Hedge Funds Report

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1 Introduction

In contrast with the traditional equity and bond asset classes, financial literature has shown a complex behaviour of hedge funds returns. Persistent non-linearity (Fung-Hsieh 2004) and strong autocorrelation (Harris and Mazibas 2011) set forth new challenges to be addressed in the field of hedge funds replications. Historically, two main approaches have been adopted: the distribution-matching and the factor-based approach. While for the first one the objective is to recreate the distributional properties of the replicated instrument through constrained optimization, the factor-based approach uses linear and, sometimes, non-linear approximation to minimise the tracking error between the hedge fund returns and the weighted average return of the factors.

In this research project, we propose an approach to replicate the monthly returns of the HRF Fund of Hedge Fund Index (FOF). Our sample refers to the period from January 1990 to September 2022, covering more than three decades and different market cycles. The proposed model builds upon the past literature of factor-based analysis and implements two types of models: one with linear risk exposures and one non-linear generalized additive model (GAM). The idea is to use the results from the linear model as baseline and check whether a better fit obtained through GAM can improve our estimation accuracy and reduce the tracking error of the replication. To achieve this result, we try to identify and dynamically replicate the empirical properties observed in hedge fund indices through a time-variant exposure to a set of market factors.

In the final section, we will then focus on the Covid crisis 2020 and use this period of market stress as solid baseline for comparing the replicating performances of the model across different market regimes. We will show that both linear and GAM model present high replicative power over the whole sample, reaching R^2 levels above 80% after the 2008 crisis. In addition, we will see how GAM model yields more volatile estimates than the linear counterpart during the Covid market crash, evidenced by much higher levels of tracking error.

2 Literature review

The paper "Factor-Based Hedge Fund Replication with Risk Constraints" by Richard D. F. Harris and Murat Mazibas from 2011 is relevant and interesting to study for our project. Indeed, they propose a method to replicate the monthly returns of hedge funds indices using a factor-based model supplemented with a series of risk and return constraints and which implicitly target all the moments of the hedge fund return distribution.

The interesting element of this paper is that they develop a replicating method which combines the factor and the distribution-matching methodologies. They use the linear component from the factor-based approach but impose a range of constraints to ensure that the replicating portfolio matches various risk measures of the hedge fund such as the Conditional Value at Risk, Conditional Drawdown at Risk and the partial moments of returns. For the estimation, the model is tested over 9 years (from 2002 to 2011).

They found that the factor-based model generates a beta value closer to one than all other models and that it is able to explain an important part of the variance in hedge funds returns. However, the constraints model is able to generate highest R-squared statistic. Therefore, the replicating portfolios in many cases display a significant improvement over the out-of-sample performance of the factor-based model.

As part of our research, we decided to test the methodology presented in this paper while adapting it slightly. We applied a factor-based replication model and added four constraints. We have the following objective function:

$$minf(x) = var(r_{hf,t} - r_{p,t}) \tag{1}$$

We run the constrained optimization on a rolling window of 120 months with one month out-of-sample test to be consistent with the methodology described in Section 3. The following constraints are imposed in the optimization:

$$\sum_{i=1}^{m} x_i = 1, i = 1, ..., m \tag{2}$$

$$x_i \geq 0 \tag{3}$$

$$\sum_{i=1}^{m} \bar{r_i} x_i = \bar{r_{hf}} \tag{4}$$

$$CVaR_p = CVaR_{hf} \tag{5}$$

Additionally, constraints 1 and 3 are fixed. We then tested with the following combinations:

1. Matching VaR and no short selling
2. No short selling
3. Matching VaR and short selling
4. Short selling

The result of the replication can be seen in *Figure 6*.

3 Data and empirical methodology

In this section, we will be discussing our data and methodology for replicating the hedge fund strategy.

3.1 Data

Our analysis cover the period Jan 1990 to Sept 2022. We collect monthly data of our target variables HFRI Fund of Hedge fund index for a total of 394 observations, and we consider the aggregated index HRFI as a benchmark. Data for our factors is collected from Bloomberg and Kenneth French website (Ref 8). Although little consensus exists on which and how many factors are optimal to correctly capture the hedge funds's returns exposure, we follow the approach of Tupitsyn 2014, who proposed an initial set of 14 factors, 6 of which are in common with Hazadovic and Lo (2007). The complete set of factors is shown in Table 1. The selection not only goes in accordance with the past literature but is also based on the economic sense as well as the availability of liquid tradable securities in the market.

