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VOLATILITY PERSISTENCE AND STOCK VALUATIONS: SOME EMPIRICAL EVIDENCE USING GARCH

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SUMMARY

In this paper issues of volatility persistence and the changing risk premium in the stock market are investigated using the GARCH estimation technique. We get a point estimate of the index of relative risk aversion of 4.5 and confirm the existence of changing equity premiums in the US during 1962–1985. The persistence of shocks to the stock return volatility is so high that the data cannot reject a non-stationary volatility process specification. The results of this paper are consistent with Malkiel and Pindyck's hypothesis that it is the unforseen rise in the investment uncertainty during 1974 that causes the market to plunge.

1. INTRODUCTION

The temporal behaviour of the stock market volatility has received increasing interest recently. The upward trend in volatility during the past three decades has been contended by Malkiel (1979) and Pindyck (1984) to be the major reason for the decline in equity prices. Pindyck argues that 'the variance of the firms' real gross marginal return on capital has increased significantly since 1965, that this has increased the relative riskiness of investors' net real returns from holding stocks, and that this in turn can explain a large fraction of the market decline'. Their explanation is different from that of others who attribute the increasing inflation rate as the cause of the market decline, e.g. Modigliani and Cohn (1979), Fama (1981), Feldstein (1980) and Summers (1981). On the other hand, their argument is in line with Black's (1976) finding that stock returns tend to be strongly negatively correlated with changes in volatility.

Poterba and Summers (1986) (henceforth P-S) examine this issue by testing the time-series property of volatility. It is argued that shocks to volatility have to persist for a very long time in order for volatility to have significant impact on the stock prices. If shocks to volatility are only transistory, no adjustment of the future discount rate will be made by the market. Thus, expected stock returns are not affected by the volatility movement. Hence by empirically rejecting the persistence property using some volatility measures they refute the above claims of Malkiel and Pindyck. The volatility measures they use are monthly averages of daily stock sample variances, and the implied variance of a CBOE index, an *ex ante* measure. Unit root tests are performed and the existence of unit roots is rejected for both cases. They estimate the serial correlation in the volatility measure, and conclude that shocks to the stock volatility are very short-lived. As a result, using the fundamental valuation formula they obtain a very small estimate for the elasticity of the stock price to volatility shocks.

French, Schwert and Stambaugh (1987) (henceforth F-S-S) report the necessity of first-order differencing in order to obtain stationarity, although no explicit tests of integration are made.

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A series of anticipated volatility is obtained from the fitted value of the ARIMA process and it is found to be weakly related to the expected risk premium. On the other hand, strong evidence is found that 'unexpected stock market returns are negatively related to the unexpected volatility of stock returns'.

Pindyck (1986) rejoins this issue by estimating a portfolio choice model. He obtains estimates of the index of relative risk aversion in the range of 3 to 4. While admitting that changes in variance do not persist long, he still claims that 'they do seem to explain more than other variables (e.g. changes in corporate profits and changes in the real interest rate), because they have been large in magnitude and because the index of risk aversion is large'. He reports that about one-third of the 1974 market decline can be attributed to volatility changes.

In this paper a recently developed time-series technique called autoregressive conditional heteroscedasticity (henceforth ARCH) is adopted to study the temporal behaviour of the stock return volatility. The ARCH model, introduced by Engle (1982), formulates time-varying conditional variances in time-series data. It proves to be an effective tool in modelling temporal behaviour of many economic variables, especially financial market data. For a survey of its applications see Engle and Bollerslev (1986).

Bollerslev (1986) extends the ARCH model to GARCH, or the generalized ARCH. The GARCH model provides a more flexible framework to capture various dynamic structures of conditional variance. Another important extension of ARCH is the 'ARCH in mean' or 'ARCH-M' model, which links the conditional variance with the mean. Engle, Lilien and Robins (1987) use this model and find strong evidence of this link between risk and return in the term structure of interest rates. Under an assumption of conditional t-distribution, Bollerslev (1987) finds the conditional standard deviations help to explain variations in the expected return of the S&P500 index. Bollerslev, Engle and Wooldridge (1988) also report significant time-varying risk premiums in a multivariate GARCH-M model.

We use a univariate GARCH-M model to study the issue of volatility persistence and its relationship with market fluctuations. Following P-S, the fundamental valuation formula is employed to evaluate the impact of volatility on the stock price. Like Pindyck (1986), we also estimate the index of relative risk aversion in a maximum-likelihood estimation framework. This parameter estimate and that determining volatility persistence are used to evaluate the impact of a changing volatility.

The main difference between this paper and others is on the estimation methodology. The estimation procedure of P-S and others (Pindyck and F-S-S) is two-stage OLS. Two-stage procedures are less efficient than maximum-likelihood methods. Further, as noted in Pagan and Ullah (1988), P-S's estimator is not even consistent. We claim and demonstrate that the GARCH-M model is a more suitable tool than the two-stage method and gives much more reliable results.

The paper is organized in the following way. Section 2 presents the economic framework of equity valuation given by P-S and establishes the GARCH-M model for estimation. Data and the estimation result are presented in section 3. In section 4 a comparison is made between P-S's methodology and ours, and in section 5 the possibility of a non-stationary variance is investigated. The last section concludes with suggested future research directions.

2. STOCK PRICE, RISK PREMIUM AND VOLATILITY PERSISTENCE

The theoretical framework of this paper follows that of P-S. The stock price is assumed to satisfy the standard valuation formula for risky assets. The expected rate of return from time t to $t+1$ of the market portfolio with dividends reinvested, R_{t+1}, is assumed to be the sum of two portions, the risk-free rate and a risk premium,

$$E_t R_{t+1} = \frac{E_t(S_{t+1}) - S_t + D_t}{S_t} = r_{f,t} + p_t, \tag{1}$$

where S_t, D_t, $r_{f,t}$, p_t are respectively the stock price, dividend paid, nominal risk-free rate, and the risk premium at period t. S_t can be solved by recursive substituion to yield a representation of the stock price.

$$S_t = E_t \sum_{i=0}^{\infty} \frac{D_{t+i}}{\prod_{s=0}^{i} (1 + r_{f,t+s} + p_{t+s})}. \tag{2}$$

Equation (2) is the familiar fundamental valuation formula that says the stock price is the expected present discount value of future dividend streams.

