Quadratic Sieve Running on Java

Intro

My name is **Ilya Gazman**, I am a dev.
Over the course of the next episodes, I will implement an **efficient** version of the Quadratic Sieve in **Java**.

To follow up you need dev skills, basic Java knowledge, and some basic Math.

If you have any questions, please comment below, or post in the Facebook group.
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Factor the number $N = A \cdot B$ where A and B are big prime numbers.

Episode 1: Fermat's factorization method

The idea is to write N as a difference of squares

$$(C + D)(C - D) = C^2 - D^2 = N$$

Then we can search for square numbers who are square minus N is also a square
$C^2 - N = D^2$

In the worse case, it is a slower then trivial division (testing all the numbers up to $\sqrt{N}$). But as we see later it has tons of optimizations that eventually turners it into the **Quadratic Sieve** algorithm!

Episode 2: Congruence of squares

Recap

In the last episode, we used Fermat's factorization method to factor numbers having factors of 22 bits in seconds. We picked random numbers to satisfy the formula below

$$C^2 - N = D^2$$

We can make a much weaker condition to look at the Congruence of squares

$$C^2 \equiv D^2 \ (mod\ N) \Rightarrow C^2 - D^2 = 0 \ (mod\ N) \Rightarrow (C + D)(C - D) = 0 \ (mod\ N)$$

It means that $C + D$ is a multiplication of N or it's factors. We can efficiently extract it using the Greatest Command Devisor function, gcd. It works in O(log(N)) time, so it's super fast. And it's also part of Java native APIs, so it's a huge plus.
gcd(N, $C + D$) will either give us the factor of N or N or 1, in which case we will try another option.

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Episode 3: Stress tool

Recap

So far we spoke about three optimizations for the Fermat's factorization method

- Looking for small mods at $\left|\sqrt{N}\right|^2$
- Using congress of squares $C^2 \equiv D^2 \ (mod\ N)$
- And multiplying N by a small integer k

We created a small program for each one of those:

- Episode 1
- Episode 2
- Episode 2.1

Now let's build a stress a tool, to compare the performance of those algorithms. And also add a trivial division as another comparison.

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Episode 4: Dixon Method - Part 1

Recap:

In the last episode, we built a stress tool for testing the performances, of our algorithms so far. A big lesson that I learned is to not record and run a stress tool at the same time lol.

Here are the true results running offline, speed of factoring 30-bit numbers over 1 sec

- Episode 2.1 speed 1631.198103012559
- Trial Division speed 680.556549352442
- Episode 2 speed 451.9630294241931
- Episode 1 speed 87.29335158954746

Progress:

- With Fermat's method, we have been looking for a difference of squares using random numbers $C^2 - N = D^2$
- Then we switched looking into sequential numbers from starting from $C = \left|\sqrt{N}\right|$
- Then we found a weeker condition of Congruence of squares, looking into $C^2 \equiv D^2 \ (mod\ N)$
- We added a small multiplier k for $C = \left|\sqrt{kN}\right|$

Now with the Dixon method, we are going to look into the even weaker condition of $C^2 \equiv E \ \ (mod\ N)$

Where E is a product of small factors bounded by some constant B, we are colling for each such combination of $C^2 \equiv E \ \ (mod\ N)$ a “relation” or a B-Smooth value. Once we found $B + 1$ such relations, we can find a combination that will form a square.
Ex:

$C_1{}^2 \equiv 2^3 11^1 23^8 \ (mod\ N)$
$C_2{}^2 \equiv 2^5 11^3 41^6 \ (mod\ N)$
$C^2 \equiv 2^8 11^4 23^8 41^6 \ \equiv (2^4 11^2 23^4 41^3)^2 \ (mod\ N)$

We can also represent the relations as vectors R over the base 2

$R_1 = 1, 1, 0, 0$
$R_2 = 1, 1, 0, 0$
$xor(R_1, R_2) = 0, 0, 0, 0$

In fact, we are going to have multiple such relations, and we can use Gaussian Elimination over base 2, to find the null space, the 0 vectors. Every 0 vector is going to be a solution, and just like with the congruence of squares, it can be a trivial solution of N or 1, or it can be a real factor of N.

