Knowledge-Based Systems 166 (2019) 170–185

Contents lists available at ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

Does Google search index really help predicting stock market volatility? Evidence from a modified mixed data sampling model on volatility

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HIGHLIGHTS

- We develop a novel multiple factors GARCH-MIDAS (MF-GARCH-MIDAS) model.
- It allows for a long-run volatility driven by multiple factors sampled at different frequencies.
- It is used to investigate the usefulness of Google trends in predicting stock market volatility.
- The empirical results show favorable evidence about our proposed model.
- We find that Google trends really matters for volatility forecasting.

ARTICLE INFO

Article history:
Received 26 June 2018
Received in revised form 18 December 2018
Accepted 20 December 2018
Available online 26 December 2018

Keywords:
Volatility forecasting
Google trends
Mixed frequency data
GARCH-MIDAS
Multiple factors GARCH-MIDAS

ABSTRACT

Accurately predicting volatility plays an important role in many financial interactions and has already attracted considerable attention from both academics and practitioners. Much effort has been devoted to looking for key determinants of volatility and developing more powerful models. In this paper, we investigate the influences of high frequency event impact and low frequency macroeconomic fundamentals on volatility. To handle the mixed frequency data, we develop a modified GARCH-MIDAS model, multiple factors GARCH-MIDAS (MF-GARCH-MIDAS), to allow for a long-run component driven by multiple factors sampled at different frequencies. We present the technical details of the MF-GARCH-MIDAS model and apply it to predict monthly volatility of the US DJIA. In our empirical illustration, we consider weekly Google trends (GT) as a proxy for event impact and use three US macroeconomic variables: quarterly GDP, monthly PPI, and monthly IP. Our empirical findings support that event impact measured by GT is also a source of volatility besides macroeconomic fundamentals. The economic implications are significant in that both event impact and macroeconomic fundamentals really matter for volatility forecasting and the combination GT+PPI+GDP in M2 performs best in predicting volatility of DJIA.

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1. Introduction

It is essential and difficult to predict stock returns in finance [1–3]. According to the Efficient Market Hypothesis (EMH), all available information is fully reflected in asset prices and it is impossible for investors to gain excess returns or beat the market [4]. By contrast, volatility, a statistical measure of the variation of trading price series over time, has already proved to be predictable. Actually, it plays a central role in practical financial decisions, such as option pricing, risk management, asset allocation, and so on. As a result, much effort has been devoted to developing accurate measures and good forecasts of volatility.

Considerable progress has been made on volatility forecasts during the last three decades and there are mainly three types of popular methods in volatility modeling. The first one depends on historical information, such as generalized autoregressive conditional heteroscedasticity (GARCH) type models of [5], and stochastic volatility (SV) type models of [6]. Second, we can use an implied volatility (IV) calculated from an option pricing model [7]. Third, realized volatility (RV) based on high frequency realized returns provides a relatively simple but accurate measure of volatility, which is very useful for many purposes including volatility forecasting and forecast evaluation [8,9]. To the best of our knowledge, SV model assumes that volatility is a latent process, which creates computational difficulties. In general, it is impossible to give a closed-form formula for IV and its accuracy depends on the performance or correctness of the preset option pricing model. As for RV, it consistently estimates the quadratic variation of return

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https://doi.org/10.1016/j.knosys.2018.12.025
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process under regular conditions. However, these conditions are often violated in financial practice. In GARCH settings, the current volatility or variance is related to previous innovations and variances, and evolves through time. GARCH models are very popular for accurately predicting volatility. In addition, they are suitable for modeling financial stylized facts, such as high kurtosis, persistence, and volatility clustering. To this end, we focus on GARCH models and study their performances on volatility forecasting.

Innovations in information science and computer technology have made the collection and storage of large dataset possible. Consequently, mixed frequency data emerges in financial markets and brings serious challenges. To this end, a Mixed Data Sampling (MIDAS) method of [10–12] can be used to model the original mixed frequency data directly. Moreover, realized power variation used as covariates in MIDAS regressions has the most desirable forecasting properties [13–16]. Recently, a copula-MIDAS model is developed in [17] to quantify the return-liquidity dependence in financial markets.

We notice that all methods mentioned above only consider and use dynamics of financial time series themselves rather than other information or exogenous variables. Recently, the existing literature documents that stock market volatility is also influenced or dominated by various macro-finance/economic variables, e.g. [18–23]. A new class of models: GARCH-MIDAS developed through combining a daily GARCH process and a MIDAS polynomial, have been successfully applied to forecast volatility. The competitive advantage of GARCH-MIDAS is that it is able to distinguish short-run and long-run volatilities. In those empirical studies, the long-run volatility component is assumed to be driven by economic fundamentals such as industrial production growth, inflation, and so on.

In addition, the predictability and economic significance of news on returns have been documented in [24–27]. In practice, Google trend/index gathering more useful and timely information is helpful to make scientific decisions. The emerging research utilizing Google search data is predominately found in economic and financial applications such as GDP growth, unemployment [28,29], tourism [30], private consumption [31], international investment [32,33], inflation expectations [34], and stock returns [35–39]. Most recently, Google index has also been successfully applied to predict stock market volatility. In [40,41], retail investors' attention to the stock market is measured by internet search queries related to the leading stock market index and is found to be not only positively correlated with, but also indeed a Granger cause of Dow Jones' realized volatility. Their findings imply that internet search data is helpful to improve volatility forecasts. In [42,43], they distinguish information demand and supply, and then apply conventional GARCH models to examine the impact of two types of information on both stock market returns and volatility. The empirical results suggest that the impact of public information of Google search on stock volatility is much more important than on stock returns. In addition, information demand, as proxied by daily Google search volume index (SVI), is found in [44] to be positively related to stock market liquidity. The SVI is also used by [45] to measure investor attention in investigating financial contagion among currency markets. Their findings show that investor attention works as a predictive variable.

It is well known that volatility forecasting mainly depends on two basic elements: input variables and the model used. The input variables determine the quantity and quality of information used for predicting, and the model determines how to make best use of the information. In this paper, we aim to forecast stock volatility using both low frequency macroeconomic variables and high frequency Google trends (GT), which has not yet been involved. To this end, we develop a modified GARCH-MIDAS model to allow a long-run stock volatility being driven by multiple factors sampled at different frequencies, which is hereinafter called multiple factors GARCH-MIDAS (MF-GARCH-MIDAS). The main contributions of this paper are threefold. First, we propose a novel MF-GARCH-MIDAS model, which is an extension of the standard GARCH-MIDAS model in that it enables to handle input variables with different frequencies. Second, we apply the MF-GARCH-MIDAS model to predict stock market volatility by using GT as a proxy for event impact and macroeconomic variables to proxy macroeconomic fundamentals. The empirical results show that the MF-GARCH-MIDAS model is robust and outperforms several popular models. Third, we find that event impact is an important source of stock market volatility and its influence varies among different market environments. In particular, the more volatile the market, the more useful GT is.

The rest of this paper proceeds as follows. In Section 2, we present related work. We develop a novel MF-GARCH-MIDAS model and elaborate its techniques details in Section 3. We conduct an empirical application to illustrate the efficacy of our model and present empirical results in Section 4. Section 5 concludes the paper.

2. Related work

In this section, we first conduct theoretical analysis on sources of volatility, then present the data and empirical observations, and finally review the standard GARCH-MIDAS model with only one exogenous variable.

2.1. Theoretical analysis

2.1.1. Volatility determinants

Existing literature has documented that Internet search is helpful in stock market analysis, which can be explained from the perspective of information demand and information supply. Hence, we make the following assumptions about the influence mechanism.

In financial markets, an event study is able to assess the impacts of a specific event on the value of a firm. The usefulness of such a study comes from the fact that, given rationality in an EMH marketplace, the effects of an event will be reflected immediately in stock market. There are a large number of events, for instance mergers and acquisitions, earnings announcements, debt or equity issues, investment decisions and corporate social responsibility, that have different influences on stock price changes, i.e., volatility.

As shown in Fig. 1, occurred events will attract investor attention in a very short time. In order to make correct response to events, investors need more information and execute internet search behavior. On the other hand, events will eventually cause stock market volatile and stock volatility changes dynamically. In addition, high stock market volatility will further attract more investor attention and raise corresponding Google trends.

The search index can response to the occurrence of events among financial and economic markets, and the search volume is definitely associated with event impacts. In general, the larger the event impact, the higher the search volume. It is rational to believe that there is a close positive relationship between internet search and stock market volatility.

2.1.2. Volatility decomposition

We all know that the same news may have different influences on financial markets depending on different market environments. Following this idea, volatility is decomposed into long-run and short-run components in the standard GARCH-MIDAS model. As shown in Fig. 2, long-run component is mainly driven by macroeconomic fundamentals, while short-run one is assumed to be generated by unobservable elements (events for example) and follows a GARCH process.

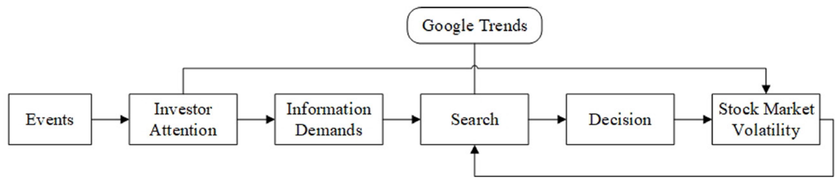

Fig. 1. Influence mechanism of events and Google trends on stock volatility.

