Machine learning introduction

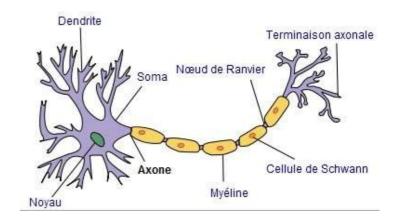
Part I – From neurons to networks

2 - The Formal Neuron

Simon Gay

Github: https://github.com/gaysimon/ARTISAN2022

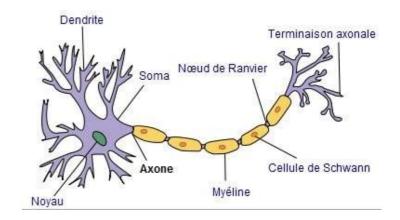
- The biological neuron:
 - Dendrites collect input signals
 - The body processes these signals
 - An axon conveys the output signal
 - Information is encoded with impulses (frequency)



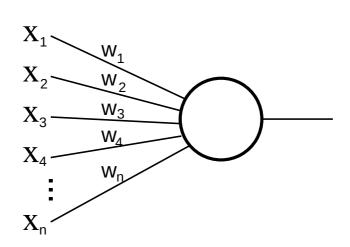
- The model must be simplified to work on a computer:
 - /!\ The model of the formal neuron is a VERY simplified model of the biological neuron



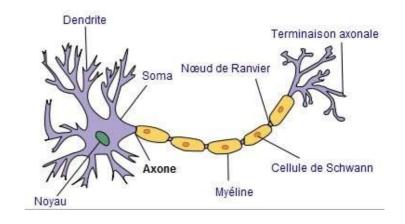
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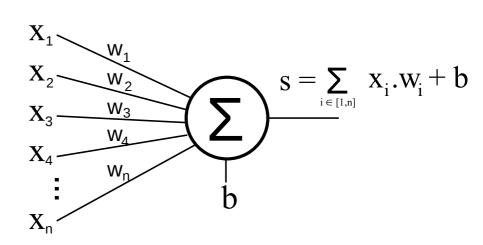
- The model must be simplified to work on a computer:
 - Frequency is replaced with a real value
 - Input: a vector of real values
 - Vector $X = \{ x_1, x_2, x_3, ..., x_n \}$
 - A vector of real values replace synapses
 - Vector W = $\{ w_1, w_2, w_3, ..., w_n \}$



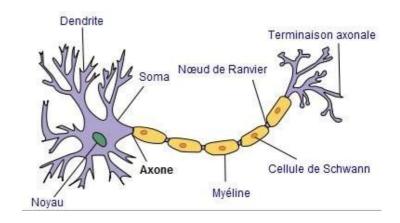
- The biological neuron:
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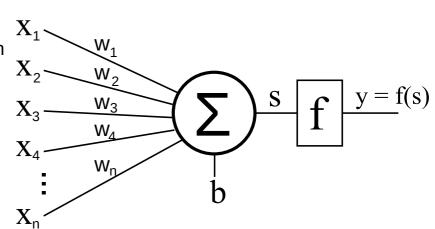
- The model must be simplified to work on a computer:
 - Input processing:
 - Sum of inputs weighted by synaptic weights
 - A bias b



- The biological neuron:
 - Dendrites collect input signals
 - The body processes these signals
 - An axon conveys the output signal
 - Information is encoded with impulses (frequency)



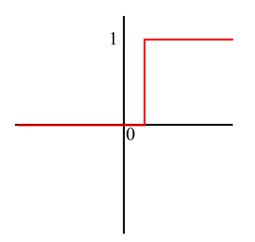
- The model must be simplified to work on a computer:
 - The output passes through an activation function
 - Limits the neuron's output value



The activation function

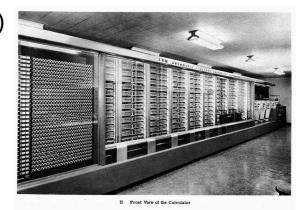
Threshold function : S = 1 when y>threshold

S = 0 otherwise



(note: The threshold value is not important due to the bias)

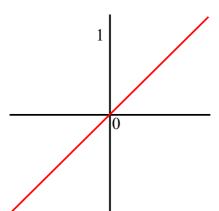
 Function used in the first neuron models (proposed by Warren McCulloch and Walter Pitts in 1943)