It is obvious from this equation that the stock price responds positively to the future dividend payment and negatively to changes in the risk-free rate and the risk premium. By assuming a constant discount factor, Shiller (1981) derives a variance bound test and concludes that the variation due to dividends is too small to be consistent with the huge variance of the stock prices. The goal of this paper, however, is to relax the restriction of the constant discount factor and study the effect of stochastically changing risk (volatility), hence a changing risk premium, on the stock price. To quantify this effect, further assumptions are necessary.

Following P-S, the expected dividends are assumed to grow at a constant rate g and the risk-free rate is assumed constant over time.* Linearizing equation (2) around the mean value of the risk premiums, $\bar{p}$, gives

$$S_t = \sum_{i=0}^{\infty} \frac{(1+g)^i D_t}{(1 + r_f + \bar{p})^{i+1}} + \sum_{s=0}^{\infty} \frac{\partial S_t}{\partial p_{t+s}} E_t(p_{t+s} - \bar{p}), \tag{3}$$

where

$$\frac{\partial S_t}{\partial p_{t+s}} = - \frac{D_t(1+g)^s}{(1 + r_f + \bar{p})^{s+1}(r_f + \bar{p} - g)}. \tag{4}$$

Next, the link between equity premium and return volatility needs to be established. With fairly general conditions Merton (1973, 1980) gives an intertemporal CAPM model which implies a linear relationship between the equity premium and the market return variance, i.e.

$$p_t = \delta V_t, \tag{5}$$

where V_t is the instantaneous variance of the market return, δ is the harmonic mean of individual investors' Pratt–Arrow measures of relative risk aversion. In the following discrete time model, V_t denotes the conditional variance of return R_{t+1} given information up to time t, i.e. $V_t = \mathrm{Var}(R_{t+1} \mid I_t)$.†

It will soon be clear that the dynamic specification for the volatility series V_t is crucial to our result. An AR(1) specification has been adopted by P-S and Pindyck (1986). In section 4 we argue that this specification and the two-stage estimation method lead to very unreliable results. Instead, a GARCH(1,1) process is assumed for the evolution of V_t,

$$V_t = \alpha_0 + \alpha e_t^2 + \beta V_{t-1}, \tag{6}$$

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* These are simplifying assumptions. According to Pindyck's (1986) result, variations in these two series do not explain much of the stock price fluctuations.

† This notation is different from most papers using ARCH where the time index for conditional variances is $t + 1$ rather than t. The change is made here to cope with the notation used by P-S.

where e_t is the forecasting error for the stock return, i.e.

$$e_t = R_t - E_{t-1}R_t.$$

Namely, the investors adjust their expectations of future variances according to past errors they made in forecasting the market returns.

By recursive substitution and the law of iterated expectations, it can be derived from equation (6) that

$$E_t(V_{t+s} - \sigma^2) = (\alpha + \beta)^s (V_t - \sigma^2), \tag{7}$$

where σ^2 is the unconditional variance of R_t given by

$$\sigma^2 = \frac{\alpha_0}{1 - (\alpha + \beta)}, \quad \text{for } (\alpha + \beta) < 1. \tag{8}$$

Namely, the GARCH(1,1) specification has a convenient property that shocks to volatility decay at a constant rate and the speed of the decay is measured by the estimate of $\alpha + \beta$. This property is shared by the AR(1) process, in which the speed of memory decay is measured by the serial correlation coefficient. In fact, it is the simplicity of this property which causes the adoption of the AR(1) specification by P-S and Pindyck (1986).

It follows from equation (7) that shocks to the current volatility remain important for long periods into the future if the parameter $\alpha + \beta$ is close to one. If unity is the true value of this parameter then shocks to volatility persist forever, and the unconditional variance is not determined by the model. Engle and Bollerslev (1986) call this type of process 'Integrated-GARCH'. In this paper we first restrict our disucssion to the stationary case, i.e. when the persistence parameter $\alpha + \beta$ is less than one. In section 5, tests and properties of the integrated-GARCH model are pursued further.

From equation (5) the mean value of the risk premium, is given by $\bar{p} = \delta\sigma^2$; and from equation (7) the expected value for future premiums follows

$$E_t(p_{t+s} - \bar{p}) = (\alpha + \beta)^s (p_t - \bar{p}). \tag{9}$$

Equation (3) can therefore be simplified to:

$$\begin{aligned} S_t &= \sum_{i=0}^{\infty} \frac{(1+g)^i D_t}{(1 + r_f + \bar{p})^{i+1}} - \sum_{i=0}^{\infty} \frac{D_t (1+g)^i (\alpha+\beta)^i (p_t - \bar{p})}{(1 + r_f + \bar{p})^{i+1} (r_f + \bar{p} - g)} \\ &= \frac{D_t}{r_f + \bar{p} - g} - \frac{1}{1 + r_f + \bar{p} - (\alpha+\beta)(1+g)} \times \frac{D_t}{r_f + \bar{p} - g} (p_t - \bar{p}) \\ &= \bar{S} + \frac{dS_t}{dp_t} (p_t - \bar{p}). \end{aligned} \tag{10}$$

The last expression can be used together with the chain rule to establish

$$\frac{dS_t}{dV_t} = -\frac{\delta}{[1 + r_f + \bar{p} - (\alpha+\beta)(1+g)]} \times \frac{D_t}{[r_f + \bar{p} - g]}. \tag{11}$$

Multiplying both sides by V_t/S_t gives the elasticity of stock price to shocks in volatility.