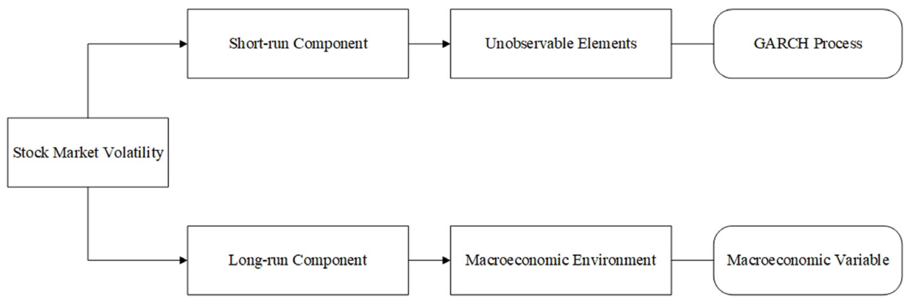

Fig. 2. Volatility decomposition without strengthening events.

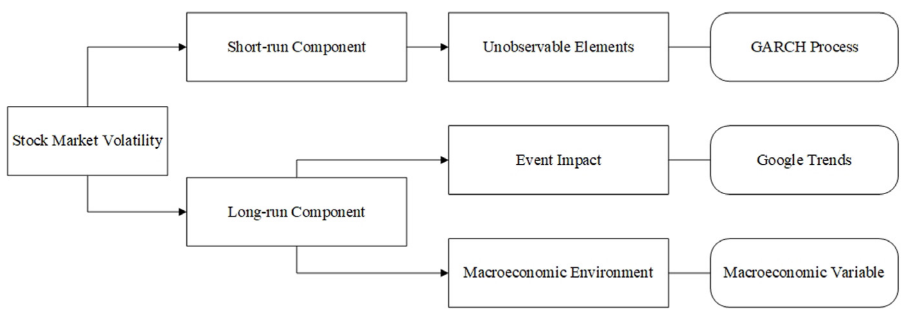

Fig. 3. Volatility decomposition with strengthening events.

This is a rather rough decomposition scheme as it treats events on market as an unobservable element. However, the truth is that the impacts of events on financial markets are often dramatic, which will change the market environment. For example, when government develops a new policy, the macroeconomic environment will not have obvious change within one or two months, but the news itself will no doubt impact the whole market and lead to the rise of volatility immediately. It is obviously irrational to ignore the impact of events when studying volatility. In this regard, we strength the importance of events and re-decompose the volatility as shown in Fig. 3. We further decompose the long-run component into two sub-elements: macroeconomic fundamentals and event impact. Accordingly, we modify the standard GARCH-MIDAS model and develop a MF-GARCH-MIDAS model in next section.

For macroeconomic fundamentals and unobservable elements, they can still be quantitatively described by macroeconomic variables and GARCH processes just like GARCH-MIDAS. Additionally, event impact can be represented as Google trends. To this end, we extract event information from Google trends.

2.2. Data and empirical observations

In this paper, we focus on the usefulness of Google trends (GT) and apply GT to predict stock market volatility. The used data contains three parts: (1) daily DJIA (Dow Jones Industrial Average) stock prices obtained from the Yahoo finance (https://finance.yahoo.com/); (2) macroeconomic fundamentals include: quarterly GDP (Gross Domestic Product) downloaded from the US Bureau of Economic Analysis (https://www.bea.gov/national/index.htm), monthly PPI (Producer Price Index) downloaded from the US Bureau of Labor Statistics (https://stats.bls.gov/news.release/ppi.toc.htm), and monthly IP (Industrial Production) downloaded from the Board of Governors of the US Federal Reserve System (https://www.federalreserve.gov/default.htm); (3) Google trends data downloaded from the Google (https://www.google.com/). We should note that GDP and IP have overlapped information and we do not use both of them at the same time in our empirical application.

Considering the data service from Google trends began in Jan, 2004, we collect data from Jan 1, 2004 to July 30, 2018. This leaves us with a complete dataset over 173 months, 765 weeks, or 3629 trading days.

2.2.1. Stock return and macroeconomic fundamentals

Our study focuses on the DJIA index from January 1, 2004 to July 30, 2018. We select its daily closing price at time t denoted as P_t and calculate its logarithm return r_t as

$$r_t = (\log(P_t) - \log(P_{t-1})) \times 100, \tag{1}$$

where $\log(\cdot)$ is the natural logarithm operator.

It has been documented that macroeconomic fundamentals play a significant role in volatility forecasting. This insight enables us to shed light on the link between stock market volatility and economic activities, such as GDP, inflation, and interest rates. In this paper, we consider GDP, PPI, and IP, and do not directly use their raw values but growth rates for two reasons. The first is that they are all nonstationary time series, which can be made stationary via differencing to avoid wrong statistical inference.

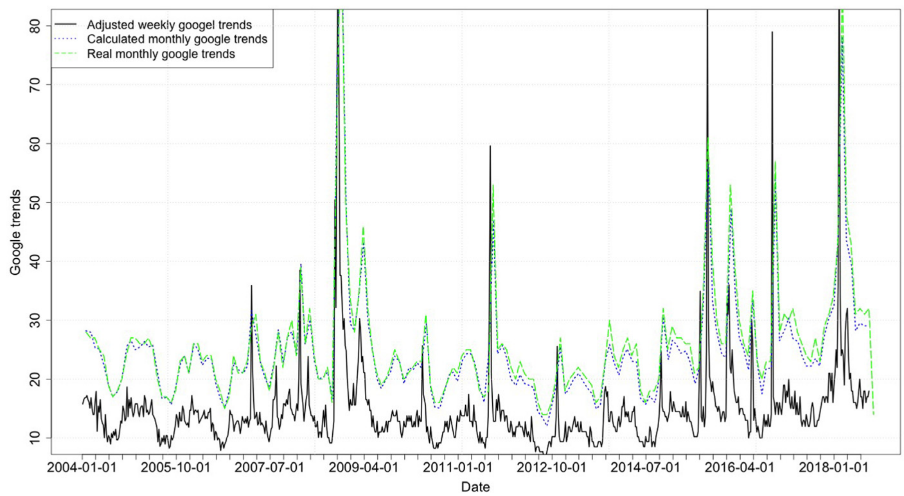

Fig. 4. Time series of three kinds of Google trends.

Another reason is that growth rate represents changes in a time series and is more strongly related to volatility than raw values. We then calculate their growth rate as

$$gr_t = (\log(x_t) - \log(x_{t-1})) \times 100, \quad (2)$$

where x_t denotes the levels of GDP, PPI, or IP, respectively.

2.2.2. Google trends

Since 2004, Google starts to provide service of Google trends search queries at https://www.google.com/trends. Since our target is stock market volatility, the keyword of search index we used in this paper is "stock" and the search behavior area of Google trends is constrained in US.

Google provides four kinds of Google trends data: (1) hourly or even higher frequency (10 min) data for time interval shorter than 30 days, (2) daily data for time interval shorter than 90 days; (3) weekly data for time interval shorter than 5 years, and (4) monthly data for longer time interval. As lower frequency data means less information and higher frequency data always have more noise, particularly high frequency periodic noise, we choose weekly Google trends for later use.

Since our sample covers a period of about 14 years (over 5 years), we can only download monthly Google trends data from Google directly. As Google trends have been adjusted by scaling each search volume to a value range of 0 to 100 based on its proportion to maximum search volume of that time interval with the same key word, we calculate full sample weekly Google trends data as below. First, we choose a 5-year period that includes the maximum search volume and download corresponding weekly Google trends data. Second, we select another period before or after the first one, and make sure that the two periods have an overlap interval about one or two years and download corresponding weekly data. Third, we denote the i-th Google search score of the first overlap time series as $overlap_i^{(\max)}$ and the other as $overlap_i^j$, and calculate the adjusted Google trends time series $\hat{S}_t$ as:

$$\hat{S}_t = S_t^j \cdot w^j, \quad (3)$$

where S_t^j denotes search score at time t in period j, and $w^j = \frac{1}{n}\sum_{i=1}^{n} \frac{overlap_i^{(\max)}}{overlap_i^j}$ is the weight. Consequently, we obtain full sample weekly Google trends data using the above 3 steps and plot the results in Fig. 4 with three Google trends time series: (1) our adjusted weekly Google trends, (2) calculated monthly Google trends aggregated from adjusted weekly Google trends, and (3) real monthly Google trends provided by Google. It is obvious that the calculated monthly Google trends is very close to the real monthly Google trends, which proves that our method is accurate for calculating weekly Google trends.

2.2.3. Empirical observations

Table 1 presents summary statistics for the DJIA returns as well as GT, PPI, IP, and GDP from Jan 1, 2014 to July 30, 2018. It shows that DJIA returns are negative skewed and fat tailed with mean of 0.02 and standard deviation of 0.08.

Fig. 5 provides time series plots of the above five variables including a target variable: DJIA, and four predictors: GT, PPI, IP, and GDP. We can see that all four predictors have a similar shape with volatility of DJIA. For example, highest volatility of DJIA happens at the maximum of GT or minimum of PPI, IP, and GDP, while lower volatility of DJIA corresponds to stable periods of GT, PPI, IP, and GDP. The main differences between Google trends and macroeconomic determinants in two main aspects. First, the stock market volatility is positively related to Google trends, but negatively to macroeconomic variables. This is because that higher volatility always means larger uncertainty, which makes people pay more attention to stock market and causes Google trends rising. Second, it is obvious that Google trends is more volatile than macroeconomic indicators. The possible reason is that Google trends is more sensitive and rapidly responds to stock market events than any other macroeconomic variables.

The above results can also be found in correlation analysis. We transform mixed frequency data into quarterly data with aggregating and conduct Pearson correlation test. In Fig. 6, the diagonals present histograms with kernel density estimations of each time series, the lower panels give scatter plots of each pair, and the upper panels report Pearson correlation coefficients with significant tests, where *, **, and *** denote significance at 10%, 5%, and 1% levels, respectively. We find that Google trends is negatively correlated with DJIA and all macroeconomic indicators.

