$249,290.00**

Maximize: H12
By changing: C8:G8, C9:G9
Subject to:
H17:H22 <= I17:I22 : Resource Availability Constraint
D26 >= 0 : Minimum value constraint
D27 <= 0 : Maximum Value constraint
C8:G9 >= 0 : Non-Negativity Constraint
C8:G8 = integer : Integer Constraint
C10:G10 <= 0 : Linking Constraint
C9:G9 binary : Binary Constraint
C9:G9 >=0 : Non - Negativity Constraint

	Amount of Resource in the products						
Resources	**Product 1**	**Product 2**	**Product 3**	**Product 4**	**Product 5**	**Amount of resource**	**Maximum Resource Available**
Resource 1	2	10	2	3	6	2486	2487
Resource 2	6	3	6	3	10	3030	3030
Resource 3	2	3	10	6	2	1026	5217
Resource 4	7	6	5	4	3	2421	4000
Resource 5	5	6	3	10	2	1905	4999
Resource 6	10	3	5	3	4	2766	2769

For X3 :		
Minimum Value	225	0
Maximum Value	325	0

Objective Function:
Here, we need to Maximise our Profit based on the quantity of products used
MAX : $510X_1 + 300X_2 + 510X_3 + 270X_4 + 810X_5 -$
$2000Y_1 - 4000Y_2 - 8000\ Y_3 - 16000Y_4 - 1000Y_5$
Constraints:
Non Negativity and Integer Constraints:
$X_1, X_2, X_3, X_4, X_5 >= 0$
X_1, X_2, X_3, X_4, X_5 are Integers
Binary Constraints:
All Y_i must be binary, where i = 1,2,3,4,5
Linking Constraints:
$X_1 <= M_1Y_1$ or $X_1 - M_1Y_1 = 0$

$X_2 <= M_2Y_2$ or $X_2 - M_2Y_2 = 0$
$X_3 <= M_3Y_3$ or $X_3 - M_3Y_3 = 0$
$X_4 <= M_4Y_4$ or $X_4 - M_4Y_4 = 0$
$X_5 <= M_5Y_5$ or $X_5 - M_5Y_5 = 0$
Here, M_i introduces an upper bound on X_i
Calculating M_i values:-
$M_1 = MIN(\frac{2487}{2}, \frac{3030}{6}, \frac{5217}{2}, \frac{4000}{7}, \frac{4999}{5}, \frac{2769}{10}) = 279.6$
$M_2 = MIN(\frac{2487}{10}, \frac{3030}{3}, \frac{5217}{3}, \frac{4000}{6}, \frac{4999}{6}, \frac{2769}{3}) = 248.7$
$M_3 = MIN(\frac{2487}{2}, \frac{3030}{6}, \frac{5217}{10}, \frac{4000}{5}, \frac{4999}{3}, \frac{2769}{5}) = 505$
$M_4 = MIN(\frac{2487}{3}, \frac{3030}{3}, \frac{5217}{6}, \frac{4000}{4}, \frac{4999}{10}, \frac{2769}{3}) = 499.9$
$M_5 = MIN(\frac{2487}{6}, \frac{3030}{10}, \frac{5217}{2}, \frac{4000}{3}, \frac{4999}{2}, \frac{2769}{4}) = 303$
Constraints as per Resource Availability:
$2X_1 + 10X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$
$6X_1 + 3X_2 + 6X_3 + 3X_4 + 10X_5 = 3030$
$2X_1 + 3X_2 + 10X_3 + 6X_4 + 2X_5 = 5217$
$2X_1 + 10 X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$
$2X_1 + 10 X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$
$2X_1 + 10 X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$

Working :
Here, we have an additional constraint.
It is given that the minimum amount of product 3 to be produced is 225.
And, the maximum amount of product 3 to be produced is 325.

In order to take into consideration these two conditions, we use the method of linking constraints and upper bounds.

Thus, I have mentioned the minimum and maximum values of X_3
Here, X_3 = Amount of Product 3 produced.
Constraint :
For X_3 :
Lower_Bound = 225
Upper_Bound = 325
Minimum Value Constraint : $X_3 - (225)(Y_3) >= 0$
Maximum Value Constraint : $X_3 - (325)(Y_3) <= 0$

Solver Observations :

From the solver, we can see that
From the solver, we can see that the :
Optimal Amount produced for Product 1 = 177
Optimal Amount produced for Product 2 = 116
Optimal Amount produced for Product 3 = 0
Optimal Amount produced for Product 4 = 0
Optimal Amount produced for Product 5 = 162

1o) Part ii

ANSWER:
Spreadsheet : See 1o
Snapshot :

	Product 1	Product 2	Product 3	Product 4	Product 5	
Units to mix (X)	**177**	**116**	**0**	**0**	**162**	
Binary Variables (Y)	1	1	0	0	1	
Linking Constraints	-99.9	-132.7	0	0	-141	
Set Up Cost	2000	4000	8000	16000	1000	**Total Profit**
Profit of the product	$510.00	$300.00	$510.00	$270.00	$810.00	**$249,290.00**

Maximize: H12
By changing: C8:G8, C9:G9
Subject to:
- H17:H22 <= I17:I22 : Resource Availability Constraint
- D26 >= 0 : Minimum value constraint
- D27 <= 0 : Maximum Value constraint
- C8:G9 >= 0 : Non-Negativity Constraint
- C8:G8 = integer : Integer Constraint
- C10:G10 <= 0 : Linking Constraint
- C9:G9 binary : Binary Constraint
- C9:G9 >=0 : Non - Negativity Constraint

	Amount of Resource in the products						
Resources	**Product 1**	**Product 2**	**Product 3**	**Product 4**	**Product 5**	**Amount of resource**	**Maximum Resource Available**
Resource 1	2	10	2	3	6	2486	2487
Resource 2	6	3	6	3	10	3030	3030
Resource 3	2	3	10	6	2	1026	5217
Resource 4	7	6	5	4	3	2421	4000
Resource 5	5	6	3	10	2	1905	4999
Resource 6	10	3	5	3	4	2766	2769

For X3 :		
Minimum Value	225	0
Maximum Value	325	0

Working:
Total Profit
= (Product 1 Amount)*(Product 1 Profit) +
(Product 2 Amount)*(Product 2 Profit) +
(Product 3 Amount)*(Product 3 Profit) +
(Product 4 Amount)*(Product 4 Profit) +
(Product 5 Amount)*(Product 5 Profit) +
= (177)*(510) + (116)*(300) + (0)*(510) + (0)*(270) + (162)*(810)
= $ 249,200

Again, we can see that adding more constraints such as the minimum and maximum values of a particular product has further decreased the value of the objective function, i.e the profit. These minimum and maximum values introduce bounds on the variables and affect their quantity produced.

Even from the Solver, we can see that the value of the objective function now is : $249,200

1p part 1:
ANSWER:
Spreadsheet : See 1p
Snapshot :

	Product 1	Product 2	Product 3	Product 4	Product 5	
Units to mix (X)	**177**	**116**	**0**	**0**	**162**	
Binary Variables (Y)	1	1	0	0	1	
Linking Constraints	-99.9	-132.7	0	0	-141	
Set Up Cost	2000	4000	8000	16000	1000	**Total Profit**
Profit of the product	$510.00	$300.00	$510.00	$270.00	$810.00	**$249,290.00**

Maximize: H12
By changing: C8:G8, C9:G9
Subject to:

H17:H22 <= I17:I22	: Resource Availability Constraint
D26 >= 0	: Minimum value constraint
D27 <= 0	: Maximum Value constraint
C8:G9 >= 0	: Non-Negativity Constraint
C8:G8 = integer	: Integer Constraint
C10:G10 <= 0	: Linking Constraint
C9:G9 binary	: Binary Constraint
C9:G9 >=0	: Non - Negativity Constraint
C29 = 0	: X3 Should be a multiple of 50

	Amount of Resource in the products						
Resources	**Product 1**	**Product 2**	**Product 3**	**Product 4**	**Product 5**	**Amount of resource**	**Maximum Resource Available**
Resource 1	2	10	2	3	6	2486	2487
Resource 2	6	3	6	3	10	3030	3030
Resource 3	2	3	10	6	2	1026	5217
Resource 4	7	6	5	4	3	2421	4000
Resource 5	5	6	3	10	2	1905	4999
Resource 6	10	3	5	3	4	2766	2769