To best deal with high number of features and avoid overfitting, we apply stepwise selection on the whole data sample. In a stepwise procedure, variables are included iteratively in the regression with the possibility of inclusion and exclusion based on the new significance level. As a result of this procedure, a new filtered set of variables is used for the model estimation (Table 2).

3.2 Estimation

Parameters estimatation is done on a 12-month rolling window and the model is evaluated on a 1-month ahead out-of-sample forecast. We consider 120 observations enough to guarantee a good non-parametric estimation of the non-linear model (Giamouridis and Paterlini 2010) and, at the same time, to capture the long-term dynamics among factors.

3.3 Linear model

The factor-based replication approach shares similar principles and form with the arbitrage pricing theory (APT) developed by Ross in 1976. We will be using the following linear multifactor model:

$$R_{i,t} = \alpha_i + \sum_{j=1}^{N} \beta_{i,j} F_{j,t} + \varepsilon_{i,t} \tag{6}$$

where R is a fund's excess return is the abnormal return, beta is the factor sensitivity, F is the risk factor proxy and epsilon is the error term, which is assumed to be normally distributed and uncorrelated to the factors or to its previous realizations.

3.4 Non-linear model

Alongside with the linear factor model, we also propose a non-linear replication approach. As shown by Fung and Hsieh (2004) the linear form represents a strong limitation as it does not accurately reflect the empirical evidence of HF returns. Indeed, many studies agree on the fact that hedge funds have both linear and non-linear exposure to systematic risk factors. Their trading strategies are dynamic in nature and may involve investment in financial securities with non-linear payoffs such as options. Thus, fitting continuous smoothing functions to the data should help to better capture the non-linear relation between risk and expected return.

For these reasons, we have chosen to use an alternative methodology, the Generalized Additive Model (GAM). We have the following formula for GAM:

$$E\left[R\right] = \alpha + \sum_{i=1}^{m} f_i(F_i) \tag{7}$$

With the GAM methodology, we replace the parametric beta coefficients in the linear multifactor model with variable-specific smoothing functions, *f(.)* which summarize the trend of a response variable (such as the return) as a function of one or more systematic risk factors *F1, ..., Fm.*
To address the tendency of GAM models to overfit the data, we apply a visual screening of variables with are deemed to inidicate non-linear patterns when plotted against HF returns - see *Figure 5.* Out of 11 factors, only three variables are fitted through spline interpolation: SP500, EQEM, GSCI. The rest of the factors is included linearly as no evidence of complex patterns can be found in the scatter plots.

3.5 Testing

We will also be comparing the replication before and during the covid crisis. According to the analysis in the paper "Mutual Fund Performance and Flows during the COVID-19 Crisis" by Pastor and Vorsatz (Ref 6), most active funds underperform passive benchmarks during the crisis, contradicting a popular hypothesis. Indeed, according to the paper, the crisis has created unusually large price dislocations in financial markets. Until liquidity improved following the interventions from the Federal Reserve, its temporary shortage created massive market disruptions. The paper chooses February 20th 2020 as the start of the covid crisis which is also the start date we have chosen. However, they choose April 30th 2020 as the end of the covid crisis and only analyze this 10-week period. We chose to align with the choice of this paper as this was the critical period of covid. Indeed, although there was still major disruptions in 2021 and 2022, this did not sufficiently reflect in the markets which is why we decided to keep the covid crisis period short.

4 Empirical results

In this section, we will be discussing the results we obtained with our replication.

4.1 Linear model and GAM

Figure 1.A. (see Appendix) shows the results obtained with the linear replication model. First, by assumining a 5% significance level, we observe a strong fluctuations over time of the rolling p-values. Except for MOM and EQEM, which show a high degree of persistency, the significance all the other factors is volatile and varies across market cycles. For example, during the financial crisis of 2008, only 3 of the factors (SMB, HML, EQEM) mantain their statistical significance. On the other hand, the regression constant show little persistence, casting doubts on the ability of HF managers to generate positive alpha over time. A confirmation of this result is provided by *Figure 1.B.* which show the evolution of R^2 measure. Our evidence demostrate how high levels of replicability could be achieved after the 2008 crisis. Possible explanations could found in the failure of many hedge funds and subsequent outflows of capital from the HF space.