2.3. The standard GARCH-MIDAS model

Based on an univariate daily GARCH process and a MIDAS polynomial applied to monthly or quarterly financial variables, the standard GARCH-MIDAS model is proposed by [18]. For a stock market, we consider its log return $r_{i,t}$ on day i of any arbitrary period t (which may be a month and has N_t days, without loss of generality N days). The unexpected returns can be assumed to follow a GARCH process as

$$r_{i,t} = \mu + \sigma_{i,t}\varepsilon_{i,t}, \quad \text{for} \quad i = 1, 2, \ldots, N, \quad (4)$$

Table 1
Summary statistics for the DJIA returns on entire sample.

Variable	Frequency	Max	3rd Q	Mean	Median	1st Q	Min	S.D.	Skewness	Kurtosis
DJIA	Daily	10.51	0.50	0.02	0.05	−0.39	−8.20	0.08	−0.15	11.61
GT	Weekly	100.00	15.36	14.62	13.00	11.08	6.82	8.00	5.94	47.73
PPI	Monthly	2.94	0.91	0.21	0.34	−0.43	−5.48	0.01	−1.24	4.41
IP	Monthly	3.72	1.05	0.00	0.00	−0.01	−5.10	0.01	−0.16	−0.07
GDP	Quarterly	1.32	0.81	0.46	0.55	0.24	−2.18	0.01	−1.93	5.78

NOTE: (1) PPI, IP, and GDP take their corresponding growth rates; (2) Max stands for maximum, 3rd Q for the 3trd quartile, 1st Q for the 1st quartile, Min for minimum, and S.D. for standard deviation.

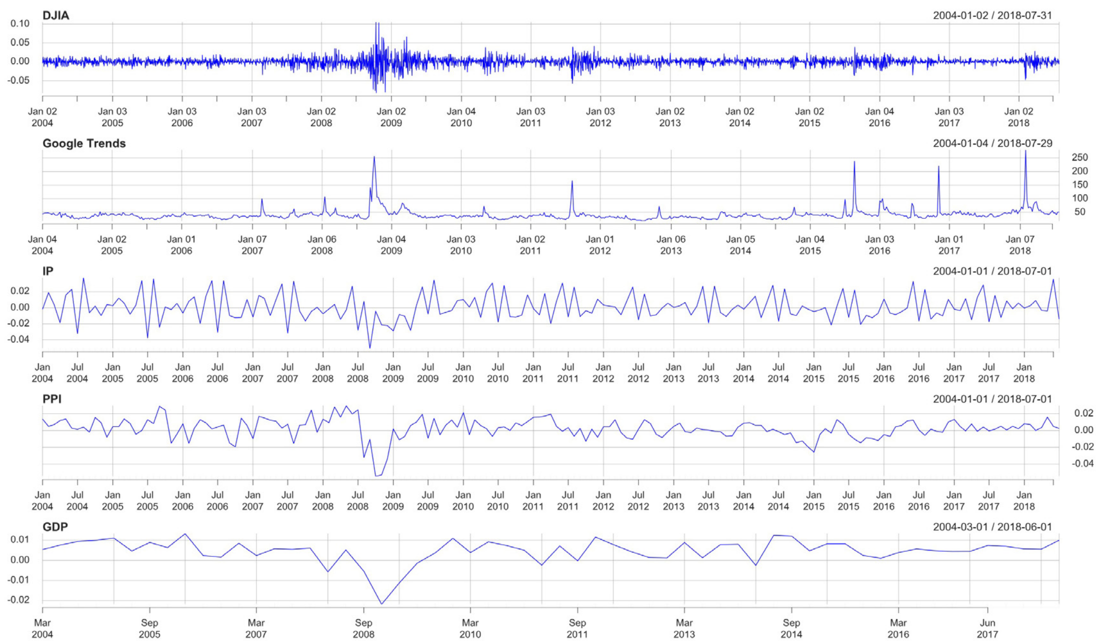

Fig. 5. Trends of time series.

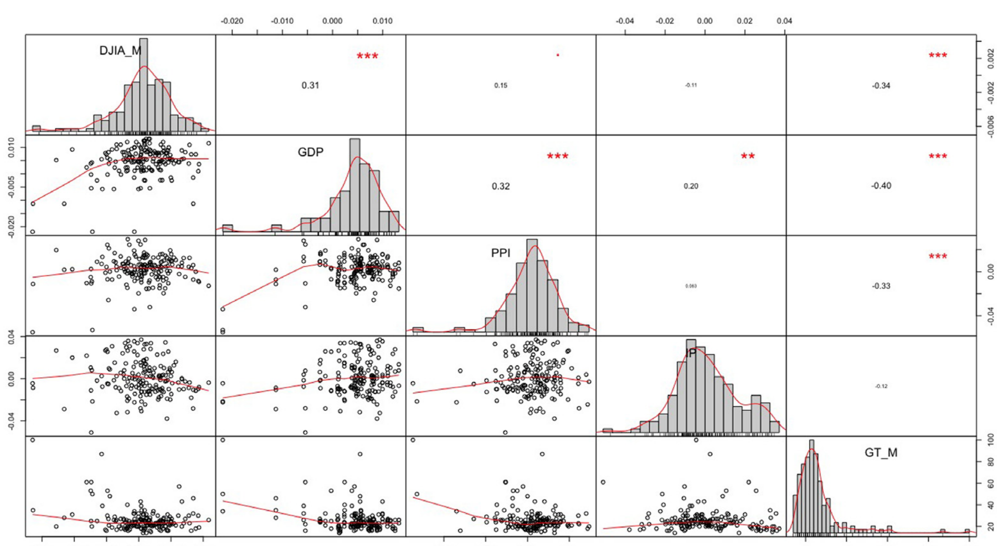

Fig. 6. Correlation analysis of quarterly data.

where μ is an unconditional mean, conditional volatility $\sigma_{i,t}$ is driven by a GARCH process, and $\varepsilon_{i,t}|\Omega_{i-1,t} \sim N(0, 1)$ with information set $\Omega_{i-1,t}$ available on day $(i-1)$ of period t. Following the idea that different news may have different impacts on financial markets, conditional volatility is divided into two components in [18], i.e. $\sigma_{i,t} = \sqrt{\tau_t \cdot g_{i,t}}$, and the model can be rewritten as

$$r_{i,t} - E_{i-1,t}(r_{i,t}) = \sqrt{\tau_t \cdot g_{i,t}}\varepsilon_{i,t}. \tag{5}$$

More specifically, conditional volatility depends on two components, namely, τ_t which accounts for a long-run component, and daily fluctuations $g_{i,t}$. In practice, the long-run component τ_t is assumed to be driven by macroeconomic factors, such as GDP growth, inflation, future expected cash flows, future discount rates, and some other low frequency variables. The short-run component $g_{i,t}$ often relates to daily liquidity concerns and some other possible short-lived factors.

In the standard GARCH-MIDAS specification, the dynamics of the component $g_{i,t}$ is assumes to be a GARCH(1,1) process

$$g_{i,t} = (1-\alpha-\beta) + \alpha\frac{(r_{i-1,t}-\mu)^2}{\tau_t} + \beta g_{i-1,t}. \tag{6}$$

In addition, the τ_t component for GARCH-MIDAS is specified by smoothing macroeconomic variables in the spirit of MIDAS regression and MIDAS filtering

$$\tau_t = \alpha_0 + \theta\sum_{k=1}^{K}\varphi_k(\omega_1,\omega_2)X_{t-k}, \tag{7}$$

where X_{t-k} denotes the kth lag of a macroeconomic variable, and the Beta function $\varphi_k(\omega_1,\omega_2)$ provides a weighting scheme for X_t and is defined as

$$\varphi_k(\omega_1,\omega_2) = \frac{(k/K)^{\omega_1-1}(1-k/K)^{\omega_2-1}}{\sum_{j=1}^{K}(j/K)^{\omega_1-1}(1-j/K)^{\omega_2-1}}. \tag{8}$$

The major advantage of GARCH-MIDAS is that it enables us link the long-run component of volatility τ_t directly to economic activities regardless of the frequency difference between input variables and daily returns.

3. The MF-GARCH-MIDAS model

In this section, we develop a modified GARCH-MIDAS model that allows us to input variables with different observed frequencies and call it multiple factors GARCH-MIDAS (MF-GARCH-MIDAS). In model setup, we consider two forms of MF-GARCH-MIDAS and discuss their relationships. In addition, we present the techniques of MF-GARCH-MIDAS in detail.

3.1. Model setup

In spite of the strength of the standard GARCH-MIDAS model, it can only handle just one exogenous variable in Eq. (7) and does not involve many variables even with different frequencies. We decide to modify the standard GARCH-MIDAS model for two reasons.

First, we need a model to incorporate multiple factors. Like other volatility decompose model, GARCH-MIDAS decompose volatility into long-run and short-run components. According to the discussion in Section 2, we have extended the definition of long-run component as macroeconomic fundamentals and event impact. In this paper, the macroeconomic fundamentals are represented by macroeconomic variables like GDP, PPI, and IP. In addition, event impact can be measured by Google trends as it is a proxy for investor attention.

Second, we need an approach to handle different frequencies. To understand the dynamics of τ_t, it is necessary to incorporate many factors observed at different frequencies. To address this issue, we develop a MF-GARCH-MIDAS model with a long-run stock market volatility being driven by multiple factors sampled at different frequencies.

