# Project: Peirce's Alpha System

## INSTRUCTIONS

1. This project must be done and submitted individually.
2. Submit one ZIP file, called `<student number>.zip` to the Luminus folder:
   "`Projects > Peirce's Alpha System> Submissions`" by **Friday 9 April, 17:00**.
3. The ZIP file contains a non longer than 8 pages report, in a file "`<student number>.pdf`", briefly presented the design, implementation, and features of the software, the code, in a subdirectory "`Code`", and a no longer than five minutes video of a commented presentation and demonstration of the software, in a file "`<student number>.mp4`".

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Peirce's Alpha system is a diagrammatic representation of propositional formulae. A well-formed formula, or well-formed diagram, is defined as follows.

- The empty slate is a well-formed diagram.
- A proposition symbol, $p$, is a well-formed diagram (we write $p$).
- If $D$ is a well-formed diagram, then so is a single cut of $D$ (we write $[D]$) .
- If $D_1$ and $D_2$ are well-formed diagrams, then their juxtaposition is a well-formed diagram (we write $D_1 D_2$).
- Nothing else is a well-formed diagram.

Every formula of propositional logic can be represented with conjunction and negation alone. Juxtaposition corresponds to conjunction and cut corresponds to negation. Every formula of propositional logic can be represented as a well-formed diagram of Peirce's Alpha system.

For instance, the diagram in Figure 1 represents the propositional formula $p \wedge \neg(p \wedge \neg q)$.

Peirce's Alpha system is a deductive system consisting of the following five inference rules.

- Erasure: Any evenly enclosed diagram may be erased.
- Insertion: Any diagram may be scribed on any oddly enclosed area.
- Iteration: If a diagram $D$ occurs in a nest of cuts, it may be scribed on any area not part of $D$, which is contained in the area containing $D$.
- Deiteration: Any diagram whose occurrence could be the result of iteration may be erased.
- Double Cut: A double cut may be inserted around or removed from any diagram.

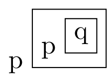


Figure 1: A well-formed diagram for the propositional formula $p \wedge \neg(p \wedge \neg q)$.

$$p \ \boxed{\boxed{q}}$$

Figure 2: A well-formed diagram for the propositional formula $p \land \neg\neg q$.

p q

Figure 3: A well-formed diagram for the propositional formula $p \land q$.

Notice that by application of the rule of erasure with the outmost $p$ and the $p$ appearing in the first cut in the diagram in Figure 1, we obtain the diagram in Figure 2 and by the application of the rule of double cut to the diagram in Figure 2, we obtain the diagram in Figure 3. The diagram in Figure 3 represents the propositional formula $p \land q$. The sequence of application of the rules is equivalent to Modus Ponens.

**Question 1** [20 marks]

The five inference rules define the valid modifications of a well-formed diagram. The basic actions are reminiscent of Xerox PARC's 'select, cut, copy, and paste' command paradigm of modern interactive interfaces. Implement, in the language of your choice, a graphical interactive proof assistant for Peirce's Alpha system with the ability to select, cut, copy, and paste according to the five rules. For convenience, add a tool that transforms a formula of propositional logic with the main connectives ($\land$, $\lor$, $\neg$, $\rightarrow$, $\leftrightarrow$, etc., written in Coq or Latex or both) into an equivalent Peirce diagram.

– END OF QUESTIONS –