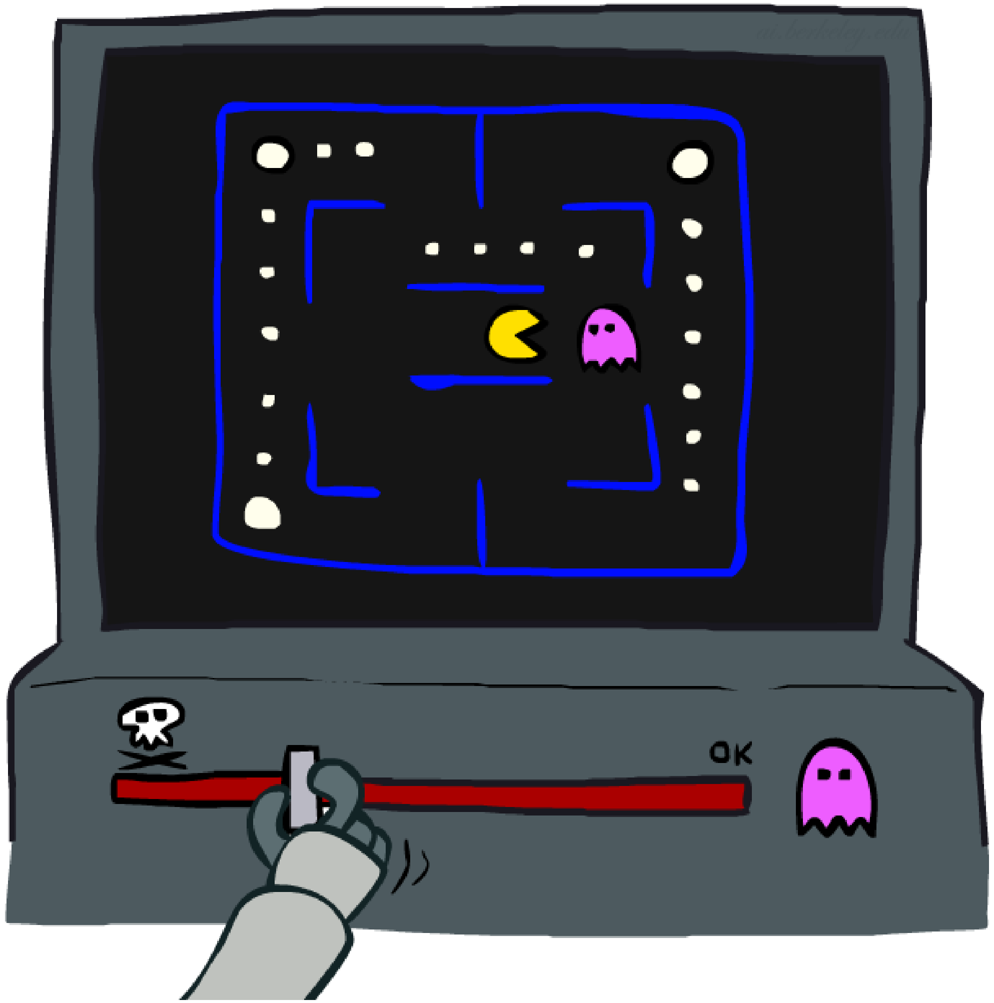
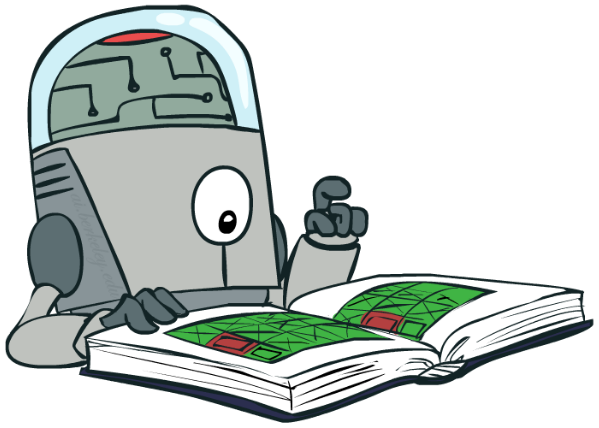
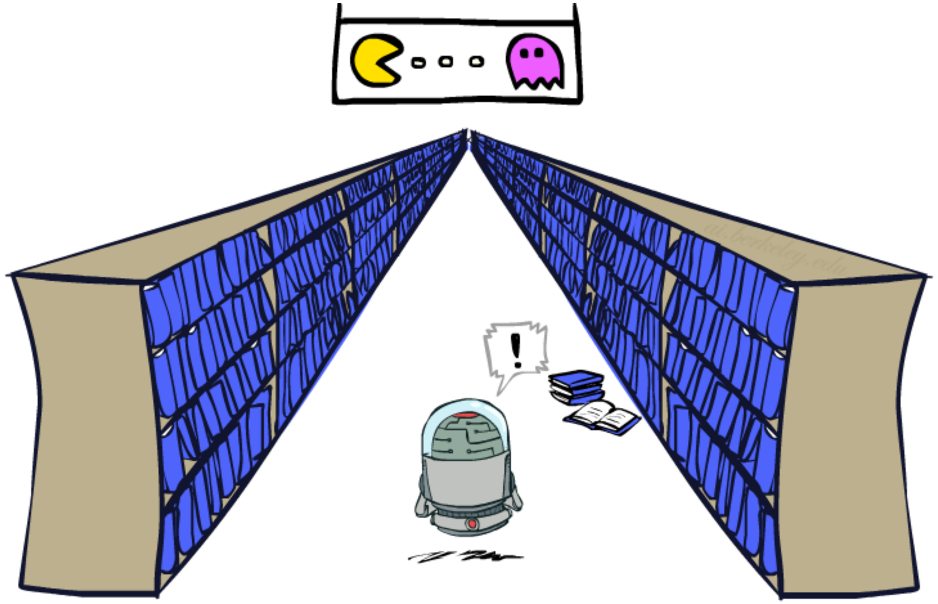
Approximate Q-Learning



Generalizing Across States

- Basic Q-Learning keeps a table of all q-values
- In realistic situations, we cannot possibly learn about every single state!
  - Too many states to visit them all in training
  - Too many states to hold the q-tables in memory
- Instead, we want to generalize:
  - Learn about some small number of training states from experience
  - Generalize that experience to new, similar situations
  - This is a fundamental idea in machine learning, and we'll see it over and over again

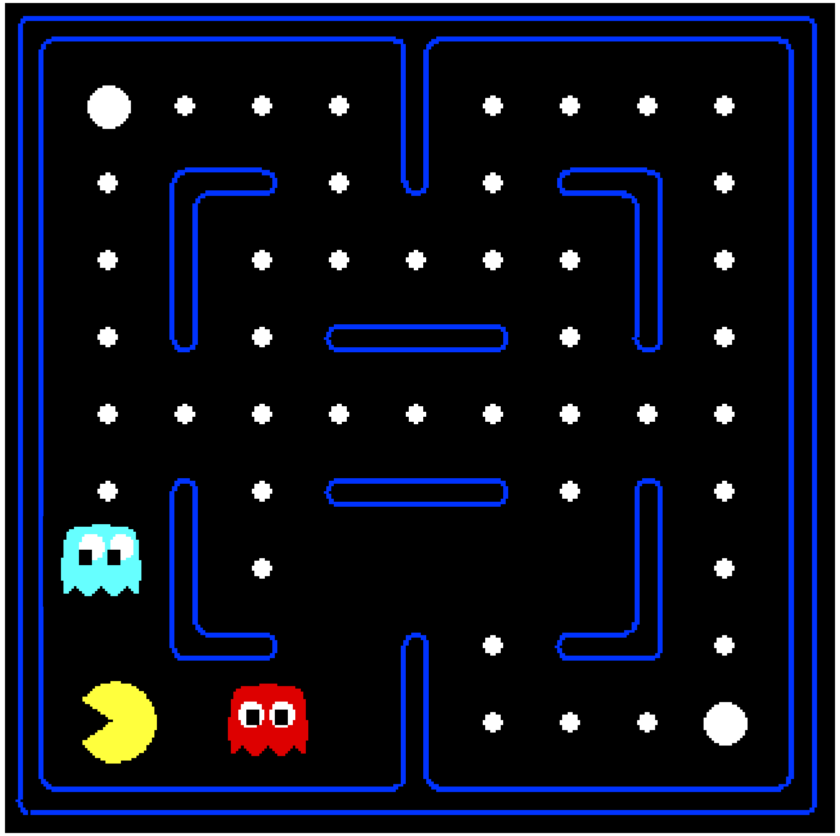




[demo – RL pacman]

Feature-Based Representations

- Solution: describe a state using a vector of features (properties)
  - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
  - Example features:
    - Distance to closest ghost
    - Distance to closest dot
    - Number of ghosts
    - 1 / (dist to dot)^2
    - Is Pacman in a tunnel? (0/1)
    - …… etc.
    - Is it the exact state on this slide?
  - Can also describe a q-state (s, a) with features (e.g. action moves closer to food)



Linear Value Functions

- Using a feature representation, we can write a q function (or value function) for any state using a few weights:

$$V(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

- Advantage: our experience is summed up in a few powerful numbers

- Disadvantage: states may share features but actually be very different in value!

Approximate Q-Learning

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

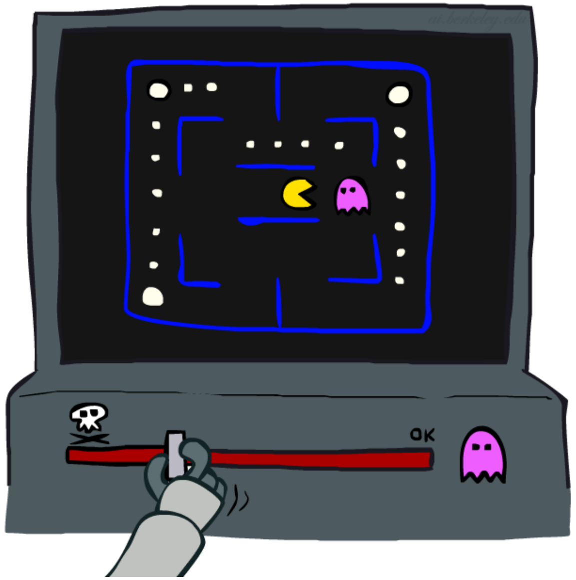
- Q-learning with linear Q-functions:

$$\text{transition } = (s,a,r,s')$$

$$\text{difference} = \left[r + \gamma \max_{a'} Q(s',a')\right] - Q(s,a)$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha\,[\text{difference}] \qquad \text{Exact Q's}$$

$$w_i \leftarrow w_i + \alpha\,[\text{difference}]\, f_i(s,a) \qquad \text{Approximate Q's}$$

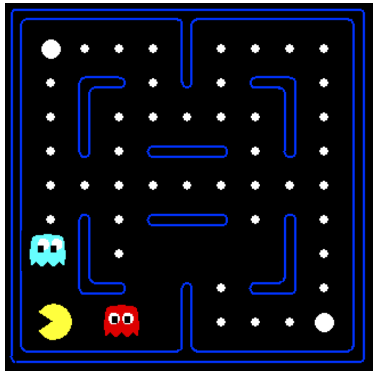


- Intuitive interpretation:
  - Adjust weights of active features
  - E.g., if something unexpectedly bad happens, blame the features that were on: disprefer all states with that state's features
- Formal justification: online least squares