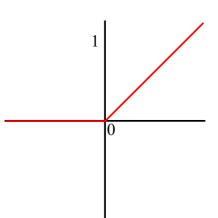


- Several limitations:
 - Neurons' outputs cannot be compared
 - In a network, output "strength" cannot be transmitted to other neurons

The activation function

- Linear functions and ReLU (Rectified Linear Unit)
 - y = x
 - y = max(0,x)



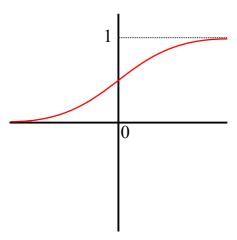


- Very simple functions
- ReLU allows removing negative values
 - Often used in deep learning approaches
- The output value can be greater than 1 (Weighted sum s must approach 1)
- Constant gradient (variations of a weight have the same effect on the output)

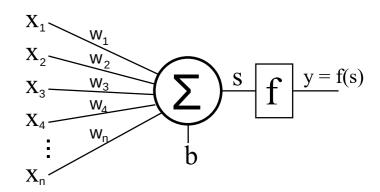
The activation function

- Sigmoid function
 - $y = \frac{1}{1 + e^{-x}}$

(note: the tanh function is also used)



- More complex functions
- High values provide an output close to 1 (or 0 for negative values)
- The gradient tends to 0 for high values
 - Stabilization of weights (variations become weaker and weaker)
 - But we lose the gradient (can be problematic when using multiple layers)
- Another great advantage (we will see this later)



Methods for supervised learning

- Hebb rule (1949): "cells that fire together, wire together" (biological observations)
 - $w_i^{t+1} = w_i^t + \alpha$. x_i . r: if result r and input x_i have same sign, weight w_i is reinforced
 - α Is the *learning rate* (limiting weights variations)
- Delta rule : weights are modified according to the output error :

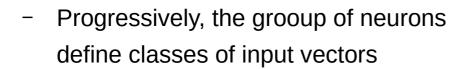
$$\Delta = r - y$$

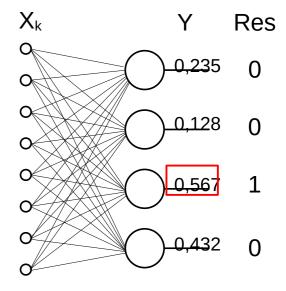
- Perceptron rule (Rosenblatt, 1957) : use threshold activation function, thus $\Delta \in \{-1; 0; 1\}$
- Widrow-Hoff rule (1960) : continuous activation functions, thus $\Delta \in \mathbb{R}$
- → Weights are modified to makes the output closer from expected result
 - principles: modify each weight w_i according to :
 - The input value (a higher value will have a greater influence on the output)
 - The difference (Δ) between output and expected result (the greater the difference, the more the weights must be modified)
 - $w_i^{t+1} = w_i^t + \alpha \cdot x_i \cdot \Delta$
- Neurons are trained on large number of examples

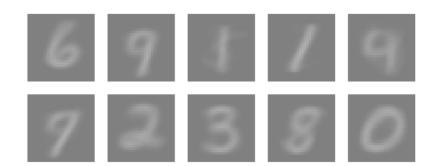
...And some for unsupervised learning

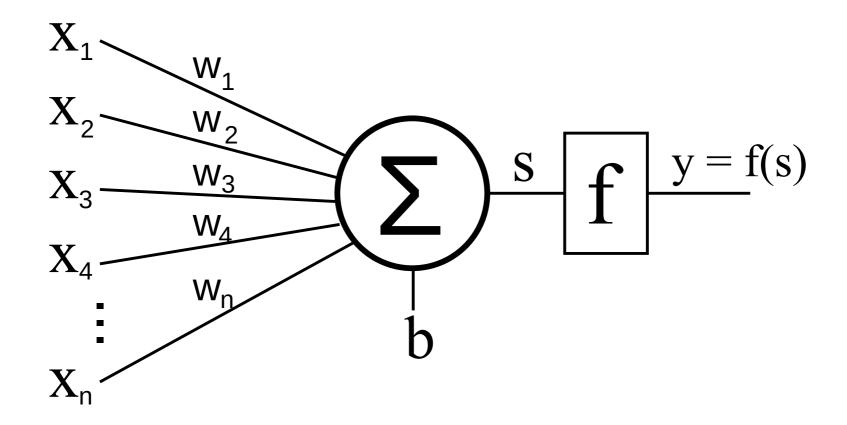
- Winner-Takes-All :
 - Multiple neurons in competition
 - The neuron with highest input inibates others
 - Weights are modified until convergence
- Grossberg rule :
 - Neuron with highest output update its weights to get closer to the input vector

$$w_i^{t+1} = w_i^t + \alpha \times (x_i - w_i^t)$$









Enough with theory, now to practice!

