Correction TD Maths EXO

EXO 3 :

1. $a|b = \bar{a}+\bar{b}$

a) $\bar{a}$

Comme $a|b = \bar{a}+\bar{b}$, alors

- $a|1 = \bar{a}+\bar{1} = \bar{a}+0 = \bar{a}$

$\boxed{\text{donc } \bar{a} = a|1}$

- $\bar{a}|\bar{b} = \bar{\bar{a}}+\bar{\bar{b}}$

$\bar{a}|\bar{b} = a+b$

$a|b = \bar{a}+\bar{b} = \overline{ab}$

$ab = \overline{a|b}$

- $a+\bar{b} = \bar{a}|b$

$\boxed{a+b = (a|1)|(b|1)}$

$\boxed{a+\bar{b} = (a|1)|b}$

$\boxed{a.b = (a|b)|1.}$

$\boxed{\bar{a} = a|1}$

b) Etudions l'associativité.

Soient $a, b, c \in A$. $(a|b)|c = a|(b|c)$.

$(a|b)|c = (\bar{a}+\bar{b})|c$

$= (\overline{\bar{a}+\bar{b}})+\bar{c}$

$= a.b+\bar{c}$

$G(a,b,c) = a.b+\bar{c}$

$a|(b|c) = a|(\bar{b}+\bar{c})$

$\bar{a}+(\overline{\bar{b}+\bar{c}})$

$F(a,b,c) = \bar{a}+bc$

Prenant $a=0$ $c=1$

$F(a,b,c) = 1$

$G(a,b,c) = 0$

$F \neq G$

L'opération de Sheffer n'est pas associatives.

$a\downarrow b = \bar{a}.\bar{b}$

$\boxed{a\downarrow a = \bar{a}.1 = \bar{a}}$

$a\downarrow b = \bar{a}.\bar{b} = \overline{a+b}$

$\overline{a\downarrow b} = \overline{\overline{a+b}} = a+b$

$\boxed{a+b = (a\downarrow b)\downarrow a}$