Gaussian Elimination is an expensive algorithm that runs in $O(N^3)$, but as we move father you will see that the bottleneck is going to be in finding B-smooth numbers,

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Episode 5: Dixon Method - Part 2

Recap:

In the last episode, we started to implement the Dixon method. We built a prime base, declared a bound B, and found B+1 relations.

Now we are going to build a matrix to extract a solution out of them. We are going to use an optimized version of Gaussian Elimination for that.

- Since we are working in a base two space, we are going to use XOR instead of multiplication
- Also since we are working in base two, we going to work with bits instead of numbers. It both saves space and speeds up the multiplications.
- Cleanup - We will remove relations who only have has one prime, that can't be eliminated.

The algorithm

- Filter out rows who only has one prime
- Build an identity solution matrix with the size of (Rows count) X (Rows count), so each time we are xoring/eliminating rows, we will also make the xor operation on the solution matrix. It will allow us to extract the solution once we find the null space, empty or zero rows.

- Iterate over `i` from 0 to `B`
    - Iterate over all the rows
        - Find a row with an `i` prime and use as an eliminator, to eliminate/xor all the other `i` primes and exclude it from future eliminations(it cannot be used as an eliminator for other primes, but it can and should be eliminated with other eliminators)

Example

0> 2^8, 3^3, 5^1, 7^2, 11^2, 13^5 -> 0,1,1,0,0,1
1> 2^1, 3^2, 5^2, 7^1, 11^0, 13^3 -> 1,0,0,1,0,1
2> 2^3, 3^0, 5^4, 7^0, 11^3, 13^2 -> 1,0,0,0,1,0 filter out, it only has one prime
3> 2^7, 3^7, 5^3, 7^2, 11^8, 13^4 -> 1,1,1,0,0,0
4> 2^0, 3^0, 5^5, 7^0, 11^0 13^7 -> 0,0,1,0,0,1
5> 2^0, 3^4, 5^2, 7^3, 11^0, 13^8 -> 0,0,0,1,0,0

```
6> 2^0, 3^5, 5^4, 7^2, 11^2, 13^1     ->      0,1,0,0,0,1
7> 2^9, 3^2, 5^0, 7^1, 11^4, 13^0     ->      1,0,0,1,0,0


  0> 0,1,1,0,0,1          0,1,1,0,0,1    |        1,0,0,0,0,0,0     1,0,0,0,0,0,0
  1> 1,0,0,1,0,1  -       1,0,0,1,0,1    |        0,1,0,0,0,0,0     0,1,0,0,0,0,0
  3> 1,1,1,0,0,0  +       0,1,1,1,0,1    |        0,0,1,0,0,0,0     0,1,1,0,0,0,0
  4> 0,0,1,0,0,1  ->      0,0,1,0,0,1    |        0,0,0,1,0,0,0  ->  0,0,0,1,0,0,0
  5> 0,0,0,1,0,0          0,0,0,1,0,0    |        0,0,0,0,1,0,0     0,0,0,0,1,0,0
  6> 0,1,0,0,0,1          0,1,0,0,0,1    |        0,0,0,0,0,1,0     0,0,0,0,0,1,0
