

Why Does Stock Market Volatility Change Over Time?

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ABSTRACT

This paper analyzes the relation of stock volatility with real and nominal macroeconomic volatility, economic activity, financial leverage, and stock trading activity using monthly data from 1857 to 1987. An important fact, previously noted by Officer (1973), is that stock return variability was unusually high during the 1929–1939 Great Depression. While aggregate leverage is significantly correlated with volatility, it explains a relatively small part of the movements in stock volatility. The amplitude of the fluctuations in aggregate stock volatility is difficult to explain using simple models of stock valuation, especially during the Great Depression.

ESTIMATES OF THE STANDARD deviation of monthly stock returns vary from two to twenty percent per month during the 1857–1987 period. Tests for whether differences this large could be attributable to estimation error strongly reject the hypothesis of constant variance. Large changes in the *ex ante* volatility of market returns have important negative effects on risk-averse investors. Moreover, changes in the level of market volatility can have important effects on capital investment, consumption, and other business cycle variables. This raises the question of why stock volatility changes so much over time.

Many researchers have studied movements in aggregate stock market volatility. Officer (1973) relates these changes to the volatility of macroeconomic variables. Black (1976) and Christie (1982) argue that financial leverage partly explains this phenomenon. Recently, there have been many attempts to relate changes in stock market volatility to changes in expected returns to stocks, including Merton (1980), Pindyck (1984), Poterba and Summers (1986), French, Schwert, and Stambaugh (1987), Bollerslev, Engle, and Wooldridge (1988), and Abel (1988). Mascaro and Meltzer (1983) and Lauterbach (1989) find that macroeconomic volatility is related to interest rates.

Shiller (1981a,b) argues that the level of stock market volatility is too high relative to the *ex post* variability of dividends. In present value models such as Shiller's, a change in the volatility of either future cash flows or discount rates

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causes a change in the volatility of stock returns. There have been many critiques of Shiller's work, notably Kleidon (1986). Nevertheless, the literature on "excess volatility" has not addressed the question of why stock return volatility is higher at some times than at others.

This paper characterizes the changes in stock market volatility through time. In particular, it relates stock market volatility to the time-varying volatility of a variety of economic variables. Relative to the 1857–1987 period, volatility was unusually high from 1929 to 1939 for many economic series, including inflation, money growth, industrial production, and other measures of economic activity. Stock market volatility increases with financial leverage, as predicted by Black and Christie, although this factor explains only a small part of the variation in stock volatility. In addition, interest rate and corporate bond return volatility are correlated with stock return volatility. Finally, stock market volatility increases during recessions. None of these factors, however, plays a dominant role in explaining the behavior of stock volatility over time.

It is useful to think of the stock price, P_t as the discounted present value of expected future cash flows to stockholders:

$$E_{t-1} P_t = E_{t-1} \sum_{k=1}^{\infty} \frac{D_{t+k}}{[1 + R_{t+k}]^k} \quad (1)$$

where D_{t+k} is the capital gain plus dividends paid to stockholders in period $t + k$ and $1/[1 + R_{t+k}]$ is the discount rate for period $t + k$ based on information available at time $t - 1$. (E_{t-1} denotes the conditional expectation.) The conditional variance of the stock price at time $t - 1$, $\text{var}_{t-1}(P_t)$, depends on the conditional variances of expected future cash flows and of future discount rates, and on the conditional covariances between these series.¹

At the aggregate level, the value of corporate equity clearly depends on the health of the economy. If discount rates are constant over time in (1), the conditional variance of security prices is proportional to the conditional variance of the expected future cash flows. Thus, it is plausible that a change in the level of uncertainty about future macroeconomic conditions would cause a proportional change in stock return volatility.² If macroeconomic data provide information about the volatility of either future expected cash flows or future discount rates, they can help explain why stock return volatility changes over time. "Fads" or "bubbles" in stock prices would introduce additional sources of volatility.

Section I describes the time series properties of the data and the strategy for modeling time-varying volatility. Section II analyzes the relations of stock and bond return volatility with the volatility of inflation, money growth, and industrial production. Section III studies the relation between stock market volatility

¹ The variance of the sum of a sequence of ratios of random variables is not a simple function of the variances and covariances of the variables in the ratios, but standard asymptotic approximations depend on these parameters.

² For a positively autocorrelated variable, such as the volatility series in Table II, an unexpected increase in the variable implies an increase in expected future values of the series for many steps ahead. Given the discounting in (1), the volatility series will move almost proportionally. See Poterba and Summers (1986) for a simple model that posits a particular ARIMA process for the behavior of the time-varying parameters in a related context.

and macroeconomic activity. Section IV analyzes the relation between financial leverage and stock return volatility. Section V analyzes the relation between stock market trading activity and volatility. Finally, Section VI synthesizes the results from the preceding sections and presents concluding remarks.

I. The Time Series Behavior of Stock and Bond Return Volatility

A. Volatility of Stock Returns

Following French, Schwert, and Stambaugh (1987), I estimate the monthly standard deviation of stock returns using the daily returns to the Standard and Poor's (S&P) composite portfolio from January 1928 through December 1987. The estimates from February 1885 through December 1927 use daily returns on the Dow Jones composite portfolio. (See Schwert (1989d) for a more detailed description of these data.) The estimator of the variance of the monthly return is the sum of the squared daily returns (after subtracting the average daily return in the month):

$$\hat{\sigma}_t^2 = \sum_{i=1}^{N_t} r_{it}^2, \quad (2)$$

where there are N_t daily returns r_{it} in month t .³ Using nonoverlapping samples of daily data to estimate the monthly variance creates estimation error that is uncorrelated through time.⁴

Daily stock return data are not readily available before 1885. Also, macroeconomic data are rarely measured more often than monthly. To estimate volatility from monthly data, I use the following procedure:

- (i) Estimate a 12th-order autoregression for the returns, including dummy variables D_{jt} to allow for different monthly mean returns, using all data available for the series,

$$R_t = \sum_{j=1}^{12} \alpha_j D_{jt} + \sum_{i=1}^{12} \beta_i R_{t-i} + \varepsilon_t. \quad (3a)$$

- (ii) Estimate a 12th-order autoregression for the absolute values of the errors from (3a), including dummy variables to allow for different monthly standard deviations,

$$|\hat{\varepsilon}_t| = \sum_{j=1}^{12} \gamma_j D_{jt} + \sum_{i=1}^{12} \rho_i |\hat{\varepsilon}_{t-i}| + u_t. \quad (3b)$$

³ French, Schwert, and Stambaugh (1987) use one lagged cross-covariance in (2), and they make no adjustment for the mean return. Their estimator is not guaranteed to be positive. Indeed, for one month in the 1885–1927 period, the French, Schwert, and Stambaugh estimate of volatility is negative. The estimates from (2) are very similar to the French, Schwert, and Stambaugh estimates, except that they are always positive.

⁴ If the data are normally distributed, the variance of the estimator $\hat{\sigma}_t$ is $\sigma_t^2/2N_t$, where σ_t^2 is the true variance (Kendall and Stuart (1969, p. 243)). Thus, for $N_t = 22$ and $\sigma_t = 0.04$, the standard error of $\hat{\sigma}_t$ is 0.006, which is small relative to the level of σ_t . Since this is a classic errors-in-variables problem, the autocorrelations of the estimates $\hat{\sigma}_t$ will be smaller than, but will decay at the same rate as, the autocorrelations of the true values σ_t .

- (iii) The regressand $|\hat{\epsilon}_t|$ is an estimate of the standard deviation of the stock market return for month t similar to $\hat{\sigma}_t$ (although it uses one rather than 22 observations). The fitted values from (3b) $|\tilde{\epsilon}_t|$ estimate the conditional standard deviation of R_t , given information available before month t .⁵

This method is a generalization of the 12-month rolling standard deviation estimator used by Officer (1973), Fama (1976), and Merton (1980) because it allows the conditional mean return to vary over time in (3a) and allows different weights for lagged absolute unexpected returns in (3b). It is similar to the autoregressive conditional heteroskedasticity (ARCH) model of Engle (1982). Davidian and Carroll (1987) argue that standard deviation specifications such as (3b) are more robust than variance specifications based on $\hat{\epsilon}_t^2$. They also argue that iterated weighted least squares (WLS) estimates, iterating between (3a) and (3b), provide more efficient estimates. Following their suggestion, I iterate three times between (3a) and (3b) to compute WLS estimates.

Figure 1 plots the predicted standard deviations from monthly returns $|\tilde{\epsilon}_{st}|$ for 1859–1987, along with the predicted standard deviations from daily returns $\hat{\sigma}_t$ (from a 12th-order autoregression for $\hat{\sigma}_t$ as in (3b)) for 1885–1987. Volatility predictions from the daily data are much higher following the 1929 and 1987 stock market crashes because there were very large daily returns in October 1929 and October 1987. Otherwise, Figure 1 shows that the predicted volatility series are similar. Stock return volatility is persistent over time.

B. Volatility of Bond Returns

If the underlying business risk of the firm rises, the risk of both the stock and the bonds of the firm should increase. Also, if leverage increases, both the stocks and the bonds of the firm become more risky. Thus, in many instances the risk of corporate stock and long-term corporate debt should change over time in similar ways.

Figure 2 plots the predicted standard deviations of long-term corporate bond returns $|\tilde{\epsilon}_{rht}|$ for 1859–1987. It also shows the predicted standard deviations of stock returns $|\tilde{\epsilon}_{st}|$ for comparison. Note that the scale of the right-hand bond return axis is about three times smaller than the scale of the left-hand stock return axis, showing that the standard deviation of monthly stock returns is about three times larger than for bond returns over this period. There are many similarities between predicted volatilities of stock and bond returns. In particular, volatility was very high from 1929 to 1939 compared with the rest of the 1859–1987 period. Moreover, bond returns were unusually volatile in the periods during and immediately following the Civil War (1861–1865). In recent times, the “OPEC oil shock” (1973–1974) caused an increase in the volatility of stock and bond returns.

Figure 3 plots the predicted standard deviations of short-term interest rates $|\tilde{\epsilon}_{rst}|$ for 1859–1987. The volatility of Int_t measures time variation in the ex ante

⁵ Since the expected value of the absolute error is less than the standard deviation from a normal distribution, $E|\tilde{\epsilon}_{st}| = \sigma_t(2/\pi)^{1/2}$, all absolute errors are multiplied by the constant $(2/\pi)^{-1/2} \approx 1.2533$. Dan Nelson suggested this correction.

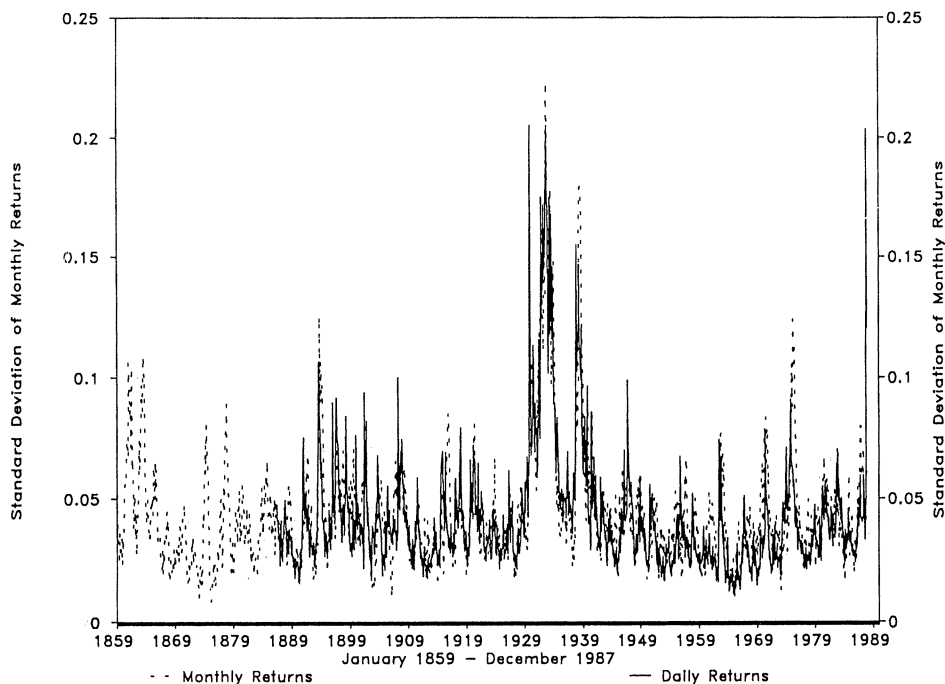


Figure 1. Predictions of the monthly standard deviation of stock returns based on monthly data (---) for 1859–1987 and on daily data (—) for 1886–1987. For monthly returns, a 12th-order autoregression with different monthly intercepts is used to model returns, and then the absolute values of the residuals are used to estimate volatility in month t . For daily returns, the returns in the month are used to estimate a sample deviation for each month. To model conditional volatility, a 12th-order autoregressive model with different monthly intercepts is used to predict the standard deviation in month t based on lagged standard deviation estimates. This plot contains fitted values from the volatility regression models.

nominal interest rate, not risk, since these securities are essentially default free.⁶ Note that the right-hand interest rate volatility scale is over 12 times smaller than the left-hand stock volatility scale. There are periods in the 19th century when short-term interest rate volatility rose for brief periods, many of which were associated with banking panics. (See Schwert (1989b).) It is clear from Figures 2 and 3 that long-term bond return and short-term interest rate volatility increased dramatically around 1979. There is not a similar increase in stock return volatility. As noted by Huizinga and Mishkin (1986), the Federal Reserve Board changed its operating procedures to focus on monetary aggregate targets at that time.