For X3 :		
Minimum Value	300	0
Maximum Value	450	0

X3_Multiple_of_50	0

Objective Function:

Here, we need to Maximise our Profit based on the quantity of products used

$$\textbf{MAX : } 510X_1 + 300X_2 + 510X_3 + 270X_4 + 810X_5 - 2000Y_1 - 4000Y_2 - 8000\,Y_3 - 16000Y_4 - 1000Y_5$$

Constraints:

Non Negativity and Integer Constraints:

$X_1, X_2, X_3, X_4, X_5 >= 0$

X_1, X_2, X_3, X_4, X_5 are Integers

Binary Constraints:

All Y_i must be binary, where i = 1,2,3,4,5

Linking Constraints:

$X_1 <= M_1Y_1$ or $X_1 - M_1Y_1 = 0$

$X_2 <= M_2Y_2$ or $X_2 - M_2Y_2 = 0$

$X_3 <= M_3Y_3$ or $X_3 - M_3Y_3 = 0$

$X_4 <= M_4Y_4$ or $X_4 - M_4Y_4 = 0$

$X_5 <= M_5Y_5$ or $X_5 - M_5Y_5 = 0$

Here, M_i introduces an upper bound on X_i

Calculating M_i values:-

$M_1 = \text{MIN}(\frac{2487}{2}, \frac{3030}{6}, \frac{5217}{2}, \frac{4000}{7}, \frac{4999}{5}, \frac{2769}{10}) = 279.6$

$M_2 = \text{MIN}(\frac{2487}{10}, \frac{3030}{3}, \frac{5217}{3}, \frac{4000}{6}, \frac{4999}{6}, \frac{2769}{3}) = 248.7$

$M_3 = \text{MIN}(\frac{2487}{2}, \frac{3030}{6}, \frac{5217}{10}, \frac{4000}{5}, \frac{4999}{3}, \frac{2769}{5}) = 505$

$M_4 = \text{MIN}(\frac{2487}{3}, \frac{3030}{3}, \frac{5217}{6}, \frac{4000}{4}, \frac{4999}{10}, \frac{2769}{3}) = 499.9$

$M_5 = \text{MIN}(\frac{2487}{6}, \frac{3030}{10}, \frac{5217}{2}, \frac{4000}{3}, \frac{4999}{2}, \frac{2769}{4}) = 303$

Constraints as per Resource Availability:

$2X_1 + 10X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$

$6X_1 + 3X_2 + 6X_3 + 3X_4 + 10X_5 = 3030$

$2X_1 + 3X_2 + 10X_3 + 6X_4 + 2X_5 = 5217$

$2X_1 + 10\,X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$

$2X_1 + 10\,X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$

$2X_1 + 10\,X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$

Working :
Here, we have an additional constraint.
Here, the minimum value of Product 3 produced is 300 and
The maximum value of Product 3 produced is 450.

In order to take into consideration these two conditions, we use the method of linking constraints and upper bounds.

Thus, I have mentioned the minimum and maximum values of X_3
Here, X_3 = Amount of Product 3 produced.
Additional Constraint :
For X_3 :
Lower_Bound = 300
Upper_Bound = 450
Minimum Value Constraint : X_3 - (300)(Y_3) >= 0
Maximum Value Constraint : X_3 - (450)(Y_3) <= 0
Also,
We also want the amount of product 3 produced to be a multiple of 50 as well.
Thus, this additional constraint has been mentioned as a separate row.
MOD(X_3, 50) = 0
Where, MOD gives us the remainder of X_3 and 50.

From the solver, we can see that
From the solver, we can see that the :
Optimal Amount produced for Product 1 = 177
Optimal Amount produced for Product 2 = 116
Optimal Amount produced for Product 3 = 0
Optimal Amount produced for Product 4 = 0
Optimal Amount produced for Product 5 = 162

1p part 2:
ANSWER:
Spreadsheet : See 1p
Snapshot :

	Product 1	Product 2	Product 3	Product 4	Product 5	
Units to mix (X)	177	116	0	0	162	
Binary Variables (Y)	1	1	0	0	1	
Linking Constraints	-99.9	-132.7	0	0	-141	
Set Up Cost	2000	4000	8000	16000	1000	Total Profit
Profit of the product	$510.00	$300.00	$510.00	$270.00	$810.00	$249,290.00

Maximize: H12
By changing: C8:G8, C9:G9
Subject to:
H17:H22 <= I17:I22 : Resource Availability Constraint
D26 >= 0 : Minimum value constraint
D27 <= 0 : Maximum Value constraint
C8:G9 >= 0 : Non-Negativity Constraint
C8:G8 = integer : Integer Constraint
C10:G10 <= 0 : Linking Constraint
C9:G9 binary : Binary Constraint
C9:G9 >=0 : Non - Negativity Constraint
C29 = 0 : X3 Should be a multiple of 50

	Amount of Resource in the products						
Resources	Product 1	Product 2	Product 3	Product 4	Product 5	Amount of resource	Maximum Resource Available
Resource 1	2	10	2	3	6	2486	2487
Resource 2	6	3	6	3	10	3030	3030
Resource 3	2	3	10	6	2	1026	5217
Resource 4	7	6	5	4	3	2421	4000
Resource 5	5	6	3	10	2	1905	4999
Resource 6	10	3	5	3	4	2766	2769

For X3 :		
Minimum Value	300	0
Maximum Value	450	0

X3_Multiple_of_50	0

Working:
Total Profit
= (Product 1 Amount)*(Product 1 Profit) +
(Product 2 Amount)*(Product 2 Profit) +
(Product 3 Amount)*(Product 3 Profit) +
(Product 4 Amount)*(Product 4 Profit) +
(Product 5 Amount)*(Product 5 Profit) +
= (177)*(510) + (116)*(300) + (0)*(510) + (0)*(270) + (162)*(810)
= $ 249,200

Clearly, adding more and more constraints decreases the feasible region of the solution and this further decreased the value of the objective function.
Even from the Solver, we can see that the value of the objective function now is : $249,200

1q) part 1
ANSWER:
Spreadsheet : See 1q
Snapshot :

Objective Function:
Here, we need to Maximise our Profit based on the quantity of products used
MAX : $510X_1 + 300X_2 + 510X_3 + 270X_4 + 810X_5 -$
$2000Y_1 - 4000Y_2 - 8000\ Y_3 - 16000Y_4 - 1000Y_5$
Constraints:
Non Negativity and Integer Constraints:
$X_1, X_2, X_3, X_4, X_5 >= 0$
X_1, X_2, X_3, X_4, X_5 are Integers
Binary Constraints:
All Y_i must be binary, where i = 1,2,3,4,5
Linking Constraints: (For only 1, 3 and 5)
$X_1 <= M_1Y_1$ or $X_1 - M_1Y_1 = 0$

$X_3 <= M_3Y_3$ or $X_3 - M_3Y_3 = 0$
$X_5 <= M_5Y_5$ or $X_5 - M_5Y_5 = 0$
Here, M_i introduces an upper bound on X_i

Calculating M_i values:-

$M_1 = MIN(\frac{2487}{2}, \frac{3030}{6}, \frac{5217}{2}, \frac{4000}{7}, \frac{4999}{5}, \frac{2769}{10}) = 279.6$

$M_3 = MIN(\frac{2487}{2}, \frac{3030}{6}, \frac{5217}{10}, \frac{4000}{5}, \frac{4999}{3}, \frac{2769}{5}) = 505$

$M_5 = MIN(\frac{2487}{6}, \frac{3030}{10}, \frac{5217}{2}, \frac{4000}{3}, \frac{4999}{2}, \frac{2769}{4}) = 303$

Constraints as per Resource Availability:

$2X_1 + 10X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$
$6X_1 + 3X_2 + 6X_3 + 3X_4 + 10X_5 = 3030$
$2X_1 + 3X_2 + 10X_3 + 6X_4 + 2X_5 = 5217$
$2X_1 + 10 X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$
$2X_1 + 10 X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$
$2X_1 + 10 X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$

Additional Constraints:

If X_2 is produced, it should be one of these quantities : [102, 103, 105, 107, 111]
If X_4 is produced, it should be one of these quantities : [320, 330, 350, 370, 410]
Thus,
Here, we will be changing only Y_1 , Y_3 and Y_5
The remaining Y_2 and Y_4 will be subjected to the above constraints.

