



Forecasting stock market volatility: Do realized skewness and kurtosis help?



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HIGHLIGHTS

- We investigate the impacts of RSK and RKU on forecasting volatility.
- RV, RSK and RKU are all multifractal and persistent using the MF-DFA method.
- RSK and RKU have significantly negative impact on future volatility.
- RSK is more predictive than RKU at mid- and long-term forecasting horizons.

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ABSTRACT

In this study, we investigate the predictability of the realized skewness (RSK) and realized kurtosis (RKU) to stock market volatility, that has not been addressed in the existing studies. Out-of-sample results show that RSK, which can significantly improve forecast accuracy in mid- and long-term, is more powerful than RKU in forecasting volatility. Whereas these variables are useless in short-term forecasting. Furthermore, we employ the realized kernel (RK) for the robustness analysis and the conclusions are consistent with the RV measures. Our results are of great importance for portfolio allocation and financial risk management.

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1. Introduction

Stock market volatility is crucial to asset pricing, portfolio allocation and risk management, especially out-of-sample volatility forecasts are of great importance for market participants to make investment decisions. More accurate forecasts help investors generate tangible economic benefits by rebalancing portfolio weights. In previous studies, many scholars aim to improve forecast accuracy of stock market volatility by considering volatility features and its components such as leverage effect, volume, and signed returns [1–3].

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Recently, Amaya et al. [4] construct daily realized measures realized skewness (RSK) and realized kurtosis (RKU) according to conventional concepts about skewness and kurtosis risks. Their results demonstrate that there is a reliable and significant negative relation between realized skewness and the future stock returns. Strategies using upside and downside volatility components as well as realized skewness are shown to reveal additional information and deliver incremental economic benefits over strategies using total realized volatility alone. Subsequently, Nolte and Xu [5] find that using realized skewness can improve portfolio performance since realized measures contain incremental information over simple realized variances. However, little information about realized skewness and kurtosis in forecasting volatility is available in extant literature. This paper attempts to examine whether realized skewness and kurtosis help forecasting stock market volatility. We are the first to incorporate realized skewness and kurtosis as additional variables into the popular volatility models based on high-frequency data.

In previous studies, high-frequency data has been used to improve portfolio allocation by estimating the full realized covariance matrix. According to Nolte and Xu [5], strategies using high-frequency data for measuring and forecasting univariate realized volatility alone can already generate statistically significant and economically tangible benefits compared to low-frequency strategies. Using high-frequency data to measure and forecast realized volatility enables us to better understand future risk, and hence may improve portfolio allocation performance for a risk-averse investor.

With the availability of high-frequency data, research on measuring and modeling volatility based on high-frequency data has taken new avenues [6–9]. The seminal work on measuring the high-frequency volatility of Andersen and Bollerslev [10] proposes the realized volatility (RV), which is robust to market microstructure effects. For a given fixed interval, RV is defined as the sum of non-overlapping squared returns of high frequency within a day, which allows us to treat volatility as an observed rather than a latent process. RV enables researchers to better gauge the current level of volatility and understand its dynamics. Among the realized volatility models, the heterogeneous autoregressive RV (HAR-RV) model proposed by Corsi [11] is one of the most popular. Although the specification of HAR-RV is simple, it can capture “stylized facts” in financial market volatility such as long memory and multi-scaling behavior. Currently, HAR-RV has become the standard benchmark for analyzing and forecasting financial volatility dynamics (e.g., [12,13,2]).

Our paper contributes to the field on two aspects. First, we use RSK and RKU to gain new insights into the empirical behavior of forecasting volatility and explore whether those variables are multifractal and persistent using the multifractal detrended fluctuation analysis (MF-DFA) proposed by Kantelhardt et al. [14]. Second, we investigate whether adding RSK, RKU and both of them as additional variables to the HAR-RV model can improve its forecasting performance which has not been addressed in the existing studies. In our analysis, we use the 5-min high-frequency data of Chinese the US stock markets to calculate daily realized market volatility. Our main results indicate that RSK can significantly improve the models’ mid- and long-term forecasting performance, which the results are consistent in Chinese and US stock markets. Moreover, our empirical results demonstrate that RSK is more powerful forecasting ability than RKU in mid- and long-term. However, at the 1-day horizon, both RSK and RKU are useless to help in forecasting. Finally, we use the realized kernel (RK) as alternative volatility measures and find that the conclusions are robust.

