

VEDIC MATHEMATICS

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DIVISION

1. Division by 5

Whenever you have to divide a number by 5, do the following steps:

- i) Multiply the number by 2 (i.e., double the figure)
- ii) Then divide the answer by 10

This method is based on $1 \div 5 = 1/5 = 2/10$.

Examples:

(1) $32 \div 5$

As per step (i), double the figure. So you get 64.

As per step (ii), divide by 10. So you get $64/10 = 6.4$ which is the final answer.

(2) $48 \div 5$

As per step (i), double the figure. So you get $48+48 = 96$

As per step (ii), divide by 10. So you get $96/10 = 9.6$ which is the final answer.

(3) $104 \div 5$

As per step (i), double the figure. So you get $104+104 = 208$.

As per step (ii), divide by 10. So you get $208/10 = 20.8$ which is the final answer.

(4) $1028 \div 5$

As per step (i), double the figure. So you get $1028+1028 = 2056$.

As per step (ii), divide by 10. So you get $2056/10 = 205.6$ which is the final answer.

2. Division by 25

Whenever you have to divide a number by 25, do the following steps:

- i) Multiply the number by 4
- ii) Then divide the answer by 100

This method is based on $1 \div 25 = 1/25 = 4/100$.

Examples:

(1) $16 \div 25$

As per step (i), multiply the figure by 4. So you get $16 \times 4 = 64$.

As per step (ii), divide by 100. So you get $64/100 = 0.64$ which is the final answer.

(2) $1000 \div 25$

As per step (i), multiply the figure by 4. So you get $1000 \times 4 = 4000$.

As per step (ii), divide by 100. So you get $4000/100 = \mathbf{40}$ which is the final answer.

(3) $1700 \div 25$

As per step (i), multiply the figure by 4. So you get $1700 \times 4 = 6800$. (You can also calculate it as $1700 + 1700 = 3400$ and $3400 + 3400 = 6800$)

As per step (ii), divide by 100. So you get $6800/100 = \mathbf{68}$ which is the final answer.

3. Division by 125

Whenever you have to divide a number by 125, do the following steps:

- i) Multiply the number by 8
- ii) Then divide the answer by 1000

This method is based on $1/125 = 2/250 = 4/500 = 8/1000$.

Examples:

(1) $1000 \div 125$

As per step (i), multiply the figure by 8. So you get $1000 \times 8 = 8000$.

As per step (ii), divide by 1000. So you get $8000/1000 = 8$ which is the final answer.

(2) $700 \div 125$

As per step (i), multiply the figure by 8. So you get $700 \times 8 = 5600$.

As per step (ii), divide by 1000. So you get $5600/1000 = 5.6$ which is the final answer.

MULTIPLICATION

4. Multiplication by 5

There are 2 cases when you have to multiply a number by 5.

A. Multiplication by 5 when the number is divisible by 2.

These are the numbers that end in 0, 2, 4, 6 and 8. In such cases, do the following steps:

- i) Divide the number by 2
- ii) Then put a 0 at the end of your answer

Examples:

(1) 50×5

Following the steps mentioned, we get $50/2 = 25$.

Now add a 0 to the answer and we get the final answer as 250.

(2) 380×5

Following the steps mentioned, we get $380/2 = 190$.

Now add a 0 to the answer and we get the final answer as 1900.

(3) 1480×5

Following the steps mentioned, we get $1480/2 = 740$.

Now add a 0 to the answer and we get the final answer as 7400.

B. Multiplication by 5 when the number is not divisible by 2.

These are the numbers that end in 1, 3, 5, 7 and 9. In such cases, do the following steps:

- i) Deduct 1 from the number
- ii) Divide the number by 2
- iii) Then put a 5 at the end of your answer

Examples:

(1) 51×5

Following the steps mentioned, we get $51 - 1 = 50$.

Then dividing by 2 we get $50/2 = 25$.

Now add a 5 to the answer and we get the final answer as 255.

(2) 391×5

Following the steps mentioned, we get $391 - 1 = 390$.

Then dividing by 2 we get $390/2 = 195$.

Now add a 5 to the answer and we get the final answer as 1955.

(3) 1279×5

Following the steps mentioned, we get $1279 - 1 = 1278$.

Then dividing by 2 we get $1278/2 = 639$.

Now add a 5 to the answer and we get the final answer as 6395.

5. Multiplication by 11

For multiplying a number by 11, do the following steps:

- i) Split the number and write the first and last digits while leaving blanks in between.
- ii) From the right to the left, add 2 digits (first the one and tens, then tens and hundreds, then hundreds and thousands etc.,) and fill in the blanks one digit at a time.

If the sum of the 2 digits (ones + tens, tens + hundreds, hundreds + thousands etc.,) is a 2 digit answer, then write the rightmost digit in the blank and carry over the leftmost digit to the next digit of the original number.

The examples will explain it clearly.

Examples:

(1) 163×11

As per first step, write down the first and last digits as it is with blank in between i.e., 1_3 .

As per second step, start adding 2 digits starting from the ones place. So from the number 163, add 3 and 6 which gives you 9. Fill it in the blank just to the left of the ones position. You will get 1_93 . Continue this process. So we now add 6 and 1 to get 7. Fill in the next blank with this number. So we get **1793**. This is the final answer as we don't have any more digits to add.

OR

$$163 \times 11$$

$$= 1_3 \quad (\text{As per step i})$$

$$= 1_93 \quad [3+6] \quad (\text{As per step ii})$$

$$= 1793 \quad [6+1] \quad (\text{Continuing step ii})$$

$$\text{Therefore, } 163 \times 11 = 1793$$

(2) 1345×11

As per first step, write down the first and last digits as it is with blank in between i.e.

$$1_5.$$

As per second step, start adding 2 digits starting from the ones place. So from the number 1345, add 5 and 4 which gives you 9. Fill it in the blank just to the left of the ones position. You will get 1_95 . Continue this process. So we now add 4 and 3 to get 7. Fill in the next blank with this number. So we get 1_795 . Continue this process. So we now add 3 and 1 to get 4. Fill in the next blank with this number. So we get 14795. This is the final answer as we don't have any more digits to add.

OR

$$\begin{aligned} &1345 \times 11 \\ &= 1_5 \quad (\text{As per step i}) \\ &= 1_95 \quad [5+4] \text{ (As per step ii)} \\ &= 1_795 \quad [4+3] \text{ (Continuing step ii)} \\ &= 14795 \quad [3+1] \text{ (Continuing step ii)} \\ &\text{Therefore, } 163 \times 11 = \mathbf{14795} \end{aligned}$$

(3) 169×11

As per first step, write down the first and last digits as it is with blank in between i.e.

$$1_9.$$

As per second step, start adding 2 digits starting from the ones place. So from the number 169, add 9 and 6 which gives you 15. Since your answer is a 2 digit number, you need to fill only the unit position and carry over the ten's position. Fill it in the blank just to the left of the ones position. So you will get 1_59 (with 1 carried over). Continue this process. So we now add 6 and 1 to get 7. Now add the carry over to this answer so you will get $7+1 = 8$. Fill in the next blank with this number. So we get 1859. This is the final answer as we don't have any more digits to add.

OR

$$\begin{aligned} &169 \times 11 \\ &= 1_9 \quad (\text{As per step i}) \\ &= 1_59 \quad [9+6] \text{ (As per step ii) with a carryover of 1} \\ &= 1859 \quad [6+1+1] \text{ (Continuing step ii and adding the carryover)} \\ &\text{Therefore, } 163 \times 11 = 1859. \end{aligned}$$

(4) 2784×11

As per first step, write down the first and last digit with blanks in between i.e., 2____4.

As per second step, start adding 2 digits starting from the ones place. So from the number 2784, add 8 and 4 which gives you 12. Since your answer is a 2 digit number, you need to fill only the unit position and carry over the ten's position. Fill it in the blank just to the left of the ones position. You will get 2_24. Continue this process. So we now add 7 and 8 to get 15. We have to add the earlier carryover of 1 to this so we get $15 + 1 = 16$. Since your answer is a 2 digit number, you need to fill only the unit position and carry over the ten's position. Fill in the next blank with this number. So we get 2_624. Continue this process. So we now add 2 and 7 to get 9. We have to add the earlier carryover of 1 to this so we get $9 + 1 = 10$. Since your answer is a 2 digit number, you need to fill only the unit position and carry over the ten's

position. Fill in the next blank with this number. So we get 20624. However we don't have any more digits to add but there is a carryover of 1 that is still there. We need to add this to the leftmost digit which in this case is 2. So that digit will now be $2+1=3$. Thus we get the final answer as 30624.

