Machine learning introduction

Part I – From neurons to networks

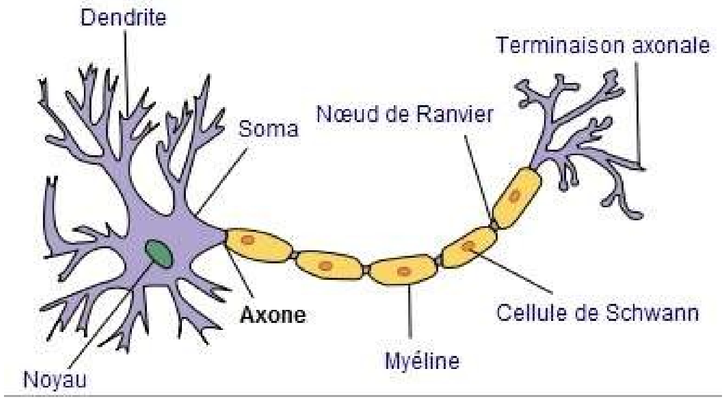
2 - The Formal Neuron

Simon Gay

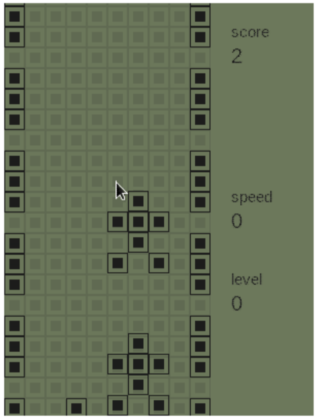
Github : https://github.com/gaysimon/ARTISAN2022

Part I-2 : the formal neuron

- **A simplified model of the neuron:**
  - The biological neuron:
    - Dendrites collect input signals
    - The body processes these signals
    - An axon conveys the output signal
    - Information is encoded with impulses (frequency)



  - The model must be simplified to work on a computer:
    - /!\ The model of the formal neuron is a VERY simplified model of the biological neuron



https://github.com/tshiamobhuda/Racing

Part I-2 : the formal neuron

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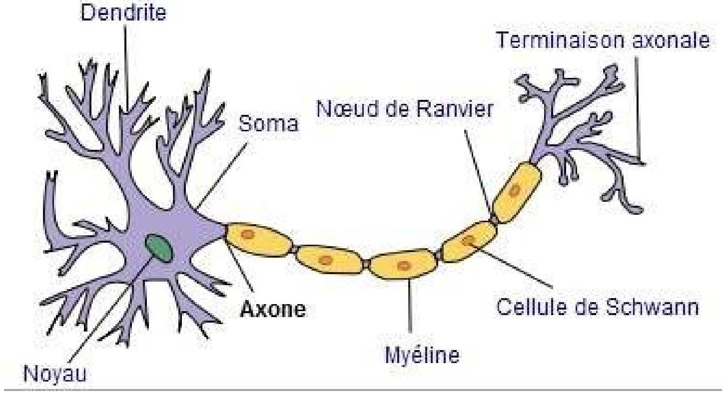
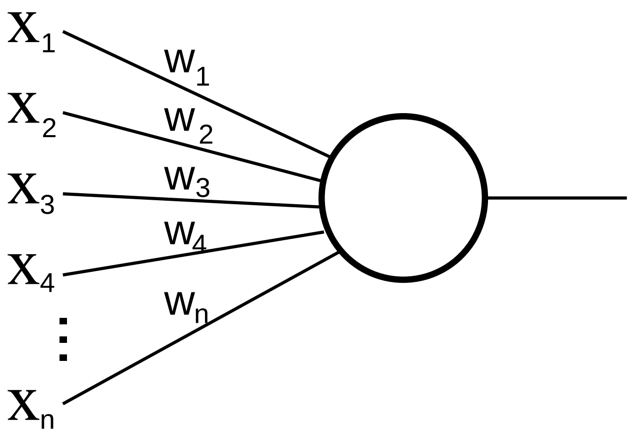


  - The model must be simplified to work on a computer:
    - Frequency is replaced with a real value
    - Input: a vector of real values
      - Vector X = { x_1, x_2, x_3, … , x_n }
    - A vector of real values replace synapses
      - Vector W = { w_1, w_2, w_3, … , w_n }



Part I-2 : the formal neuron

- **A simplified model of the neuron:**
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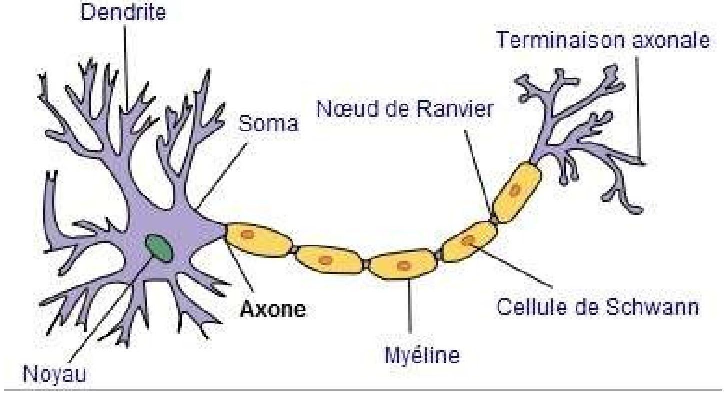
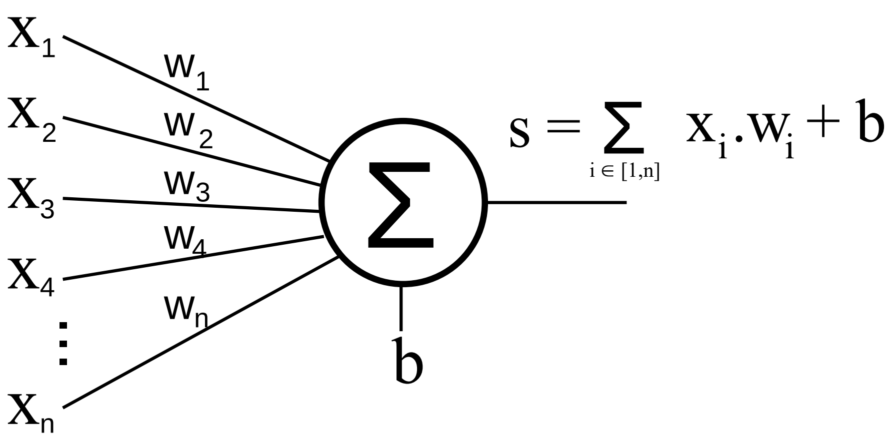


  - The model must be simplified to work on a computer:
    - Input processing:
      - Sum of inputs weighted by synaptic weights
      - A bias *b*



Part I-2 : the formal neuron

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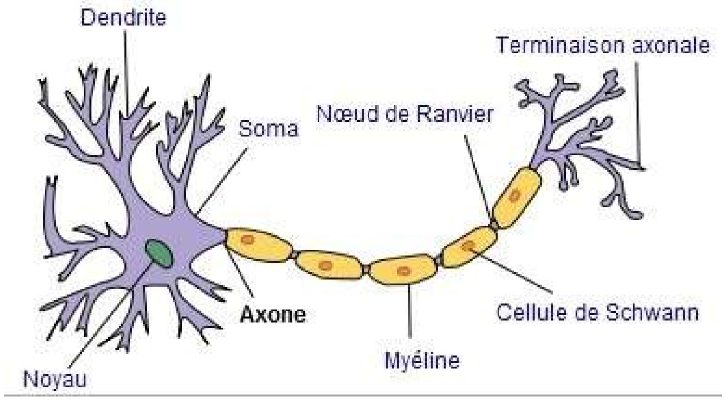
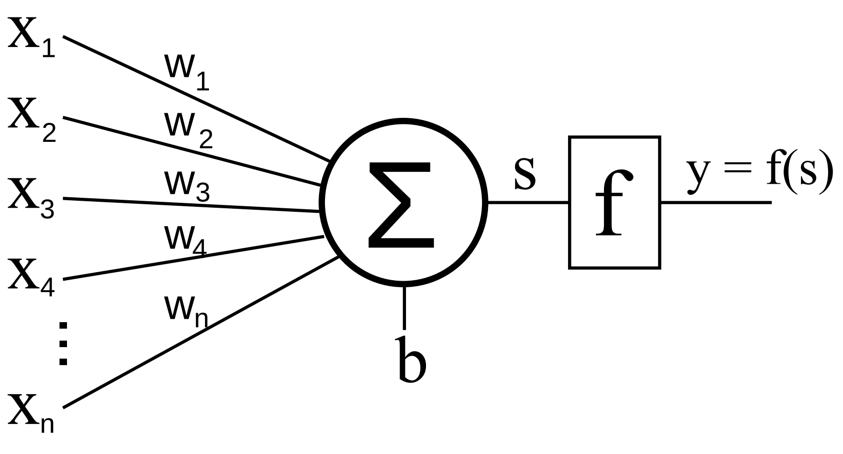
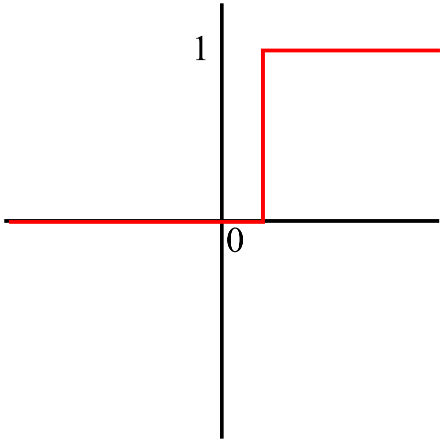