The reminder of the paper is organized as follows. Section 2 describes volatility measure and the models Section 3 provides the data description and MF-DFA analysis. The out-of-sample forecasting performance and robustness check are discussed in Section 4. Section 5 concludes the paper.

2. Methodology

2.1. Realized volatility

According to the theory of quadratic variation, the quadratic variation for the cumulative return process is defined as:

$$[r, r]_t = \int_0^t \sigma^2(s)ds + \sum_{0 < s \leq t} \kappa^2(s). \quad (1)$$

From the above equation, quadratic variation can be decomposed into its continuous and discontinuous components. To estimate these quantities, we divide the time interval $[0, 1]$ into n subintervals of length $n = 1/\Delta$. Therefore, Realized Volatility or Variation (at day t) is defined as:

$$RV_t = \sum_{j=1}^{1/\Delta} r_{(t-1)+j\Delta, \Delta}^2. \quad (2)$$

2.2. Realized skewness and kurtosis

The realized measures for skewness and kurtosis constructed using high-frequency data have not been studied until recently. Motivated by Barndorff-Nielsen et al. [15], Amaya et al. [4], Nolte and Xu [5] suggest that the realized third (RTM)

Table 1

Descriptive statistics of RV, RSK and RKU.

		Mean	Max.	Min.	Std.dev	Skewness	Kurt. (excess)	Jarque–Bera
SSEC	RV	0.000	0.004	0.000	0.000	5.105	45.130	332 021
	RSK	0.091	5.013	−5.123	0.984	0.134	1.216	240
	RKU	4.268	32.731	1.767	2.439	4.172	26.688	121 258
S&P 500	RV	0.000	0.002	0.000	0.000	4.455	28.578	159 764
	RSK	0.093	6.838	−7.705	1.070	0.683	4.121	3 407
	RKU	5.356	65.324	1.908	4.198	4.843	35.759	248 201

Notes: The Jarque–Bera statistic tests are for the null hypothesis of normality for the distribution of the series.

and fourth moments (RFM) can be written as follow:

$$\begin{aligned}
 RTM_t &= \sum_{j=1}^{1/\Delta} (r_{j,t})^3 \xrightarrow{p} \sum_{0 < s \leq t} (\Delta p_s)^3 \\
 RFM_t &= \sum_{j=1}^{1/\Delta} (r_{j,t})^4 \xrightarrow{p} \sum_{0 < s \leq t} (\Delta p_s)^4.
 \end{aligned} \tag{3}$$

To be consistent with conventional concepts about skewness and kurtosis risks, we follow Amaya et al. [4] to construct daily realized skewness (RSK) and realized kurtosis (RKU) as below:

$$RSK_t = \frac{\sqrt{1/\Delta \sum_{j=1}^{1/\Delta} r_{j,t}^3}}{\left(\sum_{j=1}^{1/\Delta} r_{j,t}^2\right)^{3/2}} \tag{4}$$

$$RKU_t = \frac{1/\Delta \sum_{j=1}^{1/\Delta} r_{j,t}^4}{\left(\sum_{j=1}^{1/\Delta} r_{j,t}^2\right)^2}. \tag{5}$$