The plots in Figures 2 and 3 are consistent with the following simple story. Short-term interest rate and long-term bond return volatility have similarities due to inflation and monetary policy. Stock and long-term bond return volatility have similarities due to real financial and business risk.

Table I contains means, standard deviations, skewness, and kurtosis coeffi-

⁶ See Fama (1976) for an analysis of the variability of short-term nominal interest rates.

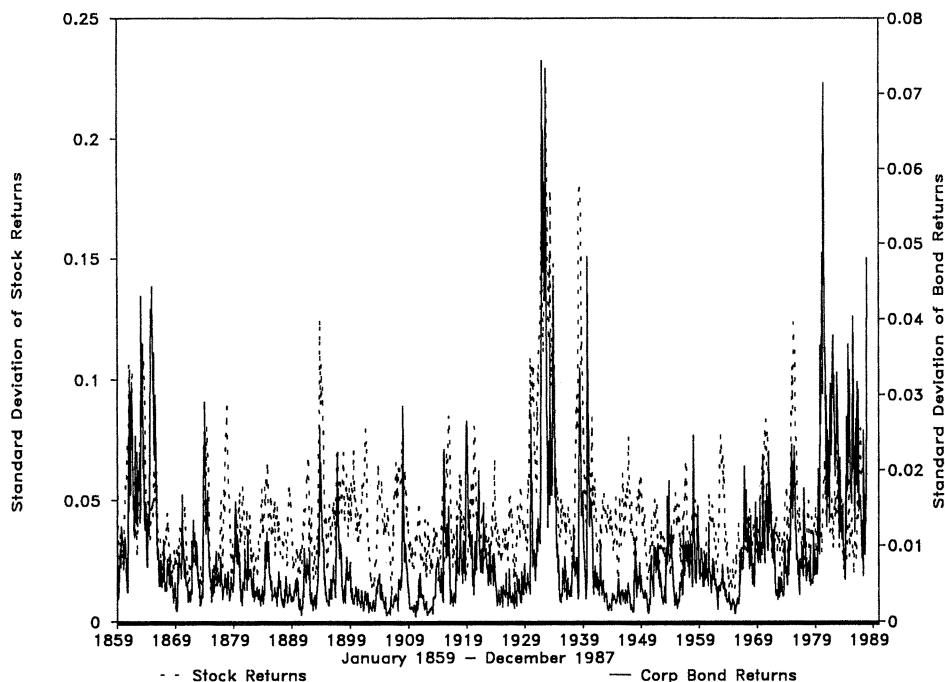


Figure 2. Predictions of the monthly standard deviations of stock returns (---) and of high-grade long-term corporate bond returns (—) for 1859–1987. A 12th-order autoregression with different monthly intercepts is used to model returns, and then the absolute values of the residuals are used to estimate volatility in month t . To model conditional volatility, a 12th-order autoregressive model with different monthly intercepts is used to predict the standard deviation in month t based on lagged standard deviation estimates. This plot contains fitted values from the volatility regression models.

coefficients and autocorrelations of the estimates of stock return volatility based on monthly and daily data, $|\hat{\epsilon}_{st}|$ and $\hat{\sigma}_t$. It also contains summary statistics for estimates of the volatility of short- and long-term bond returns, $|\hat{\epsilon}_{rst}|$ and $|\hat{\epsilon}_{rht}|$, inflation, $|\hat{\epsilon}_{pt}|$, money growth, $|\hat{\epsilon}_{mt}|$, and industrial production, $|\hat{\epsilon}_{it}|$.⁷

Table II summarizes the autoregressions used to predict volatility. The sum of the autoregressive coefficients measures the persistence of the volatility series, where a value of unity implies nonstationarity. (See Engle and Bollerslev (1986) for a discussion of integrated conditional heteroskedasticity.) The F -test measures whether there is significant deterministic seasonal variation in the average volatility estimates. The coefficient of determination R^2 and the Box-Pierce (1970) statistic $Q(24)$ measure the adequacy of the fit of the model.

As suggested by the analysis in footnote 1, the estimates of volatility from daily data have much less error than the estimates from monthly data. The sample standard deviation of $|\hat{\epsilon}_{st}|$ is about sixty percent larger than that of $\hat{\sigma}_t$ from 1885 to 1987, though the average values are similar. Moreover, the autocor-

⁷ See Table AI in the Appendix for a brief description of the variables used in this paper.

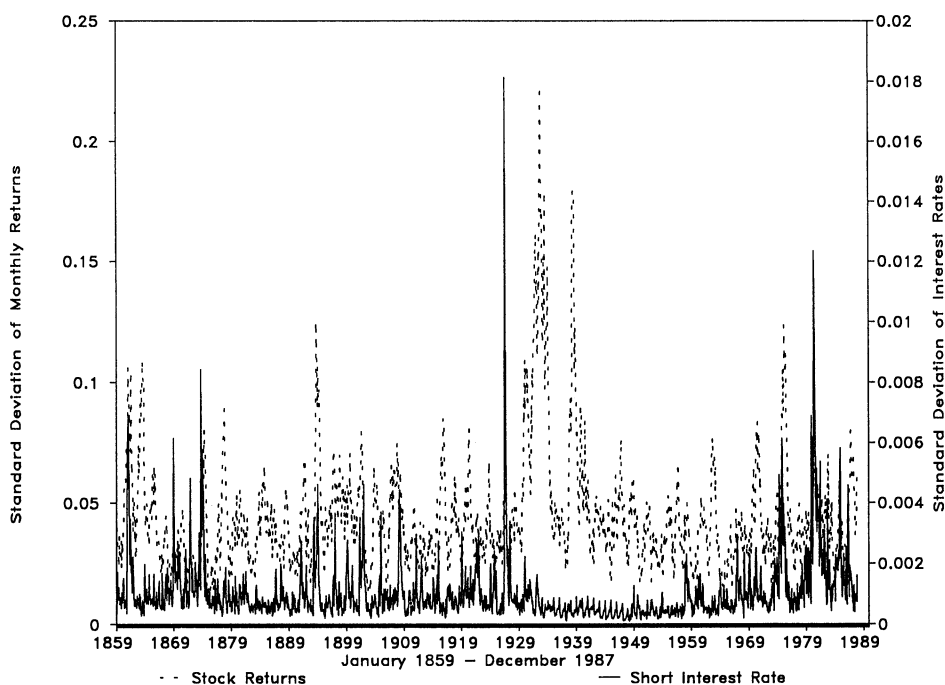


Figure 3. Predictions of the monthly standard deviations of stock returns (---) and of short-term interest rates (—) for 1859–1987. A 12th-order autoregression with different monthly intercepts is used to model returns or interest rates, and then the absolute values of the residuals are used to estimate volatility in month t . To model conditional volatility, a 12th-order autoregressive model with different monthly intercepts is used to predict the standard deviation in month t based on lagged standard deviation estimates. This plot contains fitted values from the volatility regression models.

relations of $\hat{\sigma}_t$ are much larger than those of $|\hat{\epsilon}_{st}|$, though they decay slowly for both series. This slow decay shows that stock volatility is highly persistent, perhaps nonstationary. (See Poterba and Summers (1986) and Schwert (1987) for further discussion.) The correlation between $|\hat{\epsilon}_{st}|$ and $\hat{\sigma}_t$ is 0.56 from 1885 to 1987, and the correlation between the volatility predictions $|\tilde{\epsilon}_{st}|$ and $\tilde{\sigma}_t$ is 0.78 from 1886 to 1987. The two methods of predicting volatility have similar time series properties.

The autocorrelations in Table I and the summary statistics for the estimated models in Table II are similar for all the volatility series. The autocorrelations are small (between 0.2 and 0.4), but they decay very slowly. This is consistent with conditional volatility being an integrated moving average process, so shocks to volatility have both permanent and transitory parts. The unit root tests in Table II show that most of the sums of the autoregressive coefficients are reliably different from unity using the tables in Fuller (1976). However, Schwert (1987, 1989a) shows that the Fuller critical values are misleading in situations such as this. The estimation error in the monthly volatility estimates biases the unit root

Table I
**Summary Statistics for Monthly Estimates of the Standard Deviations of Stock Returns,
 Bond Returns, and Growth Rates of the Producer Price Index, the Monetary Base, and
 Industrial Production, 1858–1987**

The summary statistics are the means, standard deviations, skewness, kurtosis, and autocorrelations at lags 1, 2, 11, and 12 of the monthly standard deviation estimates and the Box-Pierce (1970) statistic for 24 lags of the autocorrelations $Q(24)$. A 12th-order autoregression with different monthly intercepts is used to model the growth rates, and then the absolute values of the errors from this model estimate the monthly standard deviations. The exception is the estimate of stock market volatility based on daily stock returns within the month. For further details, see equations (3a) and (3b) in the text and the data appendix.

| Volatility Series | Sample Period | Sample Size | Mean | Std. Dev. | Skewness | Kurtosis | Autocorrelations | | | | | $Q(24)$ |
|---|---------------|-------------|--------|-----------|----------|----------|------------------|-------|-------|----------|----------|---------|
| | | | | | | | r_1 | r_2 | r_3 | r_{11} | r_{12} | |
| Monthly stock returns | 1858–1987 | 1560 | 0.0444 | 0.0435 | 3.06 | 17.59 | 0.21 | 0.19 | 0.24 | 0.19 | 0.16 | 913 |
| Monthly stock returns | 1885–1987 | 1235 | 0.0455 | 0.0450 | 3.16 | 18.43 | 0.20 | 0.20 | 0.25 | 0.18 | 0.17 | 807 |
| Daily stock returns | 1885–1987 | 1235 | 0.0415 | 0.0272 | 3.16 | 14.29 | 0.69 | 0.58 | 0.51 | 0.44 | 0.44 | 5711 |
| Monthly short-term interest rates | 1858–1987 | 1560 | 0.0010 | 0.0141 | 5.42 | 53.21 | 0.43 | 0.34 | 0.19 | 0.14 | 0.16 | 1053 |
| Monthly high-quality long-term bond returns | 1858–1987 | 1560 | 0.0084 | 0.0116 | 3.25 | 13.76 | 0.42 | 0.32 | 0.34 | 0.25 | 0.22 | 2589 |
| Monthly medium-quality long-term bond returns | 1920–1987 | 816 | 0.0163 | 0.0223 | 5.25 | 47.94 | 0.40 | 0.25 | 0.33 | 0.26 | 0.24 | 1256 |
| PPI inflation rates | 1858–1987 | 1560 | 0.0127 | 0.0161 | 3.33 | 16.46 | 0.48 | 0.37 | 0.28 | 0.25 | 0.24 | 2586 |
| Monetary base growth rates | 1879–1987 | 1302 | 0.0080 | 0.0102 | 3.36 | 16.01 | 0.43 | 0.34 | 0.24 | 0.30 | 0.29 | 1549 |
| Industrial production growth rates | 1890–1987 | 1175 | 0.0184 | 0.0202 | 2.20 | 6.67 | 0.41 | 0.31 | 0.30 | 0.20 | 0.19 | 1486 |

Table II

Summary Statistics for Autoregressive Predictive Models for the Volatility of Stock Returns, Bond Returns, and the Growth Rates of the Producer Price Index, the Monetary Base, and Industrial Production, 1859–1987

A 12th-order autoregression with different monthly intercepts is used to model the growth rates or returns, and then the absolute values of the errors from this model $|\hat{\epsilon}_t|$ estimate the monthly standard deviations. The exception is the estimate of stock market volatility based on daily stock returns within the month. The 12th-order autoregression for the volatility estimates is

$$|\hat{\epsilon}_t| = \sum_{j=1}^{12} \gamma_j D_{jt} + \sum_{i=1}^{12} \rho_i |\hat{\epsilon}_{t-1}| + u_t. \quad (3b)$$

This table shows the sum of the autoregressive coefficients ($\rho_1 + \dots + \rho_{12}$), indicating the persistence of volatility. A *t*-test for whether the sum equals unity, indicating nonstationarity, is in parentheses below the sum. It also shows an *F*-test for the equality of the 12 monthly intercepts ($\gamma_1 = \dots = \gamma_{12}$) and its *p*-value. Finally, it shows the coefficient of determination R^2 and the Box-Pierce (1970) $Q(24)$ statistic for the residual autocorrelations (which should be distributed as $\chi^2(12)$ in this case).

| Volatility Series | Sum of AR | <i>F</i> -Test for Equal | R^2 | $Q(24)$ |
|---|---|--|-------|---------|
| | Coefficients (<i>t</i> -test vs. one) | Monthly Intercepts (<i>p</i> -value) | | |
| Monthly stock returns | 0.8471 (−3.72) | 0.97 (0.475) | 0.132 | 45.8 |
| Daily stock returns | 0.9634 (−1.07) | 0.59 (0.838) | 0.524 | 60.2 |
| Monthly short-term interest rates | 0.7925 (−4.40) | 1.96 (0.028) | 0.371 | 19.5 |
| Monthly high-quality long-term bond returns | 0.8376 (−4.20) | 0.59 (0.835) | 0.260 | 59.4 |
| Monthly medium-quality long-term bond returns | 0.7769 (−3.47) | 6.78 (0.000) | 0.280 | 16.6 |
| PPI inflation rates | 0.8438 (−4.29) | 0.39 (0.961) | 0.271 | 53.1 |
| Monetary base growth rates | 0.7918 (−4.74) | 0.65 (0.787) | 0.220 | 37.0 |
| Industrial production growth rates | 0.8336 (−3.82) | 0.42 (0.948) | 0.219 | 46.9 |

estimates toward stationarity.⁸ The results for the estimate of stock volatility from daily data $\hat{\sigma}_t$ support this conclusion since the sum of the autoregressive coefficients is closer to unity and the test statistic is small.