OR

$$2784 \times 11$$

$$= 2_4 \quad (\text{As per step i})$$

$$= 2_24 \quad [4+8] \quad (\text{As per step ii, we get } 4+8 = 12 \text{ so write down the 2 in the blank space and carryover the 1})$$

$$= 2_624 \quad [8+7+1] \quad (\text{Continuing step ii, we get } 8+7 = 15; \text{ adding the carryover of 1 we get } 15+1=16. \text{ Again carryover the 1 and write the 6 in the blank})$$

$$= 2+10624 \quad [7+2+1] \quad (\text{Continuing step ii, we get } 7+2=9; \text{ adding the carryover of 1 we get } 9+1=10. \text{ Now carryover the 1 and write the 0 in the blank. Since there are no more digits to add, there are no more blanks. So the carryover of 1 must be added to the leftmost digit which is 2 in this case.}$$

Thus that digit will now change to $2+1=3$)

$$= 30624.$$

Therefore, $163 \times 11 = 30624$.

6. Multiplication of 2 numbers between 11 and 20

When multiplying 2 numbers between 11 and 20, the product (answer) is the number 1 followed by sum of the two ones digits and then the product of the 2 ones digits. Note that if either the sum or the product of the 2 ones digits is a 2 digit result, only the rightmost digit is written down and the remaining is carried over to the digit on the left. Examples will make it clearer to understand.

Examples:

(1) 12×13

We now write the answer as:

1 (2+3) (2x3)

156 Final answer

(2) 12×12

We now write the answer as:

1 (2+2) (2x2)

144 Final answer

(3) 15×19

We now write the answer as:

1 (5+9) (5x9)

1 14 45 (since 45 is a 2 digit result, carryover the 4 to 14)

1 14⁴ 5

1 18 5 (Since 18 is a 2 digit result, carryover the 1 to the 1 on the left)

1¹ 8 5

285 Final answer

(4) 16×18

We now write the answer as:

1 (6+8) (6x8)

1 14 48 (since 48 is a 2 digit result, carryover the 4 to 14)

1 14⁴ 8

1 18 8 (Since 18 is a 2 digit result, carryover the 1 to the 1 on the left)

1¹ 8 8

288 Final answer

(5) 19×15

We now write the answer as:

1 (9+5) (9x5)

1 14 45 (since 45 is a 2 digit result, carryover the 4 to 14)

1 14⁴ 5

1 18 5 (Since 18 is a 2 digit result, carryover the 1 to the 1 on the left)

1¹ 8 5

285 Final answer

7. Multiplication of 2 digit numbers that are close to 100

This method is useful to multiply 2 digit numbers that are close to 100. The method is as follows:

Consider the 2 numbers to be N1 and N2.

- i) Find the difference between 100 and both the numbers. Let the two answers be A1 (for N1) and A2 (for N2) with the corresponding signs (-ve if number >100 and +ve if number <100).
- ii) Now find the answer to either $N1-A2$ or $N2-A1$. The result will be the same. Let us denote it as D (for difference).
- iii) Now find the product of A1 and A2. Let the answer be denoted as P (for product).
i.e., $P = A1 \times A2$
- iv) The final answer is found by 2 different methods depending on the sign of P.
 - (1) If P is positive:
The final answer is DP. Note that P should be a 2 digit number. If it is a 3 digit number then the hundreds part of P is carried over to D to be added to it.
 - (2) If P is negative:
The final answer will be the result of $D00 + P$ irrespective of whether P is a 2 digit or 3 digit number. No carryover in this case even if P is a 3 digit number. The examples will explain it clearly.

Examples:

(1) 98×95

Here $N1 = 98$ and $N2 = 95$

$A1 = (100-98) = +2$ and $A2 = (100-95) = +5$

D = $(N1-A2)$ or $(N2-A1) = 98-5$ or $95-2 = \mathbf{93}$

P = $A1 \times A2 = +2 \times +5 = \mathbf{+10}$

Final answer is DP since P is positive

Answer = **9310**

(2) 89×96

Here $N1 = 89$ and $N2 = 96$

$A1 = (100-89) = +11$ and $A2 = (100-96) = +4$

D = $(N1-A2)$ or $(N2-A1) = 89-4$ or $96-11 = \mathbf{85}$

P = $A1 \times A2 = +11 \times +4 = \mathbf{+44}$

Final answer is DP since P is positive

Answer = **8544**

(3) 90×91

Here $N1 = 90$ and $N2 = 91$

$A1 = (100-90) = +10$ and $A2 = (100-91) = +9$

D = $(N1-A2)$ or $(N2-A1) = 90-9$ or $91-10 = \mathbf{81}$

P = $A1 \times A2 = +10 \times +9 = \mathbf{+90}$

Final answer is DP since P is positive

Answer = **8190**

(4) 105×110

Here $N1 = 105$ and $N2 = 110$

$A1 = (100-105) = -5$ and $A2 = (100-110) = -10$

$D = (N1-A2) \text{ or } (N2-A1) = 105 - (-10) \text{ or } 110 - (-5) = 115$

$P = A1 \times A2 = -5 \times -10 = +50$

Final answer is DP since P is positive

Answer = **11550**

(5) 109×119

Here $N1 = 109$ and $N2 = 119$

$A1 = (100-109) = -9$ and $A2 = (100-119) = -19$

$D = (N1-A2) \text{ or } (N2-A1) = 109 - (-19) \text{ or } 119 - (-9) = 128$

$P = A1 \times A2 = -9 \times -19 = +171$ (Note that P is of 3 digits here so we need to carryover 1 to D)

Final answer is DP since P is positive

Answer = $128^{171} = 12971$ (D now becomes $128 + 1 = 129$ due to carryover from P)

(6) 112×119

Here $N1 = 112$ and $N2 = 119$

$A1 = (100-112) = -12$ and $A2 = (100-119) = -19$

$D = (N1-A2) \text{ or } (N2-A1) = 112 - (-19) \text{ or } 119 - (-12) = 131$

$P = A1 \times A2 = -12 \times -19 = +228$ (Note that P is of 3 digits here so we need to carryover 2 to D)

Final answer is DP since P is positive

Answer = $131^{228} = 13328$ (D now becomes $131 + 2 = 133$ due to carryover from P)

(7) 104×96

Here $N1 = 104$ and $N2 = 96$

$A1 = (100-104) = -4$ and $A2 = (100-96) = +4$

$D = (N1-A2) \text{ or } (N2-A1) = 104 - (4) \text{ or } 96 - (-4) = 100$

$P = A1 \times A2 = -4 \times +4 = -16$ (Note that P is a negative number)

Final answer is D00 + P since P is negative

Answer = $10000 + (-16) = 9984$

(8) 109×81

Here $N1 = 109$ and $N2 = 81$

$A1 = (100-109) = -9$ and $A2 = (100-81) = +19$

$D = (N1-A2) \text{ or } (N2-A1) = 109 - 19 \text{ or } 81 - (-9) = 90$

$P = A1 \times A2 = -9 \times +19 = -171$ (Note that P is a negative number so we do not need a carryover even though it is of 3 digits)

Final answer is D00+P since P is negative

Answer = $9000 + (-171) = 8829$

Multiplication of 2 digit numbers that are close to 100

(9) 109×91

Here $N1 = 109$ and $N2 = 91$

$A1 = (100-109) = -9$ and $A2 = (100-91) = +9$

$D = (N1-A2) \text{ or } (N2-A1) = 109 - 9 \text{ or } 91 - (-9) = \mathbf{100}$

$P = A1 \times A2 = -9 \times +9 = \mathbf{-81}$ (Note that P is negative)

Final answer is $D00 + P$ since P is negative

Answer = $10000 + (-81) = 10000 - 81 = \mathbf{9919}$

8. Multiplication of 2 digits when the sum of unit digits equal 10 and tens place digits are the same

Examples of such digits would be:

$$52 \times 58 \text{ [} 2+8=10, 5 \text{ in both the tens places]}$$

$$64 \times 66 \text{ [} 4+6=10, 6 \text{ in both the tens place]}$$

$$91 \times 99 \text{ [} 1+9=10, 9 \text{ in both the tens place]}$$

$$22 \times 28 \text{ [} 2+8=10, 2 \text{ in both the tens place]}$$

The method is as follows:

- i) First multiply the digits of both the unit digits and write it down.
- ii) Next, multiply the digit in the tenth place with the next higher digit (if it is 5, 5×6 ; if it is 8, 8×9 etc.)
- iii) The final answer is the answer of step (ii) written next to answer of step (i).