$$\frac{d \log S_t}{d \log V_t} = -\frac{\delta V_t}{1 + r_f + \bar{p} - (\alpha+\beta)(1+g)}. \tag{12}$$

Equation (12) is identical to the formula given by P-S (equation (14)) except for the persistence parameter. In their model the parameter is the coefficient of the first-order serial correlation of the volatility series. To evaluate the elasticity of equation (12), the parameters δ and $\alpha + \beta$ need to be estimated. We proceed by estimation of the GARCH(1,1)-M model given by combining equations (1), (5) and (6),

$$\begin{aligned} R_{t+1} &= r_f + \delta V_t + e_{t+1}, \\ V_t &= \alpha_0 + \alpha e_t^2 + \beta V_{t-1}, \end{aligned} \tag{13}$$

with

$$e_{t+1} \mid I_t \sim N(0, V_t),$$

where I_t denotes the publicly available information set at date t. The advantage of this model is that the parameters of interest can be estimated simultaneously. In particular, the index of relative risk aversion, δ, is obtained as well as the persistence parameter $\alpha + \beta$ by a maximum-likelihood technique. The appendix provides a discussion of the general GARCH(p,q) and GARCH-M models together with the estimation and test procedures.

3. DATA AND ESTIMATION RESULTS

The data used are weekly returns of the NYSE value-weighted index with dividends reinvested from CRSP (Center for Research in Security Prices). Observations are constructed by taking

Figure 1. NYSE stock index

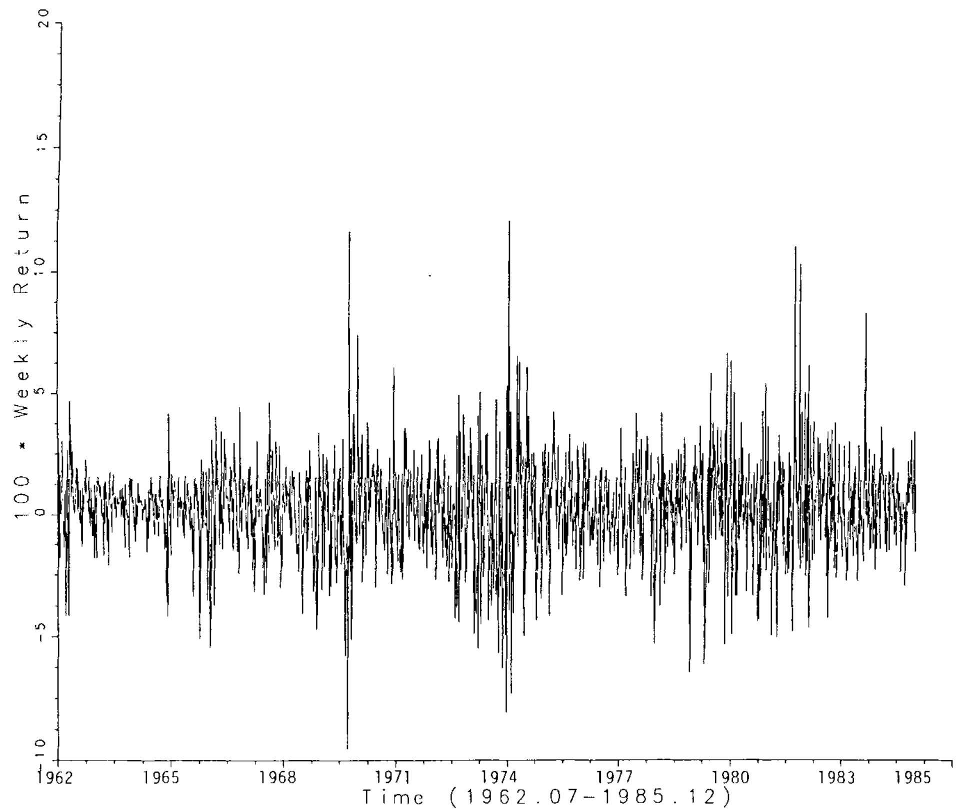

Figure 2. NYSE stock return

the difference of the logarithms of every consecutive Tuesday's closing price. For holidays, the closing prices of the trading days following Tuesday are taken. Weekly data are chosen to avoid the serial correlation problem existing in the daily data. Tuesday's prices are used to avoid complications due to the weekend effect. The sample period starts from July 1962 through December 1985 and totally 1225 observations are available.

Figures 1 and 2 give plots of the stock level and the weekly return series. The index level shows a serious plunge during the mid-1970s which is the focus of this paper. During 1974 the NYSE market index dropped from 208·6 to 152·8, giving a 27 per cent decline. If measured in real terms the decline is even more serious. In the return series the clustering of fluctuations is apparent: large changes tend to be followed by large changes, and small by small, of either sign. This is a typical phenomenon for many financial data sets. The ARCH model explicitly deals with this effect by using squared past forecasting errors to predict future variances.

Tables I and II summarize the estimation and test results. We first discuss estimation results for the full sample period. The nominal risk-free rate is estimated to be 0·122 per cent per week or 6·55 per cent annually, which is very close to the average 1-month treasury bill rate of 6·37 per cent. The value of the index of risk aversion is important in determining the size of the elasticity. The estimate we obtain is 4·5, which is marginally significant at the 5 per cent level. This value is within the reasonable range of 2 to 6 indicated by Pindyck (1986). The estimation using excess returns greatly increases the precision of this parameter and gives a t-ratio of 3·28.