C. Measurement Problems—The Effects of Diversification

Even though the set of stocks contained in the “market” portfolio changes over time, the behavior of volatility is not affected. There are few stocks in the sample

⁸ Also see Pagan and Ullah (1988) for a discussion of the errors-in-variables problem associated with models like (3b).

in 1857, and they are all railroad stocks. Nevertheless, they represent most of the actively traded equity securities at that time. Also, railroads owned a wide variety of assets at that time. I have calculated tests for changes in stock volatility around the times when major changes in the composition of the portfolio occurred, and, surprisingly, there is no evidence of significant changes. Schwert (1989d) analyzes several alternative indices of United States stock returns for the 19th century and finds that the different portfolios have similar volatility after 1834. Though the number of securities and industries included has grown over time, the plot of stock return volatility in Figure 1 does not show a downward trend.

This conclusion contrasts with the analysis of unemployment, industrial production, and gross national product data by Romer (1986a,b, 1989). Also, when the Bureau of Labor Statistics has expanded the monthly sample used to calculate the CPI inflation series, there have been noticeable reductions in the volatility of measured inflation rates. Shapiro (1988) argues that the stability of stock return volatility between the 19th and 20th centuries supports Romer's conclusions that the higher level of volatility in pre-1930 macroeconomic data is primarily due to measurement problems. Nonetheless, it is perhaps surprising that stock return volatility is not higher in the 19th century due to measurement problems.

II. Relations between Stock Market Volatility and Macroeconomic Volatility

A. Volatility of Inflation and Monetary Growth

The stock returns analyzed above all measure nominal (dollar) payoffs. When inflation of goods' prices is uncertain, the volatility of nominal asset returns should reflect inflation volatility. I use the algorithm in equations (3a) and (3b) to estimate monthly inflation volatility from 1858 to 1987 for the PPI inflation rate. Figure 4 plots the predicted PPI inflation volatility $|\tilde{\epsilon}_{pt}|$ from 1859 to 1987. Note that the right-hand PPI inflation volatility axis is about $\frac{2}{3}$ smaller than the left-hand stock volatility axis. The volatility of inflation was very high around the Civil War (1860–1869), reflecting changes in the value of currency relative to gold after the U.S. went off the gold standard in 1862. Since the U.K. remained on the gold standard, this also represents volatility in the exchange rates between U.S. and U.K. currencies. The Spanish-American War (1898), World War I and its aftermath (1914–1921), and World War II (1941–1946) are also periods of high inflation uncertainty. Another increase in inflation volatility occurred during the 1973–1974 OPEC oil crisis. While inflation volatility increased during the 1929–1939 period, this change is minor compared with the volatility that occurred during wars.

Figure 5 plots the predicted volatility of the monetary base growth rates $|\tilde{\epsilon}_{mt}|$ from 1880 to 1987. The volatility of money base growth rates rose during the bank panic and recession of 1893 and remained high until about 1900. The next sharp increase in volatility occurred during the bank panic of 1907. The period following the formation of the Federal Reserve System (1914–1923) was another period of high volatility. Finally, the period of the Great Depression (1929–1939)

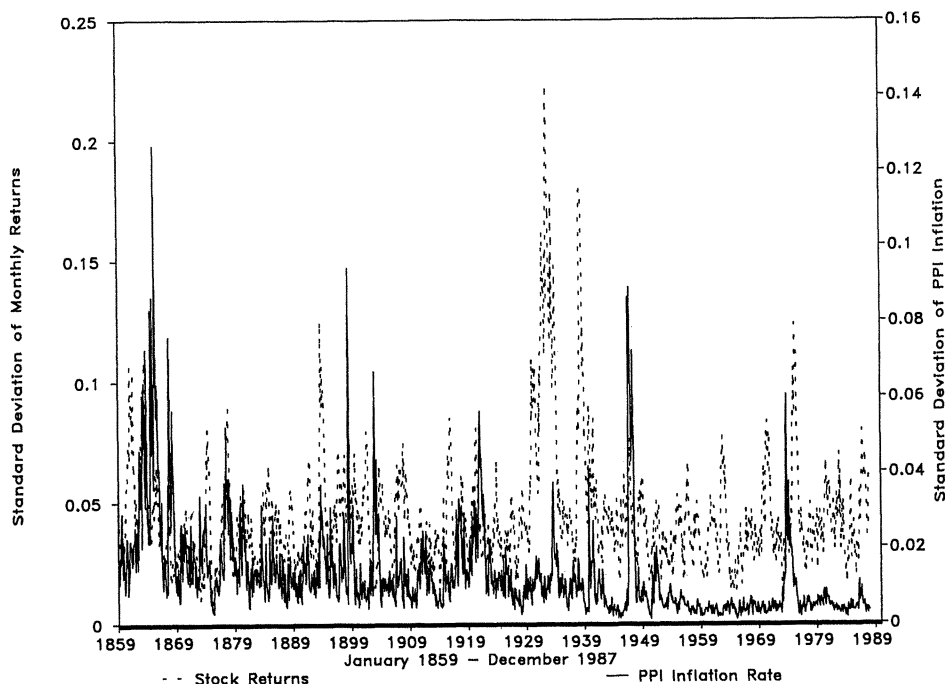


Figure 4. Predictions of the monthly standard deviations of stock returns (---) and of producer price index inflation rates (—) for 1859–1987. A 12th-order autoregression with different monthly intercepts is used to model returns or inflation rates, and then the absolute values of the residuals are used to estimate volatility in month t . To model conditional volatility, a 12th-order autoregressive model with different monthly intercepts is used to predict the standard deviation in month t based on lagged standard deviation estimates. This plot contains fitted values from the volatility regression models.

was a period of very high volatility. Since the early 1950's, the volatility of the monetary base growth rate has been relatively low and stable.⁹

Both the PPI inflation rate and the monetary base growth rate exhibit much lower levels of volatility after World War II. In each case, the sample used to measure these variables has expanded over time, and there have been major institutional changes that have been intended to dampen macroeconomic fluctuations. Without detailed analysis similar to Romer's work on industrial production, unemployment, and gross national product, it is impossible to tell how important the changes in measurement techniques have been in reducing volatility.

Table III contains tests of the incremental predictive power of 12 lags of PPI inflation volatility $|\hat{\epsilon}_{pt}|$ in a 12th-order vector autoregressive (VAR) system for

⁹ It is surprising that the pattern of volatility is so different for the money base growth rate and the PPI inflation rate. Nevertheless, I have also analyzed the volatility of money supply ($M2$) growth and the Consumer Price Index (CPI) inflation rates since 1915, and they lead to similar conclusions. The lack of relation between monetary volatility and price volatility is an interesting question for future research.

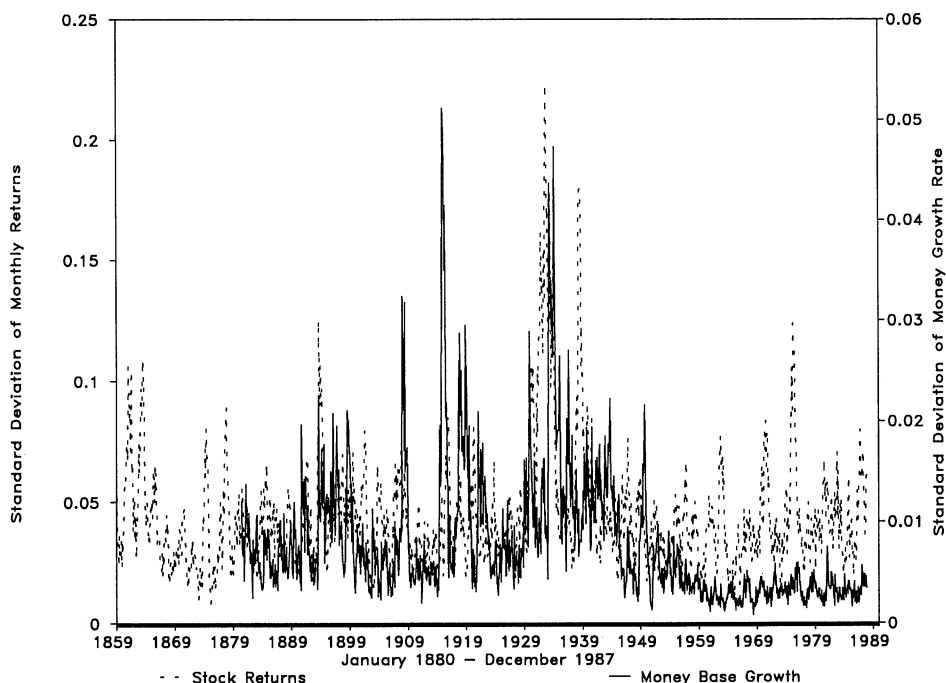


Figure 5. Predictions of the monthly standard deviations of stock returns (---) and of money base growth rates (—) for 1880–1987. A 12th-order autoregression with different monthly intercepts is used to model returns or money growth rates, and then the absolute values of the residuals are used to estimate volatility in month t . To model conditional volatility, a 12th-order autoregressive model with different monthly intercepts is used to predict the standard deviation in month t based on lagged standard deviation estimates. This plot contains fitted values from the volatility regression models.

stock volatility, high-grade bond return volatility $|\hat{\epsilon}_{rht}|$, and short-term interest volatility $|\hat{\epsilon}_{rst}|$ that allows for different monthly intercepts. The VAR model uses both the monthly measure of stock return volatility $|\hat{\epsilon}_{st}|$ and the daily measure $\hat{\sigma}_t$.¹⁰ These VAR models are generalizations of the autoregressive model in (3b), but they include lagged values of other variables to help predict volatility. The F -tests in Table III measure the significance of the lagged values of the column variable in predicting the row variable, given the other variables in the model. F -statistics that are larger than the 0.01 critical value 2.28 are indicated with asterisks.

The largest F -statistics are on the main diagonal of these matrices, and the size of the statistics decreases away from the diagonal. For example, lagged stock

¹⁰ Models using the volatility of medium-grade (Baa-rated) bond return volatility, $|\hat{\epsilon}_{rmt}|$, instead of high-grade bond return volatility, yielded similar results for the post-1920 periods. Medium-grade bond volatility is more strongly related to the stock volatility and more weakly related to the short-term interest rate volatility, but the relations with the macroeconomic volatility series are generally similar. Because these data are only available from 1920 to 1987 and the results are similar, they are not reported.

volatility is the most important variable in predicting current stock volatility. Lagged bond return volatility also helps in most sample periods, and lagged short-term interest volatility contributes less. Likewise, stock volatility helps predict bond return volatility in most periods, but it rarely improves predictions of interest rate volatility. In most sample periods, short-term interest volatility helps predict bond return volatility and vice versa. Except for monthly stock volatility from 1953 to 1987, there is little evidence that inflation volatility helps to predict future asset return volatility.

The present value relation in (1) is forward-looking. In an efficient market, speculative prices will react in anticipation of future events. Thus, it is also interesting to see whether asset return volatility helps to forecast later volatility of macroeconomic variables. Except for long-term bond returns from 1859 to 1987, there is no evidence that either stock or bond return volatility helps to predict inflation volatility. Perhaps this is because the major changes in inflation volatility occur during wars, and there seems to be little effect of wars on stock or bond return volatility.

Table IV contains tests of the incremental predictive power of 12 lags of monetary base growth volatility $|\hat{\epsilon}_{mt}|$ in a 12th-order VAR system similar to Table III. The relations among the measures of financial return volatility are similar to Table III. There is evidence that money growth volatility helps to predict the volatility of long-term bond returns from 1885 to 1919. Also, from 1885 to 1987, 1885 to 1919, and 1920 to 1952, there is evidence that money growth volatility helps to predict the volatility of stock returns measured using daily data. On the other hand, from 1920 to 1952 (and the sample periods that include this subperiod), both measures of stock return volatility help to predict the volatility of the base growth rate.

The relations between inflation or money growth volatility and the volatility of asset returns are not strong. It is surprising that these macroeconomic measures of nominal volatility are not more closely linked with the volatility of short- and long-term bond returns.

B. Real Macroeconomic Volatility

Since common stocks reflect claims on future profits of corporations, it is plausible that the volatility of real economic activity is a major determinant of stock return volatility. In the present value model (1), the volatility of future expected cash flows, as well as discount rates, changes if the volatility of real activity changes.

Figure 6 contains a plot of the predicted volatility of the growth rates of industrial production $|\tilde{\epsilon}_{it}|$.¹¹ Note that the right-hand industrial production volatility scale is about $\frac{2}{3}$ smaller than the left-hand stock volatility scale. Summary statistics for these estimates are in Tables I and II. Industrial production volatility was high during the mid-1930's, during World War I, and especially

¹¹ I also examined the volatility of bank clearings data from Macaulay (1938) and the volatility of the liabilities of business failures data from Dun and Bradstreet (Citibase (1978)). Neither of these "real activity" variables was strongly related to stock volatility.