The examples will explain it clearly.

Examples:

(1) 52×58

As per step (i), multiply the unit digits.

i.e., $2 \times 8 = 16$

As per step (ii), multiply tens digit with next higher digit

i.e., $5 \times 6 = 30$

Final answer is **3016** as per step (iii).

(2) 64×66

As per step (i), multiply the unit digits.

i.e., $4 \times 6 = 24$

As per step (ii), multiply tens digit with next higher digit

i.e., $6 \times 7 = 42$

Final answer is **4224** as per step (iii).

(3) 91×99

As per step (i), multiply the unit digits.

i.e., $1 \times 9 = 09$. (Always write this answer as a 2 digit number. So it should be written as 09 and not 9)

As per step (ii), multiply tens digit with next higher digit

i.e., $9 \times 10 = 90$

Final answer is **9009** as per step (iii).

(4) 81×89

As per step (i), multiply the unit digits.

i.e., $1 \times 9 = 09$. (Always write this answer as a 2 digit number. So it should be written as 09 and not 9)

As per step (ii), multiply tens digit with next higher digit.

i.e., $8 \times 9 = 72$

Final answer is **7209** as per step (iii).

9. Multiplication of 2 digit numbers differing by 10 and each ending in 5

This method is used to multiply two 2-digit numbers each having the number 5 in its ones place and differing by 10. Examples would be 25×15 , 85×95 etc. **Please note that the smaller number should always be written first.**

Let the smaller number be N_15 and the bigger number be N_25 . (Here $N_25 - N_15 = 10$ where N_1 and N_2 are the digits in the tens position of both numbers). The steps are as follows:

- i) The last two digits of the final answer is always 75.
- ii) The first few digits are calculated as $[N_1 + (N_1 \times N_2)]$.

Examples will make it clearer to understand.

Examples:

(1) 25×35

The last 2 digits of the final answer will be __75 as per step (i)

The first few digits are calculated as $[2 + (2 \times 3)] = 2 + 6 = 8$ (as per step (ii))

Final answer is **875**

(2) 35×45

The last 2 digits of the final answer will be __75 as per step (i)

The first few digits are calculated as $[3 + (3 \times 4)] = 3 + 12 = 15$ (as per step (ii))

Final answer is **1575**

(3) 45×55

The last 2 digits of the final answer will be __75 as per step (i)

The first few digits are calculated as $[4 + (4 \times 5)] = 4 + 20 = 24$ (as per step (ii))

Final answer is **2475**

(4) 55×65

The last 2 digits of the final answer will be __75 as per step (i)

The first few digits are calculated as $[5 + (5 \times 6)] = 5 + 30 = 35$ (as per step (ii))

Final answer is **3575**

(5) 65×75

The last 2 digits of the final answer will be __75 as per step (i)

The first few digits are calculated as $[6 + (6 \times 7)] = 6 + 42 = 48$ (as per step (ii))

Final answer is **4875**

(6) 95×85

Rewrite it as 85×95 since smaller number needs to be written first

The last 2 digits of the final answer will be __75 as per step (i)

The first few digits are calculated as $[8 + (8 \times 9)] = 8 + 72 = 80$ (as per step (ii))

Final answer is **8075**

Multiplication of 2-digit numbers where sum of tens place values is 10 and ones place value of both numbers is the same

10. Multiplication of 2-digit numbers where sum of tens place values is 10 and ones place value of both numbers is the same

There is a simple method to multiply 2 numbers where the sum of the values in the tens place of the 2 numbers is 10 and the ones place value is the same for both numbers.

Examples of such numbers would be 76×36 , 63×43 etc. So if the first number is N_1N_0 , the second number would have to be $(10-N_1)N_0$.

Then do the following steps:

- i) Multiply the common digit in the ones place of both numbers. i.e., $N_0 \times N_0$. This answer has to be written as a 2-digit number.
- ii) Next, multiply the tens place digits of both numbers i.e., $N_1 \times (10-N_1)$
- iii) Now add the common ones place digit to this answer i.e., $N_0 + [N_1 \times (10-N_1)]$
- iv) Now the final answer is written as result of step (iii) followed by step (i)

Examples will make it clearer to understand.

Examples:

(1) 76×36

As per step (i) we get $6 \times 6 = 36$

As per step (ii) $7 \times 3 = 21$

As per step (iii) we get $6 + 21 = 27$

Final answer is **2736** as per step (iv)

(2) 47×67

As per step (i) we get $7 \times 7 = 49$

As per step (ii) $4 \times 6 = 24$

As per step (iii) we get $7 + 24 = 31$

Final answer is **3149** as per step (iv)

(3) 85×25

As per step (i) we get $5 \times 5 = 25$

As per step (ii) $8 \times 2 = 16$

As per step (iii) we get $5 + 16 = 21$

Final answer is **2125** as per step (iv)

(4) 63×43

As per step (i) we get $3 \times 3 = 09$ (It has to be written as 09 and not 9)

As per step (ii) $6 \times 4 = 24$

As per step (iii) we get $3 + 24 = 27$

Final answer is **2709** as per step (iv)

11. Multiplication of 2 numbers where ones place value is 5 for both

We have a method to solve multiplication of 2 numbers where both of them have 5 in the ones place value. Both the numbers can be of any number of digits. Let us consider the first number to be N_15 and the second number to be N_25 where N_1 and N_2 can be any number of digits.

Now we have to do the following steps:

- i) Multiply N_1 and N_2 . We get $\text{Result1} = N_1 \times N_2$
- ii) Find the average of N_1 and N_2 . $\text{Result2} = (N_1 + N_2) \div 2$
- iii) Add $\text{Result1} + \text{Result2}$
- iv) Now multiply the answer in step (iii) by 100
- v) Finally, add 25 to the result of step (iv) to get the final answer

Examples will make it clearer to understand.

Examples:

(1) 35×85

As per step (i) we get $\text{Result1} = 3 \times 8 = 24$

As per step (ii) we get $\text{Result2} = (3+8) \div 2 = 11/2 = 5.5$

As per step (iii) we get $24+5.5 = 29.5$

As per step (iv) we get $29.5 \times 100 = 2950$

As per step (v) we get the final answer as $2950+25 = \mathbf{2975}$

(2) 45×95

As per step (i) we get $\text{Result1} = 4 \times 9 = 36$

As per step (ii) we get $\text{Result2} = (4+9) \div 2 = 13/2 = 6.5$

As per step (iii) we get $36+6.5 = 42.5$

As per step (iv) we get $42.5 \times 100 = 4250$

As per step (v) we get the final answer as $4250+25 = \mathbf{4275}$

(3) 35×175

As per step (i) we get $\text{Result1} = 3 \times 17 = 51$

As per step (ii) we get $\text{Result2} = (3+17) \div 2 = 20/2 = 10$

As per step (iii) we get $51+10 = 61$

As per step (iv) we get $61 \times 100 = 6100$

As per step (v) we get the final answer as $6100+25 = \mathbf{6125}$

(4) 55×65

As per step (i) we get $\text{Result1} = 5 \times 6 = 30$

As per step (ii) we get $\text{Result2} = (5+6) \div 2 = 11/2 = 5.5$

As per step (iii) we get $30+5.5 = 35.5$

As per step (iv) we get $35.5 \times 100 = 3550$

As per step (v) we get the final answer as $3550+25 = \mathbf{3575}$

Multiplication of 2 numbers where ones place value is 5 for both

(5) 125×165

As per step (i) we get $\text{Result1} = 12 \times 16 = 192$

As per step (ii) we get $\text{Result2} = (12+16) \div 2 = 28/2 = 14$

As per step (iii) we get $192+14 = 206$

As per step (iv) we get $206 \times 100 = 20600$

As per step (v) we get the final answer as $20600+25 = \mathbf{20625}$

(6) 125×2505

As per step (i) we get $\text{Result1} = 12 \times 250 = 3000$

As per step (ii) we get $\text{Result2} = (12+250) \div 2 = 262/2 = 131$

As per step (iii) we get $3000+131 = 3131$

As per step (iv) we get $3131 \times 100 = 313100$

As per step (v) we get the final answer as $313100+25 = \mathbf{313125}$

12. Multiplication of 2 numbers where one number consists of only 9s

There are simple methods to multiply a number with another number consisting of only 9s. There are various scenarios for the above mentioned multiplication. We will look at each scenario with examples.