The parameter estimates of the variance equation imply a very high persistence for shocks

Table I. Full and sub-sample estimation of the GARCH(1,1)-M model using weekly returns of NYSE value weighted index

$$R_{t+1} = r_f + \delta V_t + e_{t+1}$$
$$V_t = \alpha_0 + \alpha e_t^2 + \beta V_{t-1}$$
$$\text{with } e_{t+1} \mid I_t \sim N(0, V_t),$$

Sample period	$r_f \times 10^2$	$\alpha_0 \times 10^4$	α	β	δ	$\alpha+\beta$	$\sigma^2 \times 10^4$ [a]
July 1962–December 1985	0·122	0·096	0·151	0·835	4·50	0·986	4·074
No. obs. = 1225	(1·49) [b]	(2·83)	(7·17)	(38·85)	(1·94)		
July 1962–December 1985	—	0·099	0·151	0·834	4·56	0·985	4·090
No. obs. = 1225		(2·83)	(7·15)	(38·64)	(3·28)		
July 1962–December 1973	0·159	0·092	0·217	0·778	5·05	0·995	3·574
No. obs. = 599	(1·85)	(2·18)	(6·12)	(24·13)	(1·78)		
January 1974–December 1985	0·008	0·159	0·084	0·882	6·15	0·966	4·567
No. obs. = 626	(0·04)	(2·03)	(3·14)	(23·67)	(1·32)		

[a] The estimate is computed using the residuals of the last step iteration. This estimate is more stable than that given by equation (8) if $\alpha+\beta$ is close to one.
[b] Numbers in parentheses are t-ratios.
[c] Weekly excess returns are used for estimation hence no constant terms are necessary.

Table II. Auto-correlation for residuals and normalized squared residuals of the GARCH(1,1)-M model (1962.07–1985.12)

$$R_{t+1} = r_f + \delta V_t + e_{t+1}$$
$$V_t = \alpha_0 + \alpha e_t^2 + \beta V_{t-1}$$
$$\text{with } e_{t+1} \mid I_t \sim N(0, V_t),$$

lag	1	2	3	4	5	6	7	8	9	10
acf[a] of e_t	0·050[b]	−0·024	0·046	0·026	−0·043	0·048	0·025	−0·056	−0·007	−0·003
acf of u_t^2 [c]	0·012	0·040	−0·047	0·004	−0·011	0·006	−0·021	0·005	−0·022	−0·032
lag	11	12	13	14	15	16	17	18	19	20
acf of e_t	−0·005	0·019	0·028	0·012	−0·023	0·014	0·023	0·006	0·004	0·056
acf of u_t^2	0·008	−0·023	−0·005	0·033	0·001	−0·040	0·003	−0·036	−0·032	−0·015
lag	21	22	23	23	25	26				
acf of e_t	0·018	0·042	0·044	0·012	−0·018	0·008				
acf of u_t^2	−0·001	0·022	0·018	0·000	−0·035	0·028				

[a] acf: auto-correlation function.

[b] $2(\text{std. error of acf}) = \dfrac{2}{\sqrt{\text{no. observations}}} = \dfrac{2}{\sqrt{1225}} = 0{\cdot}057.$

[c] $u_t^2 = \dfrac{e_t^2}{V_t}$: normalized squared residuals.

[d] The Box–Pierce Q statistics for e_t and u_t^2 are respectively 29·62 and 18·12, which are both less than the 5 per cent critical value of 38·90.

[e] Lagrange multiplier test statistics for GARCH(1,2) and GARCH(2,1) are respectively 0·074 and 0·355, while the 5 per cent critical value is 3·84.

to the variance. The response function of volatility of shocks decays at a very slow rate, measured by $\alpha + \beta$ or 0·986 weekly. The proportion of shocks remains at $0{\cdot}986^{4{\cdot}3}$ or 0·941 after a month, and $0{\cdot}986^{52}$ or 0·480 after a year. Namely, even 1 year after the shock occurs, about half of the initial impact remains in effect.

The subsample estimation results are similar but with some parameter changes. The diagnostic test results seem to strongly support the specification of the GARCH(1,1)-M model. The autocorrelation functions (see Table II) of the residual and the modified residual square indicate no departure from the null of no correlations. The Box–Pierce Q statistics are significantly less than the 5 per cent critical value. Lagrange multiplier tests of GARCH(1,2) and GARCH(2,1) are insignificant, thus favouring the GARCH(1,1) specification. In general, although there are some indications of shifting structures within the sample period, the GARCH(1,1)-M model seems to be supported by the data.

Figure 3 gives a plot of the estimated weekly conditional variance series. The two spikes correspond to periods of the 1970 credit crunch and the 1974 recession. The plot confirms our previous conclusion from the parameter estimates that the volatility series can deviate from its long-term mean for long periods of time. In fact, a closer examination may raise doubts about the existence of a stationary mean. This question will be further pursued in section 5.

We proceed to use our parameter estimates to evaluate the elasticity of the stock price to volatility expressed in equation (12). Estimates of parameters from equation (13) for the full sample period are used for this evaluation. The expected growth rate of the dividends g is taken from P-S. All parameters are converted to monthly measures. The elasticity at the mean value