Github: https://github.com/gaysimon/ARTISAN2022

The Workspace

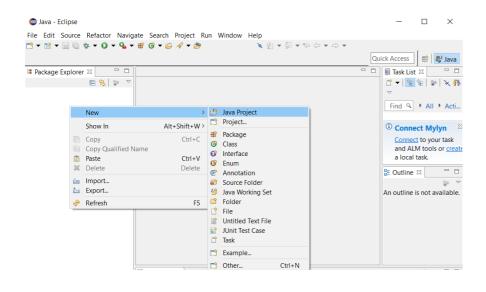
- Create a 'Workspace' folder somewhere (e.g. in 'my documents' or 'Desktop')
- Create a folder 'classes' and a folder 'numbers' beside your workspace folder
- classes
 numbers
 workspace

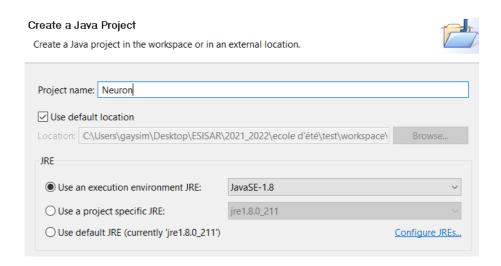
- Download the 3 files at https://github.com/gaysimon/ARTISAN2022/tree/main/classes
 - Click on green button 'Code' then 'Download Zip'
 - Unzip the three files in 'classes' folder
- Download the 4 dataset files at http://yann.lecun.com/exdb/mnist/ and unzip them in 'numbers' (/!\ be sure that the four files have the same name as below)



The Workspace

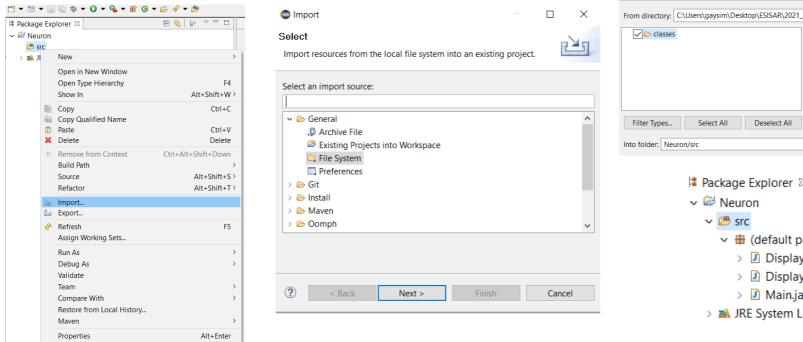
- open Eclipse and select your 'workspace' folder, then click on 'Workbench' (top right corner)
- Create a new project 'Neuron'
 - Right click in the package explorer (left part) → new → Java project
 - Name your project 'Neuron', then click 'Finish'

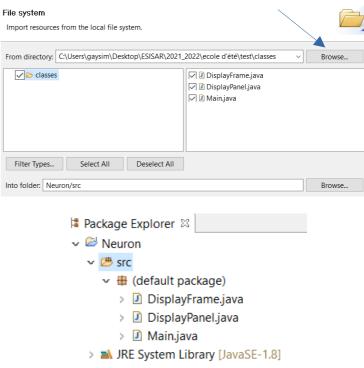




The Workspace

- Import the java file from the previously downloaded archive
 - Expand the project in package explorer
 - Right click on 'src' (left part) → import, then 'general' → 'file system'.
 - Browse to your 'classes' folder and select Main.java, DisplayFrame.java and DisplayPanel.java
 - Click on 'Finish' to complete the import

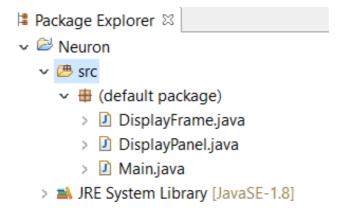




□ Package Explorer □

■ JRE System Library [JavaSE-1.8]

The Workspace



3 classes :

