Homework 8 – Intro to Probability and Statistics

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Instructions:

Due: 06/04 at 11:59PM.

What am I expecting? An R Markdown with the answers. Note: this one can be done in paper and pencil. If you want to do so, do the problem and take pictures of it. Put the pictures in a PDF and send it.

Have fun!

Question 1

A factory produces valves, with 20% chance of a given valve be broken. The valves are sold in boxes, containing ten valves in each box. If no broken valve is found, then they sell the box for $10.00. With one broken valve, the box costs $8.00. With two or three, the box is sold for $6.00. More than three valves broken, they sell the box for $2.00. What is the mean sales price for the boxes?

I have done the question 1 using binomial distribution and these are the results:

Computing the chances of the box is getting sold for $10.00:

$$P(A) = \binom{n}{k} P^k Q^{n-k}$$

$$P(A) = \binom{10}{0} 0.2^0 0.8^{10-0}$$

$$P(A) = \frac{10!}{(10-0)!\,0!} 0.2^0 0.8^{10-0}$$

$$P(A) = 0.8^{10}$$

$$P(A) = 0.1073741824$$

Computing the chances of the box get sold for $8.00:

$$P(B) = \binom{n}{k} P^k Q^{n-k}$$

$$P(B) = \binom{10}{1} 0.2^1 0.8^{10-1}$$

$$P(B) = \frac{10!}{(10-1)!\,1!} 0.2^1 0.8^{10-1}$$

$$P(B) = \frac{10.9!}{9!} 0.2^1 0.8^9$$

$$P(B) = (2)0.8^9$$

$$P(B) = 0.268435456$$

Computing the chances of the box get sold for \$6.00:

$$P(C') = \binom{n}{k} P^k Q^{n-k}$$

$$P(C') = \binom{10}{2} 0.2^2 0.8^{10-2}$$

$$P(C') = \frac{10!}{(10-2)!\,2!} 0.2^2 0.8^{10-2}$$

$$P(C') = \frac{10.9.8!}{8!\,(2)} 0.2^2 0.8^8$$

$$P(C') = (45)(0.2^2)(0.8^8)$$

$$P(C') = 0.301989888$$

$$P(C'') = \binom{10}{3} 0.2^3 0.8^{10-3}$$

$$P(C'') = \frac{10.9.8.7!}{7!\,3.2.1} 0.2^3 0.8^7$$

$$P(C'') = (120)(0.2^3)(0.8^7) = 0.201326592$$

Thus,

$$P(C) = P(C') + P(C'') = 0.50331648$$

Computing the chances of the box get sold for \$2.00:

$$P(D) = 1 - P(A) - P(B) - P(C') - P(C'')$$

$$P(D) = 1 - 0.1073741824 - 0.268435456 - 0.301989888 - 0.201326592$$

$$P(D) = 0.1208738816$$

Therefore, the mean sales price for the boxes is giving by the following:

$Mean = P(A).pa + P(B).pb + P(C).pc + P(D).pd,$

since pa, pb, pc, pd are the prices for the boxes.

So, the mean is equal to:

$$Mean = 0.1073741824(10) + 0.268435456(8) + 0.301989888(6) + 0.1208738816(2)$$

$$Mean = 5.27491256$$

$$Mean \approx \$5.2$$

Question 2

If a random variable X has distribution:

x	-2	-1	0	1	2
f(x)	1/2	1/10	1/5	1/10	1/10
F(x)	1/2	3/5	4/5	9/10	1

Compute the mean, the variance, and the standard deviation of X.

Computing the mean:

$$E(x) = \sum x_i p_i = (-2)\left(\frac{1}{2}\right) + (-1)\left(\frac{1}{10}\right) + 0\left(\frac{1}{5}\right) + 1\left(\frac{1}{10}\right) + 2\left(\frac{1}{10}\right)$$

$$E(x) = -0.8$$

Computing the variance:

$$V(x) = \sum [x_i - E(x)]^2 p_i =$$

$$= (-2 + 0.8)^2\left(\frac{1}{2}\right) + (-1 + 0.8)^2\left(\frac{1}{10}\right) + (0 + 0.8)^2\left(\frac{1}{5}\right) + (1 + 0.8)^2\left(\frac{1}{10}\right) + (2 + 0.8)^2(\frac{1}{10})$$

$$V(x) = 1.96$$

Computing the standard deviation of x:

$$SD = \sqrt{V(x)}$$

$$SD = \sqrt{1.96} = 1.4$$

Question 3

If a random variable x has distribution (note: it is the same as in the previous problem):

x	-2	-1	0	1	2
f(x)	1/2	1/10	1/5	1/10	1/10
F(x)	1/2	3/5	4/5	9/10	1

Compute the distribution of the following transformations:

• x^2 ?

	0	1	4
$f(x^2)$	1/5	1/5	3/5
$F(x^2)$	1/5	2/5	1

• $3x$?

	-6	-3	0	3	6
f(x)	1/2	1/10	1/5	1/10	1/10
F(x)	1/2	3/5	4/5	9/10	1

Question 4

Prove the following statements:

• Let two constants, $a \in R$ and $b \in R$, and a discrete random variable x. Prove that $E(aX + b) = aE(X) + b$.

We already know that $\begin{cases} E(x) = \sum x_i p_i \\ E(ax) = aE(x) \\ \sum p(x) = 1 \end{cases}$, so

$$E(ax+b) = \sum (ax+b)p\,(x)$$

$$E(ax+b) = \sum (ax)p\,(x) + \sum (b)p\,(x)$$

$$E(ax+b) = a\sum (x)p\,(x) + b\sum p\,(x)$$

$$E(ax+b) = aE(x) + b$$

• Let two constants, $a \in R$ and $b \in R$, and a discrete random variable x. Prove that $V(aX + b) = a^2V(X)$.

$$V(aX+b) = E[\big((ax+b) - E[(ax+b)]\big)^2]$$

$$V(aX+b) = E[\big((ax+b) - (aE(x)+b)\big)^2]$$

$$V(aX+b) = E[\big(ax - aE(x)\big)^2]$$

$$V(aX+b) = E[(a(x - E(x)))^2]$$

$$V(aX+b) = E\big(a^2(x - E(x)\big)^2$$

$$V(ax+b) = a^2E(x - E(x))^2$$

$$V(ax+b) = a^2V(x)$$

• Let $a \in R$ a constant. Prove that $E(a) = a$ and that $V(a) = 0$.

Given $E(ax) = aE(x)$, a is a constant and x is a variable. If there is no variance in the variable, the expectation is equal to the constant. Consequently, there is no variance.