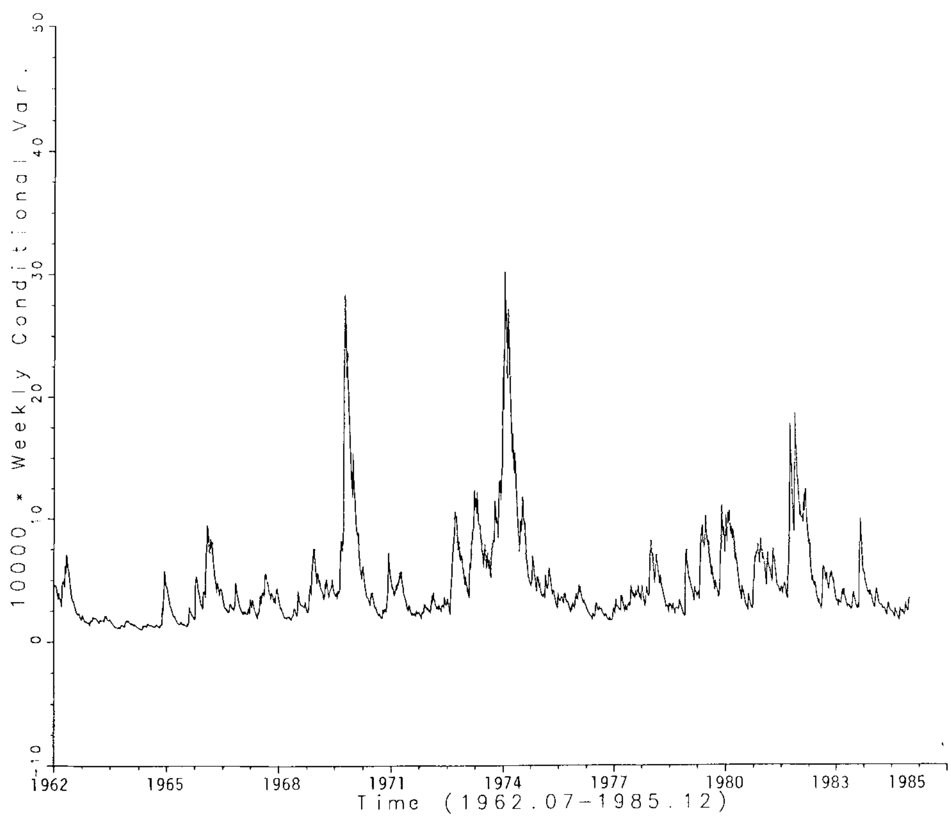

Figure 3. NYSE stock variance

of the volatility level is estimated to be $-0 \cdot 11$ (let $V_t = \sigma^2$ and evaluate equation (12)). Namely, on average a doubling of the stock volatility reduces the stock value by about 11 per cent. Based on our estimate, the average weekly volatility for the year 1974 is $2 \cdot 35$ times the unconditional variance. If the elasticity is evaluated at the average volatility level during 1974 the magnitude rises to 26 per cent. Historically, the volatility level essentially doubles from 1973 to 1974, hence our estimate predicts a drop of 26 per cent in the stock price, which is roughly the magnitude of the market decline during 1974.

P-S report an estimate of $3 \cdot 5$ for the index of risk aversion, which does not deviate much from ours. The sharp contrast of the degree of influence of volatility on stock prices is largely due to the estimate of the persistence parameter. It will be shown that our GARCH-M estimation method provides a more stable and hence more reliable estimate of this key parameter than P-S's two-stage method.

4. COMPARISON OF AR(1)-TWO-STAGE AND GARCH(1,1)-MLE METHODS

In this section we compare our method using GARCH with the two-stage method used by P-S and many authors. In particular, robustness of the persistence parameter estimate with respect to changes in the volatility measure is examined. Since volatility is not an observable variable, different approaches are taken by these two methods for measurement and to model its evolution. The GARCH method uses the conditional variance as the measure of volatility and provides a flexible structure for its dynamic property. Since the maximum-likelihood method is used for estimation, efficiency and consistency are both obtained. The two-stage method approaches the problem first by measuring the unobservable volatility series with average sample variances within given horizons. The most commonly adopted volatility measure is the monthly volatility.* The second step is to fit a Box–Jenkins model to the constructed volatility measure. An AR(1) specification is adopted by P-S and Pindyck (1986) and an IMA(1,3) process is used by F-S-S for the evolution of volatility.† The serial correlation coefficient in an AR(1) process characterizes the persistence of volatility shocks exactly as the parameter $\alpha + \beta$ does in a GARCH(1,1) specification.

While the two-stage procedure may seem plausible there are problems to be noticed. First, two-stage methods are typically less efficient than maximum-likelihood methods in estimation. Secondly, implicitly assumed by this methodology is that the variance remains constant within a given horizon (e.g. a month) and becomes variable for longer horizons. This assumption is itself not justifiable no matter what horizons are chosen. Thirdly, as will be shown in this section, the estimate of the persistence parameter out of this method is very unstable, particularly it is very sensitive to the choice of the horizon measuring volatility. This parameter is crucial in determining the magnitude of the elasticity of the stock price to volatility shocks. We will compare the estimates of this parameter for both methods using different sampling frequencies in measuring volatility.

We first estimate the GARCH(1,1)-M model with different temporal aggregates of the daily returns of the NYSE stock index. A N-trading day return to date t, denoted R_t^N is obtained by

$$R_t^N = \sum_{s=1}^{N} R_{t,s}$$

* For example, Merton (1980), P-S, F-S-S and Pindyck (1986) all use the monthly measure of volatility. In Pindyck's work the annual measure of volatility is also employed.

† Other high-order autoregressive specifications are also adopted by P-S but AR(1) is the specification used mainly for their inference.