  - The model must be simplified to work on a computer:
    - The output passes through an activation function
      - Limits the neuron's output value



Part I-2 : the formal neuron

- **The activation function**

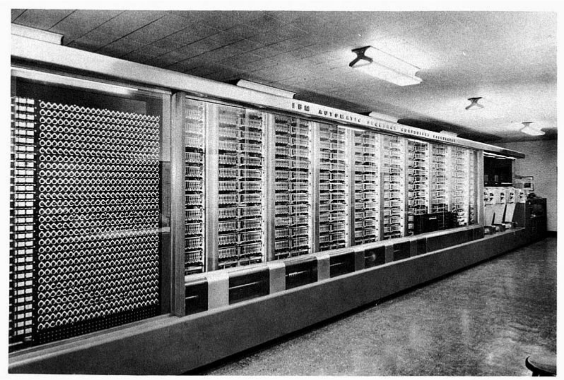


  - Threshold function : y = 1 when s>threshold

    y = 0 otherwise

    (note: The threshold value is not important due to the bias)

  - Function used in the first neuron models (proposed by Warren McCulloch and Walter Pitts in **1943**)



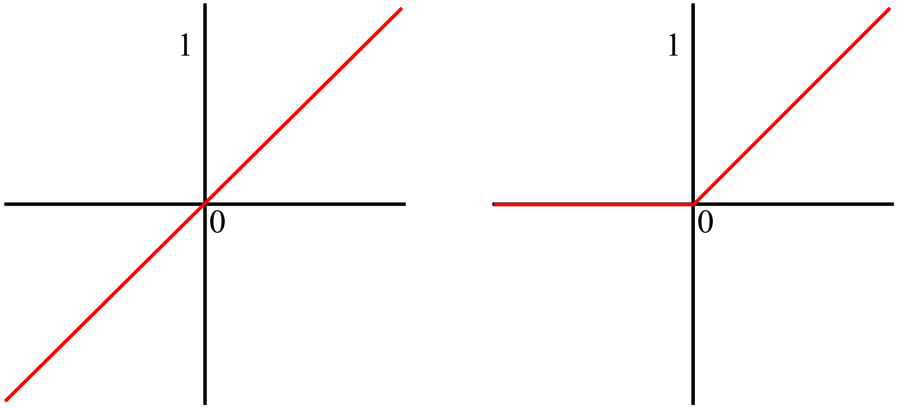
II Front View of the Calculator

  - Several limitations:
    - Neurons' outputs cannot be compared
    - In a network, output "strength" cannot be transmitted to other neurons

Part I-2 : the formal neuron

- **The activation function**

  - Linear functions and ReLU (Rectified Linear Unit)
    - y = x
    - y = max(0,x)



  - Very simple functions
  - ReLU allows removing negative values
    - Often used in deep learning approaches

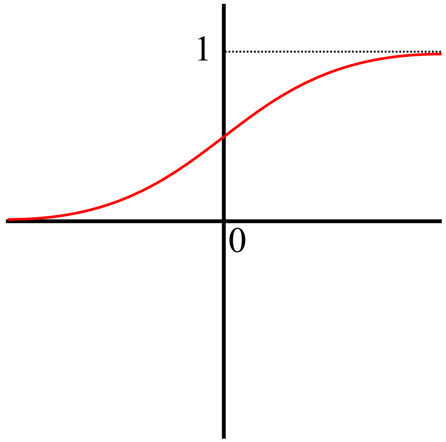
  - The output value can be greater than 1 (Weighted sum s must approach 1)
  - Constant gradient (variations of a weight have the same effect on the output)

Part I-2 : the formal neuron

- **The activation function**

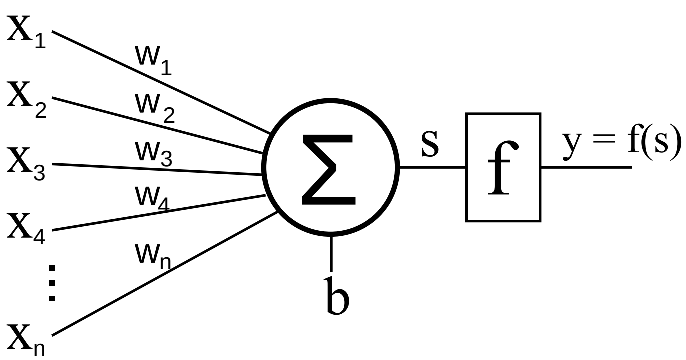
  - Sigmoid function
    - $$y = \frac{1}{1 + e^{-x}}$$

    (note: the tanh function is also used)



  - More complex functions
  - High values provide an output close to 1 (or 0 for negative values)
  - The gradient tends to 0 for high values
    - Stabilization of weights (variations become weaker and weaker)
    - But we lose the gradient (can be problematic when using multiple layers)
  - Another great advantage (we will see this later)

Part I-2 : the formal neuron



- **Methods for supervised learning**
  - Hebb rule (1949) : "*cells that fire together, wire together*" *(biological observations)*
    - $w_i^{t+1} = w_i^t + \alpha \, . \, x_i \, . \, r$: if result *r* and input x_i have same sign, weight w_i is reinforced
    - α Is the *learning rate* (limiting weights variations)
  - Delta rule : weights are modified according to the output error :

$$\Delta = r - y$$

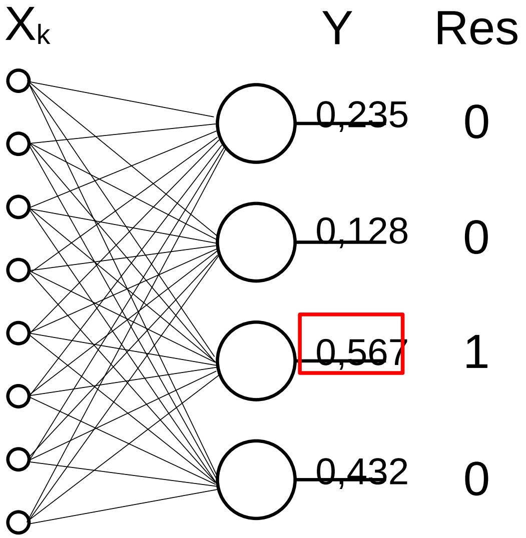
    - Perceptron rule (Rosenblatt, 1957) : use threshold activation function, thus $\Delta \in \{ -1 \,;\, 0 \,;\, 1 \}$
    - Widrow-Hoff rule (1960) : continuous activation functions, thus $\Delta \in \mathbb{R}$

  → Weights are modified to makes the output closer from expected result

    - principles: modify each weight w_i according to :
      - The input value (a higher value will have a greater influence on the output)
      - The difference (Δ) between output and expected result (the greater the difference, the more the weights must be modified)
    - $w_i^{t+1} = w_i^t + \alpha \, . \, x_i \, . \, \Delta$
  - Neurons are trained on large number of examples

Part I-2 : the formal neuron

- **...And some for unsupervised learning**

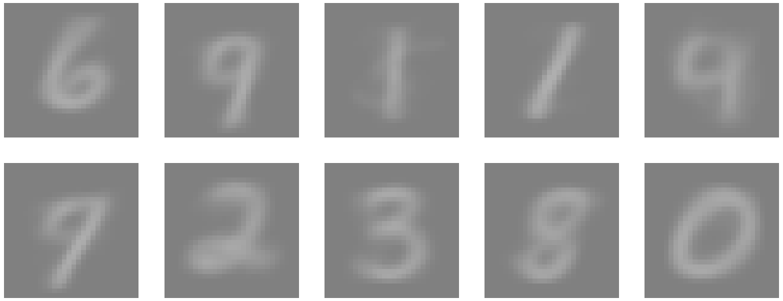


  - Winner-Takes-All :
    - Multiple neurons in competition
    - The neuron with highest input inibates others
    - Weights are modified until convergence