A. Case 1: When both numbers have same number of digits

This method is to be applied only when the number of digits in both the numbers are the same. While writing the numbers, write the number with only 9 in as its digits in the second place. The steps to be followed after that are:

- (i) Subtract 1 from the first number (number that doesn't contain 9 as all its digits) and write it down first.
- (ii) Now subtract each digit of the result from step (i) and write the corresponding answer after the result got from step (i).

Examples will make it clearer to understand.

Examples:

- (1) 999×389 (same number of digits i.e., 3)

Rewrite it as 389×999

$389 - 1 = 388$ (As per step (i))

Subtracting each digit of 388 from 9 we get $9 - 3 = 6$, $9 - 8 = 1$ and $9 - 8 = 1$ (as per step (ii))

Final answer from steps (i) and (ii) is **388610**

- (2) 1209×9999 (same number of digits i.e., 4)

$1209 - 1 = 1208$ (As per step (i))

Subtracting each digit of 1208 from 9 we get $9 - 1 = 8$, $9 - 2 = 7$, $9 - 0 = 9$ and $9 - 8 = 1$ (as per step (ii))

Final answer from steps (i) and (ii) is **12088791**

B. Case 2: When the digits of the non-9 number < digits of the number with all 9s

This method is to be applied only when the number with all 9s as its digits is bigger or greater than the other number. While writing the numbers, write the number with only 9 in as its digits in the second place. The steps to be followed are:

- (i) Since the number of digits in the non-9 number is lesser, we need to add zeroes to the beginning of it to make the number of digits the same as that of the all 9s number.
- (ii) Subtract 1 from the result of step (i) and write it down first.
- (iii) Now subtract each digit of the result from step (ii) and write the corresponding answer after the result got from step (ii).

Examples will make it clearer to understand.

Examples:

(1) 434 X 9999

434 should now be written as 0434 (to make number of digits the same; as per step (i))

$0434 - 1 = 0433$ (As per step (ii))

Subtracting each digit of 0433 from 9 we get $9 - 0 = 9$, $9 - 4 = 5$, $9 - 3 = 6$ and $9 - 3 = 6$ (as per step (iii))

Final answer from steps (ii) and (iii) is **04339566 or 4339566**

(2) 28 X 9999

28 should now be written as 0028 (to make number of digits the same; as per step (i))

$0028 - 1 = 0027$ (As per step (ii))

Subtracting each digit of 0027 from 9 we get $9 - 0 = 9$, $9 - 0 = 9$, $9 - 2 = 7$ and $9 - 7 = 2$ (as per step (iii))

Final answer from steps (ii) and (iii) is **00279972 or 279972**

C. Case 3: When the digits of the non-9 number > digits of the number with all 9s

This method is to be applied only when the number with all 9s as its digits is smaller or lesser than the other number. While writing the numbers, write the number with only 9 in as its digits in the second place.

The steps to be followed are:

- i) Since the number of digits in the non-9 number is greater, we need to add zeroes to the end of it where the number of zeroes is equal to the number of digits of the all 9s number.
- ii) Subtract the non-9 number from the result of step (i) which is the final answer.

Examples will make it clearer to understand.

Examples:

(1) 128×99

Add 2 zeroes to 128 to make it 12800 (2 zeroes as number of digits of 99 = 2; as per step (i))

Now, $12800 - 128 = 12672$ (as per step (ii))

Final answer is **12672**

(2) 7561×999

Add 3 zeroes to 7561 to make it 7561000 (3 zeroes as number of digits of 999 = 3; as per step (i))

Now, $7561000 - 7561 = 7553439$ (as per step (ii))

Final answer is **7553439**

13. Multiplication of a 2 digit number by another 2 digit number

There are 2 methods to multiply a 2 digit number by another 2 digit number.

A. Method 1

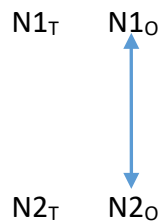
We have a method to solve multiplication of a 2-digit number by another 2-digit number in Vedic mathematics by using cross-multiplication and addition in a certain pattern. Let us denote the first 2-digit number as $N1_T N1_O$ where $N1_T$ is the tens digit of the number and $N1_O$ is the ones digit. Now let the second 2-digit number be denoted by $N2_T N2_O$ where $N2_T$ is the tens digit of the number and $N2_O$ is the ones digit.

$$\begin{array}{r} N1_T N1_O \\ \times N2_T N2_O \end{array}$$

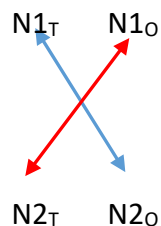
The final answer will be written in 3 blanks namely blank1 blank2 blank3. We start filling up the blanks from the right side i.e., blank3.

Now we have to do the cross-multiplication and addition in the following patterns:

i) Step 1: $N1_O \times N2_O = \text{blank3}$



ii) Step 2: $(N1_T \times N2_O) + (N1_O \times N2_T) = \text{blank2}$



iii) Step 3: $N1_T \times N2_T = \text{blank3}$



- iv) Step 4: Check the values in all the blanks starting from blank3. If they are not single digit values, retain only the ones portion of the value and carryover the remaining part to the next blank to its left till you reach blank1. That will be the final answer.

The examples will explain it clearly.

Examples:

$$\begin{array}{r} (1) \quad 78 \\ \times 61 \\ \hline \end{array}$$

Put 3 blanks. _____

_____ **8** (As per step (i), blank3 = $8 \times 1 = 8$)

_____ **55** **8** (As per step (ii), blank2 = $(7 \times 1) + (8 \times 6) = 7 + 48 = 55$)

_____ **42** **55** **8** (As per step (iii), blank1 = $7 \times 6 = 42$)

Checking for carryovers as per step (iv), we get

$42^5 5 8$

$47 5 8$

4758 is the final answer.

$$\begin{array}{r} (2) \quad 64 \\ \times 73 \\ \hline \end{array}$$

Put 3 blanks. _____

_____ **12** (As per step (i), blank3 = $4 \times 3 = 12$)

_____ **46** **12** (As per step (ii), blank2 = $(6 \times 3) + (4 \times 7) = 18 + 28 = 46$)

_____ **42** **46** **12** (As per step (iii), blank1 = $6 \times 7 = 42$)

Checking for carryovers as per step (iv), we get

$42 46^1 2$

$42^4 7 2$

$46 7 2$

4672 is the final answer.

$$\begin{array}{r} (3) \quad 81 \\ \times 21 \\ \hline \end{array}$$

Put 3 blanks. _____

_____ **1** (As per step (i), blank3 = $1 \times 1 = 1$)

_____ **10** **1** (As per step (ii), blank2 = $(8 \times 1) + (1 \times 2) = 8 + 2 = 10$)

_____ **16** **10** **1** (As per step (iii), blank1 = $8 \times 2 = 16$)

Checking for carryovers as per step (iv), we get

16^1 0 1

17 0 1

1701 is the final answer.

(4) 95

X 19

Put 3 blanks. _____

45 (As per step (i), blank3 = $5 \times 9 = 45$)

86 45 (As per step (ii), blank2 = $(9 \times 9) + (5 \times 1) = 81 + 5 = 86$)

9 86 45 (As per step (iii), blank1 = $9 \times 1 = 9$)

Checking for carryovers as per step (iv), we get

9 86^4 5

9^9 0 5

18 0 5

1805 is the final answer.

(5) 57

X 73

Put 3 blanks. _____

21 (As per step (i), blank3 = $7 \times 3 = 21$)

64 21 (As per step (ii), blank2 = $(5 \times 3) + (7 \times 7) = 15 + 49 = 64$)

35 64 21 (As per step (iii), blank1 = $5 \times 7 = 35$)

Checking for carryovers as per step (iv), we get

35 64^2 1

35^6 6 1

41 6 1

4161 is the final answer.

(6) 87

X 43

Put 3 blanks. _____

21 (As per step (i), blank3 = $7 \times 3 = 21$)

52 21 (As per step (ii), blank2 = $(8 \times 3) + (7 \times 4) = 24 + 28 = 52$)

32 52 21 (As per step (iii), blank1 = $8 \times 4 = 32$)

Checking for carryovers as per step (iv), we get

Multiplication of a 2-digit number by another 2-digit number : Method 1

$$\begin{array}{r} 3252^2 1 \\ 32^5 4 1 \\ 37 4 1 \end{array}$$

3741 is the final answer.

