

33-445 Formula Sheet

Operator Properties

$$\langle \hat{A} \rangle = \langle \psi | \hat{A} | \psi \rangle \quad \Delta \hat{A} = \sqrt{\langle \hat{A}^2 \rangle - \langle \hat{A} \rangle^2} \quad \hat{U} \hat{U}^\dagger = 1 \quad [\hat{A}, \hat{B}] = \hat{A} \hat{B} - \hat{B} \hat{A} \quad \Delta \hat{A} \Delta \hat{B} \geq \langle \frac{[\hat{A}, \hat{B}]}{2i} \rangle$$

All spins

$$\vec{n} = \sin \theta \cos \varphi \vec{i} + \sin \theta \sin \varphi \vec{j} + \cos \theta \vec{k}$$

$$z : \theta = 0, \varphi = 0 \quad x : \theta = \frac{\pi}{2}, \varphi = 0 \quad y : \theta = \frac{\pi}{2}, \varphi = \frac{\pi}{2} \quad [\hat{S}_x, \hat{S}_y] = i\hbar \hat{S}_z \quad [\hat{S}_y, \hat{S}_z] = i\hbar \hat{S}_x \quad [\hat{S}_z, \hat{S}_x] = i\hbar \hat{S}_y$$

Spin $\frac{1}{2}$ (electrons)

$$|+n\rangle = \begin{pmatrix} \cos \frac{\theta}{2} \\ \sin \frac{\theta}{2} e^{i\varphi} \end{pmatrix} \quad |-n\rangle = \begin{pmatrix} \sin \frac{\theta}{2} \\ -\cos \frac{\theta}{2} e^{-i\varphi} \end{pmatrix} \quad \hat{S}_n = \frac{\hbar}{2} \begin{pmatrix} \cos \theta & \sin \theta e^{-i\varphi} \\ \sin \theta e^{i\varphi} & -\cos \theta \end{pmatrix}$$

Spin 1 (Photons)

$$\begin{aligned} |X'\rangle &= \cos \varphi |X\rangle + \sin \varphi |Y\rangle & |R\rangle &= \frac{1}{\sqrt{2}}(|X\rangle + i|Y\rangle) & \hat{R}\varphi \vec{k} |R\rangle &= e^{i\varphi} |R\rangle & \hat{J}_z |R\rangle &= \hbar |R\rangle \\ |Y'\rangle &= -\sin \varphi |X\rangle + \cos \varphi |Y\rangle & |L\rangle &= \frac{1}{\sqrt{2}}(|X\rangle - i|Y\rangle) & \hat{R}\varphi \vec{k} |L\rangle &= e^{i\varphi} |L\rangle & \hat{J}_z |L\rangle &= -\hbar |L\rangle \end{aligned}$$

Spin 1 (General)

$$|1, 1\rangle_n = \begin{pmatrix} e^{-i\varphi} \frac{1+\cos \theta}{2} \\ \frac{\sin \theta}{\sqrt{2}} \\ e^{i\varphi} \frac{1-\cos \theta}{2} \end{pmatrix} \quad |1, 0\rangle_n = \begin{pmatrix} -e^{-i\varphi} \frac{\sin \theta}{\sqrt{2}} \\ \cos \theta \\ e^{i\varphi} \frac{\sin \theta}{\sqrt{2}} \end{pmatrix} \quad |1, -1\rangle_n = \begin{pmatrix} e^{-i\varphi} \frac{1-\cos \theta}{2} \\ -\frac{\sin \theta}{\sqrt{2}} \\ e^{i\varphi} \frac{1+\cos \theta}{2} \end{pmatrix} \quad \hat{S}_n = \hbar \begin{pmatrix} \cos \theta & \frac{e^{-i\varphi} \sin \theta}{\sqrt{2}} & 0 \\ \frac{e^{i\varphi} \sin \theta}{\sqrt{2}} & 0 & \frac{e^{-i\varphi} \sin \theta}{\sqrt{2}} \\ 0 & \frac{e^{i\varphi} \sin \theta}{\sqrt{2}} & -\cos \theta \end{pmatrix}$$

Hamiltonian

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = \hat{H} |\psi(t)\rangle \quad \frac{d\langle \hat{A} \rangle}{dt} = \frac{i}{\hbar} \langle [\hat{H}, \hat{A}] \rangle$$