- Main.java :
 - main function
 - A function to load datasets (setData)
 - Bases for training and exploiting a neuron
- DisplayFrame.java and DisplayPanel.java :
 - Create a window to display properties of the neuron

The Workspace

- Finally, double-click on the *Main* class to open it, and change the *path* value (line 9) to indicate the position of your 'numbers' folder.
 - /!\ use '\\' folder separator on windows and '/' on Linux/Mac

```
Main.java \( \text{1} \) import java.io.BufferedInputStream; \( \text{1} \)

public class Main {

public static String PATH="C:\\Users\\gaysim\\Desktop\\numbers\\"; // path to images

public static int size_x=28; // size of the image (set when image is loaded)

public static int size_y=28;
```

Test the application with 'run' button to test your installation

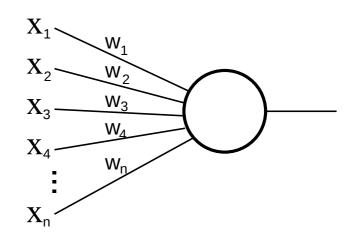




- Creating an artificial neuron
 - We will implement an artificial neuron and add it to our project
 - First, create a new class in your project
 - Right click on 'default package' → new class. Name your class 'Neuron'
 - We first implement weights and a bias

```
Main.java

1
2 public class Neuron {
3
4    public float[] synaps;
    public float bias;
6
7 }
8
```



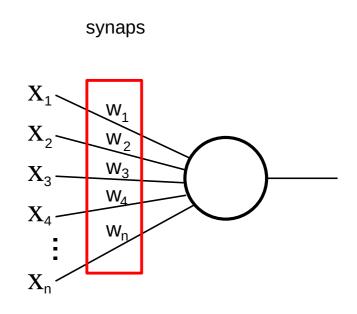
- Creating an artificial neuron
 - We then write the class constructor, and initialize the weight vector and bias
 - The parameter size gives the number of weights

```
Main.java

Neuron.java

public class Neuron {
    public float[] synaps;
    public float bias=0;

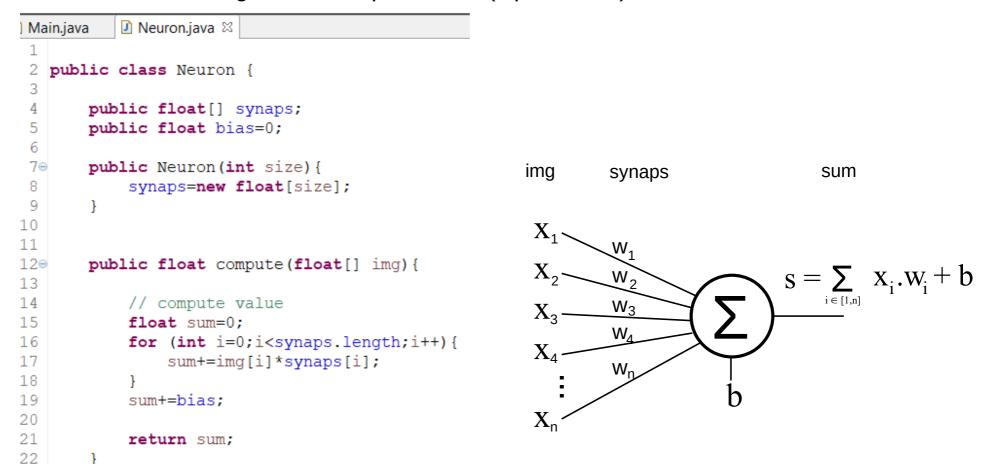
public Neuron(int size) {
        synaps=new float[size];
    }
}
```



Creating an artificial neuron

23 }

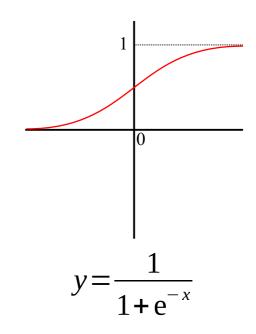
- We add a function to compute the output of the neuron
 - A vector img is used as parameter (input vector)