We consider a log version of GARCH-MIDAS to guarantee the positive of τ_t, and design the MF-GARCH-MIDAS model by extending Eq. (7) in two directions, which yields the following two versions. The first is M1 that estimates low frequency volatility using high frequency information. We introduce several exogenous variables with different higher frequencies and construct M1 from a nature extend of GARCH-MIDAS with the form

$$\text{M1:} \qquad \log(\tau_t) = \alpha_0 + \sum_{i=1}^{I}\left[\theta_i\sum_{k=0}^{Lag_i}\varphi^{(i)}(\omega_1^{(i)},\omega_2^{(i)})X_{t-k}^{(i)}\right], \tag{9}$$

where $t = 1,2,\ldots,N$, Lag_i stands for the lag of variable i and $X_{t-k}^{(i)}$ is the kth lag of ith macroeconomic variable; $\varphi^{(i)}(\omega_1^{(i)},\omega_2^{(i)})$ determines the weight scheme, θ_i is the coefficient of variable i, for $i = 1,2,\ldots,I$. In M1, we aggregate each higher frequency predictor into a lower frequency one via polynomial constraints. The other is M2 that estimates high frequency volatility using low frequency information. In this paper, we specify M2 as

$$\text{M2:} \qquad \log(\tau_{t+s/m}) = \alpha_0 + \sum_{i=1}^{I}\left[\theta_i\sum_{k=0}^{Lag_i}\varphi^{(i)}(\omega_1^{(i)},\omega_2^{(i)})X^{(i)}_{t+\lfloor\frac{s}{m/m_i}\rfloor/m_i-k/m_i}\right], \tag{10}$$

where m_i means frequency differ between X_i and X_1 with the lowest observed frequency, $m \equiv \max\{m_1,m_2,\ldots,m_I\}$ and $s = 1,2,\ldots,m$, $\lfloor\cdot\rfloor$ denotes the floor function. In M2, we apply the periodic processing to divide a regressand into m periods and alignment all predictors into the lowest frequency. For example, if we construct a weekly long-run component using quarterly GDP, monthly IP, and weekly GT, then the frequency mismatch is $m_1 = 1$, $m_2 = 3$, $m_3 = 12$ and Eq. (10) has a specific form

$$\begin{aligned}\log(\tau_{t+s/m}) = \alpha_0 &+ \theta_1\sum_{k=0}^{Lag_1}\varphi^{(1)}(\omega_1^{(1)},\omega_2^{(1)})GDP_{t-k/m_1}\\ &+ \theta_2\sum_{k=0}^{Lag_2}\varphi^{(2)}(\omega_1^{(2)},\omega_2^{(2)})IP_{t+\lfloor\frac{s}{m/m_2}\rfloor/m_2-k/m_2}\\ &+ \theta_3\sum_{k=0}^{Lag_3}\varphi^{(3)}(\omega_1^{(3)},\omega_2^{(3)})GT_{t+\frac{s}{m_3}-k/m_3}.\end{aligned} \tag{11}$$

3.2. Model discussion

The key point in building GARCH-MIDAS model is to construct long-run component τ using exogenous variables. In the standard GARCH-MIDAS model, τ is calculated as Eq. (7), which sums up weighted lag items of an input variable.

In M1, we consider different weighting schemes for variables with different frequencies. Consequently, both predictand and predictors attain the same frequency level through functional constraints that impose polynomial restrictions on frequency alignments on the right-hand. It is obvious that M1 will suffer information loss in that the frequency of τ is the same as the lowest frequency of input data. In contrast, M2 exploits periodic processing to retain high frequency nature of τ without information loss. With the help of M2, long-run component τ has the same frequency as the high frequency variables and is able to provide more useful information about volatility.

To sum up, the major difference between M1 and M2 is how to use high frequency information. We argue that M2 will outperform M1 especially in the situation where events has a huge impact on financial markets. Their performances on volatility forecasting will be compared in the subsequent empirical applications.

3.3. Model estimation

Following [18], we apply the classical quasi maximum likelihood (QMLE) method to estimate MF-GARCH-MIDAS. We start by dividing all parameters to be estimated into two groups. The first group $\boldsymbol{\Phi}^{(\tau)} \equiv (\alpha_0;\theta_1,\ldots,\theta_I;\omega_1^{(1)},\omega_2^{(1)},\ldots,\omega_1^{(I)},\omega_2^{(I)})'$ contains the parameters in the long-run component: τ defined in M1 or M2, while the second one $\boldsymbol{\Phi}^{(g)} \equiv (\mu,\alpha,\beta)'$ contains the parameters of daily fluctuation g excluding those in τ. Thus, we derive its log-likelihood function (LLF) as

$$LLF(\boldsymbol{\Phi}^{(g)}, \boldsymbol{\Phi}^{(\tau)}) = -\frac{1}{2}\sum_{t=1}^{T}\left[\log(2\pi) + \log(g_t(\boldsymbol{\Phi}^{(g)})\tau_t(\boldsymbol{\Phi}^{(\tau)})) + \frac{(r_t-\mu)^2}{g_t(\boldsymbol{\Phi}^{(g)})\tau_t(\boldsymbol{\Phi}^{(\tau)})}\right]. \tag{12}$$

To optimize Eq. (12), we adopt a kind of quasi-Newton method: BFGS. We should note that both M1 and M2 have the same form of LLF, but their specifications of τ are different from each other. Considering the gradient evaluation for the BFGS method in numerical optimization, we provide their gradients as below

(1) In M1, we have

$$\frac{\partial \tau_t}{\partial \theta_i} = \tau_t\left[\sum_{k=0}^{Lag_i}\varphi^{(i)}(\omega_1^{(i)}, \omega_2^{(i)})X_{t-k}^{(i)}\right], \tag{13}$$

$$\frac{\partial \tau_t}{\partial \omega_1^{(i)}} = \tau_t\left[\theta_i\sum_{k=0}^{Lag_i}\frac{\partial\varphi^{(i)}(\omega_1^{(i)}, \omega_2^{(i)})}{\partial\omega_1^{(i)}}X_{t-k}^{(i)}\right], \tag{14}$$

$$\frac{\partial \tau_t}{\partial \omega_2^{(i)}} = \tau_t\left[\theta_i\sum_{k=0}^{Lag_i}\frac{\partial\varphi^{(i)}(\omega_1^{(i)}, \omega_2^{(i)})}{\partial\omega_2^{(i)}}X_{t-k}^{(i)}\right], \tag{15}$$

for $i = 1, 2, \ldots, I$, with $\tau_t = \exp\left\{\alpha_0 + \sum_{i=1}^{I}\left[\theta_i\sum_{k=0}^{Lag_i}\varphi^{(i)}(\omega_1^{(i)}, \omega_2^{(i)})X_{t-k}^{(i)}\right]\right\}$.

(2) In M2, we have

$$\frac{\partial \tau_{t+s/m}}{\partial \theta_i} = \tau_{t+s/m}\left[\sum_{k=0}^{Lag_i}\varphi^{(i)}(\omega_1^{(i)}, \omega_2^{(i)})X_{t+\lfloor\frac{s}{m/m_i}\rfloor/m_i-k/m_i}^{(i)}\right], \tag{16}$$

$$\frac{\partial \tau_{t+s/m}}{\partial \omega_1^{(i)}} = \tau_{t+s/m}\left[\theta_i\sum_{k=0}^{Lag_i}\frac{\partial\varphi^{(i)}(\omega_1^{(i)}, \omega_2^{(i)})}{\partial\omega_1^{(i)}}X_{t+\lfloor\frac{s}{m/m_i}\rfloor/m_i-k/m_i}^{(i)}\right], \tag{17}$$

$$\frac{\partial \tau_{t+s/m}}{\partial \omega_2^{(i)}} = \tau_{t+s/m}\left[\theta_i\sum_{k=0}^{Lag_i}\frac{\partial\varphi^{(i)}(\omega_1^{(i)}, \omega_2^{(i)})}{\partial\omega_2^{(i)}}X_{t+\lfloor\frac{s}{m/m_i}\rfloor/m_i-k/m_i}^{(i)}\right], \tag{18}$$

for $i = 1, 2, \ldots, I$, with $\tau_{t+s/m} = \exp\left\{\alpha_0 + \sum_{i=1}^{I}\left[\theta_i\sum_{k=0}^{Lag_i}\varphi^{(i)}(\omega_1^{(i)}, \omega_2^{(i)})X_{t+\lfloor\frac{s}{m/m_i}\rfloor/m_i-k/m_i}^{(i)}\right]\right\}$.

3.4. Model evaluation

To evaluate the model's performance on volatility forecasting, one should compare the forecasted values with real values. However, volatility is an unobserved measure. To tackle this issue, we use realized volatility as a proxy for real volatility. Realized volatility has proved to be an efficient estimator of volatility and it sums up the squared returns during the period as

$$RV_t = \sum_{s=1}^{n} r_{t+s/n}^2, \tag{19}$$

where $r_{t+s/n}$ are returns on period t during interval s/n for $s = 1, 2, \ldots, n$, and n is the number of such return intervals. Here, we calculate monthly realized volatility from daily returns. According to [8,46], realized volatility is a good proxy for actual volatility.

To measure prediction accuracy of volatility, we use MSE (Mean Squared Error), MAE (Mean Absolute Error), and MAPE (Mean Absolute Percentage Error) defined, respectively, as

$$MSE = \frac{1}{T}\sum_{t=1}^{T}(\widehat{RV}_t - RV_t)^2, \tag{20}$$

$$MAE = \frac{1}{T}\sum_{t=1}^{T}|\widehat{RV}_t - RV_t|, \tag{21}$$

$$MAPE = \frac{1}{T}\sum_{t=1}^{T}\left|\frac{\widehat{RV}_t - RV_t}{\widehat{RV}_t}\right|, \tag{22}$$

with

$$\widehat{RV}_t = \frac{1}{n}\sum_{i=1}^{n}\hat{\sigma}_{i,t}^2, \tag{23}$$

where $\hat{\sigma}_{i,t}$ is the daily volatility calculated from models. As the frequency of our RV is monthly, we translate daily $\hat{\sigma}_{i,t}$ to monthly $\widehat{RV}_t$.