Here, I have assigned a binary value to each of the quantities of X_2 and X_4 produced.
Since my solver took a lot of time to run (more than 45 minutes), I paused it and saved the following scenario:

	Product 1	Product 2	Product 3	Product 4	Product 5
Units to mix (X)	37.7	111	411.8	0	0
Binary Variables (Y)	0.13615	1	0.815446	0	0

Products (1, 3 and 5)	M Value	Linking Constraint
Product 1	276.9	1.14E-13
Product 3	505	0
Product 5	303	0

Product 2 : Quantities	102	103	105	107	111
Binary Variable	0	0	0	0	1
Total Binary (<=1)	1				
Units to Mix (X2)	111				

Product 4 : Quantities	320	330	350	370	410
Binary Variable	0	0	0	0	0
Total Binary (<=1)	0				
Units to Mix (X4)	0				

						Total Profit
Set Up Cost	2000	4000	8000	16000	1000	
Profit of the product	$510.00	$300.00	$510.00	$270.00	$810.00	251749.14

Maximize H23
By changing: C4:G4, C5:G5
Subject to:
014:019 <= P14:P19: Resource Availability Constraint
C4:G4 >= 0 : Non-Negativity Constraint
C4:G4 = integer : Integer Constraint
D8:D10 <= 0 : Linking Constraint for Product 1,3 and 5
C5,E5, G5 = binary : Binary Constraint
C14 <=1 : Product 2 Constraint
C19 <=1 : Product 4 Constraint
C14 = D5 : Amount of 2 Produced
C19 = F5 : Amount of 4 Produced

	Amount of Resource in the products						
Resources	Product 1	Product 2	Product 3	Product 4	Product 5	Amount of resource	Maximum Resource Available
Resource 1	2	10	2	3	6	2009	2487
Resource 2	6	3	6	3	10	3030	3030
Resource 3	2	3	10	6	2	4526.4	5217
Resource 4	7	6	5	4	3	2988.9	4000
Resource 5	5	6	3	10	2	2089.9	4999
Resource 6	10	3	5	3	4	2769	2769

From the solver, we can see that

From the solver, we can see that the :
Optimal Amount produced for Product 1 = 37.7
Optimal Amount produced for Product 2 = 111
Optimal Amount produced for Product 3 = 411.8
Optimal Amount produced for Product 4 = 0
Optimal Amount produced for Product 5 = 0

1q) part 2

ANSWER:
Spreadsheet : See 1q
Snapshot :

	Product 1	Product 2	Product 3	Product 4	Product 5
Units to mix (X)	37.7	111	411.8	0	0
Binary Variables (Y)	0.13615	1	0.815446	0	0

Products (1, 3 and 5)	M Value	Linking Constraint
Product 1	276.9	1.14E-13
Product 3	505	0
Product 5	303	0

Product 2 : Quantities	102	103	105	107	111
Binary Variable	0	0	0	0	1
Total Binary (<=1)	1				
Units to Mix (X2)	111				

Product 4 : Quantities	320	330	350	370	410
Binary Variable	0	0	0	0	0
Total Binary (<=1)	0				
Units to Mix (X4)	0				

Set Up Cost	2000	4000	8000	16000	1000	Total Profit
Profit of the product	$510.00	$300.00	$510.00	$270.00	$810.00	251749.14

Maximize H23
By changing: C4:G4, C5:G5
Subject to:
O14:O19 <= P14:P19: Resource Availability Constraint
C4:G4 >= 0 : Non-Negativity Constraint
C4:G4 = integer : Integer Constraint
D8:D10 <= 0 : Linking Constraint for Product 1,3 and 5
C5,E5, G5 = binary : Binary Constraint
C14 <=1 : Product 2 Constraint
C19 <=1 : Product 4 Constraint
C14 = D5 : Amount of 2 Produced
C19 = F5 : Amount of 4 Produced

	Amount of Resource in the products						
Resources	Product 1	Product 2	Product 3	Product 4	Product 5	Amount of resource	Maximum Resource Available
Resource 1	2	10	2	3	6	2009	2487
Resource 2	6	3	6	3	10	3030	3030
Resource 3	2	3	10	6	2	4526.4	5217
Resource 4	7	6	5	4	3	2988.9	4000
Resource 5	5	6	3	10	2	2089.9	4999
Resource 6	10	3	5	3	4	2769	2769

Working:
Total Profit
= (Product 1 Amount)*(Product 1 Profit) +
(Product 2 Amount)*(Product 2 Profit) +
(Product 3 Amount)*(Product 3 Profit) +
(Product 4 Amount)*(Product 4 Profit) +
(Product 5 Amount)*(Product 5 Profit) +
= (37.7)*(510) + (111)*(300) + (411.8)*(510) + (0)*(270) + (0)*(810)
= $ 251, 749

Clearly, adding more and more constraints decreases the feasible region of the solution and this further decreased the value of the objective function.
Even from the Solver, we can see that the value of the objective function now is : $251, 749

1r) part 1
ANSWER:
Spreadsheet : See 1r
Snapshot :

	Product 1	Product 2	Product 3	Product 4	Product 5
Units to mix (X)	0	0	479	12	12
Binary Variables (Y)	0	0	1	1	1

Product	M Value	Linking Constraint	
Product 1	276.9	0	(<=0)
Product 2	248.7	0	(<=0)
Product 3	505	-26	(<=0)

X1 = X2			
Y1 = Y2			
X4 = X5			
Y4 = Y5			
Y1 + Y3 <= 1	1	(<=1)	
Y2 + Y3 <= 1	0	(<=1)	
Y3 = Y5			
X5 : Lower Bound	10	2	(>=0)
X5 : Upper Bound	100	-88	(<=0)

Maximise: O23
By changing: C4:G5
Subject to:
O14:O19 <= P14:P19 : Resource Availability Constraint
D22 >= 0 : Minimum value constraint
D23 <= 0 : Maximum Value constraint
C4:G5 >= 0 : Non-Negativity Constraint
C4:G4 = integer : Integer Constraint
D8:D12 <= 0 : Linking Constraint
C5:G5 binary : Binary Constraint
C5:G5 >=0 : Non - Negativity Constraint
C4 = D4 : X1 Amount = X2 Amount
C5 = D5 : Y1 = Y2
F4 = G4 : X4 Amount = X5 Amount
F5 = G5 : Y4 = Y5
E5 = G5 : Y3 = Y5
C19 <=1 : If Product 3 is produced, Product 1 is not produced
C20 <=1 : If Product 3 is produced, Product 2 is not produced

	Amount of Resource in the products						
Resources	Product 1	Product 2	Product 3	Product 4	Product 5	Amount of resource	Maximum Resource Available
Resource 1	2	10	2	3	6	1066	2487
Resource 2	6	3	6	3	10	3030	3030
Resource 3	2	3	10	6	2	4886	5217
Resource 4	7	6	5	4	3	2479	4000
Resource 5	5	6	3	10	2	1581	4999
Resource 6	10	3	5	3	4	2479	2769

Set Up Cost	2000	4000	8000	16000	1000	Total Profit
Product Profit	$510.00	$300.00	$510.00	$270.00	$810.00	$232,250.00

From the solver, we can see that

From the solver, we can see that the :

Optimal Amount produced for Product 1 = 0

Optimal Amount produced for Product 2 = 0

Optimal Amount produced for Product 3 = 479

Optimal Amount produced for Product 4 = 12

Optimal Amount produced for Product 5 = 12

1r) part 2

ANSWER:

Spreadsheet : See 1r

Snapshot :

	Product 1	Product 2	Product 3	Product 4	Product 5
Units to mix (X)	0	0	479	12	12
Binary Variables (Y)	0	0	1	1	1

Product	M Value	Linking Constraint	
Product 1	276.9	0	(<=0)
Product 2	248.7	0	(<=0)
Product 3	505	-26	(<=0)