2.3. Volatility models

The HAR-RV model accommodates some of the stylized facts found in financial asset return volatility such as long memory and multi-scaling behavior. The HAR-RV is simple to implement, as it only contains three explanatory variables: lagged daily realized volatility ($RV_{d,t}$), lagged weekly realized volatility ($RV_{w,t}$), and lagged monthly realized volatility ($RV_{m,t}$). A standard specification of HAR can be written as follows:

$$RV_{t+h} = \beta_0 + \beta_d RV_t + \beta_w RV_{w,t} + \beta_m RV_{m,t} + \varepsilon_{t+h}, \tag{6}$$

where $RV_{w,t}$ is the average RV from day $t - 5$ to day $t - 1$ and $RV_{m,t}$ is the average RV from day $t - 22$ to $t - 1$. To examine the ability of RSK, RKU, RSK and RKU to predict RV, we add the lagged daily RSK, RKU, RSK and RKU to the HAR-RV model as predictors, respectively. We abbreviate those modified models HAR-RV–RSK, HAR-RV–RKU and HAR-RV–RSK–RKU respectively, which can be written as follows:

$$RV_{t+h} = \beta_0 + \beta_d RV_t + \beta_w RV_{w,t} + \beta_m RV_{m,t} + \theta RSK_t + \varepsilon_{t+h} \tag{7}$$

$$RV_{t+h} = \beta_0 + \beta_d RV_t + \beta_w RV_{w,t} + \beta_m RV_{m,t} + \eta RKU_t + \varepsilon_{t+h} \tag{8}$$

$$RV_{t+h} = \beta_0 + \beta_d RV_t + \beta_w RV_{w,t} + \beta_m RV_{m,t} + \theta RSK_t + \eta RKU_t + \varepsilon_{t+h}. \tag{9}$$

In our analysis, we set $h = 1, 5, 22$ and naturally acquire three different forecast horizons.

3. Data

In this study, we take 5-min high frequency data of the Shanghai Stock Exchange Composite Index (SSEC) as example. The data comes from the China Centre for Economic Research (CCER) high frequency database of stock market, during the period from 1 January 2000 to 31 May 2014, which contains totally 3722 trading days. In addition, we also use 5-min price data from the S&P 500 index for the January 2, 1996–June 24, 2013 period. The data are for the trading time of each business

Table 2All-of-sample estimations results of the HAR-type and its extended models ($h = 1$).

	β_0	β_d	β_w	β_m	θ	η	Adj. R^2
SSEC							
HAR-RV	6.68e−06*** (1.62e−06)	0.388*** (0.058)	0.224*** (0.083)	0.300*** (0.062)			0.533
HAR-RV-RSU	7.65e−06*** (1.67e−06)	0.389*** (0.057)	0.223*** (0.082)	0.294*** (0.061)	−0.074*** (0.024)		0.536
HAR-RV-RKU	0.000*** (1.68e−06)	0.412*** (0.019)	0.212*** (0.032)	0.291*** (0.030)		−0.033*** (0.007)	0.540
HAR-RV-RSK-RKU	0.000*** (1.96e−06)	0.412*** (0.058)	0.213*** (0.083)	0.287*** (0.061)	−0.063*** (0.022)	−0.031*** (0.007)	0.541
S&P 500							
HAR-RV	0.000*** (6.98e−06)	−0.010 (0.098)	0.425** (0.193)	0.493*** (0.141)			0.385
HAR-RV-RSU	0.000*** (6.95e−06)	−0.011 (0.098)	0.427** (0.193)	0.491*** (0.141)	−0.135*** (0.024)		0.386
HAR-RV-RKU	0.000*** (9.84e−06)	−0.010 (0.098)	0.425** (0.193)	0.489*** (0.141)		−0.032*** (0.008)	0.386
HAR-RV-RSK-RKU	0.000*** (1.01e−05)	−0.011 (0.098)	0.426** (0.193)	0.489*** (0.141)	−0.107*** (0.027)	−0.024*** (0.009)	0.386

Notes: The parentheses are the Newey–West standard errors. RSK and RKU are divided by 10 000.

** Denote rejections of null hypothesis at 5% significance level.

*** Denote rejections of null hypothesis at 1% significance level.

day between 9:30:00 and 16:00:00, can be obtained by Thomson Reuters Tick History Database. After removing days with shortened trading sessions or too few transactions, we obtain high-frequency data for 4280 business days. All the price data are taken from the DataStream database. Table 1 shows the descriptive statistics of RV, RSK and RKU. Moreover, using the MF-DFA method, we find that the RSK and RKU are multifractal and persistent, and the details are provided in the Appendix.