4. Empirical results

In this section, we first apply the standard GARCH-MIDAS model to predict volatility of DJIA using single macroeconomic variable (IP, PPI, or GDP) as the benchmark. We then add Google trends into the MF-GARCH-MIDAS model to investigate its usefulness in volatility forecasting. We examine the predictability of each reasonable combination of Google trends and macroeconomic variables to find the determinants in stock market volatility. In addition, we perform a robustness check on the performance of MF-GARCH-MIDAS by considering structural breaks and different degree of frequency for GT.

According to the proportion of 7:3, we partition the full data into two sub-samples: (1) the in-sample covers the period from 2004-01-01 to 2013-12-31, and (2) the out-of-sample from 2014-01-01 to 2018-07-30. We apply the in-sample data to fit models and use the fitted models to predict stock volatility during the out-of-sample period. We use in Sections 4.2 and 4.3 this sample partition scheme to investigate the usefulness of GT in predicting stock market volatility. In the robustness check of Section 4.4, we consider the role of structural breaks and partition the entire sample according to the results of structural break testing.

4.1. Optimal lags

To build a standard GARCH-MIDAS or MF-GARCH-MIDAS model, the MIDAS lag period of each variable (Lag_i) should be predetermined. As an alternative way to AIC/BIC, we select the optimal MIDAS lags based on the shape of the Beta function or the values of polynomial MIDAS weights.

First, we consider the standard GARCH-MIDAS model with only one exogenous covariate. We estimate the model with different lags and choose a suitable lag for each variable according to the estimated weights approaching to zero. Second, we use the selected optimal MIDAS lags as Lag_i for each corresponding variable in the MF-GARCH-MIDAS model.

We report the shape of weight function for GT, GDP, IP, and PPI in Figs. 7–10, respectively, and summarize the results in Table 2 with column 'Zero lag' denoting the lag at which weights approach to zero. For example, we estimate the weight function of GT with a lag of 4, 8 or 12 weeks. Fig. 7 shows that the estimated weights always decay to zero around 2 weeks of lags regardless of the choice of different lags and length of MIDAS lag month. Then, we extend the zero lags and get the column 'Chosen lag', where the

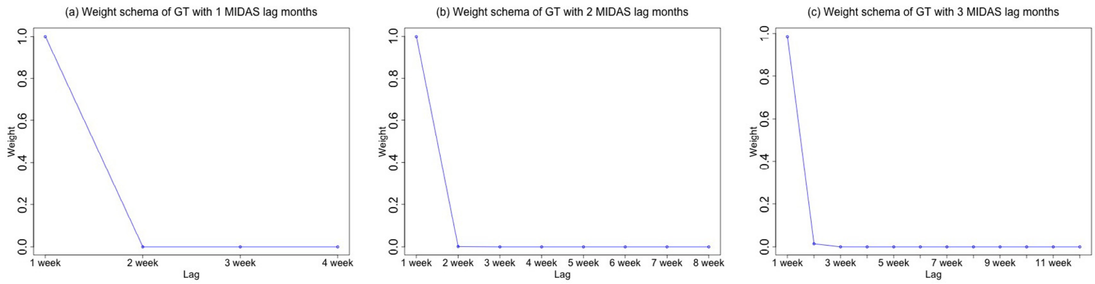

Fig. 7. Estimated polynomial MIDAS weights of GT.

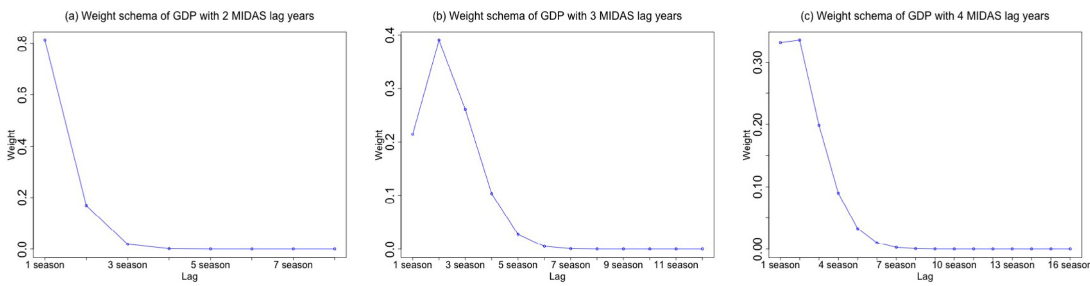

Fig. 8. Estimated polynomial MIDAS weights of GDP.

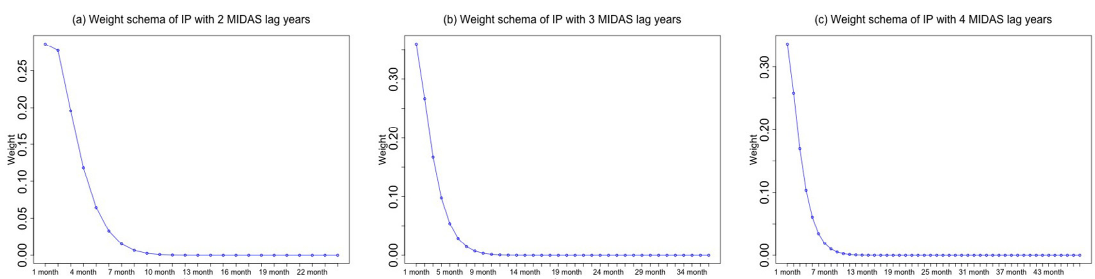

Fig. 9. Estimated polynomial MIDAS weights of IP.

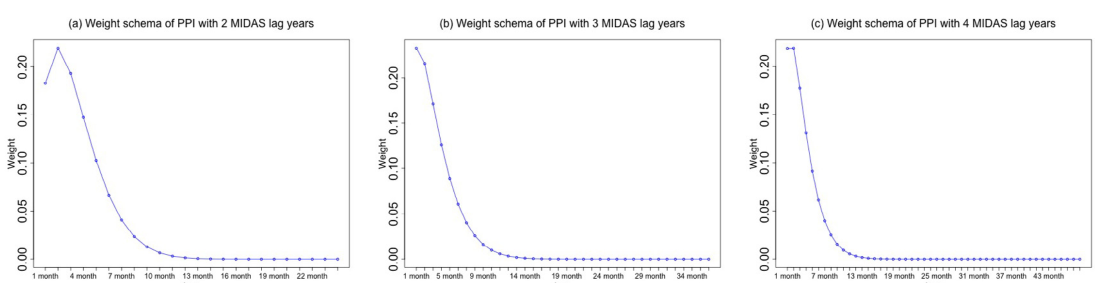

Fig. 10. Estimated polynomial MIDAS weights of PPI.

Table 2
Optimal lags selection.

Indicators	Zero lag	Chosen lag
GT	2 weeks	4 weeks
GDP	6 seasons	8 seasons
IP	10 months	16 months
PPI	12 months	16 months

lags we use so as to ensure sufficient information and not too much calculation.

The differences in estimated lags for all variables are interesting and significant. We find that the efficient lag interval of each variable depends on its nature. For example, GDP measures the whole economic status of a country and has the largest measure scope among these four variables. The valid lag interval of GDP is 6 seasons, which means that GDP helps predict stock market volatility until one and a half year ago. In contrast, Google trends, an indicator for peoples' concern, has much shorter scope. It is a kind of high quality short-run information in terms of accuracy and volume. However, it often shows short-lived contributions and its valid lag is 2 weeks. Regarding IP and PPI, these two indicators also have relatively large measure scope. Consequently, their valid lags are about one year.

Table 3
Estimation results of single factor model (standard GARCH-MIDAS).

Parameters	GT	IP	PPI	GDP
μ	0.0008	0.0007	0.0007	0.0008
α	0.1908	0.1802	0.1800	0.2074
β	0.7529	0.7400	0.7275	0.6848
θ	−0.0156	−59.152	−69.461	151.44
ω_1	6.5303	2.1336	2.3963	4.4021
ω_1	56.1508	.1479	3.9984	11.773
α_0	−8.7685	−9.6633	−9.7657	−10.597
LLF	8879.35	7578.43	7581.41	6758.16

Table 4
Estimation results of two factor model (MF-GARCH-MIDAS).

Parameters	M1			M2		
	IP+GT	PPI+GT	GDP+GT	GT+IP	GT+PPI	GT+GDP
μ	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007
α	0.1690	0.1812	0.1876	0.1864	0.1825	0.1866
β	0.7537	0.7287	0.6663	0.7350	0.7311	0.7309
θ_1	−33.3874	−60.9697	352.6863	0.0098	0.0133	0.0117
θ_2	0.0100	0.0094	0.0286	−31.4021	−68.9547	19.9256
ω_1^1	5.7150	3.6921	1.4669	18.0263	36.4054	15.6831
ω_2^1	43.7806	5.7083	1.7177	74.3588	136.3590	66.3438
ω_1^2	33.1179	11.2266	32.0427	5.3331	2.0747	45.4371
ω_2^2	44.1584	163.8896	61.3082	40.2191	3.6837	38.2349
α_0	−9.4525	−9.9998	−10.9963	−9.9417	−10.0977	−10.1533
LLF	7579.26	7582.32	6765.10	7579.59	7582.93	7576.50

Table 5
Model comparison results (one factor and two factors).