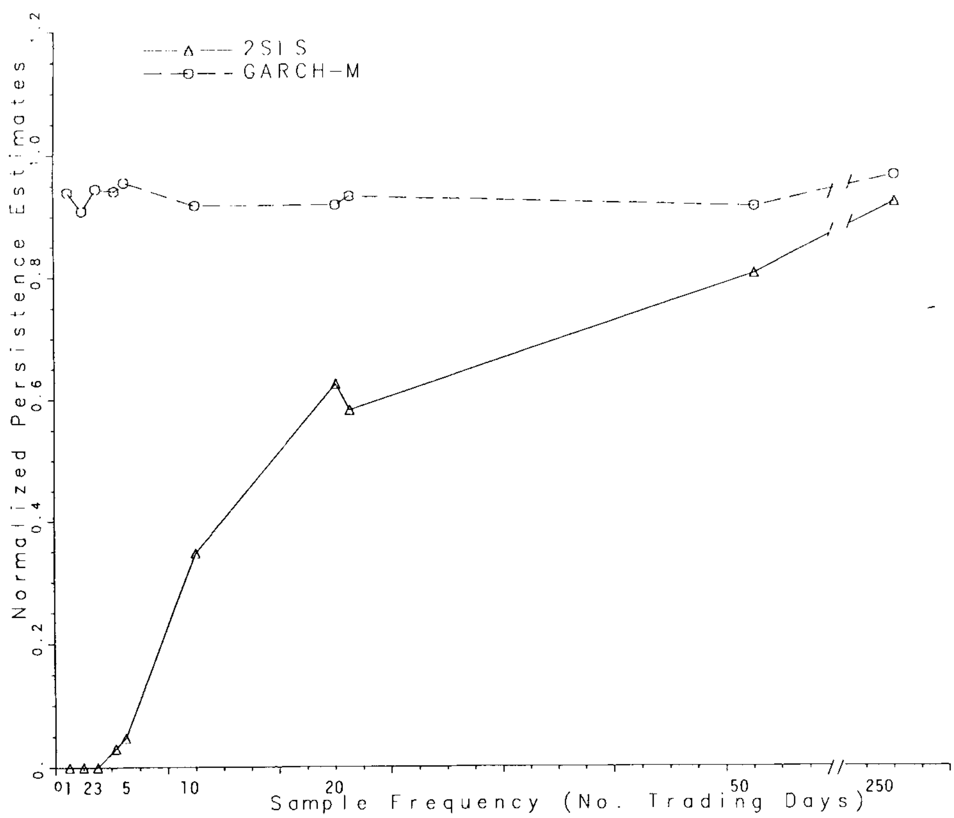

Figure 4. Estimates of the persistence parameter

The R_t^N series are constructed for $N = 1, 2, 3, 5, 10, 20, 50, 250$, together with the monthly returns. The case $N = 250$ roughly corresponds to an annual return. The GARCH(1,1)-M model of equation (13) is applied to all the data sets.* The value $(\alpha + \beta)^{21/N}$ is also computed for all N. By equation (7), this normalized persistence parameter measures the proportion of volatility shocks remaining after 1 month (21 trading days).

We perform a similar experiment on the two-stage method. Following P-S, the volatility of the N-trading-days return at period t is

$$V_{N,t} = \frac{1}{N} \sum_{s=1}^{N} R_{t,s}^2.$$

This volatility estimator is assumed to follow an AR(1) process, i.e.,

$$V_{N,t} = c + \rho_N V_{N,t-1} + u_{N,t},$$

where $u_{N,t}$ is an i.i.d. white noise. Similarly to the GARCH(1,1) process, the AR(1) process also implies a geometrically declining weight for volatility shocks. Hence $\rho^{21/N}$ also provides a 1-month memory of shocks to volatility and can be compared with the estimate $(\alpha + \beta)^{21/N}$ using GARCH method.

* In theory, temporal aggregates of a series following GARCH(1,1)-M do not follow GARCH(1,1)-M. The exact distributions are fairly complicated and are not pursued in this paper. Nonetheless, the GARCH(1,1)-M model seems to provide a very well approximation.

It is remarkable how these two methods differ in estimating the persistence parameter. As Figure 4 shows, using GARCH-M, the parameter estimate is very stable; it ranges from 0·909 to 0·964 with an average of 0·933. Our estimate from the weekly data set is well within this range, and is only slightly above the average. On the contrary, the two-stage method gives very unstable estimates. The estimate increases almost monotonically with the sampling horizon. Since there is no theory indicating a proper choice of N, the size of variation in this parameter estimate casts doubts on inferences based on any one of these estimates. In particular, the monthly volatility measure adopted by many authors seems to seriously underestimate the persistence of volatility.

The difference seems to arise from the fact that the AR(1) model is unable to model long memory dynamics which we believe are intrinsic in the volatility process. Pindyck (1986) reports that the serial correlation estimate using a monthly volatility measure is unduly low compared to that using annual data, and leads to a rejection of his model using monthly data. Higher-order autoregressive models may be able to capture the long memory property, but are not without problems. First, this methodology requires solving the problem of choosing the sampling horizon. Second, the introduction of too many lagged variables is impractical computationally. For example, if our result that the half-life of shocks to volatility is about 1-year is assumed, then 48 lags may be necessary for monthly data since 6·25 per cent of the initial shock is still in effect after 4 years. Finally, the simplicity of the elasticity expression derived by the AR(1) specification is lost; hence it becomes very difficult to gauge the impact of volatility shocks on stock prices.

So far we have been assuming the volatility to be stationary—namely, the variance always reverts to its long-term mean. This assumption may warrant further investigation since the estimates of the persistence parameter are so close to unity for all data sets. In the next section we explicitly test the possibility that the volatility may be non-stationary.

5. INTEGRATED-GARCH AND TESTING NON-STATIONARY VARIANCES

Engle and Bollerslev (1986) introduce a model within the GARCH framework but with non-stationary variances. In this model the second and fourth unconditional moments do not exist, but the conditional distribution is still well defined. They call this model the 'integrated GARCH' or 'IGARCH' model. In our GARCH(1,1) model the integration of variances arises when $\alpha + \beta = 1$. In contrast to the stationary variance case, impacts of variance shocks remain forever. This property is shared by the well-known random walk process, which has always been used to characterize the stock price movement.

Testing for IGARCH is a test for unit roots in the variance. As has been documented by Dickey and Fuller (1979), the traditional asymptotic results do not hold in the test of unit roots in the mean. The existence of unit roots seriously biases the OLS estimate towards rejection of unit roots. Hong (1987) shows that this problem does not prevail in tests of IGARCH models. Namely, all traditional test procedures are still valid in the test of IGARCH models.

We therefore proceed to test the null hypothesis that the stock returns are generated by an IGARCH(1,1)-M process. This model is given by equation (13) with the restriction $\alpha + \beta = 1$. We estimate the IGARCH model and perform likelihood ratio tests to test the restriction of integrated variances. Table III reports the estimation result of the IGARCH model for all data sets, together with the testing result.

It is interesting that the IGARCH model is supported for all cases no matter how frequently the data are measured. With only one exception, the likelihood ratio statistics cannot reject the restriction of integration in variances at the 5 per cent level. The test statistic is only marginally

Table III. Parameter estimates and tests of the IGARCH(1,1)-M model for temporal aggregated data

$$R_{t+1}^{N} = r_f + \delta V_t + e_{t+1}$$

$$V_t = \alpha_0 + \alpha e_t^2 + (1-\alpha)V_{t-1}$$

with $e_{t+1} \mid I_t \sim N(0, V_t)$,

where R_t^N is the N-trading-day return.

