33-445 Formula Sheet

Operator Properties

$$\langle \hat{A} \rangle = \langle \psi \mid \hat{A} \mid \psi \rangle \qquad \Delta \hat{A} = \sqrt{\langle \hat{A}^2 \rangle - \langle \hat{A} \rangle^2} \qquad \hat{U}\hat{U}^\dagger = 1 \qquad [\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A} \qquad \Delta \hat{A} \Delta \hat{B} \geq \langle \frac{[\hat{A}, \hat{B}]}{2i} \rangle$$

Vector Properties

$$\langle \psi \mid \psi \rangle = 1 \langle a_i \mid a_j \rangle \qquad = \delta_{ij} \hat{A} \mid \psi \rangle \qquad = \langle \psi \mid^\dagger \hat{A}^\dagger \mid \psi \rangle \qquad = \sum_n c_n \mid a_n \rangle = \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix} \langle \psi \mid \qquad = \sum_n c_n^\dagger \langle a_n \mid = \begin{pmatrix} c_1^\dagger & \dots & c_n^\dagger \end{pmatrix}$$

All spins

$$\vec{n} = \sin\theta \cos\varphi \vec{i} + \sin\theta \sin\varphi \vec{j} + \cos\theta \vec{k}$$

$$z : \theta = 0, \varphi = 0 \qquad x : \theta = \frac{\pi}{2}, \varphi = 0 \qquad y : \theta = \frac{\pi}{2}, \varphi = \frac{\pi}{2} \qquad [\hat{S}_x, \hat{S}_y] = i\hbar \hat{S}_z \qquad [\hat{S}_y, \hat{S}_z] = i\hbar \hat{S}_x \qquad [\hat{S}_z, \hat{S}_x] = i\hbar \hat{S}_y$$

Spin $\frac{1}{2}$ (Electrons)

$$\mid +n \rangle = \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} e^{i\varphi} \end{pmatrix} \qquad \mid -n \rangle = \begin{pmatrix} \sin\frac{\theta}{2} \\ -\cos\frac{\theta}{2} e^{-i\varphi} \end{pmatrix} \qquad \hat{S}_n = \frac{\hbar}{2} \begin{pmatrix} \cos\theta & \sin\theta e^{-i\varphi} \\ \sin\theta e^{i\varphi} & -\cos\theta \end{pmatrix}$$

Spin 1 (Photons)

$$\mid X' \rangle = \cos\phi \mid X \rangle + \sin\phi \mid Y \rangle \qquad \mid R \rangle = \frac{1}{\sqrt{2}}(\mid X \rangle + i \mid Y \rangle) \qquad \hat{R}\phi\vec{k} \mid R \rangle = e^{i\phi} \mid R \rangle \qquad \hat{J}_z \mid R \rangle = \hbar \mid R \rangle$$

$$\mid Y' \rangle = -\sin\phi \mid X \rangle + \cos\phi \mid Y \rangle \qquad \mid L \rangle = \frac{1}{\sqrt{2}}(\mid X \rangle - i \mid Y \rangle) \qquad \hat{R}\phi\vec{k} \mid L \rangle = e^{i\phi} \mid L \rangle \qquad \hat{J}_z \mid L \rangle = -\hbar \mid L \rangle$$

Spin 1 (General)

$$\mid 1,1 \rangle_n = \begin{pmatrix} e^{-i\varphi} \frac{1+\cos\theta}{2} \\ \frac{\sin\theta}{\sqrt{2}} \\ e^{i\varphi} \frac{1-\cos\theta}{2} \end{pmatrix} \quad \mid 1,0 \rangle_n = \begin{pmatrix} -e^{-i\varphi} \frac{\sin\theta}{\sqrt{2}} \\ \cos\theta \\ e^{i\varphi} \frac{\sin\theta}{\sqrt{2}} \end{pmatrix} \quad \mid 1,-1 \rangle_n = \begin{pmatrix} e^{-i\varphi} \frac{1-\cos\theta}{2} \\ \frac{-\sin\theta}{\sqrt{2}} \\ e^{i\varphi} \frac{1+\cos\theta}{2} \end{pmatrix} \quad \hat{S}_n = \hbar \begin{pmatrix} \cos\theta & \frac{e^{-i\varphi}\sin\theta}{\sqrt{2}} & 0 \\ \frac{e^{i\varphi}\sin\theta}{\sqrt{2}} & 0 & \frac{e^{-i\varphi}\sin\theta}{\sqrt{2}} \\ 0 & \frac{e^{i\varphi}\sin\theta}{\sqrt{2}} & -\cos\theta \end{pmatrix}$$

Hamiltonian

$$i\hbar \frac{d}{dt} \mid \psi(t) \rangle = \hat{H} \mid \psi(t) \rangle \qquad \frac{d\langle \hat{A} \rangle}{dt} = \frac{i}{\hbar} \langle [\hat{H}, \hat{A}] \rangle$$