4. Empirical results

4.1. All-of-sample estimation

Table 2 exhibits the estimation results ($h = 1$) of the HAR-RV model and its extended models over the all-sample based on the Newey–West correction, which allows for correlation up to the order of 4. All the weekly, and monthly realized-volatility parameters of each model are significant at the 1% significance level, indicating a strong persistence in the realized volatility dynamics in the Chinese and US stock market. However, the coefficient of daily realized volatility are mixed in both stock market. From the empirical results of Table 2, we find that the RSK and RKU are all significantly negative at the 1% significance level, suggesting that the RSK and RKU have the negative impacts on future volatility.¹

4.2. Out-sample analysis

To evaluate whether the additive variables can help in forecasting volatility, we use the rolling window method in our forecasting procedure. Our sample data are divided into two subgroups: (1) in-sample data for volatility modeling, covering the first 2720 (3280, S&P 500) trading days; and (2) out-of-sample data for model evaluation, covering the last 1000 (1000, S&P 500) trading days. The estimation period is then rolled forward by adding one new day and dropping the most distant day. In this way, the sample size used to estimate the models remains at a fixed length and the forecasts do not overlap. In this way, the sample size employed to estimate the model parameters is fixed, and we re-estimate the parameters of these models each day to obtain volatility forecasts.

¹ The others estimation results (for example, $h = 5$ and 22) can be obtained by the authors.

Table 3

Forecasting results of volatility models in DM test.

	SSEC			S&P 500		
	$h = 1$	$h = 5$	$h = 22$	$h = 1$	$h = 5$	$h = 22$
Loss functions: MSE						
HAR-RV vs. HAR-RV-RSK	0.0138 (0.9890)	1.8760 [*] (0.0607)	1.5470 (0.1219)	1.3670 (0.1718)	2.4670 ^{**} (0.0136)	2.8300 ^{***} (0.0047)
HAR-RV vs. HAR-RV-RKU	1.6190 (0.1053)	−2.8980 ^{***} (0.0038)	−1.3640 (0.1726)	0.6415 (0.5212)	0.1586 (0.8740)	−0.0494 (0.9606)
HAR-RV vs. HAR-RV-RSK-RKU	0.4301 (0.6671)	1.2410 (0.2145)	−0.4681 (0.6397)	0.8900 (0.3734)	0.2448 (0.8066)	−0.0677 (0.9461)
Loss functions: MAE						
HAR-RV vs. HAR-RV-RSK	−0.0849 (0.9323)	3.1770 ^{***} (0.0015)	1.9190 ^{**} (0.0550)	−3.1810 ^{***} (0.0015)	3.4780 ^{***} (0.0005)	3.6860 ^{***} (0.0002)
HAR-RV vs. HAR-RV-RKU	−1.5110 (0.1307)	−2.8770 ^{**} (0.0040)	1.0930 (0.2743)	−1.1340 (0.5212)	−2.2820 ^{**} (0.0225)	−0.8076 (0.4193)
HAR-RV vs. HAR-RV-RSK-RKU	−1.901 (0.0574)	2.4890 ^{**} (0.0128)	1.8980 ^{**} (0.0577)	−3.2810 ^{***} (0.0010)	2.2250 ^{**} (0.0261)	−0.8165 (0.4142)

Notes: The numbers in the parentheses are the p -values.^{*} Denote rejections of null hypothesis at 10% significance level.^{**} Denote rejections of null hypothesis at 5% significance level.^{***} Denote rejections of null hypothesis at 1% significance level.

We obtain volatility forecasts for the horizons of 1, 5 and 22 days. To measure forecast accuracy, we use the following two popular loss functions:

$$MSE = M^{-1} \sum_{m=1}^M (RV_m - h_m)^2, \quad (10)$$

$$MAE = M^{-1} \sum_{m=1}^M |RV_m - h_m| \quad (11)$$

where M is the number of the out-of-sample forecasts. RV_m is the actual realized volatility and h_m is the volatility forecast. In addition, we use the [16] (DM) test to examine whether the differences in the loss functions between the two models are significant.