Example: Q-Pacman

$$Q(s,a) = 4.0 f_{DOT}(s,a) - 1.0 f_{GST}(s,a)$$

s

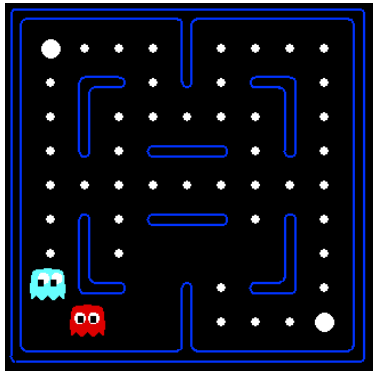


$$f_{DOT}(s, \text{NORTH}) = 0.5$$

$$f_{GST}(s, \text{NORTH}) = 1.0$$

$a = \text{NORTH}$
$r = -500$

s'



$$Q(s, \text{NORTH}) = +1$$

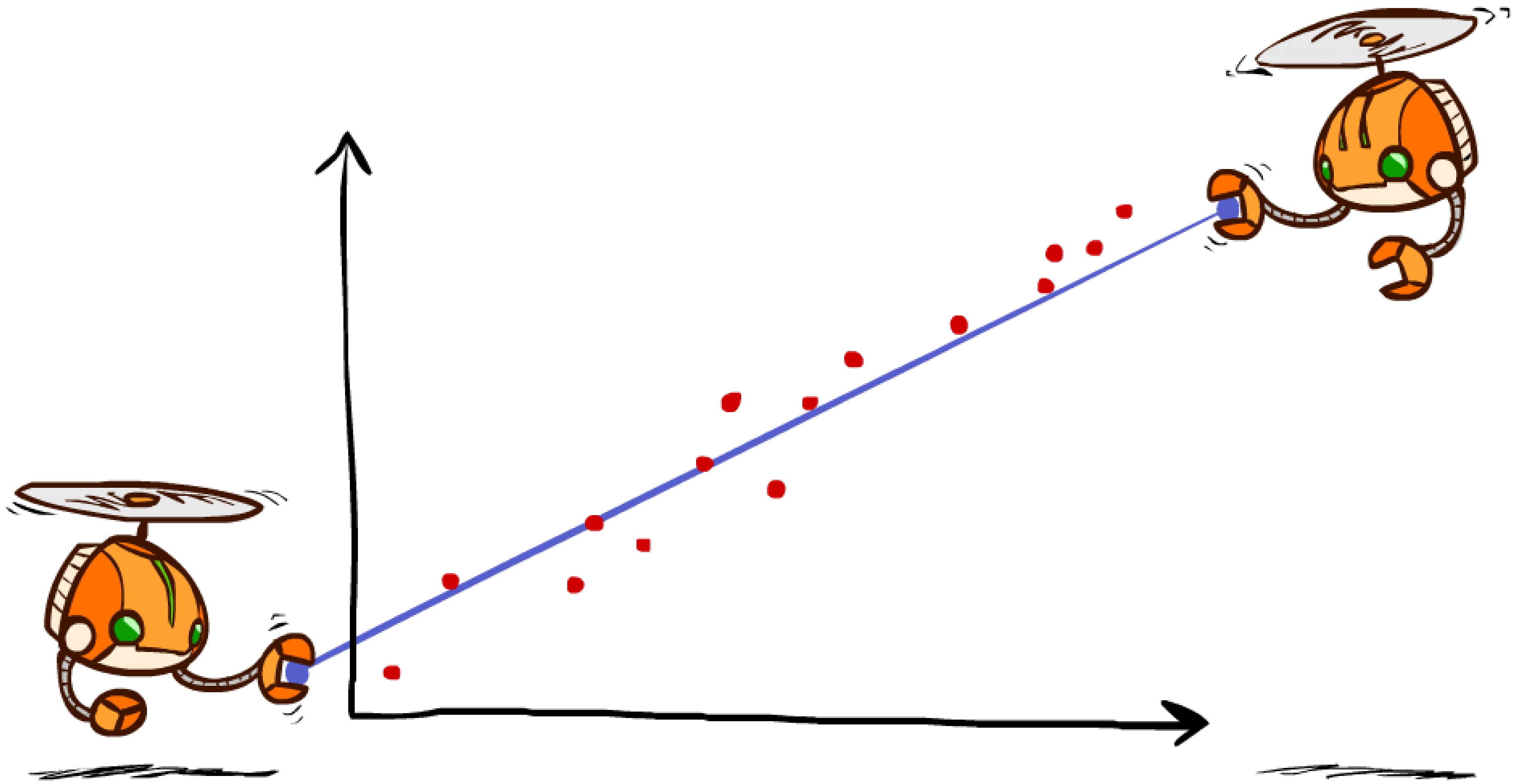
$$Q(s', \cdot) = 0$$

$$r + \gamma \max_{a'} Q(s', a') = -500 + 0$$

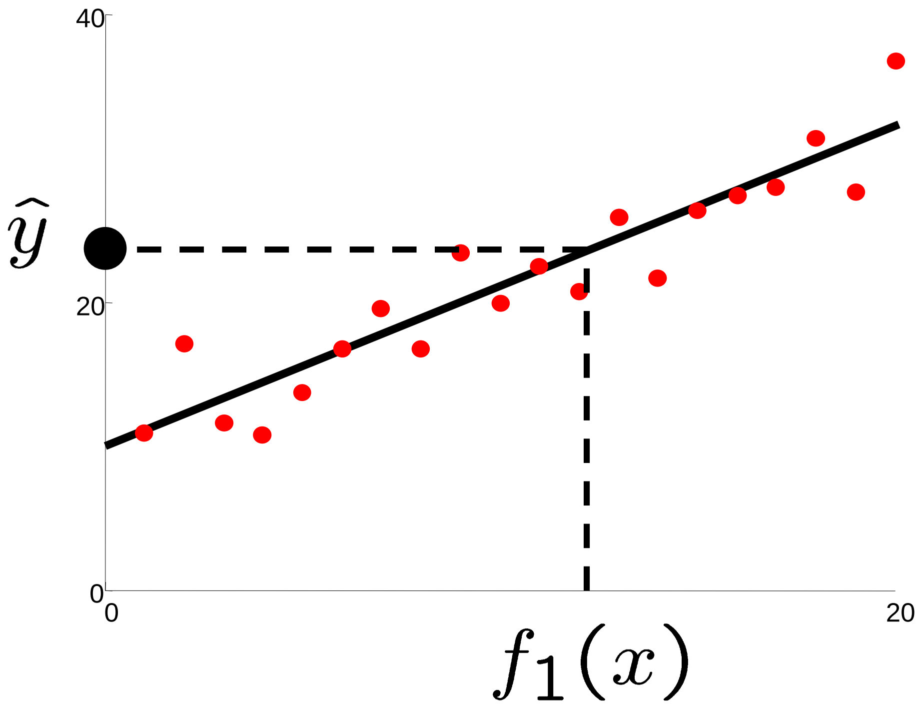
$$\text{difference} = -501 \Rightarrow \begin{array}{l} w_{DOT} \leftarrow 4.0 + \alpha\,[-501]\,0.5 \\ w_{GST} \leftarrow -1.0 + \alpha\,[-501]\,1.0 \end{array}$$

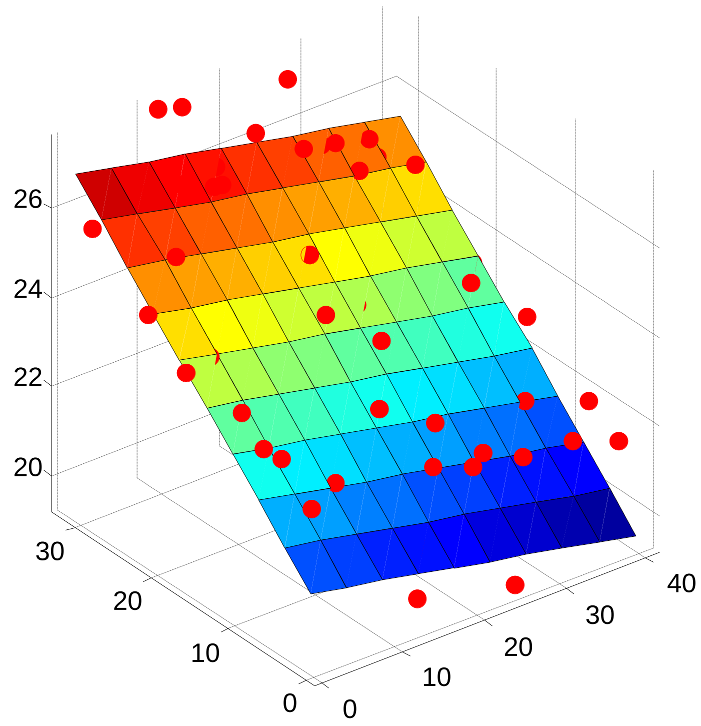
$$Q(s,a) = 3.0 f_{DOT}(s,a) - 3.0 f_{GST}(s,a)$$

Q-Learning and Least Squares



Linear Approximation: Regression





Prediction:

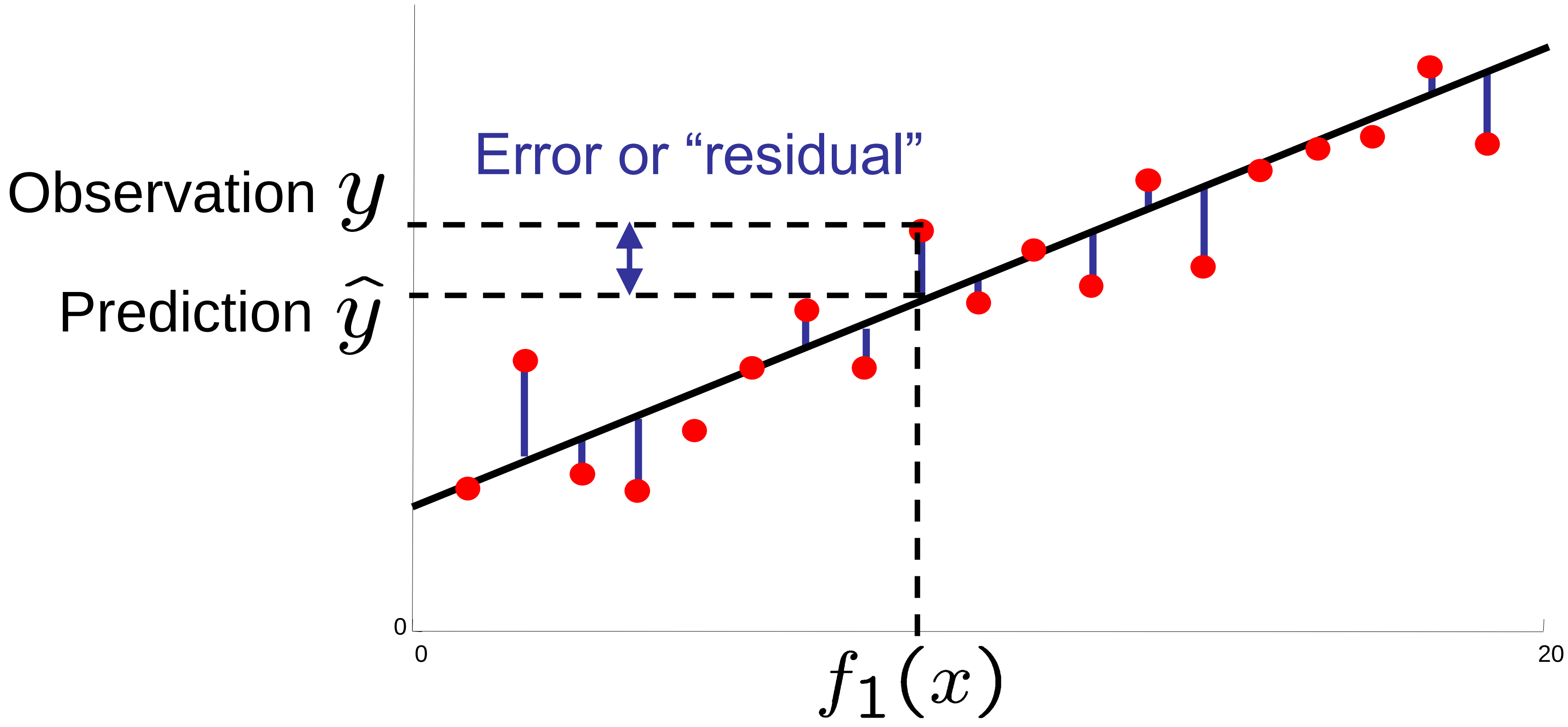
$$\hat{y} = w_0 + w_1 f_1(x)$$

Prediction:

$$\hat{y}_i = w_0 + w_1 f_1(x) + w_2 f_2(x)$$

Optimization: Least Squares

$$\text{total error} = \sum_i (y_i - \hat{y}_i)^2 = \sum_i \left(y_i - \sum_k w_k f_k(x_i) \right)^2$$



