33-445 Formula Sheet

Operator Properties

$$\langle\hat{A}\rangle = \langle\psi \mid \hat{A} \mid \psi\rangle \qquad \Delta\hat{A} = \sqrt{\langle\hat{A}^2\rangle - \langle\hat{A}\rangle^2} \qquad \hat{U}\hat{U}^\dagger = 1 \qquad [\hat{A},\hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A} \qquad \Delta\hat{A}\Delta\hat{B} \geq \langle\frac{[\hat{A},\hat{B}]}{2i}\rangle \qquad e^{\hat{A}} = \sum_{n=0}^{\infty}\frac{\hat{A}^n}{n!}$$

Vector Properties

$$\langle\psi \mid \psi\rangle = 1\langle a_i \mid a_j\rangle \qquad = \delta_{ij}\hat{A} \mid \psi\rangle \qquad = \langle\psi\mid^\dagger \hat{A}^\dagger \mid \psi\rangle \qquad = \sum_n c_n \mid a_n\rangle = \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix}\langle\psi\mid \qquad = \sum_n c_n^\dagger\langle a_n \mid = \begin{pmatrix} c_1^\dagger & \dots & c_n^\dagger \end{pmatrix}$$

All spins

$$\vec{n} = \sin\theta\cos\varphi\vec{i} + \sin\theta\sin\varphi\vec{j} + \cos\theta\vec{k}$$

$$z: \theta = 0, \varphi = 0 \qquad x: \theta = \frac{\pi}{2}, \varphi = 0 \qquad y: \theta = \frac{\pi}{2}, \varphi = \frac{\pi}{2} \qquad [\hat{S}_x,\hat{S}_y] = i\hbar\hat{S}_z \qquad [\hat{S}_y,\hat{S}_z] = i\hbar\hat{S}_x \qquad [\hat{S}_z,\hat{S}_x] = i\hbar\hat{S}_y$$

$$\hat{J}_+ = \hat{J}_x + i\hat{J}_y \qquad \hat{J}_- = \hat{J}_x - i\hat{J}_y \qquad \hat{J}_+ = \hat{J}_-^\dagger \qquad \hat{J}_- = \hat{J}_+^\dagger$$

$$\hat{J}_+ \mid j,m\rangle = \hbar\sqrt{j(j+1) - m(m+1)} \mid j,m+1\rangle \qquad \hat{J}_- \mid j,m\rangle = \hbar\sqrt{j(j+1) - m(m-1)} \mid j,m-1\rangle$$

Spin $\frac{1}{2}$ (Electrons)

$$\mid +n\rangle = \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}e^{i\varphi} \end{pmatrix} \qquad \mid -n\rangle = \begin{pmatrix} \sin\frac{\theta}{2} \\ -\cos\frac{\theta}{2}e^{-i\varphi} \end{pmatrix} \qquad \hat{S}_n = \frac{\hbar}{2}\begin{pmatrix} \cos\theta & \sin\theta e^{-i\varphi} \\ \sin\theta e^{i\varphi} & -\cos\theta \end{pmatrix}$$

Spin 1 (Photons)

$$\mid X'\rangle = \cos\phi \mid X\rangle + \sin\phi \mid Y\rangle \qquad \mid R\rangle = \frac{1}{\sqrt{2}}(\mid X\rangle + i \mid Y\rangle) \qquad \hat{R}\phi\vec{k} \mid R\rangle = e^{i\phi} \mid R\rangle \qquad \hat{J}_z \mid R\rangle = \hbar \mid R\rangle$$

$$\mid Y'\rangle = -\sin\phi \mid X\rangle + \cos\phi \mid Y\rangle \qquad \mid L\rangle = \frac{1}{\sqrt{2}}(\mid X\rangle - i \mid Y\rangle) \qquad \hat{R}\phi\vec{k} \mid L\rangle = e^{i\phi} \mid L\rangle \qquad \hat{J}_z \mid L\rangle = -\hbar \mid L\rangle$$

Spin 1 (General)

$$\mid 1,1\rangle_n = \begin{pmatrix} e^{-i\varphi}\frac{1+\cos\theta}{2} \\ \frac{\sin\theta}{\sqrt{2}} \\ e^{i\varphi}\frac{1-\cos\theta}{2} \end{pmatrix} \quad \mid 1,0\rangle_n = \begin{pmatrix} -e^{-i\varphi}\frac{\sin\theta}{\sqrt{2}} \\ \cos\theta \\ e^{i\varphi}\frac{\sin\theta}{\sqrt{2}} \end{pmatrix} \quad \mid 1,-1\rangle_n = \begin{pmatrix} e^{-i\varphi}\frac{1-\cos\theta}{2} \\ \frac{-\sin\theta}{\sqrt{2}} \\ e^{i\varphi}\frac{1+\cos\theta}{2} \end{pmatrix} \quad \hat{S}_n = \hbar\begin{pmatrix} \cos\theta & \frac{e^{-i\varphi}\sin\theta}{\sqrt{2}} & 0 \\ \frac{e^{i\varphi}\sin\theta}{\sqrt{2}} & 0 & \frac{e^{-i\varphi}\sin\theta}{\sqrt{2}} \\ 0 & \frac{e^{i\varphi}\sin\theta}{\sqrt{2}} & -\cos\theta \end{pmatrix}$$

Hamiltonian

$$i\hbar\frac{d}{dt} \mid \psi(t)\rangle = \hat{H} \mid \psi(t)\rangle \qquad \hat{H}^\dagger = \hat{H} \qquad \frac{\partial\hat{A}}{\partial t} = 0 \Rightarrow \frac{d\langle\hat{A}\rangle}{dt} = \frac{i}{\hbar}\langle[\hat{H},\hat{A}]\rangle \qquad \hat{U}(t) \mid \psi(0)\rangle = \mid \psi(t)\rangle$$

$$\frac{\partial\hat{H}}{\partial t} = 0 \Rightarrow \hat{U}(t) = e^{-i\hat{H}\frac{t}{\hbar}}$$

Constant B_0 in the $\vec{n}$ direction: $\hat{H} = -\frac{qB_0}{mc}\hat{S}_n = \omega_0\hat{S}_n$