

Sudoku Puzzles

I will explain how we can solve sudoku by using a CSP. It's surprisingly *super* easy. Like any CSP, we have to come up with the set of variables, the set of domains, and the set of constraints. Let's do that now.

Variables. $X = \{A1, A2, \dots, I8, I9\}$. We'll just have a variable for each cell in the sudoku puzzle for a total of 81 variables. To give you a sense of what this notation means, the variable $A1$ is just a variable that represents the cell in the A th row and the 1 st column. For any cell that's filled in with a value already, we'll just automatically set those to that value. Easy.

Domains. $D = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. For the domain of each variable, we'll just start with all of the numbers from 1 to 9. Keep in mind that forward checking will trim these domains over time. Note that, for a cell that has a number already, it'll just have a domain with a single number.

Constraints. We'll have three types of constraints here: a constraint that says all of the variables in each *column* has to differ, all of the variables in each *row* has to differ, and all of the variables in each *square* has to differ. Nothing too tricky. Let's formalize these types of constraints below.

1. **Column Constraints.** $AllDiff(A_1, B_1, C_1, D_1, E_1, F_1, G_1, H_1, I_1), \dots, AllDiff(A_9, B_9, C_9, D_9, E_9, F_9, G_9, H_9, I_9)$.

There will be a total of 9 column constraints because there are 9 columns.

2. **Row Constraints.** $AllDiff(A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9), \dots, AllDiff(I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8, I_9)$. Again, we'll have a total of 9 row constraints because there are 9 rows.

3. **Square Constraints.** $AllDiff(A_1, A_2, A_3, B_1, B_2, B_3, C_1, C_2, C_3) \dots, AllDiff(G_7, G_8, G_9, H_7, H_8, H_9, I_7, I_8, I_9)$.

And, surprise, we'll have 9 square constraints because there are 9 squares.

Once you've defined sudoku as a CSP, you're done! All you have to do now is feed the CSP into your backtracking search algorithm to get the answer to any sudoku puzzle. You'd be surprised how fast it is too. Even the hardest sudoku puzzles for a person can be solved in way less than a second.