Comparative Analysis of International Stock Market Volatility with ARCH Model

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Abstract

In stock markets, the price is fluctuating around its average value with the time being. One of the volatilities is the variance heteroscedasticity. It is found that Auto-regressive Conditional Heteroscedasticity (ARCH) model provides an alternative approach for the simulation of stock heteroscedasticity. In this paper, the application of ARCH and its modified models is presented for the risk analysis based on the stock index of America, Europe, China and other countries in Asia Pacific. GARCH-M model is used to test the long-term volatility self-similarity and the correlation between risk and return; TGARCH model is introduced to test the volatility leverage effect; EGARCH model is applied to verify the asymmetry heteroscedasticity of stock price fluctuation.

1. Introduction

The volatility of the stock market reflects the possibility of the actual return departure from expected yield, the higher the risk premium is, the more volatility of the stock is. It has long been recognized that stock price movements cannot be reproduced through the realm of rational expectation models [1]. Usually, the stock price is too volatile, the return are unpredictable in the short run. Investors should measure in detail the market risk and expected return both for the whole market and the individual securities.

On behalf of the uncertainty of future earnings, the volatility of the stock market uses variance and standard deviation to state in general. In describing the stock market return, traditional econometric models often make an assumption that the volatility is the same as variance of return. However, studies show that the volatility not only changes with time but also exhibits a successive high or low fluctuation in a certain period. A large number of literatures have been devoted to doing these empirical observations and to finding extensions of the model that will improve its empirical performance.

For the time series problem with phenomenon of "leptokurtosis and fat-tail, long-time memory, volatility clustering", the condition of the symmetric variance which is used in traditional econometric approaches is not longer suitable for the observation. Taking advantage of ARCH (Autoregressive Conditional Heteroscedasticity) and its derivative models, this paper makes an empirical analysis on the stock market volatility of China, the United States, Europe and Asia, trying to find out the difference of volatility characteristics of Asia, Europe and the United States stock market.

2. Modeling

2.1 ARCH model

Traditional forecast models use the mean-variance approach to describe the volatility of stock price in a certain period. However, the variance changes with time and displays some kind of similarity in different intervals. To simulate this performance, Engle[2] developed ARCH model to describe the auto-regressive heteroskedasticity and long-term memorability of stock return.

In ARCH modeling, the variance self-regressive equation is brought into the model, the expression is:

$$\begin{cases} y_t = x_t^{'}\beta + \varepsilon_t \\ h_t = Var(\varepsilon_t / \psi_{t-1}) = \alpha_0 + \alpha_1 \varepsilon_{t-1}{}^2 + \cdots \alpha_q \varepsilon_{t-q}{}^2 \end{cases} \quad (1)$$

Where, h_t denotes the variance heteroskedasticity shown in time series of return y_t, which is normally distributed with mean $x'_t\beta$ and random residual ε_t. q is conditional variance regressive degree. y_t denotes the vector of changes in the natural logarithm of stock market volatility from period 0 up to and including period t . β denotes the vector of parameters. ε denotes a stochastic disturbance term. Ψ denotes the set of regressors which is a subset of all macroeconomic and financial instruments that the investors consider to be relevant for forecasting the stock market volatility. The vector of regressors always includes a constant α_0.

The basic feature of ARCH model is that the stochastic residual ε_t of conditional variance h_t is determined by ε_{t-1}, ε_{t-2},…,ε_{t-q}. The special structure of h_t depicts a self-similarity of price fluctuation in stock market. When ε_{t-1} is relatively large, so is the variance of ε_t. A large leap of y_t in t period will lead to a large fluctuation in t+1 period, and vise versa. The larger the regressive degree q is, the longer the fluctuation lasts. This means the current volatility estimate h_t can be deducted from previous period h_{t-1}. A large previous shock leads to an increase in forecasted volatility.

2.2 GARCH model

Researches and practices show that the financial time series exhibit a high-order ARCH effect. Although the ARCH model can describe this performance, the model

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itself has the defect that it cuts off the stochastic residual terms ε when the order is higher than q. ARCH model assumes that all the coefficient α be greater than zero to ensure the conditional variance h_t is not negative, this assumption departure from the stochastic stock movements, which might result in an oscillation for the forecasting. Eagle and Robins [4] introduced a Generalized ARCH (GARCH) model to allow past conditional variance to enter the equation. The GARCH(p,q) is:

$$\begin{cases} y_t = x_t^{'}\beta_t + \varepsilon_t + \gamma\sqrt{h_t} \\ h_t = Var(\varepsilon_t / \psi_{t-1}) = \alpha_0 + \alpha_1 \varepsilon_{t-1}{}^2 + \cdots \\ \qquad + \alpha_p \varepsilon^2{}_{t-p-1} + \beta_1 h_{t-1} + \cdots + \beta_q h_{t-q-1} \end{cases} \quad (2)$$

The orders of the GARCH process are the number of lags for the squared error terms (p) and for the past variances (q). Most researchers have found that a GARCG(1,1) is generally excellent for forecasting the volatility of stock returns [1].