(7) $\begin{array}{r} 95 \\ \times 19 \end{array}$

Put 3 blanks. _____

_____ **45** (As per step (i), blank3 = $5 \times 9 = 45$)

_____ **86** **45** (As per step (ii), blank2 = $(9 \times 9) + (5 \times 1) = 81 + 5 = 86$)

9 **86** **45** (As per step (iii), blank1 = $9 \times 1 = 9$)

Checking for carryovers as per step (iv), we get

$$\begin{array}{r} 986^4 5 \\ 9^9 0 5 \\ 18 0 5 \end{array}$$

1805 is the final answer

B. Method 2

Follow the steps given below to multiply a 2 digit number by another 2 digit number. This is just a simplification of method 1.

- i) Put three blanks.
- ii) First, multiply the tens digit of both the numbers and write down the answer in the first blank.
- iii) Multiply the near digits together and far digits together and add their results. Write down this answer in the second blank. (This is cross multiplication and adding).
- iv) Multiply the ones digit of both the numbers and write down the answer in the third and final blank.
- v) Starting from the third blank, check if the answer in the blank is a single digit or double digit figure. If it is a double digit figure, retain only the ones portion of the figure and carryover the tens portion to the blank on the left.
- vi) Add the carryover (if present) to the value in the second blank and if answer is a 2 digit figure, follow the same procedure of carrying over the tens portion to the first blank and retaining only the ones portion of the result in the second blank.
- vii) Add the carryover (if present) to the value in the first blank and write down the result in the first blank.
- viii) The final answer is the digits present in the three blanks.

The examples will explain it clearly.

Examples:

(1) 64×73

Following the methods mentioned we put ____ ____ ____ (3 blanks as per step (i))

42 ____ ____ (As per step (ii) multiplying the 2 tens digits i.e., 6×7)

42 28+18 ____ (As per step (iii) multiply near and far digits and add i.e., $4 \times 7 + 6 \times 3$)

42 46 12 (As per step (iv) multiply the 2 ones digits i.e., 4×3)

42 46¹ 2 (As per step (v) carryover the tens part of 3rd blank to 2nd blank)

42⁴ 7 2 (As per step (vi) add carryover and value and again carryover tens part to blank 1)

46 7 2 (As per step (vii) add carryover and value and write down result in blank 1)

Final answer is **4672**.

(2) 81×21

Following the methods mentioned we put ____ ____ ____ (3 blanks as per step (i))

16 ____ ____ (As per step (ii) multiplying the 2 tens digits i.e., 8×2)

16 2+8 ____ (As per step (iii) multiply near and far digits and add i.e., $1 \times 2 + 8 \times 1$)

16 10 1 (As per step (iv) multiply the 2 ones digits i.e., 1×1)

16 10 1 (As per step (v) single digit result so no carryover to 2nd blank)

16¹ 1 1 (As per step (vi) carryover tens part of blank 2 to blank 1)

17 1 1 (As per step (vii) add carryover and value and write down result in blank 1)

Final answer is **1711**.

(3) 78×61

Following the methods mentioned we put ____ ____ ____ (3 blanks as per step (i))

42 ____ ____ (As per step (ii) multiplying the 2 tens digits i.e., 7×6)

42 48+7 ____ (As per step (iii) multiply near and far digits and add i.e., $8 \times 6 + 7 \times 1$)

42 55 8 (As per step (iv) multiply the 2 ones digits i.e., 8×1)

42 55 8 (As per step (v) there is no carryover to 2nd blank)

42⁵ 5 8 (As per step (vi) carryover tens part of blank 2 to blank 1)

47 5 8 (As per step (vii) add carryover and value and write down result in blank 1)

Final answer is **4758**.

(4) 95×19

Following the methods mentioned we put ____ ____ ____ (3 blanks as per step (i))

9 ____ ____ (As per step (ii) multiplying the 2 tens digits i.e., 9×1)

9 5+81 ____ (As per step (iii) multiply near and far digits and add i.e., $5 \times 1 + 9 \times 9$)

9 86 45 (As per step (iv) multiply the 2 ones digits i.e., 5×9)

9 86⁴ 5 (As per step (v) carryover the tens part of 3rd blank to 2nd blank)

9⁹ 0 5 (As per step (vi) add carryover and value and again carryover tens part to blank 1)

18 0 5 (As per step (vii) add carryover and value and write down result in blank 1)

Final answer is **1805**.

14. Multiplication of a 3-digit number by a 2-digit number

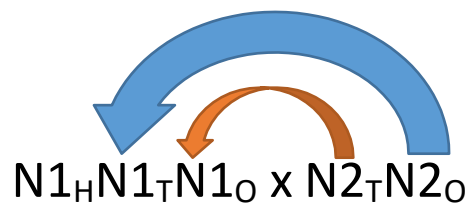
We have a method to solve multiplication of a 3-digit number by a 2-digit number in Vedic mathematics. (You can also use another method that is used to multiply two 3-digit numbers. That method is explained later in this document.)

Let us denote the 3-digit number as $N1_H N1_T N1_O$ where $N1_H$ is the hundreds digit of the number, $N1_T$ is the tens digit of the number and $N1_O$ is the ones digit. Now let the 2-digit number be denoted by $N2_T N2_O$ where $N2_T$ is the tens digit of the number and $N2_O$ is the ones digit. Always write the problem as 3-digit number multiplied by 2-digit number.

$$N1_H N1_T N1_O \times N2_T N2_O$$

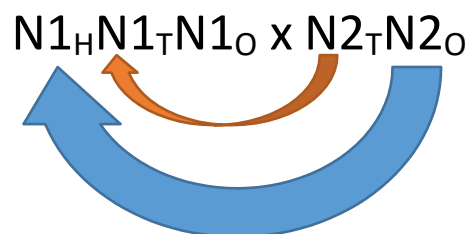
Now we have to do the following steps:

- i) First, put 4 blanks namely blank1, blank2, blank3 and blank4 i.e., _____
- ii) Blank 1 is calculated as product of the initial digits i.e., $N1_H \times N2_T$. So blanks now become $N1_H \times N2_T$ _____
- iii) Blank 4 is calculated as product of last digits i.e., $N1_O \times N2_O$. So the blanks now become $N1_O \times N2_O$ _____ $N1_O \times N2_T$
- iv) Blank 3 is calculated as the sum of 2 products as shown below:



Blank 3 = $(N2_O \times N1_T) + (N2_T \times N1_O)$. So the blanks now become
 $N1_H \times N2_O$ $(N2_O \times N1_T) + (N2_T \times N1_O)$ $N1_O \times N2_T$

- v) Blank 2 is calculated as the sum of 2 products as shown below:



Blank 2 = $(N2_O \times N1_H) + (N2_T \times N1_T)$. So the blanks now become
 $N1_H \times N2_O$ $(N2_O \times N1_H) + (N2_T \times N1_T)$ $(N2_O \times N1_T) + (N2_T \times N1_O)$ $N1_O \times N2_T$

- vi) We now have all the blanks filled with numbers. Starting from the fourth blank, check if the answer in the blank is a single digit or double/triple digit figure. If it is a double/triple digit figure, retain only the ones portion of the figure and carryover the remaining portion to the blank on the left.
- vii) Add the carryover (if present) to the value in the third blank and if answer is a 2/3 digit figure, follow the same procedure of retaining only the ones portion and carrying over the remaining portion to the next blank (to the left). Follow the procedure till you get final answer.

The examples will explain it clearly.

Examples:

(1) 121×23

Put 4 blanks as per step (i)

2 _ _ _ (1 x 2 as per step (ii))

2 _ _ 3 (1 x 3 as per step (iii))

2 _ (6)+(2) 3 (2x3 added to 1x2 giving 8 as per step (iv))

2 (3)+(4) 8 3 (1x3 added to 2x2 giving 7 as per step (v))

2783 is the final answer as there is no need of carryovers in any of the blanks

(2) 242×32

Put 4 blanks as per step (i)

6 _ _ _ (2 x 3 as per step (ii))

6 _ _ 4 (2 x 2 as per step (iii))

6 _ (8)+(6) 4 (4x2 added to 2x3 giving 14 as per step (iv))

6 (4)+(12) 14 4 (2x2 added to 4x3 giving 16 as per step (v))

6 16 14 4 (There are 2 digit number is blanks 2 & 3 so carryover to be done)

6 16¹ 4 4

6¹ 7 4 4

7744 is the final answer.