In *Figure 2.A.* (see Appendix), we can see the results obtained with the GAM replication model. We can see how the introduction of smoothing functions alters the composition of significant factors over time, resulting in more stable relationships compared with the linear clone. Surprisingly, SP500 shows little significance over time except for two short periods. The R^2 levels are in similar with what obtained in the linear regression.

If we now compare the return graphs, we can see that the results obtained are quite similar for the two models. In this case, it is interesting to compare it to the benchmark. In *Figure 4*, we can see the cumulative returns for the benchmark (HFRI), for the Fund of Fund as well as for the linear and the GAM models. We note that our two models are quite close to the benchmark and that there is no real difference between the linear and the GAM models for the overall fitting. For the Fund of Fund, both models are accurately capturing the trend but we see a tendency to overestimate the returns when testing out-of-sample.

4.2 Effect of the Covid Crisis

One goal of our analysis was also to compare the performance of the hedge fund replication before and during the covid crisis. For this, we studied the period between 2012 to 2020 as the pre-covid period and the period February 2020 to June 2022 as the covid period.
Table 4 and *Table 5* present both replication accuracy and replication clones perfomance metrics. In particular, values of mean return difference, tracking error, mean absolute error and cumulative error return are calculated on the out-of-sample forecast using FOF index as benchmark. The negative sign of the Delta R column indicates that both linear and GAM models tend to overestimate the HF returns at monthly level in the pre-Covid period. A 2.4% tracking error is evidence of a good replicative capacity over a relatively quiet 8-years period. On the contrary, *Table 5* shows higher tracking error for both models during the Covid market plunge.

5 Limitations

In this part, we will be discussing the limitations of the factor-based replication technique. The first limitation of this approach is the choice and number of risk factors included. Altough a feature selection technique is applied on the whole sample to filter out the initial set, there is no clear answer on how risk factors are priced and which proxies are relevant.
The lenght of the rolling window is another model parameter that is set arbitrarily. In our approach, we used 120 observation for our parameters estimation. Choosing a longer window may yield more precise estimates of the coefficients and hence a higher replicative power.

6 Conclusion

The synthetic replication of hedge fund strategies is an active area of research. We saw that it is possible to replicate hedge fund strategies using linear and non-linear factor-based replication.
In terms of explanatory power, the selected factors can capture up to 85% of the HF returns varibility, with a sharpe increase in R^2 measure following the 2008 crisis. We consider it to be a stricking results considering the fact that it is obtained by regressing on market factor with tradable liquid securities. On the contrary, our results show that the introduction of non-parametric smoothing function to approximate the relationship with the risk exposure does not imporove the tracking error. This result goes against the study of Baker and Filbeck (2017). In particular, we saw how tracking error metrics tends to be similar over long time period, but the forecast tends to diverge from the HF returns in period of market stress.
With our work, we can also conclude that both our linear and non-linear replication models showed better results when tracking the performance of the entire hedge fund rather than the performance of a particular index.

References

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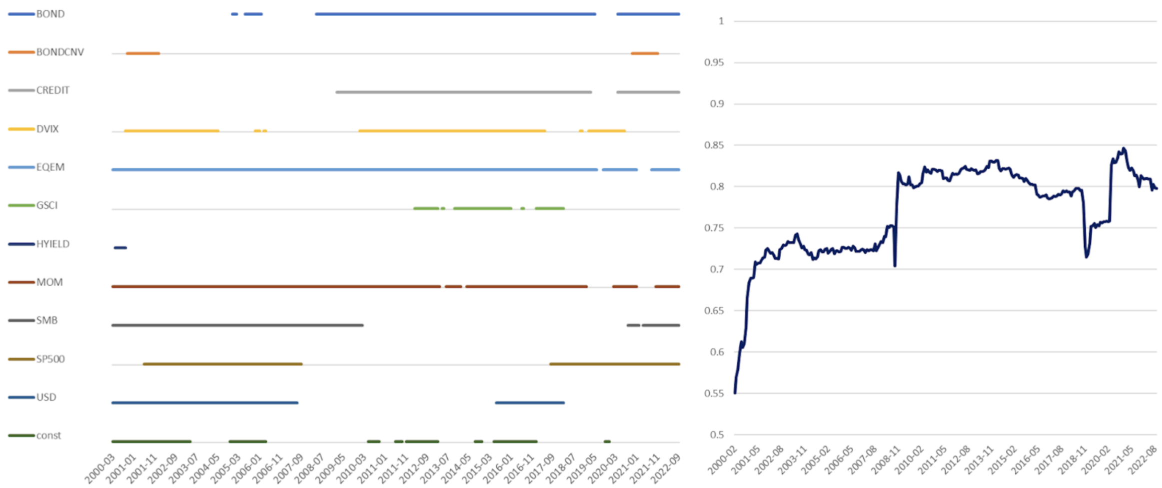