  7> 1,0,0,1,0,0  +       0,0,0,0,0,1    |        0,0,0,0,0,0,1     0,1,0,0,0,0,1

  0> 0,1,1,0,0,1  -       0,1,1,0,0,1    |        1,0,0,0,0,0,0     1,0,0,0,0,0,0
* 1> 1,0,0,1,0,1          1,0,0,1,0,1    |        0,1,0,0,0,0,0     0,1,0,0,0,0,0
  3> 0,1,1,1,0,1  +       0,0,0,1,0,0    |        0,1,1,0,0,0,0     1,1,1,0,0,0,0
  4> 0,0,1,0,0,1  ->      0,0,1,0,0,1    |        0,0,0,1,0,0,0  ->  0,0,0,1,0,0,0
  5> 0,0,0,1,0,0          0,0,0,1,0,0    |        0,0,0,0,1,0,0     0,0,0,0,1,0,0
  6> 0,1,0,0,0,1  +       0,0,1,0,0,0    |        0,0,0,0,0,1,0     1,0,0,0,0,1,0
  7> 0,0,0,0,0,1          0,0,0,0,0,1    |        0,1,0,0,0,0,1     0,1,0,0,0,0,1

* 0> 0,1,1,0,0,1  +       0,1,0,0,0,0    |        1,0,0,0,0,0,0     1,0,0,1,0,0,0
* 1> 1,0,0,1,0,1          1,0,0,1,0,1    |        0,1,0,0,0,0,0     0,1,0,0,0,0,0
  3> 0,0,0,1,0,0  ->      0,0,0,1,0,0    |        1,1,1,0,0,0,0     1,1,1,0,0,0,0
  4> 0,0,1,0,0,1  -       0,0,1,0,0,1    |        0,0,0,1,0,0,0  ->  0,0,0,1,0,0,0
  5> 0,0,0,1,0,0          0,0,0,1,0,0    |        0,0,0,0,1,0,0     0,0,0,0,1,0,0
  6> 0,0,1,0,0,0  +       0,0,0,0,0,1    |        1,0,0,0,0,1,0     1,0,0,1,0,1,0
  7> 0,0,0,0,0,1          0,0,0,0,0,1    |        0,1,0,0,0,0,1     0,1,0,0,0,0,1

* 0> 0,1,0,0,0,0          0,1,0,0,0,0    |        1,0,0,1,0,0,0     1,0,0,1,0,0,0
* 1> 1,0,0,1,0,1  +       1,0,0,0,0,1    |        0,1,0,0,0,0,0     1,0,1,0,0,0,0
  3> 0,0,0,1,0,0  -       0,0,0,1,0,0    |        1,1,1,0,0,0,0     1,1,1,0,0,0,0
* 4> 0,0,1,0,0,1  ->      0,0,1,0,0,1    |        0,0,0,1,0,0,0  ->  0,0,0,1,0,0,0
  5> 0,0,0,1,0,0  +       0,0,0,0,0,0    |        0,0,0,0,1,0,0     1,1,1,0,1,0,0
  6> 0,0,0,0,0,1          0,0,0,0,0,1    |        1,0,0,1,0,1,0     1,0,0,1,0,1,0
  7> 0,0,0,0,0,1          0,0,0,0,0,1    |        0,1,0,0,0,0,1     0,1,0,0,0,0,1

* 0> 0,1,0,0,0,0          0,1,0,0,0,0    |        1,0,0,1,0,0,0     1,0,0,1,0,0,0
* 1> 1,0,0,0,0,1 +        1,0,0,0,0,0    |        1,0,1,0,0,0,0     0,0,1,1,0,1,0
* 3> 0,0,0,1,0,0          0,0,0,1,0,0    |        1,1,1,0,0,0,0     1,1,1,0,0,0,0
* 4> 0,0,1,0,0,1 + ->     0,0,1,0,0,0    |        0,0,0,1,0,0,0  ->  1,0,0,0,0,1,0
  5> 0,0,0,0,0,0          0,0,0,0,0,0    |        1,1,1,0,1,0,0     1,1,1,0,1,0,0   solution 0,1,2,4
  6> 0,0,0,0,0,1  -       0,0,0,0,0,1    |        1,0,0,1,0,1,0     1,0,0,1,0,1,0
  7> 0,0,0,0,0,1  +       0,0,0,0,0,0    |        0,1,0,0,0,0,1     1,1,0,1,0,1,1   solution 0,1,3,5,6
```