Table III
F-Tests from Vector Autoregressive Models for Stock, Bond, and Interest Rate Volatility, Including PPI Inflation Volatility, 1859–1987

A four variable, 12th-order vector autoregressive (VAR) model is estimated for stock, bond, interest rate, and PPI inflation volatility, including dummy variables for monthly intercepts. The *F*-tests reflect the incremental ability of the column variable to predict the respective row variables, given the other variables in the model. Measures of stock return volatility based on monthly data are used in the first four columns, and measures of stock return volatility based on daily data are used in the last four columns. The 0.05 and 0.01 critical values for the *F*-statistic with 12 and 200 degrees of freedom are 1.80 and 2.28, respectively. *F*-statistics greater than 2.28 are indicated with an asterisk.

| Dependent Variable | <i>F</i> -Tests with Monthly Stock Volatility | | | | <i>F</i> -Tests with Daily Stock Volatility | | | |
|--------------------|---|--------|----------|--------|---|--------|----------|--------|
| | Stock | Bond | Interest | PPI | Stock | Bond | Interest | PPI |
| <i>1859–1987</i> | | | | | | | | |
| Stock | 2.07 | 1.46 | 1.30 | 0.50 | | | | |
| Bond | 0.80 | 10.82* | 0.94 | 1.36 | | | | |
| Interest | 0.86 | 2.65* | 12.68* | 0.94 | | | | |
| PPI | 1.93 | 6.42* | 0.68 | 5.70* | | | | |
| <i>1885–1987</i> | | | | | | | | |
| Stock | 9.33* | 2.89* | 0.89 | 0.95 | 67.92* | 6.77* | 1.65 | 2.10 |
| Bond | 5.83* | 15.74* | 1.98 | 1.14 | 2.83* | 14.06* | 3.75* | 1.27 |
| Interest | 1.67 | 3.87* | 21.99* | 0.65 | 0.84 | 2.29* | 21.39* | 0.61 |
| PPI | 2.16 | 1.04 | 0.60 | 31.91* | 1.04 | 1.13 | 0.83 | 28.83* |
| <i>1885–1919</i> | | | | | | | | |
| Stock | 1.16 | 1.24 | 0.41 | 0.72 | 8.86* | 4.31* | 2.99* | 1.26 |
| Bond | 1.41 | 8.05* | 1.25 | 0.51 | 0.71 | 6.03* | 1.20 | 0.86 |
| Interest | 2.41* | 1.51 | 3.34* | 0.48 | 0.94 | 1.20 | 4.57* | 1.95 |
| PPI | 1.33 | 1.04 | 1.65 | 3.67* | 1.14 | 0.70 | 0.59 | 3.29* |

Table IV
F-Tests from Vector Autoregressive Models for Stock, Bond, and Interest Rate Volatility, Including Monetary Base Growth Volatility, 1885–1987

A four variable, 12th-order vector autoregressive (VAR) model is estimated for stock, bond, interest rate, and monetary base growth volatility, including dummy variables for monthly intercepts. The *F*-tests reflect the incremental ability of the column variable to predict the respective row variables, given the other variables in the model. Measures of stock return volatility based on monthly data are used in the first four columns, and measures of stock return volatility based on daily data are used in the last four columns. The 0.05 and 0.01 critical values for the *F*-statistic with 12 and 200 degrees of freedom are 1.80 and 2.28, respectively. *F*-statistics greater than 2.28 are indicated with an asterisk.

| Dependent Variable | <i>F</i> -Tests with Monthly Stock Volatility | | | | <i>F</i> -Tests with Daily Stock Volatility | | | |
|--------------------|---|--------|----------|--------|---|--------|----------|--------|
| | Stock | Bond | Interest | Base | Stock | Bond | Interest | Base |
| <i>1885–1987</i> | | | | | | | | |
| Stock | 7.85* | 2.60* | 0.78 | 0.93 | 62.39* | 5.83* | 1.00 | 4.83* |
| Bond | 5.36* | 16.92* | 1.79 | 2.25 | 2.76* | 15.88* | 3.39* | 1.36 |
| Interest | 1.41 | 3.50* | 22.30* | 1.07 | 0.88 | 2.04 | 22.61* | 0.60 |
| Base | 4.80* | 1.26 | 0.55 | 21.72* | 1.40 | 1.79 | 0.85 | 18.73* |
| <i>1885–1919</i> | | | | | | | | |
| Stock | 1.19 | 0.92 | 0.44 | 0.79 | 8.80* | 3.83* | 2.95* | 2.28* |
| Bond | 1.63 | 3.52* | 1.53 | 3.71* | 1.04 | 3.09* | 1.17 | 0.85 |
| Interest | 2.26 | 1.23 | 3.43* | 0.82 | 0.96 | 1.54 | 4.61* | 1.28 |
| Base | 1.98 | 0.87 | 1.46 | 2.96* | 0.92 | 3.21* | 1.03 | 2.47* |
| <i>1920–1952</i> | | | | | | | | |
| Stock | 2.82* | 3.16* | 0.40 | 0.88 | 23.21* | 3.64* | 0.86 | 2.44* |
| Bond | 7.80* | 4.03* | 0.26 | 1.11 | 4.78* | 3.60* | 0.27 | 1.16 |
| Interest | 0.40 | 0.38 | 11.99* | 0.39 | 0.56 | 0.22 | 12.08* | 0.42 |
| Base | 4.05* | 1.64 | 0.41 | 5.23* | 3.11* | 1.72 | 0.52 | 6.22* |
| <i>1953–1987</i> | | | | | | | | |
| Stock | 1.55 | 0.96 | 1.93 | 0.95 | 9.22* | 1.02 | 0.68 | 1.15 |
| Bond | 2.00 | 3.38* | 3.20* | 1.08 | 3.11* | 3.53* | 3.45* | 1.08 |
| Interest | 1.25 | 4.82* | 6.31* | 0.63 | 0.94 | 4.77* | 5.36* | 0.90 |
| Base | 0.83 | 0.68 | 1.43 | 1.63 | 1.11 | 0.85 | 1.26 | 1.74 |

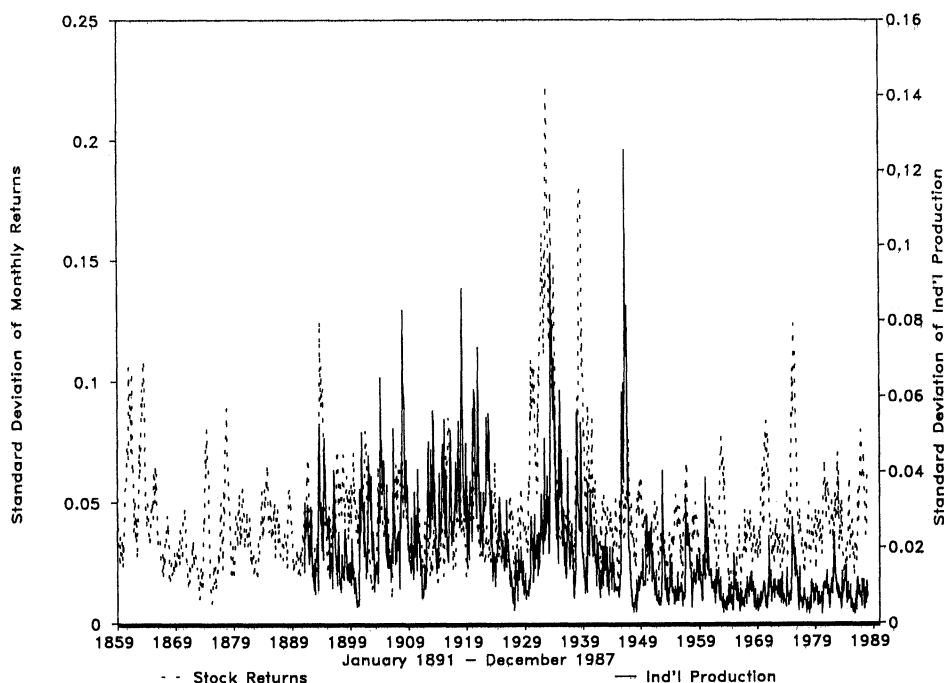


Figure 6. Predictions of the monthly standard deviations of stock returns (---) and of industrial production growth rates (—) for 1891–1987. A 12th-order autoregression with different monthly intercepts is used to model returns or industrial production growth rates, and then the absolute values of the residuals are used to estimate volatility in month t . To model conditional volatility, a 12th-order autoregressive model with different monthly intercepts is used to predict the standard deviation in month t based on lagged standard deviation estimates. This plot contains fitted values from the volatility regression models.

during the post-World War II period. There is a small increase in volatility during the 1973–1974 recession. Romer (1986b) argues that data collection procedures cause part of the higher volatility of this series before 1929.

Table V contains tests of the incremental predictive power of 12 lags of industrial production volatility $|\hat{\varepsilon}_{it}|$ in a 12th-order VAR system similar to those in Tables III and IV. The results for the financial variables are similar to those reported in Table III. The F -statistics measuring the ability of industrial production volatility to predict financial volatility are small. There is somewhat stronger evidence that stock return volatility predicts industrial production volatility for the 1891–1987 and 1920–1952 periods.

Thus, there is weak evidence that macroeconomic volatility provides incremental information about future stock return volatility. There is somewhat stronger evidence that financial volatility helps to predict macroeconomic volatility. While many of the macroeconomic volatility series are high during 1929–1939, none increases by a factor of three as stock return volatility did. This “volatility puzzle” will remain after all the subsequent analysis.

Table V
F-Tests from Vector Autoregressive Models for Stock, Bond, and Interest Rate Volatility, Including Industrial Production Growth Volatility, 1891-1987

A four variable, 12th-order vector autoregressive (VAR) model is estimated for stock, bond, interest rate, and industrial production growth volatility, including dummy variables for monthly intercepts. The *F*-tests reflect the incremental ability of the column variable to predict the respective row variables, given the other variables in the model. Measures of stock return volatility based on monthly data are used in the first four columns, and measures of stock return volatility based on daily data are used in the last four columns. The 0.05 and 0.01 critical values for the *F*-statistic with 12 and 200 degrees of freedom are 1.80 and 2.28, respectively. *F*-statistics greater than 2.28 are indicated with an asterisk.

| Dependent Variable | <i>F</i> -Tests with Monthly Stock Volatility | | | | <i>F</i> -Tests with Daily Stock Volatility | | | |
|--------------------|---|--------|----------|--------|---|--------|----------|--------|
| | Stock | Bond | Interest | IP | Stock | Bond | Interest | IP |
| <u>1891-1987</u> | | | | | | | | |
| Stock | 9.47* | 2.66* | 0.85 | 1.03 | 64.93* | 6.25* | 1.25 | 2.24 |
| Bond | 5.12* | 15.66* | 1.85 | 0.49 | 2.68* | 14.41* | 3.43* | 0.99 |
| Interest | 1.43 | 3.63* | 21.87* | 0.51 | 0.81 | 2.11 | 21.92* | 0.62 |
| IP | 3.18* | 0.76 | 0.75 | 24.07* | 1.33 | 0.89 | 0.55 | 19.79* |
| <u>1891-1919</u> | | | | | | | | |
| Stock | 0.98 | 1.59 | 0.60 | 0.98 | 7.38* | 3.57* | 2.70* | 0.35 |
| Bond | 1.26 | 6.30* | 1.24 | 1.24 | 0.96 | 4.29* | 1.20 | 0.70 |
| Interest | 2.25 | 1.46 | 2.88* | 0.37 | 0.72 | 1.22 | 3.97* | 0.58 |
| IP | 0.95 | 0.85 | 0.66 | 3.27* | 1.72 | 1.70 | 0.67 | 2.73* |
| <u>1920-1952</u> | | | | | | | | |
| Stock | 3.82* | 3.32* | 0.39 | 1.29 | 22.36* | 3.74* | 0.65 | 0.72 |
| Bond | 8.46* | 4.20* | 0.27 | 0.79 | 5.18* | 3.50* | 0.23 | 0.66 |
| Interest | 0.52 | 0.38 | 12.25* | 0.48 | 0.72 | 0.20 | 12.21* | 0.56 |
| IP | 2.65* | 1.31 | 0.57 | 5.60* | 1.08 | 1.23 | 0.58 | 4.80* |
| <u>1953-1987</u> | | | | | | | | |
| Stock | 1.72 | 0.92 | 1.64 | 1.21 | 9.77* | 1.08 | 0.72 | 0.70 |
| Bond | 1.85 | 3.54* | 2.88* | 0.49 | 3.08* | 3.66* | 3.31* | 0.61 |
| Interest | 1.22 | 4.70* | 6.65* | 1.14 | 0.71 | 4.42* | 5.67* | 1.20 |
| IP | 1.42 | 0.79 | 0.22 | 3.48* | 0.77 | 1.02 | 0.29 | 3.37* |

III. Volatility and the Level of Economic Activity

A. Volatility During Recessions

The previous tests analyzed the relations among various measures of volatility. There is also reason to believe that stock return volatility is related to the level of economic activity. For example, if firms have large fixed costs, net profits will fall faster than revenues if demand falls. This is often called “operating leverage.”¹² Table VI contains a test of the relation between stock volatility and the level of macroeconomic activity. It contains estimates of the coefficient of a dummy variable added to equation (3b) equal to unity during recessions as defined by the National Bureau of Economic Research (NBER) and equal to zero otherwise. If this coefficient is reliably above zero, the volatility of the series is larger during recessions than during expansions.¹³

Table VI shows that volatility is higher during recessions since most of the estimates are positive and none is more than 1.2 standard errors below zero. Except for 1859–1919, all the estimates for stock volatility are more than 1.8 standard errors above zero. Moreover, the estimates of the percentage increase in volatility in recessions compared with expansions, in braces below the t -statistics, are large (up to 277 percent in 1920–1952 using the daily estimates of volatility). Along with the measures of stock market volatility $|\hat{\epsilon}_{st}|$ and $\hat{\sigma}_t$, the volatility of industrial production $|\hat{\epsilon}_{it}|$ shows the most reliable increases during recessions. There is weaker evidence that bond returns, short-term interest rates, and money growth rates have higher volatility during recessions.