Models	Indicators	In-sample			Out-of-sample		
		MSE	MAE	MAPE	MSE	MAE	MAPE
GARCH(1,1)	–	5.76	2.39	2.24	5.45	1.83	1.91
GARCH(1,2)	–	5.82	2.53	2.36	5.51	1.85	1.86
GARCH(2,1)	–	5.77	2.44	2.33	5.46	1.93	1.93
GARCH(2,2)	–	5.85	2.62	2.32	5.60	1.86	1.88
GARCH-ANN	GT	5.95	3.10	2.56	5.62	1.78	1.81
	IP	5.81	2.66	2.18	5.47	1.74	1.86
	PPI	5.86	2.73	2.27	5.42	1.75	1.83
	GDP	5.80	2.65	2.16	5.66	1.80	1.88
WN's Model	GT	5.82	2.56	2.21	5.19	1.84	1.86
	IP	5.43	2.36	2.15	4.65	1.64	1.83
	PPI	5.77	2.44	2.22	4.86	1.72	1.84
	GDP	5.68	2.55	2.21	4.81	1.66	1.82
Standard GARCH-MIDAS(S)	GT	5.87	2.70	2.04	5.52	1.73	1.75
	IP	5.60	2.43	2.18	4.87	1.71	1.81
	PPI	5.63	2.53	2.16	4.85	1.69	1.82
	GDP	5.92	2.60	2.32	4.98	1.77	1.90
MF-GARCH-MIDAS(M1)	IP+GT	5.40	2.32	2.06	4.64	1.61	1.81
	PPI+GT	5.46	2.36	2.09	4.68	1.66	1.80
	GDP+GT	5.66	3.50	2.11	4.65	1.63	1.73
MF-GARCH-MIDAS(M2)	GT+IP	5.28	2.03	2.05	4.60	1.59	1.74
	GT+PPI	5.27	2.02	2.02	4.67	1.64	1.75
	GT+GDP	5.28	2.04	2.09	4.60	1.60	1.73

NOTE: (1) GT stands for the weekly GT downloaded from the Google; (2) 'S' denotes the standard GARCH-MIDAS model, 'M1' is the MF-GARCH-MIDAS model with τ_t defined in Eq. (9), and 'M2' is the MF-GARCH-MIDAS model with $\tau_{t+s/m}$ defined in Eq. (10); (3) The measurement units of MSE, MAE, and MAPE are 10^{-4}, 10^{-2}, and 10^{-1}, respectively.

4.2. Usefulness of GT from two factors model vs. single factor model

To study the usefulness of GT and its compatibility with macroeconomic variables, we use GT, IP, PPI, or GDP to predict stock market volatility based on the standard GARCH-MIDAS model and view it as the benchmark. Then we add the weekly GT data into our MF-GARCH-MIDAS model to predict stock volatility. The estimation results are shown in Tables 3 and 4.

For comparison, we also consider three types of models without using mixed frequency data information. They are: (1) conventional GARCH models of [5] including GARCH (1,1), GARCH(1,2), GARCH(2,1) and GARCH(2,2); (2) GARCH-ANN model of [47], which combines classic GARCH model and the ANN (Artificial Neural Network); (3) a GARCH model with extraneous information proposed by [48] hereinafter called WN's model. We report the comparison results for one factor and two factors in Table 5.

From Table 5, several remarkable conclusions can be drawn. First, GT is very useful in predicting stock market volatility. We introduce the weekly GT to construct the MF-GARCH-MIDAS model with two factors. Both M1 and M2 have smaller MSE, MAE, and MAPE than the single factor standard GARCH-MIDAS model for both in-sample and out-of-sample tests, which implies that GT improves the accuracy of stock market volatility forecasting. Second,

Table 6
Estimation results of two and three factors models (MF-GARCH-MIDAS).

Parameters	Two factors model			Three factors model			
	IP+PPI (S)	GDP+PPI (M1)	PPI+GDP (M2)	IP+PPI+GT (M1)	GT+IP+PPI (M2)	GDP+PPI+GT (M1)	GT+PPI+GDP (M2)
μ	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007
α	0.1801	0.2089	0.1832	0.0881	0.0900	0.0934	0.0820
β	0.7354	0.6864	0.7241	0.8911	0.8096	0.8886	0.9003
θ_1	−25.6232	−37.4556	−70.9590	−62.2052	0.0421	−35.8810	0.0423
θ_2	−40.9968	−101.8402	−13.8442	−13.0415	−39.0003	−17.6873	−27.6663
θ_3				0.0567	−16.2887	0.0169	−75.3184
ω_1^1	7.3914	94.8106	2.2161	9.6488	1.4617	18.1577	1.4312
ω_2^1	62.1724	40.1919	3.8022	16.0550	12.6903	9.0429	15.4094
ω_1^2	10.1161	1.5610	9.3554	12.3312	10.4730	1.8282	12.7022
ω_2^2	12.4365	2.3328	88.1916	29.0435	37.0112	6.8339	25.5015
ω_1^3				4.3117	12.2337	2.7530	8.6404
ω_2^3				29.8773	30.6479	2.6675	11.0817
α_0	−9.2512	−9.6120	−9.6891	−9.2774	−9.0658	−9.6860	−10.0011
LLF	12617.27	12514.19	13405.06	13638.46	13551.21	11190.57	13554.42

NOTE: (1) GT stands for the weekly GT downloaded from the Google; (2) 'S' denotes the standard GARCH-MIDAS model, 'M1' is the MF-GARCH-MIDAS model with τ_t defined in Eq. (9), and 'M2' is the MF-GARCH-MIDAS model with $\tau_{t+s/m}$ defined in Eq. (10).

Table 7
Model performance results (two factors and three factors).

Models	Types	Indicators	In-sample			Out-of-sample		
			MSE	MAE	MAPE	MSE	MAE	MAPE
Two factors model	S	IP+PPI	5.52	2.25	2.16	4.73	1.63	1.84
	M1	GDP+PPI	5.56	2.51	2.23	4.85	1.68	1.81
	M2	PPI+GDP	5.44	2.34	2.27	4.67	1.59	1.73
Three factors model	M1	IP+PPI+GT	5.29	2.16	2.10	4.32	1.58	1.77
	M2	GT+IP+PPI	5.28	2.05	2.06	4.17	1.54	1.64
	M1	GDP+PPI+GT	5.31	2.20	2.12	4.34	1.60	1.89
	M2	GT+PPI+GDP	5.14	1.98	1.56	4.06	1.51	1.56

NOTE: (1) GT stands for the weekly GT downloaded from the Google; (2) 'S' denotes the standard GARCH-MIDAS model, 'M1' is the MF-GARCH-MIDAS model with τ_t defined in Eq. (9), and 'M2' is the MF-GARCH-MIDAS model with $\tau_{t+s/m}$ defined in Eq. (10); (3) The measurement units of MSE, MAE, and MAPE are 10^{-4}, 10^{-2}, and 10^{-1}, respectively.

Table 8
Structural break test.

Breaks	Location	t-value
TB1	2007-12	0.0004
TB2	2010-08	0.0006

NOTE: 'TB' denotes the time of break.

M2 always performs better than M1 for both in-sample and out-of-sample periods. It shows that M2 could extract more information from data, i.e., the high frequency long-run component calculated from M2 contains more information than low frequency one from M1. Third, the MF-GARCH-MIDAS model is superior to those competing models in predicting volatility. As far as prediction error is concerned, it is not hard to find that the MF-GARCH-MIDAS model performs best, then followed by the WN's model, standard GARCH-MIDAS, GARCH-ANN, and GARCH models.

We also notice that all models have much smaller MSE, MAE, and MAPE for out-of-sample test. This may be due to the fact that in-sample and out-of-sample periods are in quite different market circumstances, which means that the volatility of DJIA are heterogeneous in magnitude for both in-sample and out-of-sample periods. Actually, from the top panel in Fig. 5, it is not hard to find the DJIA fluctuates largely during the in-sample period, while has small changes during the out-of-sample period. We will revisit this performance in the robustness check of Section 4.4.

4.3. Usefulness of GT from three factors model vs. two factors model

Now we consider two macroeconomic variables as the benchmark, and then introduce GT into the MF-GARCH-MIDAS model. Note that the combinations of two macroeconomic variables are: (1) IP+PPI, and (2) GDP+PPI, which indicates macroeconomic environment from two different perspectives. This ensures that we adopt new useful and relatively full information. We should note that this case requires variables observed at different frequencies, which cannot be handled by the three types of models mentioned above. So we give up the comparison with other types of models and only report the estimation results of MF-GARCH-MIDAS model in Table 6 and the performance results in Table 7.

Table 9
Summary statistics for the DJIA returns on three sub-periods.

Periods	Max	3rd Q	Mean	Median	1st Q	Min	S.D.	Skewness	Kurtosis
Period I	0.025	0.004	0.000	0.000	−0.003	−0.033	0.007	−0.330	1.602
Period II	0.105	0.007	0.000	0.000	−0.009	−0.082	0.018	0.113	5.483
Period III	0.042	0.004	0.000	0.001	−0.003	−0.057	0.009	−0.536	4.390

NOTE: Max stands for maximum, 3rd Q for the 3rd quartile, 1st Q for the 1st quartile, Min for minimum, and S.D. for standard deviation.

Table 10
Model comparison results in three sub-periods (one factor and two factors).