0> 2^8, 3^3, 5^1, 7^2, 11^2, 13^5
1> 2^1, 3^2, 5^2, 7^1, 11^0, 13^3
2> 2^3, 3^0, 5^4, 7^0, 11^3, 13^2
3> 2^7, 3^7, 5^3, 7^2, 11^8, 13^4
4> 2^0, 3^0, 5^5, 7^0, 11^0 13^7
5> 2^0, 3^4, 5^2, 7^3, 11^0, 13^8
6> 2^0, 3^5, 5^4, 7^2, 11^2, 13^1
7> 2^9, 3^2, 5^0, 7^1, 11^4, 13^0

Solution 0,1,2,4:
0> 2^8, 3^3, 5^1, 7^2, 11^2, 13^5
1> 2^1, 3^2, 5^2, 7^1, 11^0, 13^3
3> 2^7, 3^7, 5^3, 7^2, 11^8, 13^4
5> 2^0, 3^4, 5^2, 7^3, 11^0, 13^8

2^16, 3^16, 5^8, 7^8, 11^10, 13^20 = sqrt(2^8, 3^8, 5^4, 7^4, 11^5, 13^10)^2

Solution 0,1,3,5,6:
0> 2^8, 3^3, 5^1, 7^2, 11^2, 13^5
1> 2^1, 3^2, 5^2, 7^1, 11^0, 13^3
4> 2^0, 3^0, 5^5, 7^0, 11^0 13^7
6> 2^0, 3^5, 5^4, 7^2, 11^2, 13^1
7> 2^9, 3^2, 5^0, 7^1, 11^4, 13^0

2^18, 3^12, 5^12, 7^6, 11^8, 13^16

Episode 6: Hello Quadratic Sieve

During the last two episodes, we have been implementing the Dixon method.
But as it turns out it's not that fast at all.

Now we will see how two additional optimizations will turn it into a blazing fast Quadratic Sieve algorithm!

Also, this is not the last episode and more optimizations are on the way, but today we will start feeling this speed.

Sieving

Example: $292564681 = 7669 \cdot 38149$

$(\left\lfloor\sqrt{292564681}\right\rfloor + i)^2$ mod N mod $5, 7, 13, 17$

Logging approximation

$a \cdot b = log(a) + log(b)$

To find if a number is bSmooth we can test if the sum of logs is bigger than some limit, ex: $log(\left\lfloor\sqrt{N}\right\rfloor)$

Sieving process

Pick a range and starting from $\left\lfloor\sqrt{N}\right\rfloor$ iterate over "range" values at a time.
Build wheels that will help tracking the next divisor of each prime, and compute the logs over the range using all the wheels.

Than each log who is above $log(\left\lfloor\sqrt{N}\right\rfloor)$, must be a bSmooth value

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Episode 7: Sieve Of Eratosthenes

In the last episode we sow the true power of quadratic sieve.

During this episode, we will do some cleanup and make a minor optimization of Sieve Of Eratosthenes.

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Episode 8: Continued Factorization

In the last episode we did some cleanup, improved the prime base construction time, and increased the allowed reminder size that we are willing to take into our BSmooth values collections.

We now have several magic numbers in place that we need to optimize to get the best performances.

- B - The B-smooth bound that determines the largest prime size in our prime base.
- Range - The size of our logs array and the size of our sieving cycle.
- Reminder Size - The largest reminder size that we allow, to include a number in our B-Smooth values. We didn't give it a name yet so 2 for now, it's a multiplier of the biggest prime value in our prime base.
- 1.5 - The multiplier of the prime base size that determines the number of the values we are willing to collect before moving into extracting the vectors and solving the matrix.