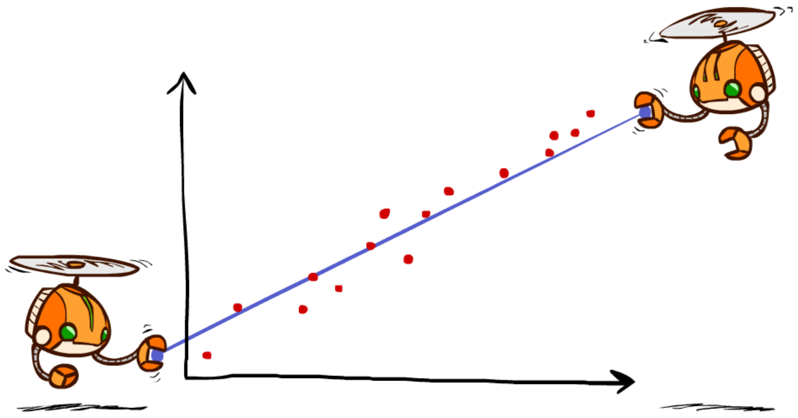
Minimizing Error

Imagine we had only one point x, with features f(x), target value y, and weights w:

$$\text{error}(w) = \frac{1}{2}\left(y - \sum_k w_k f_k(x)\right)^2$$

$$\frac{\partial\ \text{error}(w)}{\partial w_m} = -\left(y - \sum_k w_k f_k(x)\right) f_m(x)$$

$$w_m \leftarrow w_m + \alpha \left(y - \sum_k w_k f_k(x)\right) f_m(x)$$



Approximate q update explained:

$$w_m \leftarrow w_m + \alpha \left[r + \gamma \max_a Q(s', a') - Q(s,a) \right] f_m(s,a)$$

"target" "prediction"

More Powerful Function Approximation

- Linear:

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \cdots + w_n f_n(s,a)$$

- Polynomial:

$$Q(s,a) = w_{11} f_1(s,a) + w_{12} f_2(s,a)^2 + +w_{13} f_3(s,a)^3 + \cdots$$

- Neural Network:

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \cdots + w_n f_n(s,a)$$

Learn These Too

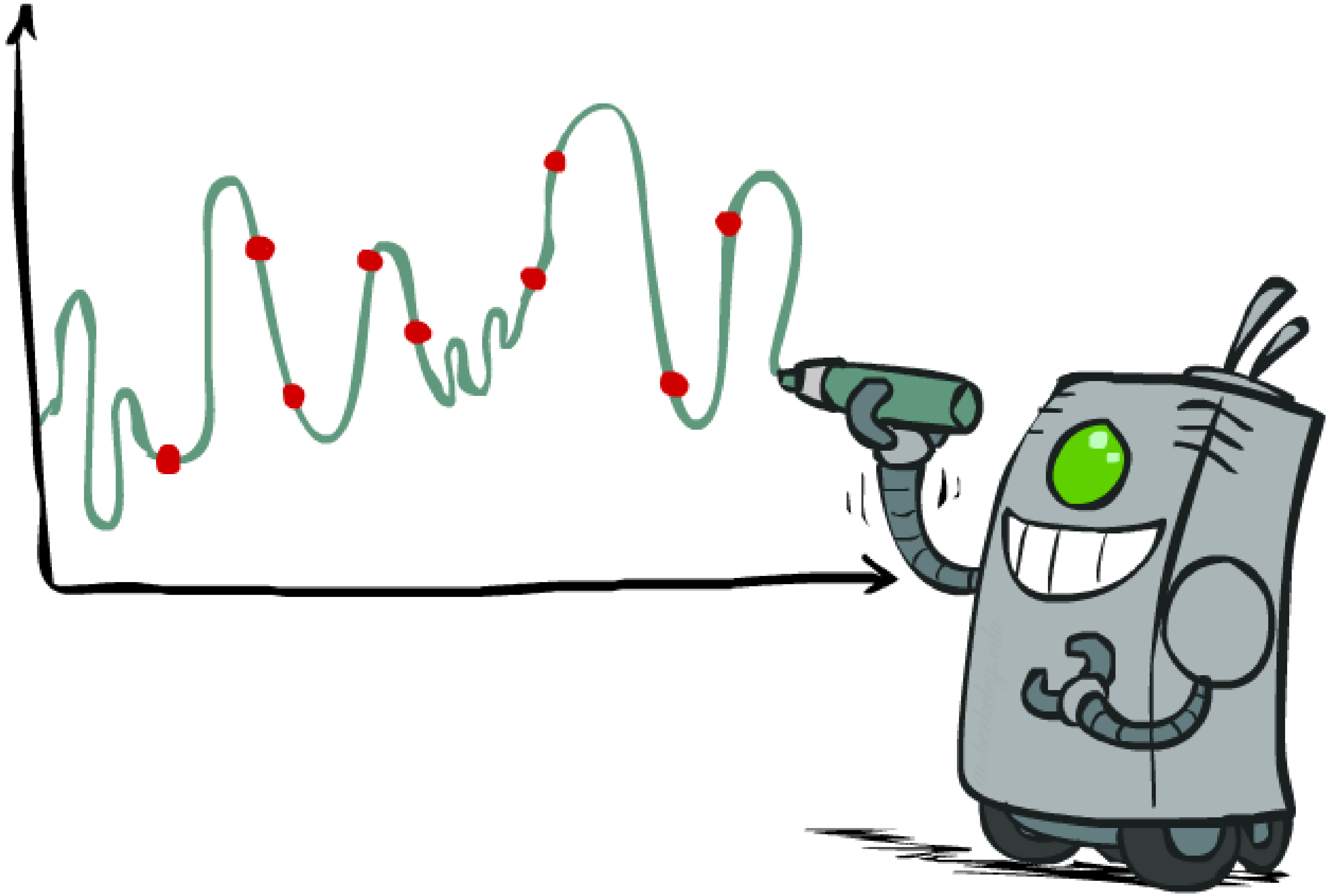
More Powerful Function Approximation

- Summary of iterative feature update equation!

$$w_m \leftarrow w_m + \alpha \left[r + \gamma \max_a Q(s', a') - Q(s, a) \right] \frac{dQ}{dw_m}(s, a)$$

$$= f_m(s, a) \text{ when linear approximation}$$

Overfitting: Why Limiting Capacity Can Help



Defining the Q-function

$$R_t = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \cdots$$

Total reward, R_t, is the discounted sum of all rewards obtained from time t

$$Q(s_t, a_t) = \mathbb{E}[R_t | s_t, a_t]$$

The Q-function captures the **expected total future reward** an agent in state, s, can receive by executing a certain action, a

How to take actions given a Q-function?

$$Q(s_t, a_t) = \mathbb{E}[R_t | s_t, a_t]$$

(state, action)

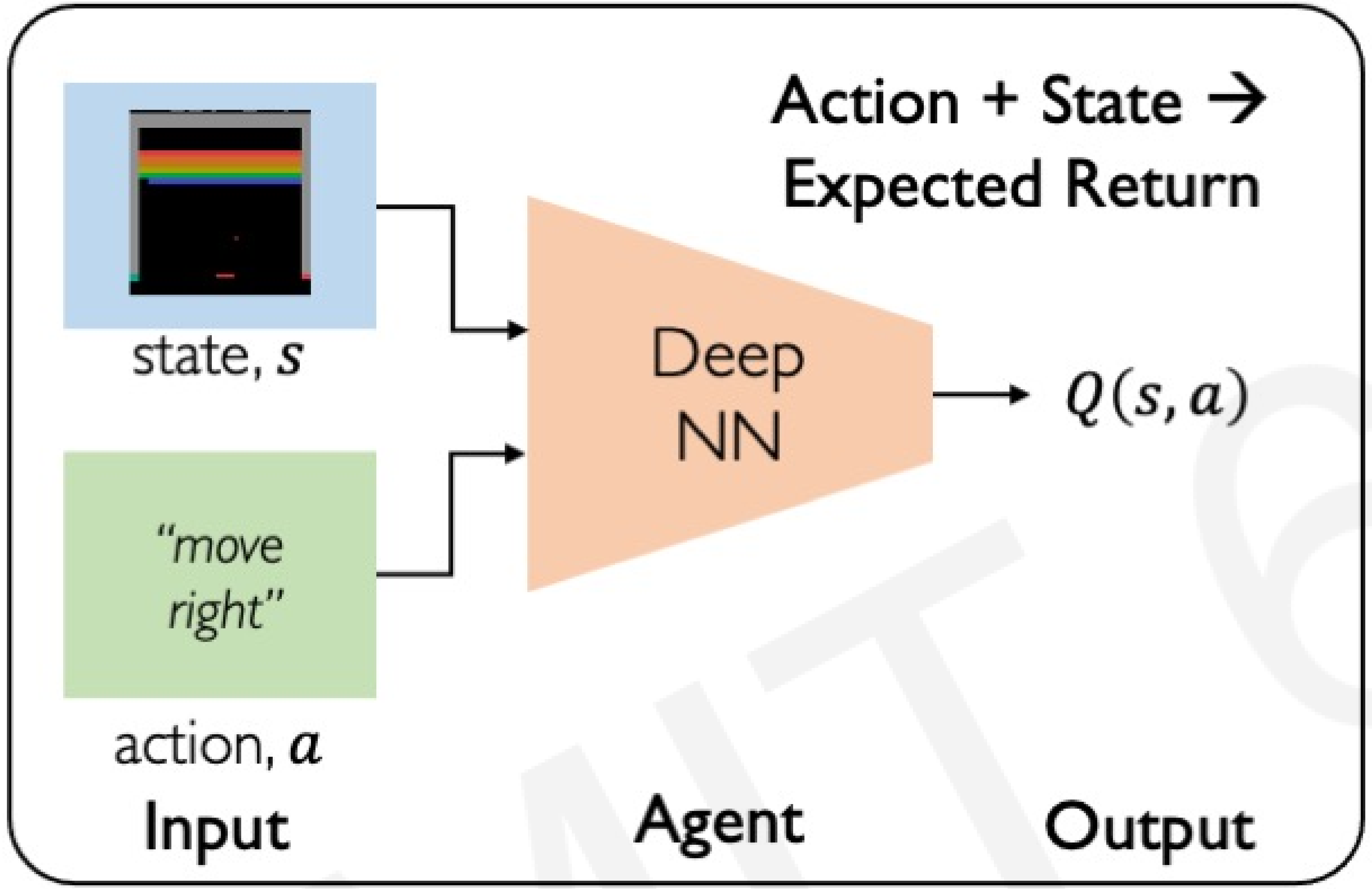
Ultimately, the agent needs a **policy** $\boldsymbol{\pi(s)}$, to infer the **best action to take** at its state, s