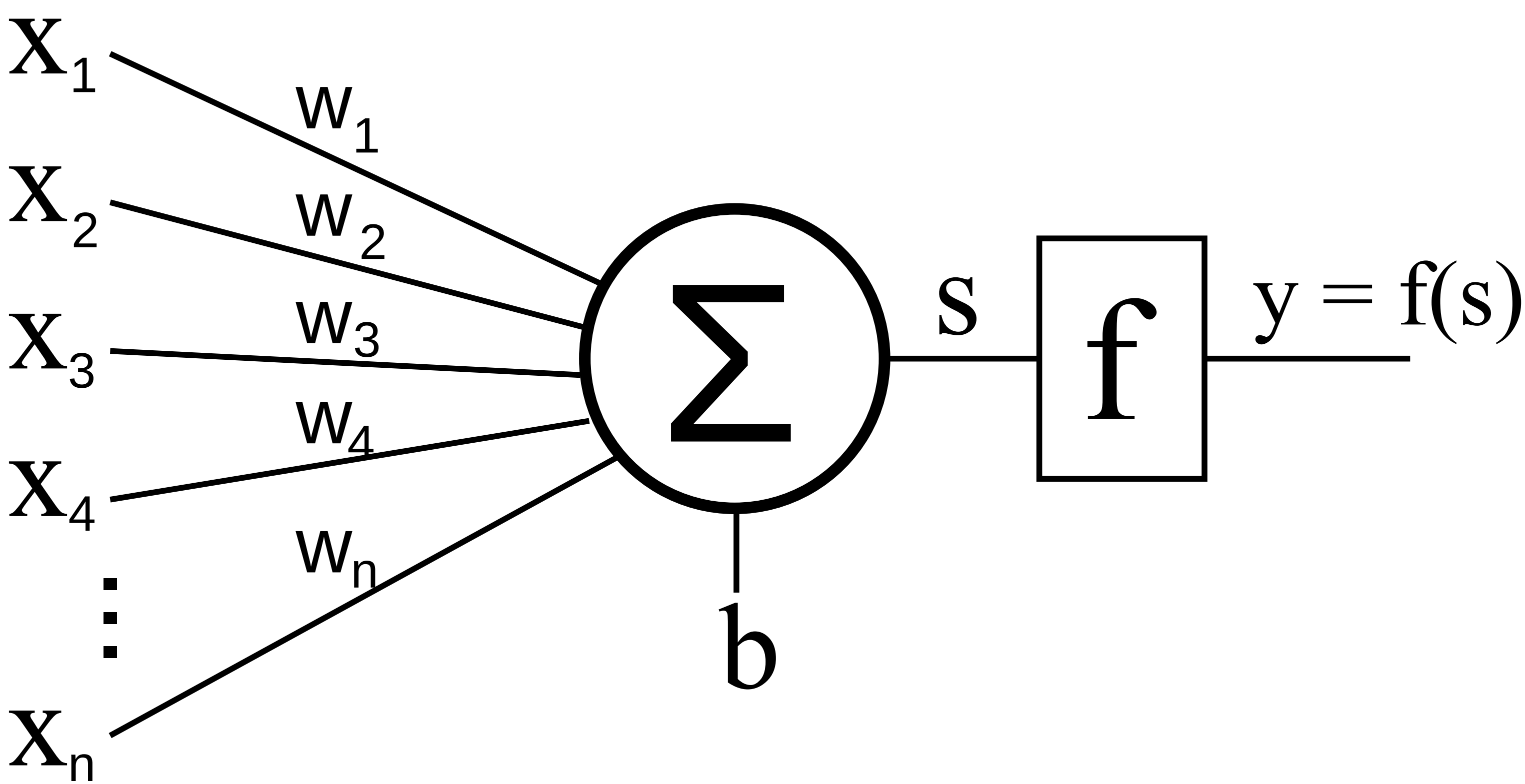
  - Grossberg rule :
    - Neuron with highest output update its weights to get closer to the input vector

$$w_i^{t+1} = w_i^t + \alpha \times (x_i - w_i^t)$$

  - Progressively, the grooup of neurons define classes of input vectors



Part I-2 : the formal neuron

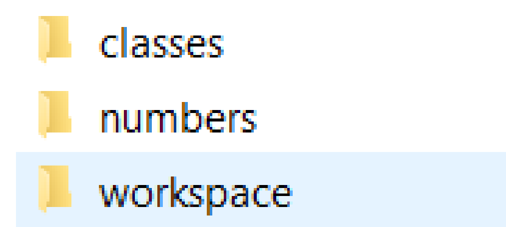


Enough with theory, now to practice!

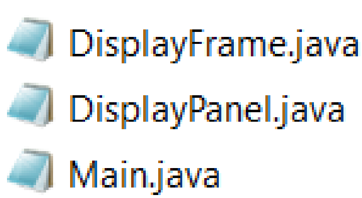
Github : https://github.com/gaysimon/ARTISAN2022

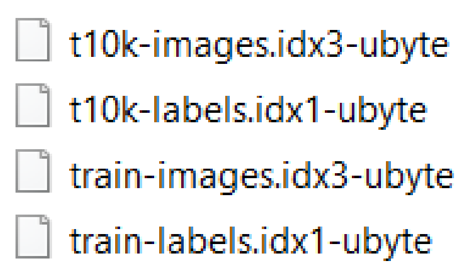
Part I-2 : the formal neuron

- **The Workspace**
  - Create a '*Workspace*' folder somewhere (e.g. in 'my documents' or 'Desktop')
  - Create a folder 'classes' and a folder 'numbers' beside your *workspace* folder



  - Download the 3 files at https://github.com/gaysimon/ARTISAN2022/tree/main/classes
    - Click on green button 'Code' then 'Download Zip'
    - Unzip the three files in 'classes' folder
  - Download the 4 dataset files at http://yann.lecun.com/exdb/mnist/ and unzip them in 'numbers' (/!\ be sure that the four files have the same name as below)

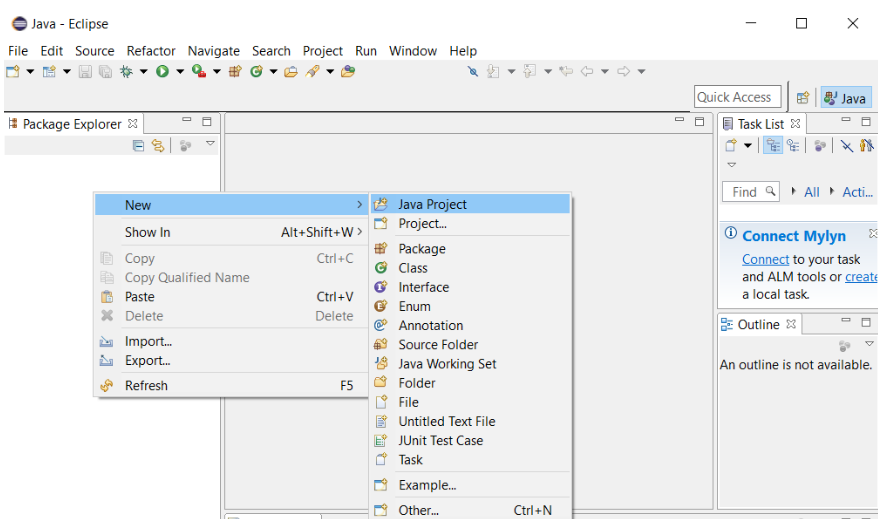


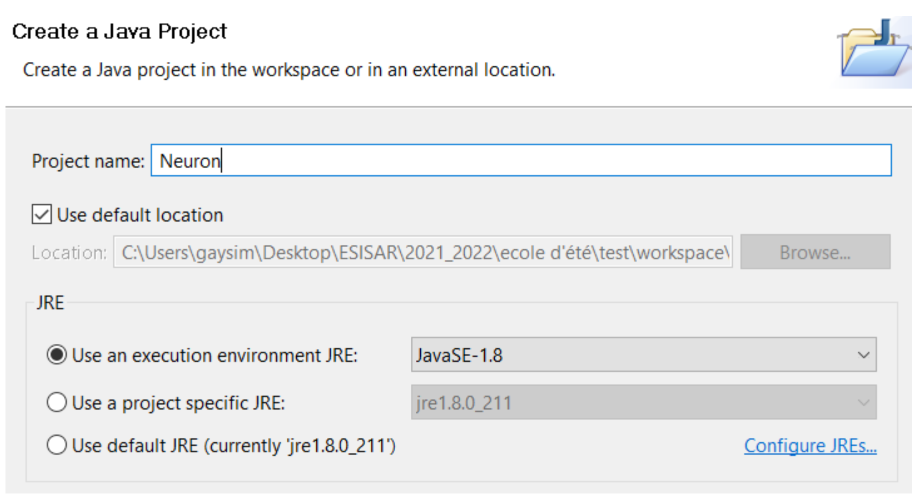


Part I-2 : the formal neuron

- **The Workspace**

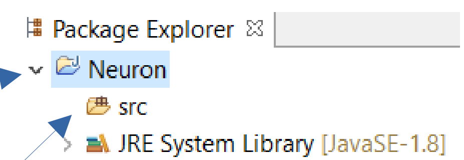
  - open Eclipse and select your ‘workspace’ folder, then click on ‘Workbench’ (top right corner)
  - Create a new project ‘Neuron’
    - Right click in the package explorer (left part) → new → Java project
    - Name your project ‘Neuron’, then click ‘Finish’