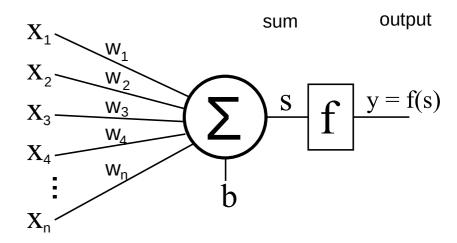


· Creating an artificial neuron

We forgot the activation function!

```
2 public class Neuron {
3
4
      public float[] synaps;
                                               We will also
      public float bias=0;
                                                record the
      public float output=0;
                                                output value
8
      public Neuron(int size) {
9⊜
          synaps=new float[size];
0
      public float compute(float[] imq){
4
          // compute value
          float sum=0;
          for (int i=0;i<synaps.length;i++) {</pre>
              sum+=img[i]*synaps[i];
9
0
          sum+=bias;
          output=activation(sum);
          return output;
4
5
6⊜
      private float activation(float x) {
7
          return (float) (1 / (1+Math.exp(-x)));
8
```





Creating an artificial neuron

- We then define a function to train the neuron :
 - Widrow-Hoff:

```
- \Delta = r - y → difference between expected result and output

- w_i^{t+1} = w_i^t + \alpha \cdot x_i \cdot \Delta → each weight is updated in proportion to related input value
```

- First, we add the delta value and a learning rate with a value of 0,01
- Then, a learning function that computes the delta value, using input vector and expected result

```
Main.java

1
2 public class Neuron {
3
4     public float learnRate=0.01f;
5
6     public float[] synaps;
7     public float bias=0;
8
9     public float output=0;
10     public float delta=0;
11
120     public Neuron(int size) {
```

```
25          output=activation(sum);
26          return output;
27     }
28
29     public void learn(float[] img, int res) {
          delta=res-output;
31
32     }
33
34     private float activation(float x) {
```

- Creating an artificial neuron
 - We then define a function to train the neuron :
 - Widrow-Hoff:

```
- \Delta = r - y → difference between expected result and output

- w_i^{t+1} = w_i^t + \alpha \cdot x_i \cdot \Delta → each weight is updated in proportion to related input value
```

- Finally, the learn function update weight according to Widrow-Hoff rule :
 - Don't forget the bias!

```
public void learn(float[] img, int res) {
    delta=res-output;

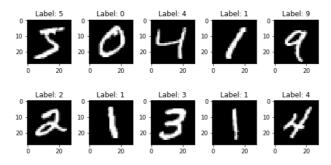
for (int i=0;i<synaps.length;i++) {
        synaps[i]+= learnRate * delta * img[i];
}

bias+=learnRate * delta;
}</pre>
```

Congratulations! Your neuron is complete!

Training and exploiting a neuron

- We will use our neuron on a dataset called MNIST number.
 - This dataset is composed of two sets of small images of 28x28 pixels of handwritten digits (0 to 9)



- A first dataset of 60 000 samples is used for training
- A second dataset of 10 000 samples is used for testing
- t10k-labels.idx1-ubyte
 train-images.idx3-ubyte
 train-labels.idx1-ubyte

t10k-images.idx3-ubyte

- Each dataset is composed of two files: a file containing the pixel values of images, and a file containing the label (number) of images
- A function is provided in main class to read these files and load samples in a matrix and labels in a vector (setData)

Training and exploiting a neuron

- In class *main*, uncomment the line adding the neuron as a variable

```
30 public Neuron neuron; // neuron
31
```

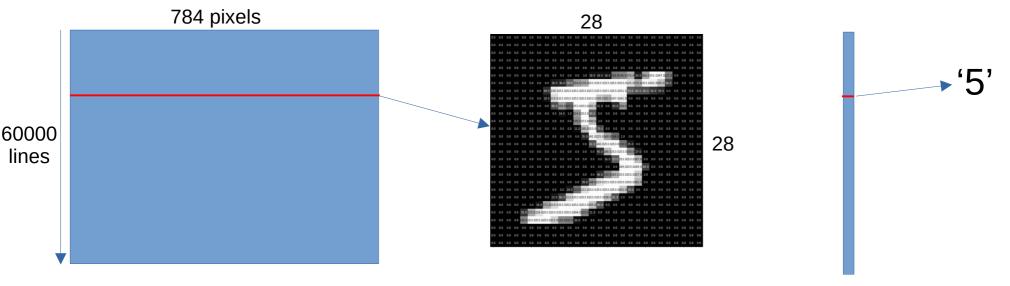
- The neuron must have a weight for each pixel of image. We thus initialize the neuron with 28x28 weights :

```
34
    public Main() {
35⊜
36
       37
38
       // initialization
       39
                                         784 inputs
       neuron=new Neuron(size x*size y); ◀
40
41
       // load test matrix
42
43
       setData("train-images.idx3-ubyte", "train-labels.idx1-ubyte");
44
       // initialize display frame
45
       display=new DisplayFrame(this);
46
```

- The lines after load the training dataset and initialize the display window

Training and exploiting a neuron

 The MNIST training dataset is loaded in a matrix (for the images) and a vector (for the labels)



- We will get sample images by reading the matrix line by line and the label by reading the same line on label vector
 - The neuron does not consider the position of pixels on the 2D image: it gets the image as a vector of pixels!