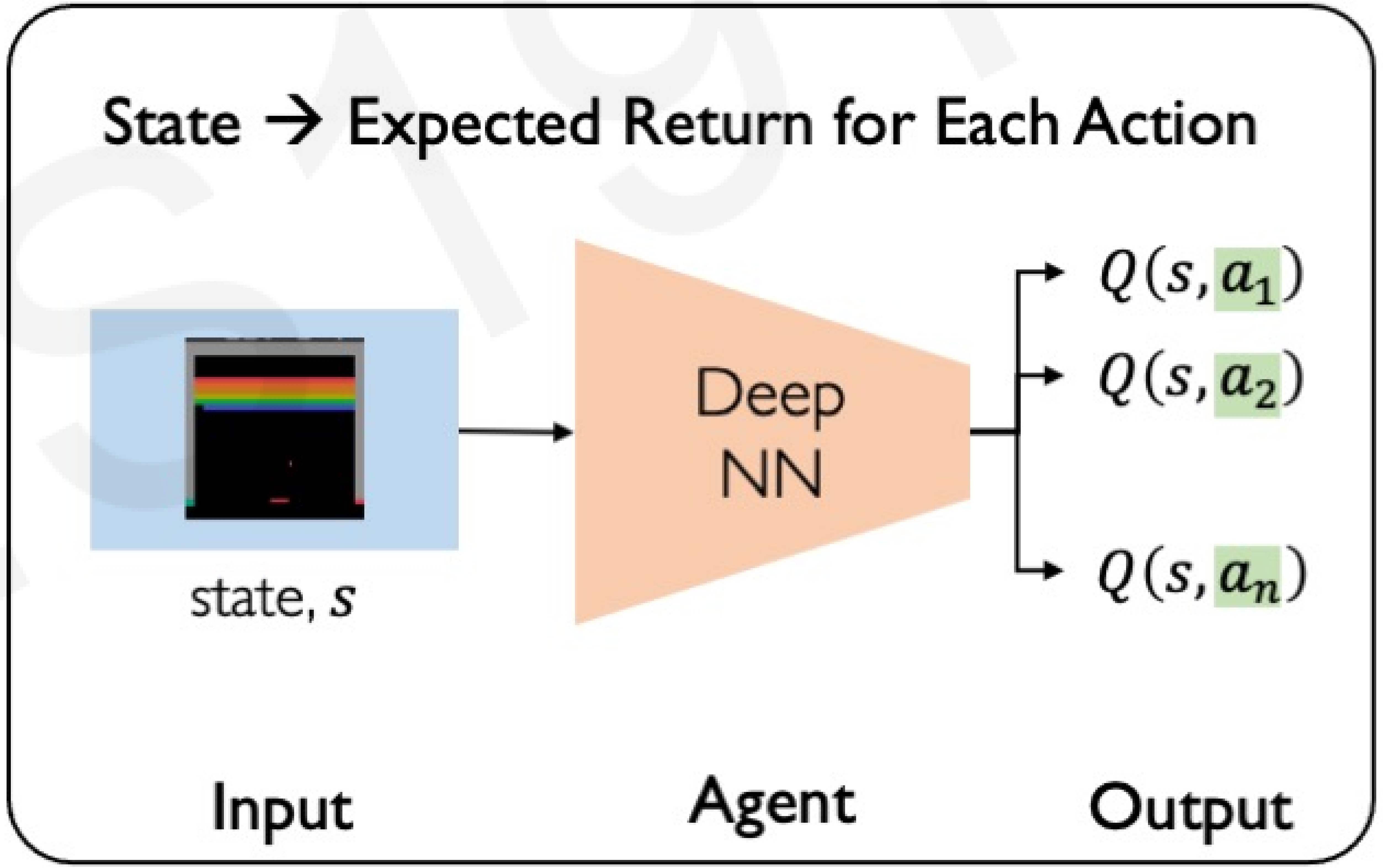
Strategy: the policy should choose an action that maximizes future reward

$$\pi^*(s) = \underset{a}{\operatorname{argmax}}\, Q(s, a)$$

Deep Q Networks (DQN)

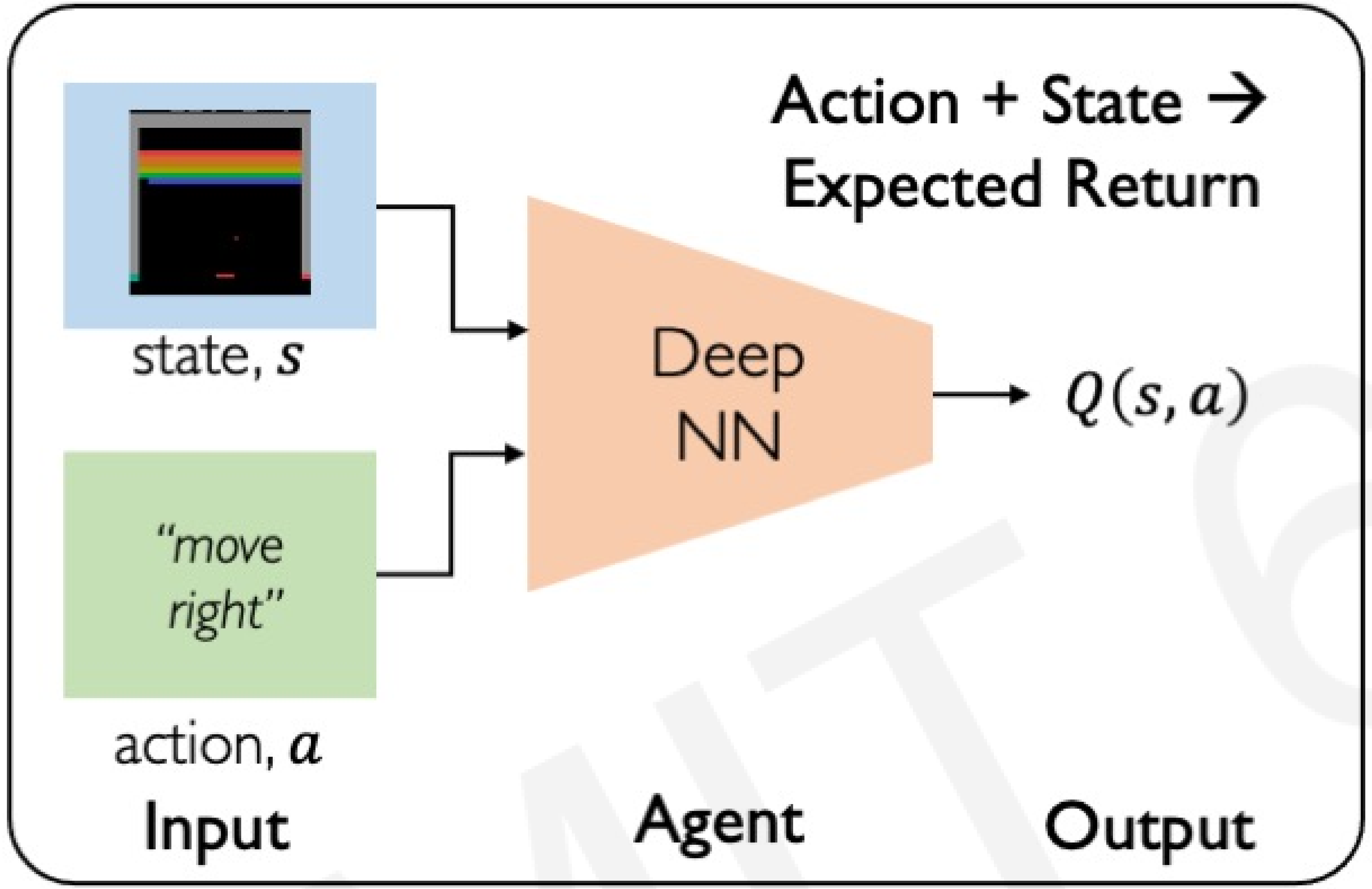
How can we use deep neural networks to model Q-functions?

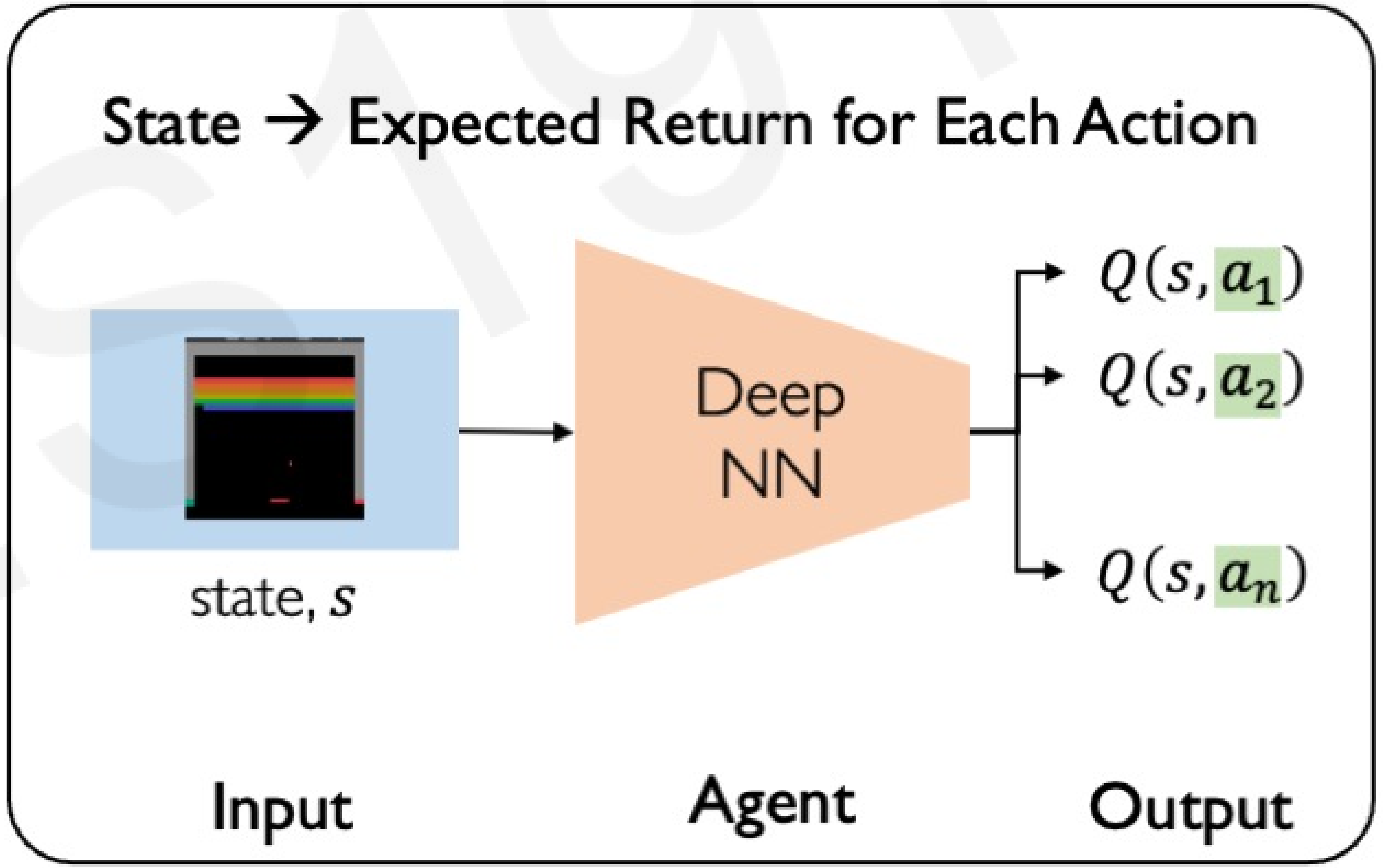




Deep Q Networks (DQN): Training

How can we use deep neural networks to model Q-functions?