Models	Indicators	Period I						Period II						Period III					
		In-sample			Out-of-sample			In-sample			Out-of-sample			In-sample			Out-of-sample		
		MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE
GARCH(1,1)	–	5.36	2.67	2.21	5.42	2.69	2.28	6.26	3.84	3.11	6.55	3.79	3.18	5.82	2.87	2.51	5.82	2.99	2.88
GARCH(1,2)	–	5.22	2.55	2.26	5.32	2.58	2.31	6.14	3.85	3.16	6.45	3.88	3.21	5.84	2.85	2.56	5.84	2.98	2.81
GARCH(2,1)	–	5.42	2.62	2.15	5.51	2.55	2.35	6.33	3.87	3.05	6.61	3.85	3.15	5.76	2.72	2.65	5.81	2.85	2.95
GARCH(2,2)	–	5.50	2.57	2.20	5.48	2.47	2.29	6.36	3.83	3.10	6.48	3.84	3.19	5.79	2.77	2.50	5.88	2.84	2.89
GARCH-ANN	GT	5.31	2.47	2.11	5.45	2.49	2.24	6.21	3.88	2.73	6.32	3.84	2.61	5.53	2.68	2.53	5.71	2.79	2.54
	IP	5.22	2.35	2.17	5.31	2.43	2.31	6.16	3.75	2.70	6.29	3.73	2.71	5.54	2.55	2.48	5.59	2.63	2.51
	PPI	5.25	2.32	2.05	5.31	2.42	2.35	6.13	3.73	2.65	6.28	3.78	2.65	5.57	2.53	2.45	5.53	2.62	2.55
	GDP	5.40	2.47	2.10	5.42	2.47	2.29	6.20	3.82	2.58	6.31	3.87	2.69	5.51	2.52	2.50	5.47	2.67	2.59
WN's Model	GT	5.40	2.46	2.12	5.57	2.51	2.21	6.22	3.81	2.71	6.28	3.76	2.72	5.64	2.56	2.42	5.67	2.66	2.53
	IP	5.23	2.36	2.15	5.33	2.44	2.26	6.36	3.83	2.77	6.34	3.84	2.67	5.55	2.66	2.55	5.63	2.74	2.51
	PPI	5.26	2.39	2.21	5.39	2.46	2.35	6.29	3.79	2.66	6.27	3.75	2.66	5.52	2.69	2.51	5.69	2.68	2.53
	GDP	5.37	2.45	2.22	5.41	2.48	2.37	6.11	3.88	2.76	6.38	3.82	2.65	5.62	2.75	2.52	5.61	2.62	2.54
Standard GARCH-MIDAS(S)	GT	4.45	2.24	1.71	4.43	2.21	1.73	6.12	3.71	2.01	6.34	3.41	2.08	5.61	2.60	1.80	5.57	2.60	1.86
	IP	4.32	2.15	1.86	4.30	2.14	1.84	6.02	3.60	2.51	6.14	3.71	2.14	5.55	2.79	1.87	5.53	2.75	1.90
	PPI	4.42	2.22	1.85	4.39	2.20	1.82	6.21	3.62	2.24	6.36	3.64	2.17	5.56	2.89	1.85	5.50	2.75	1.86
	GDP	4.50	2.27	1.80	4.47	2.25	1.88	6.09	3.56	2.21	6.44	3.69	2.21	5.30	2.87	1.91	5.42	2.83	1.98
MF-GARCH-MIDAS(M1)	IP+GT	4.21	2.09	1.73	4.24	2.18	1.75	5.90	3.65	2.12	5.42	3.45	2.13	5.41	2.40	1.80	5.50	2.50	1.96
	PPI+GT	4.33	2.20	1.74	4.39	2.15	1.74	5.92	3.70	2.16	5.41	3.57	2.12	5.52	2.59	1.81	5.49	2.65	1.97
	GDP+GT	4.36	2.21	1.72	4.40	2.22	1.77	6.03	3.52	2.20	5.72	3.61	2.13	5.31	2.40	1.81	5.43	2.55	1.91
MF-GARCH-MIDAS(M2)	GT+IP	4.20	1.97	1.72	4.26	2.13	1.72	5.88	3.56	2.11	5.36	3.24	2.12	5.37	2.43	1.76	5.40	2.49	1.81
	GT+PPI	4.23	2.04	1.71	4.33	2.11	1.72	5.92	3.67	2.01	5.39	3.29	2.11	5.42	2.59	1.78	5.50	2.55	1.82
	GT+GDP	4.25	2.06	1.70	4.31	2.06	1.67	5.99	3.41	2.13	5.62	3.55	2.11	5.31	2.29	1.80	5.42	2.47	1.89

NOTE: (1) GT stands for the weekly GT downloaded from the Google; (2) 'S' denotes the standard GARCH-MIDAS model, 'M1' is the MF-GARCH-MIDAS model with τ_t defined in Eq. (9), and 'M2' is the MF-GARCH-MIDAS model with $\tau_{t+s/m}$ defined in Eq. (10); (3) The measurement units of MSE, MAE, and MAPE are 10^{-4}, 10^{-2}, and 10^{-1}, respectively.

Table 11
Model comparison results in three sub-periods (two factors and three factors).

Models	Types	Indicators	Period I						Period II						Period III					
			In-sample			Out-of-sample			In-sample			Out-of-sample			In-sample			Out-of-sample		
			MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE
Two factors model	S	IP+PPI	4.26	2.09	1.70	4.35	2.03	1.72	5.22	3.39	2.40	5.42	3.62	2.41	4.82	2.11	2.24	4.86	2.17	2.31
	M1	GDP+PPI	4.27	2.20	1.76	4.37	2.10	1.75	5.47	3.29	2.37	5.57	3.41	2.61	4.86	2.12	2.21	4.91	2.22	2.42
	M2	PPI+GDP	4.23	1.85	1.71	4.33	1.92	1.72	5.39	3.24	2.39	5.41	3.40	2.55	4.83	1.85	2.21	4.92	2.15	2.41
Three factors model	M1	IP+PPI+GT	4.12	1.82	1.54	4.30	1.84	1.65	5.21	3.52	2.26	5.32	3.50	2.64	4.90	2.22	1.75	4.99	2.06	1.45
	M2	GT+IP+PPI	4.05	1.86	1.52	4.24	1.83	1.63	4.86	3.28	2.17	4.99	3.44	2.62	4.76	1.90	1.69	4.96	1.91	1.68
	M1	GDP+PPI+GT	4.14	1.83	1.39	4.24	1.84	1.51	5.34	3.37	2.02	5.14	3.49	2.57	4.85	1.93	1.72	4.84	1.99	1.77
	M2	GT+PPI+GDP	4.02	1.45	1.38	4.19	1.43	1.50	4.66	3.22	2.00	4.86	3.39	2.57	4.70	1.85	1.63	4.69	1.88	1.75

NOTE: (1) GT stands for the weekly GT downloaded from the Google; (2) 'S' denotes the standard GARCH-MIDAS model, 'M1' is the MF-GARCH-MIDAS model with τ_t defined in Eq. (9), and 'M2' is the MF-GARCH-MIDAS model with $\tau_{t+s/m}$ defined in Eq. (10); (3) The measurement units of MSE, MAE, and MAPE are 10^{-4}, 10^{-2}, and 10^{-1}, respectively.

Table 12
Model comparison results in three sub-periods using the monthly and quarterly GT.

Models	Types	Indicators	Period I						Period II						Period III					
			In-sample			Out-of-sample			In-sample			Out-of-sample			In-sample			Out-of-sample		
			MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE
Two factors model	S	IP+GT_M	4.23	2.10	1.75	4.25	2.20	1.76	5.96	3.64	2.26	6.02	3.49	2.14	5.48	2.45	1.85	5.53	2.50	1.90
	S	PPI+GT_M	4.35	2.20	1.76	4.39	2.18	1.78	6.00	3.61	2.22	5.46	3.57	2.13	5.52	2.62	1.83	5.50	2.66	1.80
	S	GDP+GT_Q	4.49	2.25	1.75	4.47	2.25	1.81	6.06	3.54	2.21	6.34	3.68	2.21	5.25	2.86	1.88	5.40	2.83	1.91
	M1	GDP+GT_M	4.37	2.21	1.75	4.44	2.24	1.78	6.04	3.52	2.21	6.24	3.62	2.15	5.23	2.41	1.88	5.33	2.67	1.93
	M2	GT_M+GDP	4.27	2.11	1.72	4.38	2.15	1.65	6.01	3.49	2.19	5.97	3.59	2.12	5.21	2.40	1.87	5.34	2.67	1.92
	M1	GT_Q+IP	4.35	2.24	1.86	4.32	2.37	1.83	6.00	3.81	2.50	6.11	3.69	2.26	5.52	2.69	1.86	5.60	2.61	2.03
	M2	IP+GT_Q	4.28	2.13	1.81	4.26	2.22	1.81	5.98	3.63	2.41	6.07	3.60	2.14	5.45	2.63	1.86	5.52	2.53	1.97
	M1	GT_Q +PPI	4.42	2.26	1.87	4.41	2.22	1.80	6.12	3.74	2.28	6.13	3.67	2.20	5.61	2.88	1.87	5.57	2.71	1.86
	M2	PPI+GT_Q	4.40	2.22	1.85	4.39	2.19	1.79	6.08	3.69	2.23	6.06	3.61	2.15	5.56	2.86	1.84	5.50	2.66	1.85
Three factors model	S	IP+PPI+GT_M	4.24	2.03	1.66	4.33	2.00	1.69	5.22	3.40	2.37	5.40	3.53	2.49	4.80	2.09	2.13	4.77	2.08	2.15
	M1	GDP+PPI+GT_M	4.20	2.02	1.51	4.35	2.10	1.67	5.44	3.32	2.24	5.28	3.39	2.58	4.87	2.06	2.13	4.90	2.17	2.24
	M2	PPI+GT_M+GDP	4.14	1.76	1.47	4.30	2.04	1.62	5.33	3.21	2.16	5.15	3.23	2.36	4.68	1.89	2.04	4.83	2.10	2.12
	M1	GT_Q+IP+PPI	4.26	2.07	1.67	4.35	2.10	1.74	5.31	3.44	2.41	5.41	3.60	2.41	4.85	2.11	2.23	4.80	2.15	2.19
	M2	IP+PPI+GT_Q	4.25	2.11	1.66	4.33	2.03	1.71	5.30	3.31	2.38	5.40	3.60	2.46	4.87	2.11	2.24	4.79	2.10	2.24
	M1	GT_Q+GDP+PPI	4.26	2.30	1.71	4.38	2.12	1.71	5.46	3.41	2.28	5.40	3.41	2.60	4.85	1.99	2.20	4.90	2.18	2.30
	M2	PPI+GT_Q+GDP	4.25	2.30	1.70	4.36	2.09	1.70	5.44	3.40	2.28	5.37	3.39	2.58	4.82	1.98	2.18	4.87	2.17	2.28

NOTE: (1) GT_M stands for the monthly GT downloaded from the Google, while GT_Q for the quarterly GT calculated by averaging GT_M's in the corresponding period; (2) 'S' denotes the GARCH-MIDAS model with the same frequency, 'M1' is the MF-GARCH-MIDAS model with τ_t defined in Eq. (9), and 'M2' is the MF-GARCH-MIDAS model with $\tau_{t+s/m}$ defined in Eq. (10); (3) The measurement units of MSE, MAE, and MAPE are 10^{-4}, 10^{-2}, and 10^{-1}, respectively.