(3) 32×122

Rewrite it as 122×32

Put 4 blanks as per step (i)

3 _ _ _ (3 x 1 as per step (ii))

3 _ _ 4 (2 x 2 as per step (iii))

3 _ (4)+(6) 4 (2x2 added to 2x3 giving 10 as per step (iv))

3 (2)+(6) 10 4 (1x2 added to 2x3 giving 8 as per step (v))

3 8 10 4 (There is a 2-digit number is blank 3 so carryover to be done)

3 8¹ 0 4

3 9 0 4

3904 is the final answer.

(4) 153×61

Multiplication of a 3-digit number by a 2-digit number

Put 4 blanks as per step (i)

6 _ _ _ (1 x 6 as per step (ii))

6 _ _ 3 (3 x 1 as per step (iii))

6 _ _ (5)+(18) 3 (5x1 added to 3x6 giving 23 as per step (iv))

6 (1)+(30) 23 3 (1x1 added to 5x6 giving 31 as per step (v))

6 31 23 3 (There are 2 digit number is blanks 2 & 3 so carryover to be done)

6 31¹ 3 3

6 33 3 3

6³ 3 3 3

9333 is the final answer.

(5) 124

x34

Put 4 blanks as per step (i)

3 _ _ _ (1 x 3 as per step (ii))

3 _ _ 16 (4 x 4 as per step (iii))

3 _ _ (8)+(12) 16 (2x4 added to 4x3 giving 20 as per step (iv))

3 (4)+(6) 20 16 (1x4 added to 2x3 giving 10 as per step (v))

3 10 20 16 (There are 2 digit number is blanks 2,3 & 4 so carryover to be done)

3 10 20¹ 6

3 10² 1 6

3¹ 2 1 6

4216 is the final answer.

(6) 242

x32

Put 4 blanks as per step (i)

6 _ _ _ (2 x 3 as per step (ii))

6 _ _ 4 (2 x 2 as per step (iii))

6 _ _ (8)+(6) 4 (4x2 added to 2x3 giving 14 as per step (iv))

6 (4)+(12) 14 4 (2x2 added to 4x3 giving 16 as per step (v))

6 16 14 4 (There are 2 digit number is blanks 2 & 3 so carryover to be done)

6 16¹ 4 4

6 17 1 4

6¹ 7 4 4

7744 is the final answer.

15. Multiplication of a 3-digit number by a 3-digit number

We have a method to solve multiplication of a 3-digit number by a 2-digit number in Vedic mathematics by using cross-multiplication and addition in a certain pattern. You can also multiply a 3-digit number by a 2-digit number by converting the 2 digits into 3 digits by adding a zero in the hundreds place of the 2-digit number.

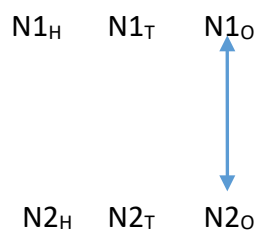
Let us denote the first 3-digit number as $N1_H N1_T N1_O$ where $N1_H$ is the hundreds digit of the number, $N1_T$ is the tens digit of the number and $N1_O$ is the ones digit. Now let the second 3-digit number be denoted by $N2_H N2_T N2_O$ where $N2_H$ is the hundreds digit of the number, $N2_T$ is the tens digit of the number and $N2_O$ is the ones digit.

$$\begin{array}{r} N1_H N1_T N1_O \\ \times N2_H N2_T N2_O \end{array}$$

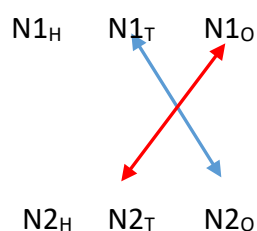
The final answer will be written in 5 blanks namely blank1 blank2 blank3 blank4 blank5. We start filling up the blanks from the right side i.e., blank5.

Now we have to do the cross-multiplication and addition in the following patterns:

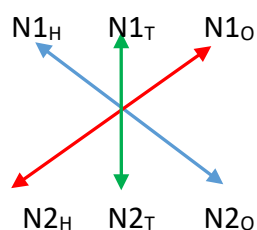
i) Step 1: $N1_O \times N2_O = \text{blank5}$



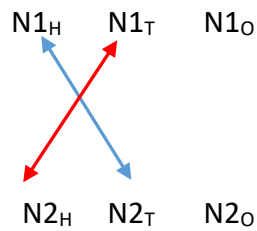
ii) Step 2: $(N1_T \times N2_O) + (N1_O \times N2_T) = \text{blank4}$



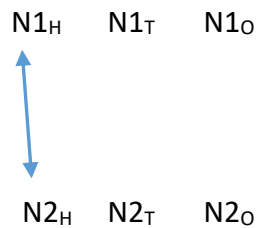
iii) Step 3: $(N1_H \times N2_O) + (N1_O \times N2_H) + (N1_T \times N2_T) = \text{blank3}$



iv) Step 4: $(N1_H \times N2_T) + (N1_T \times N2_H) = \text{blank2}$



v) Step 5: $(N1_H \times N2_H) = \text{blank1}$



vi) Step 6: Check the values in all the blanks starting from blank5. If they are not single digit values, retain only the ones portion of the value and carryover the remaining part to the next blank to its left till you reach blank1. That will be the final answer.

The examples will explain it clearly.

Examples:

(1) 358

X 452

Put 5 blanks. _____

_____ **16** (As per step (i), blank5 = $8 \times 2 = 16$)

_____ **50** **16** (As per step (ii), blank4 = $(5 \times 2) + (8 \times 5) = 10 + 40 = 50$)

_____ **63** **50** **16** (As per step (iii), blank3 = $(3 \times 2) + (8 \times 4) + (5 \times 5) = 6 + 32 + 25 = 63$)

_____ **35** **63** **50** **16** (As per step (iv), blank2 = $(3 \times 5) + (5 \times 4) = 15 + 20 = 35$)

12 **35** **63** **50** **16** (As per step (v), blank1 = $3 \times 4 = 12$)

Checking for carryovers as per step (vi), we get

12 35 63 50¹ 6

12 35 63⁵ 1 6

12 35⁶ 8 1 6

12⁴ 1 8 1 6

16 1 8 1 6

161806 is the final answer.

(2) 121×23

This can be written as

121

X 023

Put 5 blanks. _____

_____ **3** (As per step (i), blank5 = $1 \times 3 = 3$)

_____ **8 3** (As per step (ii), blank4 = $(2 \times 3) + (1 \times 2) = 6 + 2 = 8$)

_____ **7 8 3** (As per step (iii), blank3 = $(1 \times 3) + (1 \times 0) + (2 \times 2) = 3 + 0 + 4 = 7$)

_____ **2 7 8 3** (As per step (iv), blank2 = $(1 \times 2) + (2 \times 0) = 2 + 0 = 2$)

0 2 7 8 3 (As per step (v), blank1 = $2 \times 0 = 0$)

2783 is the final answer as there are no carryovers.

(3) 242×32

This can be written as

242

X 032

Put 5 blanks. _____

_____ **4** (As per step (i), blank5 = $2 \times 2 = 4$)

_____ **14 4** (As per step (ii), blank4 = $(4 \times 2) + (2 \times 3) = 8 + 6 = 14$)

_____ **16 14 4** (As per step (iii), blank3 = $(2 \times 2) + (2 \times 0) + (4 \times 3) = 4 + 0 + 12 = 16$)

_____ **6 16 14 4** (As per step (iv), blank2 = $(2 \times 3) + (4 \times 0) = 6 + 0 = 6$)

0 6 16 14 4 (As per step (v), blank1 = $2 \times 0 = 0$)

Checking for carryovers as per step (vi), we get

0 6 16¹ 4 4

0 6¹ 7 4 4

7744 is the final answer as there are no carryovers.

SQUARES

16. Squares of numbers containing only 1 as its digits

There is a simple method of finding the squares of numbers containing only 1 as its digits like 11, 111, 1111 etc.

First, count the number of digits in the number. Then, start writing from 1 to the number of digits and then in descending order. Examples will make it clear.

Examples:

(1) 11×11

There are 2 digits in 11.

So write **121** (1 to 2 and then descending till 1) as the answer.

(2) 111×111

There are 3 digits in 111.