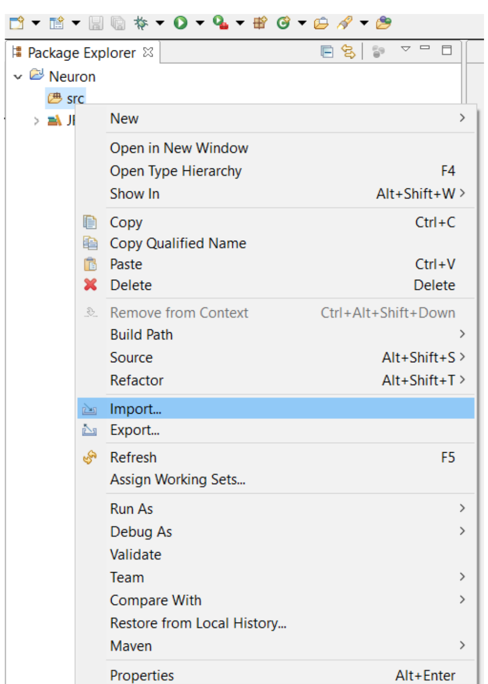


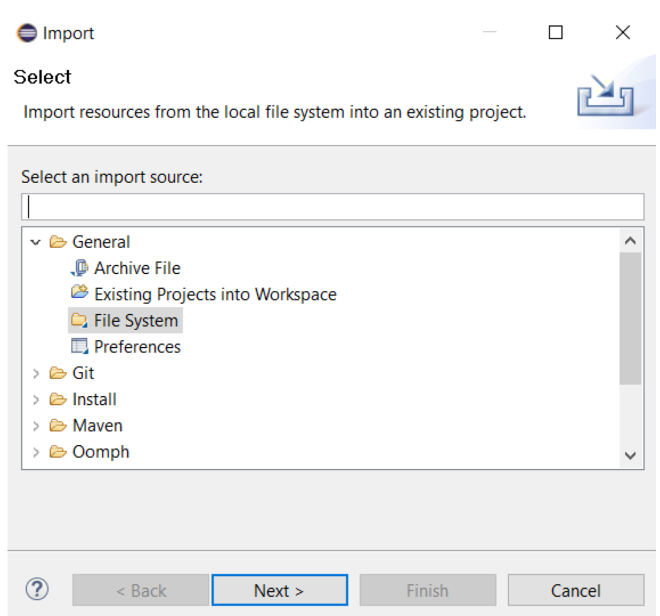


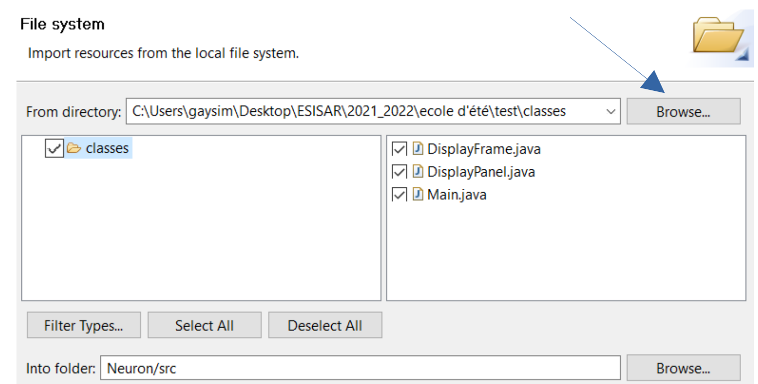
Part I-2 : the formal neuron

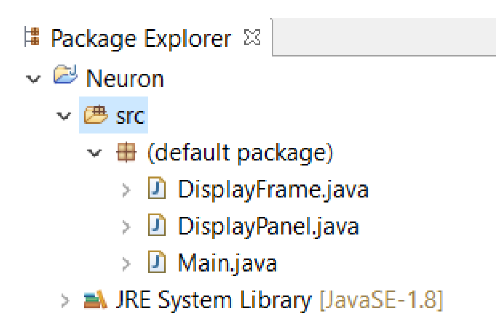
- **The Workspace**
  - – Import the java file from the previously downloaded archive
    - Expand the project in package explorer
    - Right click on ‘src’ (left part) → import, then ‘general’ → ‘file system’.
    - Browse to your ‘classes’ folder and select *Main.java*, *DisplayFrame.java* and *DisplayPanel.java*
    - Click on ‘Finish’ to complete the import





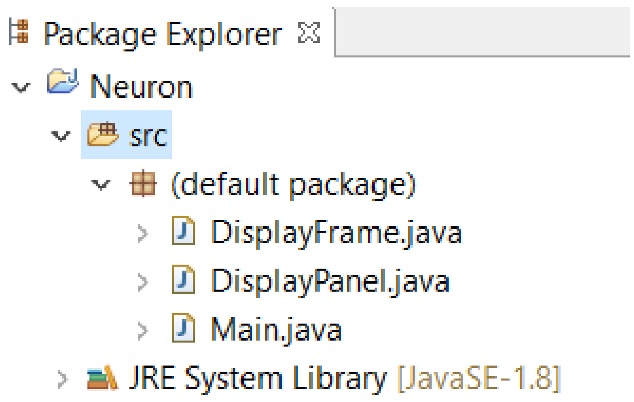






Part I-2 : the formal neuron

- **The Workspace**



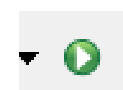
  - 3 classes :
    - **Main.java** :
      - main function
      - A function to load datasets (*setData*)
      - Bases for training and exploiting a neuron
    - **DisplayFrame.java** and **DisplayPanel.java** :
      - Create a window to display properties of the neuron

Part I-2 : the formal neuron

- **The Workspace**
  - Finally, double-click on the *Main* class to open it, and change the *path* value (line 9) to indicate the position of your ‘numbers’ folder.
    - ***/!*** *use ‘\\’ folder separator on windows and ‘/’ on Linux/Mac*

Main.java

```
 1 import java.io.BufferedInputStream;
 5
 6
 7 public class Main {
 8
 9     public static String PATH="C:\\Users\\gaysim\\Desktop\\numbers\\";  // path to images
10
11     public static int size_x=28;    // size of the image (set when image is loaded)
12     public static int size_y=28;
```

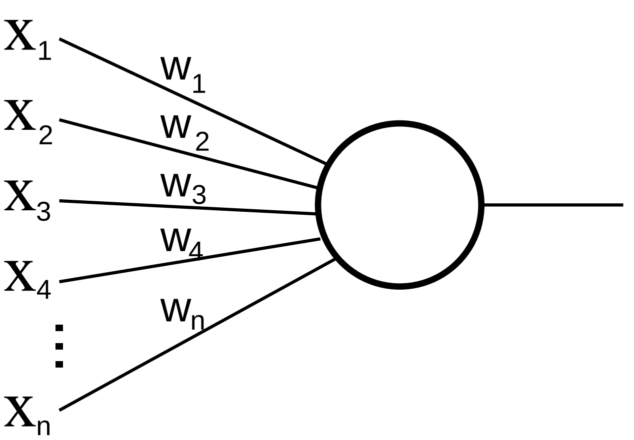
  - Test the application with ‘run’ button to test your installation 

Part I-2 : the formal neuron

- **Creating an artificial neuron**
  - We will implement an artificial neuron and add it to our project
  - First, create a new class in your project
    - Right click on ‘default package’ → new class. Name your class ‘Neuron’
  - We first implement weights and a bias

Main.java | *Neuron.java

```
public class Neuron {

	public float[] synaps;
	public float bias;

}
```

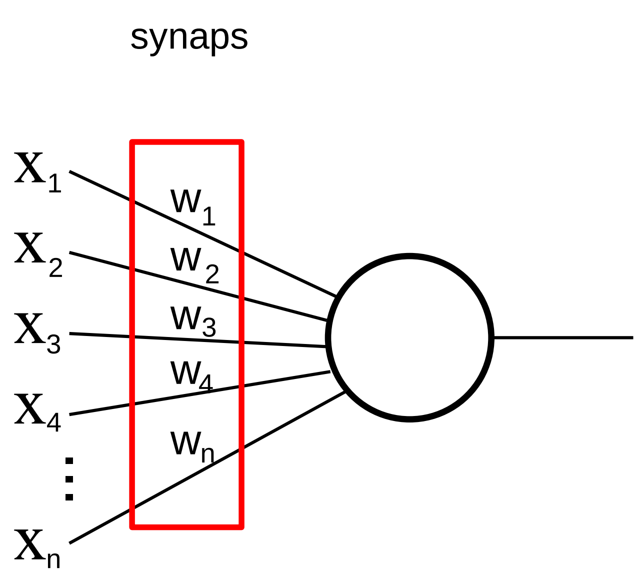


Part I-2 : the formal neuron

- **Creating an artificial neuron**
  - We then write the class constructor, and initialize the weight vector and bias
    - The parameter *size* gives the number of weights