X1 = X2			
Y1 = Y2			
X4 = X5			
Y4 = Y5			
Y1 + Y3 <= 1	1	(<=1)	
Y2 + Y3 <= 1	0	(<=1)	
Y3 = Y5			
X5 : Lower Bound	10	2	(>=0)
X5 : Upper Bound	100	-88	(<=0)

Maximise: O23
By changing: C4:G5
Subject to:
O14:O19 <= P14:P19 : Resource Availability Constraint
D22 >= 0 : Minimum value constraint
D23 <= 0 : Maximum Value constraint
C4:G5 >= 0 : Non-Negativity Constraint
C4:G4 = integer : Integer Constraint
D8:D12 <= 0 : Linking Constraint
C5:G5 binary : Binary Constraint
C5:G5 >=0 : Non - Negativity Constraint
C4 = D4 : X1 Amount = X2 Amount
C5 = D5 : Y1 = Y2
F4 = G4 : X4 Amount = X5 Amount
F5 = G5 : Y4 = Y5
E5 = G5 : Y3 = Y5
C19 <=1 : If Product 3 is produced, Product 1 is not produced
C20 <=1 : If Product 3 is produced, Product 2 is not produced

	Amount of Resource in the products						
Resources	Product 1	Product 2	Product 3	Product 4	Product 5	Amount of resource	Maximum Resource Available
Resource 1	2	10	2	3	6	1066	2487
Resource 2	6	3	6	3	10	3030	3030
Resource 3	2	3	10	6	2	4886	5217
Resource 4	7	6	5	4	3	2479	4000
Resource 5	5	6	3	10	2	1581	4999
Resource 6	10	3	5	3	4	2479	2769

Set Up Cost	2000	4000	8000	16000	1000	Total Profit
Product Profit	$510.00	$300.00	$510.00	$270.00	$810.00	$232,250.00

Method:

Non Negativity and Integer Constraints:

$X_1, X_2, X_3, X_4, X_5 >= 0$

X_1, X_2, X_3, X_4, X_5 are Integers

Binary Constraints:

All Y_i must be binary, where i = 1,2,3,4,5

Linking Constraints: (Only for Product 1, Product 2 and Product 3)

$X_1 <= M_1Y_1$ or $X_1 - M_1Y_1 = 0$

$X_2 <= M_2Y_2$ or $X_2 - M_2Y_2 = 0$

$X_3 <= M_3Y_3$ or $X_3 - M_3Y_3 = 0$

Here, M_i introduces an upper bound on X_i

Calculating M_i values:-

$M_1 = \text{MIN}(\frac{2487}{2}, \frac{3030}{6}, \frac{5217}{2}, \frac{4000}{7}, \frac{4999}{5}, \frac{2769}{10}) = 279.6$

$M_2 = \text{MIN}(\frac{2487}{10}, \frac{3030}{3}, \frac{5217}{3}, \frac{4000}{6}, \frac{4999}{6}, \frac{2769}{3}) = 248.7$

$M_3 = \text{MIN}(\frac{2487}{2}, \frac{3030}{6}, \frac{5217}{10}, \frac{4000}{5}, \frac{4999}{3}, \frac{2769}{5}) = 505$

Constraints as per Resource Availability:

$2X_1 + 10X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$

$6X_1 + 3X_2 + 6X_3 + 3X_4 + 10X_5 = 3030$

$2X_1 + 3X_2 + 10X_3 + 6X_4 + 2X_5 = 5217$
$2X_1 + 10\ X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$
$2X_1 + 10\ X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$
$2X_1 + 10\ X_2 + 2X_3 + 3X_4 + 6X_5 = 2487$

Additional Constraints :
Along with Non-Negativity, Integer, Binary, Resource and Linking constraints, we have the following constraints as well.

Here, in the question we have the following additional constraints.
Amount of Product 1 = Amount of Product 2
$X_1 = X_2$ Implies $Y_1 = Y_2$
Amount of Product 4 = Amount of Product 5
$X_4 = X_5$ Implies $Y_4 = Y_5$

If Product 3 is produced, Product 1 and Product 2 are not produced.
$Y_3 + Y_1 <= 1$
$Y_3 + Y_2 <= 1$

Also, If Product 3 is produced,
Product 5 is also produced with :
Lower Bound = 10
Upper Bound = 100
For X_5 :
Lower_Bound = 10
Upper_Bound = 100
Minimum Value Constraint : $X_3 - (10)(Y_3) >= 0$
Maximum Value Constraint : $X_3 - (100)(Y_3) <= 0$

Working:
Total Profit
= (Product 1 Amount)*(Product 1 Profit) +
(Product 2 Amount)*(Product 2 Profit) +
(Product 3 Amount)*(Product 3 Profit) +
(Product 4 Amount)*(Product 4 Profit) +
(Product 5 Amount)*(Product 5 Profit) +
= (0)*(510) + (0)*(300) + (0)*(479) + (12)*(270) + (12)*(810)
= $ 232, 250

<u>Question 2 – Transshipment and networks</u>

2a)
ANSWER:

Here :
Every Node represents a location
Supply Nodes: 1, 2
Demand Nodes: 7, 8
Transshipment Nodes: 3, 4, 5, 6

Variables:
X_{ij}, X_{jk}, X_{kl}
X_{ij} : Amount shipped from location i to location j
X_{jk} : Amount shipped from location j to location k
X_{kl} : Amount shipped from location k to location l

Thus, our variables are :
X_{13}, X_{14},
X_{23}, X_{24},
X_{35}, X_{36},
X_{45}, X_{46},
X_{57}, X_{58},
X_{67}, X_{68}

Objective Function:
Here, we want to minimize the overall shipping (transshipment) cost
The total shipping cost is the sum of shipping cost of product through every node
Hence, the function is:

Minimize(
$50X_{13}$ + 80 X_{14},
$70X_{23}$ + $40X_{24}$,
$70X_{35}$ + 50 X_{36},
$40X_{45}$ + $80X_{46}$,
$80X_{57}$ + 40 X_{58},
$60X_{67}$ + $70X_{68}$)

Where the :
Unit cost to ship the product from Node 1 to Node 3 : 50
Unit cost to ship the product from Node 1 to Node 4 : 80
Unit cost to ship the product from Node 2 to Node 3 : 70
Unit cost to ship the product from Node 2 to Node 4 : 40
Unit cost to ship the product from Node 3 to Node 5 : 70
Unit cost to ship the product from Node 3 to Node 6 : 50
Unit cost to ship the product from Node 4 to Node 5 : 40
Unit cost to ship the product from Node 4 to Node 6 : 80
Unit cost to ship the product from Node 5 to Node 7 : 80
Unit cost to ship the product from Node 5 to Node 8 : 40

Unit cost to ship the product from Node 6 to Node 7 : 60
Unit cost to ship the product from Node 6 to Node 8 : 70

2b)
ANSWER:

Number of variables: 12
The variables correspond to the amount of shipment from a given node to another.
These are the following variables:

X_{ij}, X_{jk}, X_{kl}
X_{ij} : Amount shipped from location i to location j
X_{jk} : Amount shipped from location j to location k
X_{kl} : Amount shipped from location k to location l
Thus, our variables are :
X_{13}, X_{14},
X_{23}, X_{24},
X_{35}, X_{36},
X_{45}, X_{46},
X_{57}, X_{58},
X_{67}, X_{68}

2c) ANSWER:

Spreadsheet: See 2c
Snapshot :

Ship	From	To	Unit Cost
75	1	3	$50.00
0	1	4	$80.00
0	2	3	$70.00
75	2	4	$40.00
0	3	5	$70.00
75	3	6	$50.00
75	4	5	$40.00
0	4	6	$80.00
5	5	7	$80.00
70	5	8	$40.00
75	6	7	$60.00
0	6	8	$70.00
Total Transportation Cost			$21,200.00

Nodes	Net Flow	Supply/Demand
1	-75	-75
2	-75	-75
3	0	0
4	0	0
5	0	0
6	0	0
7	80	80
8	70	70

Since Supply = Demand,
Inflow - Outflow = Supply or Demand

Minimize: E18
By changing: B6:B17
Subject to:
H6:H13=I6:I13 : Balancing the Flow Constraints
B6:B17>=0 : Non-Negativity Constraint

Formulation of LP:
From the question, every Node represents a location.
The Objective Function would be subjected to the following constraints:

Constraints :

Amount out of Node 1 : $X_{13} + X_{14} = 75$
Amount out of Node 2 : $X_{23} + X_{24} = 75$
Amount through Node 3 : $(X_{13} + X_{23}) - (X_{35} + X_{36}) = 0$
Amount through Node 4 : $(X_{14} + X_{24}) - (X_{45} + X_{46}) = 0$
Amount through Node 5 : $(X_{35} + X_{45}) - (X_{57} + X_{58}) = 0$
Amount through Node 6 : $(X_{36} + X_{46}) - (X_{67} + X_{68}) = 0$
Amount into Node 7 : $X_{57} + X_{67} = 80$
Amount into Node 8 : $X_{67} + X_{68} = 70$

Non-Negativity of the Variables: $X_{ij} >= 0$ for all i and j

From the spreadsheet,
We can see that clearly, the
Total Supply = 75 + 75 = 150
Total Demand = 80 + 70 = 150
Thus, Supply = Demand
Hence, the at every node : Inflow - Outflow = Supply or Demand

The solution is :
$X_{13} = 75$
$X_{24} = 75$
$X_{36} = 75$
$X_{45} = 75$
$X_{57} = 5$
$X_{58} = 70$
$X_{67} = 75$

The above are the edges with a non-zero flow.
Number = $e_{2c} = 7$

Here, the minimum cost = $21,200
Hence, the value of the Objective Function = $21,200

2d) ANSWER:

Spreadsheet: See 2d
Snapshot:

From	To	Unit Cost	Upper Bound	Ship	e2c	Lnking Constraint
1	3	$50.00	150	0	0	0
1	4	$80.00	150	75	1	-75
2	3	$70.00	150	0	0	0
2	4	$40.00	150	75	1	-75
3	5	$70.00	150	0	0	0
3	6	$50.00	150	0	0	0
4	5	$40.00	150	150	1	0
4	6	$80.00	150	0	0	0
5	7	$80.00	150	80	1	-70
5	8	$40.00	150	70	1	-80
6	7	$60.00	150	0	0	0
6	8	$70.00	150	0	0	0
Total Transportation		$24,200.00				

Nodes	Net Flow	Supply/Demand
1	-75	-75
2	-75	-75
3	0	0
4	0	0
5	0	0
6	0	0
7	80	80
8	70	70

Since Supply = Demand,
Inflow - Outflow = Supply or Demand

Total Used	5
e2c	7
e2c - 1	5

Here, Upper Bound = 150
Minimize: E18
By changing: G6:H17
Subject to:
L6:L13 = M6:M13 : In order to balance the flow
H6:H17 = binary : Binary Constraints
G6:G17 = integer : Integer Constraints
G6:G17 >= 0 : Integer Constraints
G23 <=6 : Nodes with Non-Zero Flow <= e2c-1 = 6
I6:I17 <= 0 : Linking Constraint

Here, the number of edges with non-zero flow = e_{2c} = 7
Implies, e_{2c} - 1 = 6

Here, the Upper Bound is taken as 150
Now, from the solver it is clear that the number of edges with a non-zero flow is = 5
Amount of flow along each of the edges giving the minimum cost:

Value of the objective function = $24,200
Edges with a non-zero flow =
Node 1- Node 4
Node 2- Node 4
Node 4- Node 5
Node 5- Node 7
Node 5- Node 8

2e)
ANSWER:
Spreadsheet: See 2e
Snapshot:

From	To	Unit Cost	Upper Bound	Ship	e2c	Lnking Constraint	Cost of using Edge
1	3	$50.00	75	0	0	0	$100,000.00
1	4	$80.00	75	75	1	-75	$100,000.00
2	3	$70.00	75	0	0	0	$100,000.00
2	4	$40.00	75	75	1	-75	$100,000.00
3	5	$70.00	150	0	0	0	$100,000.00
3	6	$50.00	150	0	0	0	$100,000.00
4	5	$40.00	150	150	1	0	$100,000.00
4	6	$80.00	150	0	0	0	$100,000.00
5	7	$80.00	80	80	1	-70	$100,000.00
5	8	$40.00	70	70	1	-80	$100,000.00
6	7	$60.00	80	0	0	0	$100,000.00
6	8	$70.00	70	0	0	0	$100,000.00
Total Transportation Cost		$24,200.00					
Total Cost Using Edges		500000					
Overall Total Cost		$524,200.00					

Nodes	Net Flow	Supply/Demand
1	-75	-75
2	-75	-75
3	0	0
4	0	0
5	0	0
6	0	0
7	80	80
8	70	70

Since Supply = Demand,
Inflow - Outflow = Supply or Demand

Total Used	5
e2c	7
e2c - 1	5

Minimize: E18
By changing: G6:H17
Subject to:
L6:L13 = M6:M13 : To balance the flow
H6:H17 = binary : Binary Constraints
G6:G17 = integer : Integer Constraints
G6:G17 >= 0 : Integer Constraints
C23 <=6 : Nodes with Non-Zero Flow <= e2c-1 = 6
I6:I17 <= 0 : Linking Constraints

Here, we have introduced a very large penalty for every edge with a non-zero flow.
The penalty value for every edge = $100,000

From the solver, we can see that the total number of edges used = 5
Hence, the total cost along each edge = 5*100,000 = $500,000
Total Transportation Cost = $24,200

Thus, overall total cost :
= Transportation Cost + Cost of transporting along every edge
= $524,200

Thus, the smallest number of edges that can have a non-zero flow for such an above solution - 5

2f)
ANSWER :
Spreadsheet: See 2f
Snapshot :

Ship	From	To	Unit Cost	Upper Bound
75	1	3	$50.00	75
0	1	4	$80.00	75
0	2	3	$70.00	75
75	2	4	$40.00	75
0	3	5	$70.00	150
75	3	6	$50.00	150
30	4	5	$40.00	30
40	4	9	$0.00	120
40	9	5	$60.00	120
5	4	6	$80.00	150
0	5	7	$80.00	80
70	5	8	$40.00	70
80	6	7	$60.00	80
0	6	8	$70.00	70
Total Transportation Cost			$22,100.00	

Nodes	Net Flow	Supply/Demand
1	-75	-75
2	-75	-75
3	0	0
4	0	0
5	0	0
6	0	0
7	80	80
8	70	70
9	0	0

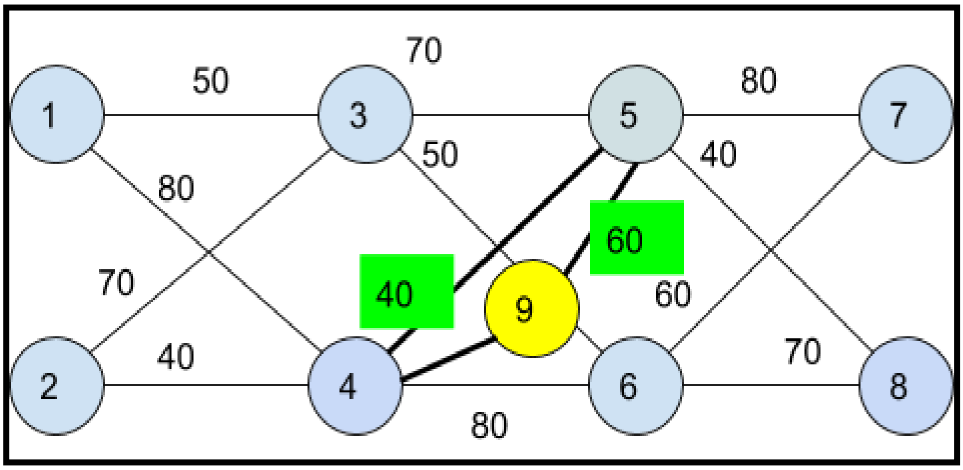
Since Supply = Demand,
Inflow - Outflow = Supply or Demand

Minimize: E20
By changing: B6:B19
Subject to:
I6:H13 = J6:J13 : In order to have balanced
B6:B19>=0 : Non-Negativity Constraints
B6:B19 : Integer Constraints
B6:B19 <=F6:F19 : To Manage the Upper Bounds

Here, we use an intermediate node, (let's name it Node 9) from Node 4 to Node 5
Since, it is given that the Unit Cost of maintaining this edge is:
Upto 30 units i.e **Upper Bound = 30**
Cost = $40/unit

Upto (150 - 30 = 120) i.e **Upper Bound = 120**
Cost = $60/unit

Thus, our new model looks like the one below:



Now, from the solver, we can see that, we have added the above constraints in the excel.