During this episode, I try to get rid of the last magic number, the prime base multiplier. Last time we factored the below number:
437485612413337193583034997070105315745309

This how it looks while I am not recording:

17> A 552330951024629792017
24> B 792071513649121280077
24> N 437485612413337193583034997070105315745309
76> built prime base of 3774/80000. Max prime 79999
2181> found 5662 bSmooth values
3303> Extracted 4573/5662 vectors
4304> Found 545
4497> 0 bad luck 1
4672> 1 bad luck 1
4855> 2 bad luck 437485612413337193583034997070105315745309
5039> 3 Oh yeah 552330951024629792017

It takes 5.039 seconds.

After playing that magic number, I found that the optimal value for this number is 4860(the total BSmooth values we wish to collect). It looks like this:

20> A 552330951024629792017
28> B 792071513649121280077
28> N 437485612413337193583034997070105315745309
89> built prime base of 3774/80000. Max prime 79999
1808> found 4860 bSmooth values
2708> Extracted 3926/4860 vectors
3457> Found 2
3634> 0 bad luck 437485612413337193583034997070105315745309
3815> 1 Oh yeah 792071513649121280077

Continued Factorization

This is my original optimization idea. It doesn't mean that I was the first to discover it, a research is needed to answer that question, but it means that I am not aware of anyplace else doing anything similar.

Instead of calculating the matrix at the end, let's calculate it for each new prime

The algorithm

Continues input: bSmooth value
Output: A factor of N

- Save bSmooth into **bSmoothList**
- Add new row for the **solutions matrix**
- For each bitIndex in **bSmooth**.vector
    - Check in **eliminators map**, if we have an eliminator that can eliminate that bit
        - eliminate/xor it if we do
- Now the bSmooth is either a solution or a new eliminator

Input: eliminator
- if eliminator vector is empty/NULL
    - Then extract a solution from the **solutions matrix** of the eliminator row.
- Otherwise find the index of the first 1 bit in the eliminator.vector
- Add the eliminator to the **eliminators map** by that index
- For each bSmooth in **bSmoothList**
    - Check if it has the eliminator index on

        - Eliminate it if it does
    - Recursively execute this method for each bSmooth that were eliminated

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Episode 9: Improving logs

In the last episode we improved our matrix to work in real time. Now we have a multithreaded system that factors our numbers.

Today we are going to update the logging, to get a better picture about how it works and fix some small bugs.

Updated log format

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Episode 10: Multi Polynomial

In the last episode we worked on getting better logs. Now let's get back for optimizations!

So far we been working on the below polinomial, searching for bSmooth values of y

$$(x + \lfloor\sqrt{N}\rfloor)^2 - N = y$$

The problem with this approach is that y will keep growing forever and so the chances for it to be smooth will decrease, and so is our sieving speed.

Peter Montgomery, offered another polynomials set of the form

$$(ax + b)^2 - N = y$$

If we choose $b^2 - N = ac$, then we can rewrite the entire thing as

$$a^2x^2 + 2ab + b^2 - N = a^2x^2 + 2ab + ac = a(a\,x^2 + 2b + c) = y$$

If we also say that a is a square, then we can ignore it while sieving and just consider $a\,x^2 + 2b + c$

To find a solution for $b^2 - N = ac$ we can pick a prime q near to $\sqrt[4]{N}$, then define a as $a = q^2$. Also $b^2 - N = ac$ will only have a solution if N is a quadratic residue of q.

To calculate b we need to compute the modulare square root of N mod a. We do that by first calculating the modulare square root of N mod q and then "lifting" the root mod q^2 using Hensel's Lemma.

The lifting is calculated as explained by Carl Schildkraut.
https://math.stackexchange.com/a/3779351/101178

The idea of a multi polynomial is to periodically pick a new q and sive over a new polynomial. It also makes it easy to sieve in parallel.

In fact q needs to be picked up near $\sqrt{\frac{\sqrt{2N}}{m}}$ where m is the sieving range, each time after sieving m values, we switch a polynomial. This way it is promised that y will always be around $m\sqrt{2n}$ for $-m < x < m$

The only problem with this optimization is that it requires rebuilding the wheels each time we change a polynomial, and that is expensive.