Training and exploiting a neuron

 We will train our neuron to recognize a digit. Choose a digit (0-9) for the number variable (you can change later)

```
public int number=2; // choose a number
```

- The training cycle works as follow:
 - For each sample of the dataset
 - Get next sample and its label
 - Compute output value of the neuron
 - Reinforce the neuron using expected result (delta)
- The neuron can be trained on the dataset multiple times to improve results. Each dataset cycle is called 'epoch'
 - We will train the neuron on 50 epoches

Training and exploiting a neuron

- First, we define the expected result: 1 when the digit is the right one, 0 otherwise

```
// learning
// apply the learning process 50 times
for (epoch=0;epoch<50;epoch++) {</pre>
   float sumdelta=0; // measure total error of the current epoch
   // for each test image
   for (test=0;test<matrixImages.length;test++) {</pre>
      // set output value
      int expected=0;
      if (matrixLabels[test] == number) expected=1;
      display.repaint();
      try {Thread.sleep(100);
      } catch (InterruptedException e) {e.printStackTrace();}
```

Training and exploiting a neuron

- Then, the neuron's output is computed on sample image(and stored in its output variable)

```
// apply the learning process 50 times
for (epoch=0;epoch<50;epoch++) {</pre>
    float sumdelta=0; // measure total error of the current epoch
    // for each test image
    for (test=0;test<matrixImages.length;test++) {</pre>
        // set output value
        int expected=0;
        if (matrixLabels[test] == number) expected=1;
        // process neuron
        neuron.compute(matrixImages[test]); // get result
        display.repaint();
        try {Thread.sleep(100);
        } catch (InterruptedException e) {e.printStackTrace();}
```

Training and exploiting a neuron

Finally, reinforce the neuron. We also record the delta to observe the evolution of predictions

```
// apply the learning process 50 times
for (epoch=0;epoch<50;epoch++) {</pre>
    float sumdelta=0; // measure total error of the current epoch
    // for each test image
    for (test=0;test<matrixImages.length;test++) {</pre>
        // set output value
        int expected=0;
        if (matrixLabels[test] == number) expected=1;
        // process neuron
        neuron.compute(matrixImages[test]); // get result
        // reinforce neuron
        neuron.learn(matrixImages[test], expected);
        // add delta to the error sum
        sumdelta+=Math.abs(neuron.delta);
        display.repaint();
        try {Thread.sleep(100);
        } catch (InterruptedException e) {e.printStackTrace();}
```

Training and exploiting a neuron

 In class DisplayPanel, uncomment the second part in PaintComponent procedure (remove '/*' on line 34) to display the neuron's weights. Then, test your application

```
g.fillRect(10+5*i, 10+5*j, 5, 5);
31
33
            for (int i=0;i<Main.size x;i++) {</pre>
34
35
                for (int j=0;j<Main.size y;j++) {</pre>
36
                     val=(int) (main.neuron.synaps[i+Main.size x*j]*50)+128;
37
                     if (val<0) val=0;</pre>
38
                     if (val>255) val=255;
39
                     g.setColor(new Color(val, val, val));
                     g.fillRect(180+3*i, 10+3*j, 3, 3);
41
            }/**/
            /*for (int n=0; n<10; n++) {
                for (int i=0;i<Main.size x;i++) {
```

If the program runs correctly, comment or remove the sleep fonction in main class.

```
display.repaint();

//try {Thread.sleep(100);

//} catch (InterruptedException e) {e.printStackTrace();}
}
```

What do you observe on neuron's weights and average delta?

Training and exploiting a neuron

- The weights form the 'average' shape of the selected digit
- The average delta first decreases quickly but seems to converge



Now, let's test our neuron on the test dataset (loaded hereafter)

- Training and exploiting a neuron
 - We simply get neuron's predictions for each sample image :
 - We can aslo count the number of prediction errors :

Not bad for 10 000 samples errors: 254

The formal neuron

- The formal neuron makes an average of positive and negative results to generate an "average image", exposing the features of the element to recognize









- This average image allows detecting the presence of the associated element on an unknown input vector.
- But Why does this works ?