Figure 7 shows the plot of predicted monthly stock volatility like Figure 1, except that the periods of NBER recessions are drawn as solid lines and expansions are drawn as dotted lines. It is clear from this plot that volatility is generally higher during recessions. This phenomenon is not limited to the Great Depression.

Thus, stock market volatility is related to the general health of the economy. One interpretation of this evidence is that it is caused by financial leverage. Stock prices are a leading indicator, so stock prices fall (relative to bond prices) before and during recessions. Thus, leverage increases during recessions, causing an increase in the volatility of leveraged stocks. The analysis below addresses this question directly.

B. Volatility and Corporate Profitability

I have also analyzed the relation between stock volatility and several measures of corporate profitability. Recently, Fama and French (1988b) and others have shown that variables such as dividend (D/P) or earnings yields (E/P) predict stock returns for horizons as far as five years into the future. Keim and Stambaugh (1986) and Fama and French (1989) show that spreads between the

¹² I am grateful to Fischer Black for suggesting this interpretation.

¹³ Since the NBER announces the timing of recessions and expansions six to nine months *after* they have begun, this evidence does not imply that the recession variable can be used to help *predict* future volatility.

Table VI

Estimates of the Relations Between Business Cycles and Financial and Macroeconomic Volatility, 1859–1987

Estimates of dummy variable coefficients are added to the autoregressive model for volatility. The *t*-statistics in parentheses use White's (1980) heteroskedasticity consistent standard errors. A dummy variable equal to one during months designated as recessions by the National Bureau of Economic Research is added to a regression containing 12 monthly dummy variables and 12 lags of volatility. The estimates represent the increase in average volatility during periods of recession. The percentage increase in volatility during recessions relative to expansions is in braces below the standard errors. The estimates in the first two columns use as much data as are available for the respective series. Coefficient estimates more than two standard errors from zero are indicated with an asterisk.

| Dependent Variable | 1859–1987 | 1859–1919 | 1920–1952 | 1953–1987 |
|------------------------------------|-----------------------------|-------------------------------|-------------------------------|-------------------------------|
| Monthly stock returns | 0.0063* (2.93) {61%} | −0.0014 (−0.55) {−6%} | 0.0195* (3.09) {234%} | 0.0139* (3.12) {68%} |
| Daily stock returns | 0.0038* (3.05) {99%} | 0.0014 (0.92) {8%} | 0.0077* (2.55) {277%} | 0.0037 (1.81) {45%} |
| High-grade long-term bond returns | 0.00065 (1.21) {42%} | 0.00019 (0.39) {14%} | 0.00234 (1.68) {161%} | 0.00160 (0.99) {70%} |
| Short-term interest rates | 0.00008 (1.22) {29%} | 0.00007 (0.88) {15%} | 0.00004 (0.33) {16%} | 0.00031 (1.41) {134%} |
| PPI inflation rates | 0.00024 (0.31) {10%} | −0.00070 (−0.58) {−13%} | −0.00067 (−0.64) {−15%} | −0.00052 (−1.16) {−57%} |
| Monetary base growth rates | 0.0015* (2.47) {125%} | 0.0017 (1.77) {54%} | 0.0010 (0.81) {42%} | −0.0002 (−0.51) {−11%} |
| Industrial production growth rates | 0.0032* (2.58) {83%} | 0.0011 (0.48) {8%} | 0.0022 (0.96) {30%} | 0.0026* (2.35) {52%} |

yields on low versus high-grade long-term corporate debt also predict stock returns. Where such variables track time-varying expected returns, they may also predict time-varying volatility.

The relations between stock volatility with either dividend or earnings yields are sometimes positive and sometimes negative. These opposite associations suggest that there is no stable relation between earnings or dividend policy and stock volatility. To limit the number of reported results, I only summarize these tests here.

Table VII contains estimates of the coefficients of the spread between the yields on Baa- versus Aa-rated corporate bonds when added to the autoregressive models summarized in Table II. All of the estimates are positive, and several are more than two standard errors above zero. Thus, the difference between the

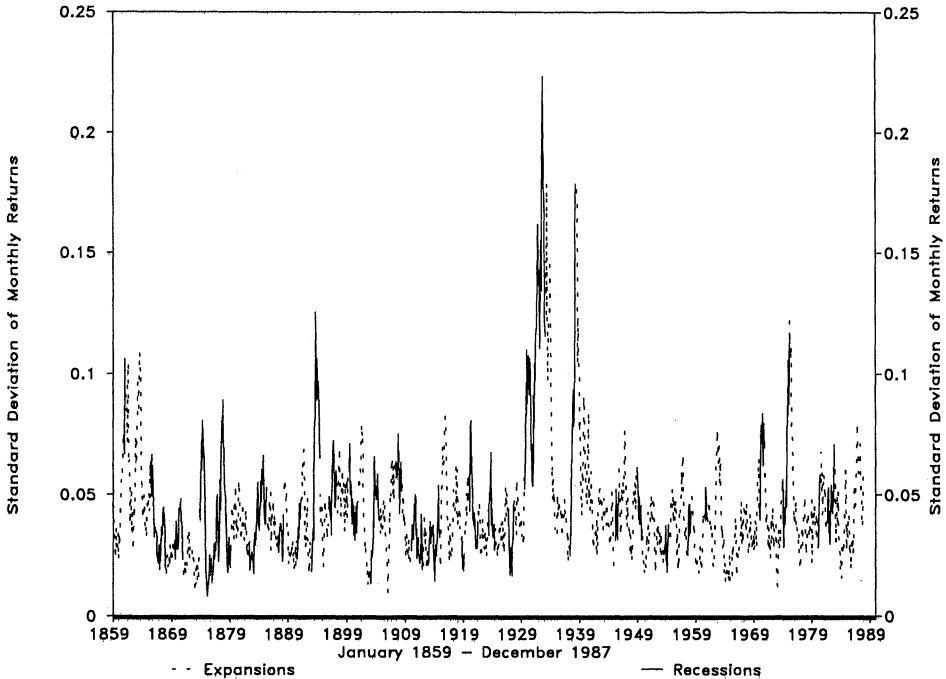


Figure 7. Predictions of the monthly standard deviation of stock returns during NBER recessions (—) and during expansions (---) for 1859–1987. A 12th-order autoregression with different monthly intercepts is used to model returns, and then the absolute values of the residuals are used to estimate volatility in month t . To model conditional volatility, a 12th-order autoregressive model with different monthly intercepts is used to predict the standard deviation in month t based on lagged standard deviation estimates. This plot contains fitted values from the volatility regression model, shown separately for recessions and expansions.

yields on bonds of different quality is directly related to subsequently observed stock volatility. This is not surprising, since the difference in promised yields on bonds of different quality should be larger in periods when default risk is high.

IV. Effects of Leverage on Stock Market Volatility

One explanation of time-varying stock volatility is that leverage changes as relative stock and bond prices change. In particular, the variance of the return to the assets of a firm σ_{vt}^2 is a function of the variances of the returns to the stock σ_{st}^2 and the bonds σ_{bt}^2 and the covariance of the returns $\text{cov}(R_{st}, R_{bt})$:

$$\sigma_{vt}^2 = (S/V)_{t-1}^2 \sigma_{st}^2 + (B/V)_{t-1}^2 \sigma_{bt}^2 + 2 (S/V)_{t-1} (B/V)_{t-1} \text{cov}(R_{st}, R_{bt}), \quad (4)$$

where $(S/V)_{t-1}$ and $(B/V)_{t-1}$ represent the fraction of the market value of the firm due to stocks and bonds at time $t - 1$. Consider a firm with riskless debt ($\sigma_{bt}^2 = \text{cov}(R_{st}, R_{bt}) = 0$), where the variance of the assets of the firm σ_v^2 is constant

Table VII
**Estimates of the Relation Between the
 Standard Deviation of Stock Returns and the
 Corporate Bond Quality Yield Spreads, 1920–
 1987**

The previous month's spread between the Moody's Baa long-term corporate bond yield and the Aa yield, $(y_{Baa} - y_{Aa})_{t-1}$, is included in an autoregressive model for volatility,

$$\sigma_{st} = \sum_{i=1}^{12} \alpha_i + \sum_{j=1}^{12} \beta_j \sigma_{st-j} + \gamma(y_{Baa} - y_{Aa})_{t-1} + u_t.$$

Only the coefficient of the yield spread γ is shown. Asymptotic standard errors are in parentheses under the coefficient estimates. Coefficient estimates more than two standard errors from zero are indicated with an asterisk.

| Sample Period | Standard Deviation from Monthly Stock Returns | Standard Deviation from Daily Stock Returns |
|---------------|---|---|
| 1920–1987 | 14.83* (5.82) | 3.937* (1.85) |
| 1920–1952 | 18.07* (8.00) | 4.256 (2.15) |
| 1953–1987 | 5.649 (8.29) | 3.950 (3.14) |

over time. The standard deviation of the stock return is $\sigma_{st} = \sigma_v (V/S)_{t-1}$. This shows how a change in the leverage of the firm causes a change in the volatility of stock returns.

Figure 8 plots the predictions of stock market volatility $\tilde{\sigma}_t$ from Figure 1 along with the estimates implied by changing leverage $(V/S)_{t-1}$ scaled to have a mean equal to the average of $\tilde{\sigma}_t$ for 1900–1987. Changing leverage explains a small portion of the increase in stock market volatility in the early 1930's and mid-1970's. It cannot explain most of the variation in $\tilde{\sigma}_t$.¹⁴

Christie (1982) proposes regression tests for the effects of changing leverage on the volatility of stock returns. He notes that, if the volatility of the value of the firm σ_v is constant, (4) implies the regression model:

$$\sigma_{st} = \alpha_0 + \alpha_1 (B/S)_{t-1} + u_t, \quad (5)$$

where $\alpha_0 = \alpha_1 = \sigma_v$, in the riskless debt case. With risky consol bonds containing protective covenants, as modeled by Black and Cox (1976), Christie shows that $\alpha_0 = \sigma_v > \alpha_1$.

Table VIII contains generalized least squares (GLS) estimates of equation (5) for 1901–1987, 1901–1952, and 1953–1987. There is strong residual autocorrelation using ordinary least squares; hence, the GLS estimates use an ARMA(1,3)

¹⁴ A plot using the monthly measure of volatility $|\hat{e}_{st}|$ yields similar conclusions.

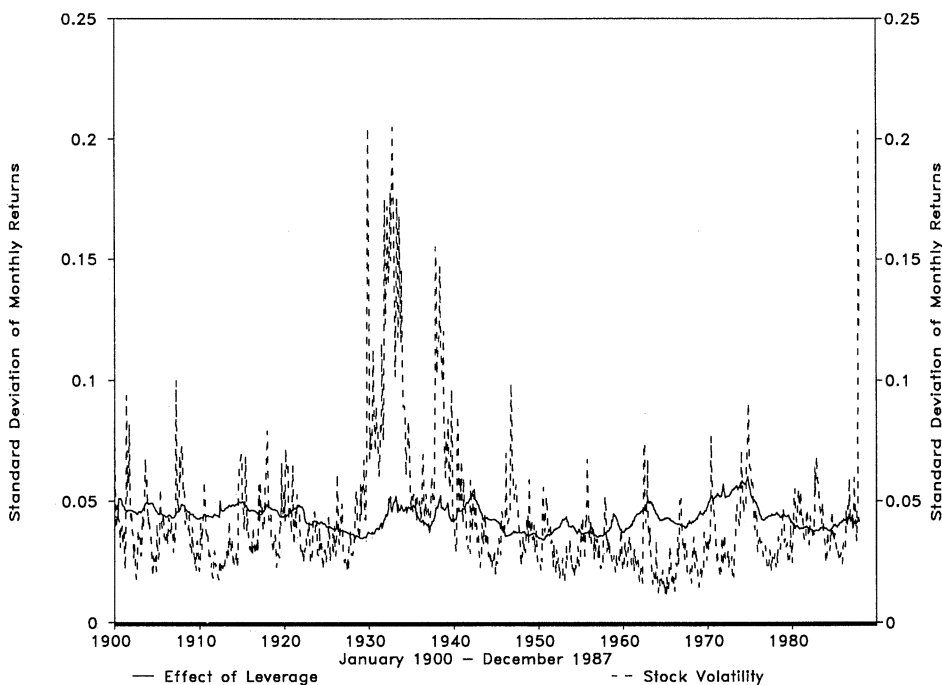


Figure 8. Predictions of the monthly standard deviation of stock returns based on daily data (---) and the level of stock return volatility implied by changing financial leverage (—) for 1900–1987. The daily returns in the month are used to estimate a sample standard deviation for each month. To model conditional volatility, a 12th-order autoregressive model with different monthly intercepts is used to predict the standard deviation in month t based on lagged standard deviation estimates. This plot contains fitted values from the volatility regression model. The effect of leverage is estimated by assuming that the volatility of the assets of the firm is constant and that debt is riskless. Then, the standard deviation of stock returns changes in proportion to the value of the firm divided by the value of the stock $(V/S)_{t-1}$. When stock prices fall relative to bond prices, stock volatility increases. Thus, the “effect of leverage” plot is a time series of aggregate firm value (stock plus bond value) divided by stock value, scaled to have the same mean as the predictions of volatility from the regression model.

model for the errors. This is similar to the French, Schwert, and Stambaugh (1987) model for $\hat{\sigma}_t$. The results depend on the volatility measure and the sample period. For the daily volatility measure $\hat{\sigma}_t$, the intercept α_0 is always greater than the slope α_1 , as predicted by the risky debt model. The estimates of α_0 are between 0.03 and 0.04 per month, and they are over three standard errors from zero. This would be the estimate of firm volatility σ_v in Christie’s model. All the estimates of α_1 are positive, showing that stock volatility rises when leverage rises. The standard errors are large, however, so the t -statistics testing $\alpha_1 = 0$, or testing $\alpha_1 = \alpha_0$, are small. The t -test in the last column of Table VIII tests the hypothesis that the slope equals the intercept ($\alpha_0 = \alpha_1$). The p -value in parentheses is for the two-sided alternative hypothesis ($\alpha_0 \neq \alpha_1$).