Table 3 displays the DM test results of the modified models and HAR-RV model. For $h = 1$, we find that the RSK, RKU and both as explanatory variables add to the HAR-RV model, which cannot improve the model's forecasting performance to the Chinese and US stock markets. Moreover, both two loss functions, RSK has a significantly positive impact on predicting the mid-term RV ($h = 5$) for the stock markets. Adding the RKU to the HAR-RV model, the new model has a worse performance than the bench model. However, the RSK and RKU, incorporated into the HAR-RV as additional variables, seems to help in forecasting realized volatility. For the 20-day forecasting horizon, RSK has significantly positive impact on forecasting volatility.

4.3. Robustness checks

It is well known that market microstructure noise plays an important role in estimating realized volatility, so we take the alternative realized measurements-realized kernel (RK)² that is robust to noise [17] to examine whether our results are robust.

We compare forecast accuracy between the volatility model with RSK, RKU, both, and neither of them and display the results in Table 4. Under the two different loss functions, the RSK significantly improves the models' performance at 5 and 20-day forecasting horizons. The results are consistent in Chinese and US stock markets. Therefore, the RSK is more powerful than RKU in forecasting mid- and long-term volatility. But at the 1-day horizon, both are not helpful in forecasting.

5. Conclusions

In this study, we add the RSK, RKU and both as additional variables to HAR-RV model and investigate whether those variables can improve forecast accuracy. Using the MF-DFA method, we find that the RV, RSK and RKU are all multifractal

² See more details about the realized kernel in [17].

Table 4

Forecasting results of volatility models in DM test based on RK.

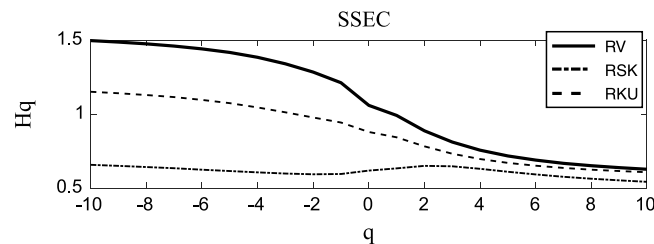
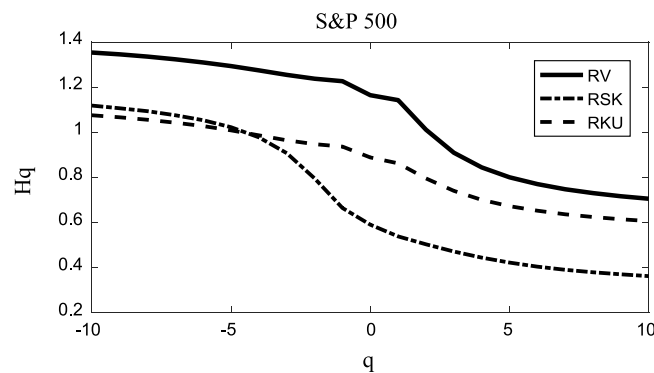
	SSEC			S&P 500		
	$h = 1$	$h = 5$	$h = 22$	$h = 1$	$h = 5$	$h = 22$
Loss functions: <i>MSE</i>						
HAR-RV vs. HAR-RV-RSK	0.5920 (0.5538)	2.6650*** (0.0077)	1.8140* (0.0696)	0.0967 (0.9230)	2.1510** (0.0315)	2.9160*** (0.0035)
HAR-RV vs. HAR-RV-RKU	0.5673 (0.5705)	−1.2030 (0.2288)	−1.0050 (0.3151)	0.7916 (0.4286)	1.1880 (0.2349)	1.2600 (0.2076)
HAR-RV vs. HAR-RV-RSK-RKU	0.6192 (0.5358)	1.0200 (0.3079)	0.1681 (0.8665)	0.2258 (0.8214)	1.4160 (0.1567)	1.6130 (0.1068)
Loss functions: <i>MAE</i>						
HAR-RV vs. HAR-RV-RSK	−0.3784 (0.7052)	2.3710** (0.0177)	2.2550** (0.0241)	−3.1810*** (0.0015)	1.6600* (0.0970)	2.4570** (0.0140)
HAR-RV vs. HAR-RV-RKU	−1.4460 (0.1481)	1.3760 (0.1687)	1.3500 (0.1771)	−1.1340 (0.2570)	0.5013 (0.6161)	0.8258 (0.4089)
HAR-RV vs. HAR-RV-RSK-RKU	−0.2880 (0.7734)	2.7610*** (0.0058)	2.5580** (0.0105)	−3.2810*** (0.0010)	0.0819 (0.9347)	1.0370 (0.2998)