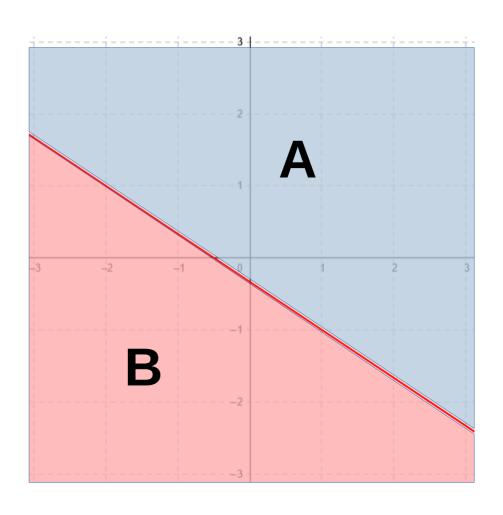
Let's consider a linear function

example:

$$3.y + 2.x + 1 = 0$$
 $(y = -(2/3).x - 1/3)$

This function separates the plane in 2:

- -3.y+2.x+1>0
 - → area A above the line
- -3.y+2.x+1<0
 - → area B below the line



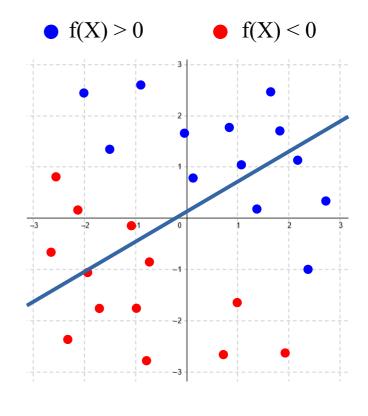
And if we do not know the function parameters?

We start with a function a.x + b.y + c = 0 with unknown parameters (a,b,c)

We have a set of points $X_i = [x_i, y_i]$ which position $f(X_i) > 0$ or $f(X_i) < 0$ is known

 \rightarrow We have to find a triplet (a , b , c) satisfying all point conditions

- We get a random triplet
- Each point is tested
 - If $f(X_i)>0$ but $ax_i+by_i+c<0$
 - ax_i+by_i+c must be increased
 - If $f(X_i)<0$ but $ax_i+by_i+c>0$
 - ax_i+by_i+c must be decreased
 - otherwise, we do not change parameters



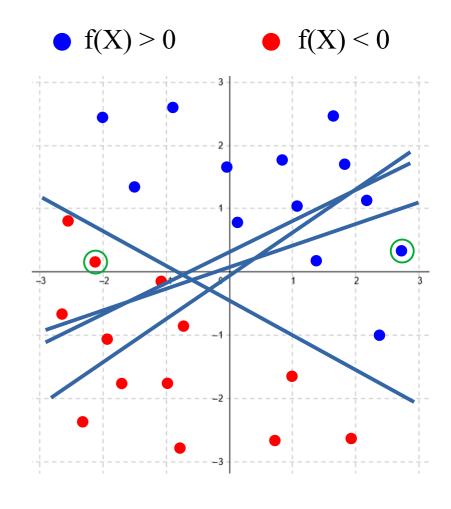
And if we do not know the function parameters?

For each parameter a, b and c, we sligtly increase or decrease the value proportionnally to, respectively, x (a), y (b) and 1 (c)

- If we note $r_i = 1$ when $f(X_i)>0$ and $r_i = -1$ when $f(X_i)<0$
 - then
 - $a \leftarrow a + \alpha \cdot r_i \cdot x_i$
 - $b \leftarrow b + \alpha \cdot r_i \cdot y_i$
 - $c \leftarrow c + \alpha \cdot r_i$
 - Points X_i are tested, until they are all on the right side of the line

The parameters will converge until finding a solution

```
• Algorithm :  errors = true  while errors do  errors = false  for each Xi do  if \ f(X_i) \cdot (a.x_i + b.y_i + c) < 0 \ do   a += \alpha \cdot r_i \cdot x_i   b += \alpha \cdot r_i \cdot y_i   c += \alpha \cdot r_i   errors = true  end if  end \ for \ each  end while
```



What has this got to do with the neuron?