Main.java | Neuron.java

```java

public class Neuron {

    public float[] synaps;
    public float bias=0;

    public Neuron(int size){
        synaps=new float[size];
    }
}

```

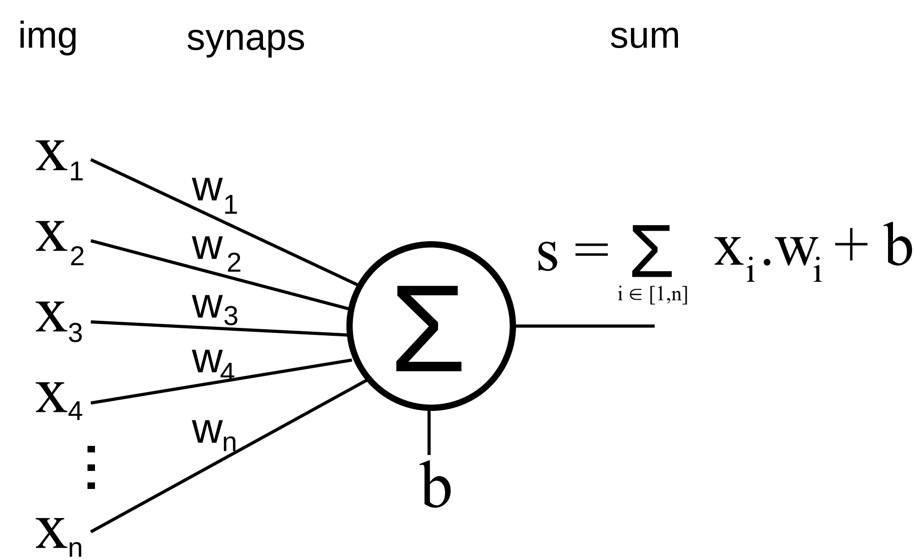


Part I-2 : the formal neuron

- **Creating an artificial neuron**
  - We add a function to compute the output of the neuron
    - A vector *img* is used as parameter (input vector)

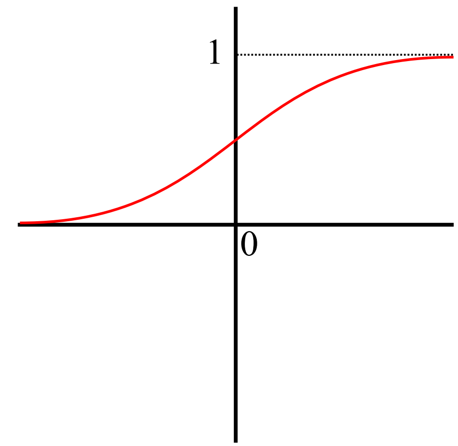
Main.java | Neuron.java

```java

public class Neuron {

    public float[] synaps;
    public float bias=0;

    public Neuron(int size){
        synaps=new float[size];
    }


    public float compute(float[] img){

        // compute value
        float sum=0;
        for (int i=0;i<synaps.length;i++){
            sum+=img[i]*synaps[i];
        }
        sum+=bias;

        return sum;
    }
}
```



Part I-2 : the formal neuron

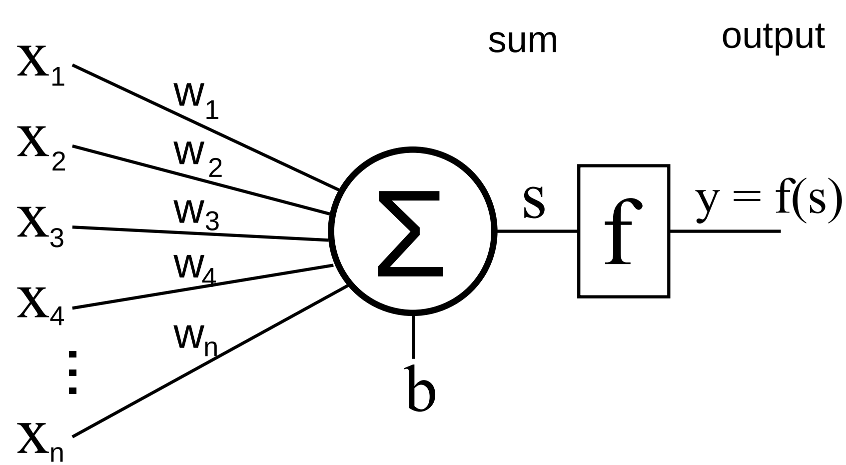
- **Creating an artificial neuron**
  - We forgot the activation function !



$$y = \frac{1}{1+e^{-x}}$$

```
public class Neuron {

    public float[] synaps;
    public float bias=0;

    public float output=0;

    public Neuron(int size){
        synaps=new float[size];
    }

    public float compute(float[] img){

        // compute value
        float sum=0;
        for (int i=0;i<synaps.length;i++){
            sum+=img[i]*synaps[i];
        }
        sum+=bias;

        output=activation(sum);
        return output;
    }

    private float activation(float x){
        return (float) (1 / (1+Math.exp(-x)));
    }
}
```

We will also record the output value



Part I-2 : the formal neuron

- **Creating an artificial neuron**
  - We then define a function to train the neuron :
    - Widrow-Hoff :
      - $\Delta = r - y$ → difference between expected result and output
      - $w_i^{t+1} = w_i^t + \alpha \,.\, x_i \,.\, \Delta$ → each weight is updated in proportion to related input value
    - First, we add the delta value and a learning rate with a value of 0,01
    - Then, a learning function that computes the delta value, using input vector and expected result

Main.java *Neuron.java

```java

public class Neuron {

    public float learnRate=0.01f;

    public float[] synaps;
    public float bias=0;

    public float output=0;
    public float delta=0;

    public Neuron(int size){
```

```java
        output=activation(sum);
        return output;
    }

    public void learn(float[] img, int res){
        delta=res-output;

    }

    private float activation(float x){
```

Part I-2 : the formal neuron

- **Creating an artificial neuron**
  - We then define a function to train the neuron :
    - Widrow-Hoff :
      - $\Delta = r - y$ → difference between expected result and output
      - $w_i^{t+1} = w_i^t + \alpha \,.\, x_i \,.\, \Delta$ → each weight is updated in proportion to related input value
    - Finally, the learn function update weight according to Widrow-Hoff rule :
      - Don't forget the bias !

```
public void learn(float[] img, int res){
    delta=res-output;

    for (int i=0;i<synaps.length;i++){
        synaps[i]+= learnRate * delta * img[i];
    }
    bias+=learnRate * delta;
}
```

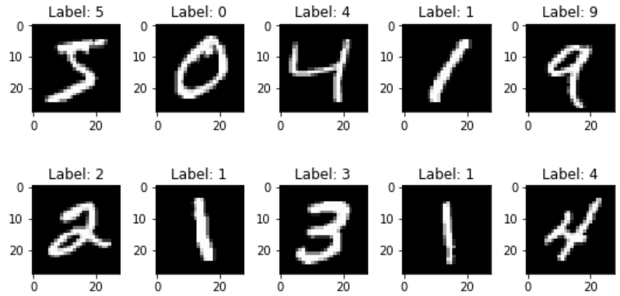
- **Congratulations ! Your neuron is complete !**

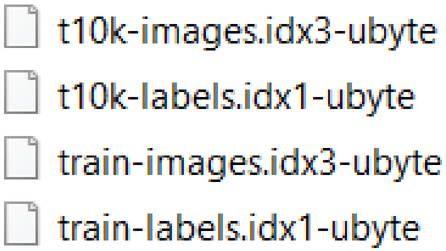
Part I-2 : the formal neuron

- **Training and exploiting a neuron**

  – We will use our neuron on a dataset called MNIST number.