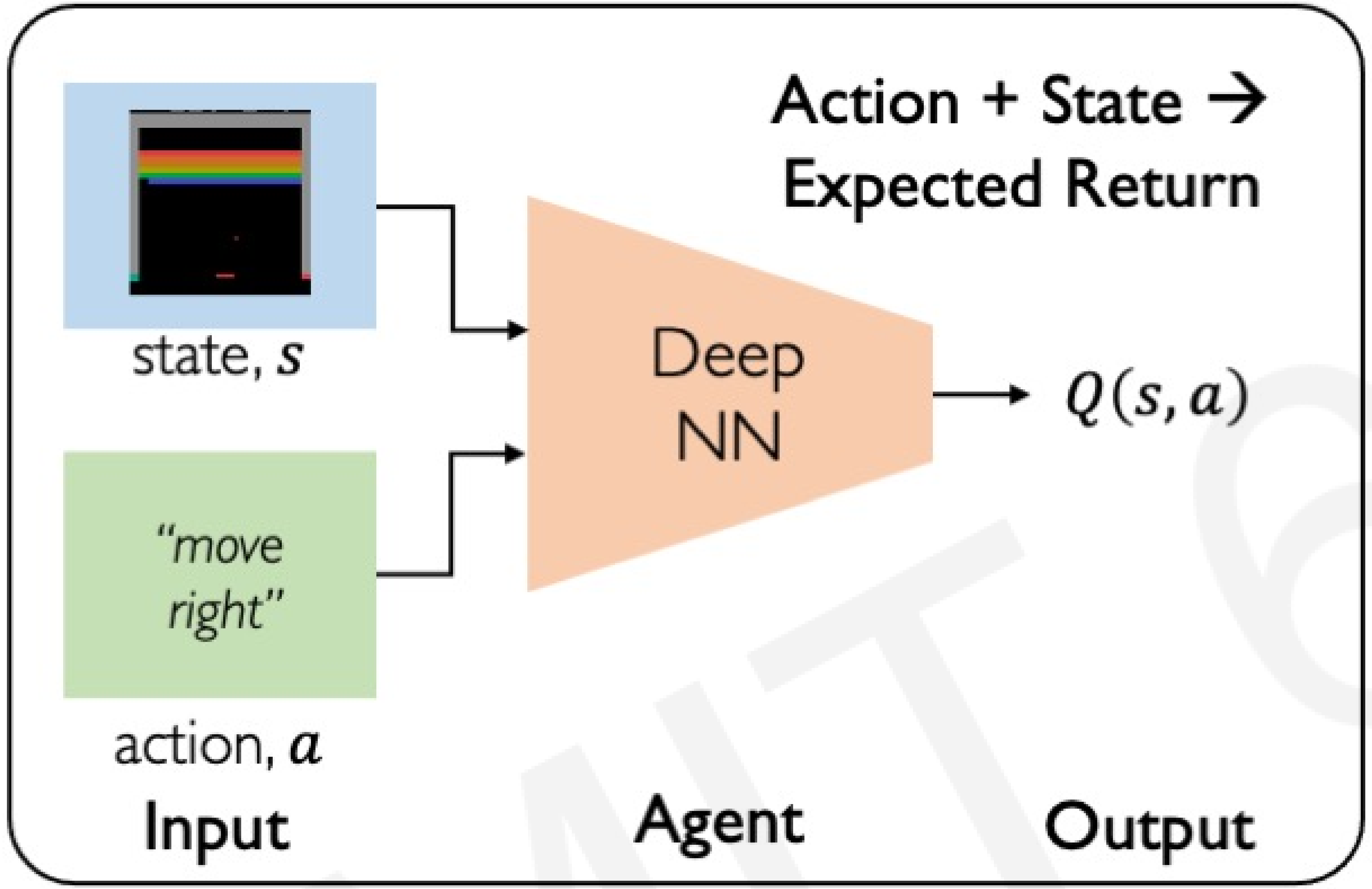


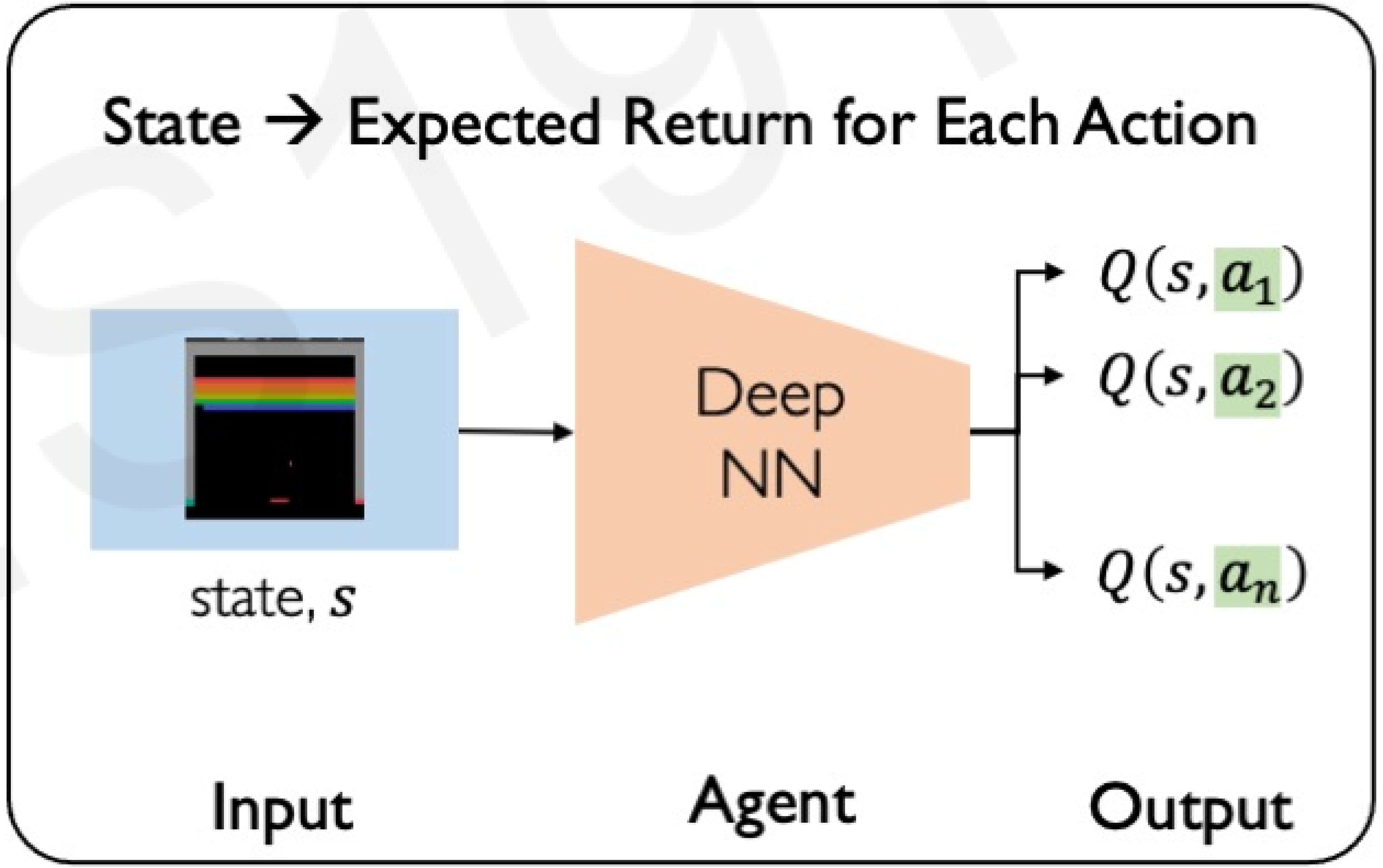
What happens if we take all the best actions?

Maximize target return → train the agent

Deep Q Networks (DQN): Training

How can we use deep neural networks to model Q-functions?