Let's write:

$$- a.x + b.y + c = 0$$
 \rightarrow $w_1.x_1 + w_2.x_2 + b = 0$

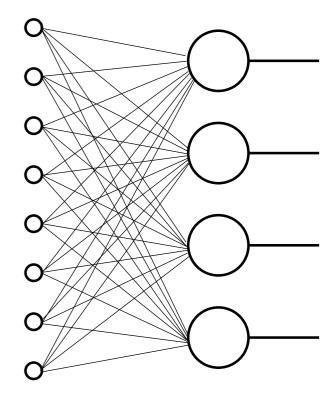
We generalize to a space with n-dimensions:

$$\sum_{k} w_{k}.x_{k}+b=0$$

- Weights of a neuron form the equation of a hyperplane
- The reinforcement modifies weights to separate space in two groups of points

- A neuron can only recognize one type of element
- But with multiple neurons, it is possible to categorize multiple elements
- With a threshold activation function:
 - The output is a binary vector
 - Conversion to binary code, ASCII code...





→ **Perceptrons**

Neurons with continual activation function (linear, ReLU, Sigmoid...)

- Each neuron is trained to recognize a category of element (e.g. a number)
 - The training process must convert labels into binary vectors
 - $'2' \rightarrow \{0,0,1,0,0,0,0,0,0,0,0,0\}$
 - Each neuron needs a binary output for training
 - Neurons are in competition: the neuron with the greatest output provide the output of the network, with the confidence of the result

Often used for object recognition

Implementing a layer of neuron

 Simple way: create a class that manage a vector of neurons and provide a vector of results as output

```
2 public class Layer {
       public Neuron[] layer;
5
6
7
8
9e
       public float[] output;
       public float delta=0;
       public Layer(int nb, int size) {
10
            layer=new Neuron[nb];
                                      // declare neuron vector
12
13
14
            for (int i=0;i<nb;i++) { // initialize each neuron</pre>
                layer[i]=new Neuron(size);
16
            output=new float[nb]; // declare the output vector
17
18 }
```

Initialize the vector of neurons

```
public float[] compute(float[] img) {

public float[] compute(float[] img) {

for (int i=0;i<layer.length;i++) {
    output[i]=layer[i].compute(img);
}

return output;
}</pre>
```

Compute output of each neuron And write their results in output vector

```
public void learn(float[] img, int[] results){

delta=0;

for (int i=0;i<layer.length;i++){
    layer[i].learn(img, results[i]);

delta+=Math.abs(layer[i].delta);
}

}</pre>
```

Reinforce neurons individually
Notice that result is a vector (result for each neuron)

- Implementing a layer of neuron
 - Training: the result vector must be obtained from labels:

```
// for each test image
for (test=0;test<matrixImages.length;test++){

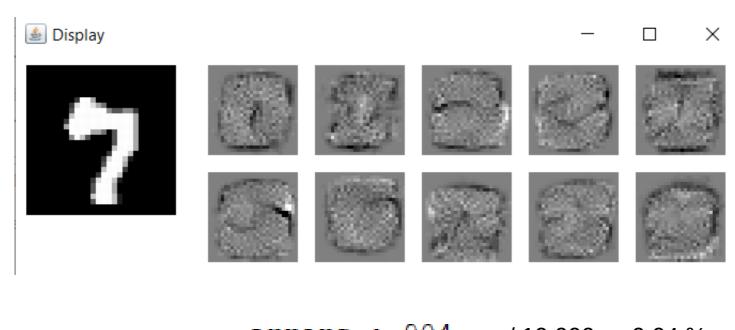
// set output value
int[] expected=new int[10];
expected[matrixLabels[test]] = 1;</pre>
```

Expected is a vector of 10 values, with only one value at 1

The result is obtained by finding the maximum output :

```
99
             // test each image of the dataset
100
             for (test=0;test<matrixImages.length;test++) {</pre>
101
102
                 // process neurons
103
                 layer.compute(matrixImages[test]);
104
105
                 float max=0;
106
                 int imax=0;
107
                 for (int i=0;i<10;i++) {
108
                     if (layer.output[i]>max) {
109
                         max=layer.output[i];
110
                         imax=i;
111
112
113
                 System.out.println(matrixLabels[test]+" -> "+imax);
114
```

• Each neuron specializes for a digit, the highest output provide the number



errors : $904 / 10000 \rightarrow 9.04\%$

ANNEX 1 provide details of this implementation