So write **12321** (1 to 3 and then descending till 1) as the answer.

(3) 1111×1111

There are 4 digits in 1111.

So write **1234321** (1 to 4 and then descending till 1) as the answer.

(4) 11111×11111

There are 5 digits in 11111.

So write **123454321** (1 to 5 and then descending till 1) as the answer.

17. Squares of 2-digit numbers ending in 1

To calculate the squares of 2-digit numbers ending in 1, we need to do just a few steps. Let us consider the number to be N1 since the digit in the ones place is always 1.

Now do the following:

- i) Put 3 blanks for the result. _____
- ii) The rightmost blank (i.e., ones position/third blank) is always 1 since $1 \times 1 = 1$.
Write it down.
- iii) The middle blank (second blank) is $(N + N)$. Write it down.
- iv) The leftmost blank (first blank) is always $(N \times N)$.
- v) Starting from the second blank (middle blank), check if the answer in the blank is a single digit or double digit figure. If it is a double digit figure, retain only the ones portion of the figure and carryover the tens portion to the blank to its left (first/leftmost blank).
- vi) Add the carryover (if present) to the value in the first blank and write down the result in the first blank.
- vii) The final answer is the digits present in the three blanks.

The examples will explain it clearly.

Examples:

(1) 11×11

____ (as per step (i))

1 2 1 (As per steps (ii), (iii) and (iv) i.e., 1×1 , $1+1$ and 1×1)

Final answer is **121** as there are no carryovers in any of the blanks.

(2) 21×21

____ (as per step (i))

4 4 1 (As per steps (ii), (iii) and (iv) i.e., 1×1 , $2+2$ and 2×2)

Final answer is **441** as there are no carryovers in any of the blanks.

(3) 31×31

____ (as per step (i))

9 6 1 (As per steps (ii), (iii) and (iv) i.e., 1×1 , $3+3$ and 3×3)

Final answer is **961** as there are no carryovers in any of the blanks.

(4) 41×41

____ (as per step (i))

16 8 1 (As per steps (ii), (iii) and (iv) i.e., 1×1 , $4+4$ and 4×4)

Final answer is **1681** as there are no carryovers in any of the blanks.

(5) 51×51

____ (as per step (i))

25 10 1 (As per steps (ii), (iii) and (iv) i.e., 1×1 , $5+5$ and 5×5)

25¹ 0 1 (As per step (v))

Final answer is **2601** as per step (vi).

(6) 81×81

____ (as per step (i))

64 16 1 (As per steps (ii), (iii) and (iv) i.e., 1×1 , $8+8$ and 8×8)

64¹ 6 1 (As per step (v))

Final answer is **6561** as per step (vi).

(7) 91×91

____ (as per step (i))

81 18 1 (As per steps (ii), (iii) and (iv) i.e., 1×1 , $9+9$ and 9×9)

81¹ 8 1 (As per step (v))

Final answer is **8281** as per step (vi).

18. Squares of numbers from 11 to 19

To calculate the squares of numbers from 11 to 19, we need to do just a few steps.

Let us consider the number to be 1N since the digit in the tens place is always 1. Now do the following:

- i) Put 3 blanks for the result. _____
- ii) The rightmost blank (i.e., ones position/third blank) is the result of $N \times N$. Write it down.
- iii) The middle blank (second blank) is $N + N$. Write it down.
- iv) The leftmost blank (first blank) is always $1 \times 1 = 1$.
- v) Starting from the third blank (rightmost), check if the answer in the blank is a single digit or double digit figure. If it is a double digit figure, retain only the ones portion of the figure and carryover the tens portion to the blank to its left.
- vi) Add the carryover (if present) to the value in the second blank and if answer is a 2 digit figure, follow the same procedure of carrying over the tens portion to the first blank and retaining only the ones portion of the result in the second blank.
- vii) Add the carryover (if present) to the value in the first blank and write down the result in the first blank.
- viii) The final answer is the digits present in the three blanks.

The examples will explain it clearly.

Examples:

(1) 11×11

____ (as per step (i))

1 2 1 (As per steps (ii), (iii) and (iv) i.e., 1×1 , $1+1$ and 1×1)

Final answer is **121** as there are no carryovers in any of the blanks.

(2) 12×12

____ (as per step (i))

1 4 4 (As per steps (ii), (iii) and (iv) i.e., 2×2 , $2+2$ and 1×1)

Final answer is **144** as there are no carryovers in any of the blanks.

(3) 13×13

____ (as per step (i))

1 6 9 (As per steps (ii), (iii) and (iv) i.e., 3×3 , $3+3$ and 1×1)

Final answer is **169** as there are no carryovers in any of the blanks.

(4) 14×14

____ (as per step (i))

1 8 16 (As per steps (ii), (iii) and (iv) i.e., 4×4 , $4+4$ and 1×1)

1 ⁸ 6 (As per step (v), the 1 of blank 3 needs to be carried over to blank 2)

1 9 6 (As per step (vi))

Final answer is **196** as there are no more carryovers.

(5) 15 x 15

____ (as per step (i))

1 10 25 (As per steps (ii), (iii) and (iv) i.e., 5x5, 5+5 and 1x1)

1 10² 5 (As per step (v), the 2 of blank 3 needs to be carried over to blank 2)

1 12 5 (As per step (vi))

1¹ 2 5 (As per step (vii))

Final answer is **225** as per step (viii).

(6) 16 x 16

____ (as per step (i))

1 12 36 (As per steps (ii), (iii) and (iv) i.e., 6x6, 6+6 and 1x1)

1 12³ 6 (As per step (v), the 3 of blank 3 needs to be carried over to blank 2)

1 15 6 (As per step (vi))

1¹ 5 6 (As per step (vii))

Final answer is **256** as per step (viii).

(7) 19 x 19

____ (as per step (i))

1 18 81 (As per steps (ii), (iii) and (iv) i.e., 9x9, 9+9 and 1x1)

1 18⁸ 1 (As per step (v), the 8 of blank 3 needs to be carried over to blank 2)

1 26 1 (As per step (vi))

1² 6 1 (As per step (vii))

Final answer is **361** as per step (viii).

19. Squares of any 2-digit numbers

To calculate the square of any 2-digit number, we need to do just a few steps. Let us consider the number to be $N_T N_O$ where N_T is the tens digit of the number and N_O is the ones digit. Now do the following:

- i) First, calculate the product of the individual digits and then multiply it by 20 i.e.,
 $\text{Product} = N_T \times N_O \times 20$.
- ii) Next, find the squares of the individual digits i.e., $Sq_T = N_T \times N_T$ and $Sq_O = N_O \times N_O$. Please note that Sq_O should always be written as a 2-digit number. So if the answer has only 1 digit, you need to add a zero to its left to make it a 2-digit answer.
- iii) The final answer is $Sq_T Sq_O + \text{Product}$. ($Sq_T Sq_O$ is a single number by joining Sq_T and Sq_O)

The examples will explain it clearly.

Examples:

(1) $(41)^2 = 41 \times 41$

Product = $4 \times 1 \times 20 = 80$ (As per step (i))

$Sq_O = 1$ to be written as 01 since it should always be a 2 digit number and $Sq_T = 16$ (1×1 and 4×4 as per step (ii))

Final answer is $1601 + 80$ (As per step (iii) i.e., $Sq_T Sq_O + \text{Product}$)
= 1681

(2) $(51)^2 = 51 \times 51$

Product = $5 \times 1 \times 20 = 100$ (As per step (i))

$Sq_O = 1$ to be written as 01 since it should always be a 2 digit number and $Sq_T = 25$ (1×1 and 5×5 as per step (ii))

Final answer is $2501 + 100$ (As per step (iii) i.e., $Sq_T Sq_O + \text{Product}$)
= 2601

(3) $(58)^2 = 58 \times 58$

Product = $5 \times 8 \times 20 = 800$ (As per step (i))

$Sq_O = 64$ and $Sq_T = 25$ (8×8 and 5×5 as per step (ii))

Final answer is $2564 + 800$ (As per step (iii) i.e., $Sq_T Sq_O + \text{Product}$)
= 3364

(4) $(97)^2 = 97 \times 97$

Product = $9 \times 7 \times 20 = 1260$ (As per step (i))

$Sq_O = 49$ and $Sq_T = 81$ (7×7 and 9×9 as per step (ii))

Final answer is $8149 + 1260$ (As per step (iii) i.e., $Sq_T Sq_O + \text{Product}$)
= 9409

20. Squares of numbers from 101 to 110

To calculate the squares of numbers between 100 and 110, we just need to do a couple of steps:

- i) First, add the number to its digit in the ones place.
- ii) Next, find the square of the digit in the ones place. Please note that this should always be written as a 2-digit number. So if the answer has only 1 digit, you need to add a zero to its left to make it a 2-digit answer
- iii) The final answer is the results of steps (i) and (ii) written next to each other.