The estimates for the monthly volatility measure $|\tilde{\varepsilon}_{st}|$ for 1953–1987 are similar

Table VIII

Estimates of the Relation Between the Standard Deviation of Stock Returns and Leverage, 1901–1987

Regressions of stock volatility on debt/equity ratios,

$$\sigma_{st} = \alpha_0 + \alpha_1 (B/S)_{t-1} + u_t, \quad (5)$$

where $(B/S)_{t-1}$ is an estimate of the debt/equity ratio for the aggregate stock market portfolio at the end of month $t - 1$. Generalized least squares estimates include an ARMA (1, 3) process for the errors. Asymptotic standard errors are in parentheses under the coefficient estimates. $S(u)$ is the standard deviation of the errors, R^2 is the coefficient of determination including the effects of estimating the ARMA (1, 3) process for the errors, and $Q(24)$ is the Box-Pierce (1970) statistic for 24 lags of the residual autocorrelations, which should be distributed as χ^2 (20). The t -test for $\alpha_0 = \alpha_1$ tests whether the riskless debt model is an adequate approximation to the effect of leverage on stock return volatility, where $\alpha_0 > \alpha_1$ is implied by the risky debt model. Coefficient estimates more than two standard errors from zero are indicated with an asterisk. The p -values for the Box-Pierce statistic and for the two-sided alternative $\alpha_0 \neq \alpha_1$ are in parentheses under the test statistics.

| Sample Period | α_0 | α_1 | $S(u)$ | R^2 | $Q(24)$ | t -test $\alpha_0 = \alpha_1$ |
|--|---------------------|---------------------|--------|-------|------------------|------------------------------------|
| <u>Standard Deviation from Monthly Returns</u> | | | | | | |
| 1901–1987 | 0.0269* (0.0101) | 0.0512* (0.0193) | 0.0424 | 0.165 | 56.2 (0.0000) | –0.87 (0.383) |
| 1901–1952 | 0.0232 (0.0157) | 0.0700* (0.0300) | 0.0475 | 0.194 | 50.3 (0.0002) | –1.08 (0.279) |
| 1953–1987 | 0.0315* (0.0066) | 0.0221 (0.0146) | 0.0336 | 0.055 | 16.9 (0.657) | 0.45 (0.651) |
| <u>Standard Deviation from Daily Returns</u> | | | | | | |
| 1901–1987 | 0.0376* (0.0093) | 0.0154 (0.0147) | 0.0187 | 0.571 | 24.6 (0.216) | 1.01 (0.311) |
| 1901–1952 | 0.0402* (0.0135) | 0.0168 (0.0225) | 0.0205 | 0.606 | 35.0 (0.020) | 0.71 (0.479) |
| 1953–1987 | 0.0317* (0.0073) | 0.0101 (0.0147) | 0.0157 | 0.296 | 12.7 (0.890) | 1.03 (0.301) |

to the daily estimates. For 1901–1952 and 1901–1987, however, the estimates of α_0 are less than the estimates of α_1 , a result that is inconsistent with all the leverage models. Again, the standard errors are large, so the t -test for $\alpha_0 = \alpha_1$ is not large. These estimates of α_1 are reliably above zero, showing that an increase in the debt/equity ratio $(B/S)_{t-1}$ leads to an increase in stock return volatility. These regressions also have strong residual autocorrelation. An obvious interpretation is that the volatility of the value of the firm σ_{vt} is *not* constant over these samples. Rather, it rose at the same time that leverage rose during the Great Depression, so the large estimates of α_1 are caused by omitting a correlated regressor. Again, this evidence shows that leverage alone cannot explain the historical movements in stock volatility.

V. Stock Market Trading and Volatility

There are at least three theories that predict a positive relation between volatility and volume. First, if investors have heterogeneous beliefs, new information will

cause both price changes and trading. Second, if some investors use price movements as information on which to make trading decisions, large price movements will cause large trading volume. Finally, if there is short-term "price pressure" due to illiquidity in secondary trading markets, large trading volume that is predominantly either buy or sell orders will cause price movements. There has been much previous research on the relation between volatility and trading volume, but most of it has focused on the behavior of individual securities. The time series behavior of volatility and trading volume for the aggregate stock market provides a different perspective on these questions.

A. Trading Days and Volatility

French and Roll (1986) show that stock volatility is higher when stock exchanges are open for trading. In particular, they find that the variance of stock returns over weekends and holidays is much less than a typical one-day variance times the number of calendar days since trading last occurred. Most peculiarly, during 1968, when the NYSE closed on Wednesdays due to the "paper-work crunch," the variance of Tuesday to Thursday returns was not much larger than a one-day variance. This occurred even though the stock exchanges were the only economic institutions taking holidays. Table IX contains regressions,

$$\hat{\sigma}_{st} = \alpha_0 + \alpha_1 \sqrt{Days_t} + u_t, \quad (6)$$

where $Days_t$ is the number of trading days the NYSE was open during month t . If variance is proportional to trading time, α_1 represents the standard deviation per trading day and α_0 should equal zero. If volatility is unrelated to trading activity, the intercept α_0 estimates the average monthly standard deviation and α_1 should equal zero. Table IX contains GLS estimates of equation (6) for 1885–1987, 1885–1919, 1920–1952, and 1953–1987. These estimates do not provide strong support for either hypothesis, but the French-Roll scenario is more consistent with the data. All but one of the estimates of the trading time coefficient α_1 are positive, and several are almost two standard errors above zero. On the other hand, many of the estimated intercepts are negative, and only one is more than two standard errors above zero. Thus, NYSE trading activity explains part of the variation in stock volatility. Nevertheless, this relation does not explain much of the variation in volatility through time.

B. Trading Volume and Volatility

Another measure of stock trading activity is share trading volume. Karpoff (1987) surveys the extensive literature on the relation between volatility and volume. Table X contains estimates of the regression

$$\hat{\sigma}_{st} = \alpha_0 + \frac{\beta}{(1-\delta L)} Vol_t + u_t, \quad (7)$$

Table IX
Estimates of the Relation Between the Standard Deviation
of Stock Returns and the Square Root of the Number of
Trading Days, 1885–1987

Regressions of stock volatility on the square root of the number of trading days per month,

$$\sigma_{st} = \alpha_0 + \alpha_1 \sqrt{Days_t} + u_t, \quad (6)$$

where $\sqrt{Days_t}$ is the square root of the NYSE trading days in the month. Generalized least squares estimates include an ARMA (1,3) process for the errors. Asymptotic standard errors are in parentheses under the coefficient estimates. $S(u)$ is the standard deviation of the errors, R^2 is the coefficient of determination including the effects of estimating the ARMA (1,3) process for the errors, and $Q(24)$ is the Box-Pierce (1970) statistic for 24 lags of the residual autocorrelations, which should be distributed as $\chi^2(20)$, with the p -value in parentheses under the test. Coefficient estimates more than two standard errors from zero are indicated with an asterisk.

| Sample Period | α_0 | α_1 | $S(u)$ | R^2 | $Q(24)$ |
|--|---------------------|---------------------|--------|-------|------------------|
| <u>Standard Deviation from Monthly Returns</u> | | | | | |
| 1885–1987 | –0.0276 (0.0357) | 0.0152* (0.0073) | 0.0418 | 0.142 | 56.1 (0.0003) |
| 1885–1919 | –0.0703 (0.0612) | 0.0224 (0.0122) | 0.0347 | 0.028 | 16.1 (0.708) |
| 1920–1952 | 0.0224 (0.0764) | 0.0065 (0.0152) | 0.0545 | 0.194 | 45.7 (0.0009) |
| 1953–1987 | –0.0514 (0.0567) | 0.0202 (0.0124) | 0.0336 | 0.055 | 16.3 (0.697) |
| <u>Standard Deviation from Daily Returns</u> | | | | | |
| 1885–1987 | 0.0341* (0.0150) | 0.0018 (0.0029) | 0.0186 | 0.538 | 21.9 (0.347) |
| 1885–1919 | 0.0251 (0.0231) | 0.0027 (0.0046) | 0.0157 | 0.226 | 10.9 (0.950) |
| 1920–1952 | 0.0632 (0.0318) | –0.0025 (0.0056) | 0.0231 | 0.622 | 38.7 (0.007) |
| 1953–1987 | –0.0038 (0.0225) | 0.0087 (0.0048) | 0.0157 | 0.300 | 13.1 (0.872) |

where Vol_t is the growth rate of volume from month $t - 1$ to month t , and the errors u_t follow an ARMA(1,3) process. This model relates stock volatility to a distributed lag of past share volume growth, where the coefficient of volume growth decreases geometrically.¹⁵ The estimates in Table X also show a positive relation between stock volatility and trading activity. All the estimates of β are more than two standard errors above zero. The estimates of δ are all positive. For the estimates of volatility based on daily data $\hat{\sigma}_t$, they are several standard

¹⁵ This model was suggested by the pattern of regression coefficients in an unrestricted regression of volatility on current and four lags of volume growth. L is the lag operator, $L^k X_t = X_{t-k}$.

Table X
Estimates of the Relation Between the Standard Deviation
of Stock Returns and Stock Market Trading Volume,
1885–1987

Distributed lag regressions of stock volatility on the growth rate of NYSE share trading volume (Vol_t),

$$\sigma_{st} = \alpha_0 + \frac{\beta}{(1 - \delta L)} Vol_t + u_t. \quad (7)$$

Generalized least squares estimates include an ARMA (1,3) process for the errors. Asymptotic standard errors are in parentheses under the coefficient estimates. The distributed lag model for the effect of current and lagged share volume growth on the monthly standard deviation of stock returns implies geometric decay. The implied coefficient for lag k is $\beta\delta^k$. L is the lag operator, $L^k X_t = X_{t-k}$. $S(u)$ is the standard deviation of the errors, R^2 is the coefficient of determination including the effects of estimating the ARMA (1,3) process for the errors, and $Q(24)$ is the Box-Pierce (1970) statistic for 24 lags of the residual autocorrelations, which should be distributed as $\chi^2(20)$, with the p -value in parentheses under the test. Coefficient estimates more than two standard errors from zero are indicated with an asterisk.

| Sample Period | α_0 | β | δ | $S(u)$ | R^2 | $Q(24)$ |
|--|---------------------|---------------------|---------------------|--------|-------|------------------|
| <u>Standard Deviation from Monthly Returns</u> | | | | | | |
| 1885–1987 | 0.0454* (0.0049) | 0.0473* (0.0038) | 0.1561 (0.0800) | 0.0394 | 0.237 | 55.4 (0.0000) |
| 1885–1919 | 0.0410* (0.0023) | 0.0331* (0.0047) | 0.3484* (0.1320) | 0.0328 | 0.127 | 19.2 (0.509) |
| 1920–1952 | 0.0545* (0.0150) | 0.0629* (0.0074) | 0.0597 (0.1188) | 0.0502 | 0.316 | 40.9 (0.004) |
| 1953–1987 | 0.0395* (0.0025) | 0.0539* (0.0092) | 0.3061 (0.1684) | 0.0324 | 0.124 | 19.9 (0.462) |
| <u>Standard Deviation from Daily Returns</u> | | | | | | |
| 1885–1987 | -0.0246 (0.0560) | 0.0168* (0.0019) | 0.9984* (0.0012) | 0.0179 | 0.568 | 21.9 (0.346) |
| 1885–1919 | 0.0372* (0.0020) | 0.0123* (0.0023) | 0.9536* (0.0299) | 0.0151 | 0.281 | 13.7 (0.845) |
| 1920–1952 | 0.0484* (0.0165) | 0.0203* (0.0037) | 0.9007* (0.1002) | 0.0223 | 0.650 | 36.0 (0.016) |
| 1953–1987 | 0.0351* (0.0041) | 0.0182* (0.0044) | 0.5952* (0.2431) | 0.0154 | 0.324 | 14.0 (0.832) |

errors above zero. For the estimates of volatility based on monthly data $|\hat{e}_{st}|$, the estimates of δ are closer to zero, though for 1885–1919 it is over two standard errors above zero. Thus, the evidence in Table X supports the proposition that stock market volatility is higher when trading activity is higher.