Notes: The numbers in the parentheses are the *p*-values.

* Denote rejections of null hypothesis at 10% significance level.

** Denote rejections of null hypothesis at 5% significance level.

*** Denote rejections of null hypothesis at 1% significance level.

**Fig. 1.** The multifractality property of RV, RSK and RKU based on SSEC index.**Fig. 2.** The multifractality property of RV, RSK and RKU based on the S&P 500 index.

and persistent. All-sample estimation results demonstrate that the RSK and RKU have significantly negative impact on the future volatility. Out-of-sample results suggest that the RSK can substantially help in mid- and long-term forecasting. However, RKU cannot improve the models' performance. Therefore, RSK is more powerful than RKU in mid- and long-term volatility forecasting. Whereas both RSK and RKU cannot improve the short-term forecast accuracy. Moreover, we also use the alternative realized kernel (RK) for robustness analysis and the empirical results are consistent with the RV measures. Our findings can help investors make better decisions in risk management, derivative pricing, and portfolio selection.

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Table A.1Generalized Hurst exponents of RV, RSK and RKU in the US and Chinese stock market with q varying from -10 to 10 using the MF-DFA.

q	SSEC			S&P 500		
	RV	RSK	RKU	RV	RSK	RKU
-10	1.497	0.657	1.151	1.354	1.119	1.076
-9	1.487	0.650	1.141	1.346	1.107	1.067
-8	1.475	0.642	1.129	1.336	1.094	1.056
-7	1.460	0.634	1.115	1.324	1.076	1.043
-6	1.441	0.624	1.096	1.310	1.054	1.028
-5	1.417	0.615	1.074	1.294	1.023	1.009
-4	1.384	0.606	1.045	1.275	0.978	0.987
-3	1.340	0.598	1.012	1.255	0.908	0.965
-2	1.284	0.593	0.977	1.238	0.796	0.948
-1	1.212	0.595	0.943	1.227	0.665	0.937
0	1.059	0.618	0.879	1.165	0.591	0.889
1	0.991	0.633	0.843	1.143	0.539	0.863
2	0.887	0.650	0.782	1.012	0.504	0.796
3	0.811	0.647	0.732	0.910	0.472	0.741
4	0.756	0.630	0.696	0.845	0.445	0.701
5	0.717	0.610	0.670	0.801	0.423	0.673
6	0.689	0.592	0.651	0.771	0.405	0.653
7	0.667	0.576	0.636	0.748	0.391	0.637
8	0.651	0.563	0.624	0.731	0.380	0.625
9	0.638	0.552	0.615	0.717	0.371	0.615
10	0.627	0.542	0.607	0.706	0.363	0.607

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Appendix

We use the multifractal detrended fluctuation analysis (MF-DFA) to explore the multifractality property of RV, RSK and RKU in US and Chinese stock market. And more details about the MF-DFA method can be seen in [14,18,19]. When $H(q)$ is constant for all q , the time series are mono-fractal. Otherwise, the series are multifractal. Figs. 1 and 2 show that different q can get the different $H(q)$, implying that those nonparametric measures are multifractal and the same conclusions can be found the [8]. Specifically, when $H(q) > 0.5$, the kinds of fluctuations related to q are persistent. An increase (decrease) is always followed by another increase. From the empirical results, we find that most of measure variables are persistent in the US and Chinese stock market. Our study first find that the RSK and RKU also have the multifractal and persistent using the MF-DFA (see Table A.1).

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