In[483]:=
```
lib = "/Users/gaze/my/bulkmin/bulkmin.dylib";

FindMinimalEnergyPhase = LibraryFunctionLoad[lib,
   "find_minimal_energy_phase", {{Real, 1}, {Real, 1}, {Real, 1}}, {Real, 1}];
```

In[485]:=
$$h = \frac{Q^2}{2\,c} + \left(-\alpha\, ej\, \text{Cos}[\phi] - n\, ej\, \text{Cos}\left[\frac{\phi ext - \phi}{n}\right]\right) + \frac{\phi 2^2}{2\,l};$$

In[486]:=
```
Solve[D[h, ϕ] == D[h, ϕ2], ϕ2][[1]];
h /. %;
u1 = % - Q^2/(2 c) // FullSimplify
```

Out[488]=
$$\frac{1}{2}\, ej \left(-2\,\alpha\, \text{Cos}[\phi] - 2\, n\, \text{Cos}\left[\frac{-\phi + \phi ext}{n}\right] + ej\, l \left(\alpha\, \text{Sin}[\phi] + \text{Sin}\left[\frac{\phi - \phi ext}{n}\right]\right)^2\right)$$

Let γ=l E_j

In[489]:=
$$u2 = \frac{2\,u1}{ej} /. ej \to \frac{\gamma}{l}$$

Out[489]=
$$-2\,\alpha\, \text{Cos}[\phi] - 2\, n\, \text{Cos}\left[\frac{-\phi + \phi ext}{n}\right] + \gamma \left(\alpha\, \text{Sin}[\phi] + \text{Sin}\left[\frac{\phi - \phi ext}{n}\right]\right)^2$$

In[490]:=
```
NJ = 3;
```

In[491]:=
$$c3[\phi\_, \alpha\_, \phi ext\_, \gamma\_] = \frac{1}{6}\, D[u2, \{\phi, 3\}] /. n \to NJ;$$
$$c4[\phi\_, \alpha\_, \phi ext\_, \gamma\_] = \frac{1}{24}\, D[u2, \{\phi, 4\}] /. n \to NJ;$$

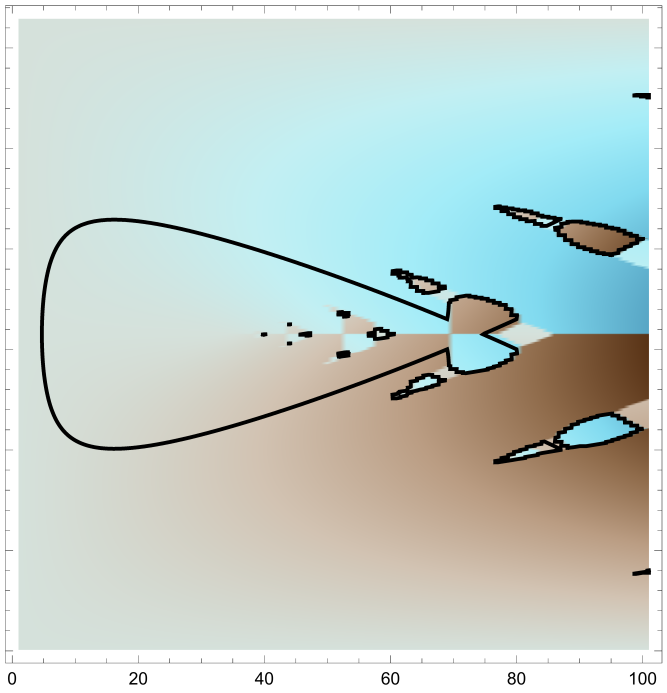
In[493]:=
```
dims = Table[{α, ϕext, 100}, {α, 0, 1, 0.01}, {ϕext, 0, 2 * π, 0.01}, {γ, 0, 0, 1}];
```

In[494]:=
```
permaxes = Transpose[dims, InversePermutation[{4, 1, 2, 3}]];
axes = Flatten /@ %;
phase = FindMinimalEnergyPhase @@ axes;
```

In[497]:=
```
ArrayReshape[Join[{phase}, axes], Dimensions[permaxes] + {1, 0, 0, 0}];
params = Transpose[%, {4, 1, 2, 3}];
m3 = Map[c3 @@ # &, params, {3}];
m4 = Map[c4 @@ # &, params, {3}];
```

In[504]:=
```
epi = ListContourPlot[m4[[ ;; ,  ;; , 1]] // Transpose,
      ContourShading → None, Contours → {{0, Thick}}][[1]];
ListDensityPlot[m3[[ ;; ,  ;; , 1]] // Transpose,
 ColorFunction → "BrownCyanTones", Epilog → epi]
ListDensityPlot[m4[[ ;; ,  ;; , 1]] // Transpose,
 ColorFunction → "BrownCyanTones", Epilog → epi]
```

Out[505]=



Out[506]=