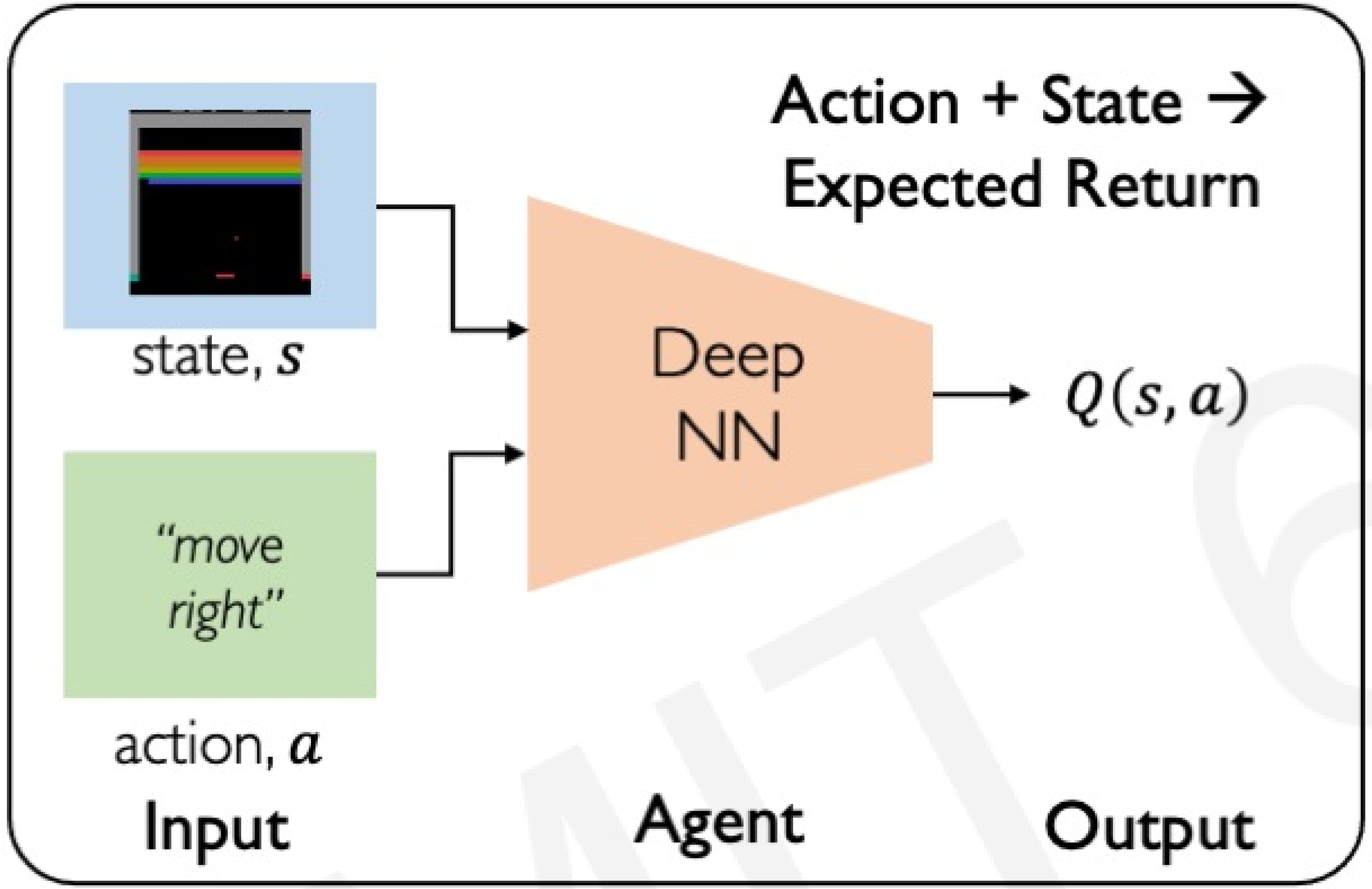


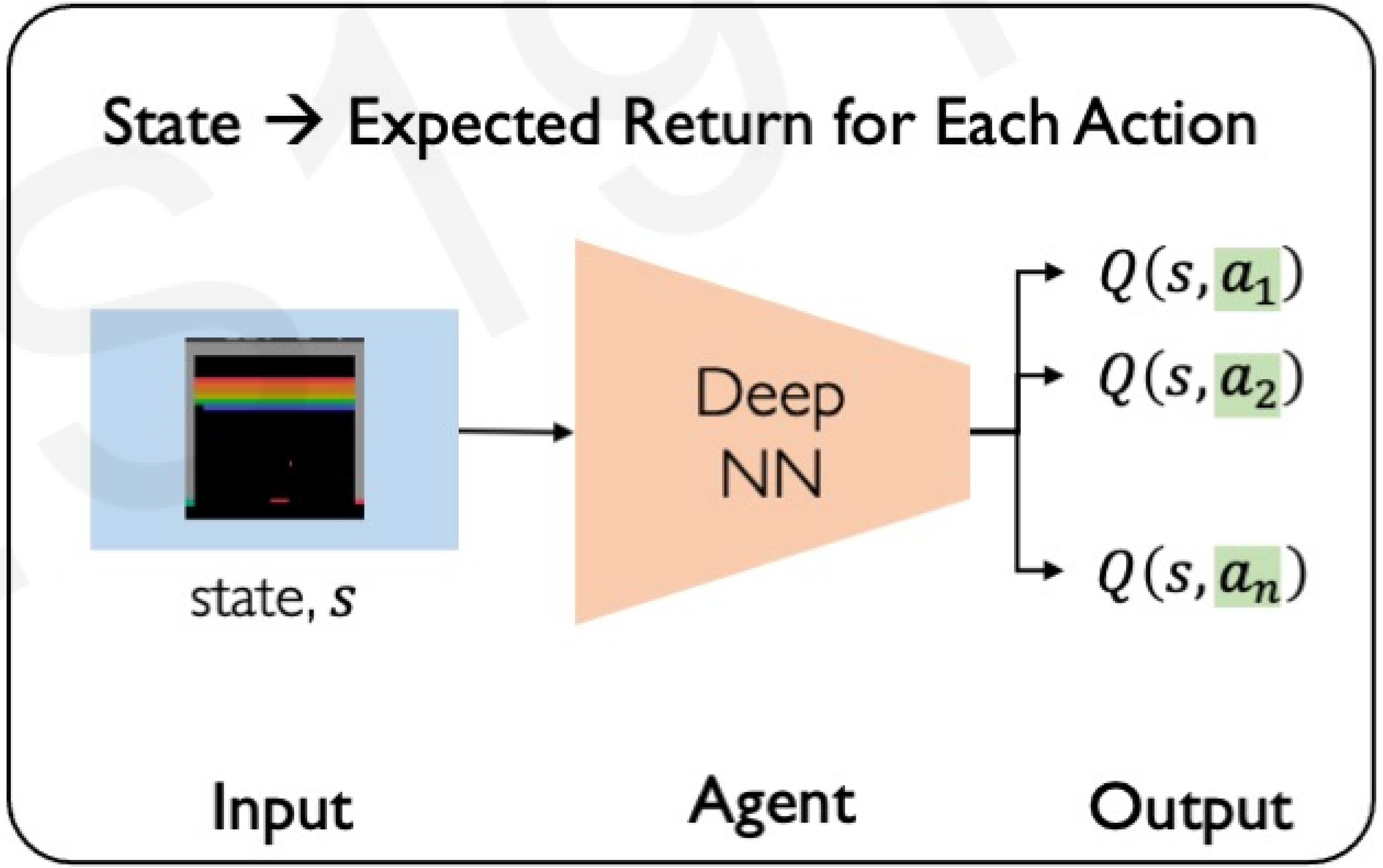
$$\overbrace{\left(r + \gamma \max_{a'} Q(s', a')\right)}^{\text{target}}$$

Take all the best actions → target return

Deep Q Networks (DQN): Training

How can we use deep neural networks to model Q-functions?



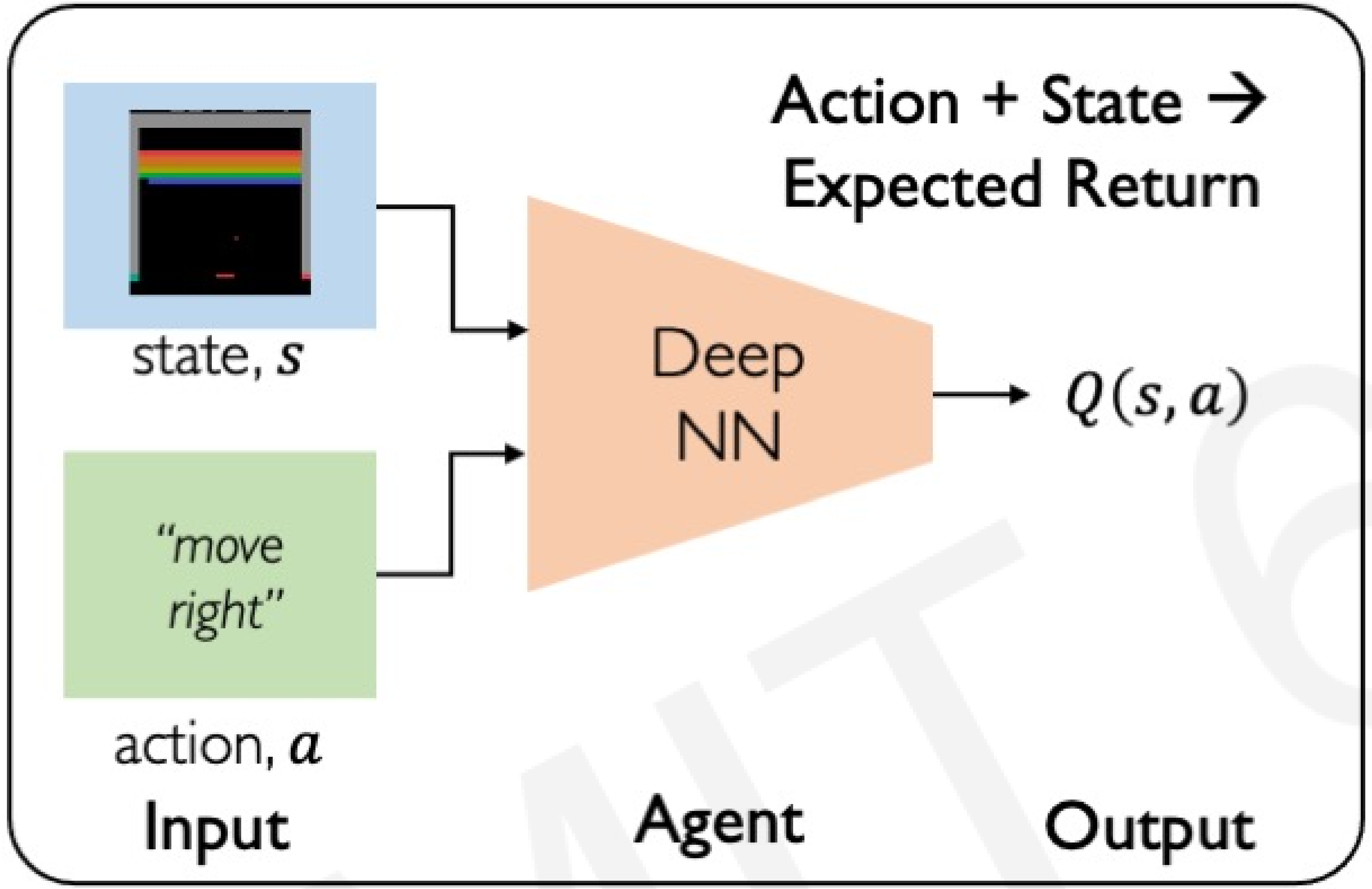


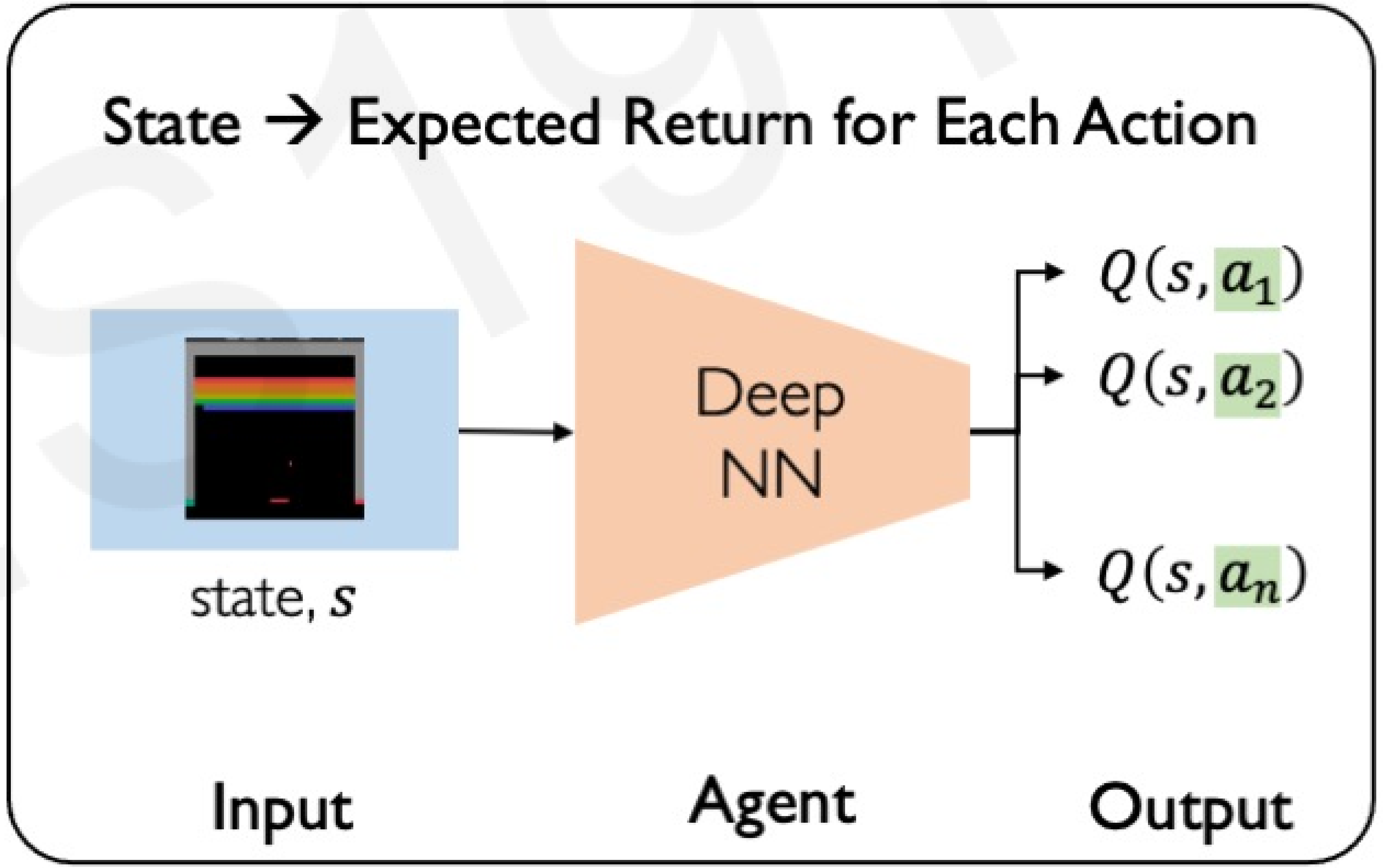
$$\underbrace{\left(r + \gamma \max_{a'} Q(s',a')\right)}_{\text{target}} \quad \underbrace{Q(s,a)}_{\text{predicted}}$$