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Episode 11: Big Primes

In the previous episode we switched using a Multi Polynomial based on Peter Montgomery offer. We also improved logging and optimized the magic numbers. We can factor 200 bit numbers in ~226 sec.

In this episode we will see how we can increase our reminder to include values outside our prime base.

So far we have been searching for smooth values b, now let's make the condition more loose and also include the values qb where q is a small prime number outside our prime base. Once we find two such values qb_1 and $q\,b_2$ we can multiply them together to get a smooth value $q^2b_1b_2$

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Episode 12: Cleanup + Better Matrix

In the previous episode we used Big Primes optimization to get our 200 bit execution performance to just 126 sec. And while our performance soure up, our code becomes more and more messy. So now it's about time to switch into a more reasonable representation of it.

Prepare yourself for a massive clean up!

But that's not all, there are some Matrix optimizations as well ;)

1. We don't need to make a recursive call as we did in episode 8. It only adds noise, the algorithm works without it as well
2. During solution extraction we almost run out of memory when computing BigIntegers. We can use the below observation to keep our calculation within the bounds of N^2. It saves about 5 sec for a solution extraction time and about 2 GB in memory usage lol. It unblocks us in targeting some really big numbers. Our goal is to calculate $A^2 - N = B^2$
    a. $ab \equiv c \ (mod \ N) \ \Rightarrow (a \ mod \ N)(b \ mod \ N) \equiv c \ (mod \ N)$ -> This will help with A calculations since we have access for A, but we don't have B until we multiply all of our relationis and perform a sqrt on the final result.
    b. We do have a vector of primes with an odd power. So we can write B in each of the relations as: $B`v = B$ where $v$ is the product of all the primes in the vector and $B` = \frac{B}{v}$, that also implies that $B`$ is a square, since we divide it by all the odd primes, so now it must be. We can now use `a.` optimization for $B`$. As for v we will create an array where we will track the power of all those vectors as we multiply the relations. At the end all of the powers must be even and we will multiply B by those primes mod N.

One of the biggest things that I noticed during the cleanup, is that we were working with Executor, that required a new Runnable creation each time we switch threads. It caused a thread overhead. So now I switched to working with BlockingQueue that have predefined data objects to pass jobs between threads. It solves the thread overhead problem and gives better performance.

The new structure is as follow:

- PolyMiner - generates polynomials for the Siever
- Siever - finds potential relations for VectorExtractor
- VectorExtractor - extract vectors for valid relations and handle them to the matrix
- Matrix - Extract the solution out of the found relations in real time

Each one of the above works in it's own thread, and we can add multiple Sievers as we wish, so the solution now adds parallelism to our work.

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Episode 13: The (-1) prime

The previous episode cleanup was awesome. We have now a pretty code, and it becomes much easier to debug and optimize it.

I was hoping to make this episode about double prime optimization. Just like we did in episode 11 with big primes, but taking two primes instead of one. It creates a few challenges

1. How are we separating two and one primes?
2. Once we understand that those are two primes, how do we factor it? What would be the most efficient method for it?
3. Once we obtained a new relation that has two big primes with all the other primes in our prime base. How do we eliminate it?

I was trying to solve 3, by including the two big primes in the prime base, just for the matrix calculation, since it's already having an elimination logic. But when I did that the memory exploded.

The matrix worked amazingly fast, easily digesting 200k+ variables. But then it ran out of juice and everything became super slow until some time later it crashed on out of memory.

So I turn into the heavy debug tools. I started to profile the memory, examine the threads, and consider new code strategies. I ended up with a bunch of small optimizations that nearly cut the memory usage by 300%, but still, I couldn't feed the matrix monster lol.

So now I will show you all the small optimizations that I came out with, especially the (-1) prime. I hope that in future episodes I will be able to find a solution for the double prime optimization.