In addition to several conclusions similar to Section 4.2, we now present several new findings.

First, adding GT into the MF-GARCH-MIDAS model can always better its performance of volatility forecasting. We therefore argue that GT can enhance model performance no matter what macroeconomic information is used. This implies that GT is not a kind of supplements of macroeconomic but a necessary information source for stock market volatility forecasting. In other words, the result confirms that event impact is also a source of volatility besides macroeconomic fundamentals.

Second, GT+PPI+GDP in M2 performs best in predicting volatility of DJIA in terms of MSE, MAE and MAPE. It outperforms all other combinations and models (S and M1) for both in-sample and out-of-sample tests. In addition, it is also better than all single factor models. We therefore conclude that single factor model with GT is not enough to predict stock market volatility and so does model with only macroeconomic variables. Actually, both GT and macroeconomic variables provide information from different aspects of the stock market. This result confirms that event impact and macroeconomic variables affect stock volatility in an essential and independent way.

4.4. Robustness checks

It is well known that structural break is an important phenomenon in financial time series. This motivates us to test the performance of MF-GARCH-MIDAS by considering structural breaks. In addition, GT can be observed at different frequencies, for instance weekly, monthly, or quarterly. We also illustrate the MF-GARCH-MIDAS model is robust to different degree of frequency for GT.

4.4.1. Results for structural breaks

In volatility modeling, we should consider structural breaks, see [49–52]. To this end, we adopt the method of [52] to test structural breaks and extract break dates. The test results in Table 8 show that there has two break points located in TB1 and TB2. We therefore partition the full data into three periods, and define 2004-01 to 2007-12 as period I, 2007-12 to 2010-08 as period II, 2010-08 to 2018-07 as period III. From Fig. 5, we find that stock returns fluctuate largely in period II than the other two periods, which is also confirmed by the summary statistics in Table 9. It is obvious that period II has larger standard deviation and kurtosis. In addition, the positive skewness of period II implies that a greater chance of extremely positive outcomes. To sum up, we denote the status of period II as 'unstable', while period I and period III as 'stable'.

Similar to Sections 4.2 and 4.3, we repeat the whole calculation procedure. However, we consider in this section three sub-samples or periods rather than the entire sample. In each period, there is no structural break.

To save space, we do not report model estimation results, while present performance results of each period in Tables 10 and 11. We find that the previous conclusions still hold. For example, all models perform better when incorporating GT. This proves once again the usefulness of GT in predicting volatility. The MF-GARCH-MIDAS model is optimal in each period, which also illustrates its robustness. In addition, we have made several new findings.

First, in each period without break points, the in-sample performance is improved and close to the out-of-sample performance for all models. Taken together, the superiority performance of out-of-sample over in-sample in Sections 4.2 and 4.3 is due to the data instead of the model.

Second, the prediction error in period II is larger than those in period I and period III. The main reason is that the stock market is more volatile in period II, see also Table 9.

4.4.2. Results for different degree of frequency

To further illustrate the robustness of the MF-GARCH-MIDAS model, we consider different degree of frequency for GT. Specifically, we repeat the whole calculation using the monthly and quarterly GT in addition to the weekly GT, and present the comparison results in Table 12. Comparing Table 12 to Tables 10 and 11, we find that the main conclusions are still hold for the frequency change. First, the MF-GARCH-MIDAS model with GT is superior to the single factor GARCH-MIDAS model and conventional models, such as the WN's model, the GARCH-ANN model, and GARCH models. Second, M2 always performs better than M1 for both in-sample and out-of-sample tests. Third, GT is very useful in predicting stock market volatility in that it improves the MF-GARCH-MIDAS model in volatility forecasting. In addition, some new empirical findings on different degree of frequency for GT appear.

First, the weekly GT outperforms both the monthly and quarterly GT in predicting volatility of DJIA. The weekly GT provides more useful information since it has a higher frequency. It can quickly capture investor attention on the changes of the stock market and reflect the details of the changes. As a result, the weekly GT performs best in predicting volatility of DJIA, followed by the monthly GT and the quarterly GT. Second, the quarterly GT also contributes to volatility forecasting, but only to a limited extent. The possible reason is that the quarterly GT is observed at the same frequency as macroeconomic fundamentals such as GDP. At this time, the information of stock market volatility reflected by the quarterly GT is also recorded by low frequency macroeconomic fundamentals. Therefore, the quarterly GT has limited ability to improve volatility forecasting on DJIA.

4.5. Further research

Besides, we are going to figure out the usefulness of Google trends (GT) under different market environments. In this section, we want to know how much of expected volatility can be explained by GT. To quantify this contribution, we compute the ratio

$$CR(GT|GT+PPI+GDP) = \frac{Var(\log(\tau))}{Var(\log(\tau)\times g)} \times \frac{Var(\log(GT))}{Var(\log(\tau))}. \quad (24)$$

where $Var(\cdot)$ denote the variance operator.

We are now able to calculate the contribution ratio (CR) under different market environments to test whether the influence of GT is heterogeneous across different market circumstance. To this end, we divide the full sample period into three sub-periods based on the structural break test. In addition, we consider different degree of frequency for GT and present in Table 13 the CR of GT observed at a weekly, monthly, or quarterly frequency.

From Table 13, it can be seen that the weekly GT contributes larger than the monthly and quarterly GT in predicting volatility of DJIA. During the unstable period, the contribution of the weekly GT to volatility forecasting approached to 50%, while the contribution of the quarterly GT is less than 20%. We therefore recommend using the weekly GT for volatility forecasting in practice. In addition, the contribution of GT increases with the stock market volatility, i.e., period II > period III > period I. When market fluctuates and its uncertainty raises, people tends to search the internet for relative information driven by an event impact on stock market. In this case, the relationship between internet search and stock volatility becomes so close that GT is helpful to improve volatility forecasts. On the contrary, stock volatility mainly depends macroeconomic fundamentals and the forecast power of GT declines, no matter

Table 13
Contribution ratio of Google trends (%).

Periods	Time intervals	Status	Weekly GT		Monthly GT		Quarterly GT	
			M1	M2	M1	M2	M1	M2
Period I	2004-01–2007-12	Stable	5.92	6.12	4.80	4.99	4.12	4.10
Period II	2007-12–2010-08	Unstable	42.64	49.05	35.43	40.17	16.17	17.41
Period III	2010-08–2018-07	Stable	30.01	35.87	22.71	26.45	12.45	11.69

NOTE: The values are the average contribution of Google trends during each period.

which model we use. Based on these results, we are now able to conclude that: (1) GT with a higher observed frequency tends to be more useful in volatility forecasting; (2) the more volatile the stock market, the more useful the GT; (3) in a stable market environment, the market volatility is relatively easier to forecast.

5. Conclusion

This paper develops the novel MF-GARCH-MIADS model to investigate the usefulness of Google search index in stock market volatility forecasting. The proposed MF-GARCH-MIADS model allows for a long-run stock market volatility being driven by multiple factors sampled at different frequencies, which has not yet been involved in the standard GARCH-MIADS model.

In empirical analysis on predicting daily volatility of DJIA, we consider the impacts of weekly Google trends (GT) and three macroeconomic variables: quarterly GDP, monthly PPI and monthly IP of US. We find that Google trends contains useful information for stock market volatility forecasting and its contribution can be enhanced by combining with other macroeconomic variables. More specifically, the combination GT+PPI+GDP in M2 performs best in predicting volatility of DJIA. To illustrate the efficacy of the MF-GARCH-MIDAS model, we conduct robustness checks on structural breaks and different degree of frequency for GT. The numerical results are robust to show that the MF-GARCH-MIDAS model really works and the weekly GT outperforms both the monthly and quarterly GT in predicting volatility of DJIA. In general, GT with a higher observed frequency tends to be more useful in volatility forecasting. In addition, the contribution of Google trends on stock volatility prediction varies on different stock market environments. The higher the stock market volatility is, the more GT accounts for the volatility of DJIA. Thus, we conclude that: (1) event impact is an important source of stock market volatility; (2) event impact is different from macroeconomic fundamentals in that it can investigate sudden fluctuations; (3) the influence of event impact varies among different market environments, in particular, the more volatile the market, the more useful the GT.

Acknowledgments

The authors are grateful to the Editor-in-Chief and four anonymous referees for their helpful comments and constructive guidance. This work was supported by the National Natural Science Foundation of PR China (71671056, 71490725) and the National Social Science Foundation of PR China (15BJY008).

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