(a) Betas significance over time

(b) Rolling R Squared

Figure 1: Linear clone

Factor	Ticker	Asset
USD	DXY	US Dollar Index return
BOND	C0A2 Index	bond returns
CREDIT	COA4 - G0Q0	credit spread
SP500	SPX Index	equity returns
GSCI	SPGSCI	commodity returns
DVIX	VIX Index	volatility
EQINT	MXWOU	international equities
EQEM	MXEF	emerging market equities
HYIELD	LF98TRUU Index	high-yield bonds
BONDINT	SBWGU Index	international bonds
BONDCNV	VXAO Index	convertible bonds
SMB	from KF	size
HML	from KF	book-to-market
MOM	from KF	Carhart's momentum

Table 1: Factors description

	Ann Mean	Ann Std	SR	Skew	Kurtosis	JB p-value	ACF(1)	Selected
Target variable								
FOF	0.061092672	0.05608	1.08948	-0.7614	4.51939	0.00	0.30217	
HRFX	0.032941812	0.04641	0.70986	-2.388	13.9842	0.00	0.37223	
Benchmark								
HRFI	0.089255218	0.06781	1.31632	-0.7174	3.27681	0.00	0.26204	
Factors								
USD	0.008817319	0.07981	0.11048	0.35315	1.03523	0.00	0.07512	x
BOND	0.06212987	0.06001	1.03531	-1.4768	8.84643	0.00	0.15982	x
CREDIT	0.039482192	0.06696	0.58967	2.073	18.6078	0.00	0.15964	x
SP500	0.082105868	0.1486	0.55255	-0.5544	1.07085	0.00	0.00252	x
GSCI	0.056542717	0.21571	0.26213	-0.3325	2.2462	0.00	0.16922	x
DVIX	0.259953158	0.76436	0.34009	1.53903	5.22379	0.00	-0.1649	x
EQINT	0.030305994	0.16723	0.18122	-0.4497	1.2451	0.00	0.0583	
EQEM	0.067808584	0.21976	0.30855	-0.6092	1.89645	0.00	0.16043	x
HYIELD	0.076135729	0.08793	0.86588	-0.9897	8.21173	0.00	0.26492	x
BONDINT	0.044453559	0.06591	0.67443	-0.0243	0.73344	0.00	0.16743	
BONDCNV	0.095963696	0.12426	0.77225	-0.7529	3.68138	0.02	0.17548	x
SMB	1.307175573	10.9183	0.11972	0.62248	7.25073	0.00	-0.0324	x
HML	1.773129771	11.2097	0.15818	0.23256	2.52011	0.00	0.17848	
MOM	5.830534351	16.271	0.35834	-1.4419	10.4595	0.00	0.05899	x

Table 2: Descriptive statistics

	Delta R	Tracking Error	MAE	CER	Total return	Std	SR	ES
FOF					0.78	0.052	0.39	-0.036
Linear Clone	-0.018	0.03	0.006	-0.398	1.12	0.051	0.59	-0.035
Non-Linear Clone	-0.019	0.034	0.007	-0.43	1.15	0.048	0.63	-0.030

Table 3: Overall performance

	Delta R	Tracking Error	MAE	CER	Total return	Std	SR	ES
FOF					0.25	0.035	0.53	-0.022
Linear Clone	-0.003	0.024	0.005	-0.022	0.28	0.032	0.64	-0.021
Non-Linear Clone	-0.005	0.024	0.005	-0.043	0.30	0.031	0.71	-0.020