    - This dataset is composed of two sets of small images of 28x28 pixels of handwritten digits (0 to 9)





    - A first dataset of 60 000 samples is used for training
    - A second dataset of 10 000 samples is used for testing
    - Each dataset is composed of two files : a file containing the pixel values of images, and a file containing the label (number) of images
    - A function is provided in *main* class to read these files and load samples in a matrix and labels in a vector (*setData*)

Part I-2 : the formal neuron

- **Training and exploiting a neuron**

  – In class *main*, uncomment the line adding the neuron as a variable

```java
    public Neuron neuron;           // neuron
```

  – The neuron must have a weight for each pixel of image. We thus initialize the neuron with 28x28 weights :

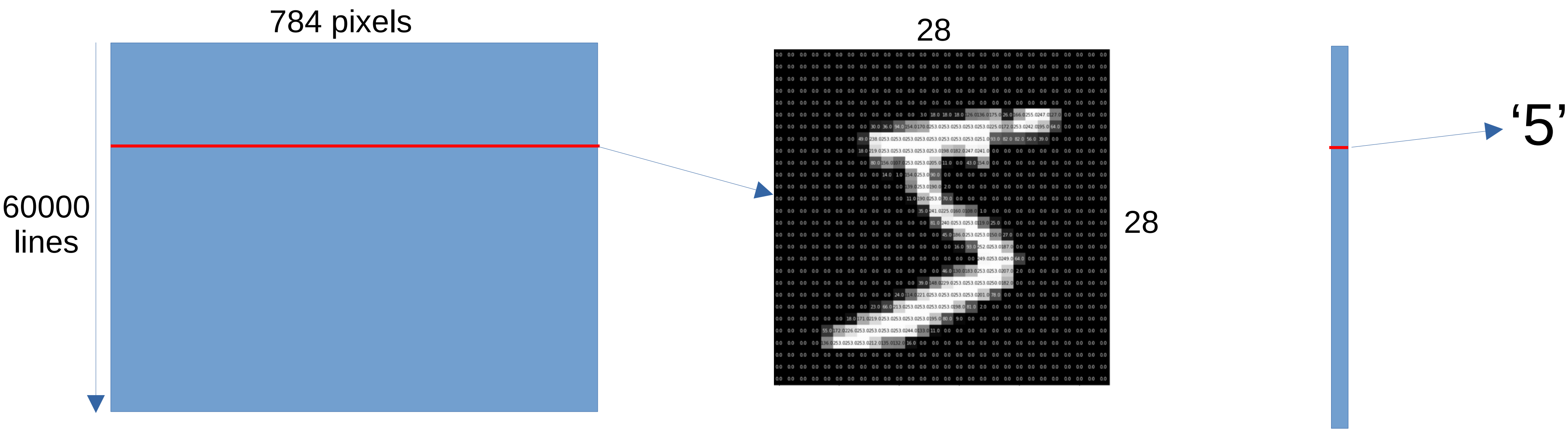
```java
    //////////////
    public Main(){

        ///////////////////////////////////////////////////
        // initialization
        ///////////////////////////////////////////////////
        neuron=new Neuron(size_x*size_y);          784 inputs

        // load test matrix
        setData("train-images.idx3-ubyte","train-labels.idx1-ubyte");

        // initialize display frame
        display=new DisplayFrame(this);
```

  – The lines after load the training dataset and initialize the display window

Part I-2 : the formal neuron

- **Training and exploiting a neuron**

  - The MNIST training dataset is loaded in a matrix (for the images) and a vector (for the labels)



  - We will get sample images by reading the matrix line by line and the label by reading the same line on label vector
    - The neuron does not consider the position of pixels on the 2D image : it gets the image as a vector of pixels !

Part I-2 : the formal neuron

- **Training and exploiting a neuron**

  – We will train our neuron to recognize a digit. Choose a digit (0-9) for the number variable (you can change later)

```java

    public int number=2;    // choose a number

```

  – The training cycle works as follow :
    - For each sample of the dataset
      – Get next sample and its label
      – Compute output value of the neuron
      – Reinforce the neuron using expected result (delta)

  – The neuron can be trained on the dataset multiple times to improve results. Each dataset cycle is called ‘epoch ‘
    - We will train the neuron on 50 epoches

```java
// apply the learning process 50 times
for (epoch=0;epoch<50;epoch++){

    float sumdelta=0;   // measure total error of the current epoch

    // for each test image
    for (test=0;test<matrixImages.length;test++){

        //////////
        // TODO //
        //////////


        display.repaint();

        try {Thread.sleep(100);
        } catch (InterruptedException e) {e.printStackTrace();}
    }


    // display mean error of the epoch
    System.out.println("epoch n°"+epoch+" : "+(sumdelta/nbTests));
}

System.out.println("learning completed");
```

Part I-2 : the formal neuron

- **Training and exploiting a neuron**

  – First, we define the expected result : 1 when the digit is the right one, 0 otherwise

```
//////////////////////////////////////////////////
// learning
//////////////////////////////////////////////////

// apply the learning process 50 times
for (epoch=0;epoch<50;epoch++){

    float sumdelta=0;   // measure total error of the current epoch

    // for each test image
    for (test=0;test<matrixImages.length;test++){

        // set output value
        int expected=0;
        if (matrixLabels[test] == number) expected=1;


        display.repaint();

        try {Thread.sleep(100);
        } catch (InterruptedException e) {e.printStackTrace();}
    }
```

Part I-2 : the formal neuron

- **Training and exploiting a neuron**
  - Then, the neuron's output is computed on sample image(and stored in its output variable)

```
// apply the learning process 50 times
for (epoch=0;epoch<50;epoch++){

    float sumdelta=0;   // measure total error of the current epoch

    // for each test image
    for (test=0;test<matrixImages.length;test++){

        // set output value
        int expected=0;
        if (matrixLabels[test] == number) expected=1;

        // process neuron
        neuron.compute(matrixImages[test]); // get result


        display.repaint();

        try {Thread.sleep(100);
        } catch (InterruptedException e) {e.printStackTrace();}
    }
```

Part I-2 : the formal neuron

- **Training and exploiting a neuron**

  - Finally, reinforce the neuron. We also record the delta to observe the evolution of predictions

```
// apply the learning process 50 times
for (epoch=0;epoch<50;epoch++){

    float sumdelta=0;   // measure total error of the current epoch

    // for each test image
    for (test=0;test<matrixImages.length;test++){

        // set output value
        int expected=0;
        if (matrixLabels[test] == number) expected=1;

        // process neuron
        neuron.compute(matrixImages[test]); // get result

        // reinforce neuron
        neuron.learn(matrixImages[test], expected);

        // add delta to the error sum
        sumdelta+=Math.abs(neuron.delta);

        display.repaint();

        try {Thread.sleep(100);
        } catch (InterruptedException e) {e.printStackTrace();}
    }
```

Part I-2 : the formal neuron

- **Training and exploiting a neuron**

  - In class DisplayPanel, uncomment the second part in PaintComponent procedure (remove ‘/*’ on line 34) to display the neuron’s weights. Then, test your application

```
30                  g.fillRect(10+5*i, 10+5*j, 5, 5);
31              }
32          }
33
34          for (int i=0;i<Main.size_x;i++){
35              for (int j=0;j<Main.size_y;j++){
36                  val=(int)(main.neuron.synaps[i+Main.size_x*j]*50)+128;
37                  if (val<0) val=0;
38                  if (val>255) val=255;
39                  g.setColor(new Color(val,val,val));
40                  g.fillRect(180+3*i, 10+3*j, 3, 3);
41              }
42          }/**/
43
44          /*for (int n=0;n<10;n++){
45              for (int i=0;i<Main.size_x;i++){
```

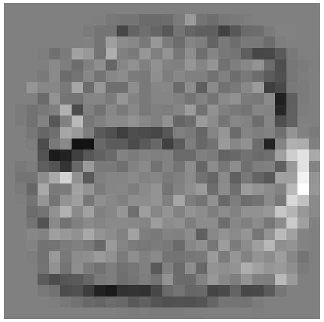
  - If the program runs correctly, comment or remove the sleep fonction in main class

```
        display.repaint();

        //try {Thread.sleep(100);
        //} catch (InterruptedException e) {e.printStackTrace();}
    }
```

- **What do you observe on neuron’s weights and average delta ?**

Part I-2 : the formal neuron

- **Training and exploiting a neuron**
  - The weights form the ‘average’ shape of the selected digit
  - The average delta first decreases quickly but seems to converge



  - Now, let’s test our neuron on the test dataset (loaded hereafter)

```
        /////////////////////////////////////////////////////
        // exploitation
        /////////////////////////////////////////////////////

        // test neuron on test dataset
        int errors=0;

        // load test matrix
        setData("t10k-images.idx3-ubyte","t10k-labels.idx1-ubyte");
```

Part I-2 : the formal neuron

- **Training and exploiting a neuron**

  - We simply get neuron’s predictions for each sample image :

  - We can aslo count the number of prediction errors :

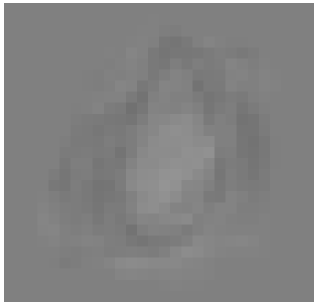
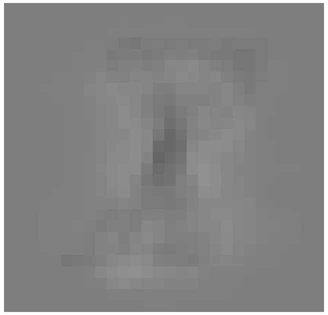
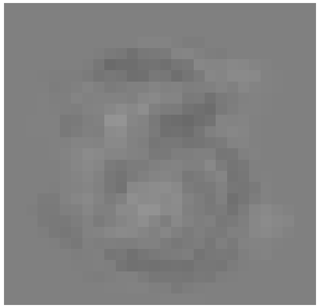
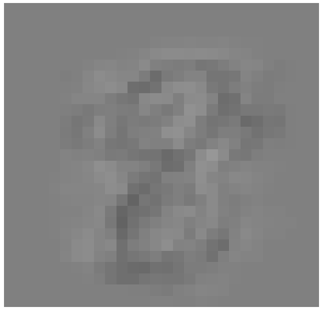
```
// test each image of the
for (test=0;test<matrixImages.length;test++){

    // process neuron
    float res=neuron.compute(matrixImages[test]);

    if (res<0.001) res=0;   // avoid low values displayed as "9.9586 x 10^-5"
    System.out.println(matrixLabels[test]+" -> "+res);


    // count errors
    if (matrixLabels[test]==number && res<0.5 || matrixLabels[test]!=number && res>0.5) errors++;

}
```

  - Not bad for 10 000 samples `errors : 254`

Part I-2 : the formal neuron

- **The formal neuron**

  - The formal neuron makes an average of positive and negative results to generate an “average image”, exposing the features of the element to recognize

  - This average image allows detecting the presence of the associated element on an unknown input vector.