Thus, the new solution to this problem is given as follows:

We can see that clearly, the
Total Supply = 75 + 75 = 150
Total Demand = 80 + 70 = 150
Thus, Supply = Demand
Hence, the at every node : Inflow - Outflow = Supply or Demand

The solution is :
$X_{13} = 75$
$X_{24} = 75$
$X_{36} = 75$
$X_{45} = 30$
$X_{49} = 30$
$X_{95} = 40$
$X_{46} = 5$
$X_{57} = 5$
$X_{58} = 70$
$X_{67} = 80$
Total Minimum Transportation Cost = $22,100

2g)
ANSWER:
Spreadsheet: 2g
Snapshot:

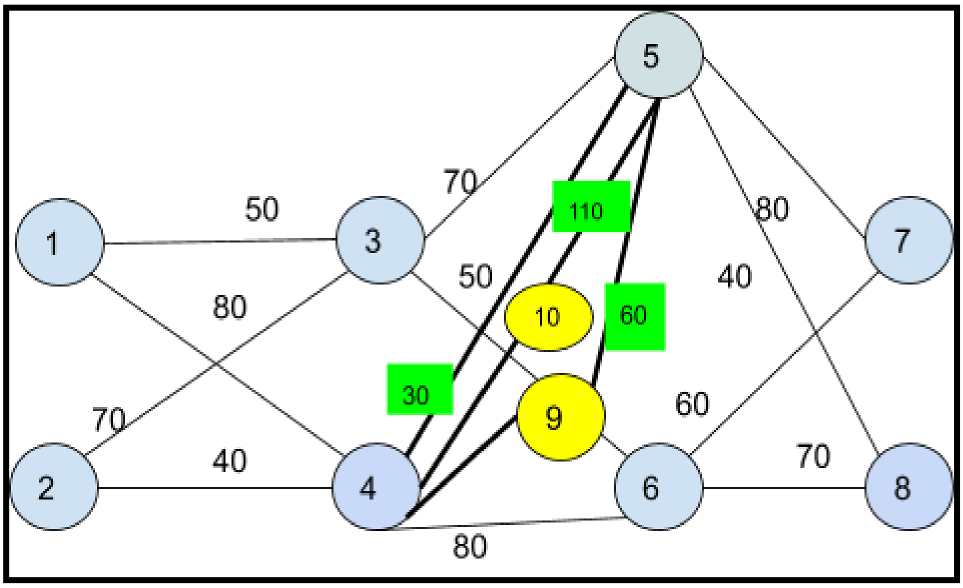
Ship	From	To	Unit Cost	Upper Bound
75	1	3	$50.00	75
0	1	4	$80.00	75
0	2	3	$70.00	75
75	2	4	$40.00	75
15	3	5	$70.00	150
60	3	6	$50.00	150
30	4	5	$40.00	30
25	4	9	$0.00	25
0	4	10	$0.00	95
25	9	5	$60.00	25
0	10	5	$110.00	95
20	4	6	$80.00	150
0	5	7	$80.00	80
70	5	8	$40.00	70
80	6	7	$60.00	80
0	6	8	$70.00	70
Total Transportation Cost			$22,700.00	

Nodes	Net Flow	Supply/Demand
1	-75	-75
2	-75	-75
3	0	0
4	0	0
5	0	0
6	0	0
7	80	80
8	70	70
9	0	0
10	0	0

Since Supply = Demand,
Inflow - Outflow = Supply or Demand

Minimize: E22
By changing: B6:B21
Subject to:
I6:H15 = J6:J15 : In order to have a balanced flow
B6:B21>=0 : Non-Negativity Constraints
B6:B21 = integer : Integer Constraints
B6:B21 <=F6:F21 : To Manage the Upper Bounds

Here, we use two intermediate nodes, (let's name it Node 9 and Node 10) from Node 4 to Node 5
Since, it is given that the Unit Cost of maintaining this edge is:
Upto 30 units **:** Cost = $40/unit
Upto 55 units **:** Cost = $60/unit
Further : Cost = $110/unit
Thus, our new model looks like the one below:



2h)

ANSWER:

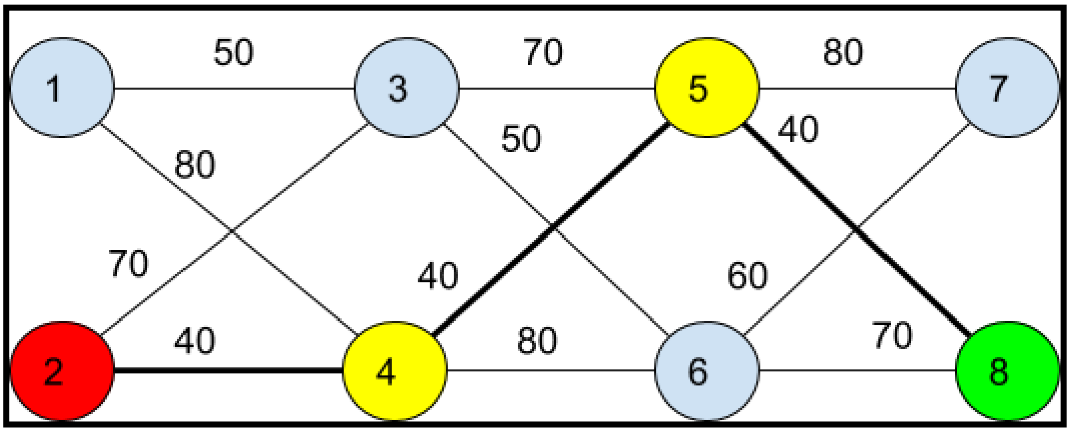
Spreadsheet: See 2h

Snapshot :

Select Route?	From	To	Distance
0	1	3	50
0	1	4	80
0	2	3	70
1	2	4	40
0	3	5	70
0	3	6	50
1	4	5	40
0	4	6	80
0	5	7	80
1	5	8	40
0	6	7	60
0	6	8	70
		Total	**120**

Nodes	Net Flow	Supply/Demand
1	0	0
2	-1	-1
3	0	0
4	0	0
5	0	0
6	0	0
7	0	0
8	1	1

Minimize: E19
By changing: B6:B17
Subject to: H6:H13=I6:I13
B6:B17 >= 0



Here, from the figure and solver we can see that

The shortest path from node 2 to node 8 is : 2 --- 4 --- 5 --- 8

Length of the path : 40 + 40 + 40 = 120

2i part 1)

ANSWER:

Spreadsheet : See 2i part 1

Snapshot:

Select Route?	From	To	Distance
0	1	3	50
0	1	4	80
0	2	3	70
1	2	4	40
0	3	5	70
0	3	6	50
1	4	5	40
0	4	6	80
0	5	7	80
1	5	8	40
0	6	7	60
0	6	8	70
		Total	120

Nodes	Net Flow	Supply/Demand
1	0	0
2	-1	-1
3	0	0
4	0	0
5	0	0
6	0	0
7	0	0
8	1	1

Minimize: E19
By changing: B6:B17
Subject to:
H6:H13=I6:I13 : Net Flow = Supply/Demand
B6:B17 >= 0 : Non-Negativity Constraint
B12 = 1 (Passing through Node 5)

We need to go through Node 5 in order to traverse from Node 2 to Node 8

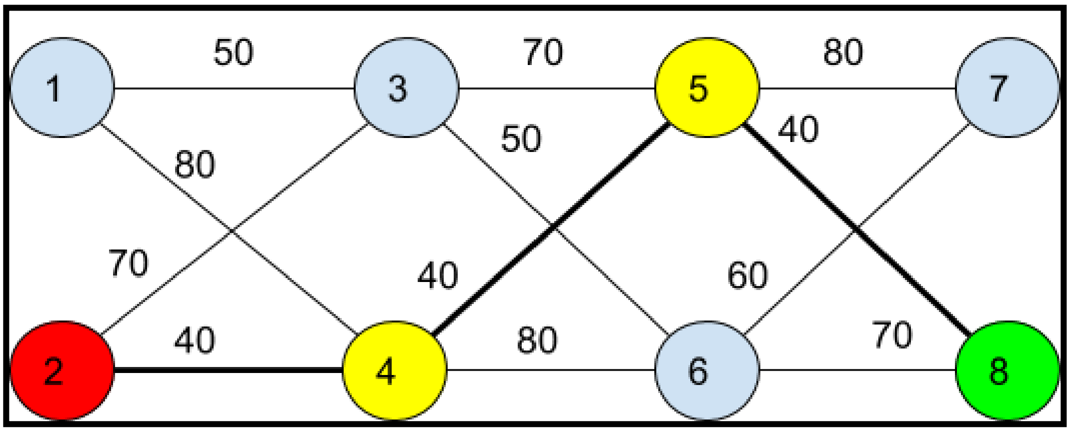
Here, the path to go from Node 2 to Node 8 would remain the same as the above shortest path obtained from 2h

Solver:

Here, in the solver we need to add one additional constraint.

