Algorithm Analysis and Design

Ghazale Kharadpoor 9513012

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First of all, we guess two functions for $g(n)$ and $h(n)$. They can be $g(n) = sin(x)$ and $h(x) = 0$. Now we prove that none of them is member of big-O(another). For the first case as an example, If we can not find any constant like $c > 0$ and any $n_0 > 1$ that for every $n \geq n_0$, $g(n) \leq c.h(n)$, our proof will be done.

We know that $sin(x)$ function is a periodic function. Sometimes It has positive amounts and sometimes negative amounts.

Assume that $g(n) \in O(h(n))$; so there is a $c > 0$ and $n_0 > 1$ such that for every $n \in \mathbb{N}$ and $n \geq n_0$:

$$g(n) \leq c.h(n)$$

As we see, there is no actual c and n_0 that $c.h(n)$ bounds $g(n)$ from up because $c.h(n)$ amounts is always zero; So that because of periodic positive and negative amounts of $g(n)$, $c.h(n)$ can not bounds $g(n)$ from top ($h(n)$ exactly goes through the $g(n)$ with every $c \geq 0$).

Now we focus on $h(n) \in O(g(n))$. $h(n)$ is a linear function with 0 gradient. Assume that $h(n) \in O(g(n))$; so there is a $c > 0$ and $n_0 > 1$ such that for every $n \in \mathbb{N}$ and $n \geq n_0$:

$$h(n) \leq c.g(n)$$

Because $g(n)$ is periodic function with both negative and positive amounts, we can not declare a c and n_0 in which $c.g(n)$ bounds $h(n)$ from top. Some parts of $g(n)$ are below of $h(n)$ and some other parts are above the $h(n)$.

So both of question's statement is wrong with above proofs.