  - **But Why does this works ?**

Part I-2 : the formal neuron

- **Let’s consider a linear function**

  example :

  $3.y + 2.x + 1 = 0$ ($y = -(2/3).x - 1/3$)

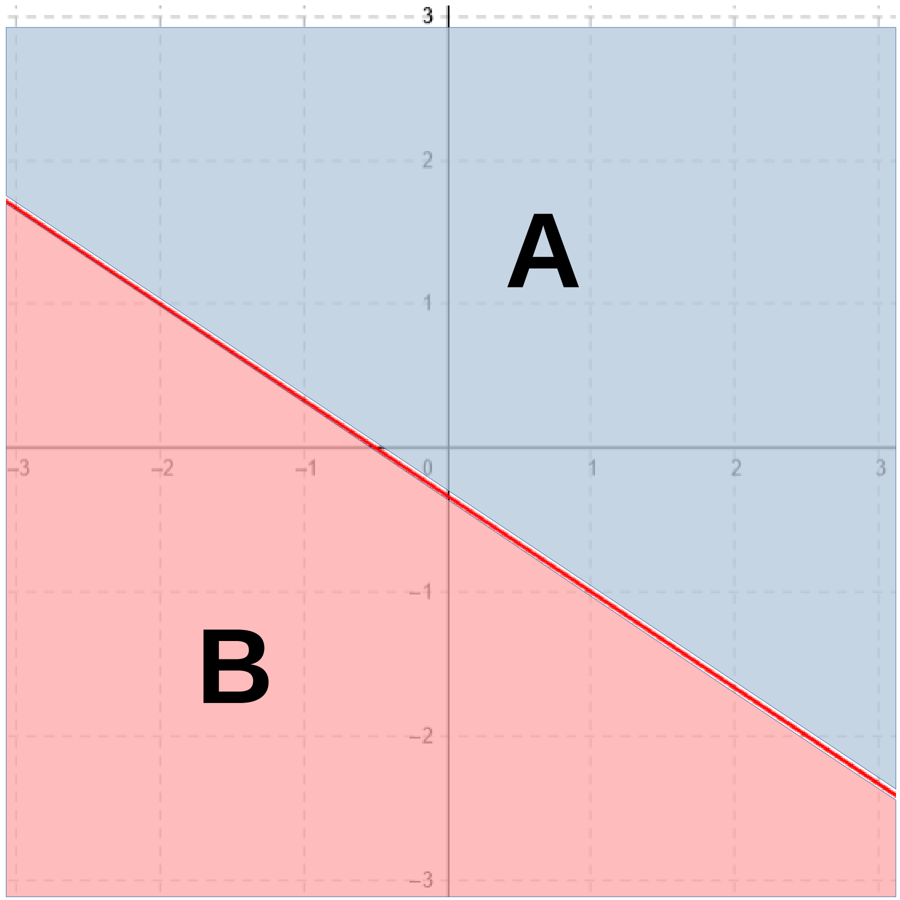
  This function separates the plane in 2:

  - $3.y+2.x+1 > 0$

    → area A above the line

  - $3.y+2.x+1 < 0$

    → area B below the line



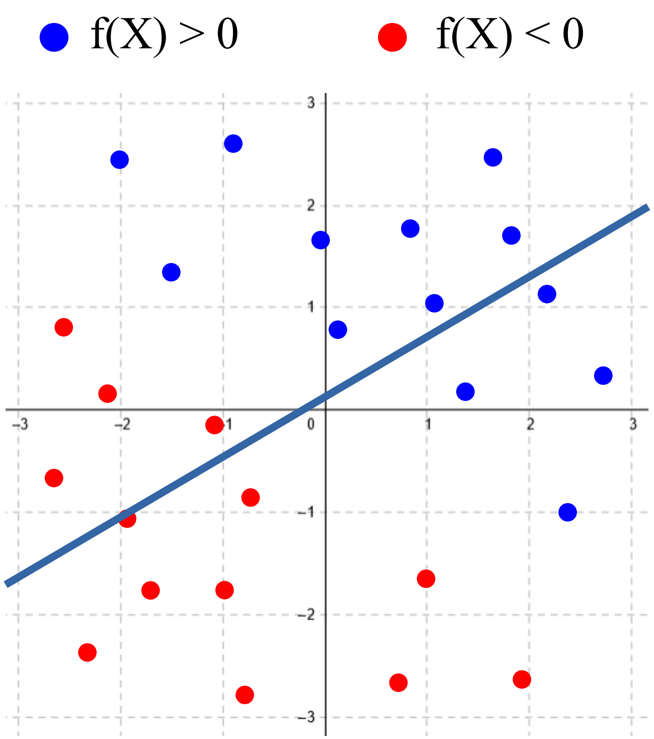
Part I-2 : the formal neuron

- **And if we do not know the function parameters ?**

  We start with a function a.x + b.y + c = 0 with unknown parameters (a,b,c)

  We have a set of points X_i = [x_i , y_i] which position $f(X_i) > 0$ or $f(X_i) < 0$ is known

  → We have to find a triplet (a , b , c) satisfying all point conditions



- We get a random triplet
- Each point is tested
  - If $f(X_i)>0$ but $ax_i+by_i+c<0$
    - ax_i+by_i+c must be increased
  - If $f(X_i)<0$ but $ax_i+by_i+c>0$
    - ax_i+by_i+c must be decreased
  - otherwise, we do not change parameters

Part I-2 : the formal neuron

- **And if we do not know the function parameters ?**

  For each parameter a, b and c, we sligtly increase or decrease the value proportionnally to, respectively, x (a), y (b) and 1 (c)

  $a \leftarrow a + \alpha \, . \, x_i$ or $a \leftarrow a - \alpha \, . \, x_i$

  $b \leftarrow b + \alpha \, . \, y_i$ or $b \leftarrow b - \alpha \, . \, y_i$

  $c \leftarrow c + \alpha \, . \, 1$ or $c \leftarrow c - \alpha \, . \, 1$

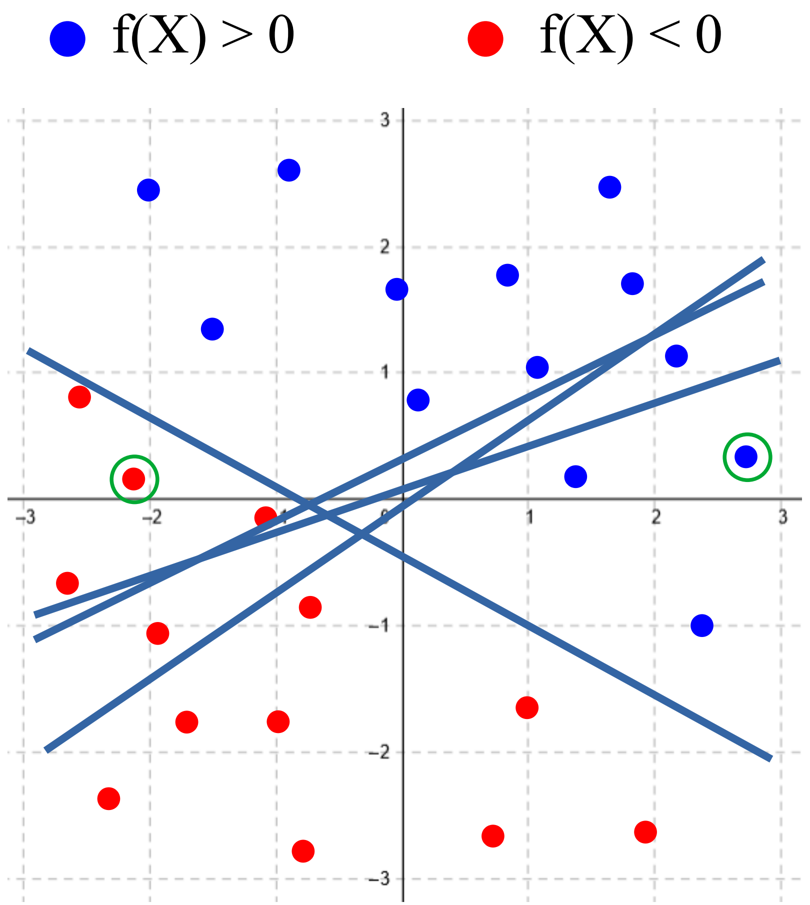
- **If we note $r_i = 1$ when $f(X_i)>0$ and $r_i = -1$ when $f(X_i)<0$**
  - then
    - $a \leftarrow a + \alpha \, . \, r_i \, . \, x_i$
    - $b \leftarrow b + \alpha \, . \, r_i \, . \, y_i$
    - $c \leftarrow c + \alpha \, . \, r_i$
  - Points X_i are tested, until they are all on the right side of the line

