Gayeon Jang: CSCI 104 HW 1 Q3

a)

```
void f1(int n)
{
    int i=2;
    while(i < n){
        /* do something that takes O(1) time */ Θ(1)
        i = i*i; Θ(1)
    }
}
```

i
i
i^2
i^3
i^4
$\vdots$
$i^{k-1} < n$
$i^k \geq n$ terminate

$K = \log_i n$

$$T(n) = 2\sum_{i}^{\log_i n} \Theta(1) = \Theta(\log n)$$

b)

```
void f2(int n)
{
    for(int i=1; i <= n; i++){
        if( (i % (int)sqrt(n)) == 0){
            for(int k=0; k < pow(i,3); k++) {
              /* do something that takes O(1) time */ Θ(1)
            }
        }
    }
}
```

same as example in class

$$T(n) = \sum_{i=1}^{n}(\Theta(1) + O(\sum_{k=0}^{i^3}\Theta(1))) = \underbrace{\sum_{i=1}^{n}\Theta(1)}_{\Theta(n)} + \underbrace{\sum_{1}^{n}\sum_{0}^{i^3}\Theta(1)}_{\sum_{1}^{n}\Theta(i^3)} = \Theta(n) + \sum_{k=1}^{\sqrt{n}}\Theta(n^3) = \Theta(n) + \Theta(n^3\sqrt{n})$$

↳ upper bound n^3

$$T(n) = \Theta(n^3\sqrt{n})$$

c)

```
for(int i=1; i <= n; i++){
  for(int k=1; k <= n; k++){
    if( A[k] == i){
      for(int m=1; m <= n; m=m+m){
        // do something that takes O(1) time
        // Assume the contents of the A[] array are not cha
      }
    }
  }
}
```

$\Theta(1)$ $\Theta(n/2)$

m
m
2m
4m
8m
⋮
$2 \cdot km \geq n$
$k = \frac{n}{2m} = \frac{n}{2}$

$$T(n) = \sum_{i=0}^{n-1} \sum_{k=0}^{n-1} (\Theta(1) + O(n/2)) = \sum_{i=0}^{n-1} \sum_{k=0}^{n-1} \Theta(1) + \sum_{i=0}^{n-1} \sum_{k=0}^{n-1} O(n/2) = \sum_{i=0}^{n-1} \Theta(n) + \sum_{i=0}^{n-1} \Theta(n^2/2) = \Theta(n^2) + \Theta(n^3/2) = \Theta(n^3/2)$$

d)

```
int f (int n)
{
  int *a = new int [10];
  int size = 10;
  for (int i = 0; i < n; i ++)
     {
        if (i == size)
          {
            int newsize = 3*size/2;
            int *b = new int [newsize];
            for (int j = 0; j < size; j ++) b[j] = a[j];
            delete [] a;
            a = b;
            size = newsize;
          }
        a[i] = i*i;
     }
}
```

$\Theta(n)$ $\Theta(1)$ $\Theta(1)$ $\Theta(size)$

$\frac{3}{2}k \cdot size \leq n$

$k = \frac{2n}{3size} \approx n$

$$T(n) = \Theta(1) + \sum_{0}^{n-1} \sum_{0}^{n-1} \Theta(1) = \Theta(n^2)$$