If it's necessary for the path to go through Node 5, then we set the corresponding path Binary Variable to 1.

Even from the solver, we get the same answer as that of 2h.



Hence,
Shortest Path : 2 --- 4 --- 5 --- 8
Length of the path : 40 + 40 + 40 = 120

2i part 2)
ANSWER:

Spreadsheet : See 2i part ii

Snapshot :

Select Route?	From	To	Distance
0	1	3	50
0	1	4	80
0	2	3	70
1	2	4	40
0	3	5	70
0	3	6	50
0	4	5	40
1	4	6	80
0	5	7	80
0	5	8	40
0	6	7	60
1	6	8	70

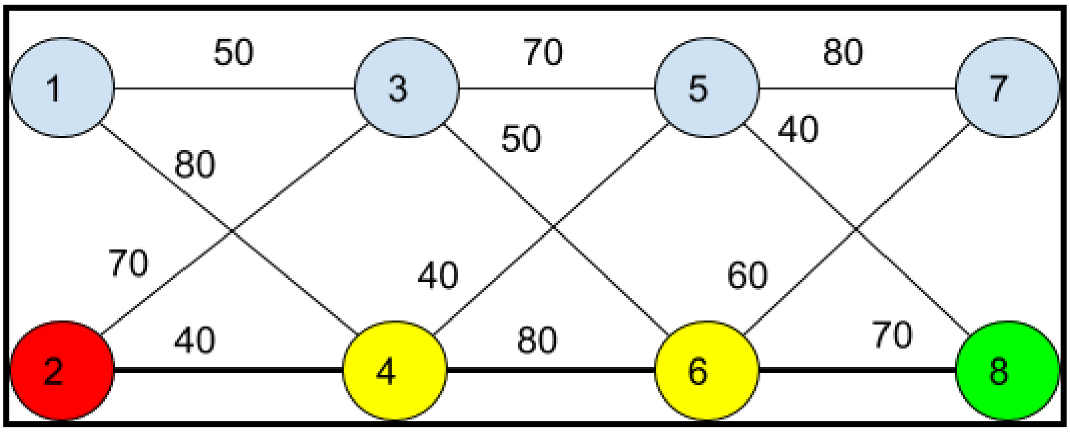
	Total	190

Nodes	Net Flow	Supply/Demand
1	0	0
2	-1	-1
3	0	0
4	0	0
5	0	0
6	0	0
7	0	0
8	1	1

Minimize: E19
By changing: B6:B17
Subject to:
H6:H13=I6:I13 : Total Supply = Supply/Demand
B6:B17 >= 0 : Non - Negativity Constraint
B13 = 1 (Passing through Node 6)

Now, we need to go through Node 6.
Clearly, it will not be one of the shortest paths obtained.

Solver:
Here, in the solver we need to add one additional constraint.
If it's necessary for the path to go through Node 5, then we set the corresponding path Binary Variable to 1.



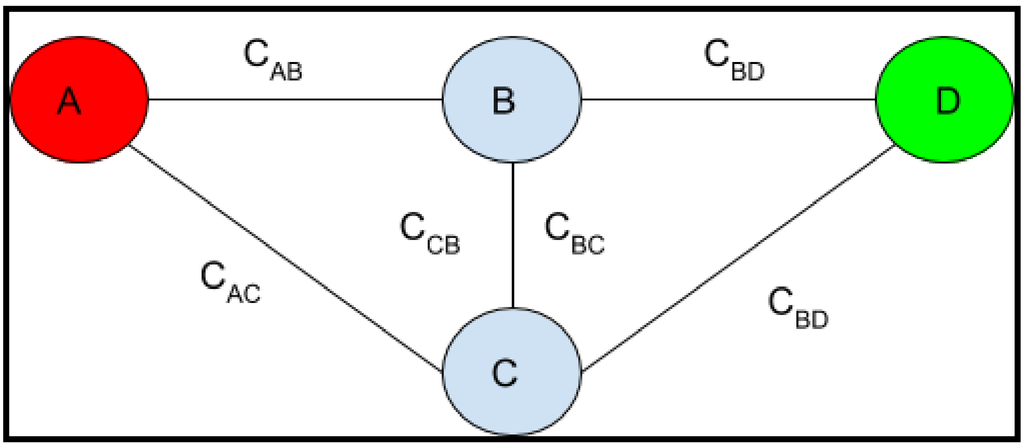
Hence,
Now, Path : 2 --- 4 --- 6 --- 8
Length of the path : 40 + 80 + 70 = 190

2j)
ANSWER:

Given:
Start Node : A
Destination Node : D
Intermediate Nodes : B, D

To Find:
Designing an linear programming problem

Method:
Now, it is given that we can go from A --- B --- C --- D and
We can go from A --- C --- B --- D
Thus, we can visualise the above problem as follows:



Formation of LP:

Variables:
X_{ij} = Amounts shipped from Node i to Node j
Here, we have defined the following variables :
X_{AB} = Amount shipped from Node A to Node B
X_{AC} = Amount shipped from Node A to Node C
X_{BC} = Amount shipped from Node B to Node C
X_{CB} = Amount shipped from Node C to Node B
X_{BD} = Amount shipped from Node B to Node D
X_{CD} = Amount shipped from Node C to Node D

C_{ij} = Cost of transportation from Node i to Node j
Here,
C_{AB} = Cost of transportation from Node A to Node B
C_{AC} = Cost of transportation from Node A to Node c
C_{BC} = Cost of transportation from Node B to Node C
C_{CB} = Cost of transportation from Node C to Node B
C_{BD} = Cost of transportation from Node B to Node D
C_{CD} = Cost of transportation from Node C to Node D

Y_i = 1 if X_i >= 0, else Y_i = 0
Y - indicates whether a particular path has been used or not

Constraints:
Non-Negativity Constraints:
X_{AB} , X_{AC} , X_{BC} , X_{CB} , X_{BD} , X_{CD} >= 0

Also, since we can either follow Route : A --- B --- C --- D or A --- C --- B --- D,
There can be a flow from either B --- C or from B --- C at a given time!
Hence, the following constraint
If $X_{BC} > 0$, $X_{CB} = 0$
or
If $X_{CB} > 0$, $X_{BC} = 0$

Assumption:
The transportation cost from B to C is same as the cost from C to B
Implies, $C_{BC} = C_{CB}$

Let S : Total Supply supplied by Node A
And D : Total Demand of Node D
Thus, at every node :
Total Inflow - Outflow = Supply/Demand