Memory optimizations and more

- It turns out that ⅔ of the time we are working with negative numbers. During the sieve we are trying to compute a log of a negative number, it returns a Nan that fails all of our estimations. So by fixing it and adding -1 into our prime base, we gain about 300% speedup for our sieving
- We invest 400MB into the wheels. After throwing them into a dynamic cache, the memory usage dropped to just 10 MB.
- But it's not as big as what we spent on BigIntegers, those use int[] arrays behind the scene and can easily grow into a couple of GB. By replacing `mod` and `divide` with `devideAndReminder` it cuts the BigInteger memory usage by half.
- The biggest memory monster is the Matrix, you can barely spot it while not using the double prime optimization, but I figured that we should not eliminate eliminators. It's both a big memory and a performance gain.

After all of those optimizations, the buttle neck moved from the Siever to the Vector extractor.

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Episode 14: Perfect M

During the last episode we entered the mini optimizations phase, those are small hacks with big impacts. So more of those small candies are coming up today as well.

Siever

- Merged with VectorExtractor - To create a better resources balance in the system I placed the VectorExtractor on the same thread as the Siever.
- Replaced double with bytes - It turns out that our bSmooth threshold became so big, so it's sufficient to use bytes instead of doubles.

PolyMinder

- Cached `MathUtils.modSqrt(N, x)` - the heaviest part of the PolyMiner is the composition of the wheels and `MathUtils.modSqrt(N, x)` was responsible for about 60% of it.
- Calculated the delta sieve for the perfect range. Due to the rounding errors in our a calculations, in $a(a\,x^2 + 2b\ +\ c)$ where a is chosen as the closest integer to $\frac{\sqrt{2N}}{m}$ we ended up with a non perfect range. So to fix it I calculated the perfect range as: $p = \frac{\sqrt{b^2 - ac} - b}{a}$ and then the delta is equal to: $p - \frac{m}{2}$.

VectorExtractor

- Fast 2 extraction - Since we are working in base two, we can quickly extract all the two powers by checking the last set bit index, and then shift right to remove it.

Due to all of those optimizations, I had to play again with the magic numbers. I moved away from using B and switched into using the prime base size. With a little bit guessing, I was able to find a combination that yielded 100% improvement in the overall speed.

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Episode 14: Receiving

Mini optimizations continue! Today, an improvement of 60% by receiving.

I actually made tons of changes to the code, but the boost is coming primarily from the receiving optimization!

- Proguard + Gradle - The debug code is starting to grow, so I wanted to clean it out using proguard. Since it's much easier to integrate it using Gradle, that's what I did. However it yielded no performance gain. Still it's a nice to have feature as now we can be more aggressive with debug and analysis checks.
- Moved wheels into Siever - I noticed that the siever threads are stuck waiting for the PolyMiner to produce more work. It puzzled me for a while, since the siever is much slower than Polyminer, but then I figured that we have many sievers and just one Polyminer, so the resources distribution is not so good. I solved it by taking 97% of the polyminer work to the Siever, the wheels generation part. I think it improved the performance by 2%-3%, I am not sure. But again, it feels like a really good change, since it makes the bottle necks much more visible.
- Analytics - I moved the analytics logic from the Logger to the new Analytics class and added a percentage counting, so it's easy to see what portion of the code takes the most resources.
- Receiving - This is where the performance boost is coming from. When checking if x is smooth, we used to try and divide it by all the primes in the prime base. Now I made two optimizations
    - First, I check if the x index in the sieving array is divisible by one of the primes. This way we don't have to work with big Integers, instead we only compute mod on Java ints.
    - After finding a prime that is divisible by x, I don't divide x by it, but instead build a "Java long" multiplication and switch to a BigInteger after the long running out of space, so we are working on smaller values. Finally when I computed a BigInteger that is the product of all the primes in the prime base that divides x, I divide x by it and convert it to long. I then continue factoring that long.

  So basically with this optimization, we are almost not working with BigIntegers anymore, all is done on ints and longs.

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