Part I-2 : the formal neuron

- **The parameters will converge until finding a solution**

- Algorithm :

errors = true
while errors do
    errors = false
    for each Xi do
        if $f(X_i) . (a.x_i+b.y_i+c) < 0$ do
            $a \mathrel{+}= \alpha . r_i . x_i$
            $b \mathrel{+}= \alpha . r_i . y_i$
            $c \mathrel{+}= \alpha . r_i$
            errors = true
        end if
    end for each
end while



Part I-2 : the formal neuron

- **What has this got to do with the neuron ?**

- Let's write:
  - $a . x + b . y + c = 0 \rightarrow w_1 . x_1 + w_2 . x_2 + b = 0$

- We generalize to a space with n-dimensions:

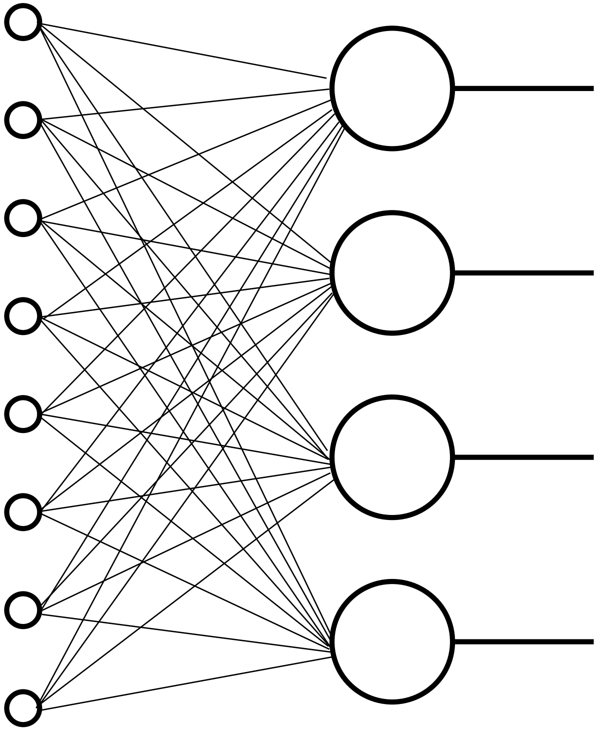
$$\sum_k w_k . x_k + b = 0$$

- Weights of a neuron form the equation of a hyperplane

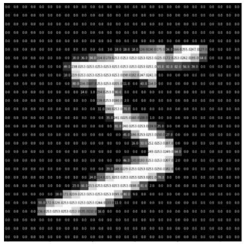
- The reinforcement modifies weights to separate space in two groups of points

Part I-2 : the formal neuron

- **A neuron can only recognize one type of element**
- **But with multiple neurons, it is possible to categorize multiple elements**



- **With a threshold activation function :**
  - The output is a binary vector
    - Conversion to binary code, ASCII code…



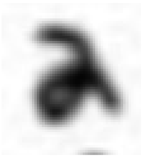
{ 0 , 1 , 0 , 1 }

→ **Perceptrons**

Part I-2 : the formal neuron

- **Neurons with continual activation function (linear, ReLU, Sigmoid…)**

- **Each neuron is trained to recognize a category of element (e.g. a number)**
  - The training process must convert labels into binary vectors
    - ‘2’ → {0,0,1,0,0,0,0,0,0,0,0}
    - Each neuron needs a binary output for training
  - Neurons are in competition : the neuron with the greatest output provide the output of the network, with the confidence of the result



(0: 0.21) (1: 0,01) (2: 0,65) (3: 0,55) (4: 0,35) (5: 0,11) (6: 0,09)...

- **Often used for object recognition**

Part I-2 : the formal neuron

- **Implementing a layer of neuron**
  - Simple way : create a class that manage a vector of neurons and provide a vector of results as output

```java
public class Layer {

    public Neuron[] layer;
    public float[] output;

    public float delta=0;

    public Layer(int nb, int size){

        layer=new Neuron[nb];   // declare neuron vector
        for (int i=0;i<nb;i++){ // initialize each neuron
            layer[i]=new Neuron(size);
        }

        output=new float[nb];   // declare the output vector
    }
}
```

Initialize the vector of neurons

```java

    public float[] compute(float[] img){

        for (int i=0;i<layer.length;i++){
            output[i]=layer[i].compute(img);
        }

        return output;
    }
```

Compute output of each neuron
And write their results in output vector

```java
    public void learn(float[] img, int[] results){

        delta=0;

        for (int i=0;i<layer.length;i++){
            layer[i].learn(img, results[i]);

            delta+=Math.abs(layer[i].delta);
        }
    }
```

Reinforce neurons individually
Notice that result is a vector (result for each neuron)

Part I-2 : the formal neuron

- **Implementing a layer of neuron**
  - Training : the result vector must be obtained from labels :

```
        // for each test image
        for (test=0;test<matrixImages.length;test++){

            // set output value
            int[] expected=new int[10];
            expected[matrixLabels[test]] = 1;
```

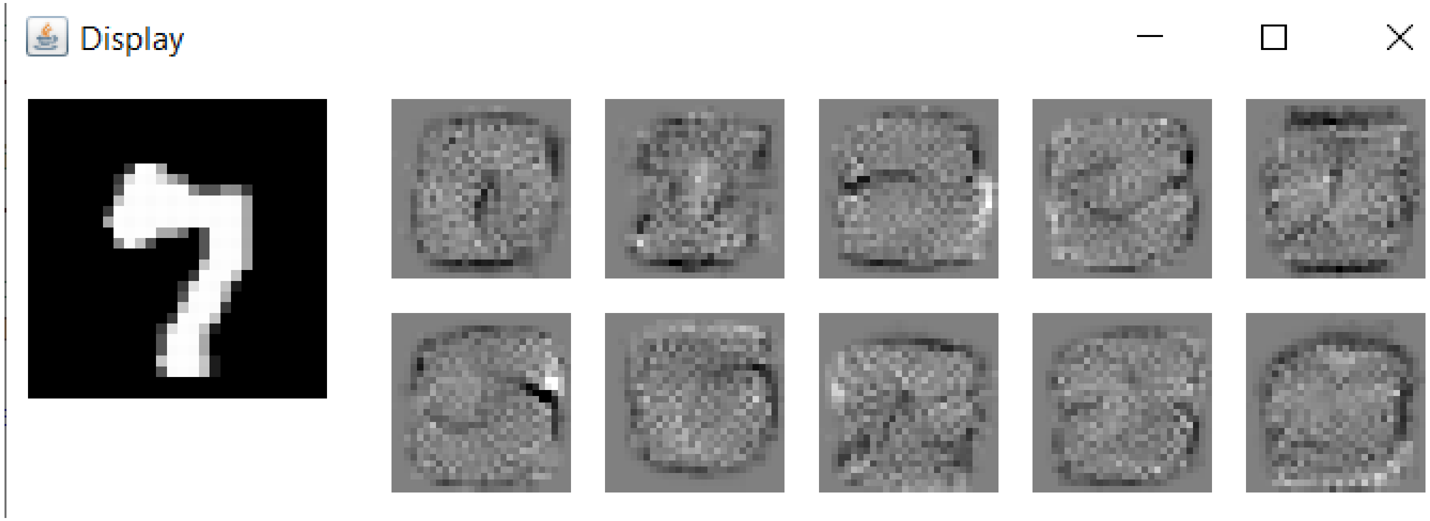
Expected is a vector of 10 values, with only one value at 1

  - The result is obtained by finding the maximum output :

```
        // test each image of the dataset
        for (test=0;test<matrixImages.length;test++){

            // process neurons
            layer.compute(matrixImages[test]);

            float max=0;
            int imax=0;
            for (int i=0;i<10;i++){
                if (layer.output[i]>max){
                    max=layer.output[i];
                    imax=i;
                }
            }

            System.out.println(matrixLabels[test]+" -> "+imax);
```

Part I-2 : the formal neuron

- **Each neuron specializes for a digit, the highest output provide the number**



errors : 904 / 10 000 → 9.04 %

ANNEX 1 provide details of this implementation