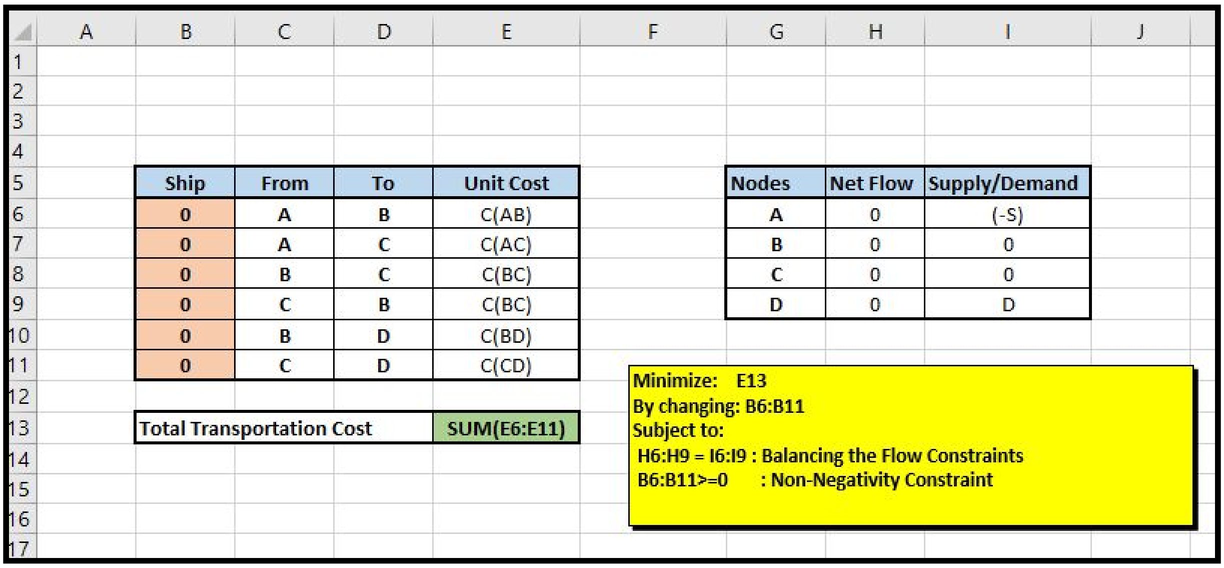
Amount out of Node A	: $(X_{AB} + X_{AC}) = S$
Amount through Node B	: $(X_{AB} + X_{CB}) - (X_{BC} + X_{BD}) = 0$
Amount through Node C	: $(X_{AC} + X_{BC}) - (X_{CB} + X_{CD}) = 0$
Amount into Node D	: $(X_{BD} + X_{CD}) = D$

Objective Function:
Here, we want to minimize the total transportation cost.

Hence, function:

$$\mathbf{MIN(}\quad X_{AB}\,C_{AB} + X_{AC}\,C_{AC} + X_{CB}\,C_{CB} + X_{CB}\,C_{CB} + X_{BD}\,C_{BD} + X_{CD}\,C_{CD}\ )$$

Solver Representation:
We can denote the above problem in solver as follows:



Ship	From	To	Unit Cost
0	A	B	C(AB)
0	A	C	C(AC)
0	B	C	C(BC)
0	C	B	C(BC)
0	B	D	C(BD)
0	C	D	C(CD)

Total Transportation Cost	SUM(E6:E11)

Nodes	Net Flow	Supply/Demand
A	0	(-S)
B	0	0
C	0	0
D	0	D

Minimize: E13
By changing: B6:B11
Subject to:
H6:H9 = I6:I9 : Balancing the Flow Constraints
B6:B11>=0 : Non-Negativity Constraint

Question 3 - Economic Order Quantity

Given :

Deterministic Annual Demand (A) = 1000

Ordering Cost (k) = $21

Holding Cost (h) = 25%

Purchase Cost :

When the quantity ordered is between 0 and 794:

- Here, there is no discount offered
- c1 = $4

When the quantity ordered is between 795 and 1099:

- Here, discount = 5%
- c2 = $3.80

When the quantity ordered is between 1100 and 1859:

- Here, discount = 8%
- c3 = $3.60

When the quantity ordered is between 1860 or more:

- Here, discount = 15%
- c4 = $3.40

To Find :

1. Optimal Order Quantity
2. Optimal Total Cost

Solution:

Snapshot:

	A	B	C	D	E
3					
4					
5	**Annual Demand(A)**	1000		**Lowest Optimal Cost = $3588.94**	
6	**Ordering Cost (k)**	$21			
7	**Holding Cost (h)**	25%		**Corresponding Optimal Quantity = 1860**	
8					
9	**Case 1 : No Discount**			Purchase Cost (Ac1)	$4,000
10	Order Quantity (Between 0 and 794)			Ordering Cost (Ak/Q1*)	$102.47
11	Purchase Cost (c1)	$4		Holding Cost (Q1*c1h/2)	$102.47
12	Optimal Order Quantity (Q1*)	**204.939**		**Total Cost (Ac1 + Ak/Q1* + Q1*ch/2)**	**$4,204.94**
13					
14	**Case 2 : 5% Discount**			Purchase Cost (Ac2)	$3,800
15	Order Quantity (Between 795 and 1099)			Ordering Cost (Ak/Q2*)	$99.87
16	Purchase Cost (c2 - after discount)	$3.80		Holding Cost (Q2*c2h/2)	$99.87
17	Optimal Order Quantity (Q2*)	**210.263**		**Total Cost (Ac2 + Ak/Q1* + Q1*c2h/2)**	**$3,999.75**
18					
19	**Case 3 : 8% Discount**			Purchase Cost (Ac3)	$3,680
20	Order Quantity (Between 1100 and 1859)			Ordering Cost (Ak/Q3*)	$98.29
21	Purchase Cost (c3 - after discount)	$3.68		Holding Cost (Q3*c3h/2)	$98.29
22	Optimal Order Quantity (Q3*)	**213.6637**		**Total Cost (Ac3 + Ak/Q3* + Q1*c3h/2)**	**$3,876.57**
23					
24	**Case 4 : 15% Discount**			Purchase Cost (Ac4)	$3,400
25	Order Quantity (1860 or more)			Ordering Cost (Ak/Q4*)	$94
26	Purchase Cost (c4 - after discount)	$3.40		Holding Cost (Q4*c4h/2)	$94.47
27	Optimal Order Quantity (Q4*)	**222.2876**		**Total Cost (Ac4 + Ak/Q4* + Q4*c4h/2)**	**$3,588.94**

Step 1:
We first calculate the smallest feasible Q* under each pricing structure
Then, we choose the Q* that results in the smallest annual total cost

Calculating the Q* under each pricing structure:

Case 1: c1 = $4 (Here, there is no discount offered)

$Q1^* = \sqrt{\frac{2Ak}{c1h}} = \sqrt{\frac{2(1000)21}{(4)(0.25)}} = 204.94$

Case 2: c2 = $3.80 (Here, discount = 5%)

$Q2^* = \sqrt{\frac{2Ak}{c2h}} = \sqrt{\frac{2(1000)21}{(3.80)(0.25)}} = 210.26$

The most economical, feasible quantity for c2 is 795

Case 3: c3 = $3.68(Here, discount = 8%)

$Q3^* = \sqrt{\frac{2Ak}{c3h}} = \sqrt{\frac{2(1000)21}{(3.60)(0.25)}} = 213.6637$

The most economical, feasible quantity for c3 is 1100
Extra = 1100 - 1000 = 100

Case 4: c4 = $3.40(Here, discount = 15%)

$Q4^* = \sqrt{\frac{2Ak}{c4h}} = \sqrt{\frac{2(1000)21}{(3.40)(0.25)}} = 222.29$

The most economical, feasible quantity for c3 is 1860
Extra = 1860 - 1000 = 860

Total Cost Comparison:
Total Cost (TCi)
= purchase + ordering + holding
$= Aci + \frac{Ak}{Q^*} + \frac{Q^* cih}{2}$

$TC1 = (1000)(4) + \frac{1000(21)}{204.94} + \frac{(204.94)(4)(0.25)}{2} = \$4204.93$

$TC2 = (1000)(3.80) + \frac{1000(21)}{210.26} + \frac{(210.26)(3.80)(0.25)}{2} = \$3999.75$

$TC3 = (1000)(3.68) + \frac{1000(21)}{216.02} + \frac{(216.02)(3.68)(0.25)}{2} = \$3876.57$

$TC4 = (1000)(3.40) + \frac{1000(21)}{222.29} + \frac{(222.29)(3.40)(0.25)}{2} = \$3588.94$

Answer:
It is given that there is no limit on the number of goods that can be held in an inventory, we don't have to scrap out any items!
Clearly, from the above calculations, we can see that the lowest total cost would be for Case 4, where the cost is $3588.94
And the corresponding optimal quantity is around 1860