2.3 TARCH model

In GARCH model, the conditional variance is a function of weighted average of p past squared forecast errors, so the sign of error does not affect the fluctuation. However, this is not true in reality because the observation to the variance in stock market is not symmetric. Based on GARCH model, Zakoian[5] introduced the TGARCH model to explain the leverage effect in stock market:

$$h_t = a_0 + \alpha\varepsilon^2{}_{t-1} + \beta h_{t-1} + \gamma\varepsilon^2{}_{t-1} d_{t-1} \quad (3)$$

Where, if $\varepsilon_t<0$, $d_t = 1$; otherwise, $d_t = 0$.

In this model, good news ($\varepsilon_t>0$) and bad news ($\varepsilon_t<0$) have different impacts on the conditional variance. Good news has a strike of α, and bad news has a stick of α+γ. If γ≠0, the information is dissymmetrical; if γ>0, it says that there exists a leverage effect, and the major effect (bad news) enlarges the fluctuation, so bad news has more impact on stock fluctuation than good news; if γ < 0, vice versa.

2.4 EARCH model

To ensure h_t is non-negative, Nelson[6] proposed the EGARCH model. The model assumes that all coefficients in conditional variance formula are positive, and the conditional variance is expressed in logarithm as:

$$\log(h_t) = \alpha_0 + \alpha\left|\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}\right| + \beta\log(h_{t-1}) + \varphi\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \quad (4)$$

The left hand side of equation (4) is the logarithm of conditional variance, which means the leverage effect is exponential rather than quadratic, so the estimation of conditional variance is definitely non-negative. The existence of leverage effect can be verified through the assumption of φ<0.

3. Empirical analysis

To make a comparative analysis of the stock market volatility, the Shanghai Composite Index, Nasdaq composite index, British FTSE100 index, France Paris Index (CAC40), Germany Frankfurt Index (DAX), Hong Kong Hang Seng Index, Taiwan weighted index, Nikkei index are selected for the study. The data sampling is based on 1000 trading days before November 9, 2007.

3.1 ARCH analysis

Firstly, we make an analysis of the statistical feature for the eight stock indexes and list the results in Table 1. It can be seen that all the market daily return skewness are less than zero, and all the kurtosis are more than 3.0. There exists a "leptokurtosis and fat-tail" feature and it is left-skewed comparing to the normal distribution. The test statistics of Jarque-Bera are significantly higher than the critical value, which reject the hypothesis of normal distribution.

Table 1: Statistical parameters and test results

Index	Skewness	Kurtosis	J—B test		ADF	ARCH-LM
			Statistic	Aprox Prob		probability-test
Sh	-0.9680	4.6565	115.8896	0.E+00	-15.5448	0.E+00
Nasdq	-0.3164	3.7225	38.4299	0.E+00	-25.0584	0.E+00
FTSE100	-0.3826	4.9284	179.3403	0.E+00	-22.0528	0.E+00
CAC40	-0.3238	4.4814	108.7989	0.E+00	-22.9636	0.E+00
DAX	-0.4710	3.9218	72.1877	0.E+00	-25.4284	0.E+00
Hsi	-0.0871	6.1765	421.6641	0.E+00	-15.8816	0.E+00
Twii	-0.8365	7.6728	1026.2950	0.E+00	-21.7140	0.E+00
N225	-0.4726	4.2584	103.1085	0.E+00	-26.1872	0.E+00

Secondly, we make a further unit root test (ADF test) for the daily return series of the samples. The result of Augmented Dickey-Fuller (ADF) test is listed in Table 1. Under the significant level of 1%, the ADF t-statistics of Shanghai Stock index's daily return is much smaller than the MacKinnon critical value, rejecting the hypothesis that the stock market daily return did not have an ARCH effect. The change of the return series is smooth, so it can be simulated by the auto-regressive model.

Thirdly, through a serial correlation test on the residual squared series, it is found that the series has an obvious self-correlated. After making an ARCH-LM test, it is also found that the probability is still close to zero even if the lag number is 3 and higher, rejecting the hypothesis that series has not an ARCH residual effect. All the market return series have obvious high-order ARCH effect, so the ARCH model can be applied for the volatility analysis.