N	$r_f \times 10^2$	$\alpha_0 \times 10^4$	α	δ	llf1	llf2[a]	LR[b] (p-value)
1	0·04	0·003	0·089	6·09	−6089·23	−6088·49	1·48
	(3·30)[c]	(6·41)	(18·14)	(2·52)			(0·224)
2	0·07	0·012	0·104	4·72	−4402·61	−4401·17	2·88
	(2·40)	(5·51)	(12·03)	(1·96)			(0·089)
3	0·10	0·017	0·097	4·03	−3416·68	−3415·68	2·00
	(2·24)	(3·56)	(8·37)	(1·80)			(0·158)
5	0·13	0·053	0·139	3·87	−2407·68	−2407·24	0·88
	(1·75)	(3·28)	(6·82)	(1·74)			(0·348)
10	0·17	0·182	0·148	4·13	−1446·91	−1445·31	3·20
	(0·97)	(2·51)	(4·73)	(1·83)			(0·074)
20	0·25	0·793	0·210	3·62	−847·70	−846·09	3·22
	(0·64)	(1·88)	(3·22)	(1·59)			(0·073)
50[b]	—	5·870	0·406	3·64	−388·09	−386·80	2·58
		(1·30)	(2·26)	(3·77)			(0·108)
250	—	—	—	—	—	−93·22	−0·55[e]
							(0·250)
Weekly	0·130	0·072	0·166	4·09	−2493·98	−2493·49	0·98
	(1·69)	(2·83)	(7·94)	(1·87)			(0·322)
Monthly	—	0·427	0·138	3·36	−801·92	−799·96	3·92
		(1·57)	(2·78)	(2·75)			(0·048)
Monthly	0·92	0·507	0·157	0·96	−2136·41	−2125·31	2·20
(1926–1985)	(4·29)	(3·04)	(8·22)	(1·20)			(0·276)

[a] llf1 and llf2 are respectively the log likelihood function estimates for the IGARCH-M and GARCH-M model.
[b] The likelihood ratio test statistic, $LR = 2 \times (\text{llf2} - \text{llf1})$.
[c] Numbers in parentheses are t-ratios except for the last column where p-values are given.
[d] For monthly and $N = 50$, 250 cases, the excess returns are used hence the model is estimated without the constant term r_f. Negative estimates for the risk-free rate are obtained if total returns are used.
[e] The estimation for IGARCH does not converge for this data set hence a Wald-t statistic is used.

significant at the 5 per cent level for monthly returns. However, if a longer monthly data set (1926–1985) is used, the unit root restriction again cannot be rejected. The evidence of integrated variances seems quite strong based on this test result.

The non-stationary variance result is consistent with Ibbotson and Sinquefield's (1982) observation that the dynamic behaviour of the equity premium can best be characterized by a random walk. If the variance is indeed non-stationary, the impact of volatility shocks on stock prices will be much higher than the estimates we give in section 3. F-S-S use an IMA(1,3), a non-stationary process to model the standard deviations of stock returns. P-S estimate the elasticity measured at the mean volatility level to be 22·5 per cent. The IGARCH result is used together with other parameter estimates to compute the elasticity given by equation (12).

The impact of the IGARCH result on the elasticity is phenomenal. Evaluated at the unconditional variance the elasticity measure increased to 59·9 per cent, or more than five times of the estimate of the stationary result. If the non-stationary result is true then instead of asking 'Why is the market so low in 1974?' we would ask: 'Why is the market so "high" in 1974?'

If this inference is extended further, then the excess volatility puzzle raised by Shiller may be resolved. Note that most of the literature on this issue has been assuming either a constant risk premium or that it is time-varying but stationary (mean-reverting). In the light of our analysis the non-mean-reverting dynamic behaviour of the risk premium may be an important factor underlying the stock fluctuations. Future research along this direction may be fruitful.

6. CONCLUSION

In this paper issues of voltility persistence and the changing risk premium in the stock market are investigated using the GARCH estimation technique. We get a point estimate of the index of relative risk aversion of 4·5 and confirm the existence of changing equity premiums in US during 1962–1985. The persistence of shocks to the stock return volatility is so high that the data cannot distinguish whether the volatility process is stationary or not. Assuming stationarity, the half-life of volatility shocks is about 1 year. The parameter estimates and the non-stationary test result are both robust to changes in the frequency of data measurements. Using monthly data the persistency result is also maintained for a longer sample period, 1926–1985.

Our finding is greatly different from that of Poterba and Summers (1986). They claim that shocks to the volatility are transitory and hence cannot have much impact on the market. The deviation stems from the difference in estimation methodologies. We claim that their methodology is limited in its nature and hence may give very misleading results. In particular, it is demonstrated that their parameter estimates are very sensitive to the frequency of volatility measurements. The monthly measure they use in their work tends to seriously underestimate the persistence parameter. On the contrary, our model using the GARCH-M specification and its maximum-likelihood estimation method, is immune to this problem.

The economic implication of our finding is far-reaching. The hypothesis raised by Malkiel and Pindyck that it is the unforseen rise in the investment uncertainty during 1974 that causes the market to plunge seems to be confirmed. Based on our estimation result, the doubling in stock volatility in 1974 causes a 26 per cent drop in the market index, which is very close to the actual value. In fact, if the IGARCH result is taken to be true, our analysis implies an even deeper drop in the stock price. With a non-stationary (or close-to-non-stationary) process for the discount factor the rational valuation formula might be sufficient to explain the tremendous variation of asset prices. Our finding also implies that the identification of the sources of uncertainty is important. Particularly, the long-term impact of trading behaviours causing sizeable disturbances such as takeovers and computer programmed tradings, should receive more serious attention.

