

## Project: Symbolic Computation

### INSTRUCTIONS

1. This is an individual project.
2. Submit one ZIP file, called `<student number>.zip` to the Luminus folder:  
 "Projects > Symbolic Computation > Submissions" by **Friday 19 February, 17:00**.
3. The ZIP file contains a report, the code and some example input files.
  - The report presents your answers. It consists of a section for each answer and a subsection for each part of each question that discuss the main design, algorithmic, and implementation issues with relevant source code snippets and example. Use the following LaTeX template to write the report. The report is a file `<student number>.pdf`.  
 "Projects > Symbolic Computation > Report Sample > `report.tex`".
  - The code is a commented SWI Prolog program. The code is a single file `<student number>.pl`.
  - In addition, provide some example input files in a directory called `Examples`.
4. There is no possibility of late submission.

Symbolic computation, computer algebra, or algebraic computation, is the design, implementation, and analysis of algorithms and software for manipulating mathematical expressions. In this project we design and implement in Prolog symbolic computation algorithms for univariate polynomials with real coefficients and for elementary functions.

#### Question 1 [10 marks]

We consider the polynomial ring formed by polynomials with real coefficients equipped with addition (and subtraction) and multiplication to form a commutative algebra.

- (a) Write a definite clause grammar that recognises and parses polynomials with real coefficients. We use the LaTeX mathematical notation (see [https://www.overleaf.com/learn/latex/List\\_of\\_Greek\\_letters\\_and\\_math\\_symbols](https://www.overleaf.com/learn/latex/List_of_Greek_letters_and_math_symbols)) to input and output the polynomials and the letter `x` to represent the indeterminate.

For instance the following polynomial is written in LaTeX as follows.

$$5.2 \times x^{22} - 3.44 \times x^3 + 25$$

`5.2 \times x^{22} - 3.44 \times x^3 + 25`

A possible parse tree for the above polynomial is the following Prolog term.

`'-'(' \times '(5.2, ''(x, 22)), '+'(' \times '(3.44, ''(x, 3), 25))`

Indicate in your report which additional shorthands and notations (e.g. associativity, recognising  $5.2 x^{22}$  as  $5.2 \times x^{22}$ , etc.) you choose to implement.

- (b) Extend the definite clause grammar so that it recognises and parses sums (and differences) and products of polynomials with real coefficients, and combinations thereof.
- (c) Write a Prolog procedure `start/2` that repeatedly reads algebraic expressions on the ring of polynomials with real coefficients (sums, and differences, and products of polynomials with real coefficients) and returns canonical representations as sums of powers of  $x$  in decreasing order or error messages. `start(+File1, +File2)` reads the input expressions from `File1` and outputs the results in `File2`.

- (d) Consider now the derivative of a polynomial with real coefficients. The algebraic structure is still closed under this operation. We use the variant of Leibniz's notation  $\frac{df(x)}{dx}$  noted `\diff{f(x)}{x}`. Extend `start/2` above so that it recognises, parses and simplifies expressions involving sums, products, as well as derivatives of polynomials with real coefficients, and combinations hereof.

**Question 2** [5 marks]

Extend the work above to a larger class of elementary functions. Clearly present and discuss the requirements that you choose and implement.

– END OF QUESTIONS –