3.2 GARCH-M(1,1) Analysis

The GARCH-M (1, 1) model is applied for the volatility analysis on the eight stock market indexes, and the results are list in Table 2. From the table we can find:

Table 2: The parametric results of GARCH(1,1)-M model

GARCH	Sh	Nasdq	FTSE100	CAC40	DAX	Hsi	Twii	N225
α_0	4.29E-06	2.16E-06	1.82E-06	2.96E-06	3.61E-06	6.83E-07	4.33E-06	3.07E-06
(Z statistic)	-2.6994	2.527849	3.81453	4.267155	3.046878	1.603229	3.640183	3.134413
α	0.087907	0.065402	0.136641	0.093099	0.096115	0.057417	0.091093	0.066363
(Z statistic)	-5.3388	3.862433	5.122174	5.178828	4.374615	5.499735	7.886459	4.285631
β	0.889086	0.893413	0.803572	0.855078	0.840957	0.937288	0.865765	0.898612
(Z statistic)	-40.9344	30.21676	25.96502	41.56929	24.59155	71.87714	43.4669	40.27346
γ	0.078629	0.058889	0.084145	0.097216	0.116192	0.091134	0.061871	0.061605
AIC	-5.9586	-7.058473	-7.709992	-7.03993	-7.00304	-6.659161	-6.551657	-6.530591
SC	-5.975166	-7.033914	-7.685434	-7.01537	-6.97848	-6.634603	-6.527099	-6.506033

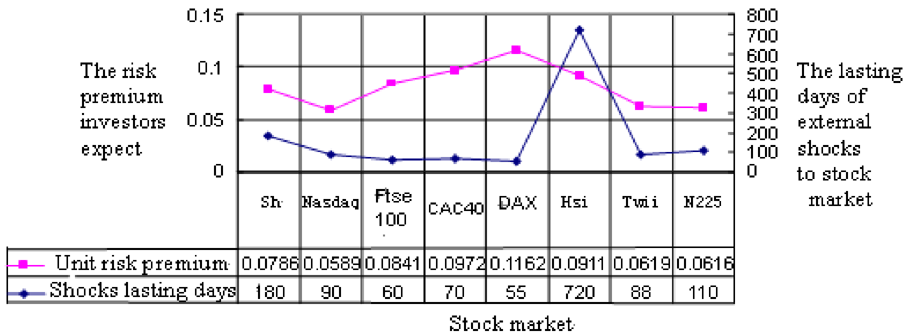

Figure 1：The fluctuation duration to external shocks and the risk premium investors expected

Decay factor（α+β）in GARCH-M model is close to 1.0, this means all the eight indexes have strong fluctuation duration, and the fluctuation responding to external shock is decreasing slowly. The duration is: Hangseng > Shanghai composite > nikkei225 > Taiwan-weighted > NASDAQ > CAC > FTSE 100 > DAX, with duration of 720 days, 180 days, 110 days ,90 days, 88days, 70 days,60 days and 55 days, respectively, which can also be seen in Figure 1. The durability of Asian stock market is stronger than that of European market, especially in Hong Kong market. A strong duration implies that it would take longer time to recover from an external shock, so the Asian stock markets have a fierce volatility, stronger speculation, and high risk.

In GARCH-M model, the coefficient (ɣ) of sqrt (h_t) denotes the investor's expected return rate increase in accordance to the percentage increase of risk. The coefficients listed in Table 2 from high to low are: DAX > CAC > Hangsheng > FTSE 100 >Shanghai composite > Nikkei 225 > Taiwan weighted > NASDAQ. It can be inferred that, under an equivalent increasing of the risk, the investor's expected return of these eight markets decrease in order shown above，the investor's speculation increases and the rationality decreases in the same sequence.

Based on the comprehensive observations of duration and risk/return ratio, the investors in European markets are relatively more rational. The fluctuation duration in the markets of Hong Kong and mainland China market is higher than that of other regions. The China bull market in these two years implies a high uncertainty and risk undoubtedly.