APPENDIX: GARCH, GARCH-M, ESTIMATION AND TEST PROCEDURES

In this appendix we briefly discuss the GARCH and GARCH-M models, estimation and testing procedures. For a complete treatment see Engle (1982), Bollerslev (1986) and Engle, Lilien and Robins (1987). Consider the following data generating process

$$y_t = X_t b + e_t\ , \tag{A1}$$

$$h_t = \alpha_0 + \sum_{i=0}^{q} \alpha_i e_{t-1}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}\ ,$$

$$e_t \mid I_{t-1} \sim N(0, h_t),$$

where X_t is a vector of variables which may include lagged dependent variables and contemporaneous exogenous variables, I_{t-1} denotes the information set up to time $t-1$.* The difference between this model and the usual OLS model is that the error term is not an i.i.d. sequence. Instead, it has an ARCH error with a GARCH(p, q) representation. The normality assumption is adopted for most papers using ARCH. This assumption is convenient for estimation although other distributional assumptions can also be assumed. Bollerslev (1987) claims that for some data the fat-tailed property can be approximated more accurately by a conditional Student-t-distribution. In this paper, it is found that although the likelihood can be improved by assuming a conditional t-distribution, the interested parameters are not affected much by this specification.

The empirical fact that variations of similar sizes tend to group together for some variables is captured by the specification of the variance equation. Namely, the future variance depends on the current information set directly through past forecast error squares and indirectly through past conditional variances. The special case that $\alpha_i = \beta_j = 0$ for all i and j corresponds to the conventional linear model. Engle's ARCH(q) process is also a special case with $\beta_j = 0$, for all j.

It is well known that the correlogram and partial correlogram of the residuals can be used to help identify the autoregressive and moving average orders of a Box–Jenkins ARMA process. Bollerslev (1988) demonstrates that, in a similar fashion, the correlogram and partial correlogram of the square residuals can be used in specifying the GARCH orders, p and q. This becomes more obvious by defining a variance innovation term $\nu_t = e_t^2 - h_t$, and rewriting the GARCH equation in terms of e_t^2,

$$e_t^2 = \alpha_0 + \sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i) e_{t-i}^2 + \nu_t + \sum_{j=1}^{p} \beta_{t-j} \nu_{t-j}. \tag{A.2}$$

Thus, the square residual has an ARMA representation in the innovation term, ν_t. Hence the correlation structure of e_t^2 corresponds to the GARCH orders p and q. Note, however, ν_t is not an i.i.d. white noise but is heteroscedastic with a time-varying support. Although the GARCH(p,q) model encompasses a lot of dynamic structures, empirical studies frequently suggest that a GARCH(1,1) is adequate in modelling conditional variance, e.g. Bollerslev (1986, 1987), Engle and Bollerslev (1986). The Box–Pierce Q statistic for the normalized squared residuals (i.e. e_t^2/h_t) can be used as a diagnostic test against higher-order specifications for the variance equation.

The GARCH-M(ean) model is an extension of GARCH by adding the conditional variance as an explanatory variable in the mean equation. This model can be viewed as a statistical implementation of the mean-variance analysis in finance. As described in section 1, this model is used to study the linkage between risk and return in the bonds market by Engle, Lilien and Robins (1987). If the evolution of the variance of the market return can be reasonably approximated by GARCH, then the GARCH-Mean model provides a unified framework to estimate the volatility and the time-varying risk premium in the stock market.

The estimation of the model can be achieved by the maximum-likelihood method. Letting ϕ be the vector of parameters in the mean and variance equations, with a sample of T observations, the log-likelihood function is (apart from a constant) given by

$$L_T(\phi) = \sum_{t=1}^{T} \log f_t(\phi), \tag{A.2}$$

$$f_t(\phi) = -\tfrac{1}{2} \log h_t - \tfrac{1}{2} e_t^2 h_t^{-1}.$$

* Note that here we adopt the convention in denoting the time subscript for the conditional variance for ease of discussion for higher order GARCH processes.

Maximum-likelihood estimates of ϕ can be obtained from the Berndt, Hall, Hall and Hausman (1974) (BHHH) algorithm. For each step the parameter estimates are given by

$$\phi^{i+1} = \phi^i + \lambda_i (S'S)^{-1} S\iota, \tag{A.4}$$

where S, the scoring matrix, is evaluated at ϕ^i, and λ_i controls for step length by maximizing the likelihood in the given direction. Note that the directional vector is easily obtained from an OLS regression of the $T \times 1$ unit vector ι on the scoring matrix. Instead of finding analytical derivatives, numerical derivatives are calculated for scores,* thus providing extra flexibility to changes in specifications.

Standard inference procedures are available for the GARCH and GARCH-M model. The Lagrange multiplier (LM) test is particularly attractive if estimation under the alternative is complicated since it requires only estimation under the null. By assuming asymptotic normality for the maximum likelihood estimator, the LM test can be constructed as

$$\mathrm{LM} = \iota' S(S'S)^{-1} S' \iota' \sim \chi^2(k), \tag{A.5}$$

with k being the number of restrictions. By (A.5), convenient test statistics can be computed to test higher-order ARCH or GARCH specifications. The statistic is computed as TR^2 of the first step OLS iteration of the BHHH algorithm for the general model with initial parameter values given by estimates under the null. This test statistic is used to test GARCH(1,2) and GARCH(2,1) specifications as alternatives to the maintained GARCG(1,1) specification. If estimations are done both under the null and under the alternative, likelihood ratio tests can be obtained by

$$\mathrm{LR} = -2(L_T(\phi_0) - L_T(\phi_a)) \sim \chi^2(k), \tag{A.6}$$

with ϕ_0 and ϕ_a estimated under the null and under the alternative respectively. In this paper this statistic is used for testing non-stationary variances. The null hypothesis is the unity restriction of the sum of coefficients (other than the constant term) in the variance equation.

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