Network prediction

Deep Q Networks (DQN): Training

How can we use deep neural networks to model Q-functions?



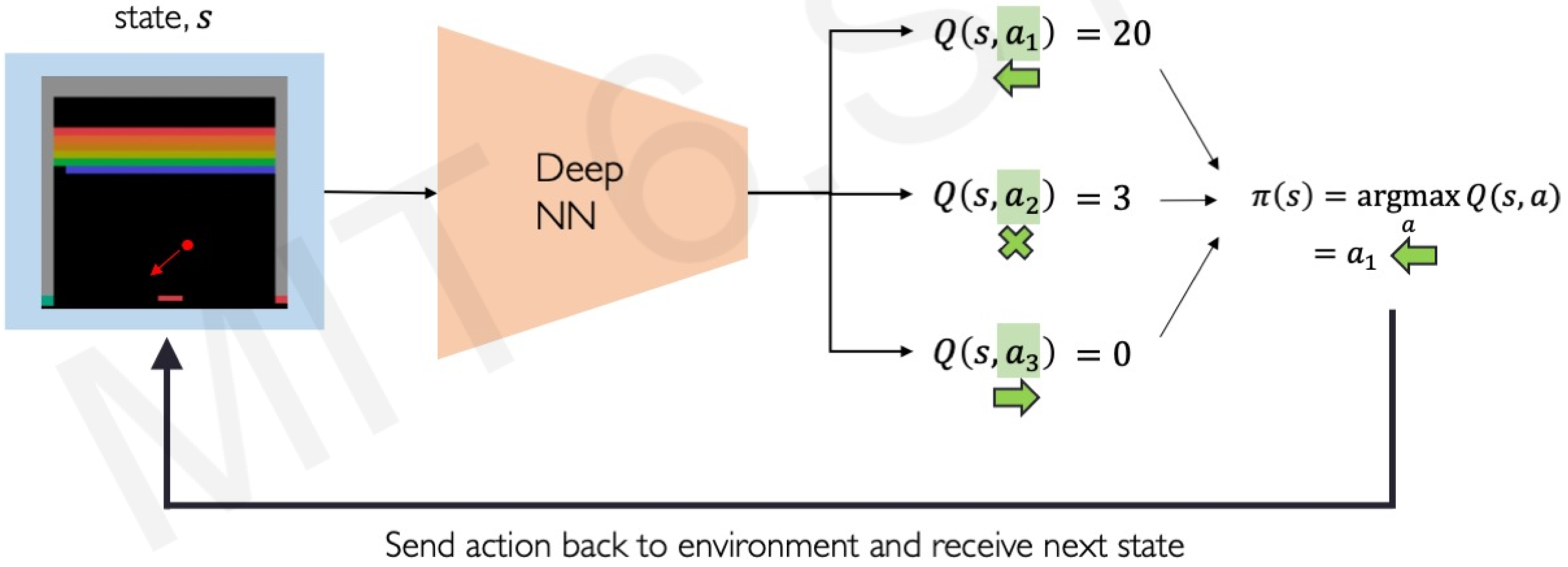


$$\mathcal{L} = \mathbb{E}\left[\left\| \overbrace{\left(r + \gamma \max_{a'} Q(s',a')\right)}^{\text{target}} - \overbrace{Q(s,a)}^{\text{predicted}} \right\|^2\right]$$

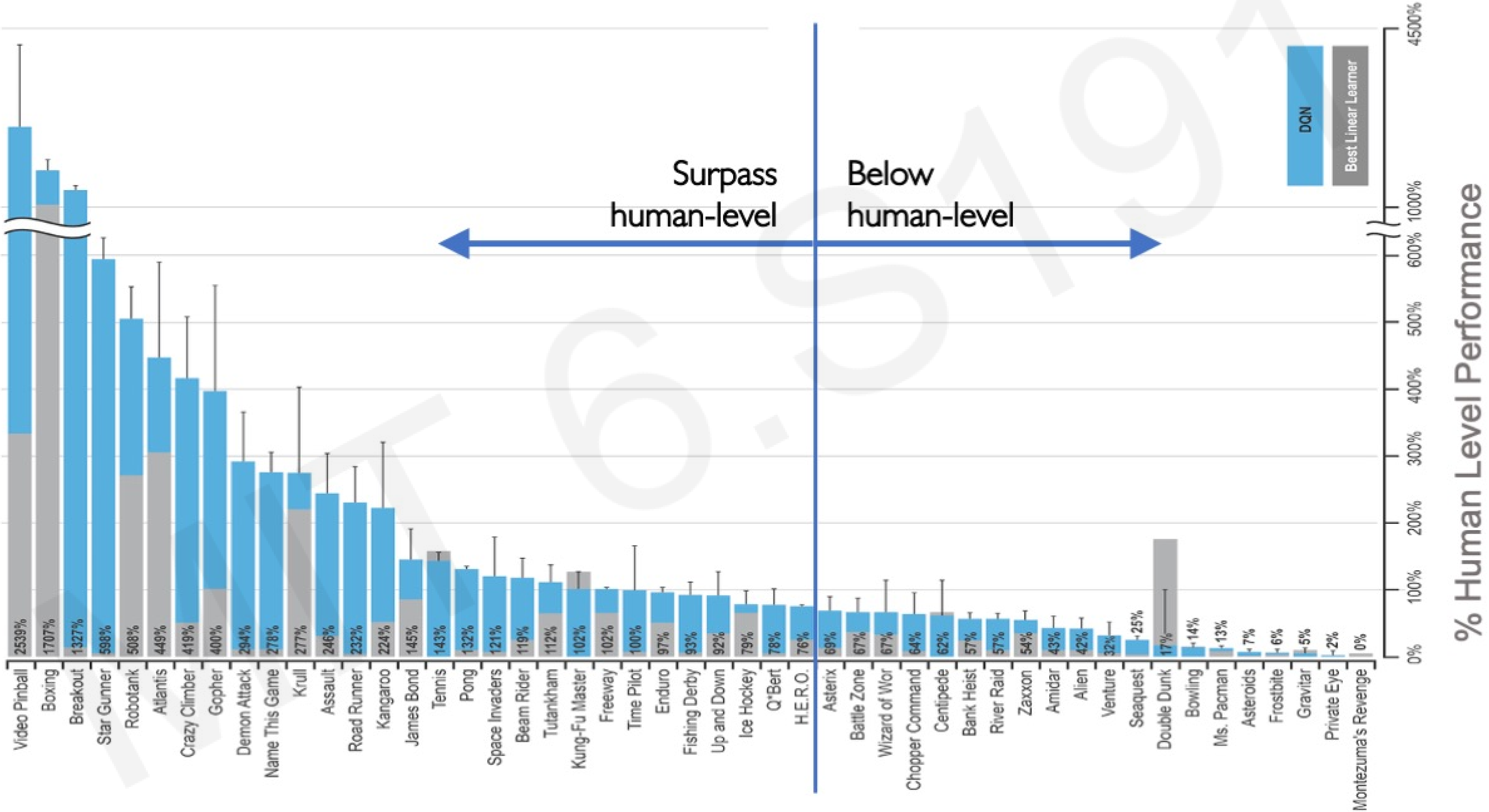
Q-Loss

Deep Q Network Summary

Use NN to learn Q-function and then use to infer the optimal policy, $\pi(s)$



DQN Atari Results



Downsides of Q-learning

Complexity:
- Can model scenarios where the action space is discrete and small
- Cannot handle continuous action spaces

Flexibility:
- Policy is deterministically computed from the Q function by maximizing the reward → cannot learn stochastic policies

To address these, consider a new class of RL training algorithms:
Policy gradient methods

Deep Reinforcement Learning Algorithms

Value Learning

Find $Q(s, a)$

$$a = \underset{a}{\operatorname{argmax}}\, Q(s, a)$$

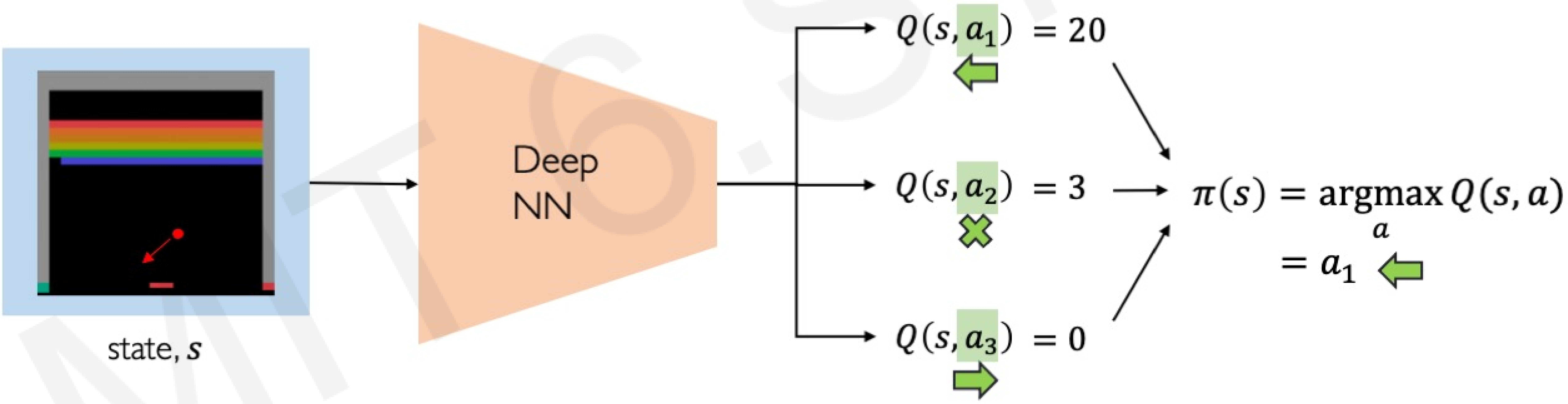
Policy Learning

Find $\pi(s)$

Sample $a \sim \pi(s)$

Deep Q Networks (DQN)