Table XI contains tests of the incremental predictive power of 12 lags of NYSE share volume growth Vol_t in a 12th-order VAR system for stock volatility, high-grade bond return volatility $|\hat{e}_{rht}|$, and short-term interest volatility $|\hat{e}_{rst}|$ that

allows for different monthly intercepts. This model is similar to those used in Tables III, IV, and V. The F -statistics measuring the ability of share volume growth to predict financial volatility are small, except for 1885–1919 and 1885–1987 using the daily measure of stock volatility $\hat{\sigma}_t$. There is little evidence that financial volatility helps to predict future trading volume growth, except for stock volatility from 1920 to 1952.

The main difference between the distributed lag models in Table X and the VAR models in Table XI is that the distributed lag models include the correlation of contemporaneous volume and volatility and the VAR models do not. The strong relations in Table X and the weak ones in Table XI point to a strong

Table XI
 F -Tests from Vector Autoregressive Models for Stock, Bond, and Interest Rate Volatility, Including Growth in NYSE Share Trading Volume, 1885–1987

A four variable, 12th-order vector autoregressive (VAR) model is estimated for stock, bond, and interest rate volatility, and NYSE share trading volume growth (*Vol*), including dummy variables for monthly intercepts. The F -tests reflect the incremental ability of the column variable to predict the respective row variables, given the other variables in the model. Measures of stock return volatility based on monthly data are used in the first four columns, and measures of stock return volatility based on daily data are used in the last four columns. The 0.05 and 0.01 critical values for the F -statistic with 12 and 200 degrees of freedom are 1.80 and 2.28, respectively. F -statistics greater than 2.28 are indicated with an asterisk.

| Dependent Variable | F -tests with Monthly Stock Volatility | | | | F -tests with Daily Stock Volatility | | | |
|--------------------|--|--------|----------|------------|--|--------|----------|------------|
| | Stock | Bond | Interest | <i>Vol</i> | Stock | Bond | Interest | <i>Vol</i> |
| <u>1885–1987</u> | | | | | | | | |
| Stock | 10.29* | 2.75* | 0.84 | 1.55 | 77.94* | 6.97* | 1.38 | 7.65* |
| Bond | 5.26* | 17.70* | 2.01 | 0.87 | 2.59* | 17.37* | 3.47* | 0.86 |
| Interest | 1.62 | 3.68* | 22.33* | 0.70 | 0.87 | 2.22 | 22.36* | 0.45 |
| <i>Vol</i> | 1.97 | 1.11 | 0.63 | 11.25* | 1.30 | 1.60 | 0.66 | 10.94* |
| <u>1885–1919</u> | | | | | | | | |
| Stock | 1.42 | 1.37 | 0.38 | 1.11 | 9.40* | 4.35* | 2.29* | 3.97* |
| Bond | 1.37 | 8.78* | 1.22 | 0.83 | 1.09 | 6.14* | 1.17 | 0.90 |
| Interest | 2.05 | 1.48 | 3.24* | 0.96 | 0.87 | 1.18 | 4.45* | 0.98 |
| <i>Vol</i> | 1.42 | 1.36 | 0.97 | 4.12* | 1.86 | 1.10 | 0.76 | 3.55* |
| <u>1920–1952</u> | | | | | | | | |
| Stock | 4.07* | 3.80* | 0.54 | 2.13 | 22.39* | 3.35* | 0.63 | 1.01 |
| Bond | 8.88* | 4.06* | 0.33 | 1.14 | 6.08* | 3.40* | 0.30 | 1.48 |
| Interest | 0.51 | 0.52 | 11.87* | 0.52 | 0.62 | 0.35 | 11.87* | 0.51 |
| <i>Vol</i> | 2.28* | 2.13 | 0.62 | 4.20* | 2.71* | 0.99 | 0.59 | 3.53* |
| <u>1953–1987</u> | | | | | | | | |
| Stock | 2.18 | 1.06 | 2.23 | 1.58 | 10.10* | 1.18 | 0.75 | 0.54 |
| Bond | 1.87 | 3.57* | 3.20* | 0.39 | 2.85* | 3.58* | 3.58* | 0.36 |
| Interest | 1.59 | 5.15* | 6.02* | 1.17 | 0.84 | 5.17* | 5.66* | 0.99 |
| <i>Vol</i> | 0.50 | 0.46 | 0.64 | 7.25* | 1.58 | 0.49 | 0.82 | 7.50* |

correlation between the “shocks” to volume and volatility. Unexpected changes in volatility and volume are highly positively correlated. Given the history of volatility, there is not much correlation between volatility and lagged values of trading volume.

In general, high trading activity and high volatility occur together. Of course, these regressions cannot show whether this relation is due to “trading noise” or to the flow of information to the stock market.

VI. Summary and Conclusions

This paper analyzes many factors related to stock volatility, but it does not test for *causes* of stock price volatility. Rather, the hypotheses involve associations between stock volatility and other variables. For example, the analysis of the volatility of bond returns, inflation rates, money growth, and industrial production growth, along with stock volatility, seeks to determine whether these aggregate volatility measures change together through time. In most general equilibrium models, fundamental factors such as consumption and production opportunities and preferences would determine all these parameters (e.g., Abel (1988)). Nevertheless, the process of characterizing stylized facts about economic volatility helps to define the set of interesting questions, leading to tractable theoretical models.

A. Joint Effects of Leverage and Macroeconomic Volatility

Most of the tests above analyze stock volatility along with one other factor. To summarize all these relations, Table XII contains estimates of the multiple regression:

$$\begin{aligned} \ln \tilde{\sigma}_{st} = & \alpha_e + \alpha_r D_{rt} + \beta_1 \ln | \tilde{\epsilon}_{pt} | + \beta_2 \ln | \tilde{\epsilon}_{mt} | \\ & + \beta_3 \ln | \tilde{\epsilon}_{it} | \\ & + \gamma \ln (V/S)_{t-1} + u_t. \end{aligned} \quad (8)$$

In (8), α_e represents the constant term during expansions, and $(\alpha_e + \alpha_r)$ represents the constant term during recessions. The slope coefficients β_1 through β_3 represent the elasticities of stock return volatility with predicted inflation volatility, predicted money growth volatility, and predicted industrial production volatility, respectively. The coefficient γ measures the effect of financial leverage on volatility. Table XII shows estimates of equation (8) for both measures of stock return volatility. There is no correction for autocorrelation in the errors from (8), although the standard errors use Hansen’s (1982) heteroskedasticity and autocorrelation consistent covariance matrix.¹⁶

Equation (8) measures the contributions of macroeconomic conditional vola-

¹⁶ Since many of the regressors in (8) are fitted values from first stage regressions (3b), the “generated regressors” problem discussed by Pagan (1984) is relevant here. In brief, to the extent that there are omitted variables that could be used to help to predict the volatility of some of these series, the coefficients of all of these second stage regressors will be biased. Experimentation with instrumental variables estimation, the technique recommended by Pagan, yielded similar results.

Table XII

Estimates of the Relation of the Standard Deviation of Stock Returns to the Predicted Volatility of Macroeconomic Variables, and the Effect of Leverage, 1900–1987

The regression model,

$$\ln \bar{\sigma}_{st} = \alpha_c + \alpha_r D_{rt} + \beta_1 \ln |\tilde{\epsilon}_{pt}| + \beta_2 \ln |\tilde{\epsilon}_{mt}| + \beta_3 \ln |\tilde{\epsilon}_{it}| + \gamma \ln (V/S)_{t-1} + u_t, \quad (8)$$

includes a constant α_c (not shown in this table), a dummy variable D_{rt} equal to unity during NBER recessions, the logarithms of the predicted standard deviations of PPI inflation $|\tilde{\epsilon}_{pt}|$, of money base growth $|\tilde{\epsilon}_{mt}|$, and of industrial production $|\tilde{\epsilon}_{it}|$, and the logarithm of leverage $(V/S)_{t-1}$. The predicted standard deviations are fitted values from the autoregressive models in Table II. The logarithm of the stock return volatility measures are the regressands. Asymptotic standard errors are in parentheses under the coefficient estimates. All tests use Hansen's (1982) heteroskedasticity and autocorrelation consistent covariance matrix, using 12 lags and leads and a damping factor of 0.7. R^2 is the coefficient of determination and $Q(24)$ is the Box-Pierce (1970) statistic for 24 lags of the residual autocorrelations, which should be distributed as $\chi^2(24)$ in this case, with the p -value in parentheses under the test. The column labeled Sum contains the sum of the coefficients of predicted volatilities. Coefficient estimates more than two standard errors from zero are indicated with an asterisk.

| Sample Period | Recessions α_r | Predicted Macroeconomic Volatility | | | | Leverage γ | R^2 | $Q(24)$ |
|---|--------------------------|------------------------------------|-------------------|-------------------|-------------------|----------------------|-------|-----------------|
| | | PPI β_1 | Base β_2 | IP β_3 | Sum | | | |
| Standard Deviation from Monthly Returns | | | | | | | | |
| 1900–1987 | 0.256* (0.120) | −0.035 (0.076) | 0.103 (0.072) | 0.079 (0.059) | 0.147 (0.088) | 0.275 (0.440) | 0.022 | 70.5 (0.000) |
| 1900–1952 | 0.193 (0.148) | −0.035 (0.096) | 0.261* (0.088) | 0.145 (0.079) | 0.371* (0.131) | −0.370 (0.634) | 0.027 | 44.7 (0.006) |
| 1953–1987 | 0.479* (0.082) | 0.112 (0.128) | −0.183 (0.133) | 0.180* (0.083) | 0.109 (0.179) | 0.256 (0.644) | 0.050 | 38.1 (0.034) |
| Standard Deviation from Daily Returns | | | | | | | | |
| 1900–1987 | 0.182 (0.096) | 0.087 (0.045) | 0.210* (0.062) | 0.031 (0.043) | 0.328* (0.068) | 0.091 (0.316) | 0.208 | 2905 (0.000) |
| 1900–1952 | 0.177 (0.125) | 0.077 (0.057) | 0.273* (0.080) | 0.099 (0.052) | 0.450* (0.116) | −0.273 (0.465) | 0.168 | 1795 (0.000) |
| 1953–1987 | 0.248* (0.109) | 0.151* (0.053) | 0.119 (0.111) | −0.009 (0.052) | 0.262 (0.137) | 0.047 (0.316) | 0.120 | 540 (0.000) |

tility factors, along with leverage, in explaining the time series variation in stock return volatility. From (4), $\sigma_{st}^2 \approx (V/S)_{t-1}^2 \sigma_{vt}^2$ since the variance of bond returns and the covariance of bond returns with stock returns will be much smaller than σ_{vt}^2 . Thus, equation (8) is an approximation of (4), where the predicted volatilities of the macroeconomic factors affect firm volatility σ_{vt}^2 . The sum of the elasticities $(\beta_1 + \beta_2 + \beta_3)$ measures the response of firm volatility to a one percent increase in the volatility of all the macroeconomic factors. The elasticity with leverage should be $\gamma = 1$.

The average level of volatility is much higher during recessions (consistent with Table VI). The column labeled "Recessions" in Table XII contains estimates

of α_r , the differential intercept during recessions. They are between 0.17 and 0.50 across the different measures of stock volatility and different periods, and many are reliably above zero. If the recession dummy variable proxies for variation in operating leverage, it is interesting that it remains important for stock volatility even when other factors are included.

The effect of financial leverage is small. The estimates using daily returns are reliably below unity. Perhaps this reflects the imperfect proxies for this and other regressors and the collinearity among them.

Most of the estimates of the predicted macroeconomic volatility coefficients are positive, and some are reliably above zero. For example, using the stock volatility measure from daily data $\ln \hat{\sigma}_t$ for 1900–1952, all these coefficients are at least 1.35 standard errors above zero. The sum of these coefficients is 0.45, with a standard error of 0.12. Thus, if the volatility of inflation rates, money growth, and industrial production all increase one percent, stock volatility increases by 0.45 percent. Across both monthly and daily measures of stock volatility and across all subperiods, the coefficient estimates of predicted money base growth volatility are reliably positive most often.

B. Summary

Many economic series were more volatile in the 1929–1939 Great Depression. Nevertheless, stock volatility increased by a factor of two or three during this period compared with the usual level of the series. (See Figure 1.) There is no other series in this paper that experienced similar behavior.

Second, there is evidence that many aggregate economic series are more volatile during recessions (Table VI). This is particularly true for financial asset returns and for measures of real economic activity. One interpretation of this evidence is that “operating leverage” increases during recessions.

Third, there is weak evidence that macroeconomic volatility can help to predict stock and bond return volatility (Tables III, IV, and V). The evidence is somewhat stronger that financial asset volatility helps to predict future macroeconomic volatility. This is not surprising since the prices of speculative assets should react quickly to new information about economic events.