The examples will explain it clearly.

Examples:

(1) $(101)^2 = 101 \times 101$

$101 + 1 = 102$ (As per step (i))

$1 \times 1 = 01$ (As per step (ii); you need to write it as 01 as it has to be 2 digit number)

Final answer = **10201**

(2) $(102)^2 = 102 \times 102$

$102 + 2 = 104$ (As per step (i))

$2 \times 2 = 04$ (As per step (ii); you need to write it as 04 as it has to be 2 digit number)

Final answer = **10404**

(3) $(103)^2 = 103 \times 103$

$103 + 3 = 106$ (As per step (i))

$3 \times 3 = 09$ (As per step (ii); you need to write it as 09 as it has to be 2 digit number)

Final answer = **10609**

(4) $(104)^2 = 104 \times 104$

$104 + 4 = 108$ (As per step (i))

$4 \times 4 = 16$ (As per step (ii))

Final answer = **10816**

(5) $(105)^2 = 105 \times 105$

$105 + 5 = 110$ (As per step (i))

$5 \times 5 = 25$ (As per step (ii))

Final answer = **11025**

(6) $(106)^2 = 106 \times 106$

$106 + 6 = 112$ (As per step (i))

$6 \times 6 = 36$ (As per step (ii))

Final answer = **11236**

(7) $(107)^2 = 107 \times 107$
 $107 + 7 = 114$ (As per step (i))
 $7 \times 7 = 49$ (As per step (ii))
Final answer = **11449**

(8) $(108)^2 = 108 \times 108$
 $108 + 8 = 116$ (As per step (i))
 $8 \times 8 = 64$ (As per step (ii))
Final answer = **11664**

(9) $(109)^2 = 109 \times 109$
 $109 + 9 = 118$ (As per step (i))
 $9 \times 9 = 81$ (As per step (ii))
Final answer = **11881**

SQUARE ROOT

21. Square root of perfect squares (3 or 4 digits)

There is a method to find the square root of perfect squares. For that we first need to note the squares of single digits:

$$1^2 = 1$$

$$2^2 = 4$$

$$3^2 = 9$$

$$4^2 = 16$$

$$5^2 = 25$$

$$6^2 = 36$$

$$7^2 = 49$$

$$8^2 = 64$$

$$9^2 = 81$$

From this we can see that none of the above squares end with the digits 2, 3, 7 or 8. This implies that numbers ending in 2, 3, 7 or 8 are never perfect squares.

It also implies that

- For numbers ending in 1, the last digit of its square root is either 1 or 9
- For numbers ending in 4, the last digit of its square root is either 2 or 8
- For numbers ending in 5, the last digit of its square root is either 5
- For numbers ending in 6, the last digit of its square root is either 4 or 6
- For numbers ending in 9, the last digit of its square root is either 3 or 7

The method of finding square root can be explained with the help of examples.

Examples:

(1) $\sqrt{7921}$

- Take the last 2 digits of the number i.e., 21 in this case. Now look at the last digit which is 1 here. Since the digit is 1, it means that the last digit of $\sqrt{7921}$ is either **1** or **9**.
- Now take the remaining digits of the number (number is 7921) which in this case would be 79 (since have to remove the last 2 digits).
- We have to now check what the squares that are closest to this number are. In this case we get $8^2 = 64$ and $9^2 = 81$ which are the squares closest to 79. We have to select the smaller number which in this case would be **8**.
- So the final answer will be 81 or 89 (since we got the last digit as either 1 or 9 in step (a))
- To figure out whether the answer is 81 or 89, we have to multiply the 8 from 81/89 with 9 (which is $8+1$. Always add 1 and then multiply).
- We get the result as $8 \times 9 = 72$. Compare it with the 79 that we got in step (ii). If the number from step (ii) is bigger, then the answer is the bigger number. In this case $79 > 72$ so our final answer is **89** and not 81.

(2) $\sqrt{625}$

Last 2 digits are 25 and last digit is 5. Hence last digit of the square root is **5**.

Taking out the last 2 digits, we are left with 6.

The squares closest to 6 are $2^2 = 4$ and $3^2 = 9$. Since we have to take the smaller number, we will get **2**.

Final answer is **25**.

(3) $\sqrt{729}$

Last 2 digits are 29 and last digit is 9. Hence last digit of the square root is either **3** or **7**.

Taking out the last 2 digits, we are left with 7.

The squares closest to 7 are $2^2 = 4$ and $3^2 = 9$. Since we have to take the smaller number, we will get **2**.

Final answer is 23 or 27. To figure it out, we multiply $2 \times 3 = 6$ which is < 7 so the final answer is **27**.

(4) $\sqrt{5041}$

Last 2 digits are 41 and last digit is 1. Hence last digit of the square root is either **1** or **9**.

Taking out the last 2 digits, we are left with 50.

The squares closest to 50 are $7^2 = 49$ and $8^2 = 64$. Since we have to take the smaller number, we will get **7**.

Final answer is either 71 or 79. To figure it out, we multiply $7 \times 8 = 56$ which is > 50 so the final answer is **71**.

(5) $\sqrt{7569}$

Last 2 digits are 69 and last digit is 9. Hence last digit of the square root is either **3** or **7**.

Taking out the last 2 digits, we are left with 75.

The squares closest to 75 are $8^2 = 64$ and $9^2 = 81$. Since we have to take the smaller number, we will get **8**.

Final answer is either 83 or 87. To figure it out, we multiply $8 \times 9 = 72$ which is < 75 so the final answer is **87**.

Exceptions

(6) $\sqrt{1236}$

Last 2 digits are 36 and last digit is 6. Hence last digit of the square root is either **4** or **6**.

Taking out the last 2 digits, we are left with 12.

The squares closest to 12 are $3^2 = 9$ and $4^2 = 16$. Since we have to take the smaller number, we will get **3**.

Square Root of perfect squares (3 or 4 digits)

Final answer is either 34 or 36. To figure it out, we multiply $3 \times 4 = 12$ which is $= 12$ so then we need to multiply and see. $34 \times 34 = 1156$ and $36 \times 36 = 1296$ so the final answer is **36**.

22. Square root of imperfect squares

There is a method to find the square root of some simple imperfect squares. To do that we need to first find the nearest perfect square. The method is easier explained with examples.

Examples:

(1) $\sqrt{10}$

- i. This can be rewritten as $\sqrt{9+1}$ where 9 is the perfect square of 3.
- ii. This can now be written as $3 + 1/(3 \times 2)$. Note that the square root now becomes a perfect square +/- a fraction. **The denominator of the fraction is the value of twice the nearest perfect square that we used in step (i).**
- iii. Answer is $3 + (1/6) = 3 + 0.16 = \mathbf{3.16}$

(2) $\sqrt{35}$

- i. This can be rewritten as $\sqrt{36-1}$ where 36 is the perfect square of 6.
- ii. This can now be written as $6 - 1/(6 \times 2)$. Note that the square root now becomes a perfect square +/- a fraction. The denominator of the fraction is the value of twice the nearest perfect square that we used in step (i).
- iii. Answer is $6 - (1/12) = 6 - 0.08 = \mathbf{5.92}$

(3) $\sqrt{90}$

- i. This can be rewritten as $\sqrt{100-10}$ where 100 is the perfect square of 10.
- ii. This can now be written as $10 - 10/(10 \times 2)$. Note that the square root now becomes a perfect square +/- a fraction. The denominator of the fraction is the value of twice the nearest perfect square that we used in step (i).
- iii. Answer is $10 - (10/20) = 10 - 0.5 = \mathbf{9.5}$

(4) $\sqrt{65}$

- i. This can be rewritten as $\sqrt{64+1}$ where 64 is the perfect square of 8.
- ii. This can now be written as $8 + 1/(8 \times 2)$. Note that the square root now becomes a perfect square +/- a fraction. The denominator of the fraction is the value of twice the nearest perfect square that we used in step (i).
- iii. Answer is $8 + (1/16) = 8 + 0.06 = \mathbf{8.06}$

END