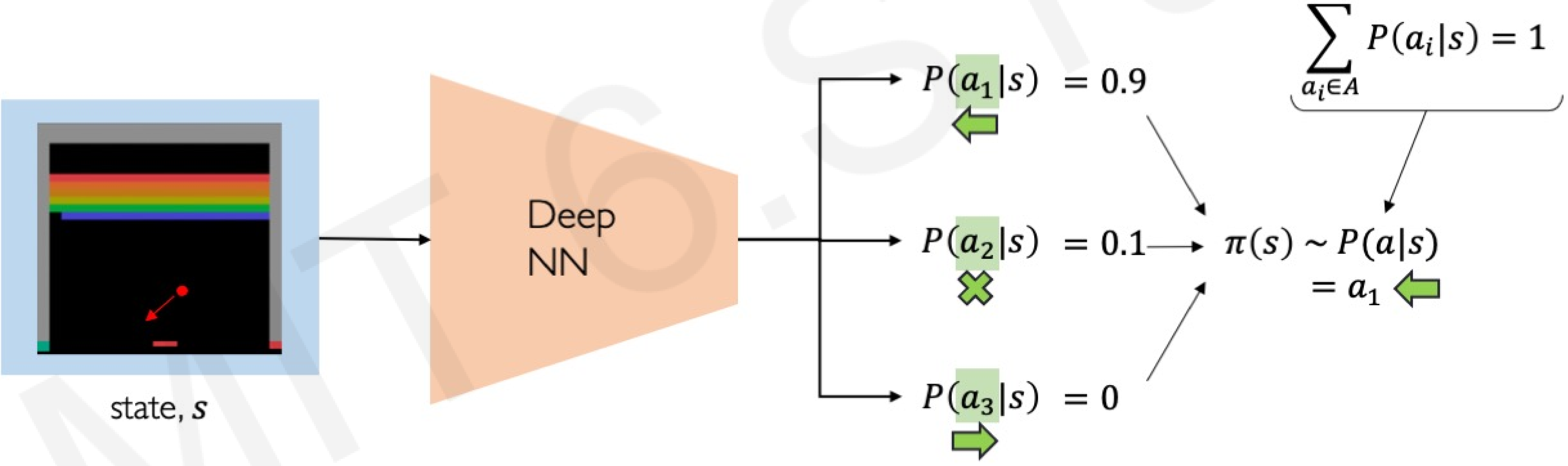
DQN: Approximate Q-function and use to infer the optimal policy, $\pi(s)$



Policy Gradient (PG): Key Idea

DQN: Approximate Q-function and use to infer the optimal policy, $\pi(s)$

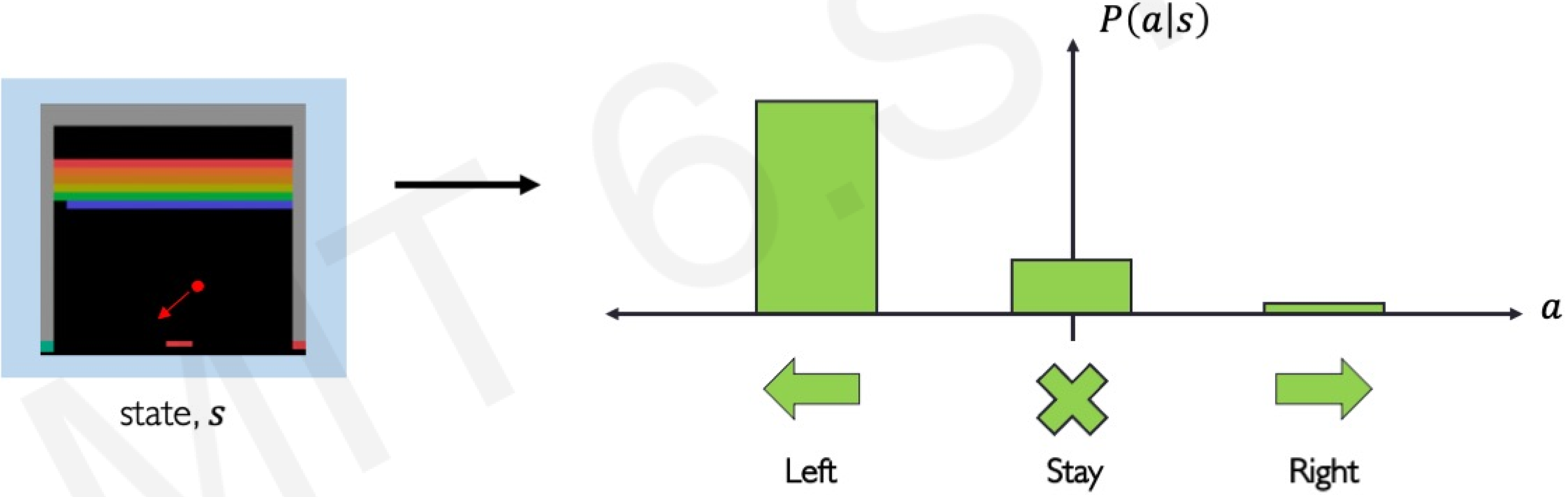
Policy Gradient: Directly optimize the policy $\pi(s)$



🤔 What are some advantages of this formulation?

Discrete vs Continuous Action Spaces

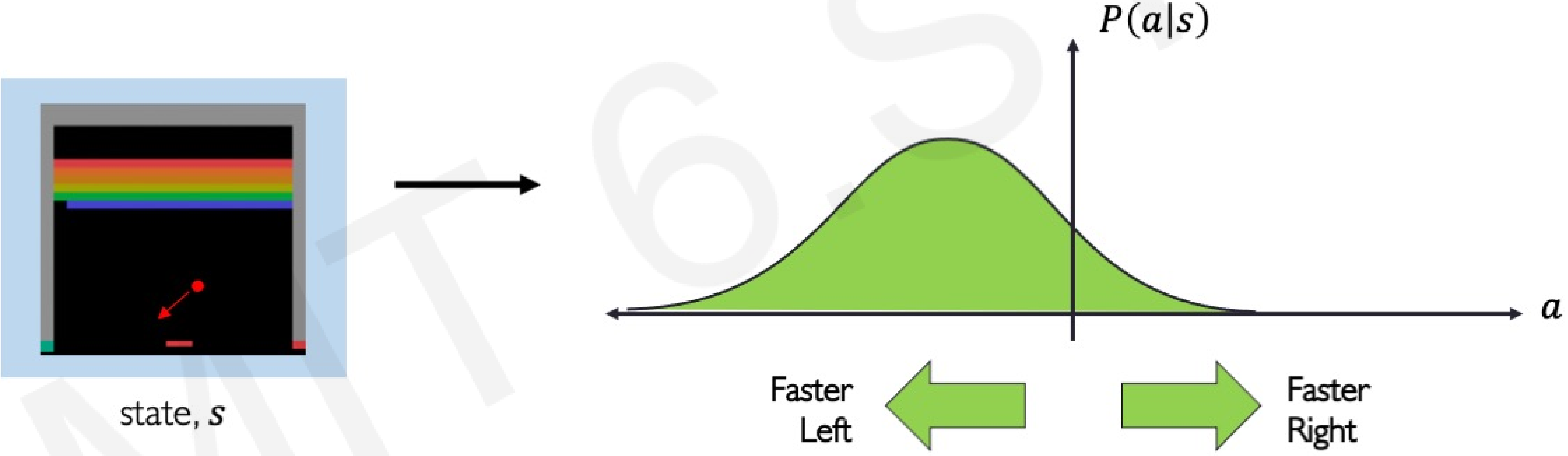
Discrete action space: which direction should I move?



Discrete vs Continuous Action Spaces

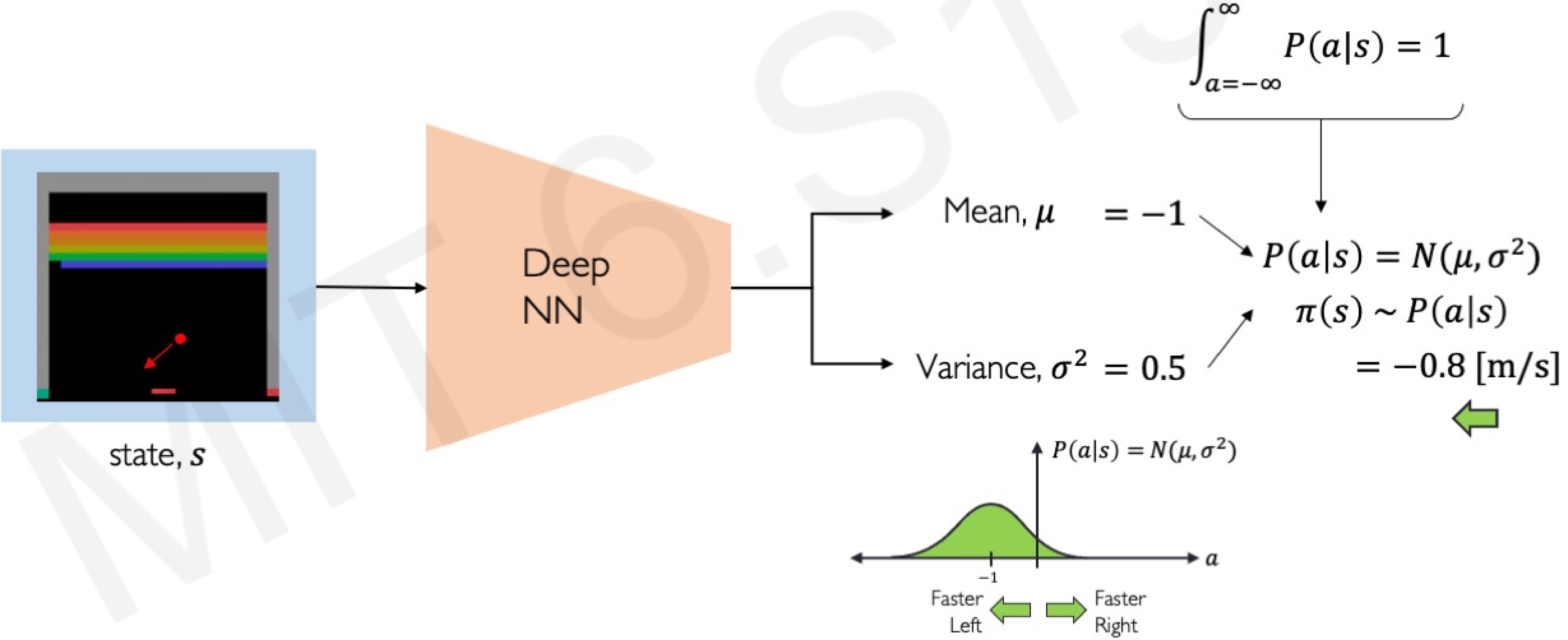
Discrete action space: which direction should I move?

Continuous action space: how fast should I move?



Policy Gradient (PG): Key Idea

Policy Gradient: Enables modeling of continuous action space



Deep Reinforcement Learning: Summary

Foundations

- Agents acting in environment
- State-action pairs → maximize future rewards
- Discounting

Q-Learning

- Q function: expected total reward given **s, a**
- Policy determined by selecting action that maximizes Q function

Policy Gradients

- Learn and optimize the policy directly
- Applicable to continuous action spaces