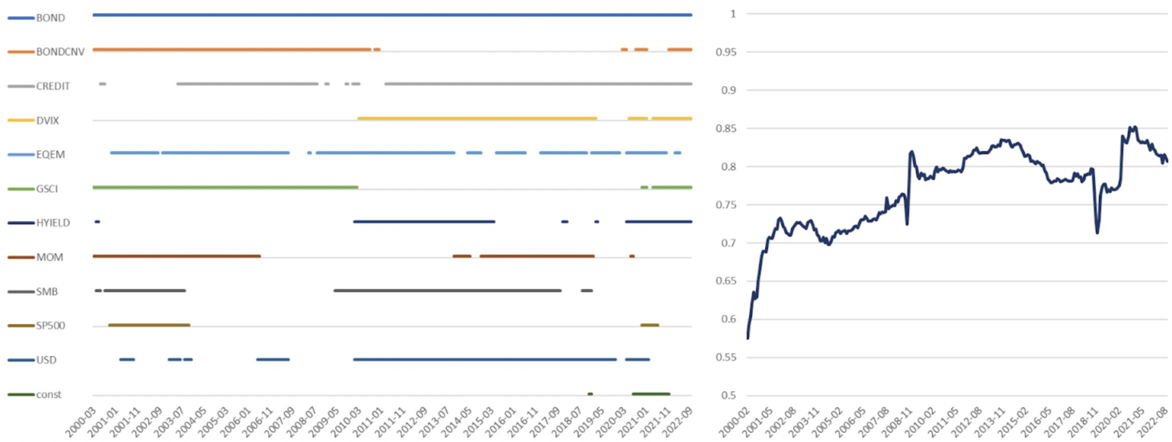
Table 4: Before covid (Jan 2012-Jan 2020) performance

	Delta R	Tracking Error	MAE	CER	Total return	Std	SR	ES
FOF					-0.019	0.210	-0.22	-0.076
Linear Clone	0.009	0.028	0.006	0.002	-0.021	0.186	-0.28	-0.062
Non-Linear Clone	-0.067	0.134	0.027	-0.016	-0.002	0.080	-0.08	-0.023

Table 5: During covid (Feb 2020-May 2020) performance

	count	ann_mean	ann_std	ann_SR	skew	kurtosis	min	max	autocorr
FOF	273	0.03487	0.052391	0.665565	-1.14019	4.768091	-0.07629	0.0521	0.239451
CVaR_NoSS	273	0.058935	0.047244	1.247467	-0.3891	0.72131	-0.05045	0.040354	0.120024
NoSS	273	0.05335	0.051669	1.032537	-1.01804	4.183526	-0.07958	0.044843	0.111376
CVaR and SS	273	0.083878	0.053039	1.581433	-0.5095	1.601218	-0.06662	0.044964	0.203941
SS	273	0.047483	0.05102	0.930671	-1.03398	3.750119	-0.07538	0.045898	0.07919

