12.

$$u'' = 1, \qquad x \in (0,1) \qquad \begin{array}{l} u(0) = 0 \\ u'(1) = 0 \end{array}$$

$$\varphi_i(x) = \sin((i+1)\pi x/2), \qquad i = 0,1,2,\dots,N$$

$$w(x) = \sum_{j=0}^{N} c_j \varphi_j(x)$$

In the Galerkin approximation the problem of finding $w$ is formulated as

$$(w'', v) = (1, v), \qquad \forall v \in V,$$

where $V = \text{span}\{\varphi_i(x),\ i = 0,1,\dots,N\}$.

If we do not perform integration by parts, we get

$$\sum_{j=0}^{N} (\varphi_j'', \varphi_i) c_j = (1, \varphi_i).$$

For $N=0$, this equation becomes

$$(\varphi_0'', \varphi_0) c_0 = (1, \varphi_0).$$

$$\varphi_0''(x) = -\frac{\pi^2}{4} \sin(\pi x/2)$$

$$\varphi_0(x) = \sin(\pi x/2)$$

$$(\varphi_0'', \varphi_0) = -\frac{\pi^2}{4} \int_0^1 dx \sin^2(\pi x/2)$$

$$= -\frac{\pi^2}{4} \cdot \frac{1}{2} = -\frac{\pi^2}{8}$$

$$(1, \varphi_0) = \frac{2}{\pi} \qquad \Rightarrow \qquad c_0 = \frac{\frac{2}{\pi}}{-\frac{\pi^2}{8}} = -\frac{16}{\pi^3},$$

Thus, we get the same result with and without integration by parts.