Fourth, financial leverage affects stock volatility. When stock prices fall relative to bond prices, or when firms issue new debt securities in larger proportion to new equity than their prior capital structure, stock volatility increases (Table VIII). However, this effect explains only a small proportion of the changes in stock volatility over time (Figure 8).

Fifth, there seems to be a relation between trading activity and stock volatility. The number of trading days in the month is positively related to stock volatility, especially in 1953–1987 (Table IX). This reinforces the evidence in French and Roll (1986). Also, share trading volume growth is positively related to stock volatility (Tables X and XI).

C. The Volatility Puzzle

Major episodes in United States economic history are associated with larger volatility, such as the Civil War, World War I, the Great Depression, World War

II, the OPEC oil shock, and the post-1979 period. The puzzle highlighted by the results in this paper is that stock volatility is not more closely related to other measures of economic volatility. For example, the volatility of inflation and money growth rates is very high during war periods, as is the volatility of industrial production. Yet the volatility of stock returns is not particularly high during wars.¹⁷ There were many “financial crises” or “bank panics” during the 19th century in the United States that caused very high and volatile short-term interest rates. Schwert (1989b) shows that stock volatility increases for brief periods during and immediately following the worst panics, but there were no long-term effects on volatility.

On the other hand, the evidence in this paper reinforces the argument made by Officer (1973) that the volatility of stock returns from 1929 to 1939 was unusually high compared with either prior or subsequent experience. For many years macroeconomists have puzzled about the inability of their models to explain the data from the Great Depression. The results in this paper pose a similar challenge to financial economists. Moreover, based on evidence in Fama and French (1988a) and Poterba and Summers (1988), the 1929–1939 period plays a crucial role in the evidence for “mean reversion” in stock prices. I suspect that an analysis of Shiller’s (1981a,b) variance bounds tests would reveal that the 1929–1939 period is responsible for the inference of “excess volatility” of stock prices. Indeed, the spirit of the preceding discussion suggests that stock volatility was inexplicably high during this period. I am hesitant to cede all this unexplained behavior to social psychologists as evidence of fads or bubbles.

Robert Merton has suggested that the Depression was an example of the so-called “Peso problem,” in the sense that there was legitimate uncertainty about whether the economic system would survive. The Russian Revolution occurred only 12 years before the 1929 stock market crash, and there were major political and economic upheavals occurring throughout Europe in the interim. With the benefit of hindsight, we know that the U.S. and world economies came out of the Depression quite well. At the time, however, investors could not have had such confident expectations. Uncertainty about whether the “regime” had changed adds to the fundamental uncertainty reflected in past and future volatility of macroeconomic data. Hamilton’s (1988) regime-switching model formalizes this notion. Schwert (1989b) and Turner, Startz, and Nelson (1989) use Hamilton’s model to represent stock return volatility. It is not possible, however, to determine whether volatility was “too high” during the Depression without some model of the possible outcomes that did not occur. Thus, there remains a challenge to both theorists and empiricists to explain why this episode was so unusual.

Appendix: Data Series Used in This Paper

A. Common Stock Returns, 1857–1987

I use the monthly stock return index from Schwert (1989d). For 1926–1987, I use the returns including dividends to the value-weighted portfolio of all New

¹⁷ If investors knew that the wars would have only short-term effects, it is likely that stock volatility would be affected less than the volatility of inflation or other macroeconomic variables.

Table AI
Variables Used in This Paper

| Series | Description (Source) | Sample Period, Size |
|----------------|--|------------------------------|
| Stock | Monthly return to a value-weighted portfolio of New York Stock Exchange stocks (CRSP/Cowles/Macaulay/Smith and Cole) | 1/1857–12/1987 $T = 1572$ |
| σ_t | Volatility of monthly stock returns from daily returns in the month (Dow Jones/Standard & Poor's) | 2/1885–12/1987 $T = 1235$ |
| Interest | Short-term interest rate on low risk debt instrument (CRSP/Macaulay) | 1/1857–12/1987 $T = 1572$ |
| y_{Aaa} Bond | Yield or return on high-grade long-term corporate debt (Moody's Aa/Macaulay) | 1/1857–12/1987 $T = 1572$ |
| y_{Baa} | Yield on medium-grade long-term corporate debt (Moody's Baa) | 1/1919–12/1987 $T = 828$ |
| PPI | Inflation of producer price index for all commodities (BLS/Warren and Pearson) | 1/1857–12/1987 $T = 1572$ |
| Base | Growth rate of monetary base (high-powered money) (Friedman and Schwartz/NBER/Federal Reserve) | 7/1878–12/1987 $T = 1314$ |
| IP | Growth rate of the index of industrial production (seasonally adjusted - Federal Reserve) | 2/1889–12/1987 $T = 1187$ |
| V/S | Market value of firm divided by the value of stock for S&P composite (Holland and Myers) | 1/1900–12/1987 $T = 1056$ |
| Vol | NYSE share trading volume (S&P/NYSE) | 4/1881–12/1987 $T = 1280$ |
| $Days$ | Number of NYSE trading days per month (Dow Jones/S&P) | 2/1885–12/1987 $T = 1235$ |

York Stock Exchange (NYSE) stocks constructed by the Center for Research in Security Prices (CRSP) at the University of Chicago. For 1885–1925, I use the capital gain returns to the Dow Jones composite index (1972) and add the dividend yield from the value-weighted portfolio of NYSE stocks constructed by the Cowles Commission (1939, pp. 168–169), as corrected by Wilson and Jones (1987, p. 253, with erratum). For 1871–1885, I use the Cowles returns, corrected for the effects of time-averaging by Schwert (1989d). For 1857–1870, I use Macaulay's (1938, pp. A142–A161) index of railroad stock prices to calculate capital gain returns and then add an estimate of the dividend yield from Schwert (1989d). This is equivalent to adding a dividend yield of 0.56 percent per month (6.7 percent per year) to the percent changes in railroad stock prices.

B. Short-Term Interest Rates, 1857–1987

For 1926–1987, I use the monthly yields on the shortest term U.S. Government security (with no special tax provisions) which matures after the end of the month from the Government Bond File constructed by CRSP. For 1857–1925, I use the four to six month commercial paper rates in New York from Macaulay (1938, Table 10, pp. A141–A161). The commercial paper yields are adjusted so that the level of the series is comparable to the Treasury yields, using the

regression of CRSP yields on Macaulay yields from 1926 to 1937:

$$CRSP_t = -0.000761 + 0.9737368 \text{ Macaulay}_t + u_t, \\ (.000085) \quad (0.0309330)$$

where standard errors are in parentheses under the coefficient estimates. This is equivalent to subtracting an average risk premium of 0.076 percent per month (0.91 percent per year) from the Macaulay yields to reflect a small default premium in commercial paper. The correlation between the CRSP and the Macaulay yields is 0.94 for 1926–1937.

C. Long-Term Interest Rates, 1857–1987

The high-grade corporate bond yield for 1919–1987 is the Moody's Aa bond yield (Federal Reserve (1976a, Table 128, pp. 468–471) for 1919–1940, Federal Reserve (1976b, Table 12.12, pp. 720–721) for 1941–1947, and Citibase (1978) for 1948–1987). For 1857–1918, I use Macaulay's (1938, Table 10, pp. A141–A161) railroad bond yield index, adjusted to splice with the Moody's series using the average ratio of the yields during 1919, (RR/Aa) = 0.964372.

D. Returns to Long-Term Corporate Bonds, 1857–1987

The capital gain or loss from holding the bond during the month is estimated from yields assuming that, at the beginning of the month, the bond has a 20-year maturity, a price equal to par, and a coupon equal to the yield, using the conventional bond pricing formula to calculate beginning and ending prices. The monthly income return is assumed to be one twelfth of the coupon. Since the Moody's yields are averages of the yields within the month, these returns are not comparable to returns based on end-of-month data. To correct for this problem, I estimate a first-order moving average process for the returns:

$$R^*_{bt} = \alpha + \varepsilon_t - \theta \varepsilon_{t-1},$$

and then the "corrected" returns are defined as $R_{bt} = \alpha + \varepsilon_t$. This correction eliminates the positive autocorrelation at lag one induced by the within-month aggregation of yields. (See Working (1960).) Note, however, that corrected returns are not good estimates of actual returns based on end-of-month prices since their cross-correlations with other variables are still affected by time aggregation of the yields.¹⁸

E. Inflation Rates, 1857–1987

For 1890–1987, I use the Bureau of Labor Statistics' Producer Price Index (PPI) inflation rate, not seasonally adjusted. For 1857–1889, I use the inflation rate of the Warren and Pearson (1933) index of producer prices. I am grateful to Grant McQueen for making these data available to me.

¹⁸ Schwert (1989d) develops a correction similar to this one for returns calculated from indexes of time-averaged stock prices.

F. Stock Market Share Trading Volume, 1881–1987

Standard & Poor's (1986, p. 214) reports monthly NYSE share trading volume for 1883–1985.¹⁹ Citibase (1978) contains similar data for 1986–1987. The NYSE provided data from April 1881 through 1882. I measure the number of trading days per month for 1885–1987 from the daily data on the Dow Jones indexes in Dow Jones (1972) and on the Standard & Poor's composite index in Standard & Poor's (1986, pp. 134–187).

G. Financial Leverage, 1900–1987

Taggart (1986) discusses many estimates of the equity to total capital ratio (S/V) for public corporations in the United States for 1900–1979. Holland and Myers (1979) estimate the capital structure of corporations using National Income Accounts data on dividend and net interest payments from nonfinancial corporations. They capitalize these flows using the S&P dividend yield and the Moody's Baa bond yield, respectively. These data are available annually for 1929–1945 and quarterly for 1946–1987. For 1926, I use the estimate from Ciccolo and Baum (1986), based on the market value of debt and preferred and common stock for a sample of about 50 manufacturing firms. For 1900, 1912, and 1922, I multiply estimates of the book value of S/V from Goldsmith, Lipsey, and Mendelson (1963, Tables III-4, and III-4b, pp. 140–141, 146–147) by the average ratio of these estimates divided by the Holland-Myers estimates for the years 1929, 1933, 1939, and 1945–1958, $(HM/Goldsmith) = 1.226$. Thus, I have annual estimates of S/V for 1900, 1912, 1922, 1926, and 1929–1945 and quarterly estimates for 1946–1987.

I create a monthly series $(S/V)_t$ using the rates of return to the stock portfolio, R_{st} , described above and the returns to corporate bonds from Ibbotson (1986), R_{bt} . Before 1926, I estimate corporate bond returns using the yields on high-grade long-term bonds described above. I interpolate forward,

$$(S/V)_t^+ = \{S_{t-1}(1 + R_{st})/[S_{t-1}(1 + R_{st}) + B_{t-1}(1 + R_{bt})]\},$$

and backward,

$$(S/V)_t^- = \{S_{t+1}/(1 + R_{st+1})/[S_{t+1}/(1 + R_{st+1}) + B_{t+1}/(1 + R_{bt+1})]\}$$

and then use the average of these estimates for the monthly leverage estimate,

$$(S/V)_t = \{(S/V)_t^+ + (S/V)_t^-\}/2.$$

H. Stock Return Volatility, 1885–1987

Following French, Schwert, and Stambaugh (1987), I use the daily returns to the Standard & Poor's composite portfolio for 1928–1987 to estimate the standard deviation of monthly stock returns. The estimate of the monthly standard

¹⁹ The New York Stock Exchange was closed from August through mid-December, 1914 due to the outbreak of World War I. For purposes of this paper, I interpolate share volume growth during this period.

deviation is

$$\hat{\sigma}_t = \left\{ \sum_{i=1}^{N_t} r_{it}^2 \right\}^{1/2},$$

where r_{it} is the return to the S&P portfolio on day i in month t (after subtracting the sample mean for the month) and there are N_t trading days in month t . For 1885–1927 I use a comparable estimator based on the daily values of the Dow Jones composite portfolio. See Schwert (1989c,d) for more information about the daily stock returns and volatility estimates.

I. Industrial Production, 1889–1987

For 1919–1987, I use the index of industrial production from the Federal Reserve Board (1986) and Citibase (1978). For 1889–1918, I use Babson's Index of the physical volume of business activity from Moore (1961, p. 130), adjusted to splice with the industrial production data using the average ratio of Babson to adjusted industrial production for 1919–1939 (7.372662). I am grateful to Grant McQueen for providing these data.

J. Money Supply, 1867–1987

I use the monetary base (called high-powered money in Friedman and Schwartz (1963)). For 1867–1960, I use data from Friedman and Schwartz (1963, Table B-3, column (1), pp. 799–808) for the base. For 1961–1987, I use the seasonally adjusted monetary base reported by the Federal Reserve Board from Citibase (1978). These series are spliced using the average ratio of the respective series during 1960. Thus, the base data since 1960 are multiplied by 1.127538. The Friedman and Schwartz data are reported on a monthly basis beginning in May 1907. From June 1878 through April 1907, I use a monthly monetary base series from the National Bureau of Economic Research (NBER), multiplied by the average ratio of the Friedman and Schwartz series to the NBER series for 1878–1914, 1.006948. These data were provided by Professor Robert Barro. Thus, there are monthly data on growth rates of the base for 1878–1987.

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