Table 6: Factor-based replication with constraints



(a) Betas significance over time

(b) Rolling R Squared

Figure 2: GAM clones

Figure 3: Returns replication

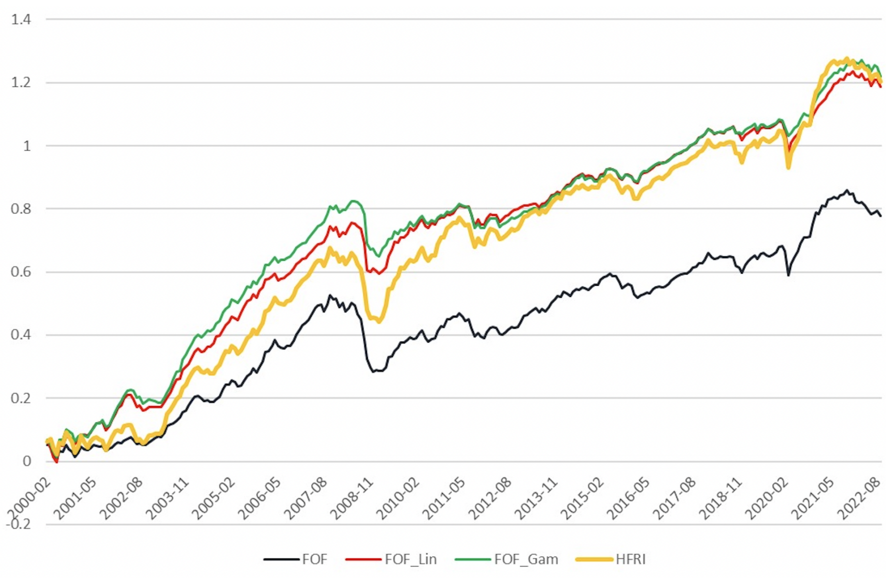


Figure 4: Benchmark and replicated cumulative returns

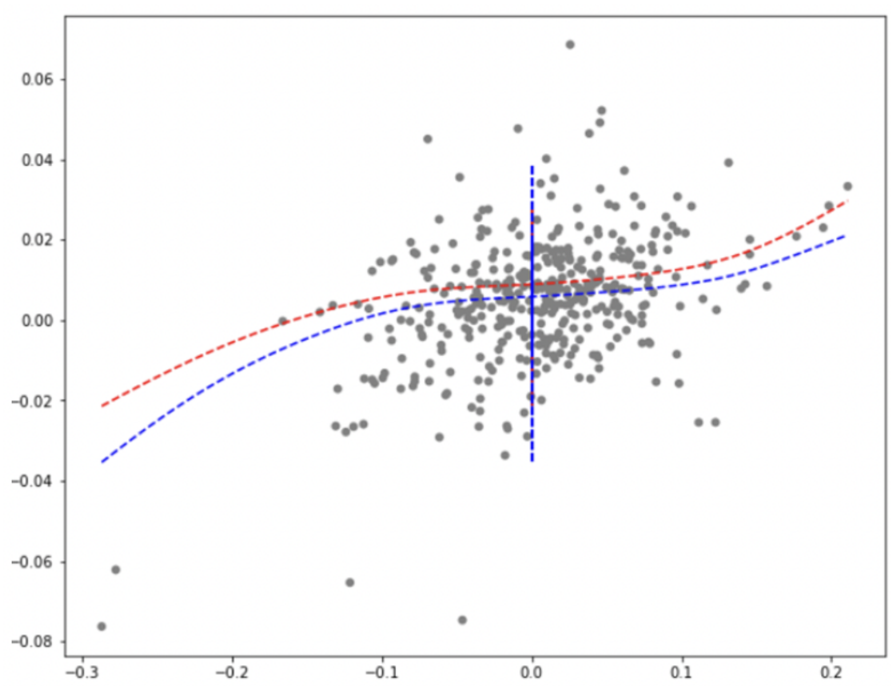


Figure 5: Example of spline interpolation for GSCI

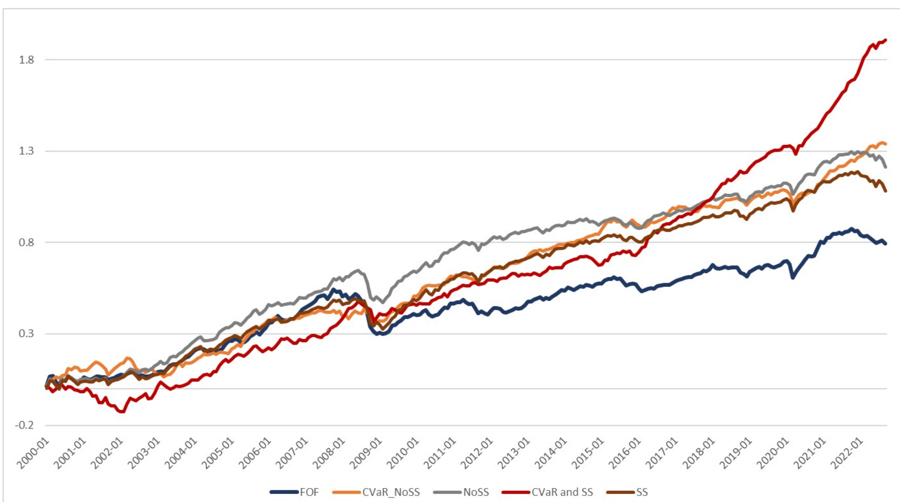


Figure 6: Factor-based replication with constraints