3.3 TGARCH

Table 3 lists the parameters calculated from TGARCH (1, 1) model. According to the TGARCH model, if ɣ>0，a negative price change will result in a higher fluctuation than that from a positive price change; If ɣ<0, good news will lead to a larger fluctuation shock than bad news with a same scale. Except Shanghai, the stock markets exhibit a positive leverage effect, from large to small, they are: Germany, America, Japan, France, Taiwan, Britain and Hong Kong. The leverage effect is not simply corresponding to the maturity of stock market, more or less, it reflects the investors' rationality. Although Hong Kong has shown a positive leverage effect, but it is only 0.055, while the least in other countries (Britain) is 0.19. Hong Kong stock market exhibits a low leverage effect due to the strong correlation between Hong Kong and mainland China. To some extent, the results explain why the China stock market is a bull market continuously and investment upsurges enthusiastically. A bad news can not influence on the investment enthusiasm, while a good news can strengthen the investors' expectation in China stock market, leading to a blind investing surge and high volatility[7].

Table 3: The parametric results of TGARCH model

TARCH	Sh	Nasdq	FTSE100	CAC40	DAX	Hsi	Twii	N225
α_0	0.00000348	0.00000541	2.28E-06	0.00000316	4.75E-06	0.00000092	0.0000078	0.00000929
(Z statistic)	-2.5973	4.720444	4.310828	5.844491	5.356969	2.271327	5.738549	6.220419
α	0.080211	-0.049688	0.01015	-0.041427	-0.05944	0.029421	-0.024788	-0.04932
(Z statistic)	-4.6213	-2.673172	0.403135	-4.251268	-3.86314	1.943626	-2.423776	-2.926252
β	0.903296	0.818761	0.814873	0.875176	0.840957	0.936921	0.836465	0.822125
(Z statistic)	-46.2691	22.91667	22.91748	45.89531	24.59155	64.03407	29.35636	33.15965
γ	-0.007133	0.272208	0.199391	0.22909	0.328474	0.055021	0.208337	0.253554
AIC	-5.9934	-7.09106	-7.722325	-7.091451	-7.04832	-6.658255	-6.577568	-6.560323
SC	-5.968863	-7.09106	-7.697767	-7.066892	-7.02376	-6.633697	-6.55301	-6.535764

Table 4: The parametric results of EGARCH model

EARCH	Sh	Nasdq	FTSE100	CAC40	DAX	Hsi	Twii	N225
α_0	-0.32233	-0.993985	-0.932749	-0.778109	-1.02232	-0.184606	-0.905123	-1.192033
(Z statistic)	-3.6231	-4.879602	-5.072662	-7.821322	-6.23571	-3.374498	-5.274355	-6.482269
α	0.164279	0.090558	0.193669	0.050233	0.109341	0.138121	0.171648	0.116343
(Z statistic)	-6.205	2.736936	4.890057	1.77524	3.163796	5.725175	6.684856	3.25723
β	0.97788	0.906776	0.925619	0.924698	0.904084	0.991241	0.916926	0.882553
(Z statistic)	-112.461	46.51361	55.53275	93.67857	56.60743	204.5351	51.7061	46.70204
φ	0.00864	-0.187775	-0.145655	-0.241105	-0.25262	-0.035339	-0.138287	-0.203773
AIC	-5.9818	-7.095343	-7.720896	-7.110779	-7.06228	-6.652377	-6.570118	-6.569332
SC	-5.968863	-7.070784	-7.696338	-7.086221	-7.03772	-6.627819	-6.545559	-6.544774

3.4 EGARCH analysis

The piecewise linearity of EGARCH model generates a dissymmetrical conditional variance，the EGARCH can be used to depict the influence of return on stock price. Table 4 lists the parameters simulated from EGARCH model.

Parameter φ<0 means the stock market has a leverage effect. When stock price goes down, the stock holders and purchasers both have an expectation of higher fluctuation, leading to a continuous price change. In Table 4, only the φ in Shanghai market is positive, again it proves the Chinese investors’ irrationality. If dissymmetry is measured by （□φ□/α）(assuming good news strike is α), from large to small, the countries are France, Germany, American, Japan, Taiwan, Britain and Hong Kong, respectively, which is almost in the same order as in TGARCH model.

4．Conclusion

From the empirical analysis based on ARCH models, it is observed that all the stock markets have return and risk volatilities, the stock market’s return series have significant "leptokurtosis and fat-tail" feature and the existence of volatility clustering, and all the stock market fluctuations show a long-term memorability. Hong Kong stock market has the longest fluctuation duration and the German stock market has the shortest. Except Shanghai stock market, the others exhibit fluctuations in a positive leverage effect, which means the volatility caused by bad news is larger than that caused by the same level of good news. This can be explained from the fact that the China bullish stock market enhances the investors’ confidence, and any bad news can not shake the investors’ enthusiasm. The China bull market in these two years implies a high uncertainty and risk.

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