Câu 3. *(2 điểm)*

Cho p(x), q(x) $\in P_2[x]$, chứng minh rằng $<p, q> = \int_{-1}^{1} p(x)q(x)dx$ là một tích vô hướng trong $P_2[x]$. Hãy trực chuẩn hóa cơ sở $\{1, x-1, x^2-1\}$.

$$S = \{u_1(1,0,0), u_2(-1,1,0), u_3(-1,0,1)\}$$

$$\downarrow$$

$$S_\perp = \{v_1 = ?, v_2 = ?, v_3 = ?\}$$

$$\downarrow$$

$$S_e = \{e_1 = ?, e_2 = ?, e_3 = ?\}$$

$$\forall u, v, w \in V,\ \alpha \in R$$

$$\downarrow$$

$$\langle u, v \rangle = \alpha$$

$$\uparrow$$

1. $\langle u, v \rangle = \langle v, u \rangle$
2. $\langle u + w, v \rangle = \langle u, v \rangle + \langle w, v \rangle$
3. $\langle \beta u, v \rangle = \beta \langle v, u \rangle, \forall \beta \in R$
4. $\langle u, u \rangle \geq 0,\ \langle u, u \rangle = 0 \leftrightarrow u = 0_V$

$$\langle p, q \rangle = \int_{-1}^{1} pqdx = \alpha \in R$$

$$\langle p, q \rangle = \int_{-1}^{1} pqdx = \int_{-1}^{1} qpdx = \langle q, p \rangle$$

$$\langle p + r, q \rangle = \int_{-1}^{1} (p + r)qdx = \int_{-1}^{1} (pq + rq)dx$$

$$= \int_{-1}^{1} pqdx + \int_{-1}^{1} rqdx = \langle q, p \rangle + \langle r, q \rangle,\ r \in P_2$$

$$\langle \beta p, q \rangle = \int_{-1}^{1} \beta pqdx = \beta \int_{-1}^{1} qpdx = \beta \langle q, p \rangle,\ \beta \in R$$

$$\langle p, p \rangle = \int_{-1}^{1} ppdx = \int_{-1}^{1} p^2dx \geq 0,\ do\ p^2 \geq 0$$

$$\langle p, p \rangle = 0 \Leftrightarrow p = 0$$